Antisymmetric galaxy cross-correlations

Eleonora Vanzan

Supervisors: Prof. Alvise Raccanelli, Prof. Nicola Bartolo

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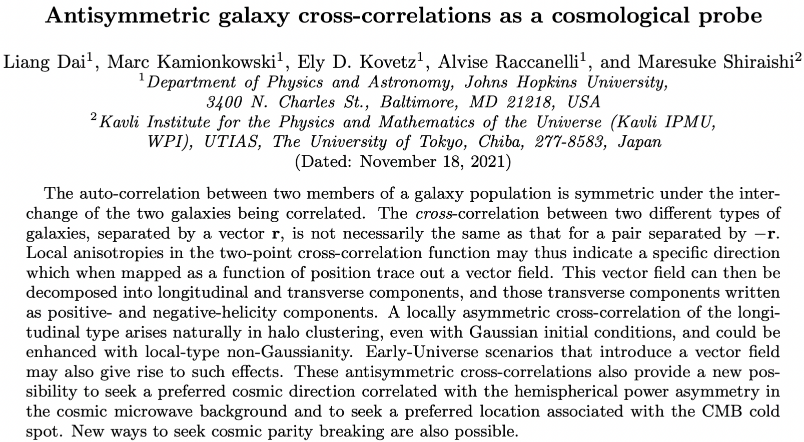
arXiv: 1507.05618



Antisymmetric galaxy cross-correlations as a cosmological probe

Liang Dai[1], Marc Kamionkowski[1], Ely D. Kovetz[1], Alvise Raccanelli[1], and Maresuke Shiraishi[2]
[1] *Department of Physics and Astronomy, Johns Hopkins University,*
3400 N. Charles St., Baltimore, MD 21218, USA
[2] *Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU,*
WPI), UTIAS, The University of Tokyo, Chiba, 277-8583, Japan

(Dated: November 18, 2021)

The auto-correlation between two members of a galaxy population is symmetric under the interchange of the two galaxies being correlated. The *cross*-correlation between two different types of galaxies, separated by a vector $\mathbf{r}$, is not necessarily the same as that for a pair separated by $-\mathbf{r}$. Local anisotropies in the two-point cross-correlation function may thus indicate a specific direction which when mapped as a function of position trace out a vector field. This vector field can then be decomposed into longitudinal and transverse components, and those transverse components written as positive- and negative-helicity components. A locally asymmetric cross-correlation of the longitudinal type arises naturally in halo clustering, even with Gaussian initial conditions, and could be enhanced with local-type non-Gaussianity. Early-Universe scenarios that introduce a vector field may also give rise to such effects. These antisymmetric cross-correlations also provide a new possibility to seek a preferred cosmic direction correlated with the hemispherical power asymmetry in the cosmic microwave background and to seek a preferred location associated with the CMB cold spot. New ways to seek cosmic parity breaking are also possible.

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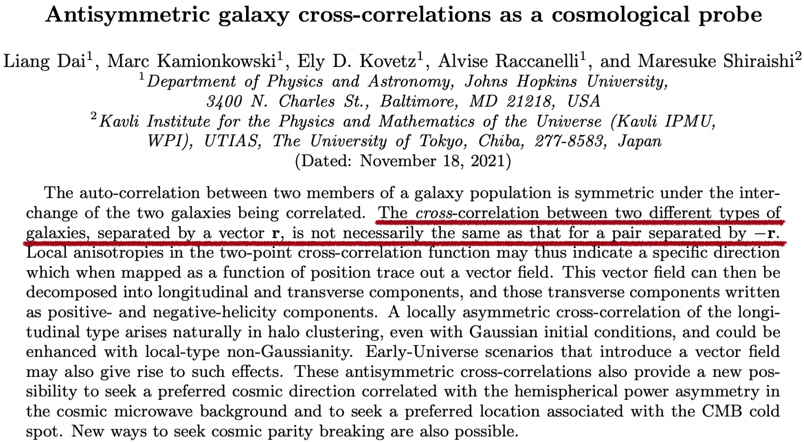

