Exploring the Role of Self-Interacting Scalar Dark Matter: Dynamical Friction and GW Emission

Alexis Boudon
With: Patrick Valageas & Philippe Brax

[Phys. Rev. D 106, 043507] and up-coming papers

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Why Self-Interacting Scalar Dark Matter ?

Tensions in standard model of cosmology
(e.g., Core-cusp problem, missing satellites, Fornax Globular Cluster timing problem)

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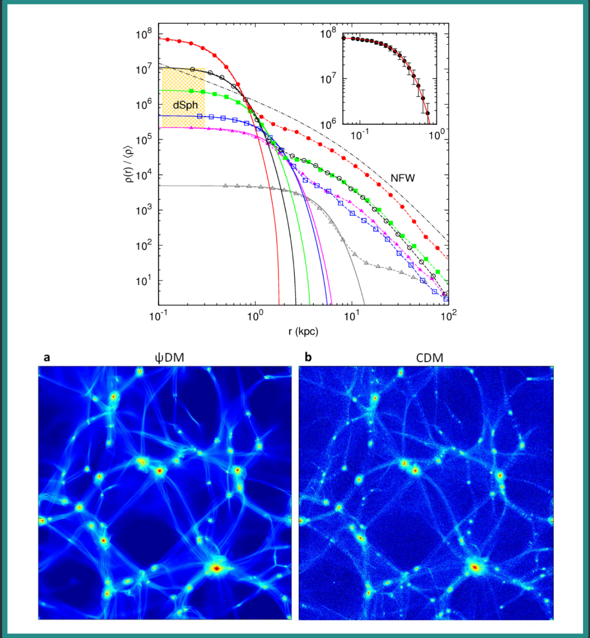
Thus: Self-interacting scalar dark matter

- Dark matter is composed of bosons within $10^{-22}\mathrm{eV} < m < \mathrm{eV}$
- Form **stable equilibrium configurations**: self-gravity and effective pressure
$\longrightarrow$ different behavior at galactic scales



[**Shive et al.** Nature Phys 10, 496 - 499 (2014)]

Dynamical Friction

Definition

The dynamical friction is referring **to the loss of momentum** of moving objects through gravitational interactions with the environment

Here, we refer to:

Mass accretion and gravitational drag

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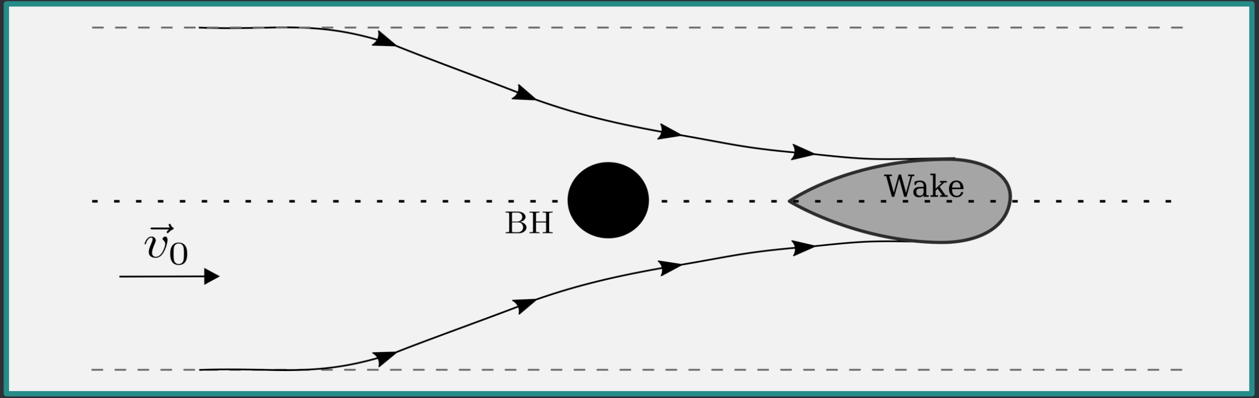
A Schwarzschild (non-spinning) **black hole in motion in a self-interacting scalar field equilibrium state** (i.e., solitonic solution), in steady state



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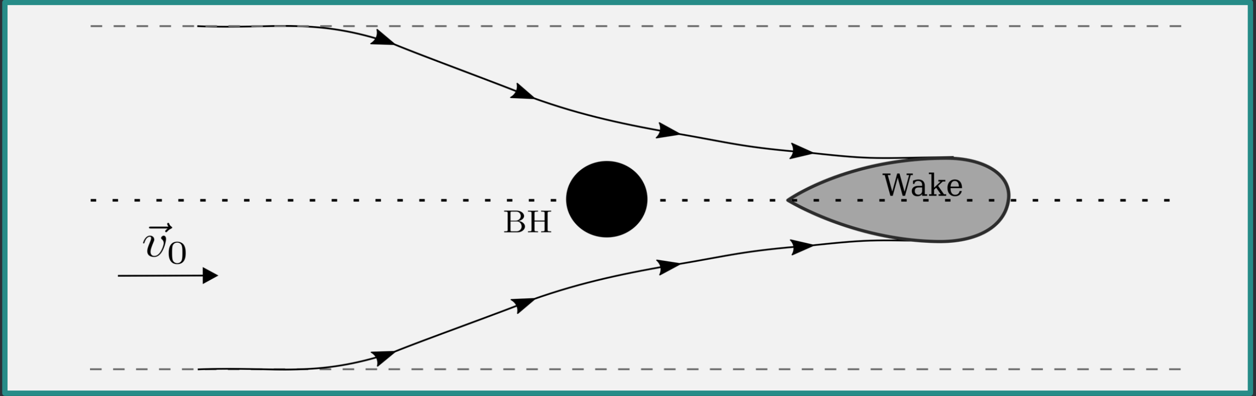

