Strong electroweak phase transition and simplified dark matter models

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S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models,* JCAP **10** (2022) 044 [2207.12207], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

Motivation

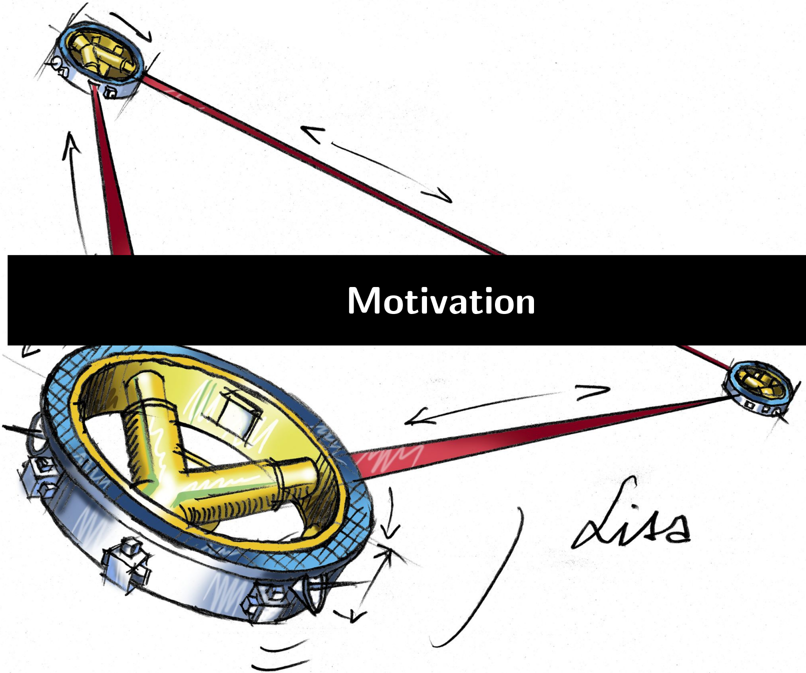

figure by ESA-C. Vijoux

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- Baryogenesis — Baryon asymmetry of the universe
- Colliding bubbles — Gravitational wave (GW) production

In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

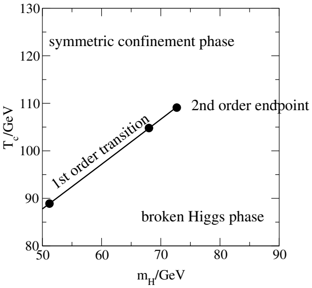

figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

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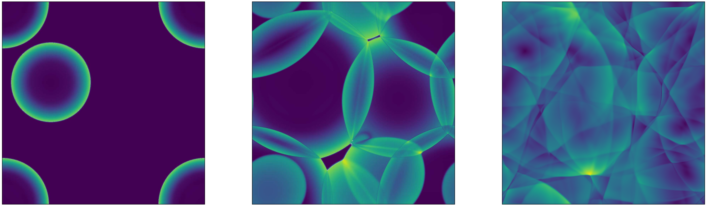
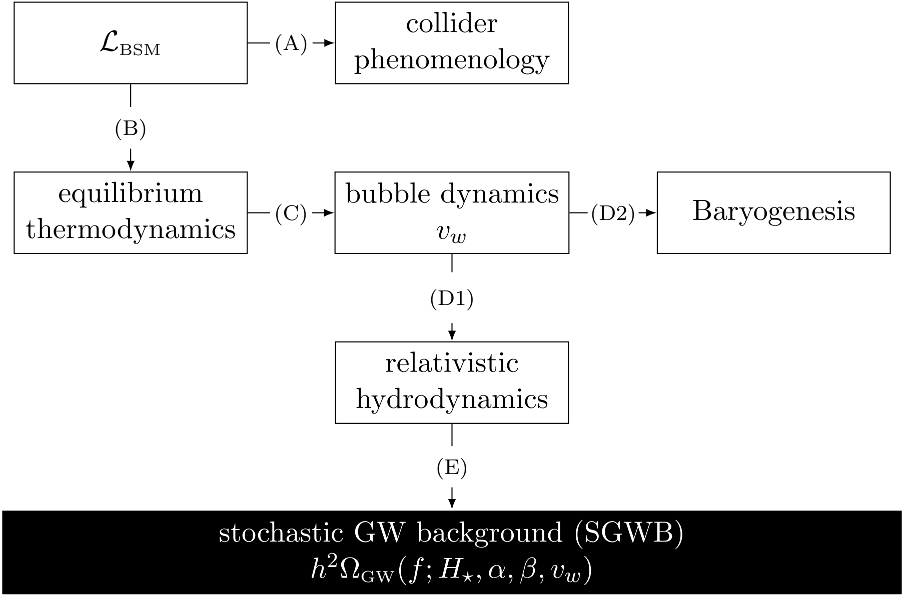
In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions. Study **BSM physics** near EW scale in context of phase transitions:

- Light fields strongly coupled to Higgs
- Comply with DM energy density $h^2\Omega_{\mathrm{DM}} = 0.1200 \pm 0.0012$[1]



[1] **Planck** Collaboration, N. Aghanim *et al.*, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. **641** (2020) A6 [1807.06209], figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

