

# Cosmological tests of Einstein and Euler

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LP, M. Raveri, K. Koyama, M. Martinelli, A. Silvestri, G.-B. Zhao, J. Li, S. Peirone, A. Zucca, arXiv:2107.12992, Nature Astronomy  
M. Raveri, LP, K. Koyama, M. Martinelli, A. Silvestri, G.-B. Zhao, arXiv:2107.12990, JCAP  
C. Bonvin, LP, arXiv:2209.03614, Nature Astronomy (accepted)

# The phenomenology of modified gravity

Expansion history:  
effective dark energy density  $X(a)$   
( $X(a) = 1$  in LCDM)

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_M}{a^3} + \Omega_{DE} X(a)$$

Linear perturbations:  
Modified Einstein equations  
( $\mu = \Sigma = \gamma = 1$  in LCDM)

$$\begin{aligned} -k^2 \Psi &= 4\pi \mu(a, k) G a^2 \delta\rho \\ \Phi &= \gamma(a, k) \Psi \\ -k^2 \left( \frac{\Phi + \Psi}{2} \right) &= 4\pi \Sigma(a, k) G a^2 \delta\rho \end{aligned}$$

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$$\begin{aligned} -k^2 \Psi &= 4\pi \mu(a, k) G a^2 \delta\rho && \text{“}G_{matter}\text{”} \\ \Phi &= \gamma(a, k) \Psi \\ -k^2 \left( \frac{\Phi + \Psi}{2} \right) &= 4\pi \Sigma(a, k) G a^2 \delta\rho && \text{“}G_{light}\text{”} \end{aligned}$$

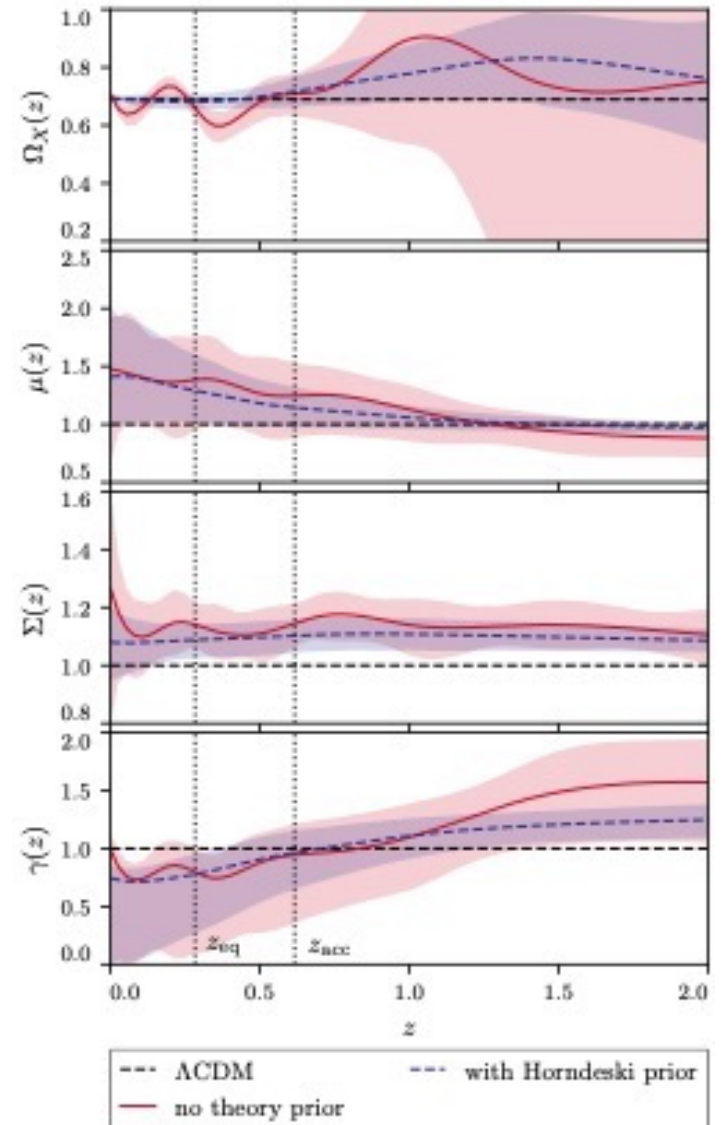
A smoking gun of new gravitational physics:

$$G_{matter} \neq G_{light} \quad \text{or} \quad \Phi \neq \Psi$$

(the “gravitational slip”  $\gamma$  is also known as  $\eta$ )

