Real Scalar Phase Transitions: Bubble Nucleation, Nonperturbatively

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PONT

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Phase transitions in the early universe

- Grand Unified Theories, Electroweak, QCD...
- In the Standard Model (SM) the electroweak PT is a crossover
- SM is incomplete → beyond SM (BSM) physics
- Things to look for: topological defects, bubbles from EWPT, ... ?
- Something we could possibly detect: Gravitational waves?

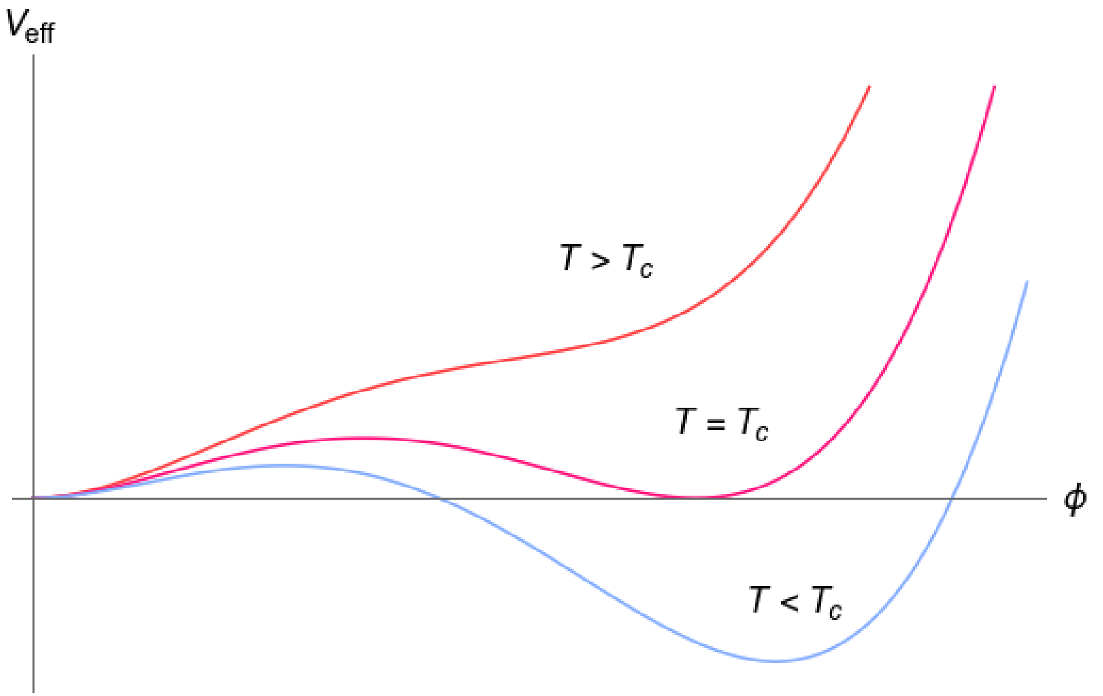
Fate of the false vacuum

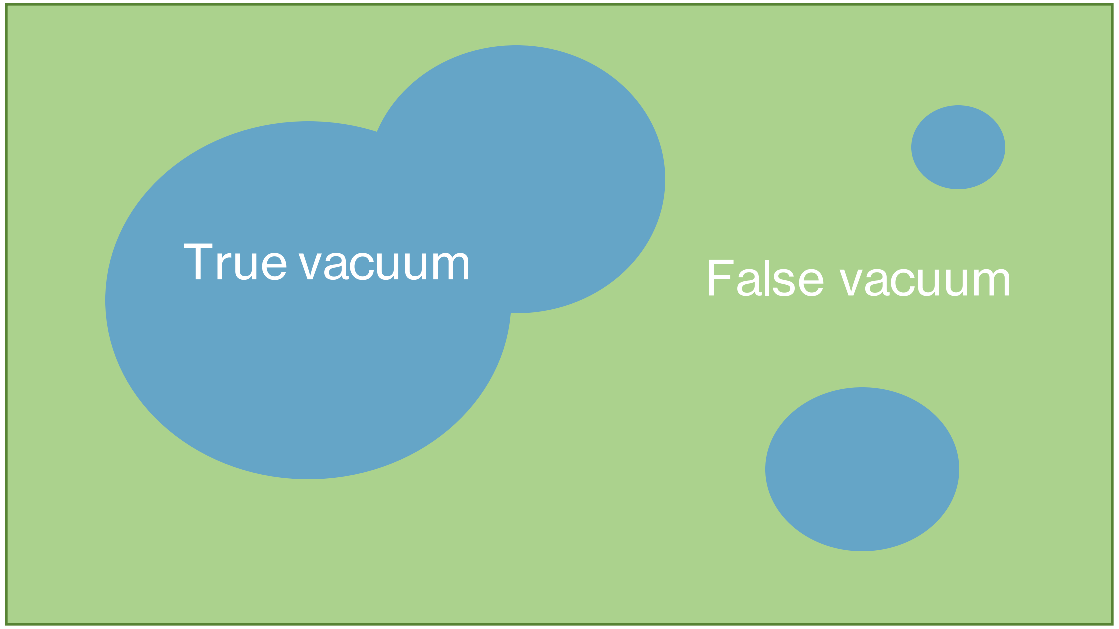
- Semiclassical solution (Coleman 1977)

3D action

Volume averaged nucleation rate $\frac{\Gamma}{V} = A\, e^{-S_{3,b}(T)/T}$

Prefactor





Fate of the false vacuum

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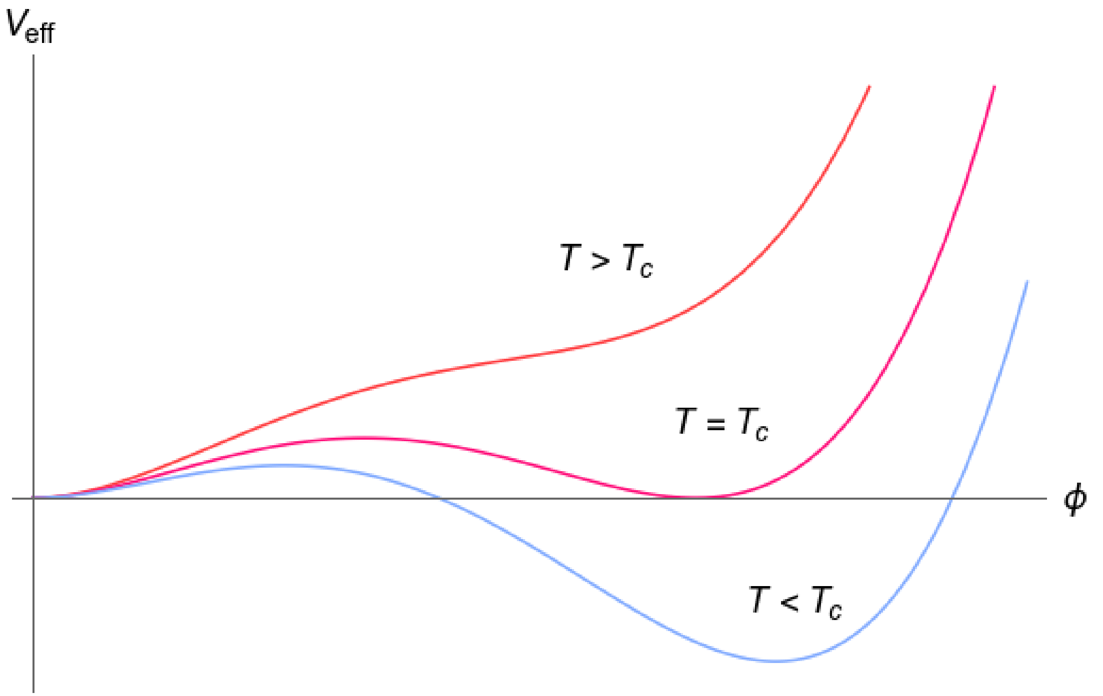
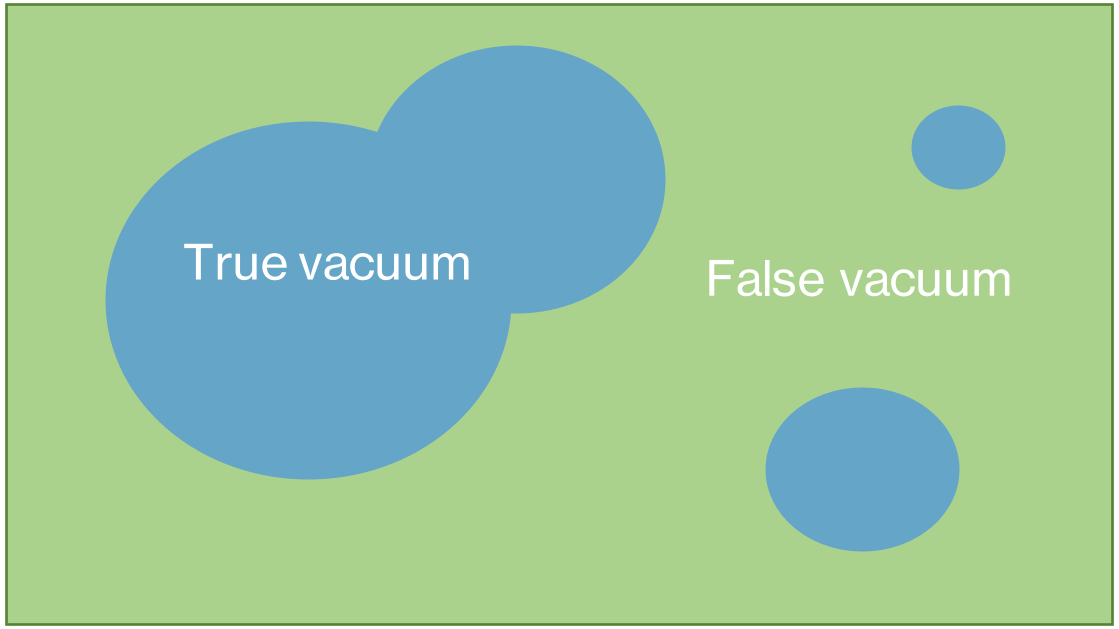
3D action