Impact on: Gravitational waves emission (phase shift) for binary black holes

$$\Psi(f) = 2\pi f t_c - \Phi_c - \frac{\pi}{4} + \Psi_{\rm GR}(f) + \boldsymbol{\Psi_{\rm env}(f)}$$

[**Kocsis et al.** PhysRevD.84.024032 (2011), **Barausse et al.** PhysRevD.89.104059 (2014), **Cardoso & Maselli** A&A 664 (2020), ...]

Action and Field Solution (Large-Mass Limit)

Action: $$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$\mathrm{V}(\phi) = \frac{m^2}{2}\phi^2 + V_{\mathrm{I}}(\phi)\,, \qquad V_{\mathrm{I}}(\phi) = \frac{\lambda_4}{4}\phi^4$$ (first term obeys $\rho \propto a^{-3}$)

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The Klein-Gordon equation:
[**Brax et al.** PhysRevD. 101 023521 (2020)]

$$\boxed{\frac{\partial^2\phi}{\partial t^2} - \sqrt{\frac{f}{h^3}}\vec{\nabla}\cdot\left(\sqrt{fh}\vec{\nabla}\phi\right) + f\frac{\partial V(\phi)}{\partial\phi} = 0}$$

At small scales:

$$f = \left(\frac{1-\frac{r_{\mathrm{s}}}{4r}}{1+\frac{r_{\mathrm{s}}}{4r}}\right)^2$$

$$h = \left(1+\frac{r_{\mathrm{s}}}{4r}\right)^4$$

At large radii:

$$h = 1 - 2\Phi_{\mathrm{N}}$$

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Using **local approximation** (neglecting at zeroth order the partial derivatives), we obtain **the Duffing equation**:

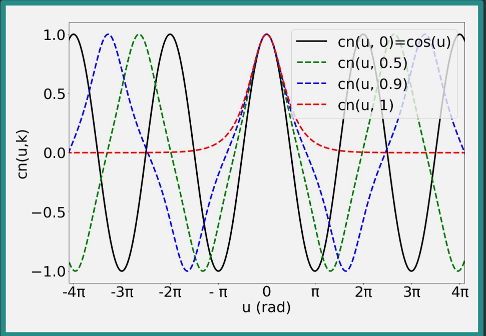
Amplitude

Phase (related to the velocity)

$$\phi = \phi_0(r,\theta)\mathrm{cn}[\omega(r,\theta)t - \mathbf{K}(k)\beta(r,\theta), k(r,\theta)]$$

Angular frequency

Modulus (nonlinear oscillator)

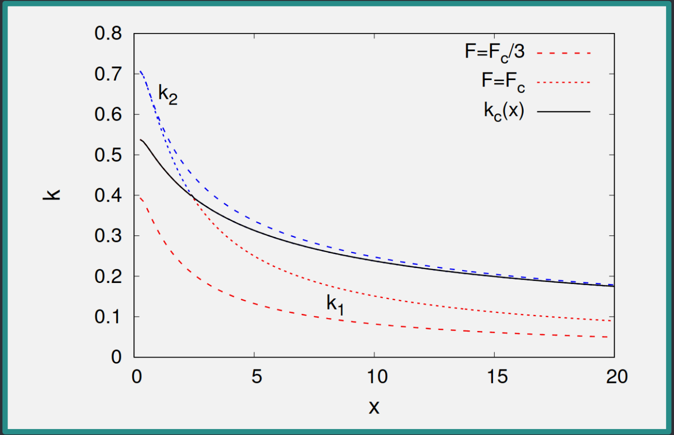


Hydrodynamical Infall?

Radial accretion

The effective continuity equation can be **integrated at once** (only radial derivatives)

- 2 Solutions for k as for hydrodynamical infall [**Bondi** (1952), **Michel** (1972)]



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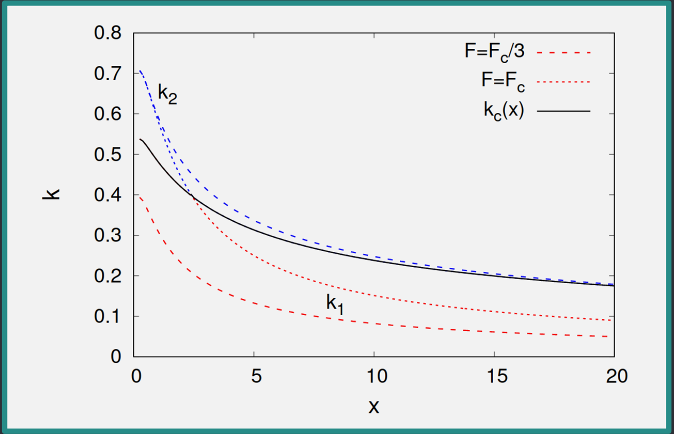

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In our case

In subsonic case:
same k near the black hole
+ need to solve at large-radii along k_2