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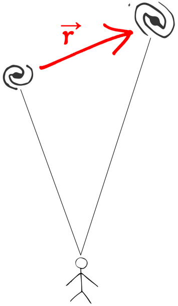
The auto-correlation between two members of a galaxy population is symmetric under the interchange of the two galaxies being correlated. The *cross*-correlation between two different types of galaxies, separated by a vector $\mathbf{r}$, is not necessarily the same as that for a pair separated by $-\mathbf{r}$. Local anisotropies in the two-point cross-correlation function may thus indicate a specific direction which when mapped as a function of position trace out a vector field. This vector field can then be decomposed into longitudinal and transverse components, and those transverse components written as positive- and negative-helicity components. A locally asymmetric cross-correlation of the longitudinal type arises naturally in halo clustering, even with Gaussian initial conditions, and could be enhanced with local-type non-Gaussianity. Early-Universe scenarios that introduce a vector field may also give rise to such effects. These antisymmetric cross-correlations also provide a new possibility to seek a preferred cosmic direction correlated with the hemispherical power asymmetry in the cosmic microwave background and to seek a preferred location associated with the CMB cold spot. New ways to seek cosmic parity breaking are also possible.

Idea



Cross-correlation between **different** tracers may not be symmetric under exchange $\boldsymbol{r} \longmapsto -\boldsymbol{r}$, e.g. if they have different bias parameters.

$$\delta_i = b_i \delta + c_i \delta^2 + \dots$$

tracer i

dark matter

$$\boxed{P^{\mathrm{A}} = (b_2 c_1 - b_1 c_2) \frac{\partial P(k_1)}{\partial k_1} P(k_3) \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_3}{k_1} \qquad k_3 \ll k_1}$$

k_3 long-wavelength mode, k_1 short-wavelength mode

- Add redshift space distortions.
- Add primordial non-Gaussianity f_{NL}.

Most generic parametrization of the power-spectrum: symmetric and antisymmetric cases

Symmetric case

Assume we have a new field with Fourier modes $h_p(\boldsymbol{k}_3)$ and polarization p. Global statictical isotropy requires that the new field induces a correlation:

$$\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{k}_3)} = f_p(\boldsymbol{k}_1, \boldsymbol{k}_2) h_p^*(\boldsymbol{k}_3) \epsilon_{ij}^p k_1^i k_2^j \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$

$\epsilon_{ij}^p(\boldsymbol{k})$?

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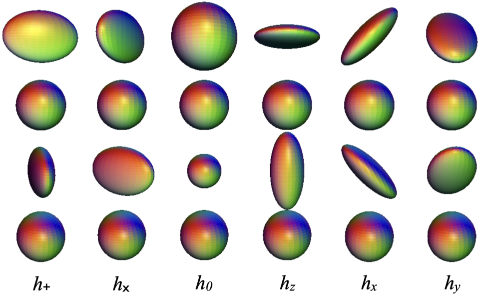
$\epsilon_{ij}^p(\boldsymbol{k})$? Most general 3×3 symmetric tensor can be decomposed into 6 orthogonal polarization states:

$$p = \{+, \times, 0, z, x, y\} \qquad\qquad \epsilon_{ij}^p \epsilon^{p',ij} = 2\delta_{pp'}$$



Jeong *et al.*, 1203.0302

Extension to antisymmetric case

More general case: $\epsilon_{ij}^{p}(\boldsymbol{k}_3)$ may be antisymmetric $\Longrightarrow$ nine degrees of freedom! Three new polarizations $p = \{L, x, y\}$.

$$\begin{aligned}
\langle\delta_1(\boldsymbol{k}_1)\delta_2(\boldsymbol{k}_2)\rangle =& P(k_1)\delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2) \\
&+ \sum_{\boldsymbol{k}_3}\sum_{p} f_p(k_1, k_2, \mu) h_p^*(\boldsymbol{k}_3)\epsilon_{ij}^{p}(\boldsymbol{k}_3) k_1^i k_2^j \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \\
&+ \sum_{\boldsymbol{k}_3, p} f_p(k_1, k_2, \mu) h_p^*(\boldsymbol{k}_3)\hat{\epsilon}_p \cdot (\boldsymbol{k}_1 - \boldsymbol{k}_2)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)
\end{aligned}$$