Prefactor

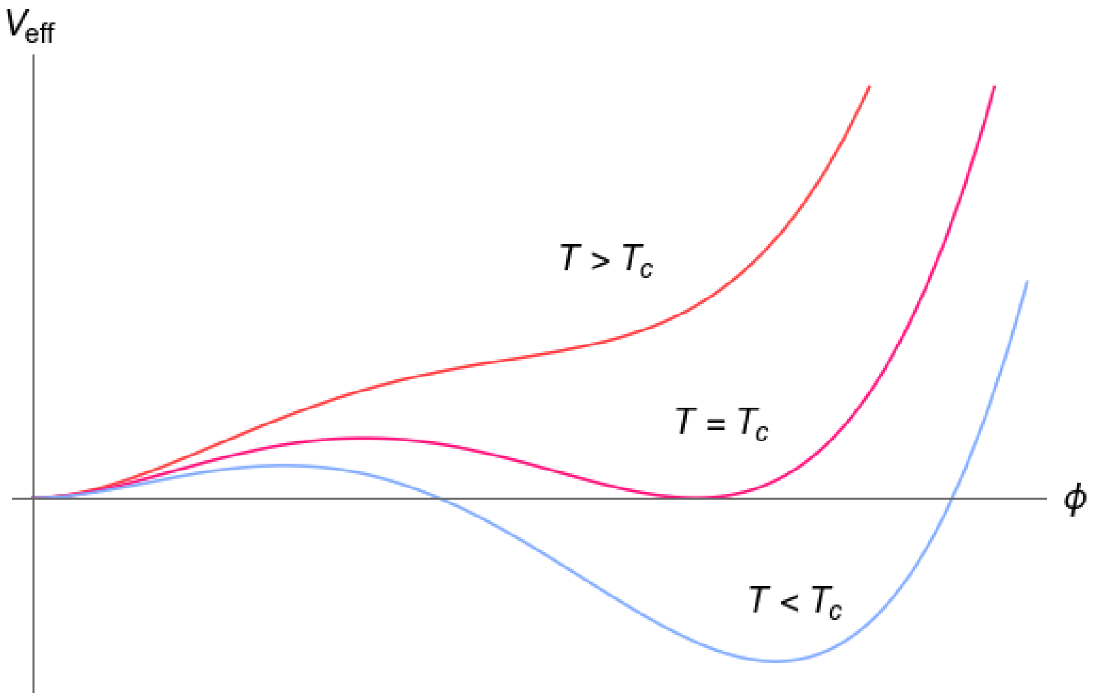
$$A = T\left(\frac{S_3(T)}{2\pi T}\right)^{3/2}\left|\frac{\det'[-\nabla^2 + V''(\phi_b)]}{\det[-\nabla^2 + V''(0)]}\right|^{-1/2}$$

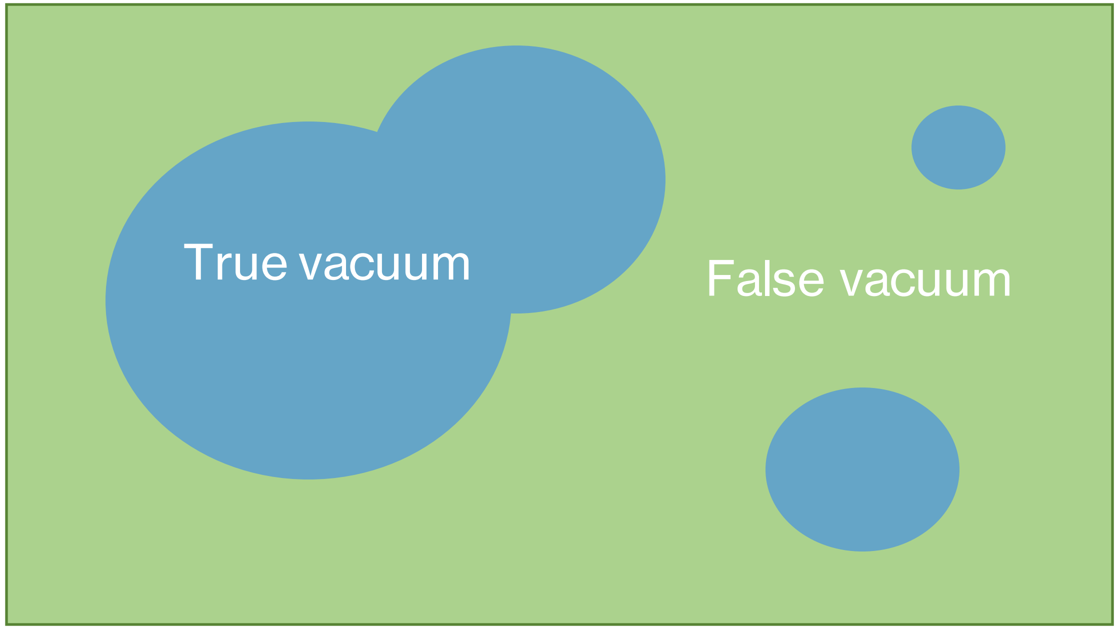




Fate of the false vacuum

- Semiclassical solution (Coleman 1977)
- Determining the prefactor analytically difficult → perturbation theory