# Reconstructing gravity from Planck+DES+RSD+BAO+SN

- First simultaneous reconstruction of  $\mu(a)$ ,  $\Sigma(a)$  and  $\Omega_x(a)$
- Did it with and without a Horndeski prior: a way to separate features consistent with theory from potential systematics
- Current data can constrain 15 eigenmodes
- Late-time modified gravity is unlikely to resolve the tensions
- Implications for scalar-tensor theories



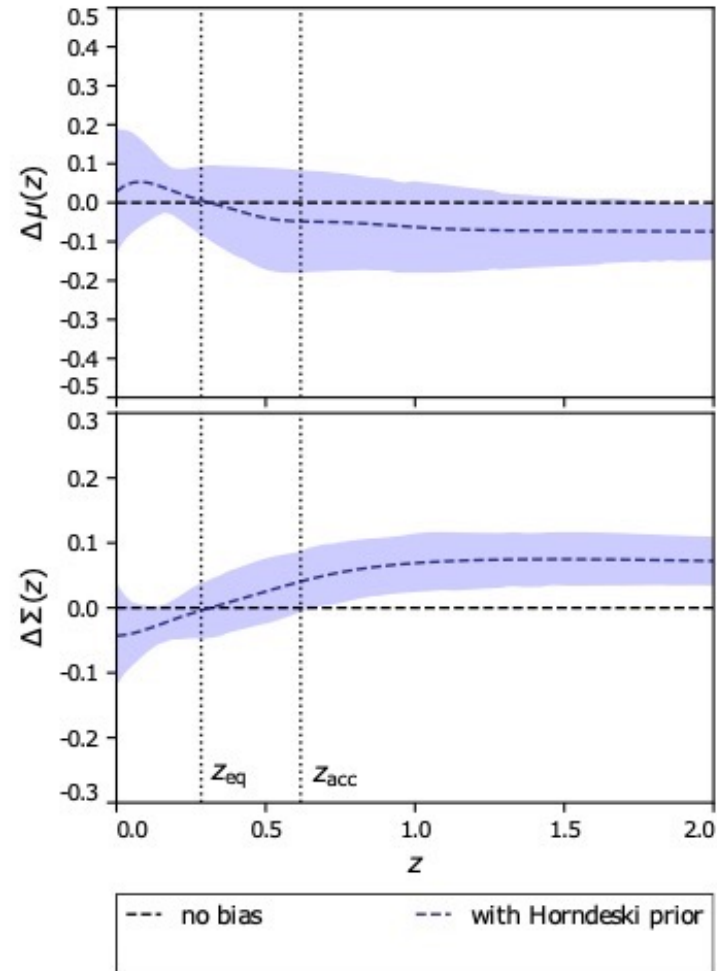
*Imprints of cosmological tensions in reconstructed gravity*, LP, M. Raveri, K. Koyama, M. Martinelli, A. Silvestri, G.-B. Zhao, J. Li, S. Peirone, A. Zucca, arXiv:2107.12992, Nature Astronomy (2022)

*Principal reconstructed modes of dark energy and gravity*, M. Raveri, LP, K. Koyama, M. Martinelli, A. Silvestri, G.-B. Zhao, arXiv:2107.12990, JCAP