Uncertainties of the gravitational wave pipeline



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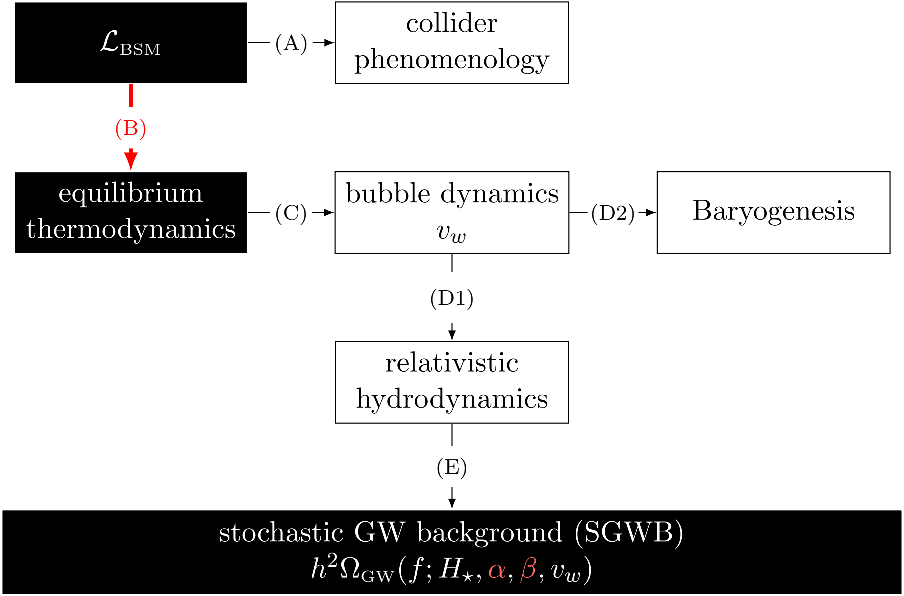
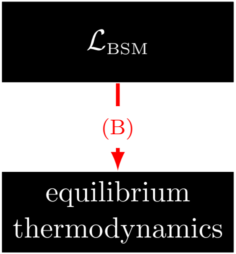
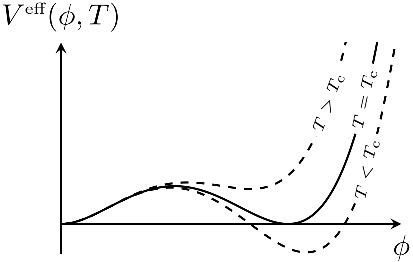


The effective potential in perturbation theory[2]





(B): Equilibrium thermodynamics as a function of BSM theory parameters. Encoded in **effective potential**, $V^{\rm eff}$. Origin of uncertainty.

The interface between **particle physics** and **cosmology**.

☆cf. **talks** by **C. Caprini Tue 11:50** and **A. Kormu Tue 17:10**

[2] R. Jackiw, *Functional evaluation of the effective potential*, Phys. Rev. D **9** (1974) 1686

Theoretical predictions are not robust

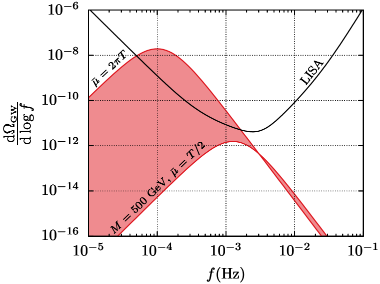
$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[3] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, T_*, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_*)^2}{T_*^8}$$

- ▷ Ensure (improve) quantitative precision at finite T?

Minimal SM extensions e.g.:

- ▷ SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$



[3] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

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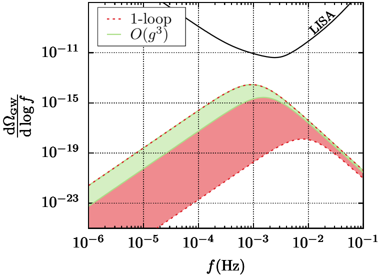
- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- xSM: SM + singlet



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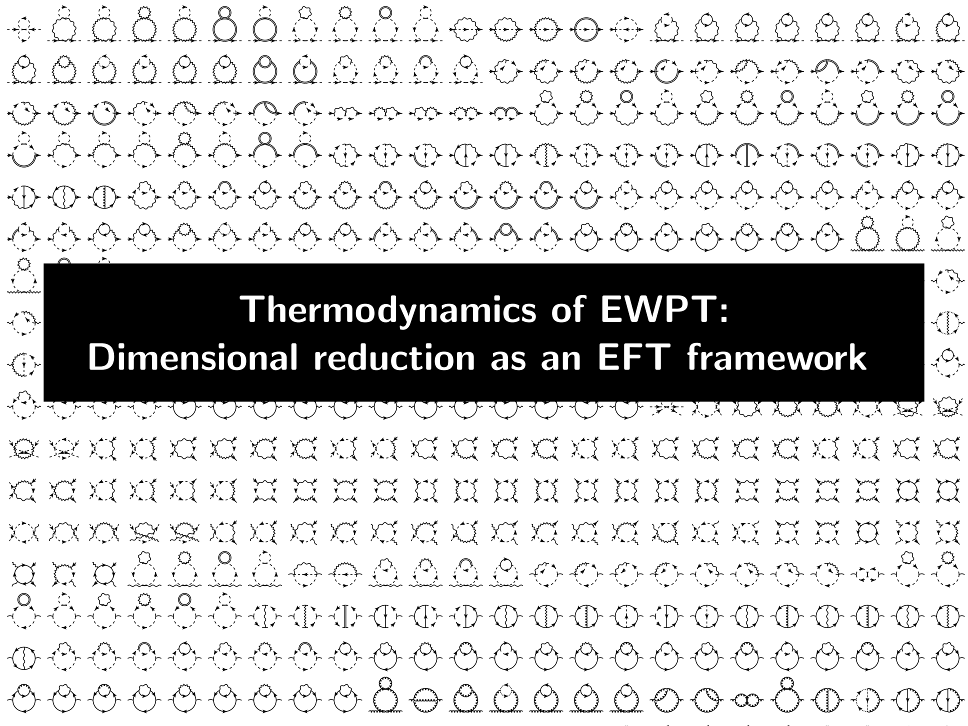


Thermodynamics of EWPT: Dimensional reduction as an EFT framework

Multi-scale Hierarchy in hot gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}}|p| \sim g^2/2$.