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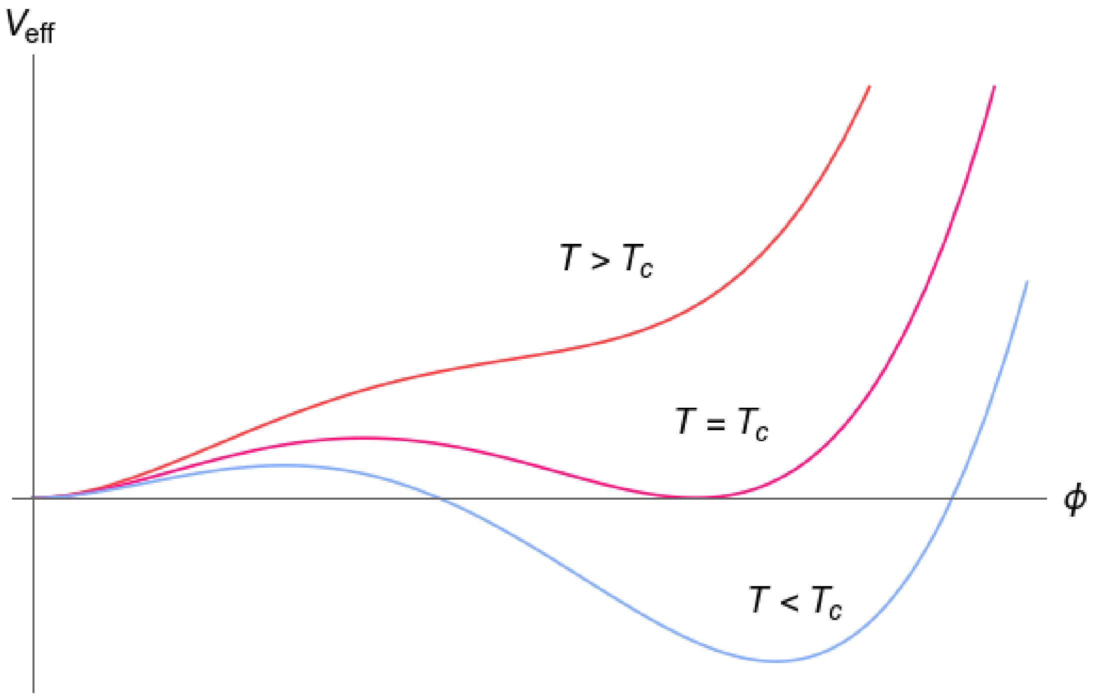
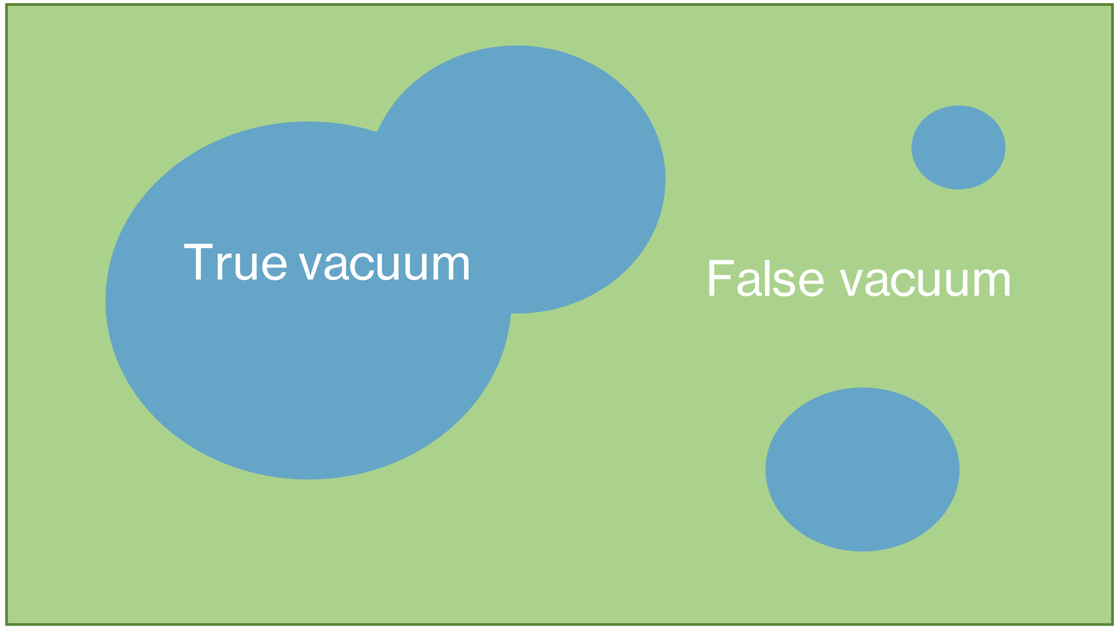
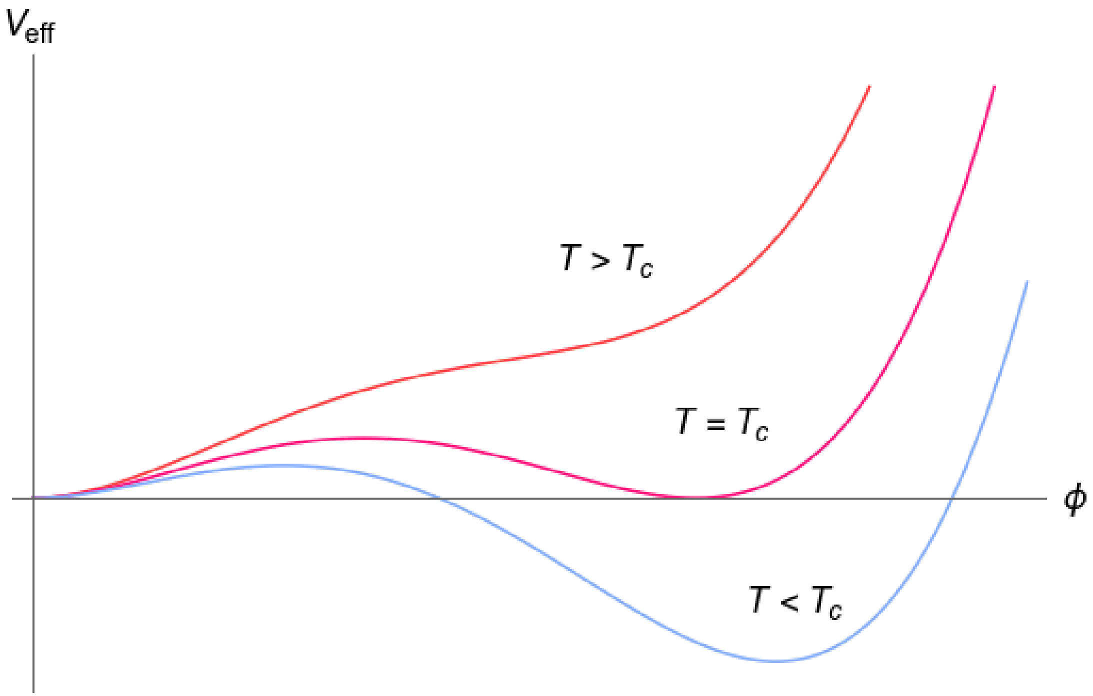
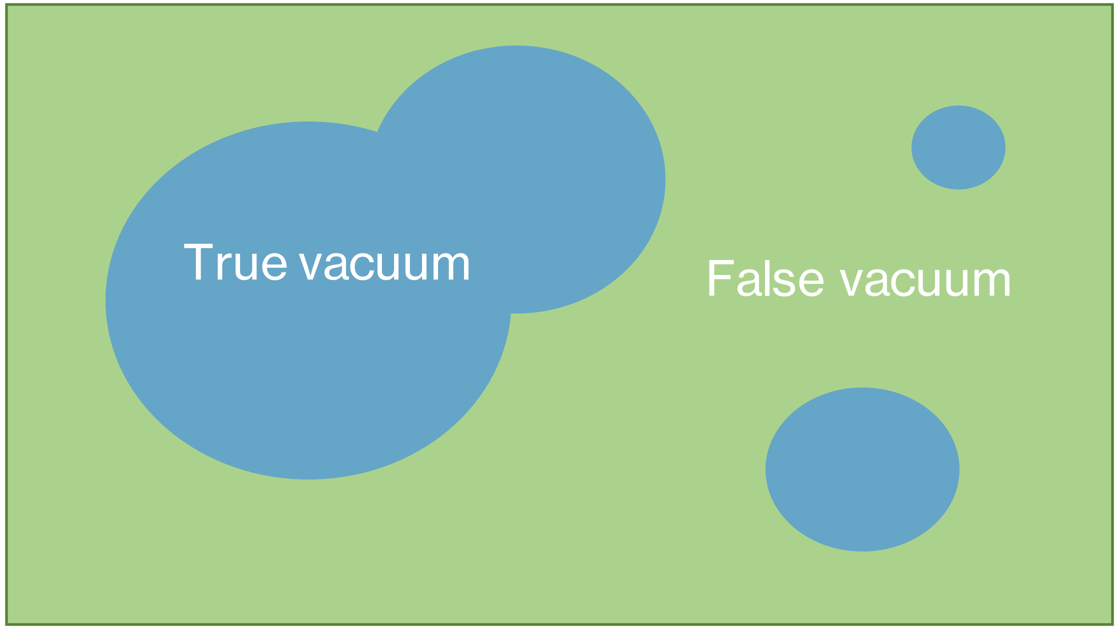