Extension to antisymmetric case

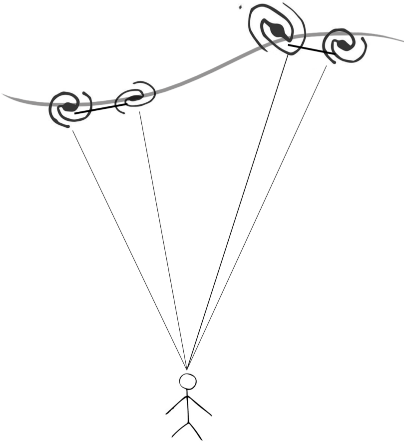
More general case: $\epsilon^{p}_{ij}(\boldsymbol{k}_3)$ may be antisymmetric $\Longrightarrow$ nine degrees of freedom! Three new polarizations $p = \{L, x, y\}$.

$$\begin{aligned}\langle \delta_1(\boldsymbol{k}_1)\delta_2(\boldsymbol{k}_2)\rangle =& P(k_1)\delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2) \\ &+ \sum_{\boldsymbol{k}_3} \sum_{p} f_p(k_1, k_2, \mu) h^{*}_p(\boldsymbol{k}_3)\epsilon^{p}_{ij}(\boldsymbol{k}_3) k^{i}_1 k^{j}_2 \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \\ &+ \sum_{\boldsymbol{k}_3, p} f_p(k_1, k_2, \mu) h^{*}_p(\boldsymbol{k}_3)\hat{\epsilon}_p \cdot (\boldsymbol{k}_1 - \boldsymbol{k}_2)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\end{aligned}$$

Parametrization allows for a global preferred direction (exotic new physics).
In our case: **in any small volume, the cross-correlation could "point" in some given direction and this direction could be spatially dependent, in such a way that global statistical isotropy is still preserved on sufficiently large scales**.

Case of biased halo clustering

Consider two tracers sitting on top of a long-wavelength dark matter mode.



Case of biased halo clustering

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$$\delta_i = b_i \delta + c_i \delta^2 + \dots$$

$$\langle \delta_1(\boldsymbol{k}_1)\delta_2(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle = 2P(k_3)\left[b_2 c_1 P(k_2) + b_1 c_2 P(k_1)\right]\delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$

Squeeze $K \ll k_1, k_2$ and antisymmetrize in $\boldsymbol{k}_1$, $\boldsymbol{k}_2$: now $k_1 \simeq k_2 \equiv k$, $k_3 \equiv K$

$$P^{\mathrm{A}} = (b_2 c_1 - b_1 c_2)\frac{\partial P(k)}{\partial k} P(K) \frac{\boldsymbol{k}\cdot\boldsymbol{K}}{k} \qquad\qquad K \ll k$$

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Consider two tracers sitting on top of a long-wavelength dark matter mode.

$$\delta_i = b_i \delta + c_i \delta^2 + \ldots$$

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$$P^{\mathrm{A}} = (b_2 c_1 - b_1 c_2)\frac{\partial P(k)}{\partial k} P(K) \frac{\boldsymbol{k} \cdot \boldsymbol{K}}{k} \qquad K \ll k$$

Compare with general parametrization: longitudinal mode only $p = L$, with

$$f_L = \frac{1}{2}\left(b_2 c_1 - b_1 c_2\right) \frac{\partial P(k)}{\partial k} \frac{K}{k}$$

Small scales ⟶ we can take $P(k)$ to be a powerlaw $P(k) \propto k^n$ with $n = -3$.

$$\frac{P^{\mathrm{A}}}{P^{\mathrm{S}}} = \frac{(b_2 c_1 - b_1 c_2)}{(b_2 c_1 + b_1 c_2)} \frac{n}{2} \frac{K}{k}$$

Adding RSD and $f_{\rm NL}$ (I)

- Bias at second order:

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + b_{K^2}K^2 \qquad \text{with } K_{ij}(\boldsymbol{k}) = \frac{2}{3\Omega_m\mathcal{H}^2}\partial_i\partial_j\Phi - \frac{1}{3}\delta_{ij}\delta = \left[\frac{k_i k_j}{k^2} - \frac{1}{3}\delta_{ij}\right]\delta(\boldsymbol{k})$$

