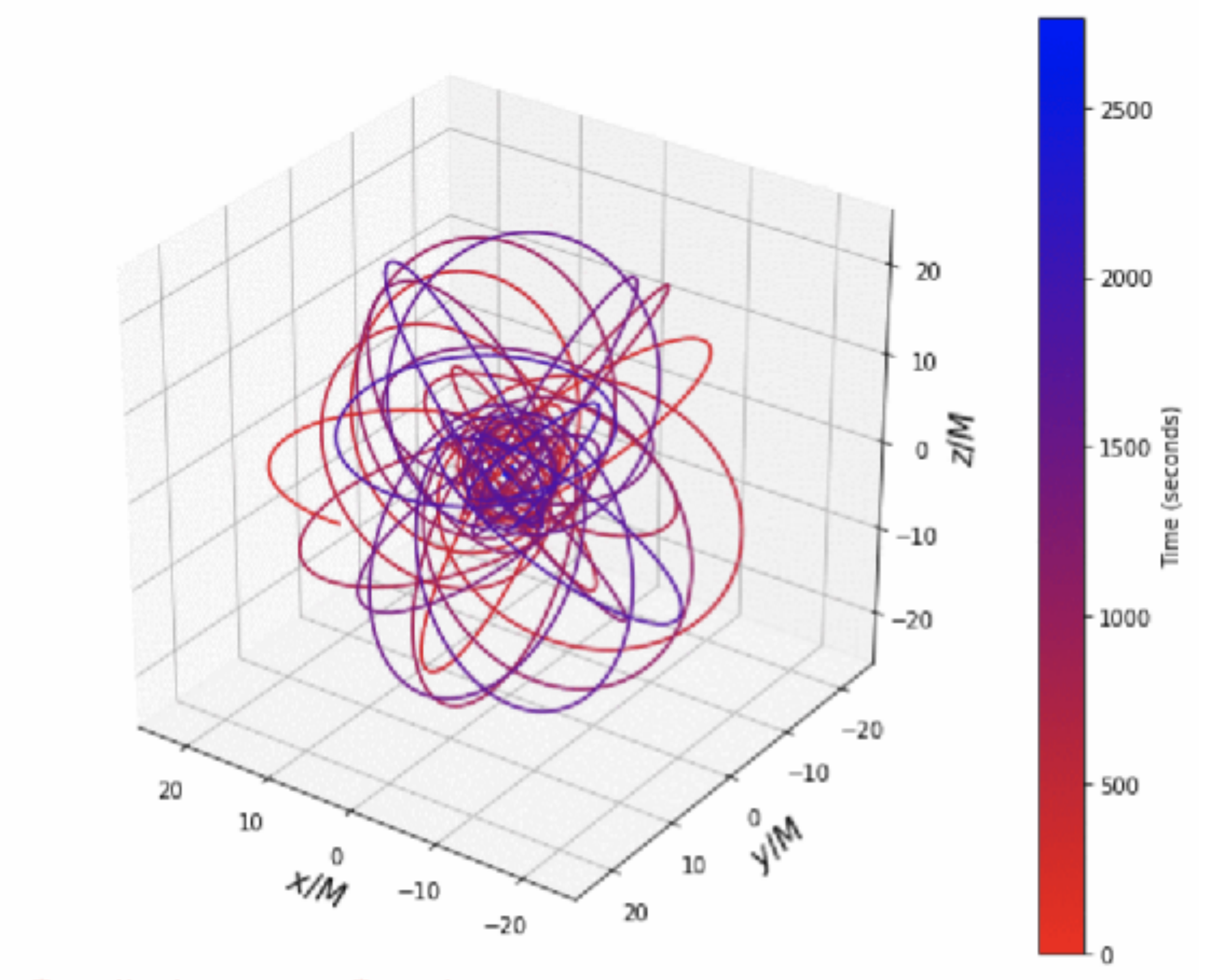
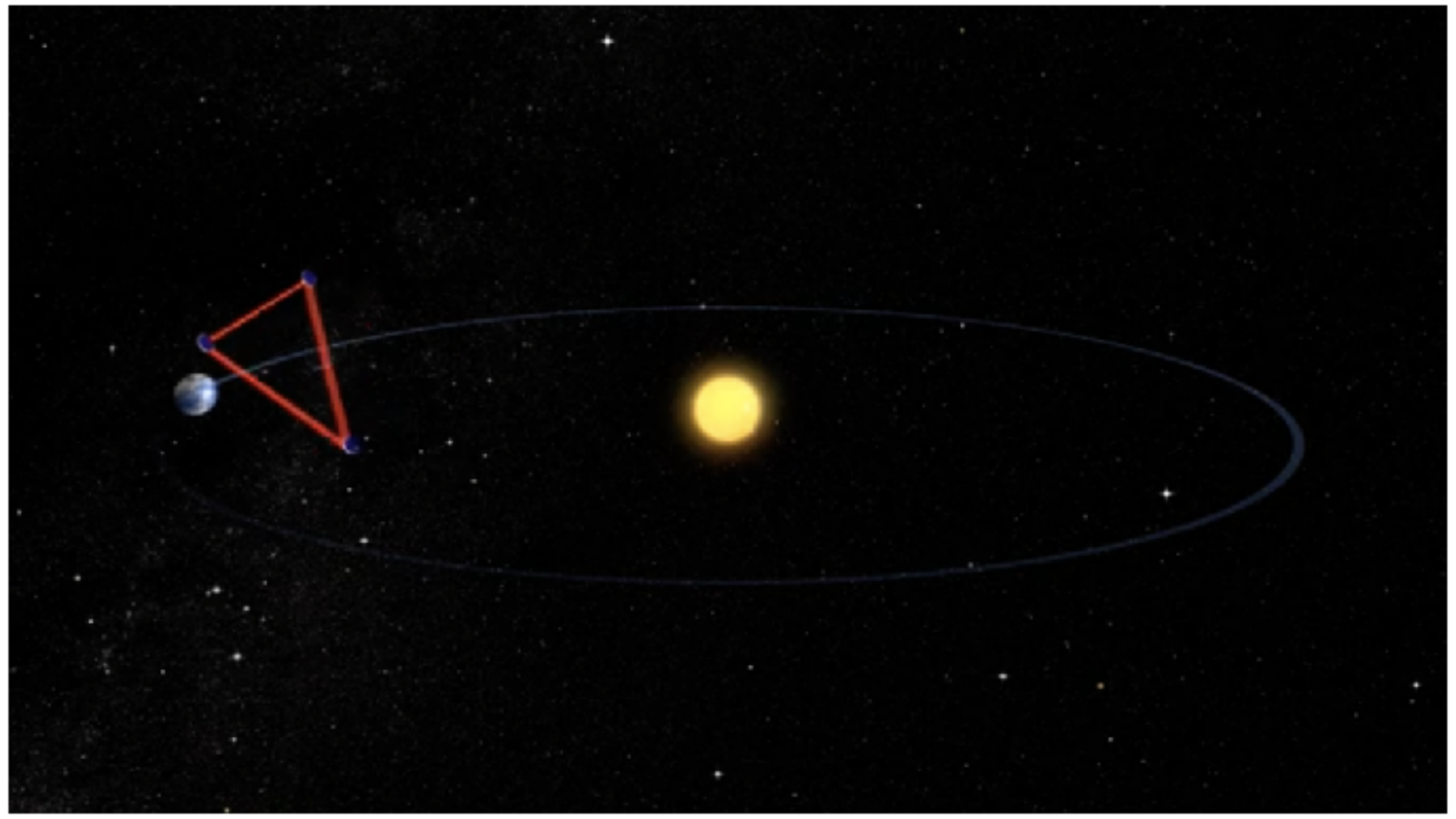
Constraining modified GW propagation with extreme mass-ratio inspirals



Credit: Lorenzo Speri

Chang Liu



In collaboration with D. Laghi, N. Tamanini et al.