Fate of the false vacuum

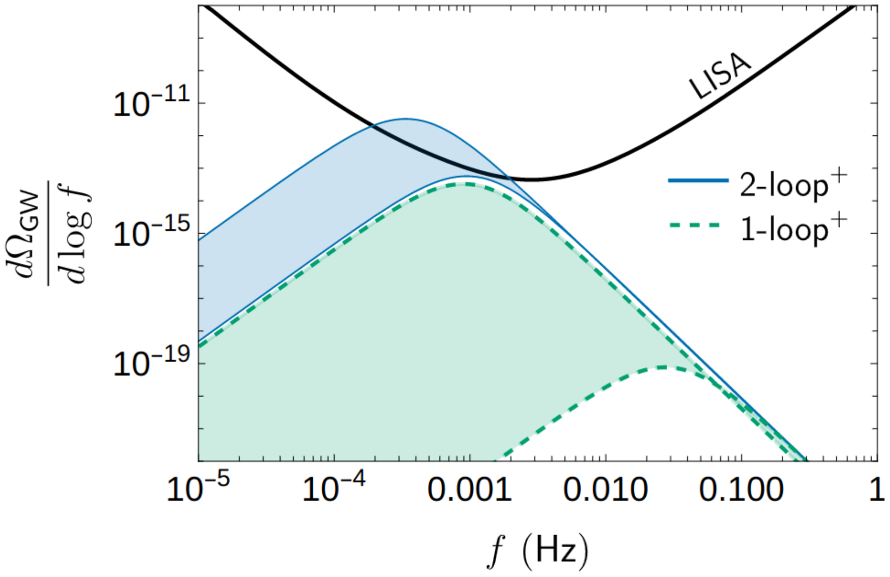
- Semiclassical solution (Coleman 1977)
- Determining the prefactor analytically difficult → perturbation theory
- Perturbation theory suffers from the so-called infrared problem
- Moore, Rummukainen & Tranberg introduce a simulation method (hep-lat/0103036, hep-ph/0009132)



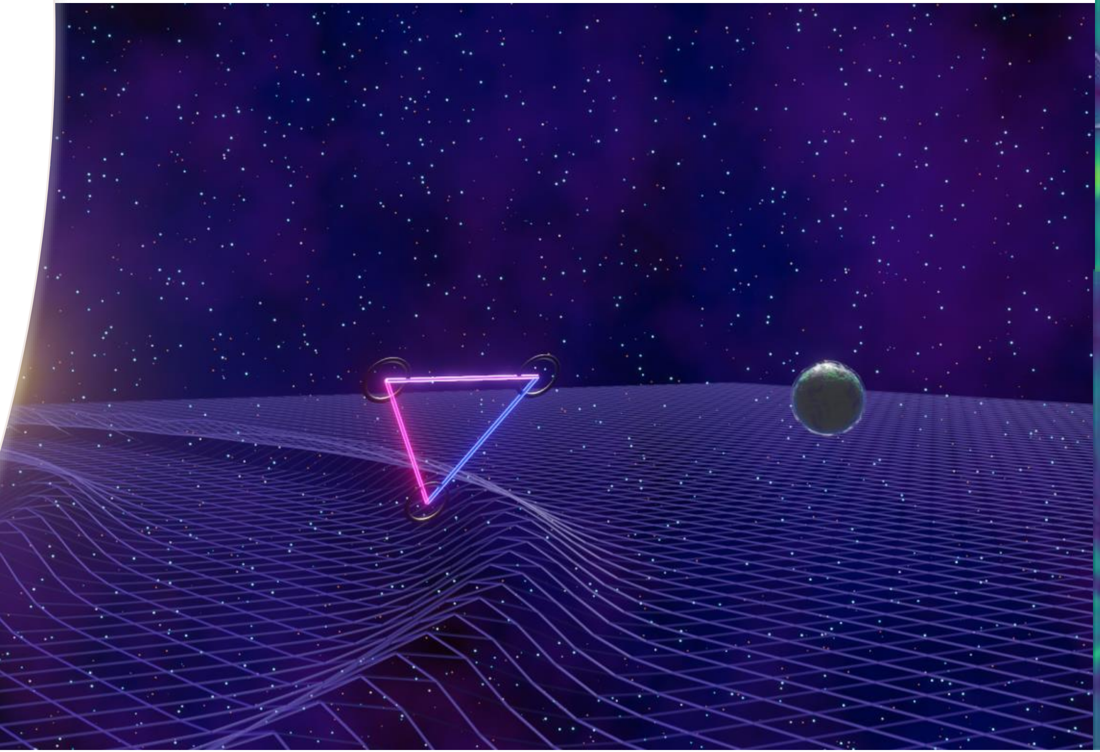


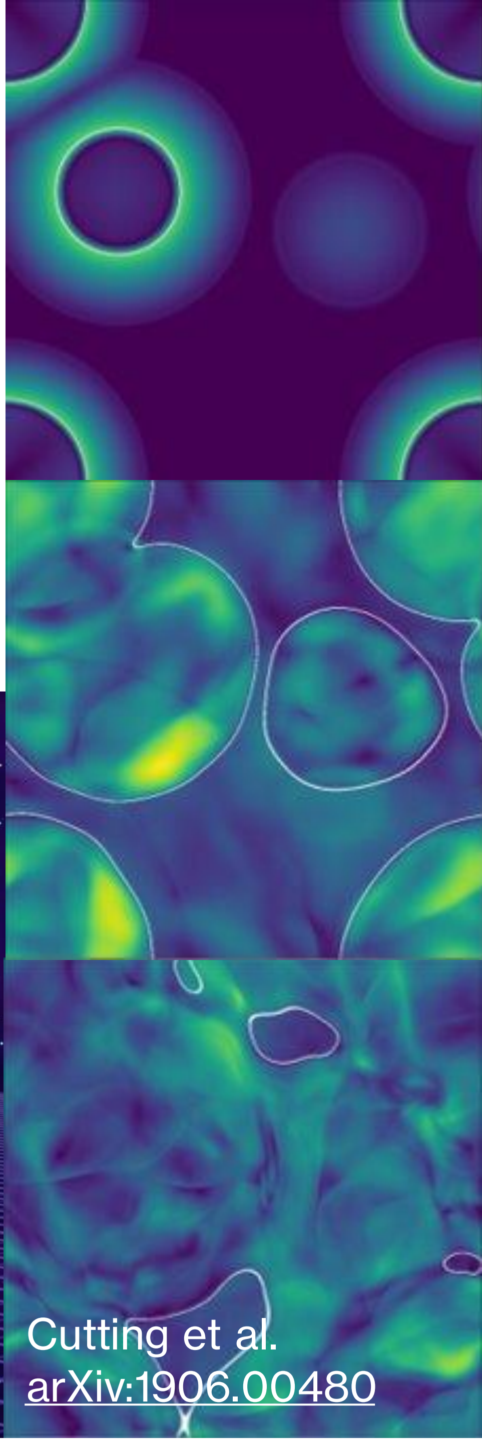
Phase transitions & gravitational waves

- Bubbles source the GWs in the early universe plasma $\longrightarrow$ stochastic GW background?
- Baryogenesis?
- Uncertainty in perturbative calculations?