- RSD at second order:

$$\begin{aligned}\delta_s \simeq\ & \delta - \frac{1}{\mathcal{H}}\partial_r v - \frac{1}{\mathcal{H}r}\left(\frac{r}{\bar{n}}\frac{\partial\bar{n}}{\partial r} + 2\right)v \\ & + \frac{1}{2\mathcal{H}^2}\partial_r^2 v^2 - \frac{1}{\mathcal{H}}\partial_r(\delta v) \\ & - \frac{1}{\mathcal{H}r}\left(\frac{r}{\bar{n}}\frac{\partial\bar{n}}{\partial r} + 2\right)\delta v + \frac{1}{\mathcal{H}^2 r^2}\left(\left(\frac{r}{\bar{n}}\frac{\partial\bar{n}}{\partial r}\right)^2 - \frac{r^2}{2\bar{n}}\nabla^2\bar{n} + 2\frac{r}{\bar{n}}\frac{\partial\bar{n}}{\partial r} + 3\right)v^2 \\ & + \frac{1}{\mathcal{H}^2}\partial_r\left[\left(\frac{r}{\bar{n}}\frac{\partial\bar{n}}{\partial r} + 2\right)\frac{v^2}{r}\right]\end{aligned}$$

Adding RSD and f_{NL} (I)

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Neglecting Doppler term and selection effects, the new kernels are:

$$\begin{aligned}Z_1(\boldsymbol{k}) =& b_1 + f\mu^2 \\ Z_2(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_2) =& \frac{b_2}{2} + b_1F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + b_{K^2}\left(\mu_{12}^2 - \frac{1}{3}\right) + f\mu^2G_2(\boldsymbol{k}_1, \boldsymbol{k}_2) \\ &+ \frac{k\mu f}{2}\left(\frac{\mu_1}{k_1}\left(b_1 + f\mu_2^2\right) + \frac{\mu_2}{k_2}\left(b_1 + f\mu_1^2\right)\right)\end{aligned}$$

Adding RSD and $f_{\rm NL}$ (II)

$$\begin{aligned}
P^{\rm A} =& P(k_1)P(k_3)\left(1+f\mu_3^2\right)\Bigg\{\Bigg[2\left(b_1^{(2)}b_{K^2}^{(1)}-b_1^{(1)}b_{K^2}^{(2)}\right)\mu_{13}\left(-\mu_{13}^2+1\right)+\\
&+\left(b_2^{(1)}-b_2^{(2)}\right)f\mu_1\frac{k_3}{k_1}\left(-\mu_{13}\mu_1+\mu_3\right)-\left(b_{K^2}^{(1)}-b_{K^2}^{(2)}\right)\frac{2f\mu_1}{3}\frac{k_3}{k_1}\left(6\mu_{13}^3\mu_1-4\mu_{13}\mu_1-3\mu_{13}^2\mu_3+\mu_3\right)\\
&-\left(b_1^{(1)}-b_1^{(2)}\right)f\mu_1\frac{k_3}{k_1}\left[f\mu_1\mu_{13}\left(-\mu_1^2+\mu_3^2\right)+\left(\mu_{13}\mu_1-\mu_3\right)\left(-f\mu_1^2+2F_2(-\boldsymbol{k}_2,-\boldsymbol{k}_3)+2G_2(-\boldsymbol{k}_1,-\boldsymbol{k}_3)\right)\right]\Bigg]\\
&-\left(b_1^{(1)}-b_1^{(2)}\right)f\mu_1^2\left(F_2(-\boldsymbol{k}_1,-\boldsymbol{k}_3)-F_2(-\boldsymbol{k}_2,-\boldsymbol{k}_3)-G_2(-\boldsymbol{k}_1,-\boldsymbol{k}_3)+G_2(-\boldsymbol{k}_2,-\boldsymbol{k}_3)\right)\Bigg\}\\
&+\frac{\partial P(k_1)}{\partial k_1}P(k_3)\mu_{13}k_3\left(1+f\mu_3^2\right)\Bigg\{\frac{1}{2}\left(b_1^{(2)}b_2^{(1)}-b_1^{(1)}b_2^{(2)}\right)+\left(b_1^{(1)}b_{K^2}^{(2)}-b_1^{(2)}b_{K^2}^{(1)}\right)\left(\frac{1}{3}-\mu_{13}^2\right)+\frac{1}{2}\left(b_2^{(1)}-b_2^{(2)}\right)f\mu_1^2\\
&+\left(b_{K^2}^{(1)}-b_{K^2}^{(2)}\right)f\mu_1^2\left(\mu_{13}^2-\frac{1}{3}\right)\\
&-\frac{1}{2}\left(b_1^{(1)}-b_1^{(2)}\right)f\mu_1^2\left(f\mu_1^2-f\mu_3^2-2F_2(-\boldsymbol{k}_2,-\boldsymbol{k}_3)+2G_2(-\boldsymbol{k}_2,-\boldsymbol{k}_3)\right)\Bigg\}
\end{aligned}$$