# How good/bad are common parametrizations?

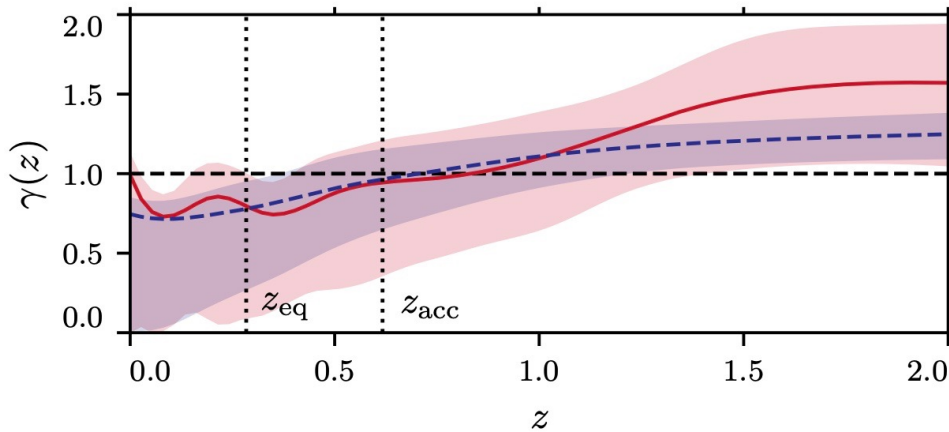
$$\mu(a) = 1 + \mu_0 \Omega_{\text{DE}}(a)$$

$$\Sigma(a) = 1 + \Sigma_0 \Omega_{\text{DE}}(a)$$



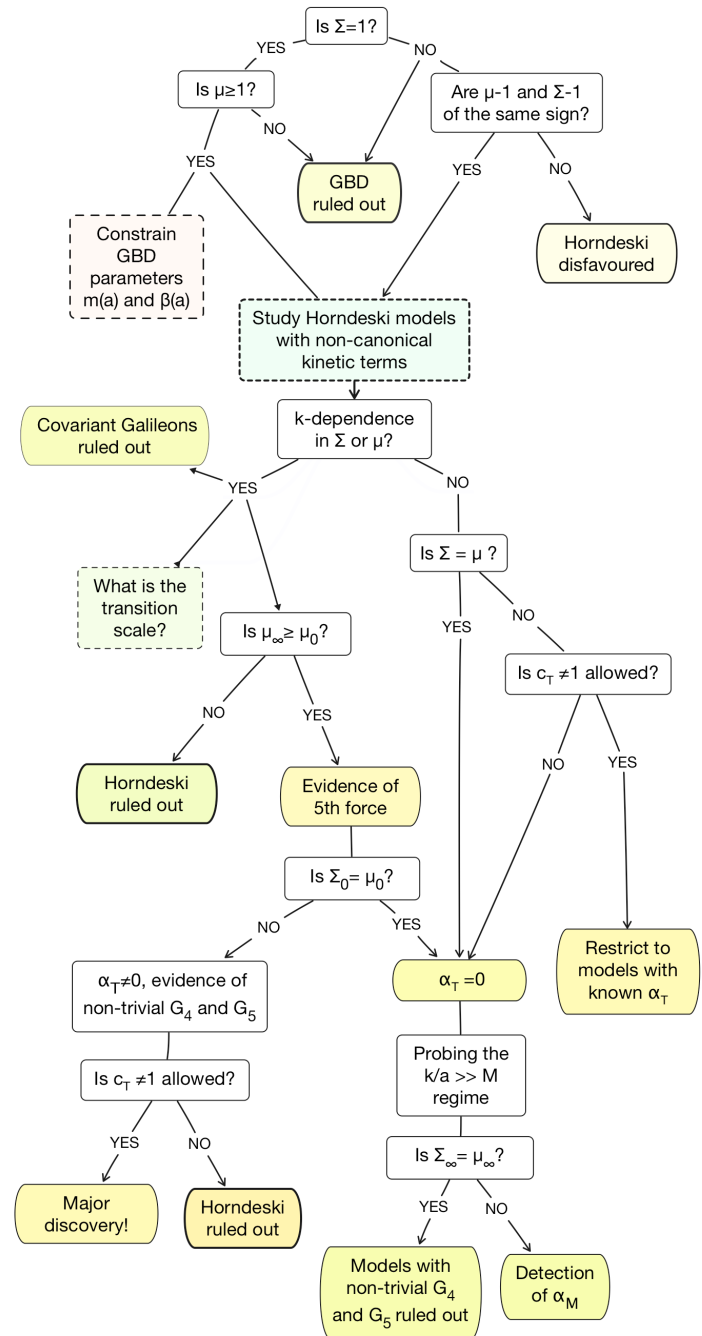
# What can cosmology tell us about gravity?