In supersonic case:
A little bit more complicated
(depends on bow shock)

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Rescaled quantities:

$$\hat{r} = \frac{r}{r_{\rm s}}$$

$$\hat{\beta} = \frac{\pi}{2mr_{\rm s}}\beta$$

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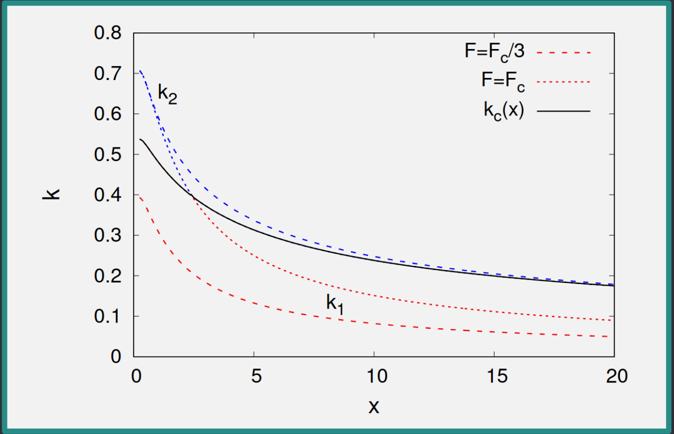

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$$(\hat{\nabla}\hat{\beta})^2 = \frac{3}{2}\left[k_+(\hat{r})^2 - k^2\right]$$ where we defined $$k_+(\hat{r})^2 = k_0^2 + \frac{2}{3}v_0^2 + \frac{2}{3\hat{r}}$$

Enthalpy/Soliton density **Black hole contribution** **Relative velocity term**

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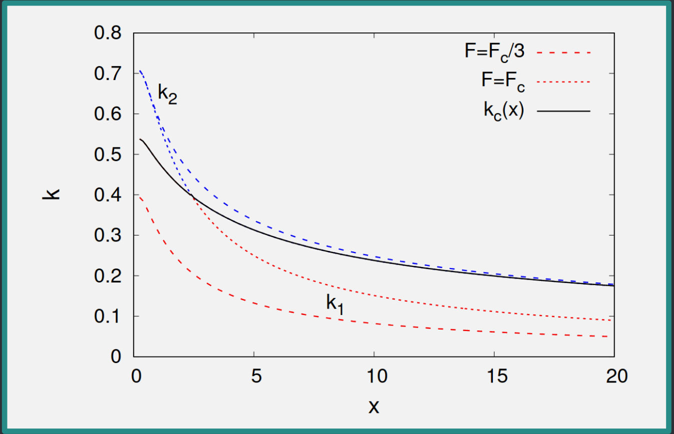

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Relative velocity term

Conservation equation: $$\boxed{\hat{\nabla}\cdot\left[\left(k_+(x)^2 - \frac{2}{3}(\hat{\nabla}\hat{\beta})^2\right)\hat{\nabla}\hat{\beta}\right] = 0}$$

Velocity and Density Fields in Subsonic Regime

(Supersonic Regime Up-Coming!)

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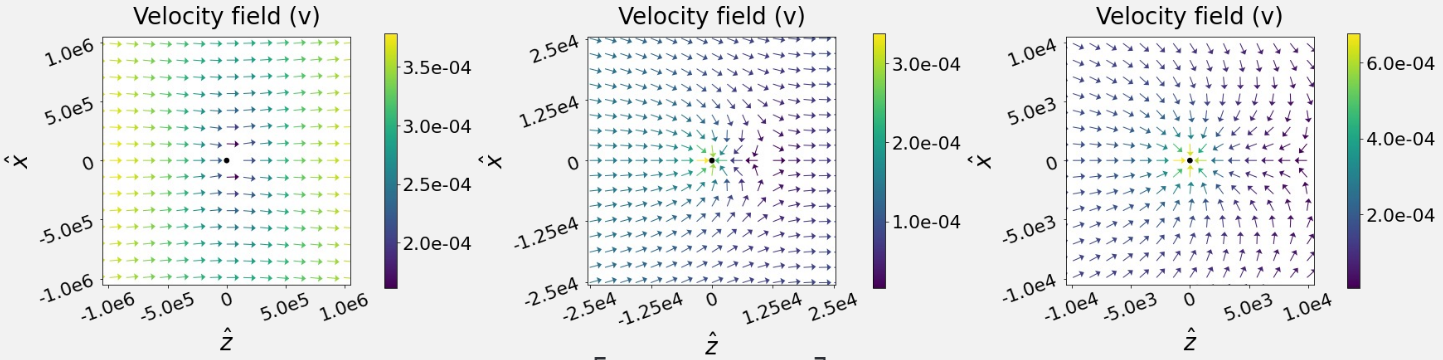