...and main contribution from $f_{\rm NL}$:

$$\begin{aligned}
&-\frac{1}{k_1k_3}\frac{3}{4}\frac{f_{\rm NL}H_0^2\Omega_{\rm m,0}}{D_{\rm md}(\tau)\left(T(k_1)\right)^2}P(k_3)\left(1+f\mu_3^2\right)\Bigg\{\Bigg[\frac{\partial P(k_1)}{\partial k_1}k_1\mu_{13}^2\left(\left(b_\phi^{(1)}b_1^{(2)}-b_\phi^{(2)}b_1^{(1)}\right)+2\left(b_\phi^{(1)}b_{K^2}^{(2)}-b_\phi^{(2)}b_{K^2}^{(1)}\right)+\left(b_\phi^{(1)}-b_\phi^{(2)}\right)f\mu_1^2\right)\\
&+P(k_1)\left(2\left(b_1^{(1)}b_\phi^{(2)}-b_1^{(2)}b_\phi^{(1)}\right)\mu_{13}-8\left(b_\phi^{(1)}b_{K^2}^{(2)}-b_\phi^{(2)}b_{K^2}^{(1)}\right)\mu_{13}-2\left(b_\phi^{(1)}-b_\phi^{(2)}\right)f\mu_1\left(2\mu_1\mu_{13}-\mu_3\right)\right)\Bigg]T(k_1)\\
&+P(k_1)k_1\mu_{13}\left(\left(b_\phi^{(1)}b_1^{(2)}-b_\phi^{(2)}b_1^{(1)}\right)-2\left(b_\phi^{(1)}b_{K^2}^{(2)}-b_\phi^{(2)}b_{K^2}^{(1)}\right)+\left(b_\phi^{(1)}-b_\phi^{(2)}\right)f\mu_1^2\right)\frac{\partial T(k_1)}{\partial k_1}\Bigg\}
\end{aligned}$$

Primordial non-Gaussianity $f_{\rm NL}$

In the presence of **local-type primordial non-Gaussianity**, the Eulerian basis of operators in the bias expansion must be augmented by additional terms: $f_{\rm NL}\phi(\boldsymbol{q})$ at first order and $f_{\rm NL}\delta(\boldsymbol{x})\phi(\boldsymbol{q})$ at second order, with ϕ the Bardeen potential.

The redshift space kernels in Fourier space become:

$$Z^{\rm tr}_{1,f_{\rm NL}}(\boldsymbol{k}) = f_{\rm NL} b_\phi \mathcal{M}^{-1}(k)$$

$$\begin{aligned} Z^{\rm tr}_{2,f_{\rm NL}}(\boldsymbol{k},\boldsymbol{k}_1,\boldsymbol{k}_2) =& f_{\rm NL} b_\phi \frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2}\left(\frac{1}{k_1^2}\mathcal{M}^{-1}(k_2)+\frac{1}{k_2^2}\mathcal{M}^{-1}(k_1)\right)\\ &+ f_{\rm NL} b_{\phi\delta}\frac{1}{2}\left(\mathcal{M}^{-1}(k_1)+\mathcal{M}^{-1}(k_2)\right)\\ &+ f_{\rm NL} b_\phi \frac{k\mu f}{2}\left(\frac{\mu_2}{k_2}\mathcal{M}^{-1}(k_1)+\frac{\mu_1}{k_1}\mathcal{M}^{-1}(k_2)\right) \end{aligned}$$

$$\delta^{(1)}(\boldsymbol{k},\tau) = \mathcal{M}(k,\tau)\phi(\boldsymbol{k}) \qquad \mathcal{M}(k,\tau) = \frac{2}{3}\frac{k^2 T(k) D_{\rm md}(\tau)}{\Omega_{\rm m,0}H_0^2}$$