Theory separates scales rigorously:

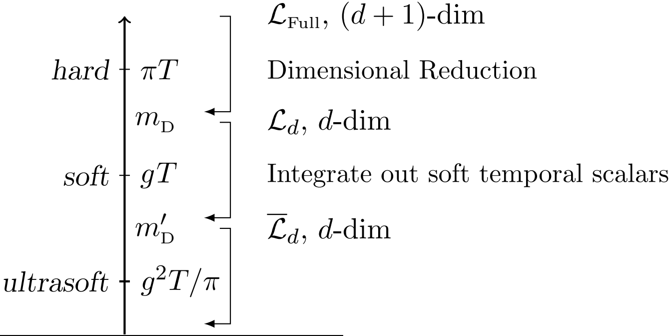
$$|p| \sim \begin{cases} \pi T & \textit{hard} \text{ scale} \\ gT & \textit{soft} \text{ scale} \\ g^{3/2} T & \boxed{\textit{supersoft} \text{ scale} \quad \text{symmetry breaking}} \\ g^2 T/\pi & \textit{ultrasoft} \text{ scale} \end{cases}$$

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[4]

[4] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas,* Phys. Lett. B **96** (1980) 289

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.
Incorporates an all order thermal resummation to by-pass IR problem.
Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[5] using e.g. `DRalgo`[6]



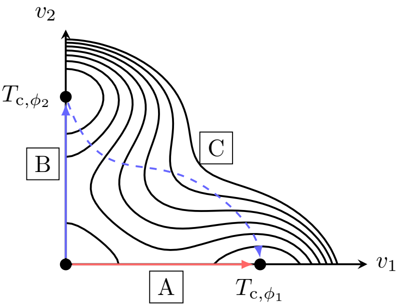
[5] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [`hep-ph/9508379`], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [`hep-lat/9510020`]

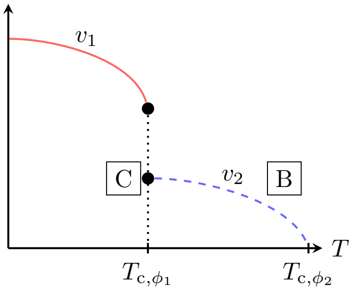
[6] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [`2205.08815`]

The effective potential in perturbation theory

requires **2-loop calculation** for 1-loop RG improvement at finite-T,[7] receives thermal corrections $\Pi_T \sim g^n T^2$. Close to crit. temperature T_c:

$$V^{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \Pi_T)\phi^2 + \frac{1}{2}\lambda\phi^4 + \#\phi^3 + \dots \ .$$





[7] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

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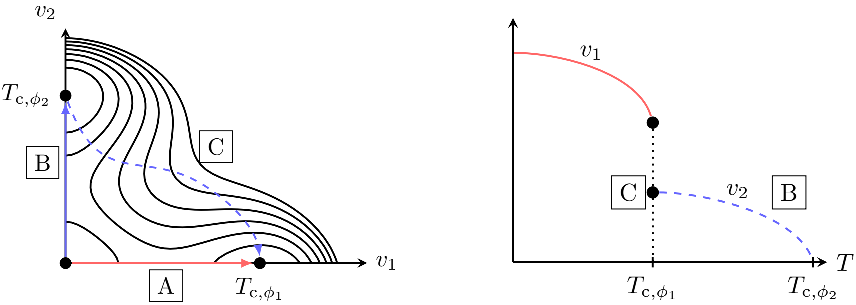
$$(-\mu^2 + g^n T^2) \sim \underbrace{0 \times (gT)^2}_{\text{soft}} + \underbrace{0 \times (g^{3/2}T)^2}_{\text{supersoft}} + \underbrace{\#(g^2T)^2}_{\text{ultrasoft}}.$$



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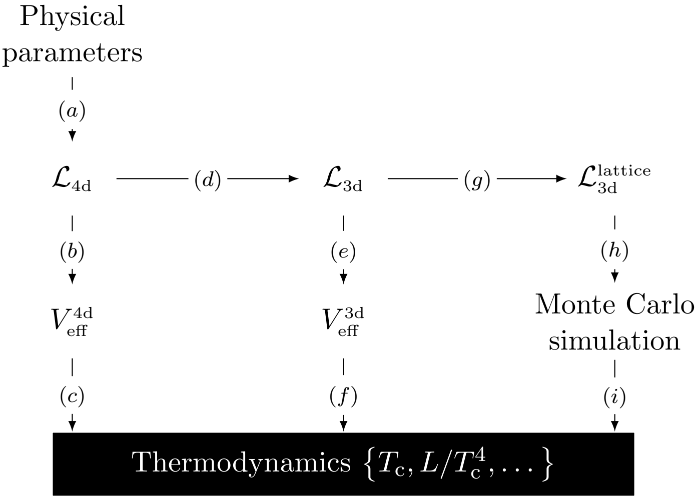
Thermodynamics of electroweak phase transition



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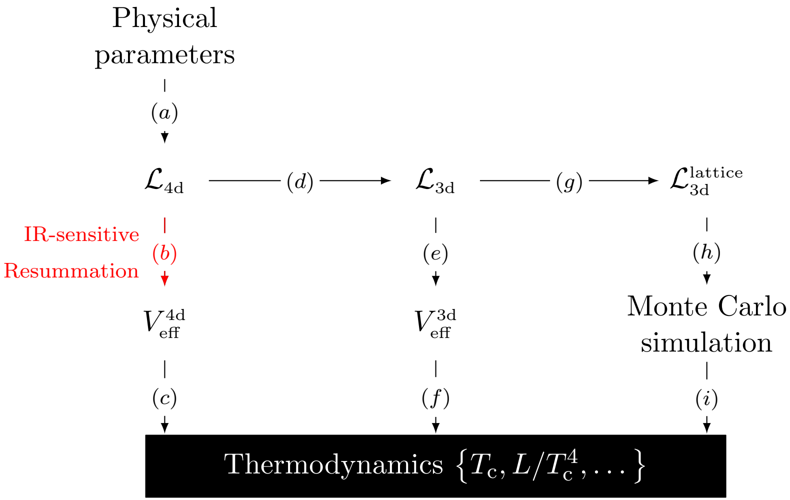