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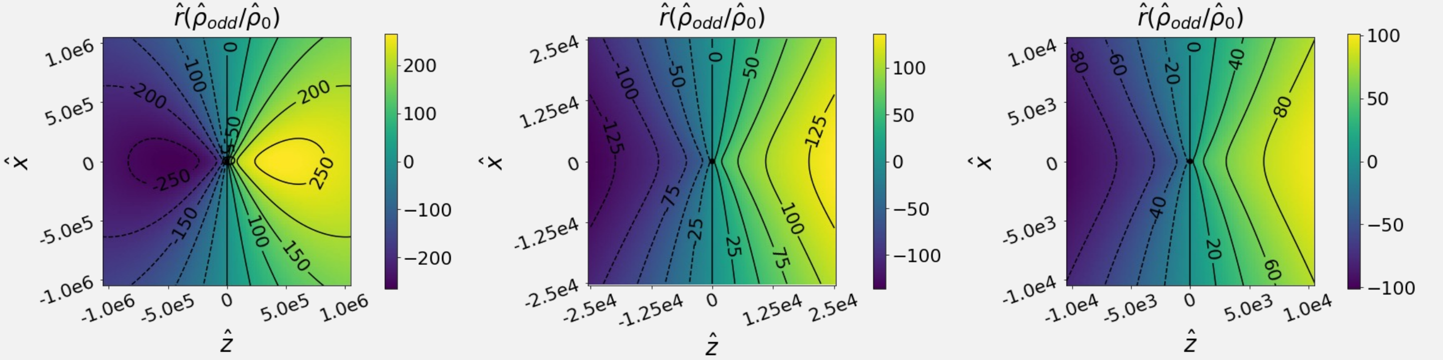

Associated Dynamical Friction

- **Mass conservation** gives the rate of mass accretion of the black hole:

$$\dot{\hat{M}}_{\mathrm{BH}} = -\int_{\hat{S}} d\vec{\hat{S}} \cdot \hat{\rho}\vec{v} = 2\pi \int_0^{\hat{b}_-} d\hat{b}\,\hat{b}\,\hat{\rho} v_z\big|_{\hat{z}_-} - 2\pi \int_0^{\hat{b}_+} d\hat{b}\,\hat{b}\,\hat{\rho} v_z\big|_{\hat{z}_+}$$

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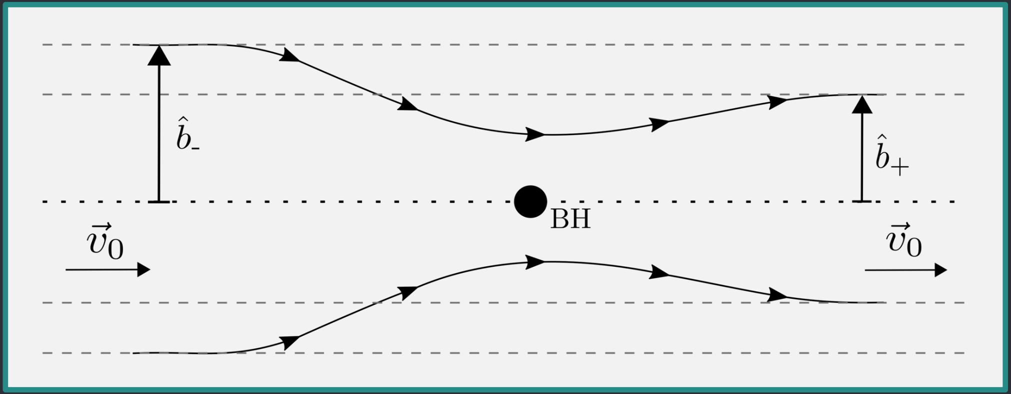

Finding $\hat{b}_+$ from the streamlines:

$$\dot{M}_{\rm BH} \sim \rho_0 r_{\rm s}^2/c_{\rm s}^2 \sim \rho_0 \mathcal{G}^2 M_{\rm BH}^2/c_{\rm s}^2$$

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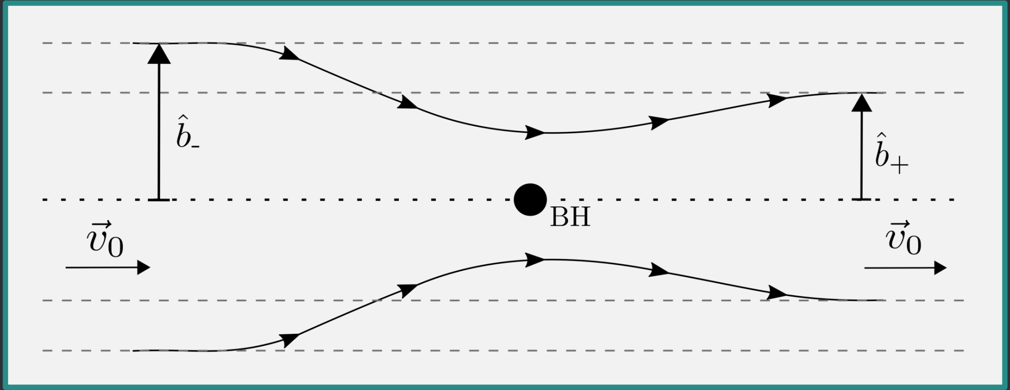

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- **Momentum conservation** gives the force (at large radii + perturbatively):

$$F_z = \frac{dp_z}{dt} = -\int_{S_{\mathrm{out}}} d\vec{S} \cdot \rho\vec{v} v_z - \int_{S_{\mathrm{out}}} d\vec{S} \cdot P\vec{e}_z = \boxed{\dot{M}_{\mathrm{BH}} v_0}$$

Comparison with Fuzzy and Cold Dark Matter

Our result: $\boxed{F_{\mathrm{z}} \sim \frac{\mathcal{G}^2 M_{\mathrm{BH}}^2 \rho_0 v_0}{c_{\mathrm{s}}^2}}$