Laboratoire des 2 Infinis & Peking University

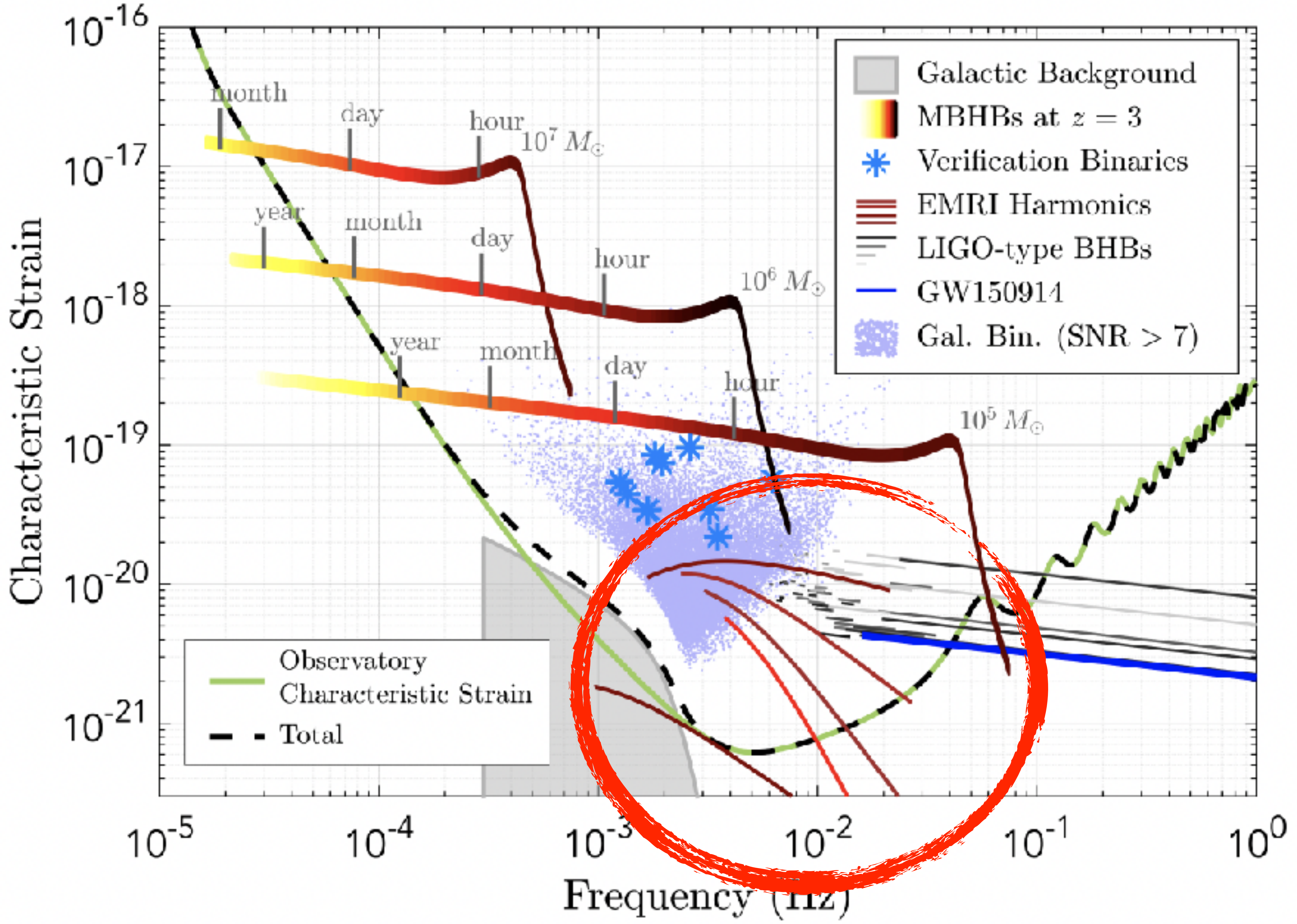
PONT2023 02/05/2023

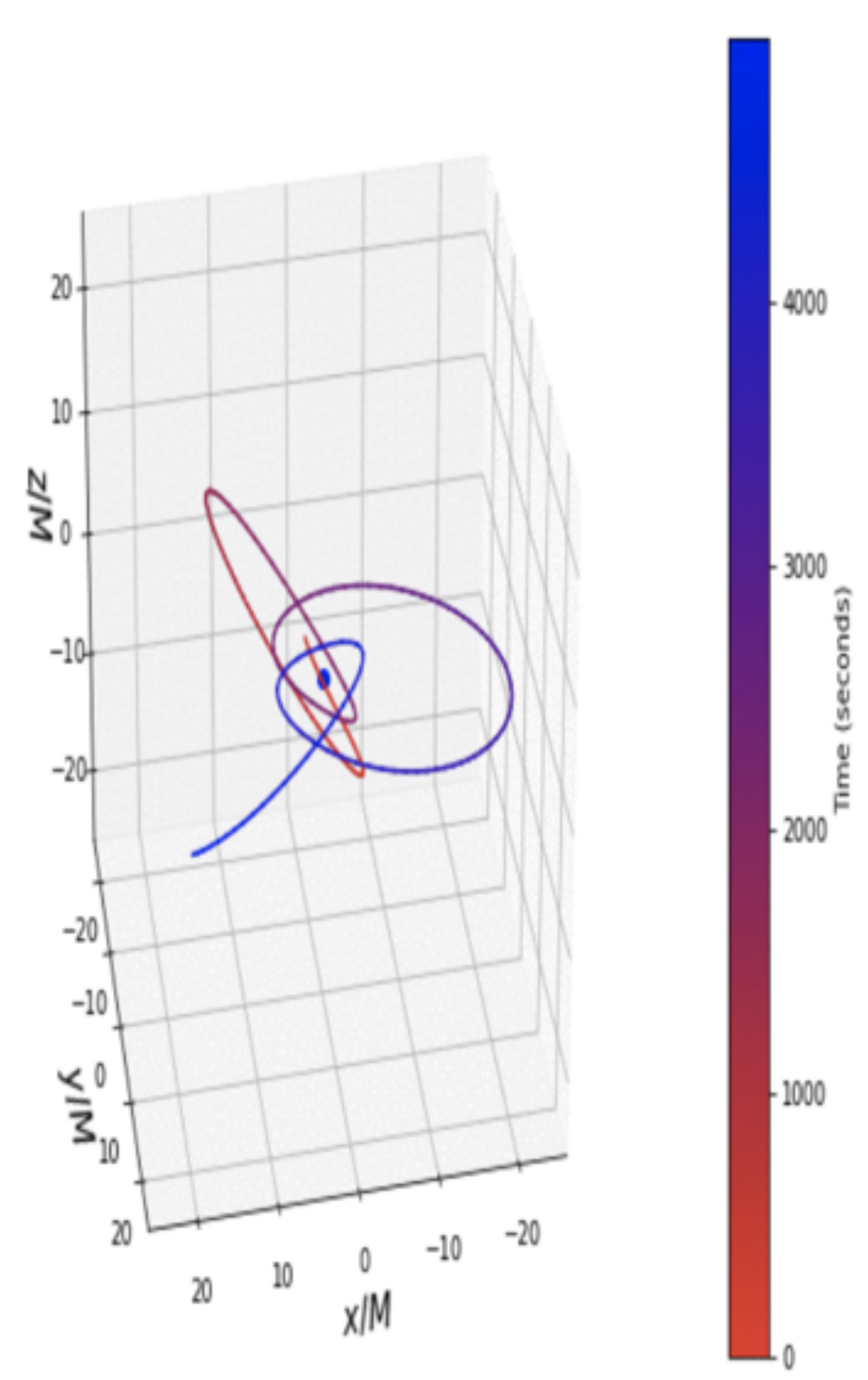
Outline

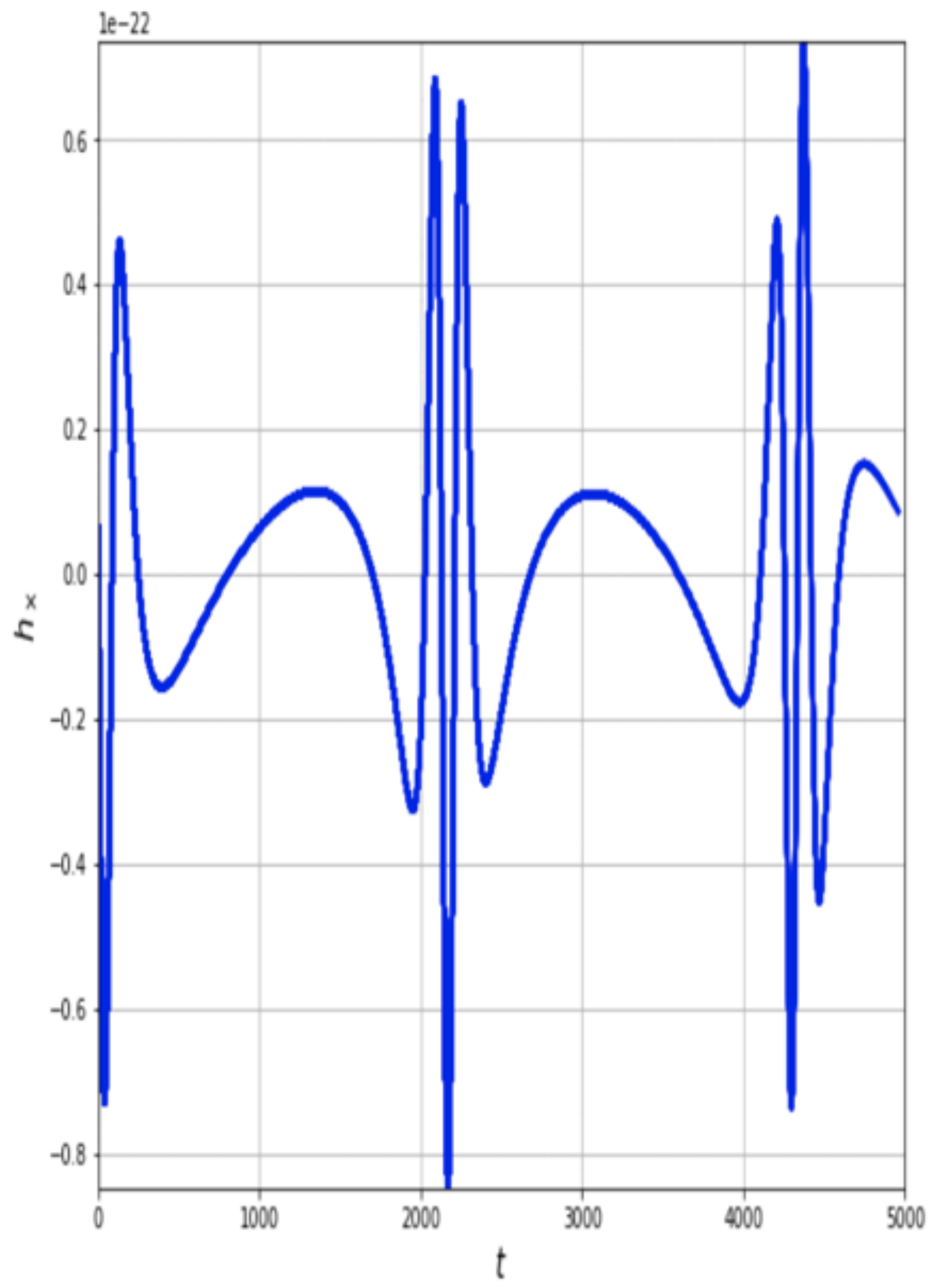
- LISA and extreme mass-ratio inspirals (EMRIs)
- Cosmology and modified GW propagation
- Inference with LISA dark sirens
- Preliminary results
- Conclusion and future prospects

Extreme Mass-Ratio Inspirals (EMRIs)

Credit: Lorenzo Speri, Ollie Burke







Mass of the Kerr Black Hole
$M \sim 10^5$ - $10^7 M_\odot$

Mass Compact Object
$\mu \approx 10\, M_\odot$

Mass Ratio
$\eta \approx 10^{-6}$ - 10^{-4}

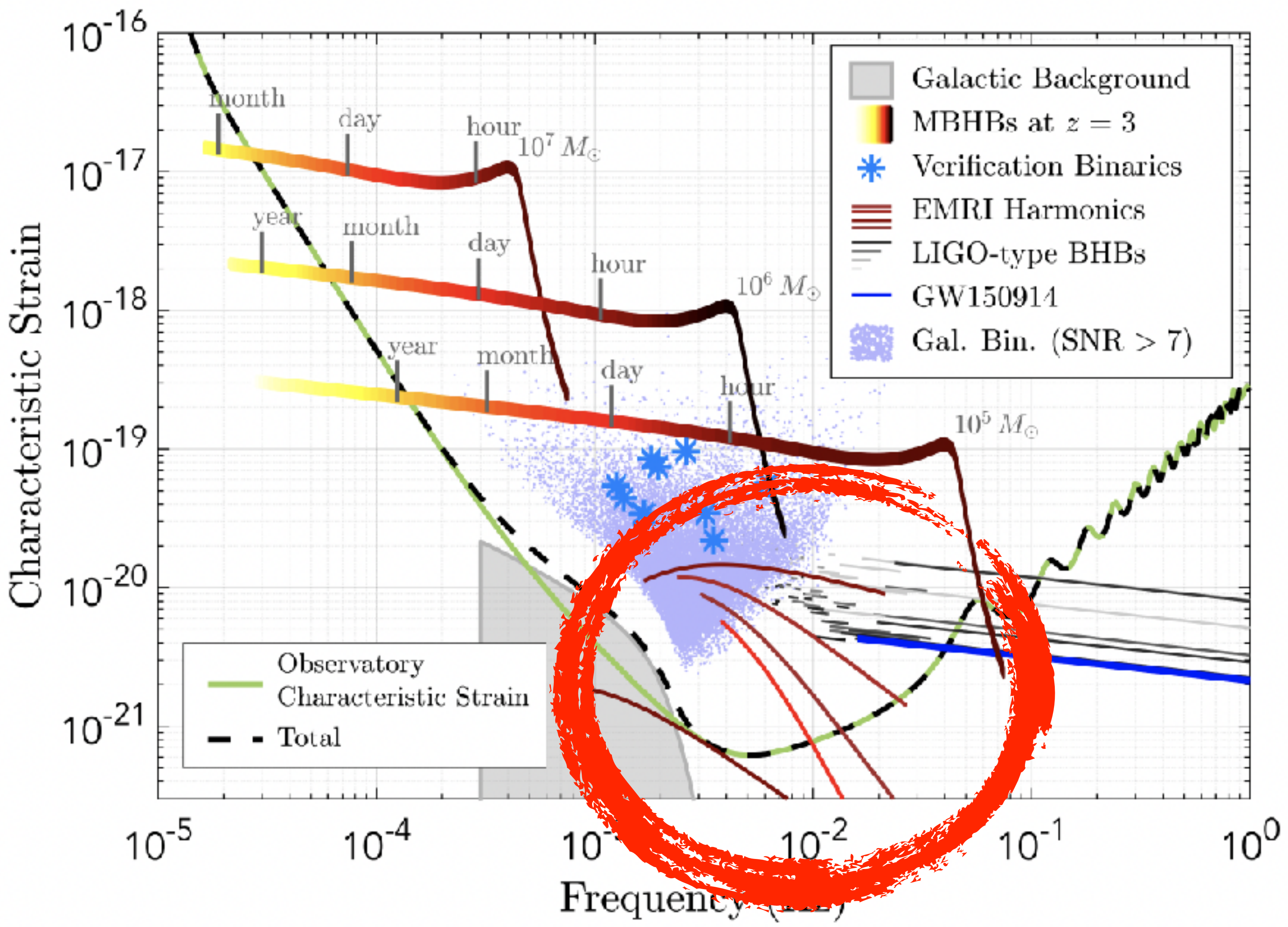
Evolution Scale
$T_{\rm ev} \sim \dfrac{1}{\eta}$

GW Frequency
$f \approx 10^{-4}$ - 10^{-2} Hz

Observation of EMRIs with LISA

[Babak et al. 2017]

- EMRI waveform: AKK (Analytical Kludge Kerr, optimistic)
- Sensitivity curve: 2.5 Gm LISA configuration, 2017
- Observation time: 10 yrs
- Parameter estimation: based on EMRI catalog M1 by Fisher Matrix analysis (SNR>20)



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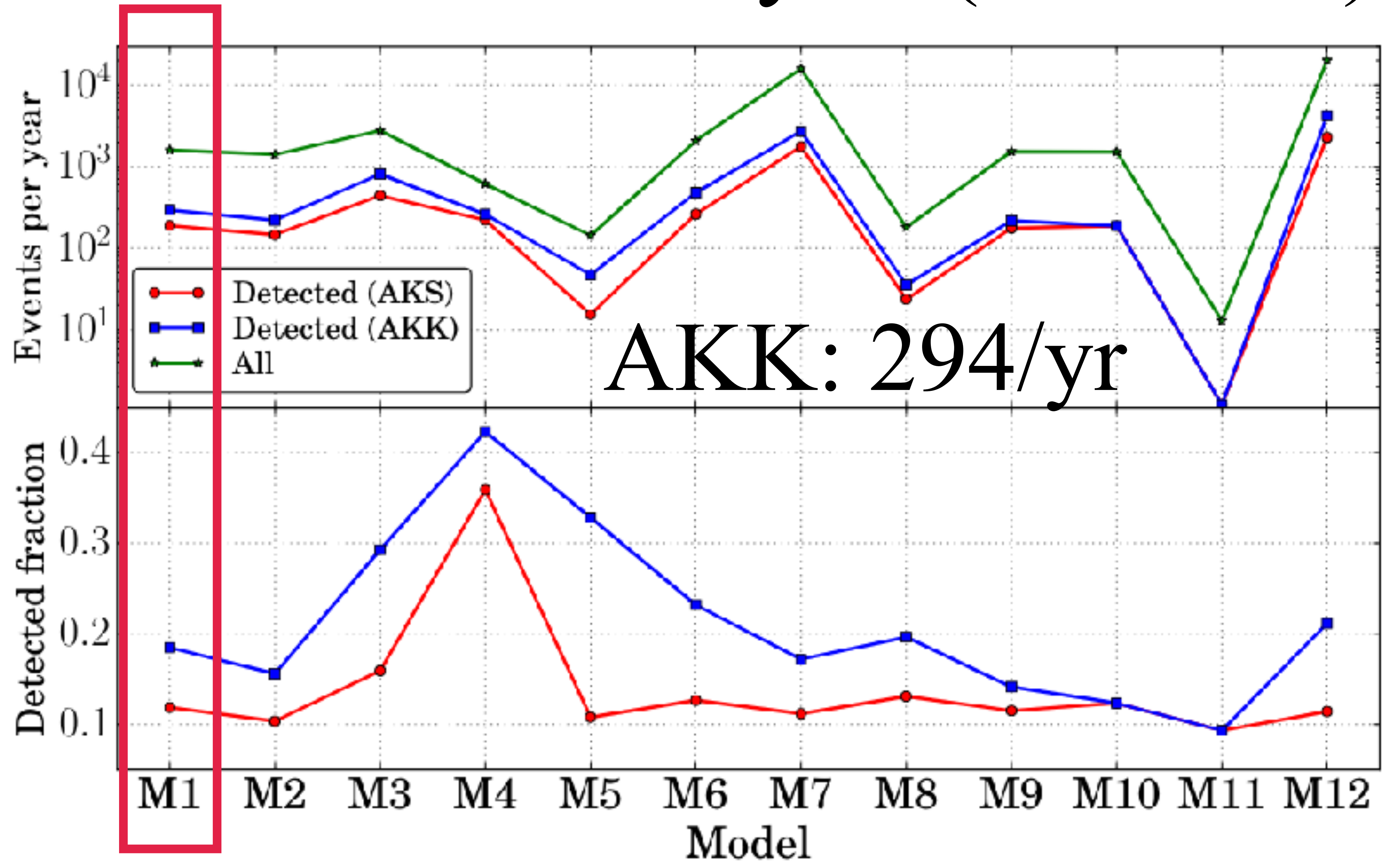
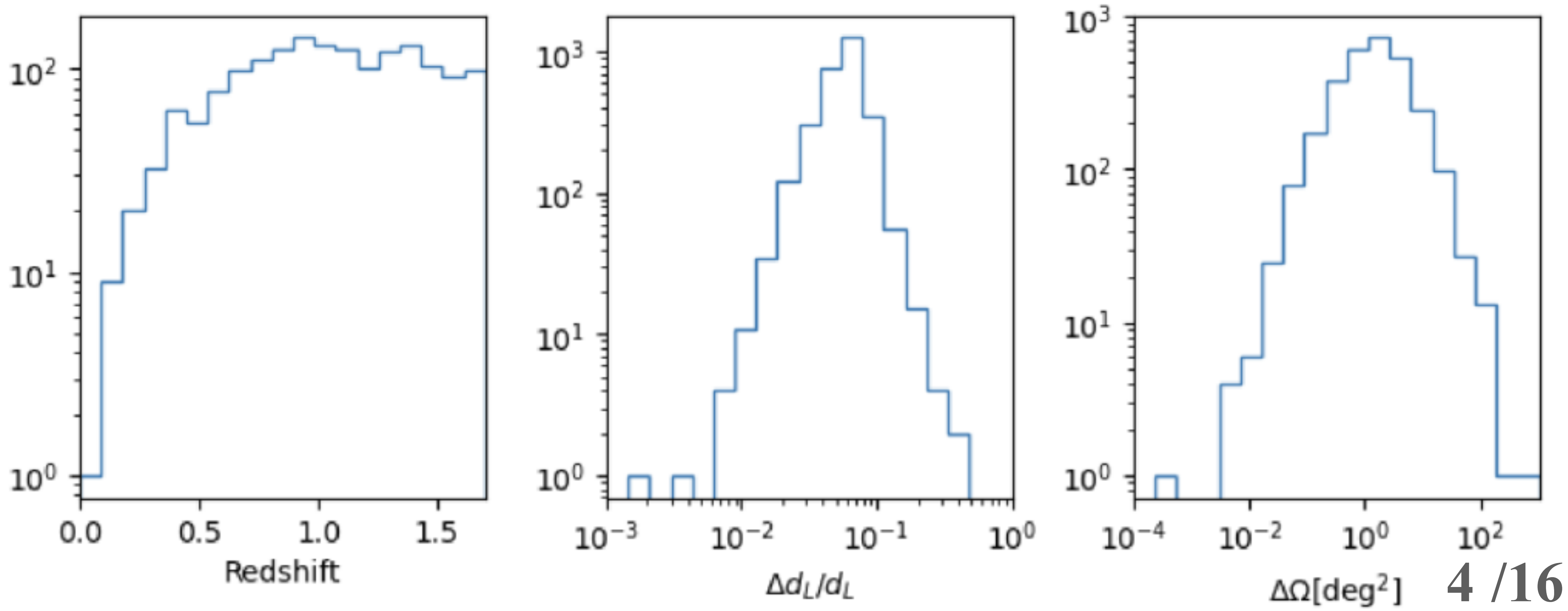


M1 Parameter distribution of the detected sources:



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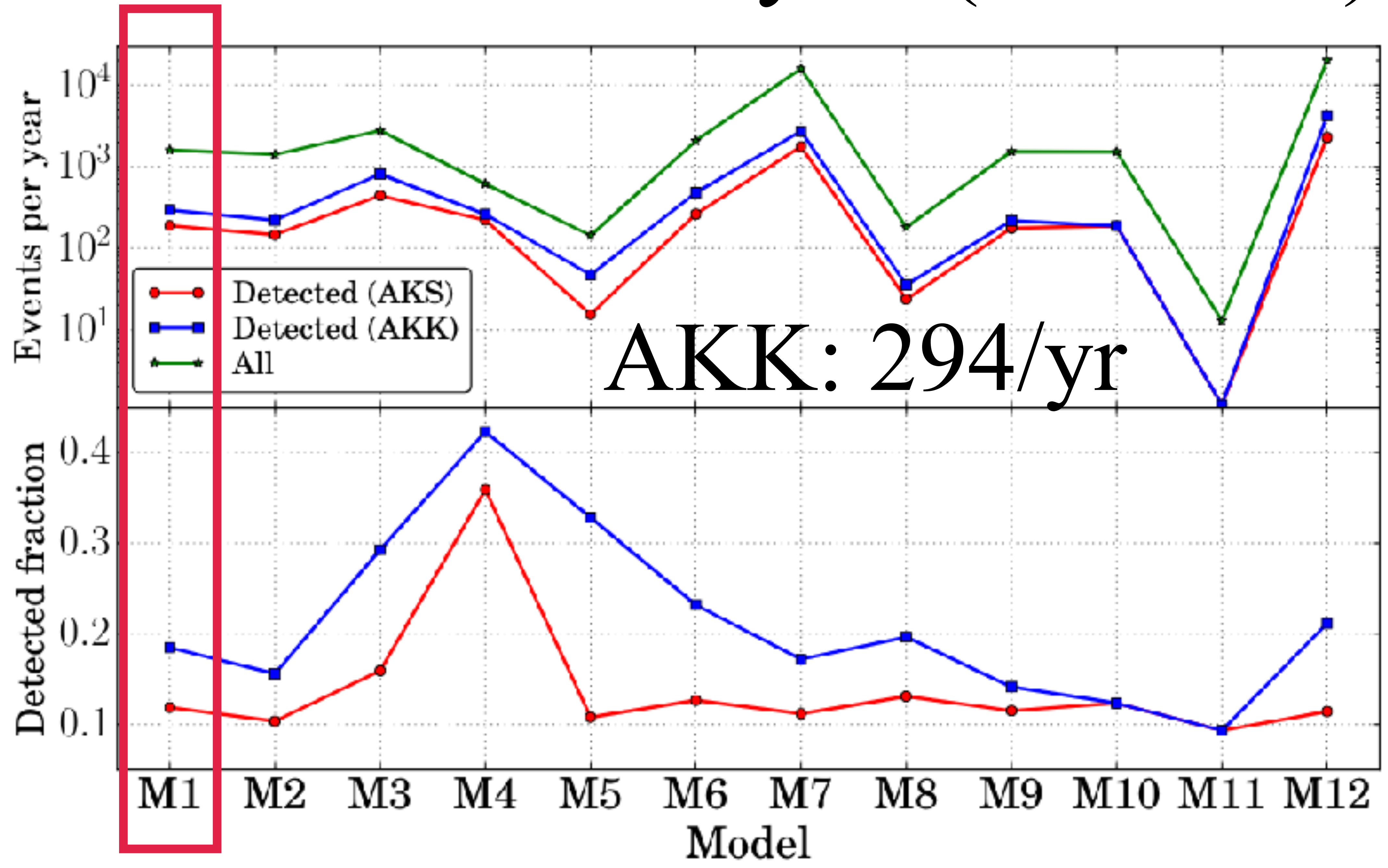

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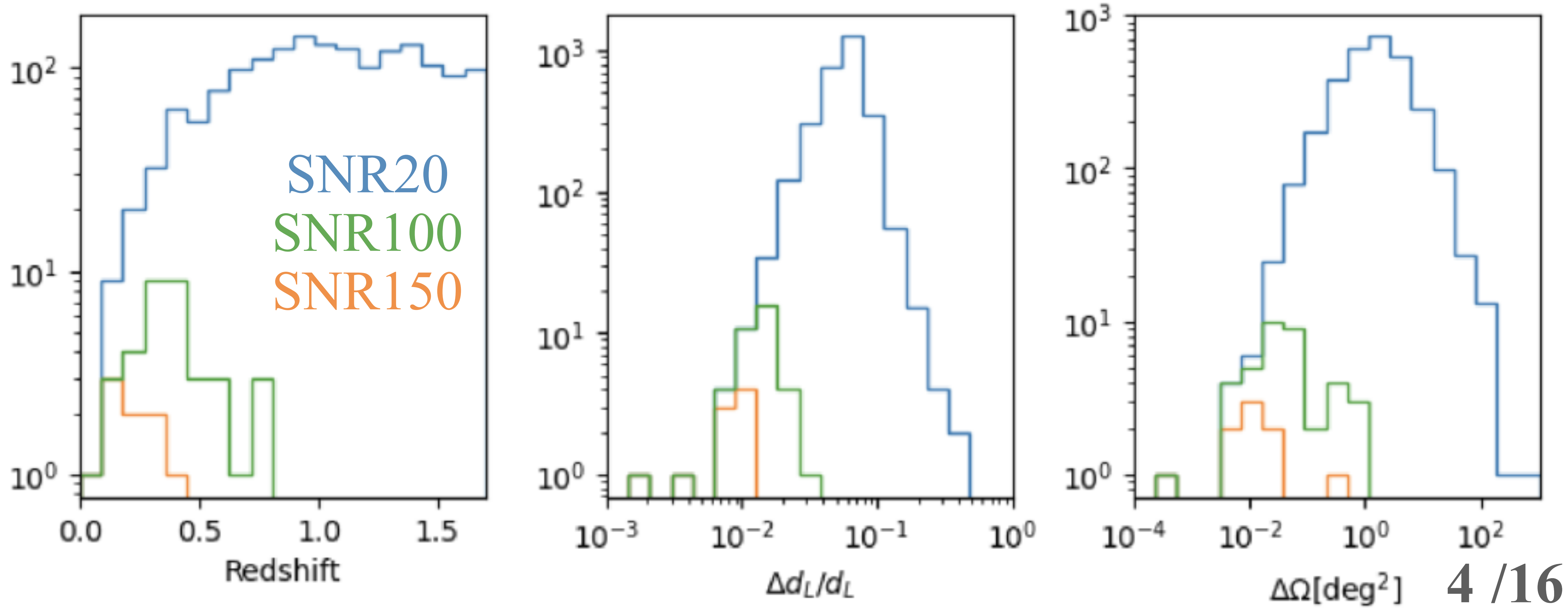


Cosmology with GWs

[Schutz 1986]

From GW:

$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

From EM: z { Counterpart: Bright sirens / No counterpart: Dark sirens

$$d_L(z) = \frac{1+z}{H_0}\int_0^z \frac{d\tilde{z}}{E(\tilde{z})},$$

$\xrightarrow{\Lambda\mathrm{CDM}}$ $H_0, \Omega_m, \Omega_\Lambda$

$$E(z) = \sqrt{\Omega_M(1+z)^3 + \rho_{\mathrm{DE}}(z)/\rho_0},$$

Modified GW propagation

Considering how GWs propagate across cosmological distances, the free propagation of tensor perturbations over FRW is governed by the equation:

$$\tilde{h}''_A + 2\mathcal{H}\tilde{h}'_A + c^2k^2\tilde{h}_A = 0$$

$\tilde{h}'_A$: derivative with respect to cosmic time η

Friction term

- Affect amplitude

$$\tilde{h}''_A + 2\mathcal{H}[1-\delta(\eta)]\tilde{h}'_A + c^2k^2\tilde{h}_A = 0$$

$$d_L^{\rm gw}(z) = d_L^{\rm em}(z)\exp\left\{-\int_0^z \frac{dz'}{1+z'}\,\delta(z')\right\}$$

GW speed

- Affect speed

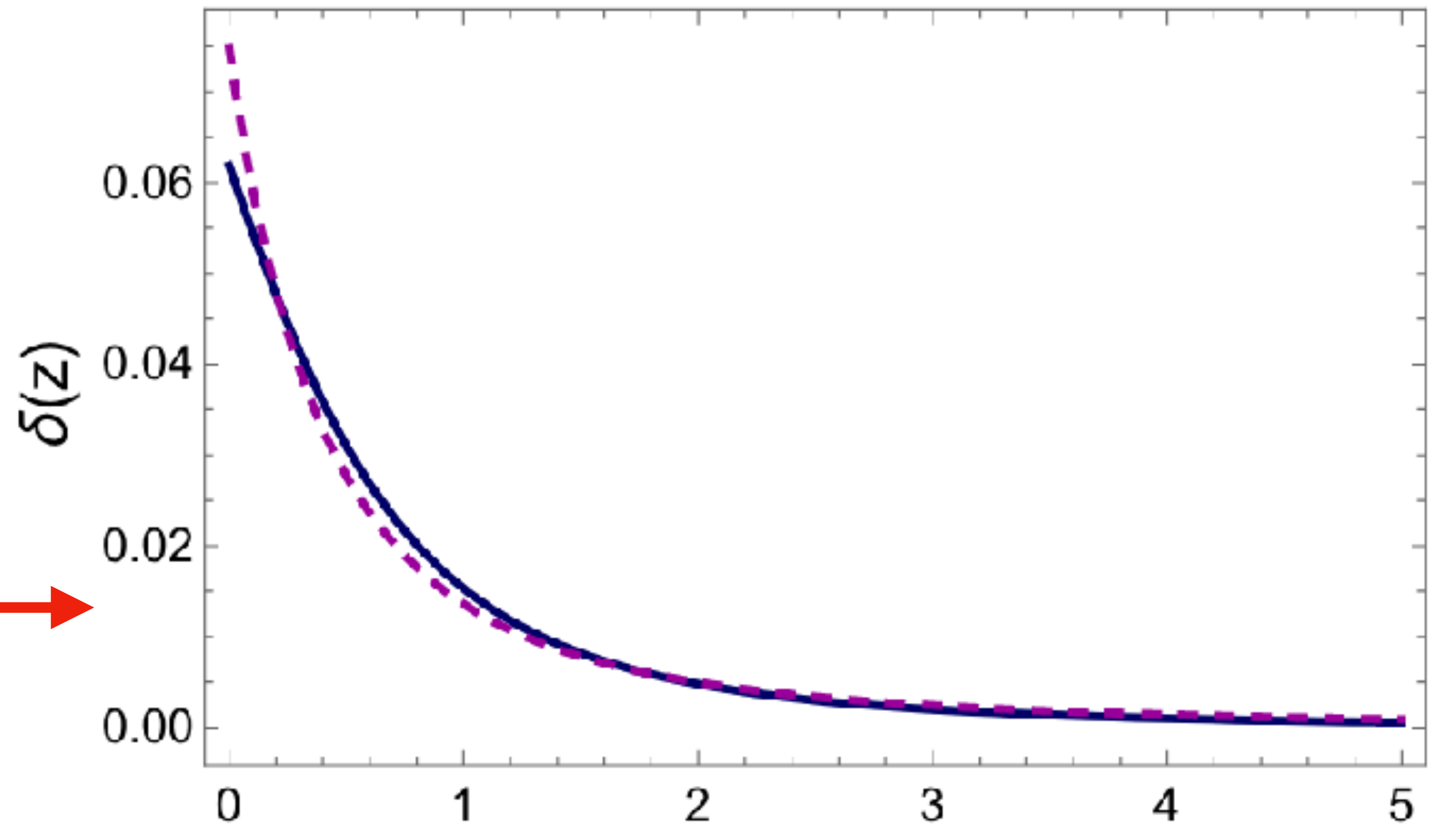
From GW170817

$$|c_{\rm gw}-c|/c < O(10^{-15})$$

Modified gravity: an example

[Belgacem et al., 2018]

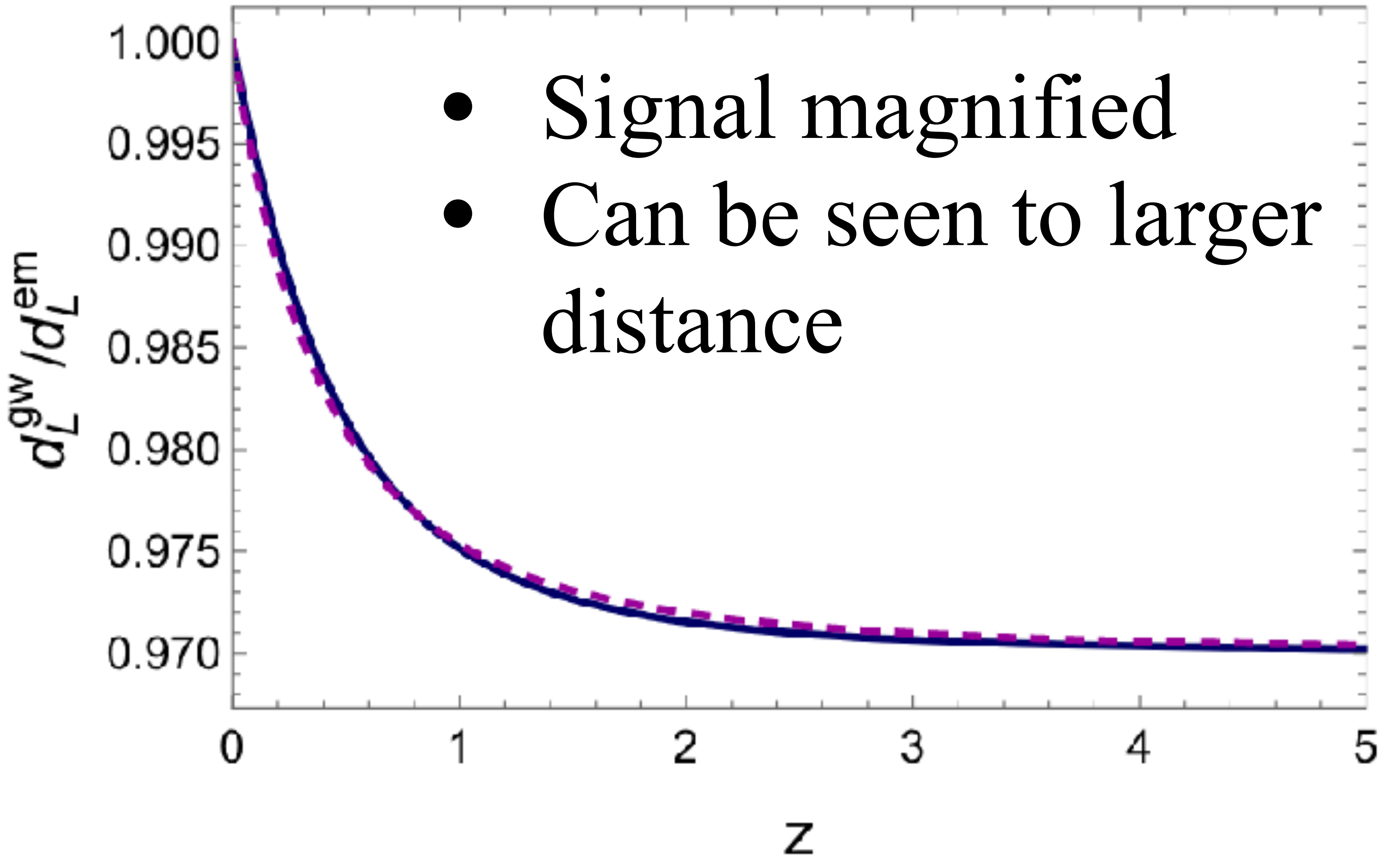
The function δ(η) is predicted explicitly by the RR model:



$$d_L^{\mathrm{gw}}(z) = d_L^{\mathrm{em}}(z) \exp\left\{ -\int_0^z \frac{dz'}{1+z'}\, \delta(z') \right\}$$