▷ $4d$ approach: $(a) \to (b) \to (c)$

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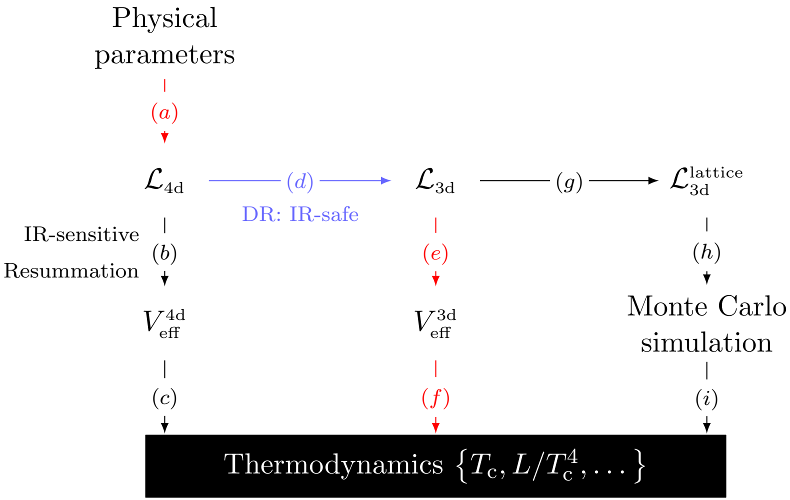

- Perturbative 3d approach: $(a) \to (d) \to (e) \to (f)$

Combining EWPT and DM at improved precision

Simplified DM models

extend the SM by dark Majorana fermion (χ) and complex scalar (η)[8] with $D_\mu \eta = (\partial_\mu - i g_1 \frac{Y_\eta}{2} B_\mu)\eta$:

$$\mathcal{L}_{\rm 4d} = \mathcal{L}_{\rm SM} + \mathcal{L}_\chi + \mathcal{L}_\eta - \mathcal{L}^{\rm portal} \,, \qquad \mathcal{L}^{\rm portal}_{\rm Yukawa} = y\, \eta\, \bar{\chi} P_{\rm R} \ell_{\rm R} + {\rm h.c.} \,,$$

$$\mathcal{L}^{\rm portal}_{\rm scalar} = \lambda_3 (\eta^\dagger \eta)(\phi^\dagger \phi) \,.$$

	$\mathrm{SU(3)_c}$	$\mathrm{SU(2)_L}$	$\mathrm{U(1)_Y}$	Z_2
ϕ	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1
η	$\mathbf{1}$	$\mathbf{1}$	$-Y_e/2$	-1
χ	$\mathbf{1}$	$\mathbf{1}$	0	-1

Chiral projector $P_{\rm R/L} = (\mathbb{1} \pm \gamma_5)/2$ and $\ell_{\rm R}$ is a right-handed SM lepton. Odd Z_2-symmetric χ and η stabilise DM candidate.

[8] M. Garny, A. Ibarra, and S. Vogl, *Signatures of Majorana dark matter with t-channel mediators*, Int. J. Mod. Phys. D **24** (2015) 1530019 [1503.01500]

Finite-temperature phases and DM production

After dimensional reduction[9] of the model use background field method

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_\phi + h + iz) \end{pmatrix} , \qquad \eta = \frac{1}{\sqrt{2}}(v_\eta + s + iA) .$$

Both Higgs ϕ and scalar η condense. Assess via strong FOPT criterion

$$\frac{v_{\mathrm{c},\phi}}{T}(M_\chi, M_\eta, y, \lambda_3) \gtrsim 1 .$$

DM energy density both freeze-out and freeze-in. Extract DM energy density via freeze-out mechanism[10]

$$h^2\Omega_{\mathrm{DM}}(M_\chi, M_\eta, y, \lambda_3) = 0.1200 \pm 0.0012 .$$

Treat EWPT and DM as separate events since $T_{\mathrm{EWPT}} > T_{\text{freeze-out}}$.

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[9] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions*, Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

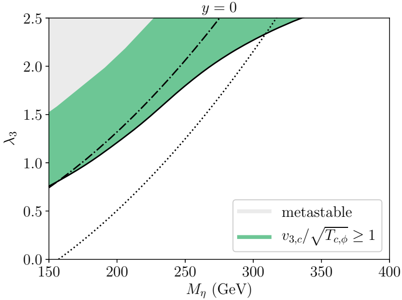
[10] **Planck** Collaboration, N. Aghanim *et al.*, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. **641** (2020) A6 [1807.06209]

The EWPT depends on the Majorana fermion

Determine perturbatively via discontinuous background fields at $T_{\mathrm{c},\phi}$. Strong first-order phase transition (FOPT) for regions above contours

$$\frac{v_{\mathrm{c},\phi}}{T_{\mathrm{c},\phi}} > 1 \text{ (solid)} , \quad \mu_\eta^2 = 0 \text{ (dash-dotted)} , \quad \mu_\eta^2 = 0.5\pi T \text{ (dotted)} .$$

Non-trivial y and M_χ-dependence. The y-dependence enters as NLO effect as χ and ϕ interact indirectly.