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  [**Chandrasekhar** (1943)]

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r_{a} : Soliton size

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Before going further, supersonic result: $F_\mathrm{DF} = \dot{M}_\mathrm{BH} v_0 + \dfrac{4\mathcal{G}^2 M_{BH}^2 \rho_0}{3 v_0^2} \log\left(\dfrac{e(v_0^2 - c_\mathrm{s}^2)^{\frac{3}{2}} r^2}{18 c_\mathrm{s}^3 r_\mathrm{sg}^2}\right)$

Gravitational Wave Phase Shift and Fisher Analysis

Considering corrections from dynamical friction as perturbations, we obtain at **leading order**:

$$\boxed{\Psi_+ \simeq 2\pi f_{\rm gw} t_{\rm c} - \Phi_0 - \frac{\pi}{4} + \Psi_{\rm gw} + \Psi_{\rm accr} + \Psi_{\rm chandra}}$$

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$$\Psi_{\rm chandra} \simeq -\frac{50\left[\mathcal{C}_1(f)+\mathcal{C}_2(f)\right]}{361\nu^2}\mathcal{G}^3 M_{\rm c}^2\rho_0\left(\mathcal{G}M_{\rm c}8\pi f_{\rm gw}\right)^{-\frac{16}{3}}$$

$$\mathcal{C}_{\rm i}(f) = \left(\frac{m_{\rm i}}{M}\right)^3\left[105 + 304\log\left(\frac{f_{\rm gw}}{\mathfrak{f}}\frac{r_{\rm a}^2}{R_{\rm i,c}^2}\right)\right]\Theta(v_{\rm i} > v_{\rm i,c}) \ , \qquad \mathfrak{f} = \frac{c_{\rm s}^3}{\mathcal{G}M\pi} \ , \qquad R_{\rm i,c} = 6\sqrt{\frac{2}{e}}\frac{\mathcal{G}m_{\rm i}^{\frac{5}{2}}}{\mu^{\frac{3}{2}}c_{\rm s}^2} \ , \qquad v_{\rm i,c} = c_{\rm s}\max\left[1, \left(6\sqrt{\frac{2}{e}}\frac{\mathcal{G}m_{\rm i}}{r_{\rm a}c_{\rm s}^2}\right)^{\frac{2}{3}}\right]$$

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$$\rho_{\rm a} = \frac{\rho_0}{c_{\rm s}^2}$$

Fisher matrix: $\Gamma_{\rm ij} = \int_{f_{\rm min}}^{f_{\rm max}} \frac{\mathcal{A}^2}{S_{\rm n}(f_{\rm gw})}\frac{\partial \Psi_+}{\partial \theta_{\rm i}}\frac{\partial \Psi_+}{\partial \theta_{\rm j}} df_{\rm gw} \ , \quad \theta_{\rm i} = \{\log(M_{\rm c}), \log(\nu), t_{\rm c}, \phi_{\rm c}, \rho_0, \rho_{\rm a}\}$

Covariance: $\Sigma_{\rm ij} = \left(\Gamma^{-1}\right)_{\rm ij}$, Standard deviation: $\sigma_{\rm i} = \sqrt{\Sigma_{\rm ii}}$

Some Preliminary Results (1/2)

Dark matter in Solar neighborhood: $\rho_{\rm DM} \sim 6,7.10^{-23}\mathrm{g/cm^3}$

White lines: Limit on logarithm

Green lines: Subsonic limit for largest BH

Yellow lines: Subsonic limit for smallest BH

Baryonic densities in:

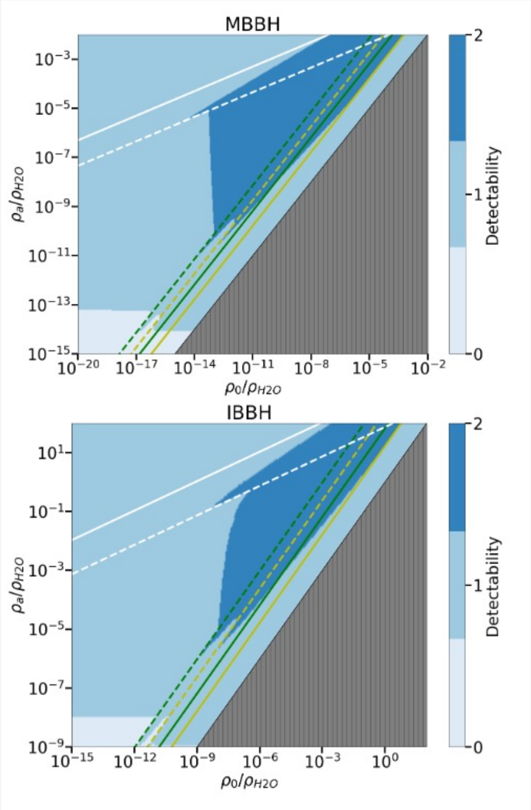
- Thin disks:

$\rho \leq 10\mathrm{g/cm^3}$

- Thick disks:

$\rho \leq 10^{-7}\mathrm{g/cm^3}$

[**Barausse,et al.** Phys.Rev.D. 89, 104059 (2014)]



MBBH:

$$m_1 = 10^6\mathrm{M}_\odot$$
$$m_2 = 5.10^5\mathrm{M}_\odot$$
$$\mathrm{SNR} = 3.10^4$$