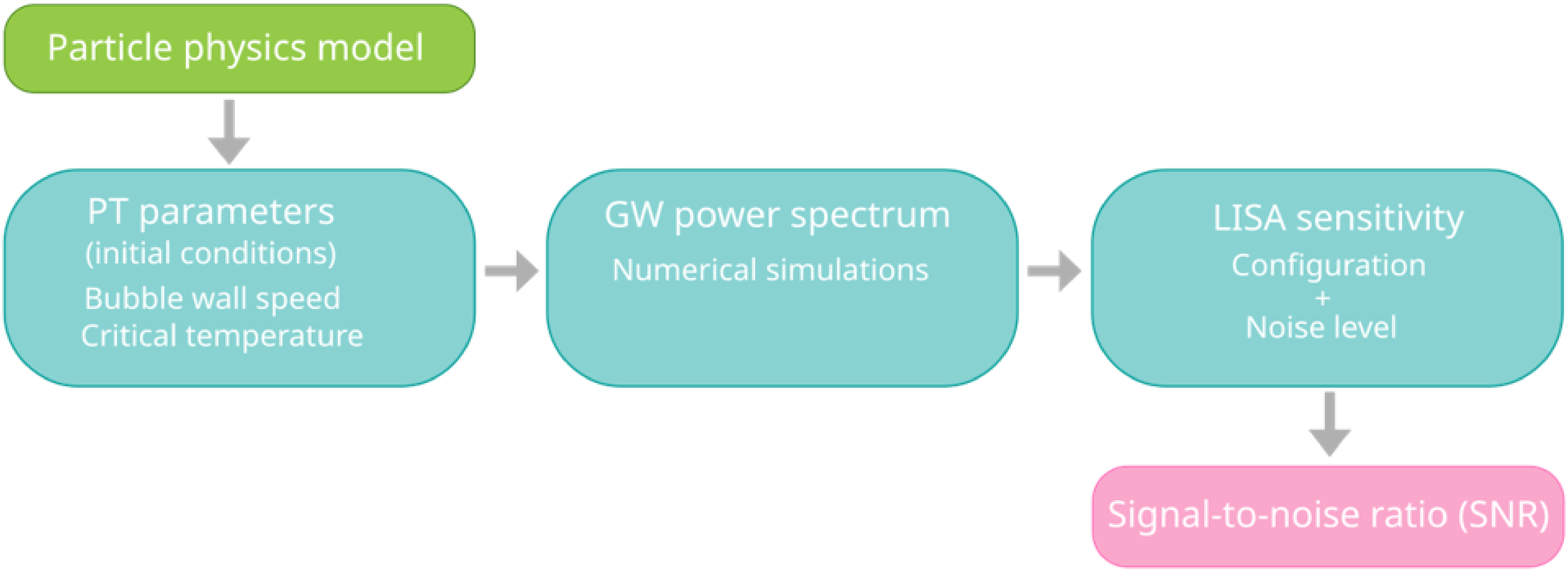
Gould, Tenkanen arXiv:2104.04399





Cutting et al. arXiv:1906.00480

Gravitational waves & LISA



Modified from Caprini et al. arXiv:1910.13125

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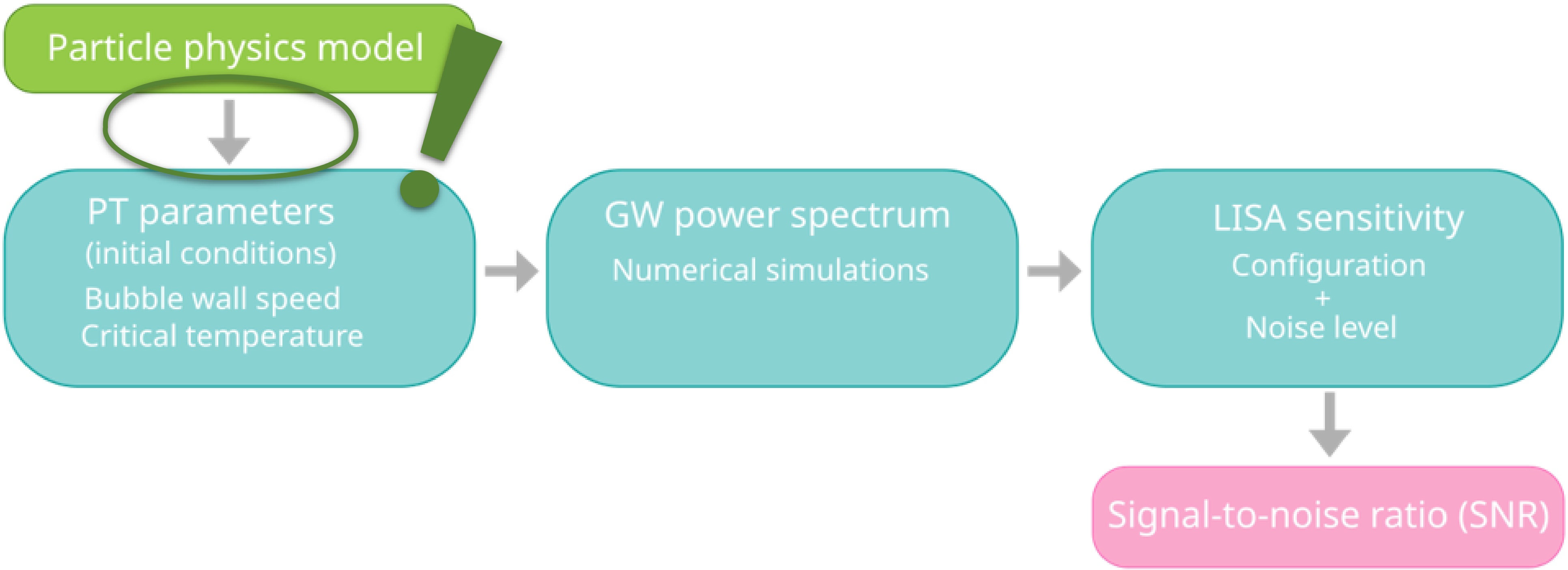

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The real scalar theory

Gould, arXiv:2101.05528

- Toy model possessing key features of BSM models
  - Potential has a tree-level barrier
  - Compared to the previously studied cubic anisotropy model, this model would have a stronger PT
- Dimensional reduction 4D cont → 3D cont → 3D lattice (imaginary time, high temp)

$$S_{\rm lat} = \sum_x a^3 \left[-\frac{1}{2} Z_\phi \phi_x (\nabla^2_{\rm lat}\phi)_x + \sigma_{\rm lat}\phi_x + \frac{1}{2} Z_\phi Z_m m^2_{\rm lat}\phi_x^2 + \frac{1}{3!} g_{\rm lat}\phi_x^3 + \frac{1}{4!} Z_\phi^2 \lambda_{\rm lat}\phi_x^4 \right]$$

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Bubble nucleation, nonperturbatively

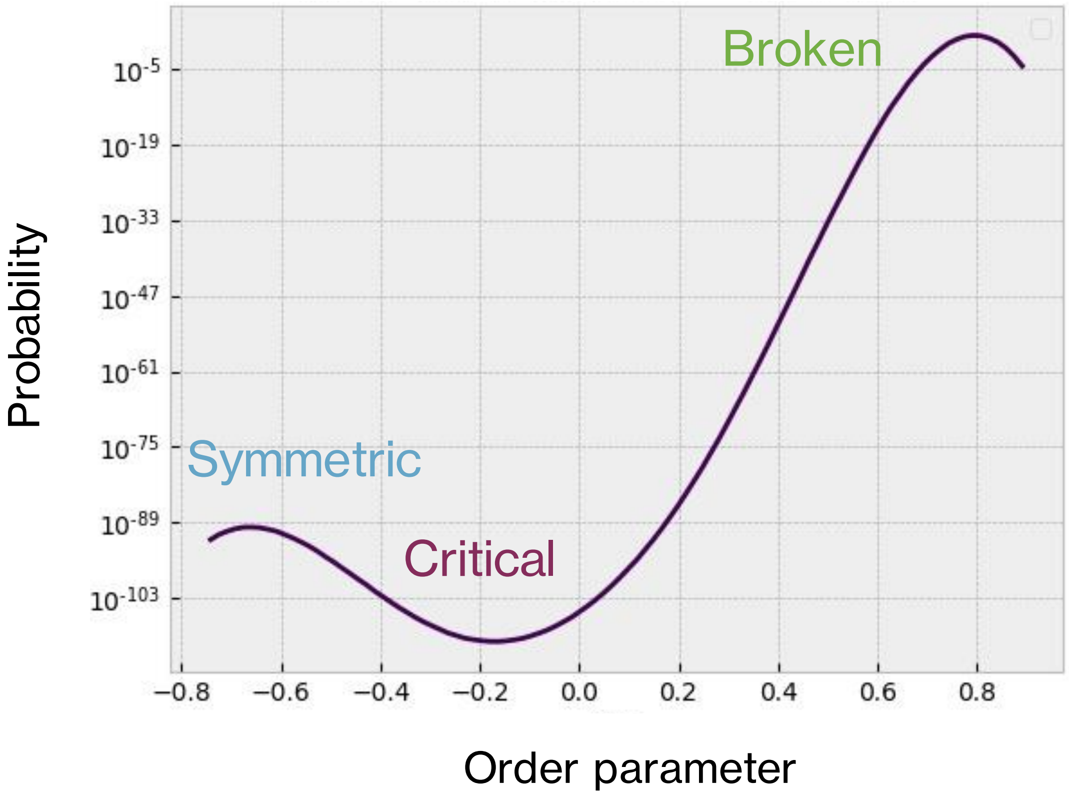
1. Pick an order parameter that behaves differently in the two phases