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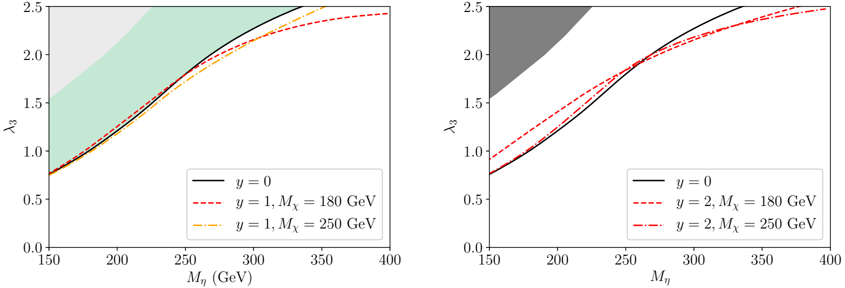


DM energy density via freeze-out

Co-annihilation of η/χ states encoded by single Boltzmann equation are effective for mass splits $\Delta M/M_\chi \lesssim 0.2$ ($\Delta M_T = M_\eta - M_\chi$):

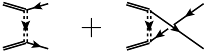
$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\rm eff}\, v\rangle (n^2 - n_{\rm eq}^2)\,, \quad n_{\rm eq} = \int_{\mathbf{p}} e^{-E_{p,\chi}/T}\left[2 + 2e^{-\Delta M_T/T}\right].$$

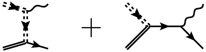
Effective thermally averaged cross-section:

$$\langle \sigma_{\rm eff} v\rangle = \sum_{i,j} \frac{n_i^{\rm eq}\, n_j^{\rm eq}}{(\sum_k n_k^{\rm eq})^2} \langle \sigma_{ij} v\rangle$$

$$\approx \langle \sigma_{\chi\chi} v\rangle + \langle \sigma_{\chi\eta} v\rangle e^{-\Delta M_T/T} + \langle \sigma_{\eta\eta^\dagger} v\rangle e^{-2\Delta M_T/T}\,.$$

Diagrams for the DM pair annihilation and co-annihilation:

$\mathcal{M}_{\chi\chi\to\ell_{\rm R}\bar{\ell}_{\rm R}} =$ 

$\mathcal{M}_{\chi\eta^\dagger\to\ell_{\rm R} Z(\gamma)} =$ 

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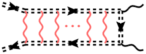
Repeated soft γ or Z-exchanges affect cross-section non-perturbatively:

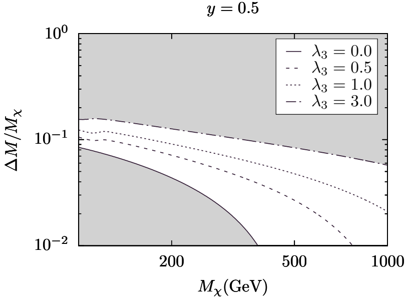
- Sommerfeld-enhancement (SE)
- Bound-state-formation (BSF)

$\mathcal{M}_{\eta\eta^\dagger\to\gamma\gamma} \supset$ 

The (constrained) $h^2\Omega_{\rm DM}$ parameter space

At LO, annihilations $\langle\sigma_{\chi\eta}v\rangle \sim y$ and $\langle\sigma_{\eta\eta^\dagger}v\rangle \sim \lambda_3$ are relevant at small y with λ_3 affecting $\Delta M/M_\chi \subset \langle\sigma_{\eta\eta^\dagger}v\rangle$ at $\mathcal{O}(1)$.

ATLAS collaboration search $2\ell_{\rm R} + \not{E}_T$.[11] Drell-Yan production of $\eta\eta^\dagger$ and subsequent $\eta \to \chi + \ell_{\rm R}$ decays. Most (least) constrained from $\mu(\tau)$.



[11] **ATLAS** Collaboration, G. Aad *et al.*, *Search for electroweak production of charginos and sleptons decaying into final states with two leptons and missing transverse momentum in* $\sqrt{s} = 13$ *TeV pp collisions using the ATLAS detector*, Eur. Phys. J. C **80** (2020) 123 [1908.08215], **ATLAS** Collaboration, G. Aad *et al.*, *Search for direct stau production in events with two hadronic* τ*-leptons in* $\sqrt{s} = 13$ *TeV pp collisions with the ATLAS detector*, Phys. Rev. D **101** (2020) 032009 [1911.06660]

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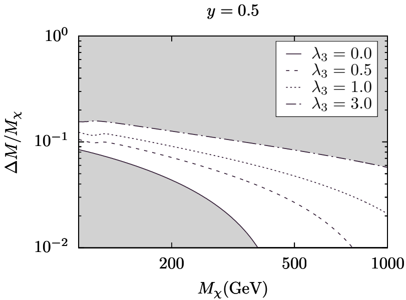

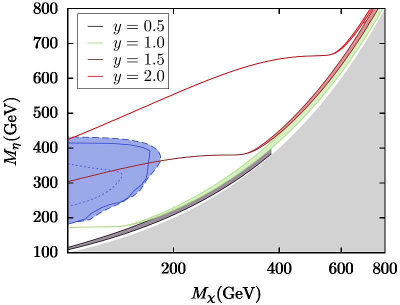

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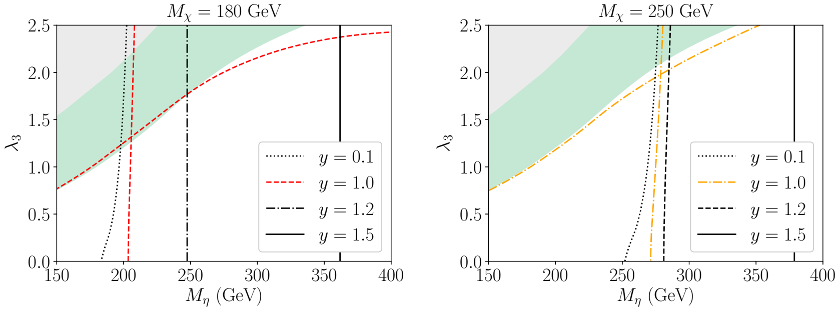
Combining EWPT and DM:[12] the (M_η, λ_3) and (M_χ, y)-plane

Non-trivial dependence of FOPT in (M_χ, y, λ_3)-space

- smaller $y \to$ larger $h^2\Omega_{\rm DM}(\lambda_3)$-dependence
- larger $M_\chi \to$ larger $M_\eta \to$ smaller FOPT–DM parameter space