IBBH:

$$m_1 = 10^4\mathrm{M}_\odot$$
$$m_2 = 5.10^3\mathrm{M}_\odot$$
$$\mathrm{SNR} = 708$$

Some Preliminary Results (1/2)

Dark matter in Solar neighborhood: $\rho_{\rm DM} \sim 6,7.10^{-23}{\rm g/cm^3}$

White lines: Limit on logarithm

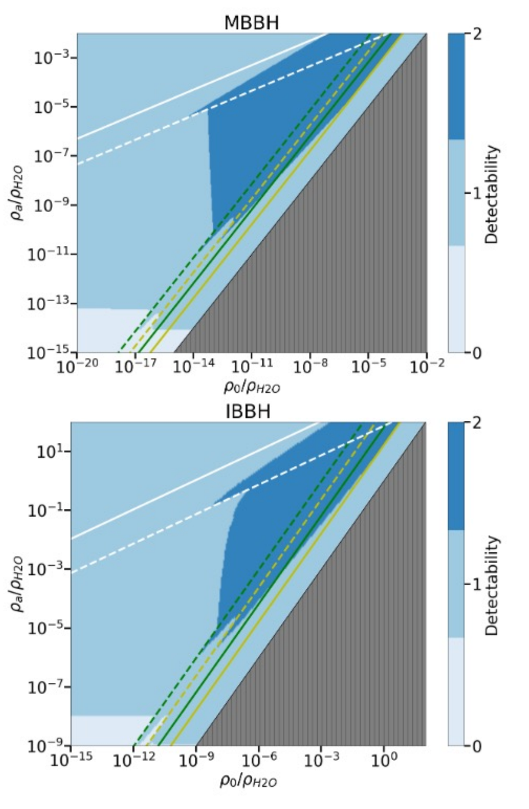
Green lines: Subsonic limit for largest BH

Yellow lines: Subsonic limit for smallest BH

Baryonic densities in:

- Thin disks: $\rho \leq 10{\rm g/cm^3}$
- Thick disks: $\rho \leq 10^{-7}{\rm g/cm^3}$

[**Barausse,et al.** Phys.Rev.D. 89, 104059 (2014)]



MBBH:

$$m_1 = 10^6 {\rm M_\odot}$$
$$m_2 = 5.10^5 {\rm M_\odot}$$
$${\rm SNR} = 3.10^4$$

IBBH:

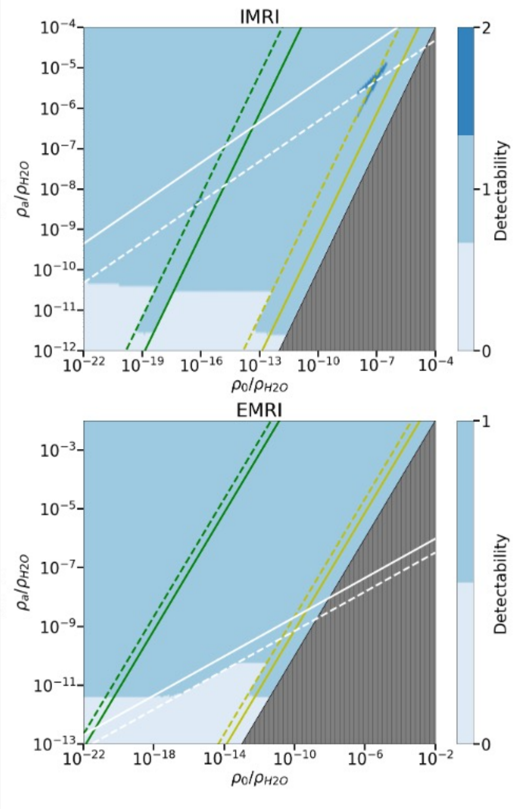
$$m_1 = 10^4 {\rm M_\odot}$$
$$m_2 = 5.10^3 {\rm M_\odot}$$
$${\rm SNR} = 708$$

IMRI:

$$m_1 = 10^4 {\rm M_\odot}$$
$$m_2 = 10 {\rm M_\odot}$$
$${\rm SNR} = 22$$

EMRI:

$$m_1 = 10^5 {\rm M_\odot}$$
$$m_2 = 10 {\rm M_\odot}$$
$${\rm SNR} = 64$$



Some Preliminary Results (2/2)

Dark matter in Solar neighborhood: $\rho_{\mathrm{DM}} \sim 6,7.10^{-23}\mathrm{g/cm}^3$

GW150914:

$m_1 = 35,6\mathrm{M}_\odot$

$m_2 = 30,6\mathrm{M}_\odot$

GW170608:

$m_1 = 11\mathrm{M}_\odot$

$m_2 = 7,6\mathrm{M}_\odot$

Baryonic densities in:

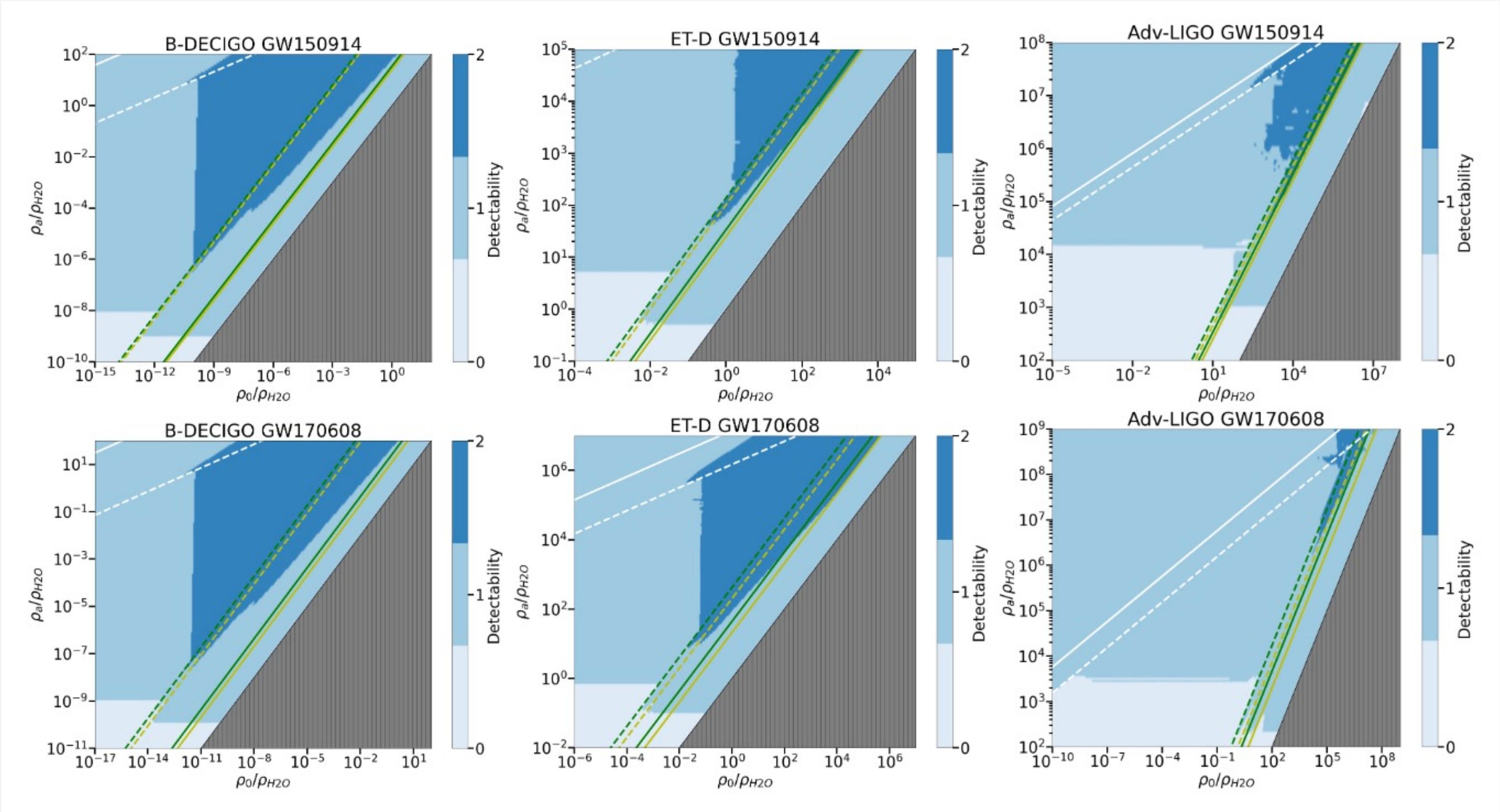
- Thin disks:

$\rho \leq 10\mathrm{g/cm}^3$

- Thick disks:

$\rho \leq 10^{-7}\mathrm{g/cm}^3$

[**Barausse,et al.** Phys.Rev.D. 89, 104059 (2014)]



A. Boudon

Conclusion and Prospects

Smaller friction and mass accretion than other models

Gravitational waves phase shift:

- -4PN for mass accretion term
- -5.5PN for Chandrasekhar-like term

Detection with the next generation interferometers **seems unlikely**
→ except for very dense dark matter medium

What needs to be done:

- Making more realistic assumptions (Kerr black hole, eccentricity?)
- **Including stars** in the calculation (neutron stars in binaries and application to cosmological tensions)
- What happens if we consider another kind of self-interaction?

Thanks!

ANY QUESTIONS?

Bernouilli, Non-Harmonic and Conservation Equations

3 equations (to solve 3 unknowns), at **leading order**:

Relativistic Bernoulli equation: $$(\nabla\beta)^2 = \frac{h}{f}\left(\frac{2\omega_0}{\pi}\right)^2 - \frac{hm^2}{(1-2k^2)\mathbf{K}^2}$$

Deviation from harmonic oscillator: $$\frac{\lambda_4\phi_0^2}{m^2} = \frac{2k^2}{1-2k^2}$$

- At large radii: $$k \to k_0 \quad \text{with} \quad k_0^2 \simeq \frac{\lambda_4\phi_0^2}{2m^2} = \frac{\lambda_4\rho_0}{m^4}$$

$$\phi = \frac{1}{\sqrt{2m}}\left(e^{-imt}\psi + e^{+imt}\psi\right)$$

$$\psi = \sqrt{\frac{\rho}{m}}e^{is}$$

Conservation equation: $$\langle \nabla_\mu T^\mu_0 \rangle = 0$$

Taking the **average over the oscillations** (to ensure steady state)