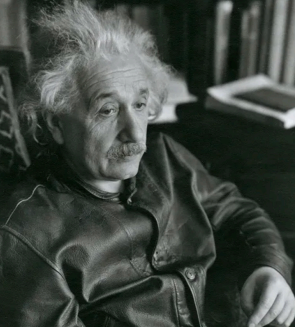
## Constraining Horndeski with $\Sigma$ and $\mu$



### Hints from the reconstruction:

- LCDM is under some tension (but we knew it already)
- $\gamma > 1$  would rule out Brans-Dicke theories
- $\Sigma \neq \mu$ , or  $\gamma \neq 1$ , can only be due to  $c_T \neq 1$  or a fifth force, would rule out Cubic Galileons
- No violation of  $(\Sigma - 1)(\mu - 1) \geq 0$  expected in Horndeski theories





## Modified Einstein vs modified Euler



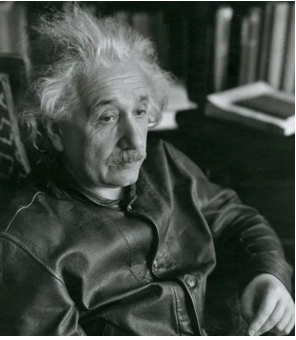
The above reconstruction assumed that all matter (CDM and baryons) follow the same geodesics, *i.e.* the modified gravity affects all matter universally

What if gravity was not modified, but there was a force acting only on CDM?  
Could we tell the difference?

A case study: Generalized Brans-Dicke vs Coupled Quintessence

$$S^{\text{GBD}} = \int d^4 \sqrt{-g} \left[ \frac{A^{-2}(\phi)}{16\pi G} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \mathcal{L}_m(\psi_{\text{DM}}, \psi_{\text{SM}}, g_{\mu\nu}) \right]$$

$$S^{\text{CQ}} = \int d^4 \sqrt{-g} \left[ \frac{1}{16\pi G} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \mathcal{L}_{\text{SM}}(\psi_{\text{SM}}, g_{\mu\nu}) + \mathcal{L}_{\text{DM}}(\psi_{\text{DM}}, A^2(\phi)g_{\mu\nu}) \right]$$



# Modified Einstein vs modified Euler



(quasistatic approximation)

Generalized Brans-Dicke (GBD)

$$k^2\Phi = -4\pi G a^2 (\rho_b \delta_b + \rho_c \delta_c) - \beta k^2 \delta\phi \quad (21)$$

$$k^2(\Phi - \Psi) = -2\beta k^2 \delta\phi \quad (22)$$

$$\dot{\delta}_b + \theta_b = 0 \quad (23)$$

$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi \quad (24)$$

$$\dot{\delta}_c + \theta_c = 0 \quad (25)$$

$$\dot{\theta}_c + \mathcal{H}\theta_c = k^2\Psi \quad (26)$$

$$\delta\phi = -\frac{\beta(\rho_c \delta_c + \rho_b \delta_b)}{m^2 + k^2/a^2} \quad (27)$$

$$\square\phi = V_{,\phi} + \beta(\rho_c + \rho_b) \equiv V^{\text{eff}}_{,\phi} \quad (28)$$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} = 4\pi G a^2 \rho \delta \left[ 1 + \frac{2\tilde{\beta}^2 k^2}{a^2 m^2 + k^2} \right] \quad (29)$$

Coupled Quintessence (CQ)

$$k^2\Phi = -4\pi G a^2 (\rho_b \delta_b + \rho_c \delta_c) \quad (30)$$

$$k^2(\Phi - \Psi) = 0 \quad (31)$$

$$\dot{\delta}_b + \theta_b = 0 \quad (32)$$

$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi \quad (33)$$

$$\dot{\delta}_c + \theta_c = 0 \quad (34)$$

$$\dot{\theta}_c + (\mathcal{H} + \beta\dot{\phi})\theta_c = k^2\Psi + k^2\beta\delta\phi \quad (35)$$