Parametrization

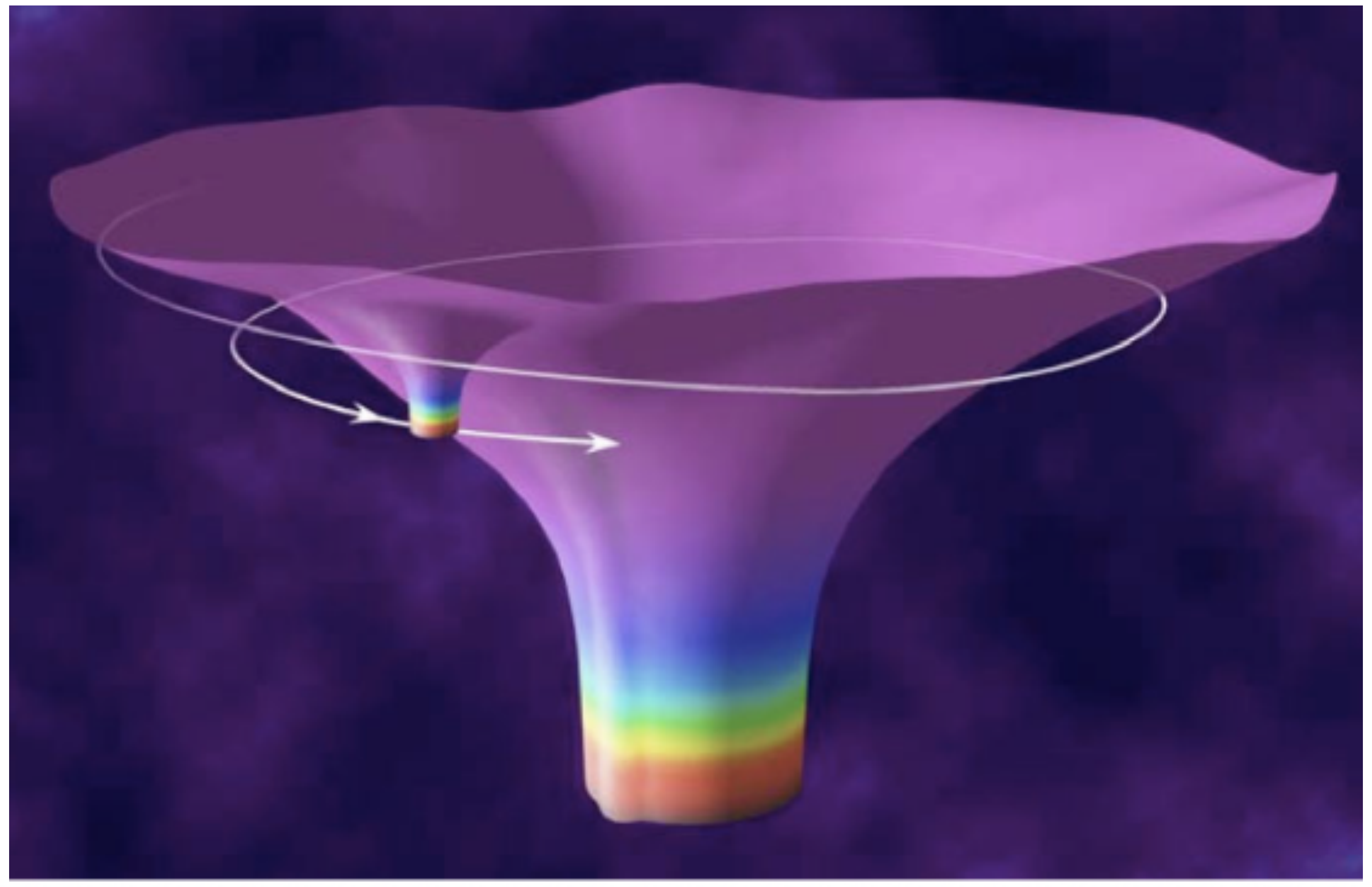
$$\frac{d_L^{\mathrm{gw}}(z)}{d_L^{\mathrm{em}}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$

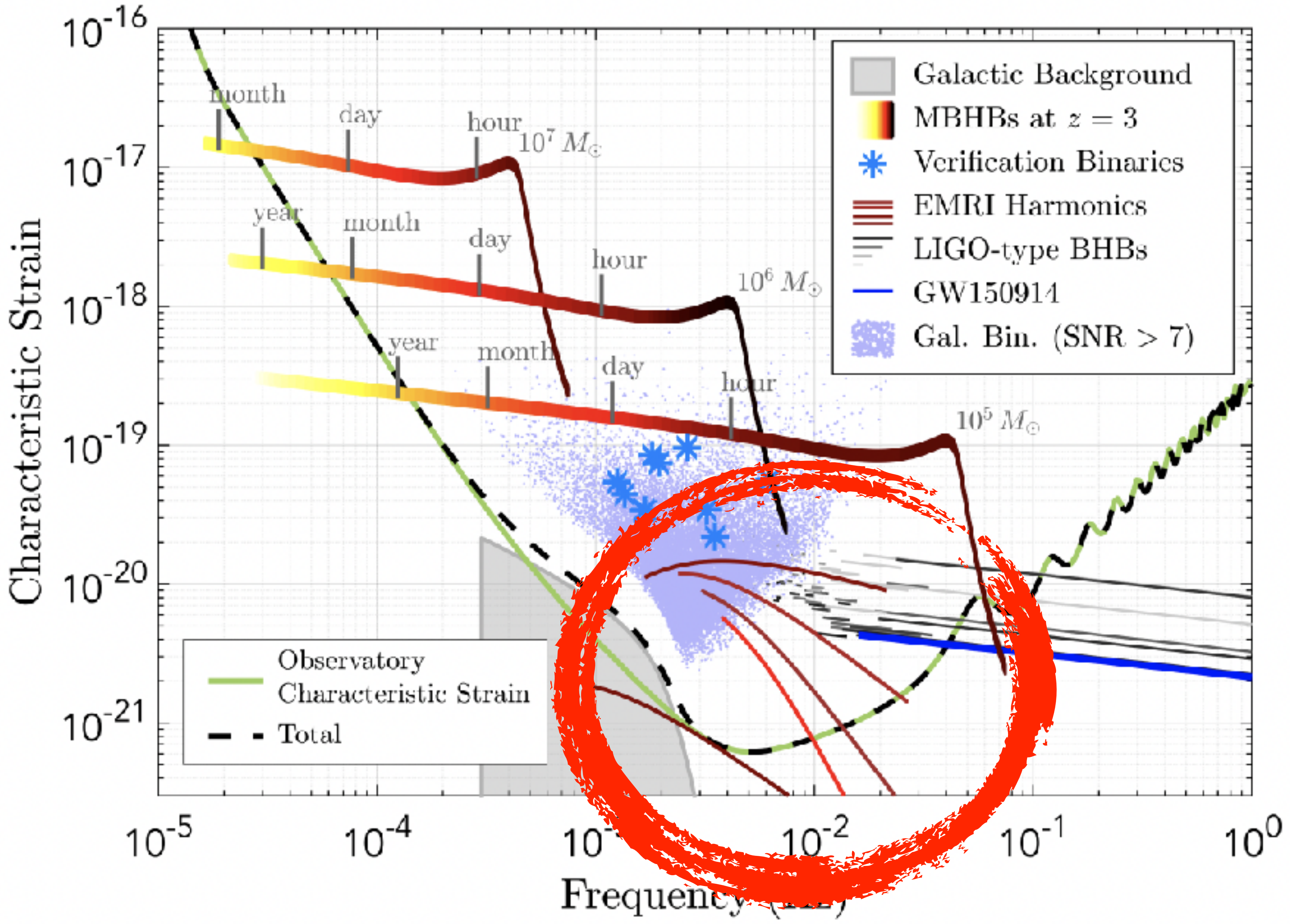


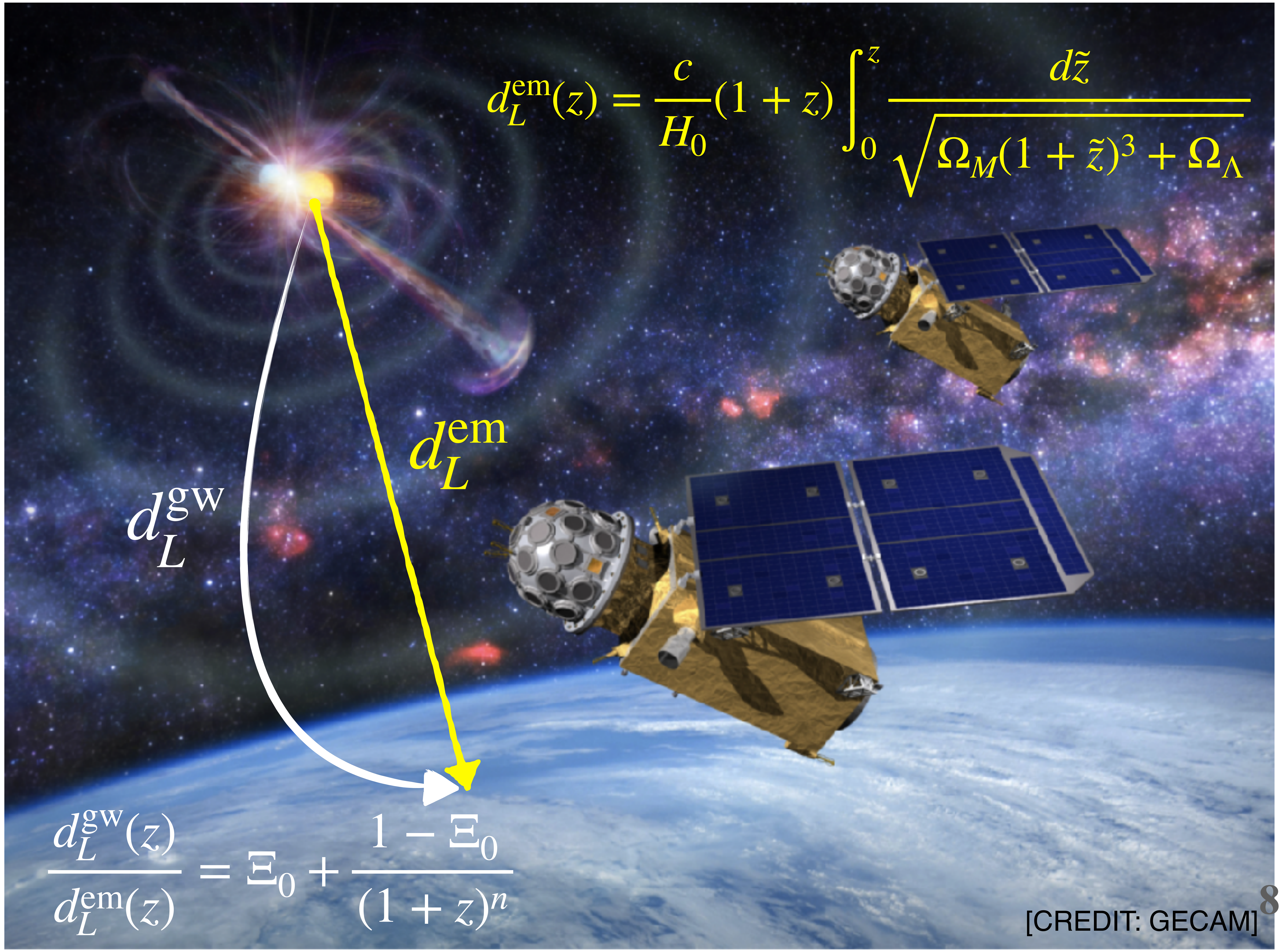
The model predicts $\delta(z=0) = 0.062$ [n=5/2 and Ξ_0=0.970]

- $d_L^{\mathrm{GW}}(z=0)/d_L^{\mathrm{EM}}(z=0) = 1$
- $d_L^{\mathrm{GW}}(z)/d_L^{\mathrm{EM}}(z)$ saturates to a constant Ξ_0
- Ξ_0: crucial parameter, fix the asymptotic value of $d_L^{\mathrm{GW}}(z)/d_L^{\mathrm{EM}}(z)$ at large z
- n only determines the precise shape of the function that interpolates from z = 0 and large z

Constraining **modified GW propagation** with **extreme mass-ratio inspirals**







Bayesian theorem in cosmology

$D \equiv \{D_1, \ldots, D_N\}$ i_{th}GW data → $D_i = \left\{\hat{d}_L, \hat{\theta}_{gw}, \hat{\phi}_{gw}\right\}_i$ + errors

$\Omega \equiv \{H_0, \Omega_m, \Xi_0, n\}$

Likelihood

Prior

Posterior

$$p(\Omega \mid D\,\mathcal{H}\,I) = p(\Omega \mid \mathcal{H}\,I)\,\frac{p(D \mid \Omega\,\mathcal{H}\,I)}{p(D \mid \mathcal{H}\,I)}$$

Calculated based on cosmological model and the EM information

Cosmological model:
Defines the relation between
$d_L^{\rm gw}$, z, Ω

$\Xi_0 = [0.3 , 2.0]$
$n = [0.5 , 3.0]$
$H_0 = [60, 86]$
$\Omega_M = [0.04, 0.5]$

1. EMRI Catalog M1

2. Parameter estimation $\rightarrow d_L, \Delta d_L, \Delta\theta, \Delta\phi$

3. Apply modified GW propagation relation $\rightarrow z$

From GW:

$d_L^{\mathrm{gw}}, \Delta d_L^{\mathrm{gw}}, \Delta\theta_{gw}, \Delta\phi_{gw}$

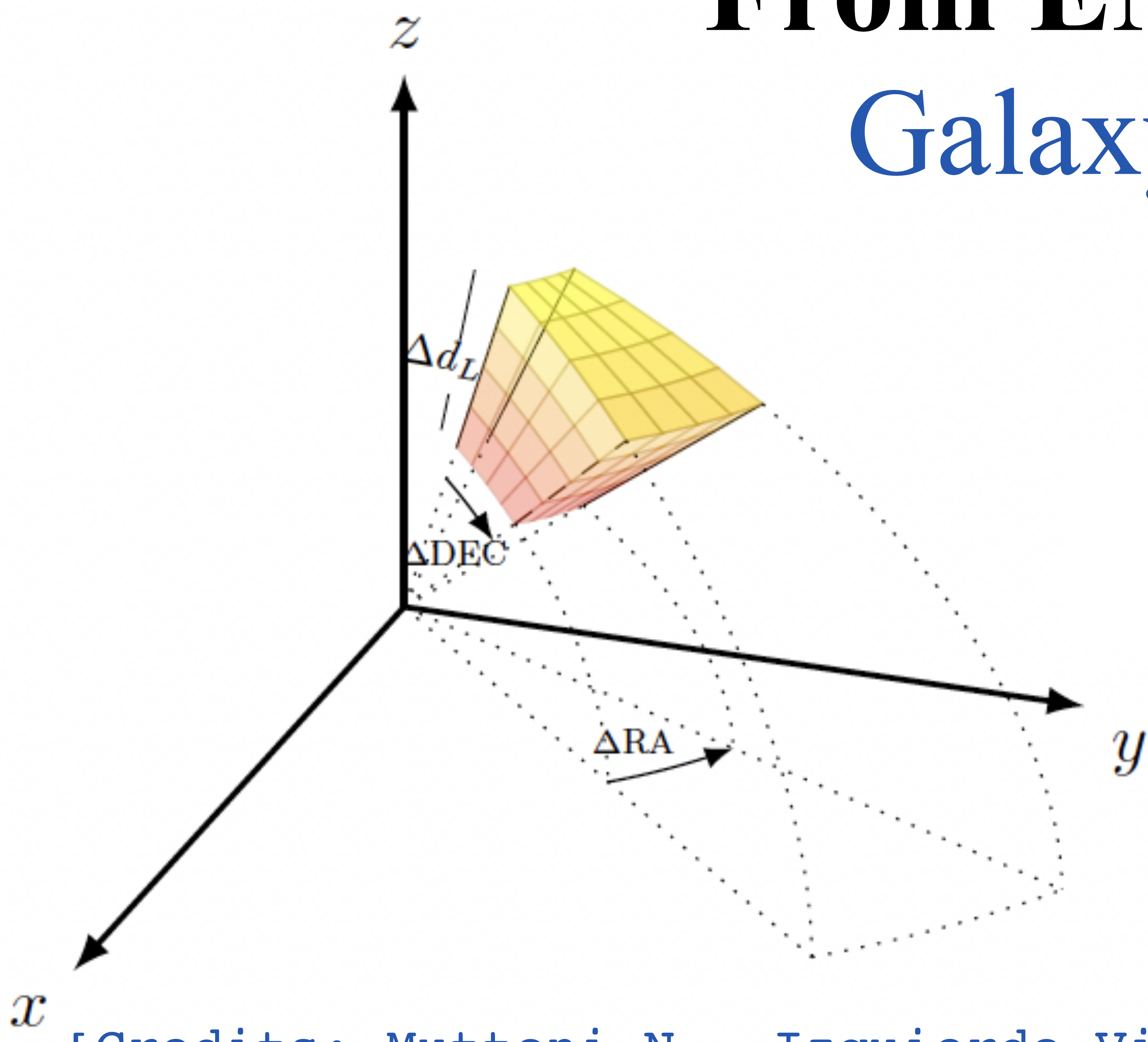
From EM:

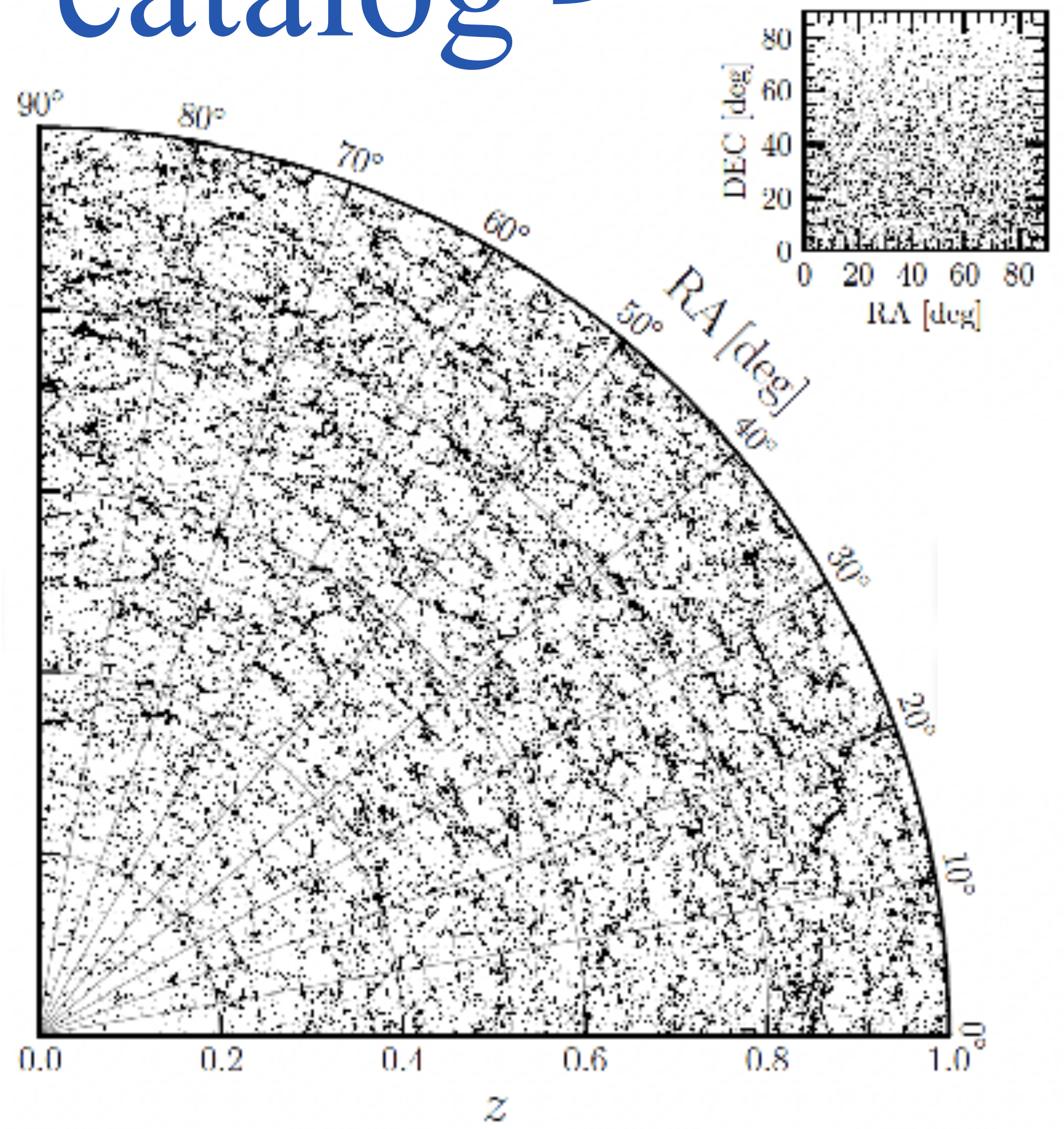
Galaxy catalog

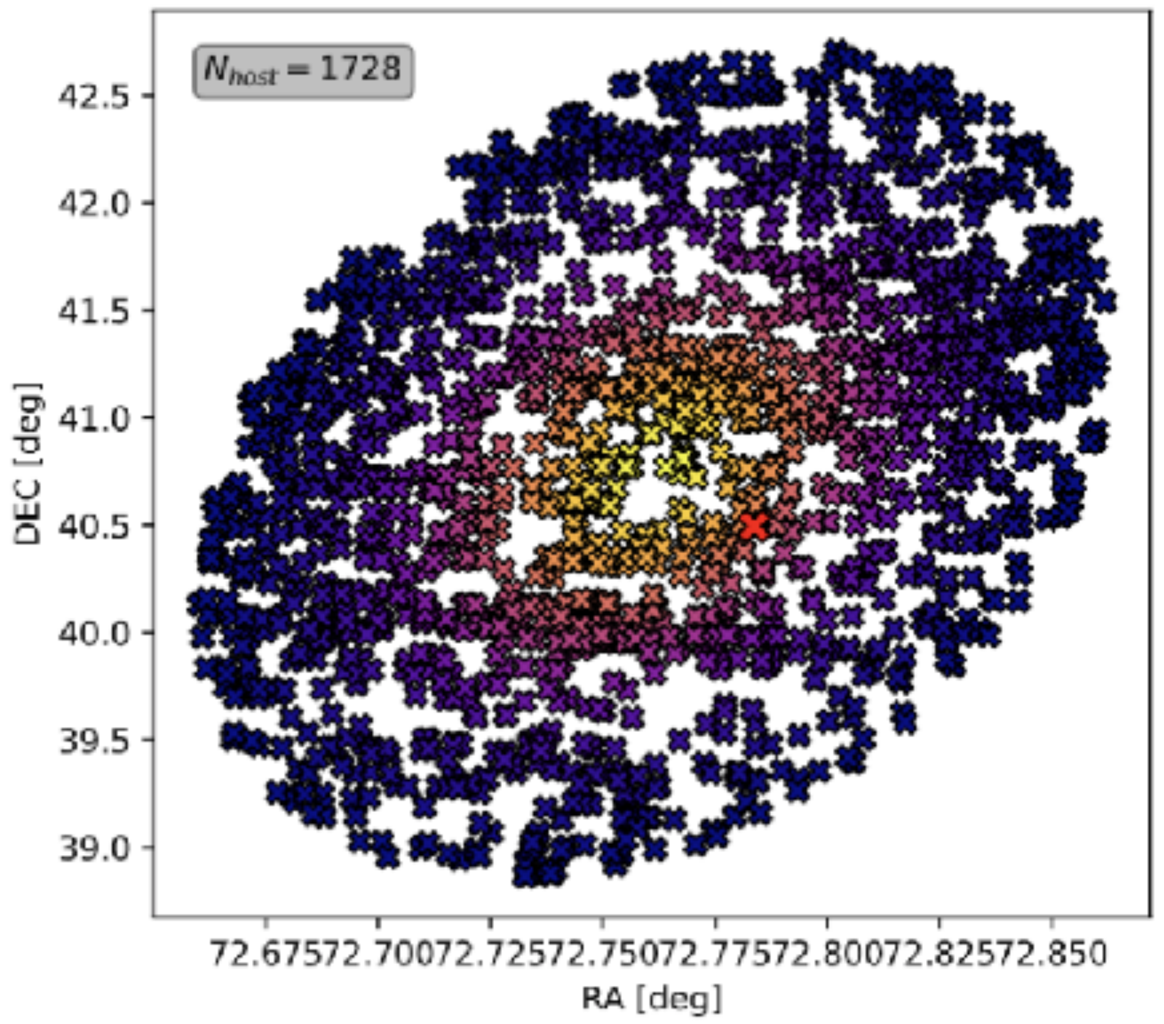
} + prior:

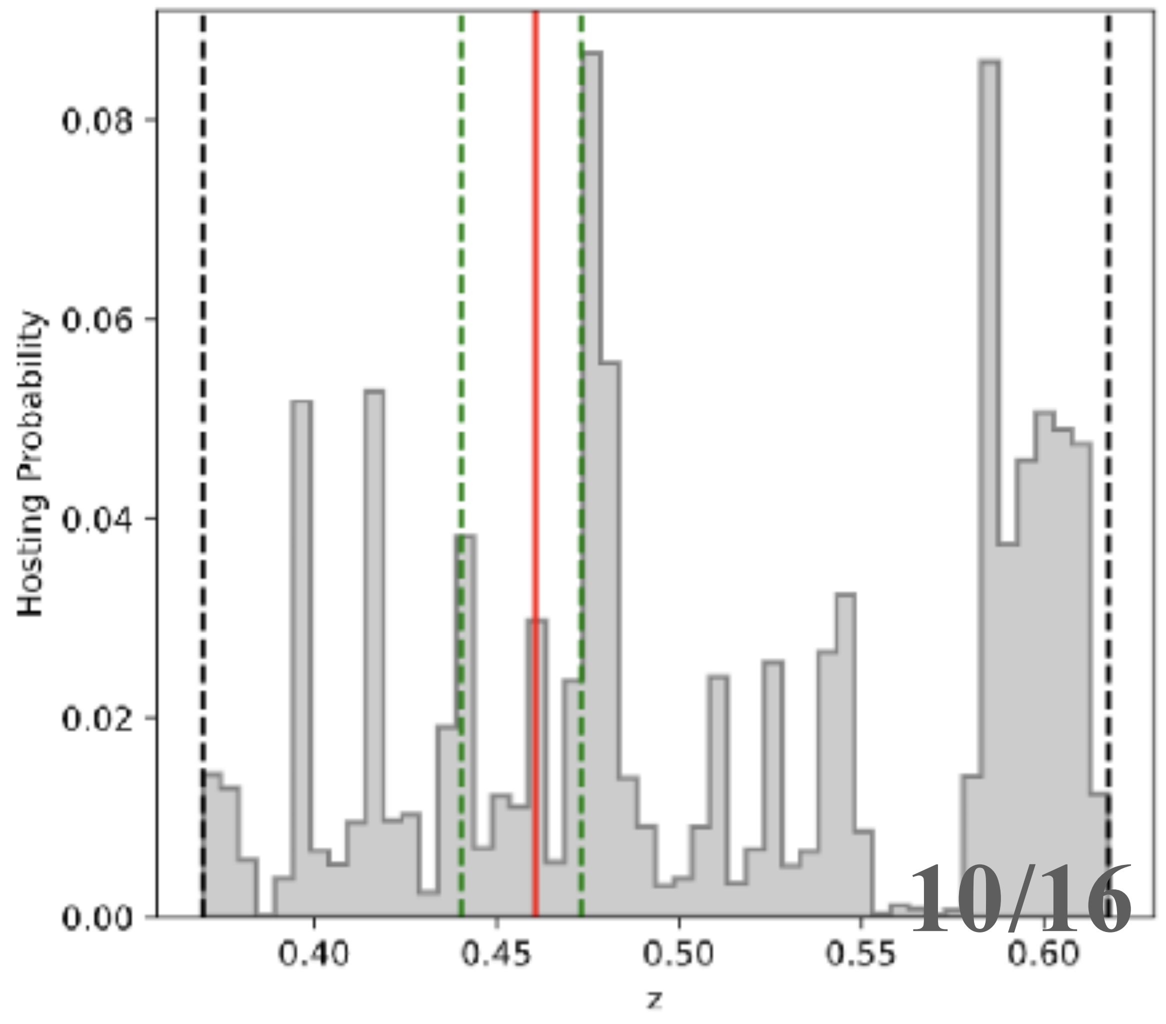
4. 3σ Error box construction:

Statistical information on: z, σ_z









[Credits: Muttoni N., Izquierdo Villalba D.]

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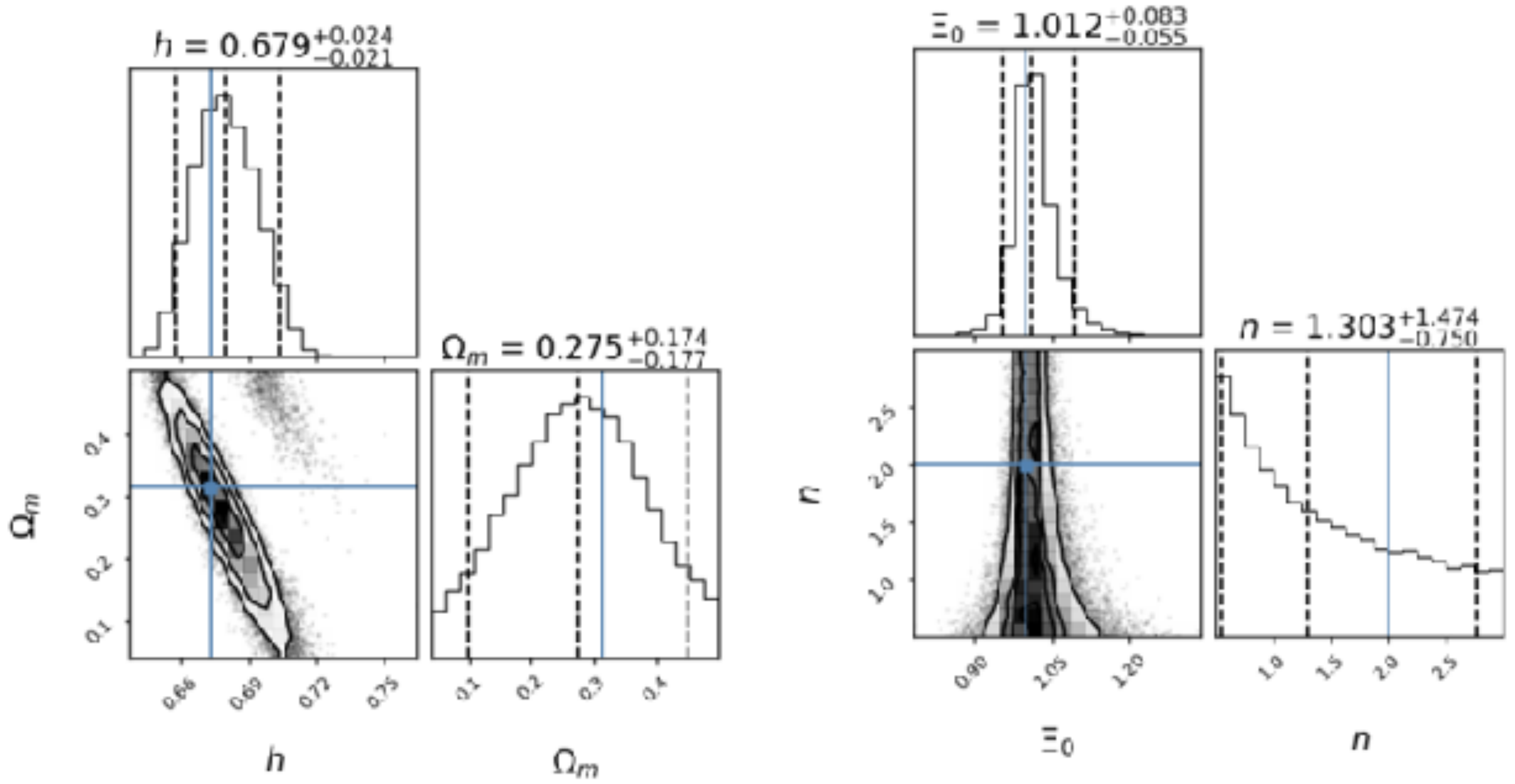