Small changes on λ_3 are

- important for EWPT thermodynamics
- irrelevant for DM



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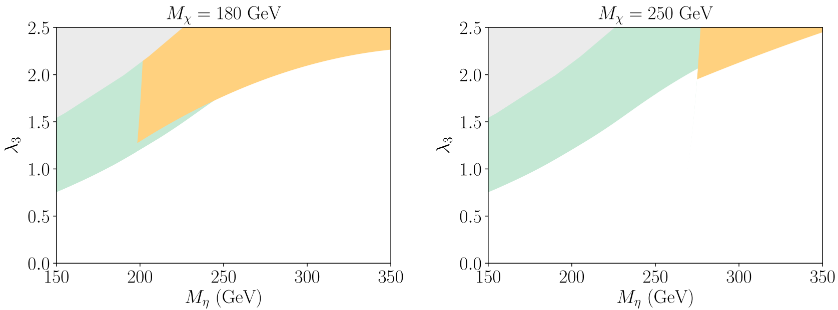

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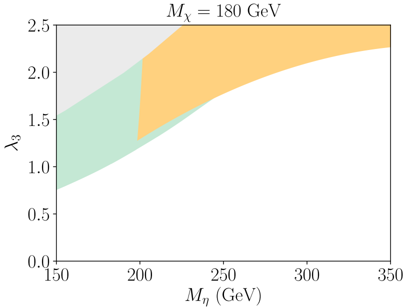
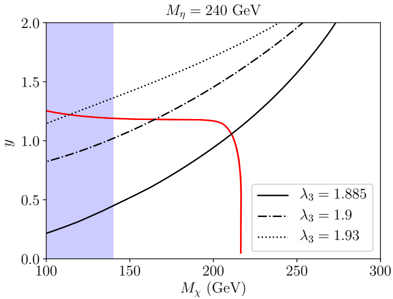
- important for EWPT thermodynamics
- irrelevant for DM





[12] S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models*, JCAP **10** (2022) 044 [2207.12207]

Conclusions

▹ Precision thermodynamics of BSM theories:

- reliable description of cosmological FOPT and GW production,
- practical approach: **dimensionally reduced 3d EFT**[13]
  automated all-order high-T resummation, analytic fermions,
  lattice treatment for 3d EFT, **universality**.

▹ BSM physics to combine DM and EWPT

- induce strong first-order EWPT,
- provide FOPT and correct DM energy density

$$h^2\Omega_{\rm DM} = 0.1200 \pm 0.0012 \quad \text{for} \quad 180\ \text{GeV} < M_\chi \lesssim 300\ \text{GeV}\ .$$

▹ Investigate next-to-minimal models e.g. DM simplified models

☆ Lift high-T expansion to extend analysis to larger M_η and M_χ.

[13] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,* JHEP **04** (2021) 055 [2009.10080]

Backup

Equilibrium Thermodynamics: Imaginary Time Formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \rightarrow \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \rightarrow -i\tau$,

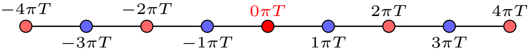
$$\mathcal{Z} = C \int_{\text{b.c.}} \mathcal{D}\phi \, \exp\left[-\int_0^{\beta=1/T} \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\text{E}} \right] \, , \quad \phi(0,\mathbf{x}) = \pm\phi(\beta,\mathbf{x}) \, .$$

(Anti-)periodic bosonic(fermionic) fields at boundaries $\rightarrow$ compact time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with **Matsubara frequencies**

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

Absent zero mode $\omega_{n=0}$ for fermions:



Resummation

Dynamically generated masses through collective plasma effects

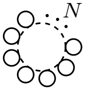
$$m_T = g^n T + m \; .$$

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Cure IR-sensitive contributions at $m_T \sim gT$ by thermal resummation:

$$V^{\mathrm{eff}} \supset$$


$$\propto \quad g^{2N} \left[{m_T}^{3-2N} T \right] \left[\frac{T^2}{12} \right]^N \propto m^3 T \left[\frac{gT}{m_T} \right]^{2N}$$

For $m_T \leq g^2 T$ weak expansion breaks down. At finite T, light bosons are non-perturbative.

Dimensionally reduced effective theory

Describe theory by $3d$ EFT[14]. Super-renormalisable "Electrostatic BSM" (E-BSM) to study high-T thermodynamics

$$\begin{aligned}
\mathcal{L}^{3\mathrm{d}} &= \mathcal{L}^{3\mathrm{d}}_{\mathrm{SM}} + \mathcal{L}^{3\mathrm{d}}_{\eta} + \mathcal{L}^{3\mathrm{d}}_{\mathrm{temp}}\;, \\
\mathcal{L}^{3\mathrm{d}}_{\eta} &= (D_\mu \eta)^\dagger (D_\mu \eta)\;, \\
V^{3\mathrm{d}}(\phi,\eta) &= \mu^2_{\phi,3}\phi^\dagger\phi + \lambda_{1,3}(\phi^\dagger\phi)^2 \\
&\quad + \frac{1}{2}\mu^2_{\eta,3}\,\eta^\dagger\eta + \frac{1}{4}\lambda_{2,3}\,(\eta^\dagger\eta)^2 + \frac{1}{2}\lambda_{3,3}\,(\eta^\dagger\eta)(\phi^\dagger\phi)\;.
\end{aligned}$$

Broken Lorentz symmetry induces temporal-scalar coupling to singlet

$$\mathcal{L}^{3\mathrm{d}}_{\mathrm{temp}} = \frac{1}{2}m^2_{\mathrm{D}}\,A^a_0 A^a_0 + \cdots + y_3\,\eta^\dagger\eta A^a_0 A^a_0\;.$$

Truncate operators at high T:

$$S^{3\mathrm{d}} = \frac{1}{T}\int_{\mathbf{x}}\left\{\mathcal{L}^{3\mathrm{d}} + \sum_{n\geq 5}\frac{\mathcal{O}_n}{(\pi T)^n}\right\}\;.$$

[14] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

EFT step 1: SM $\to$ E-SM

Inspect Higgs potential: $V(\phi) \supset \mu_\phi^2 \phi^\dagger\phi + \lambda_1(\phi^\dagger\phi)^2$.