$$\delta\phi = -\frac{\beta\rho_c \delta_c}{m^2 + k^2/a^2} \quad (36)$$

$$\square\phi = V_{,\phi} + \beta\rho_c \equiv V^{\text{eff}}_{,\phi} \quad (37)$$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} = 4\pi G a^2 \rho \delta \left[ 1 + \frac{2\tilde{\beta}^2 k^2}{a^2 m^2 + k^2} \left( \frac{\rho_c}{\rho} \right)^2 \left( \frac{\delta_c}{\delta} \right) \right] \quad (38)$$

$$G_{\text{eff}}^{\text{GBD}} = G \left[ 1 + \frac{2\tilde{\beta}^2 k^2}{a^2 m^2 + k^2} \right]$$

$$\Sigma = A^2 \simeq 1, \mu > 1, \eta < 1$$

$$G_{\text{eff}}^{\text{CQ}} = G \left[ 1 + \frac{2\tilde{\beta}^2 k^2}{a^2 m^2 + k^2} \left( \frac{\rho_c}{\rho} \right)^2 \left( \frac{\delta_c}{\delta} \right) \right]$$

$$\Sigma = \mu = \eta = 1$$



# Theory vs practice

In theory, equations suggest that  $\eta$  could be the smoking gun

- Weak lensing probes  $\Phi + \Psi$
- Redshift space distortions probe  $\theta_b$ , hence  $\Psi$

$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi$$

- Combine WL and RSD to measure  $\eta$

# Theory vs practice

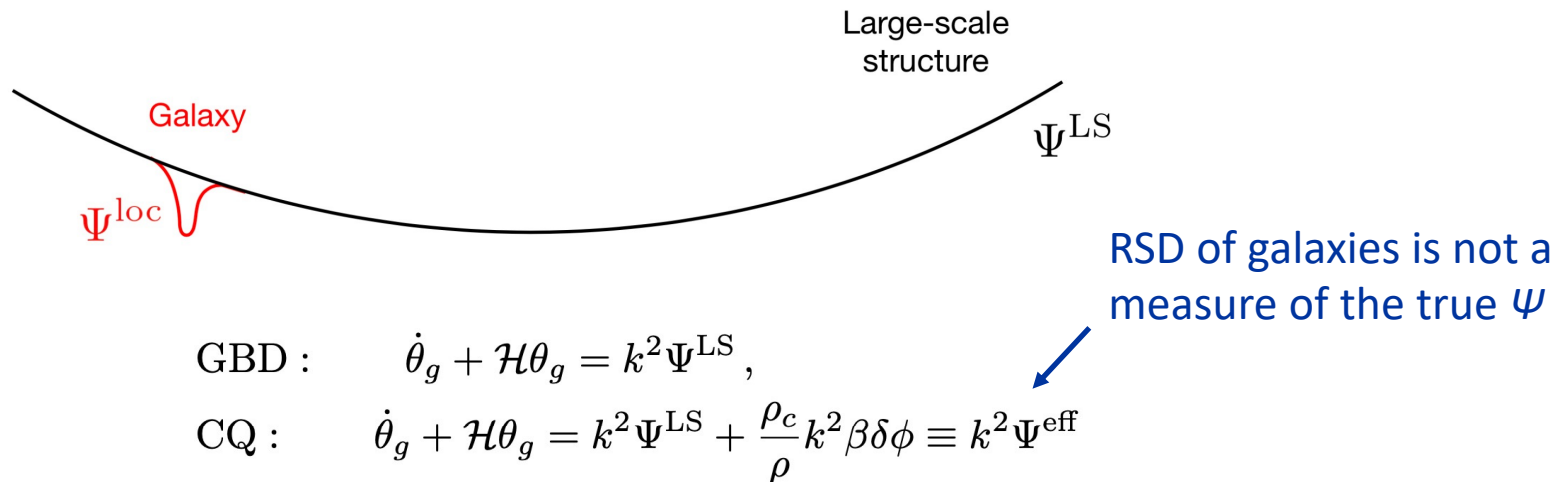
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- Combine WL and RSD to measure  $\eta$

In practice, the baryons we see are confined to galaxies made mostly of CDM



## Fitting $\mu$ and $\Sigma$ to RSD+WL (assuming Euler is valid)

RSD: 
$$P^{\text{gal}}(k, \mu_k, z) = (b^2 + \mu_k^2 f)^2 P_{\delta\delta}(k, z)$$