6. Constrain on cosmological parameters

From GW:

$d_L^{gw}, \Delta d_L^{gw}, \Delta\theta_{gw}, \Delta\phi_{gw}$

5. Bayesian theorem [nested sampling by cosmolisa]

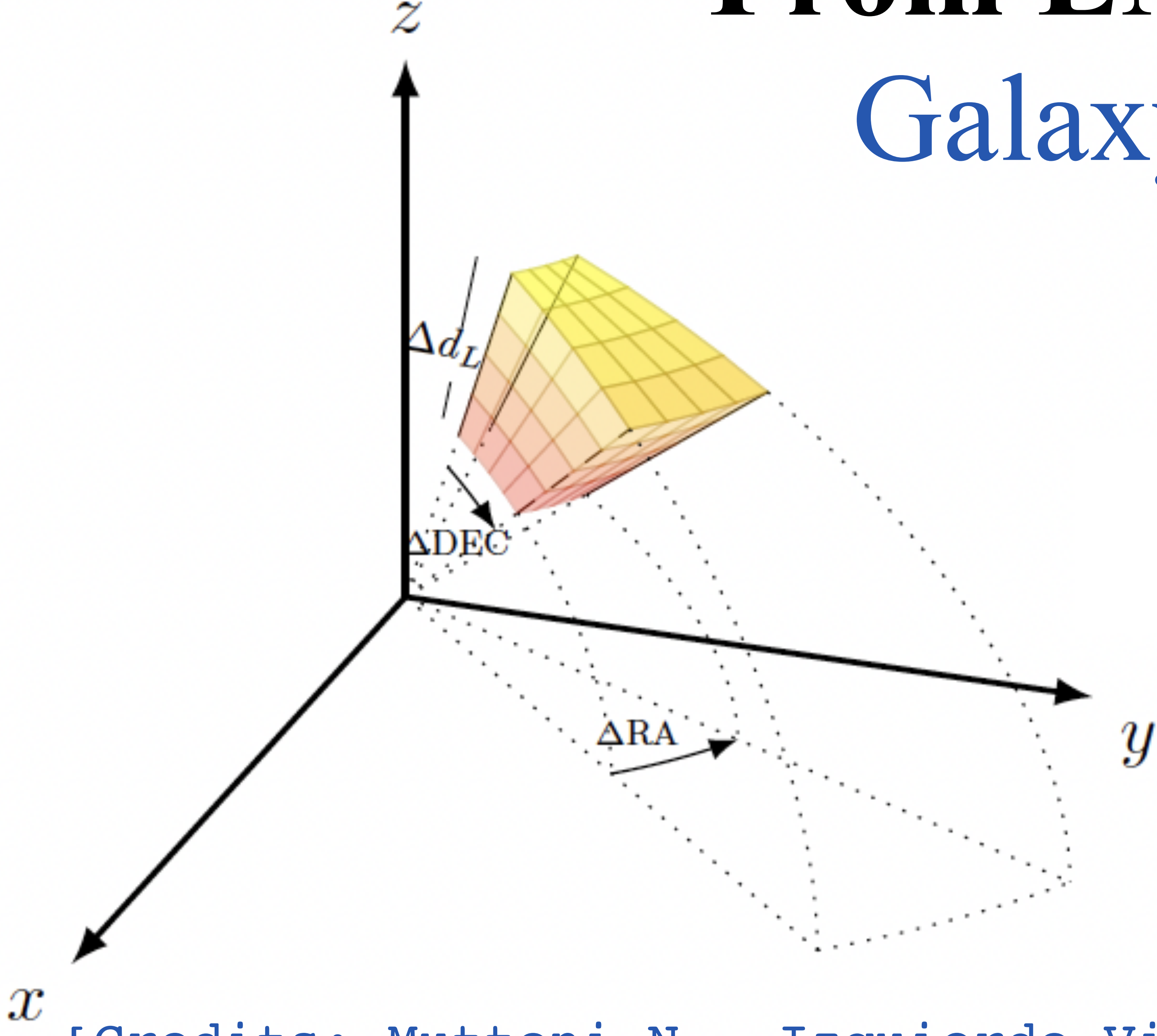
From EM:

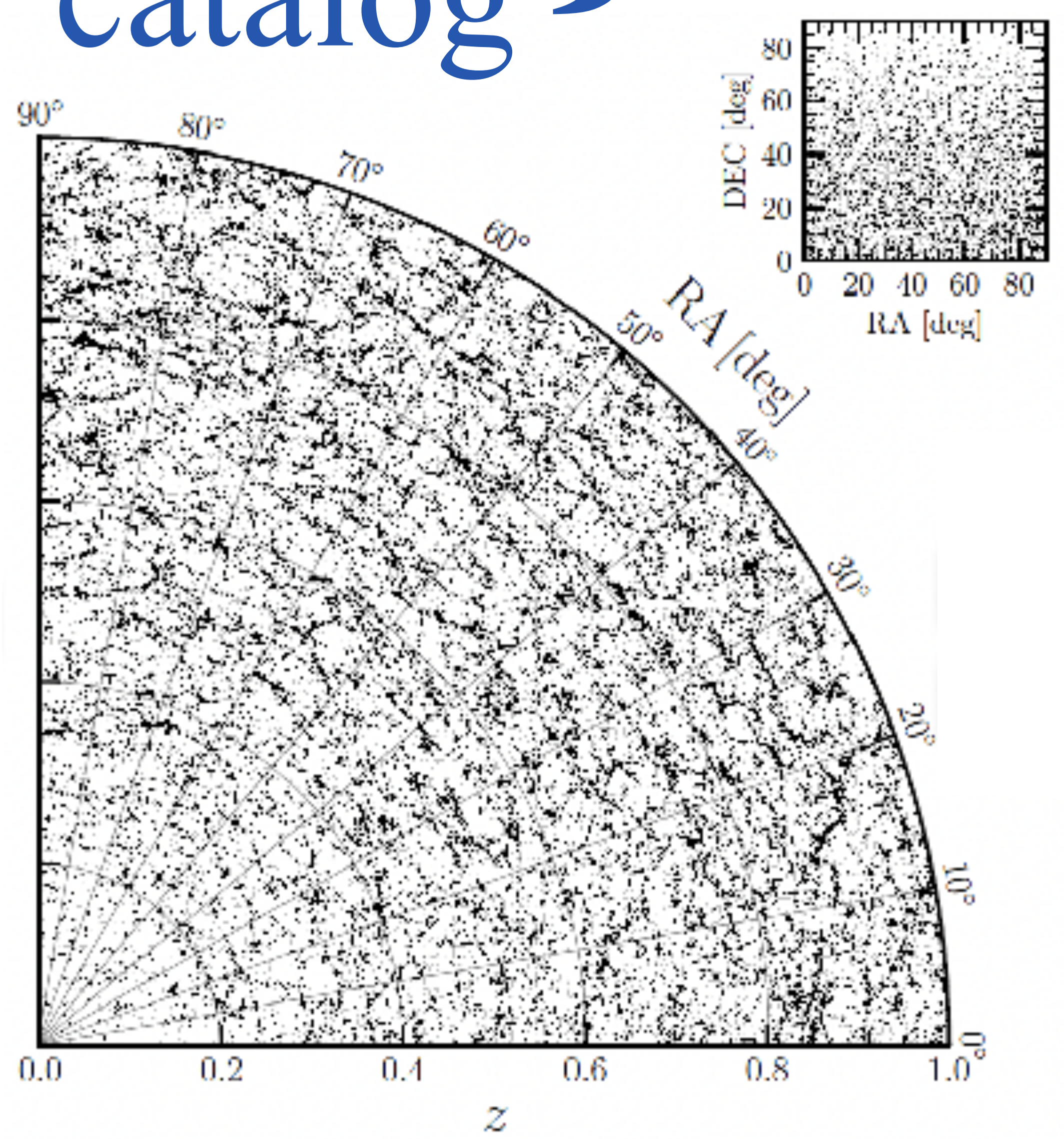
Galaxy catalog

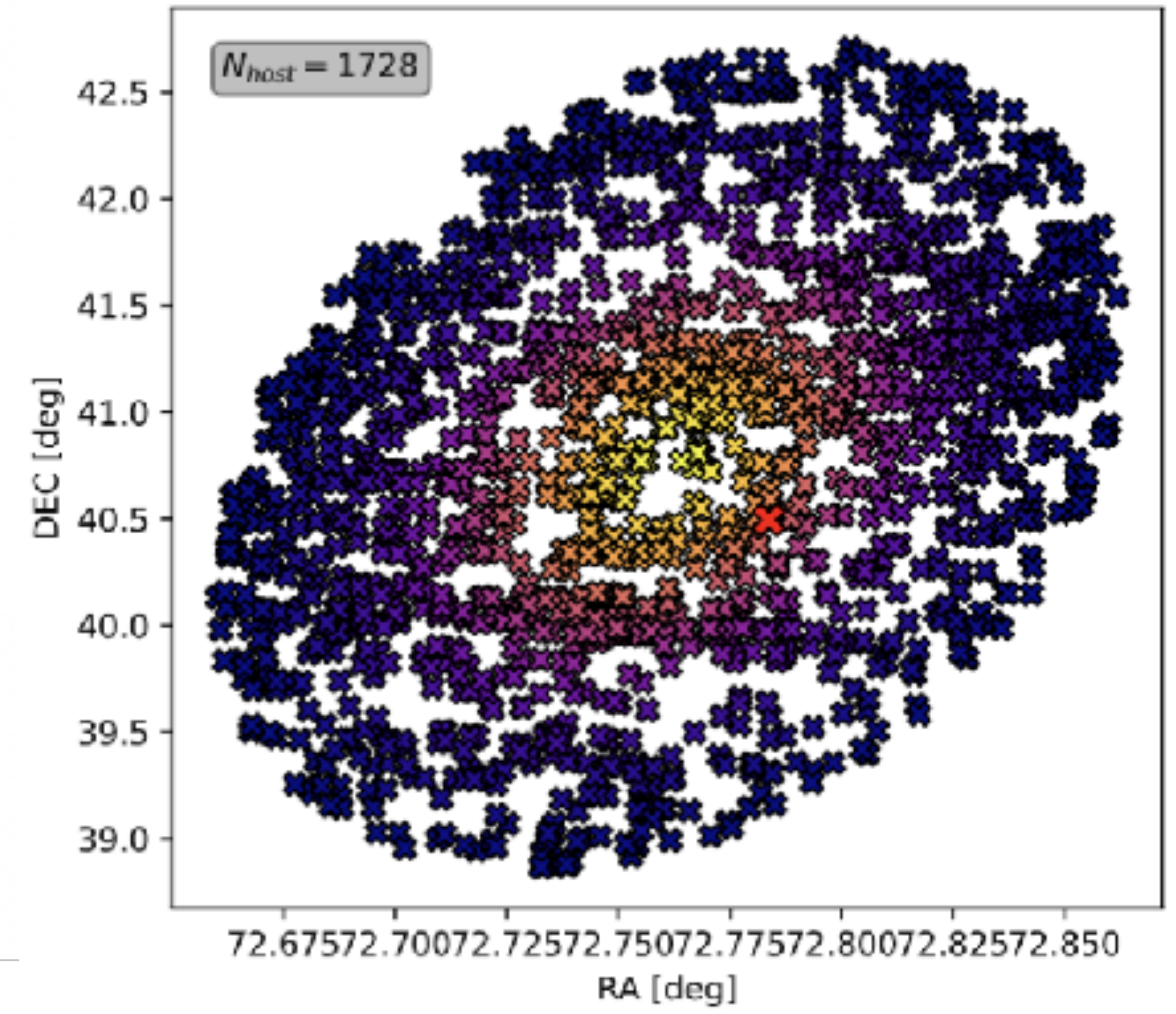
+ prior:

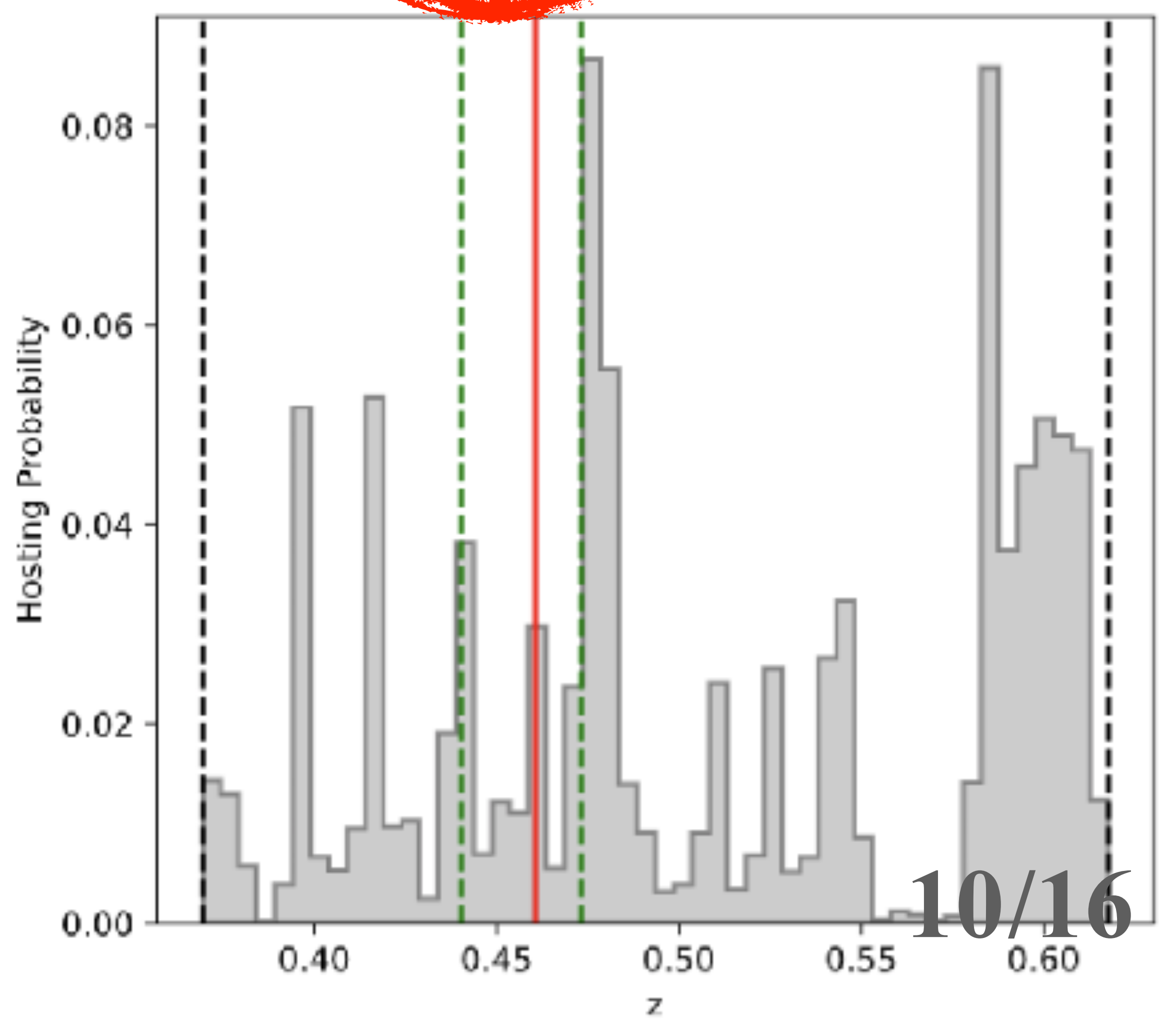
4. 3σ Error box construction:

Statistical information on: z, σ_z





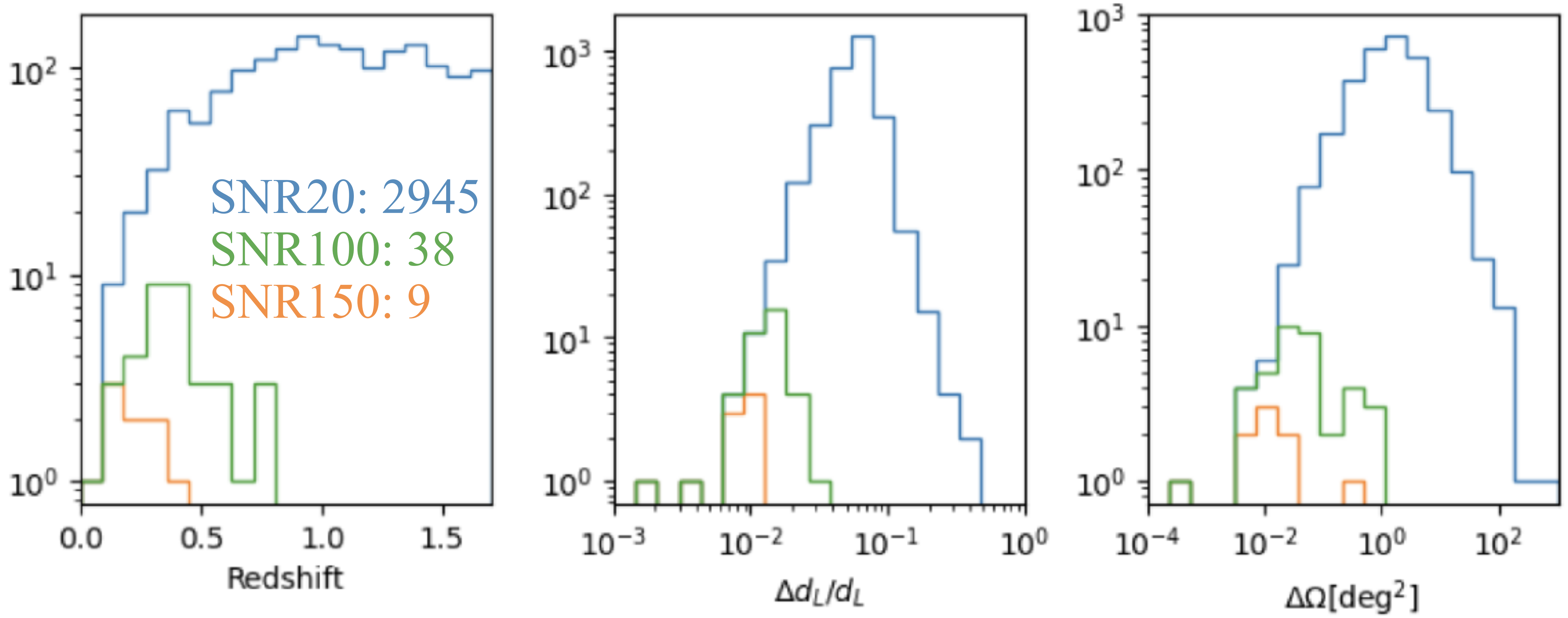




[Credits: Muttoni N., Izquierdo Villalba D.]

Preliminary results: number of detected EMRIs

- 10 yrs observation
- low SNR events tend to produce a bias in the estimation



Number of events used in the analysis ($z < 1$)

$\Xi_0 = [0.3\ , 2.0]$
$n = [0.5\ , 3.0]$
$H_0 = [60,\ \ 86\ ]$
$\Omega_M = [0.04, 0.5]$

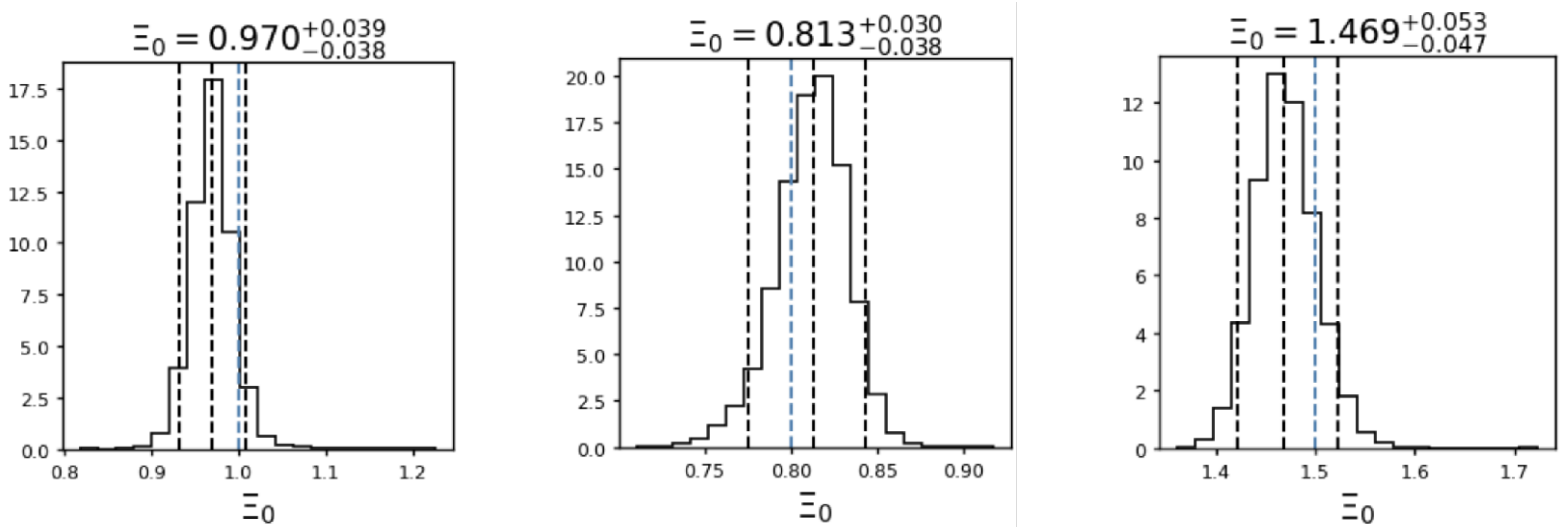
M1	$H_0 + \Omega_M + \Xi_0 + n$	$H_0 + \Omega_M + \Xi_0$	$\Xi_0 + n$	Ξ_0	$H_0 + \Omega_M$
SNR >= 150	8	8	9	9	9

Preliminary results: Ξ_0

Injected value and [prior]:
$\Xi_0 =$ **1.0/0.8/1.5** [0.3 , 2.0]
$n = 2$
$H_0 = 67.3$
$\Omega_M = 0.315$

- 90% CI
- Median of 5 realizations

$\Delta\Xi_0/\Xi_0 \sim 4\,\%$



Liu et al. in prep

Preliminary results: $\Xi_0 + n$

- 90% CI
- Median of 5 realizations
- **Strongly correlated $\Xi_0 + n$**

Injected value and [prior]:
Ξ_0 = **1.0/0.8/1.5** [0.3 , 2.0]
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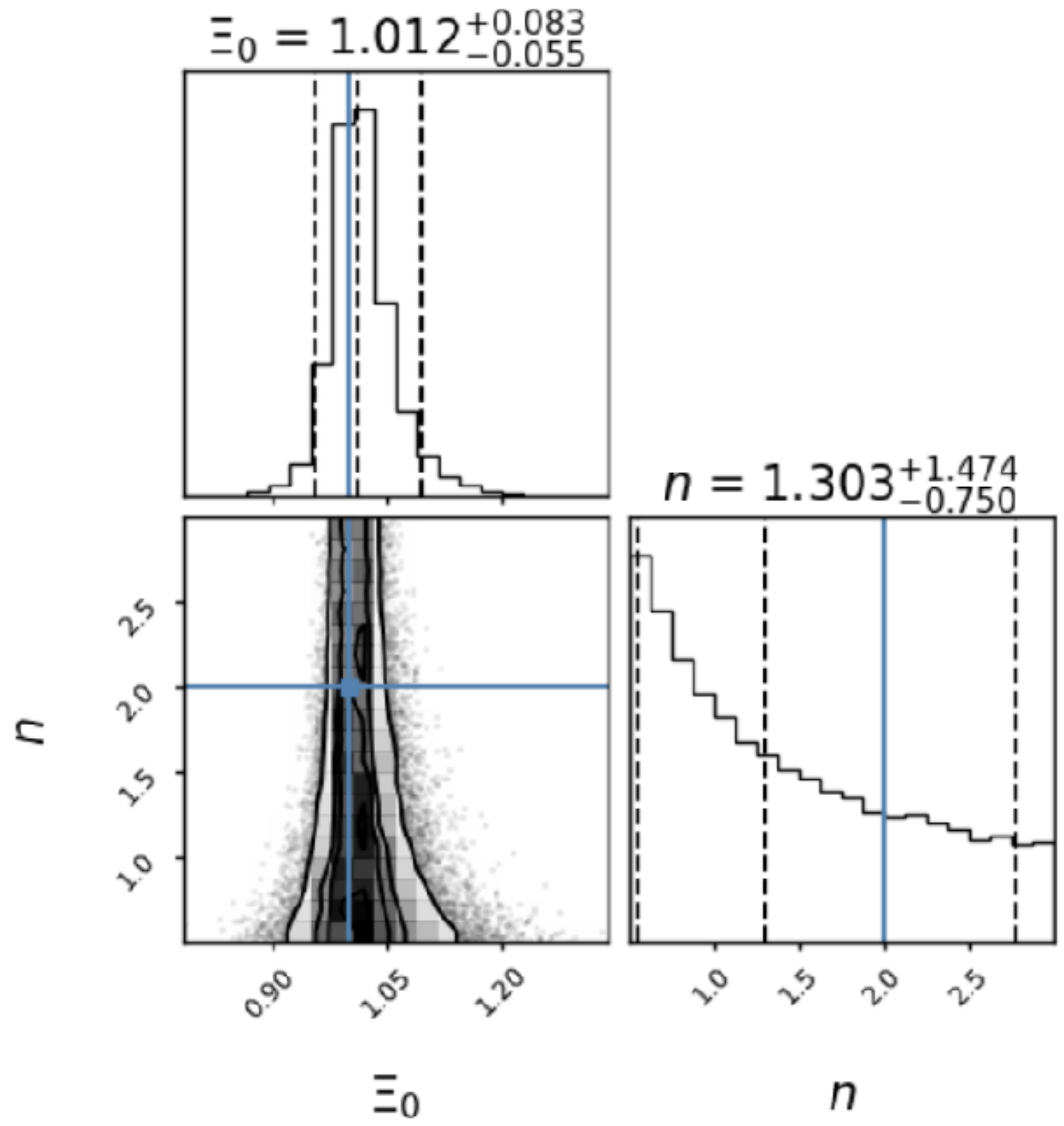