- Gives an **effective continuity equation**:

$$\nabla\cdot(\rho_{\text{eff}}\nabla\beta) = 0\,, \qquad \rho_{\text{eff}} = \sqrt{fh\phi_0^2}\,\omega\mathbf{K}\langle \mathrm{cn}'^2\rangle$$

Description of binary black holes

$v = v_1 - v_2$

Mass accretion term:

$$F_{\rm accr} = \frac{\dot{\mu}}{\mu} v$$

Chandrasekhar-like term:
(from supersonic regime)

$$F_{\rm chandra} = \frac{1}{v^2}\left[A + B\log\left(\frac{v}{c_{\rm s}}\right)\right]$$

$$A = \frac{8\pi\mathcal{G}^2\rho_0}{3\mu^2}\left[m_1^3\log\left(\frac{r_{\rm a}}{R_{1,\rm c}}\right)\Theta(v_1 > v_{1,\rm c}) + m_2^3\log\left(\frac{r_{\rm a}}{R_{2,\rm c}}\right)\Theta(v_2 > v_{2,\rm c})\right], \quad B = \frac{4\pi\mathcal{G}^2\rho_0}{\mu^2}\left[m_1^3\Theta(v_1 > v_{1,\rm c}) + m_2^3\Theta(v_2 > v_{2,\rm c})\right]$$

Critical velocity: $v_{\rm i,c} = c_{\rm s}\max\left[1, \left(6\sqrt{\frac{2}{e}}\frac{\mathcal{G}m_{\rm i}}{r_{\rm a}c_{\rm s}^2}\right)^{\frac{2}{3}}\right]$ and characteristic radii: $R_{\rm i,c} = 6\sqrt{\frac{2}{e}}\frac{\mathcal{G}m_{\rm i}^{\frac{5}{2}}}{\mu^{\frac{3}{2}}c_{\rm s}^2}$

Total and reduced masses:

$M = m_1 + m_2$

$\mu = \frac{m_1 m_2}{M}$

Impact on orbital decay and eccentricity

- Mass accretion:

$$<\dot{a}>_{\rm accr} = -\left(\frac{\dot{M}}{M} + 2\frac{\dot{\mu}}{\mu}\right)a$$

$$<\dot{e}>_{\rm accr} = 0$$

- Gravitational drag:

$$<\dot{a}>_{\rm chandra} = -2a\left(\frac{a}{\mathcal{G}M}\right)^{\frac{3}{2}}\left[A + B\log\left(\sqrt{\frac{\mathcal{G}M}{a}}\frac{1}{c_{\rm s}}\right)\right]$$

$$<\dot{e}>_{\rm chandra} = 3\mathfrak{e}\left(\frac{a}{\mathcal{G}M}\right)^{\frac{3}{2}}\left[A + B\log\left(\sqrt{\frac{\mathcal{G}M}{a}}\frac{1}{e^{\frac{1}{3}}c_{\rm s}}\right)\right]$$

Different Events for Different Interferometers

Details on masses and spins of considered events

Event \ Properties	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_1	χ_2	$\chi_{\rm eff}$
MBBH	10^6	5.10^5	0, 9	0, 8	0, 87
IBBH	10^4	5.10^3	0, 3	0, 4	0, 33
IMRI	10^4	10	0, 8	0, 5	0, 80
EMRI	10^5	10	0, 8	0, 5	0, 80
GW150914	35, 6	30, 6	0, 13	0, 05	0, 09
GW170608	11	7, 6	0, 13	0, 50	0, 28

Sound-Noise Ratio (SNR) depending on the detector

Event \ Detector	LISA	B-DECIDO	ET-D	Adv-LIGO
MBBH	3.10^4	×	×	×
IBBH	708	×	×	×
IMRI	22	×	×	×
EMRI	64	×	×	×
GW150914	×	2815	615	40
GW170608	×	2124	502	35

Minimum and maximum detection frequencies for each detector

Relativistic correction:

$$\gamma^2(1+v^2)^2$$

Detector \ Frequency	$f_{\rm min}$(Hz)	$f_{\rm max}$(Hz)
Adv-LIGO	10	f_{1M}
ET-D	3	f_{1M}
LISA	$\max(2.10^{-5}, f_{\rm obs})$	$\min(10^2, f_{1M})$
B-DECIGO	10^{-2}	$\min(1, f_{1M})$

We also use $f(v_{\rm rel})$ to stay agnostic on relativistic effects

$$v_{\rm rel} \sim 0,37$$

The considered events and detectors: [**Cardoso and Maselli**, AstronAstrophys. 644, A147 (2020)]
(not the same spectral functions, they evolved!)

[**Ajith et al.**, PhysRevLett. 106, 241101 (2011)]

- PhenomB inspiral-merger transition value:

$$f_{1M} = \frac{1}{\mathcal{G}M\pi}\left[1 - 4,4547(1-\chi_{\rm eff})^{0.217} + 3,521\,(1-\chi_{\rm eff})^{0.26} + \mathcal{O}(\nu)\right]$$

- Frequency value of the binary 4 years before the merger:

$$f_{\rm obs} = 4,149.10^{-5}\left[\frac{M_c}{10^6 M_\odot}\right]^{-\frac{5}{8}}\left[\frac{T_{\rm obs}}{1{\rm yr}}\right]^{-\frac{3}{8}}$$ [**Berti et al.**, PhysRevD. 71, 084025 (2005)]