2. Simulate the probability of being in the critical bubble configuration

3. Perform real time evolution to determine whether the critical bubble tunnels or not

4. Calculate the total nucleation rate, dynamical prefactor × probability info

Bubble nucleation, nonperturbatively



3

Perform real time evolution to determine whether the critical bubble tunnels or not

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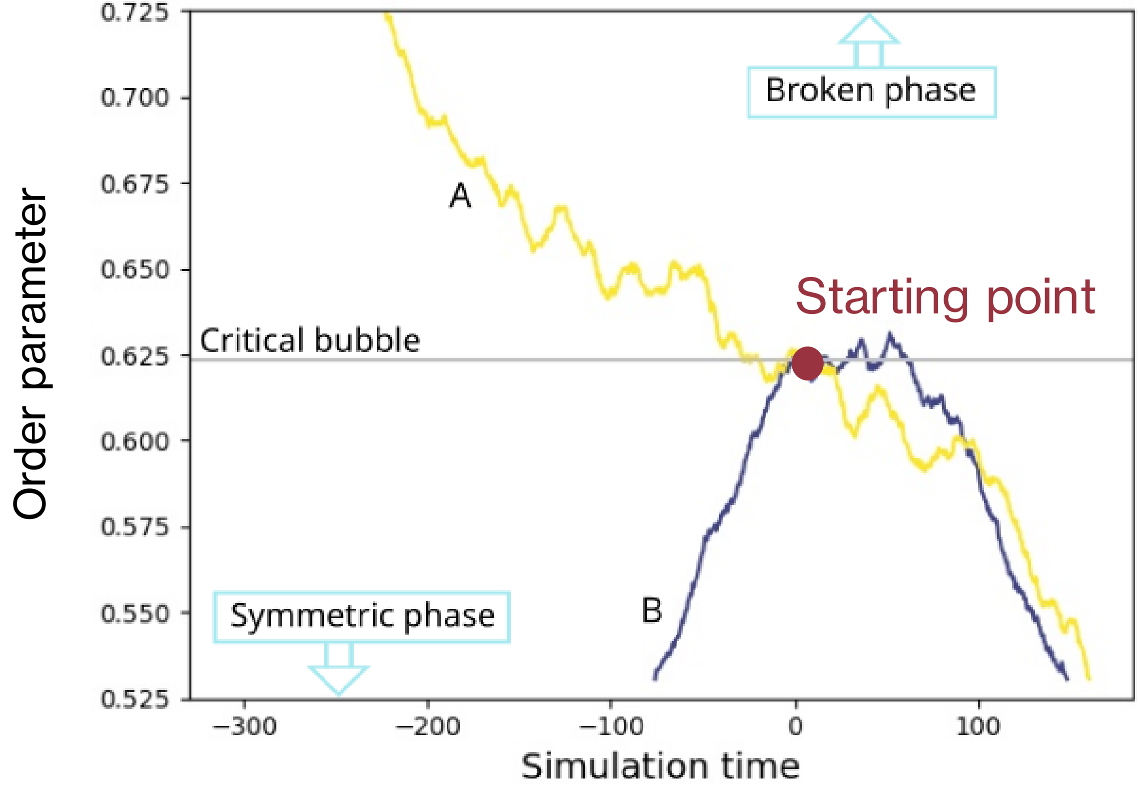

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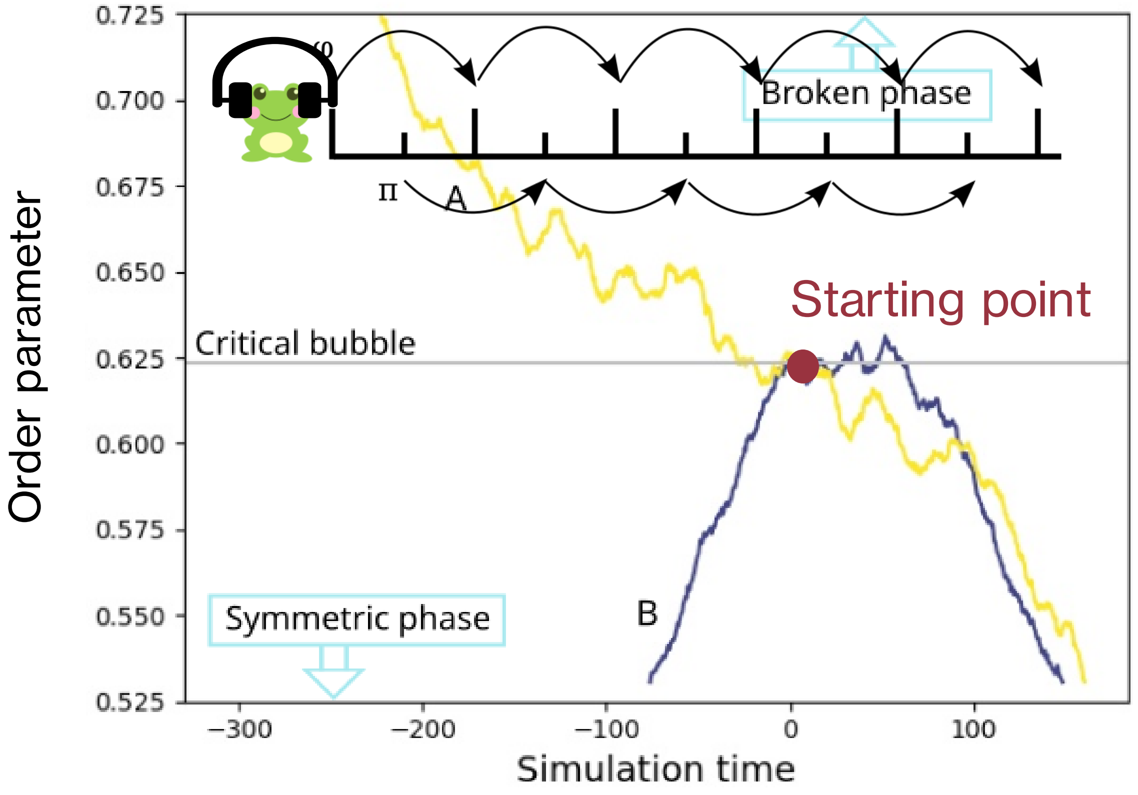
Bubble nucleation, nonperturbatively

Stochastic Hamiltonian eqs + Langevin



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Bubble nucleation, nonperturbatively

1

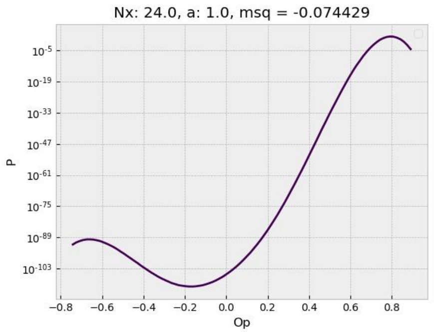
Pick an order parameter that behaves differently in the two phases

$$\Gamma V = \frac{1}{2} P_C^{\epsilon} \left\langle \left| \frac{\Delta\theta(\alpha)}{\Delta t} \right| \times \mathbf{d}^{\alpha} \right\rangle$$