Estimator

Estimator for the Fourier amplitude $\delta(K)$

Jeong *et al.* 1203.0302 & Dai *et al.* 1507.05618

$$\frac{1}{2}\left[\delta_1(\boldsymbol{k}_1)\delta_2(\boldsymbol{k}_2)-\delta_1(\boldsymbol{k}_2)\delta_2(\boldsymbol{k}_1)\right]=V_s\delta_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{K}}\,\delta^*(\boldsymbol{K})f_L^{\rm A}(\boldsymbol{k}_1,\boldsymbol{k}_2)\hat{\boldsymbol{K}}\cdot(\boldsymbol{k}_1-\boldsymbol{k}_2)$$

Each pair $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ provides an estimator:

$$\widehat{\delta(\boldsymbol{K})}=\frac{1}{2}\left[\delta_1(\boldsymbol{k}_1)\delta_2(\boldsymbol{k}_2)-\delta_1(\boldsymbol{k}_2)\delta_2(\boldsymbol{k}_1)\right]\left[f_L^{\rm A}(\boldsymbol{k}_1,\boldsymbol{k}_2)\hat{\boldsymbol{K}}\cdot(\boldsymbol{k}_1-\boldsymbol{k}_2)\right]^{-1}$$

with variance

$$\frac{V_s}{2}\left[f_L^{\rm A}(\boldsymbol{k}_1,\boldsymbol{k}_2)\hat{\boldsymbol{K}}\cdot(\boldsymbol{k}_1-\boldsymbol{k}_2)\right]^{-2}\left(P_1(k_1)P_2(k_2)+P_1(k_2)P_2(k_1)-2P_{12}(k_1)P_{12}(k_2)\right)$$

Minimum variance estimator obtained by summing over all $(\boldsymbol{k}_1,\boldsymbol{k}_2)$ with inverse-variance weighting:

$$\widehat{\delta(\boldsymbol{K})}=P_n(K)\sum_{\boldsymbol{k}}\frac{\left[f_L^{\rm A}(\boldsymbol{k}_1,\boldsymbol{k}_2)\hat{\boldsymbol{K}}\cdot(\boldsymbol{k}_1-\boldsymbol{k}_2)\right]}{\frac{V_s}{2}\left(P_1(k_1)P_2(k_2)+P_1(k_2)P_2(k_1)-2P_{12}(k_1)P_{12}(k_2)\right)}\frac{1}{2}\left[\delta_1(\boldsymbol{k}_1)\delta_2(\boldsymbol{k}_2)-\delta_1(\boldsymbol{k}_2)\delta_2(\boldsymbol{k}_1)\right]$$

$$P_n(K)=\left[\sum_{\boldsymbol{k}}\frac{\left[f_L^{\rm A}(\boldsymbol{k}_1,\boldsymbol{k}_2)\hat{\boldsymbol{K}}\cdot(\boldsymbol{k}_1-\boldsymbol{k}_2)\right]^2}{\frac{V_s}{2}\left(P_1(k_1)P_2(k_2)+P_1(k_2)P_2(k_1)-2P_{12}(k_1)P_{12}(k_2)\right)}\right]^{-1}$$

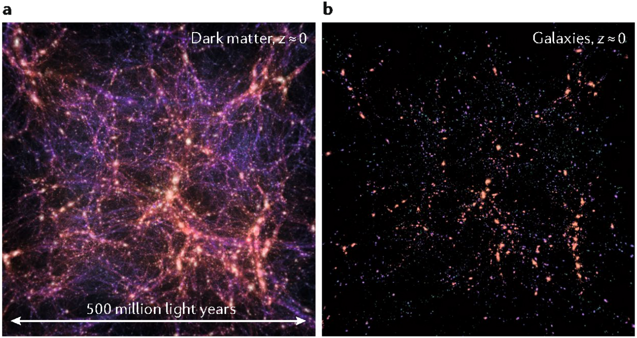
Estimator for the amplitude A

Since $\langle \left|\widehat{\delta(\boldsymbol{K})}\right|^2 \rangle = V_s \left(P(K) + P_n(K)\right)$, if one parametrizes $P(K) = AP_f(K)$, each $\boldsymbol{K}$ provides an estimator for the amplitude:

$$\hat{A}_{\boldsymbol{K}} = P_f(K)^{-1} \left(V_s^{-1} \left|\widehat{\delta(\boldsymbol{K})}\right|^2 - P_n(K) \right)$$

$$\hat{A} = \sigma^2 \sum_{\boldsymbol{K}} \frac{P_f(K)}{2\left(P_n(K)\right)^2} \left(V_s^{-1} \left|\widehat{\delta(\boldsymbol{K})}\right|^2 - P_n(K) \right)$$

$$\sigma^{-2} = \sum_{\boldsymbol{K}} \frac{\left(P_f(K)\right)^2}{2\left(P_n(K)\right)^2}$$



Robertson *et al.*, Galaxy formation and evolution science in the era of the Large Synoptic Survey Telescope

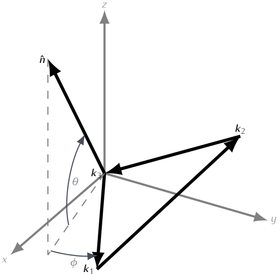
Thank you for your attention.

Literature

- clustering fossils: Jeong & Kamionkowski 2012 **1203.0302**
- antisymmetric: Dai *et al.* 2015 **1507.05618**

SNR

$$\left(\frac{S}{N}\right)^2 = \sum_{(k_1,k_2,k_3)} \int_{-1}^{+1} d\mu \int_0^{2\pi} d\phi \, \frac{1}{s_B V_s} \frac{\left(P^{\mathrm{A}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right)^2}{\mathrm{Var}\left(P^{\mathrm{A}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right)} \prod_{i=1}^{3}\left(\frac{dk_i\Delta k_i}{k_{\mathrm{f}}^2}\right) \times \begin{cases}\pi & k_i = k_j + k_k \\ 2\pi & \text{otherwise}\end{cases}$$



Covariance

$$\widehat{P^{\mathrm{A}}}(\boldsymbol{k},\boldsymbol{K}) \equiv \frac{1}{2}\left(\delta_1(\boldsymbol{k})\delta_2(\boldsymbol{K}-\boldsymbol{k}) - \delta_1(\boldsymbol{K}-\boldsymbol{k})\delta_2(\boldsymbol{k})\right)$$

under the null hypothesis:

$$\mathrm{Cov}(\boldsymbol{k},\boldsymbol{k}')_{\boldsymbol{K}} = \frac{1}{2}\left[P_{11}(k)P_{22}(k) - P_{12}(k)P_{12}(k)\right]\left[\delta^D_{\boldsymbol{k}+\boldsymbol{k}'} - \delta^D_{\boldsymbol{k}-\boldsymbol{k}'}\right]$$

but since $P^{\mathrm{A}}(-\boldsymbol{k}) = -P^{\mathrm{A}}(\boldsymbol{k})$, one can consider only one emisphere in $\boldsymbol{k}$ space and then combine the contribution from both $\boldsymbol{k}$ and $-\boldsymbol{k}$ mode:

$$\left[\langle P^{\mathrm{A}}(\boldsymbol{k})^2\rangle - \langle P^{\mathrm{A}}(\boldsymbol{k})\rangle^2\right] - \left[\langle P^{\mathrm{A}}(\boldsymbol{k})P^{\mathrm{A}}(-\boldsymbol{k})\rangle - \langle P^{\mathrm{A}}(\boldsymbol{k})\rangle\langle P^{\mathrm{A}}(-\boldsymbol{k})\rangle\right]$$

$$\boxed{\mathrm{Cov}^{\mathrm{emi}}(\boldsymbol{k})_{\boldsymbol{K}} = \frac{1}{2}\Big(P_{11}(k)P_{22}(|\boldsymbol{K}-\boldsymbol{k}|) + P_{11}(|\boldsymbol{K}-\boldsymbol{k}|)P_{22}(k)\Big) - P_{12}(k)P_{12}(|\boldsymbol{K}-\boldsymbol{k}|)}$$