WL: 
$$P^{(\Phi+\Psi)}(k, z) = 9H_0^4 \Omega_m^2 (1+z)^2 \Sigma^2(k, z) P_{\delta\delta}(k, z)$$

### Generalized Brans-Dicke

$$G_{\text{eff}}^{\text{GBD}} = G \left[ 1 + \frac{2\tilde{\beta}^2 k^2}{a^2 m^2 + k^2} \right]$$

$$\mu^{\text{fit}} = \frac{G_{\text{eff}}^{\text{GBD}}}{G} = \mu^{\text{GBD}} > 1,$$

$$\eta^{\text{fit}} = \frac{2\Sigma^{\text{fit}}}{\mu^{\text{fit}}} - 1 = \frac{2}{\mu^{\text{fit}}} - 1 = \eta^{\text{GBD}} < 1$$

### Coupled Quintessence

$$G_{\text{eff}}^{\text{CQ}} = G \left[ 1 + \frac{2\tilde{\beta}^2 k^2}{a^2 m^2 + k^2} \left( \frac{\rho_c}{\rho} \right)^2 \left( \frac{\delta_c}{\delta} \right) \right]$$

$$\mu^{\text{fit}} = \frac{G_{\text{eff}}^{\text{CQ}}}{G} > 1$$

$$\eta^{\text{fit}} = \frac{2\Sigma^{\text{fit}}}{\mu^{\text{fit}}} - 1 = \frac{2}{\mu^{\text{fit}}} - 1 < 1$$

One would measure  $\eta < 1$  in both cases!

Is it possible to measure the true  $\Psi$  ?

# Observed galaxy distribution

Redshift-space distortion

The “standard” terms:

$$\Delta(\mathbf{n}, z) = \delta_g - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V}_b \cdot \mathbf{n})$$

Leading order corrections:

$$\Delta^{\text{rel}}(\mathbf{n}, z) = \frac{1}{\mathcal{H}} \partial_r \Psi + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \left( 1 - 5s + \frac{5s - 2}{\mathcal{H}r} - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + f^{\text{evol}} \right) \mathbf{V} \cdot \mathbf{n} + \dots$$

Gravitational redshift

*Yoo, Fitzpatrick, Zaldarriaga, Phys. Rev. D80, 083514 (2009)*

*Yoo, Phys. Rev. D82, 083508 (2010)*

*Bonvin, Durrer, Phys. Rev. D 84, 063505 (2011)*

*Challinor, Lewis, Phys. Rev. D84, 043516 (2011)*

# Observed galaxy distribution

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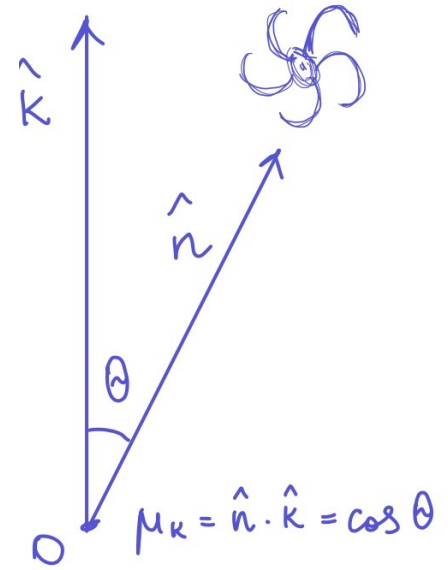
$$\Delta(\mathbf{n}, z) = \delta_g - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V}_b \cdot \mathbf{n})$$

In Fourier space:

$$\Delta(\mathbf{k}, z) = b \delta(\mathbf{k}, z) - \frac{1}{\mathcal{H}} \mu_k^2 \theta_b(\mathbf{k}, z)$$

Leading order corrections:

Even power of  $\mu_k$



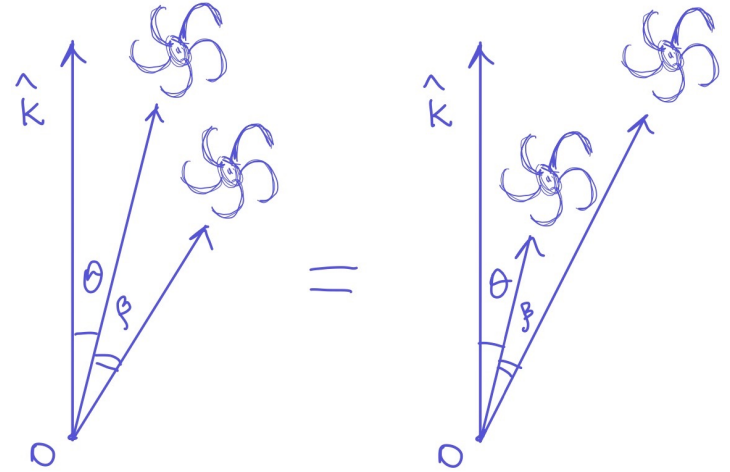
$$\Delta^{\text{rel}}(\mathbf{n}, z) = \frac{1}{\mathcal{H}} \partial_r \Psi + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \left( 1 - 5s + \frac{5s - 2}{\mathcal{H}r} - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + f^{\text{evol}} \right) \mathbf{V} \cdot \mathbf{n} + \dots$$

$$\Delta^{\text{rel}}(\mathbf{k}, z) = i\mu_k \left[ -\frac{k}{\mathcal{H}} \Psi(\mathbf{k}, z) + \left( 1 - 5s + \frac{5s - 2}{\mathcal{H}r} - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + f^{\text{evol}} \right) \frac{\theta_g(\mathbf{k}, z)}{k} + \frac{\dot{\theta}_g(\mathbf{k}, z)}{k\mathcal{H}} \right] + \dots$$

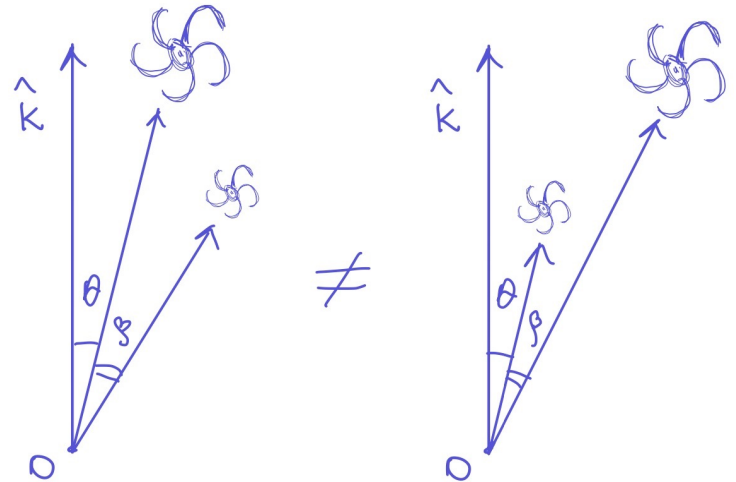
Odd power of  $\mu_k$

# Can one isolate the dipolar distortion?

No, if galaxies are indistinguishable



Yes, if galaxies are distinguishable



# Multipole expansion of correlation between two galaxy populations: B (bright) and F (faint)

$$P_{\text{BF}}^{\text{gal}}(k, \mu_k, z) = \sum_{\ell} P_{\text{BF}}^{(\ell)}(k, z) \mathcal{L}_{\ell}(\mu_k)$$

monopole:  $P_{\text{BF}}^{(0)}(k, z) = \left[ b_{\text{B}} b_{\text{F}} + \frac{1}{3}(b_{\text{B}} + b_{\text{F}}) f_m + \frac{1}{5} f_m^2 \right] P_{\delta\delta}(k, z),$

quadrupole:  $P_{\text{BF}}^{(2)}(k, z) = \left[ \frac{2}{3}(b_{\text{B}} + b_{\text{F}}) f_m + \frac{4}{7} f_m^2 \right] P_{\delta\delta}(k, z),$

hexadecapole:  $P_{\text{BF}}^{(4)}(k, z) = \frac{8}{35} f_m^2 P_{\delta\delta}(k, z),$

dipole:  $P_{\text{BF}}^{(1)}(k, z) = i \alpha \left( f_m, \dot{f}_m, \Theta_{\text{B}}, \Theta_{\text{F}} \right) \frac{\mathcal{H}}{k} P_{\delta\delta}(k, z) + i(b_{\text{B}} - b_{\text{F}}) \frac{k}{\mathcal{H}} P_{\delta\Psi}(k, z),$

octupole:  $P_{\text{BF}}^{(3)}(k, z) = i \beta \left( f_m, \Theta_{\text{B}}, \Theta_{\text{F}} \right) \frac{\mathcal{H}}{k} P_{\delta\delta}(k, z),$