DR step 1 fixes high-T E-SM. EFT for Electrostatic modes ($D_i = \partial_i - ig_3 A_i - ig'_3 B_i$). Describes SM IR dynamics and contains UV in matching coefficients:

$$\mu_{\phi,3}^2 = \underbrace{\boxed{\overset{\text{tree-level}}{\mu_\phi^2} + \overset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2\mu_\phi^2} + \overset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6) \; ,$$

$$\lambda_{1,3} = \underbrace{\boxed{\overset{\text{tree-level}}{T\lambda_1}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6) \; .$$

EFT step 2: E-SM → M-SM

Inspect Higgs potential: $V(\phi) \supset \mu_\phi^2 \phi^\dagger\phi + \lambda_1(\phi^\dagger\phi)^2$.

DR step 2 fixes high-T M-SM. EFT for Magnetostatic modes aka 3d pure gauge with dynamical Higgs ($D_i = \partial_i - i\bar{g}_3 A_i - i\bar{g}_3' B_i$). Describes E-SM IR dynamics and contains UV in matching coefficients:

$$\bar{\mu}_{\phi,3}^2 = \underbrace{\boxed{\underset{\text{tree-level}}{\mu_\phi^2} + \underset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\underset{\text{1-loop}}{\#g^2\mu_\phi^2} + \underset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\underset{\text{1-loop}}{\#g^2 m_\mathrm{D}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\underset{\text{2-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\ ,$$

$$\bar{\lambda}_{1,3} = \underbrace{\boxed{\underset{\text{tree-level}}{T\lambda_1}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\underset{\text{1-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6) + \underbrace{\boxed{\underset{\text{1-loop}}{\#\frac{g^4}{m_\mathrm{D}}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\underset{\text{2-loop}}{\#\frac{g^6}{m_\mathrm{D}^2}}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\ .$$

EFT step 2: E-SM $\to$ M-SM

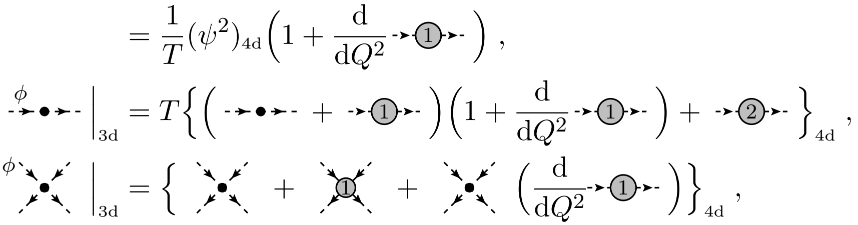
Inspect Higgs potential: $V(\phi) \supset \mu_\phi^2 \phi^\dagger \phi + \lambda_1 (\phi^\dagger \phi)^2$.

DR step 2 fixes high-T M-SM. EFT for Magnetostatic modes aka 3d pure gauge with dynamical Higgs ($D_i = \partial_i - i\bar{g}_3 A_i - i\bar{g}_3' B_i$). Describes E-SM IR dynamics and contains UV in matching coefficients:

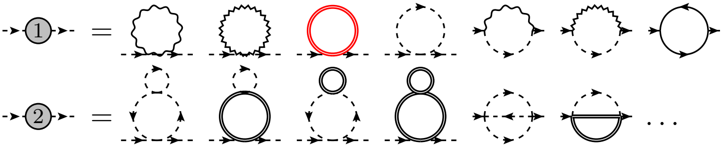
$$
\begin{aligned}
\mathcal{L}_{3\text{d}}^{\text{ultrasoft}} \equiv\ & \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{4} F_{ij} F_{ij} \\
& + (D_i \phi)^\dagger (D_j \phi) + (D_i \eta)^\dagger (D_j \eta) \\
& + \bar{\mu}_{\phi,3}^2 \phi^\dagger \phi + \bar{\lambda}_{1,3} (\phi^\dagger \phi)^2 + \frac{1}{2} \bar{\mu}_{\eta,3} \eta^\dagger \eta + \frac{1}{4} \bar{\lambda}_{2,3} (\eta^\dagger \eta)^2 \\
& + \frac{1}{2} \bar{\lambda}_{3,3} \, (\eta^\dagger \eta)(\phi^\dagger \phi) \ .
\end{aligned}
$$

EFT setup: Matching correlators at NLO

$$(\psi^2)_{3\text{d}} = \frac{1}{T}(\psi^2)_{4\text{d}} Z_\psi^{-1}$$



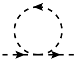
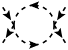
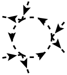
where



The thermal effective potential at LO

$$V^{\text{eff}} = V^{\text{eff}}_{\text{tree}} + V^{\text{eff}}_{1\ell}\ .$$

At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$V^{\text{eff}}_{1\ell} =$  $+\frac{1}{2}$  $+\frac{1}{3}$  $+\dots\Big|_{Q_i=0}$

$$= \frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$V^{\text{eff}}_{1\ell} = \underbrace{\frac{1}{2}\int_P \ln(P^2 + m^2)}_{\equiv V_{\text{CW}}(m)} \underbrace{-T\int_p \ln\Big(1 \mp n_{\text{B/F}}(E_p, T)\Big)}_{\equiv V_{T,b/f}\left(\frac{m^2}{T^2}\right)}$$

$$= \underbrace{\frac{T}{2}\int_p \ln(p^2 + m^2)}_{\equiv T V_{\text{soft}}(m)} + \underbrace{\frac{1}{2}\sum\!\!\!\!\!\!\!\int_{P/\{P\}}{}' \ln(P^2 + m^2)}_{\equiv V_{\text{hard}}(m)}\ .$$