4

Calculate the total nucleation rate, dynamical prefactor × probability info

Bubble nucleation, nonperturbatively



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Pick an order parameter that behaves differently in the two phases

Flux, rate of change of op

$$\Gamma V = \frac{1}{2} P_C^{\epsilon} \left\langle \left| \frac{\Delta\theta(\alpha)}{\Delta t} \right| \times \mathbf{d}^{\alpha} \right\rangle$$

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Calculate the total nucleation rate, dynamical prefactor ×

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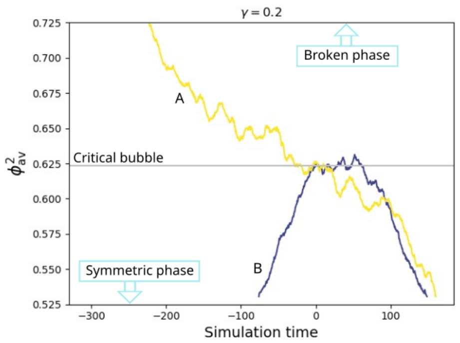
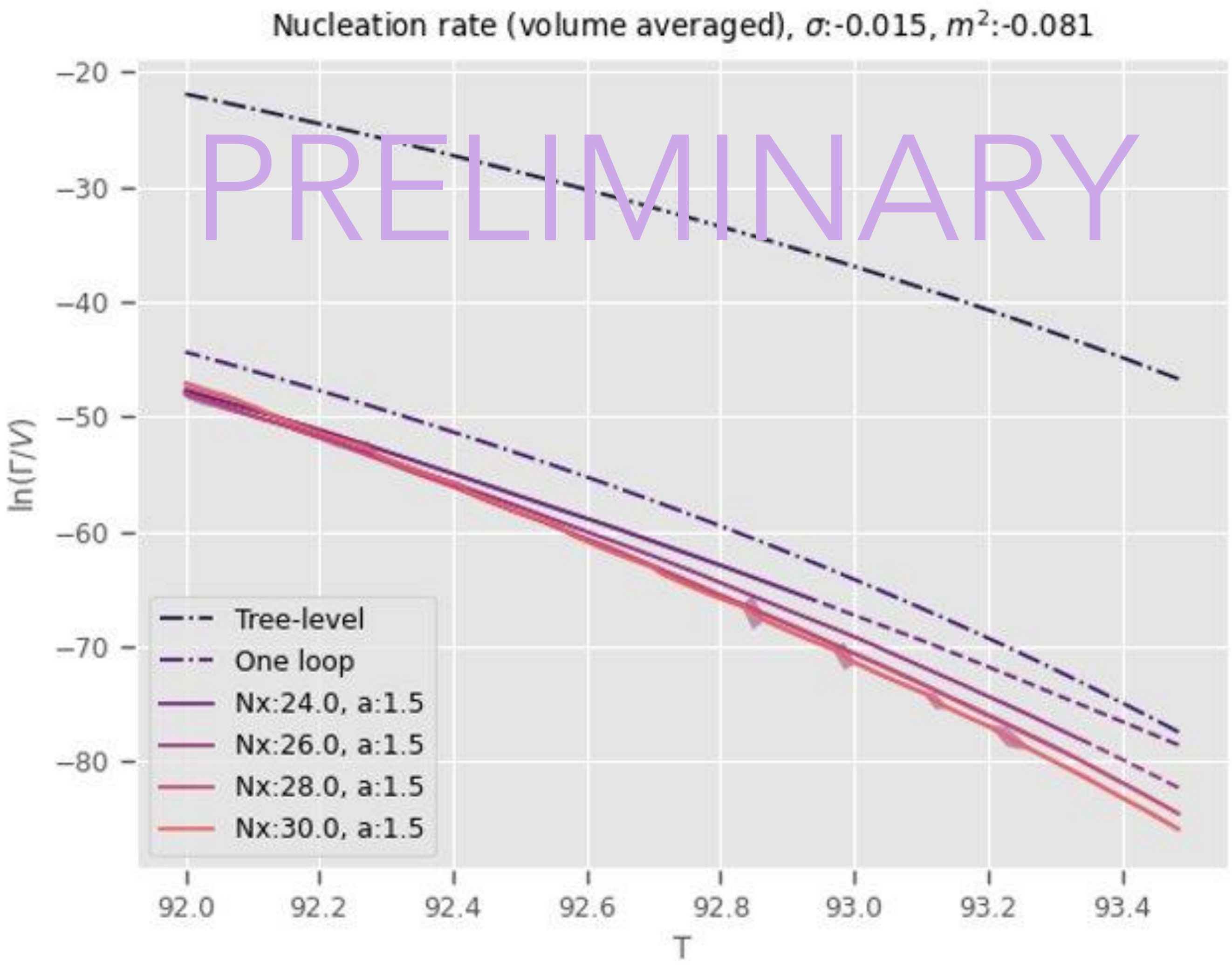
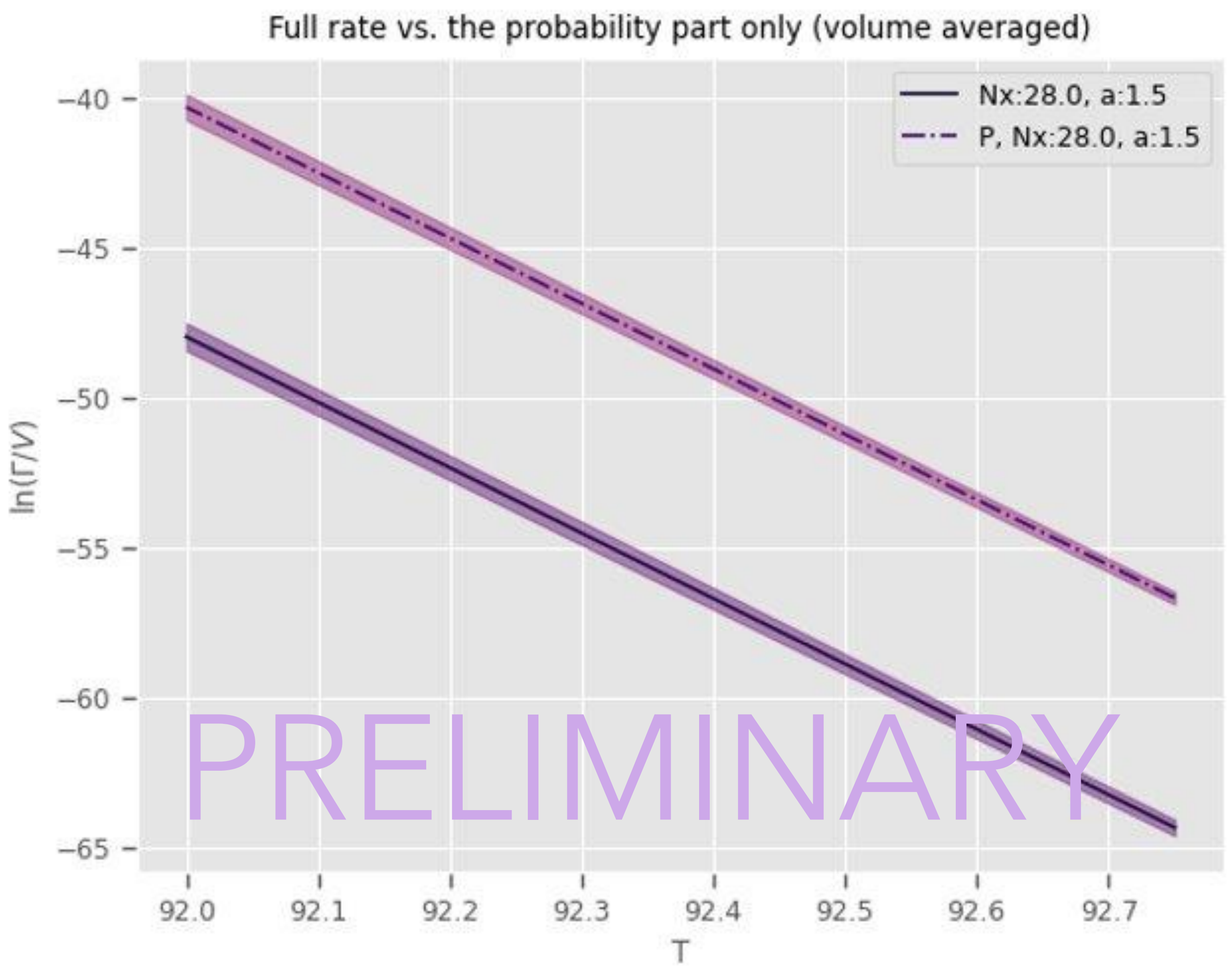