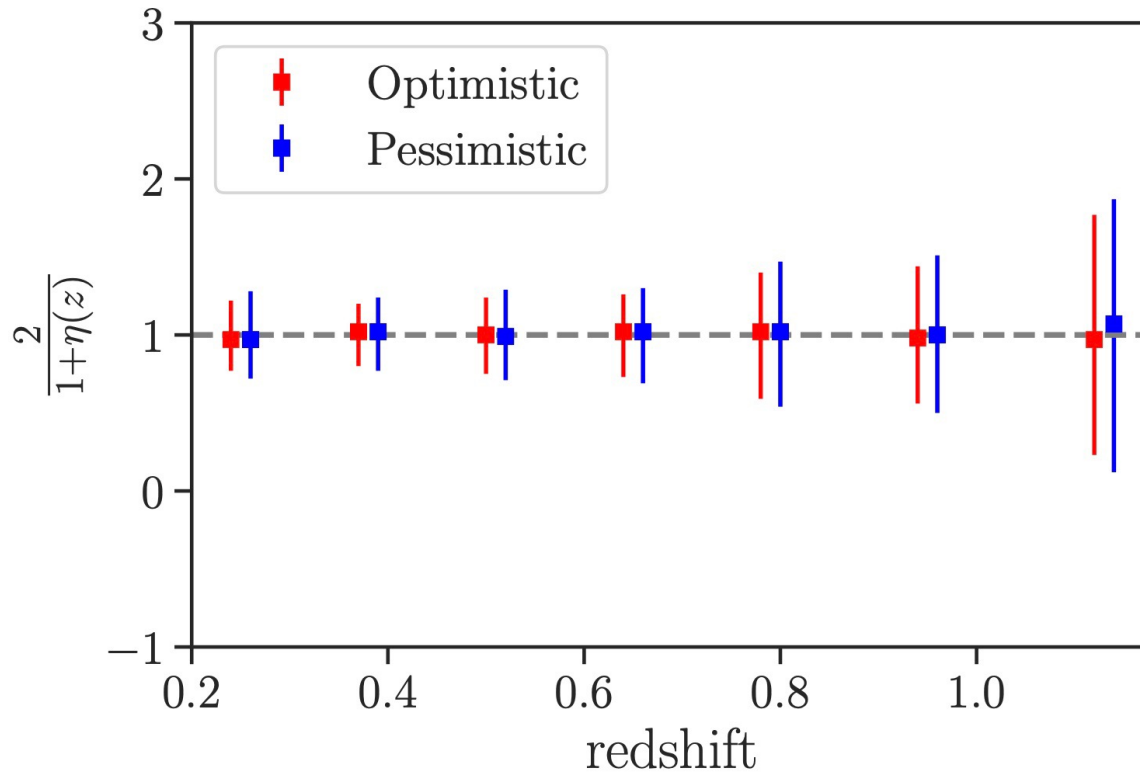
Galaxy – galaxy lensing correlation

Gravitational slip:

$$\frac{P_{\delta(\Phi+\Psi)}(k, z)}{P_{\delta\Psi}(k, z)} = 1 + \eta(k, z)$$

Galaxy dipole:  $P_{\text{BF}}^{(1)}$

# LSST+SKA forecast



# Summary

One can learn a lot more from today's data than  $w_0, w_a, \Sigma_0, \mu_0$   
(the new version of MGCAMB includes the spline implementation)

Need to measure relativistic corrections (gravitational redshift) to distinguish a modification of gravity from a dark matter force. This may be possible with DESI, more likely with SKA, combined with LSST.