Renormalization scale (in)dependence at finite T

Zero temperature

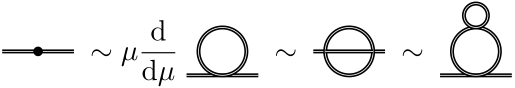
$$V^{\text{eff}}(\phi,\bar{\mu}) = \boxed{\underset{\mathcal{O}(g^2)}{V^{\text{eff}}_{\text{tree}}}} + \boxed{\underset{\mathcal{O}(g^4)}{V_{\text{CW},1\ell}}}\,, \qquad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Big(V^{\text{eff}}_{\text{tree}} + V_{\text{CW},1\ell}\Big) = 0\,.$$

At finite temperature[15]

$$V^{\text{eff}}_{\text{res.}}(\phi,T,\bar{\mu}) = \boxed{\underset{\mathcal{O}(g^2)}{V^{\text{eff}}_{\text{tree}}}} + \boxed{\underset{\mathcal{O}(g^3)}{V_{\text{res.,soft}}}} + \boxed{\underset{\color{red}{\mathcal{O}(g^2T^2)}+\mathcal{O}(g^4)}{V_{\text{hard}}}}\,,$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.

Automatically included in dimensionally reduced $3d$-approach:

$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}$  $\sim \mathcal{O}(g^4T^2)$

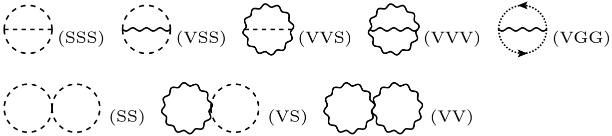
[15] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

The effective potential at NLO

$$V^{\text{eff}} = V^{\text{eff}}_{\text{tree}} + V^{\text{eff}}_{1\ell} + V^{\text{eff}}_{2\ell} \ .$$

Computing V^{eff} up to 2-loop[16] straightforward with vacuum integrals in $3d$ theory:

$V^{\text{eff}}_{2\ell} \supset$



[16] K. Farakos, K. Kajantie, K. Rummukainen, and M. E. Shaposhnikov, *3-D physics and the electroweak phase transition: Perturbation theory,* Nucl. Phys. B **425** (1994) 67 [hep-ph/9404201], M. Laine, *The Two loop effective potential of the 3-d SU(2) Higgs model in a general covariant gauge,* Phys. Lett. B **335** (1994) 173 [hep-ph/9406268], L. Niemi, M. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Thermodynamics of a two-step electroweak phase transition,* [2005.11332], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467]

A minimal scheme for gauge invariance and resummation

1. Determine 3d EFT at NLO (gauge-invariant)
2. Compute $V_{3\mathrm{d}}^{\mathrm{eff}}$ within 3d EFT at 1-loop level
3. Determine T_{c}, condensates $\langle\phi^\dagger\phi\rangle$, and latent heat

Minimum of V^{eff} is gauge parameter independent (Nielsen identities[17]); use $\hbar$-expansion. Improve previous schemes.[18]

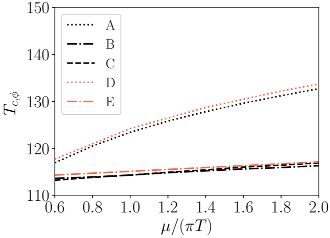
[17] N. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, Nucl. Phys. B **101** (1975) 173

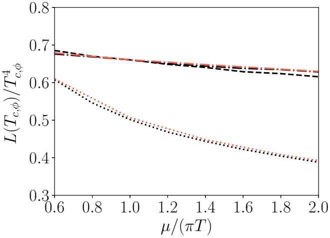
[18] PRM scheme: H. H. Patel and M. J. Ramsey-Musolf, *Baryon Washout, Electroweak Phase Transition, and Perturbation Theory*, JHEP **2011** (2011) 29 [1101.4665]

Increasing accuracy to $\mathcal{O}(g^4)$: cxSM (complex singlet)

Augment SM with **complex singlet scalar**[19], $\mathbb{S} \to v_{\mathbb{S}} + \mathbb{S} + iA$ at

Benchmark	$M_{\mathbb{S}}$	M_A	λ_p	$\lambda_{\mathbb{S}}$
BM1	62.5 GeV	62.5 GeV	0.55	0.5





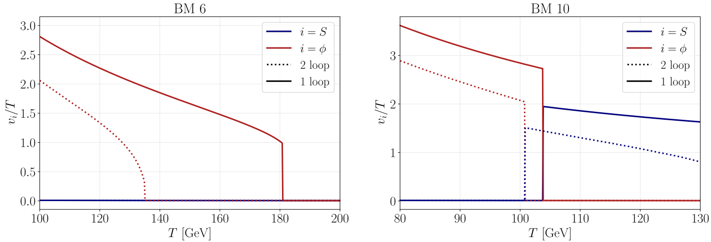
- A, D: 1-loop level dimensional reduction
- B, E: 2-loop level dimensional reduction
- C: as B, with varying $\mu_3 = \mu/(\pi T) g_3^2$

[19] P. Schicho, T. V. I. Tenkanen, and G. White, *Combining thermal resummation and gauge invariance for electroweak phase transition,* JHEP **11** (2022) 047 [2203.04284], C.-W. Chiang, M. J. Ramsey-Musolf, and E. Senaha, *Standard Model with a Complex Scalar Singlet: Cosmological Implications and Theoretical Considerations,* Phys. Rev. D **97** (2017) 1 [1707.09960]

Transitions in the xSM (real singlet)

Monitor Higgs (v) and **real singlet** (x) VEV after shift $\mathbb{S} \to x + \mathbb{S}$. 2-loop corrections are significant.[20]

Benchmark	$M_{\mathbb{S}}$	λ_p	$\lambda_{\mathbb{S}}$
BM6	350 GeV	3.5	0.3
BM10	325 GeV	3.5	0.3



[20] Plots courtesy of Daniel Schmitt as well as L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]