Results

Volume averaged nucleation rate vs. the perturbative calculation results as a function of temperature T

Clarification
Tree-level = bounce action
One loop = bounce + fluctuation determinant

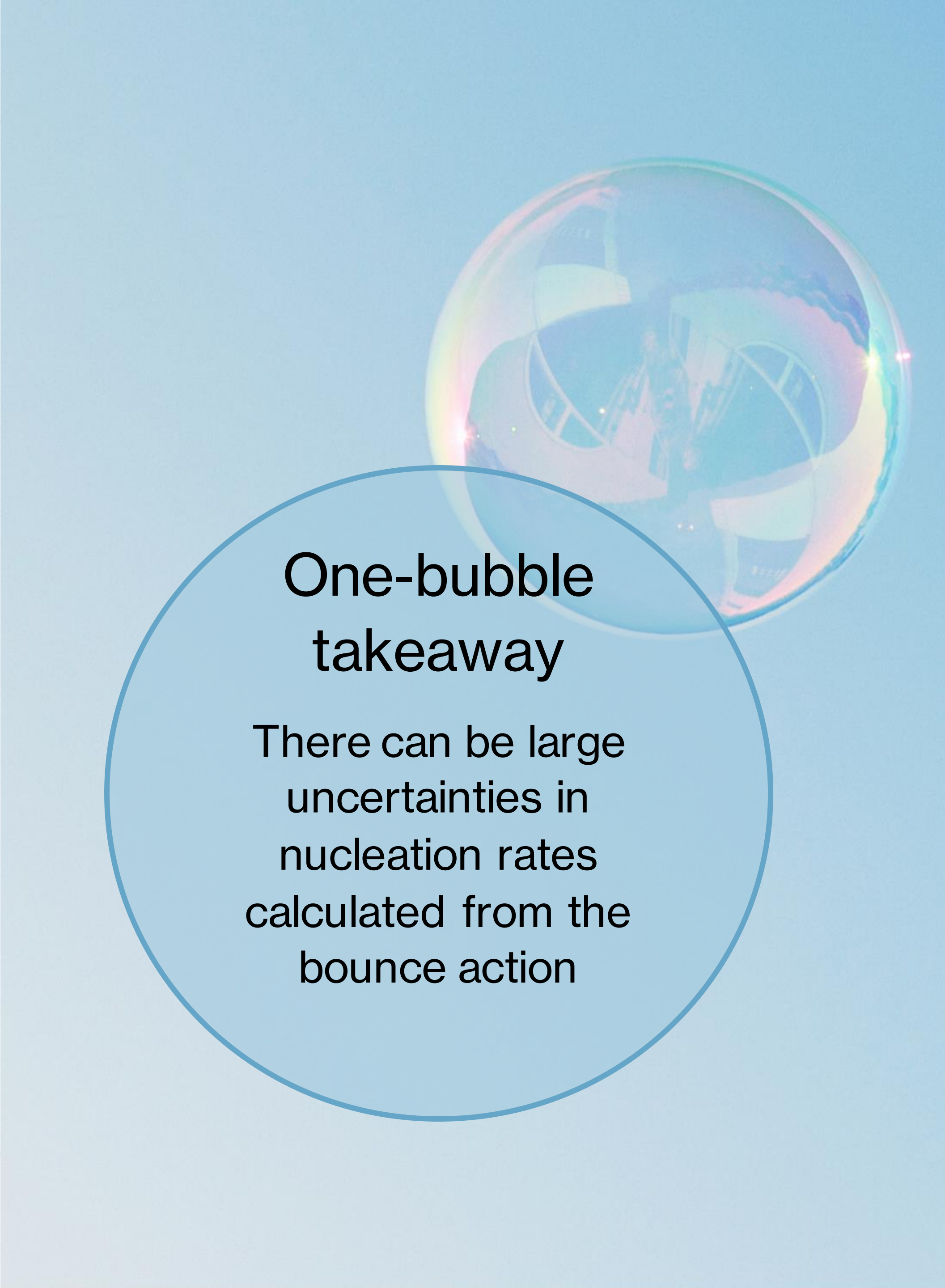


Results

Volume averaged probability part only vs. full volume averaged nucleation rate with the prefactor as a function of temperature T

Why does this matter?

- Allows us to calibrate the uncertainty in PT parameters when obtained from perturbative results
- Our simulations show us a suppression of the nucleation rate by a factor of 20 compared to the one loop estimate
- Accurate computations of the nucleation rate are crucial for calculating e.g. the GW power spectrum
- Method and results can be applied to other theories



One-bubble takeaway

There can be large uncertainties in nucleation rates calculated from the bounce action

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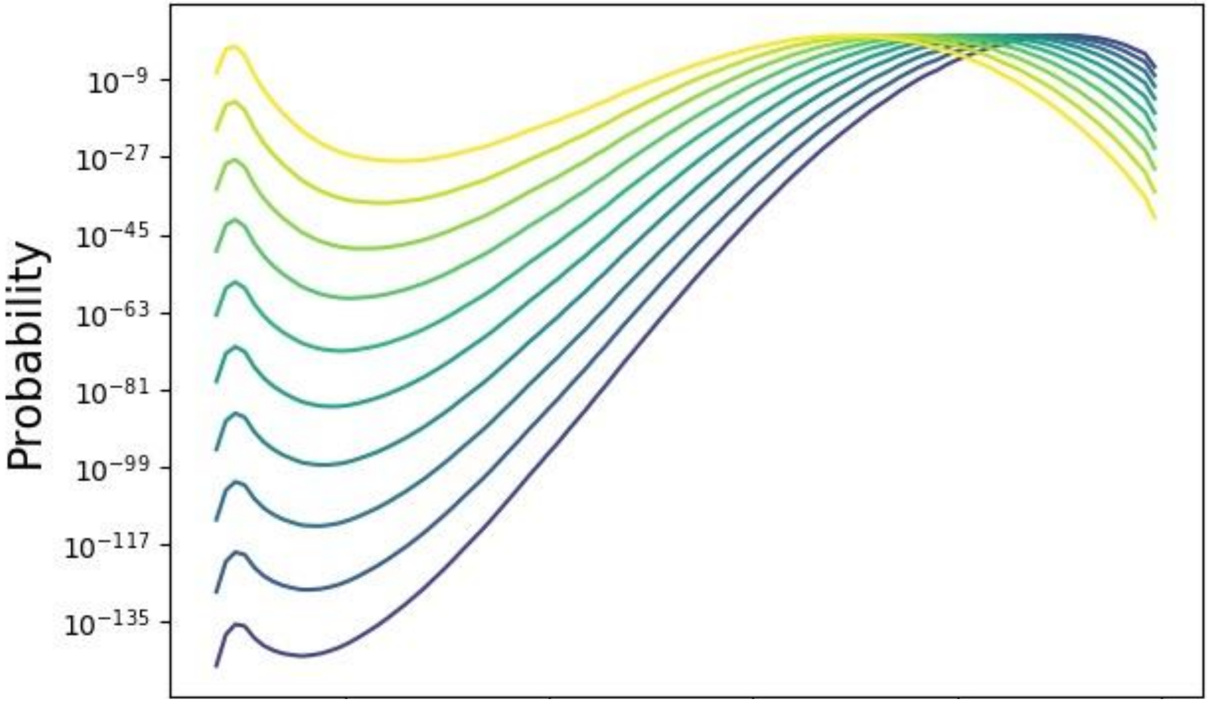
Thank you! Questions?

Contact:
anna.kormu@helsinki.fi

Backup: Reweighting



Order parameter

- Simulations are computationally expensive → use reweighting the order parameter histogram at different parameter points
- In our case we reweight in two parameters