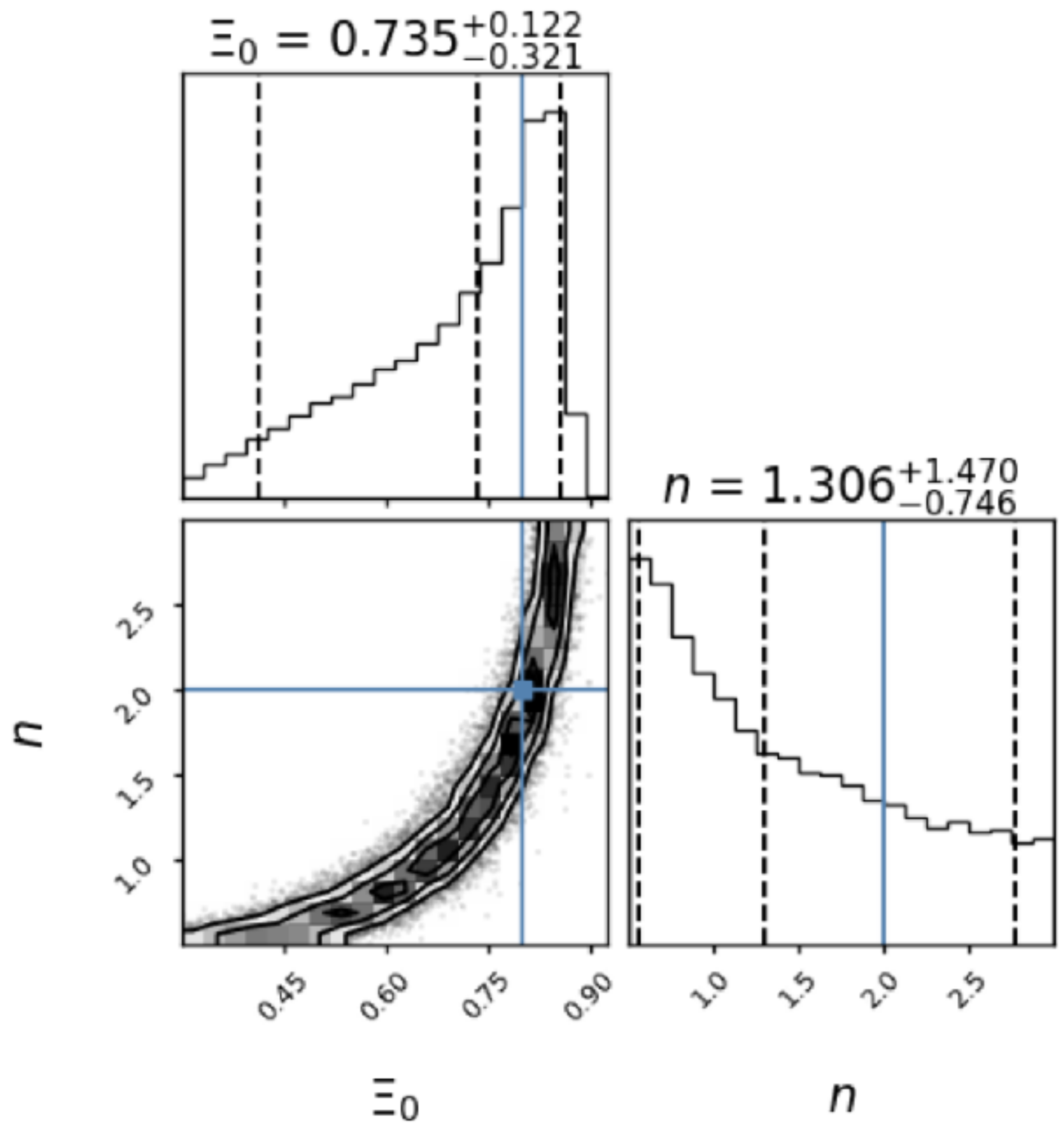

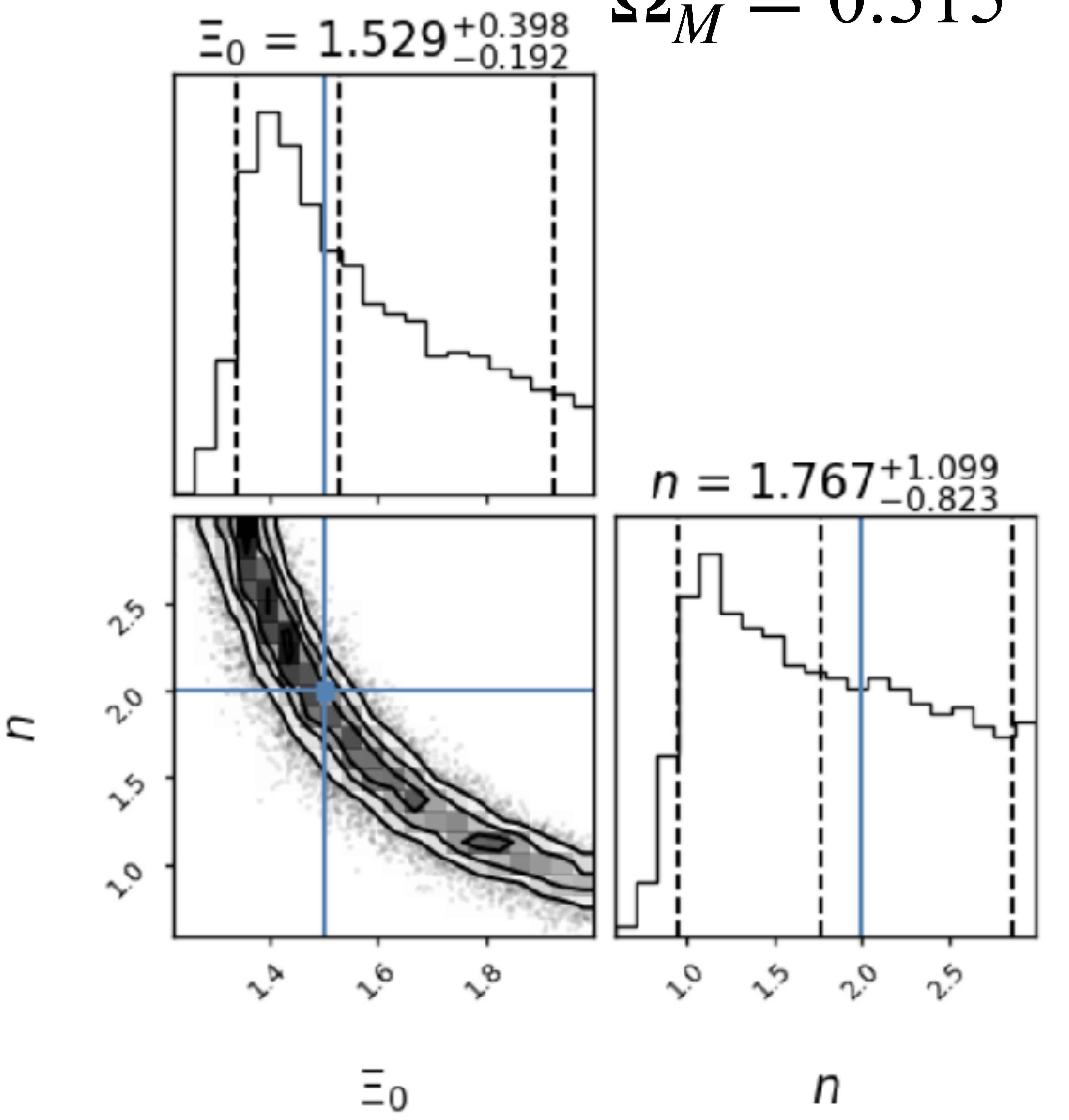

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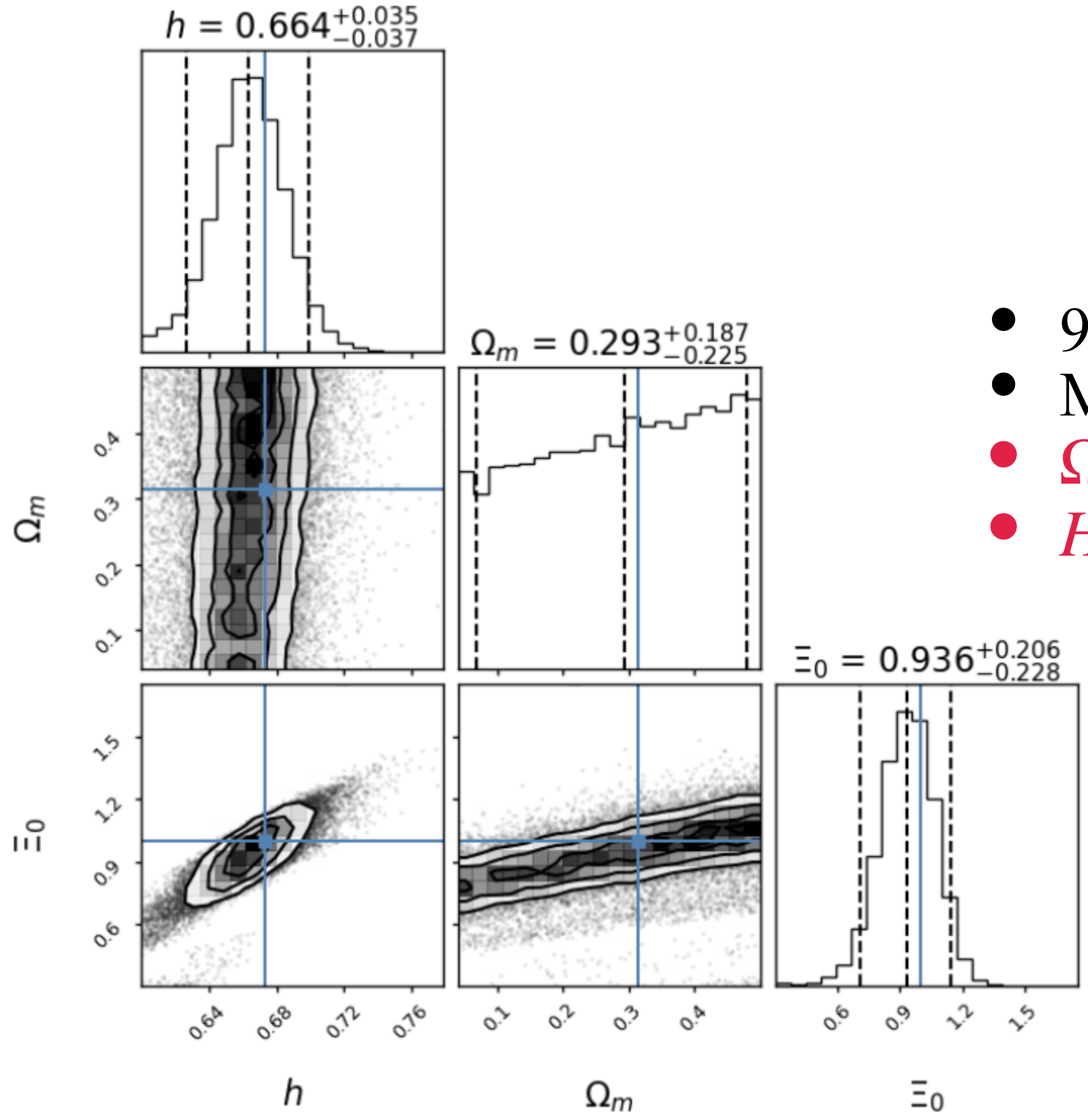
Preliminary results: $H_0 + \Omega_M + \Xi_0$



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$\Xi_0 =$ **1.0** [0.3 , 2.0]
$H_0 =$ 67.3 [60, 86]
$\Omega_M =$ 0.315[0.04,0.5]

- 90% CI
- Median of 5 realizations
- **Ω_m can not be measured**
- $H_0 \sim 5\,\%$

$$\Delta\Xi_0/\Xi_0 \sim 21\,\%$$
$$\Delta H_0/H_0 \sim 5\,\%$$

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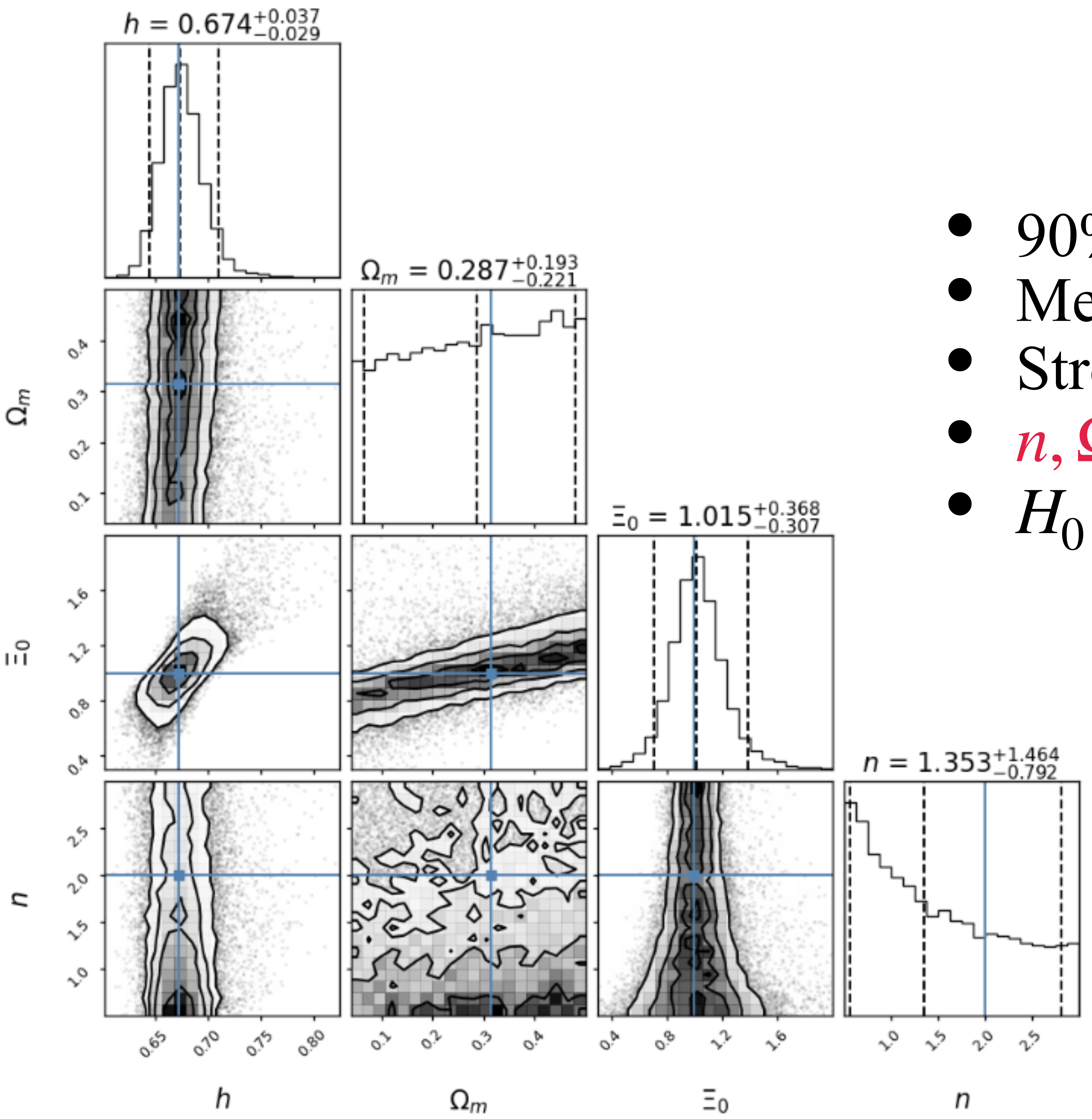
Preliminary results: $H_0 + \Omega_M + \Xi_0 + n$



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$n =$ [0.5 , 3.0]
$H_0 =$ 67.3 [60, 86]
$\Omega_M =$ 0.315[0.04,0.5]

- 90% CI
- Median of 5 realizations
- Strongly correlated $\Xi_0 + n$
- n, Ω_m can not be measured
- $H_0 \sim 5\,\%$

$$\Delta\Xi_0/\Xi_0 \sim 33\,\%$$
$$\Delta H_0/H_0 \sim 5\,\%$$

Liu et al. in prep

Conclusion & future prospects

- Conclusion:
- Ξ_0 alone $\sim 4\,\%$
- Ξ_0 and n: Strongly correlated
  - Different trend when $\Xi_0 > 1, = 1, < 1$
- When also considering other parameters, $\Delta\Xi_0/\Xi_0 > \sim 20\,\%$
- $H_0 \sim$ few percent

- Future prospects:
- New waveform model:
  - Augmented Analytic Kludge with 5PN trajectories
- New sensitivity curve + Full response TDI
- Selection effect

Backups

Modified gravity theory

RR model:

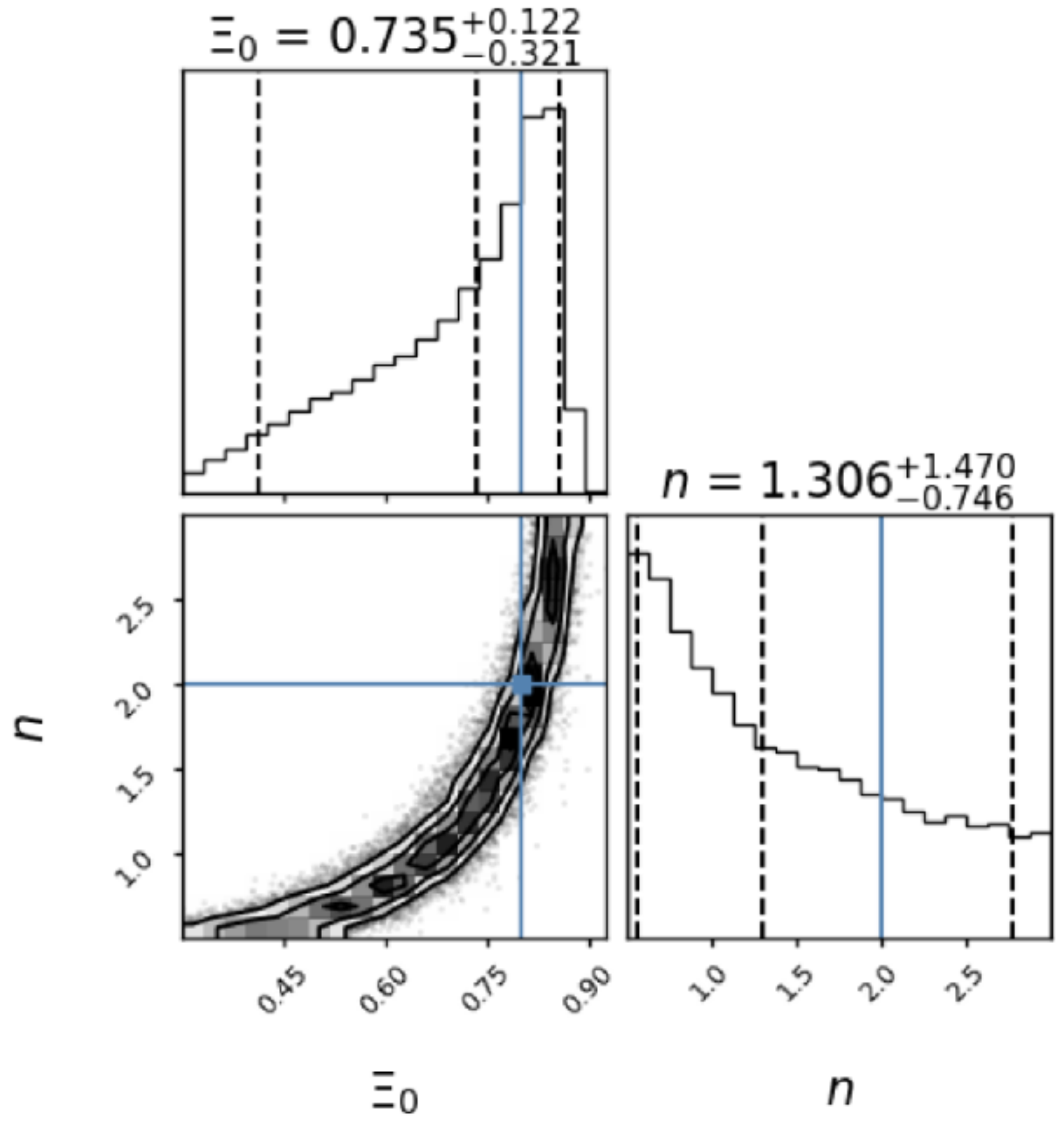
Gravity is modified by the addition of a nonlocal term

$$\Gamma_{\rm RR} = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{1}{6} m^2 R \frac{1}{\Box^2} R \right]$$

Other applicable theories: Horndeski, DHOST theories, RT non-local gravity model ($\Xi_0^{\rm max} = 1.8$, $n = 1.91$)

[Finke et al. 2021]

Explanation of the correlation



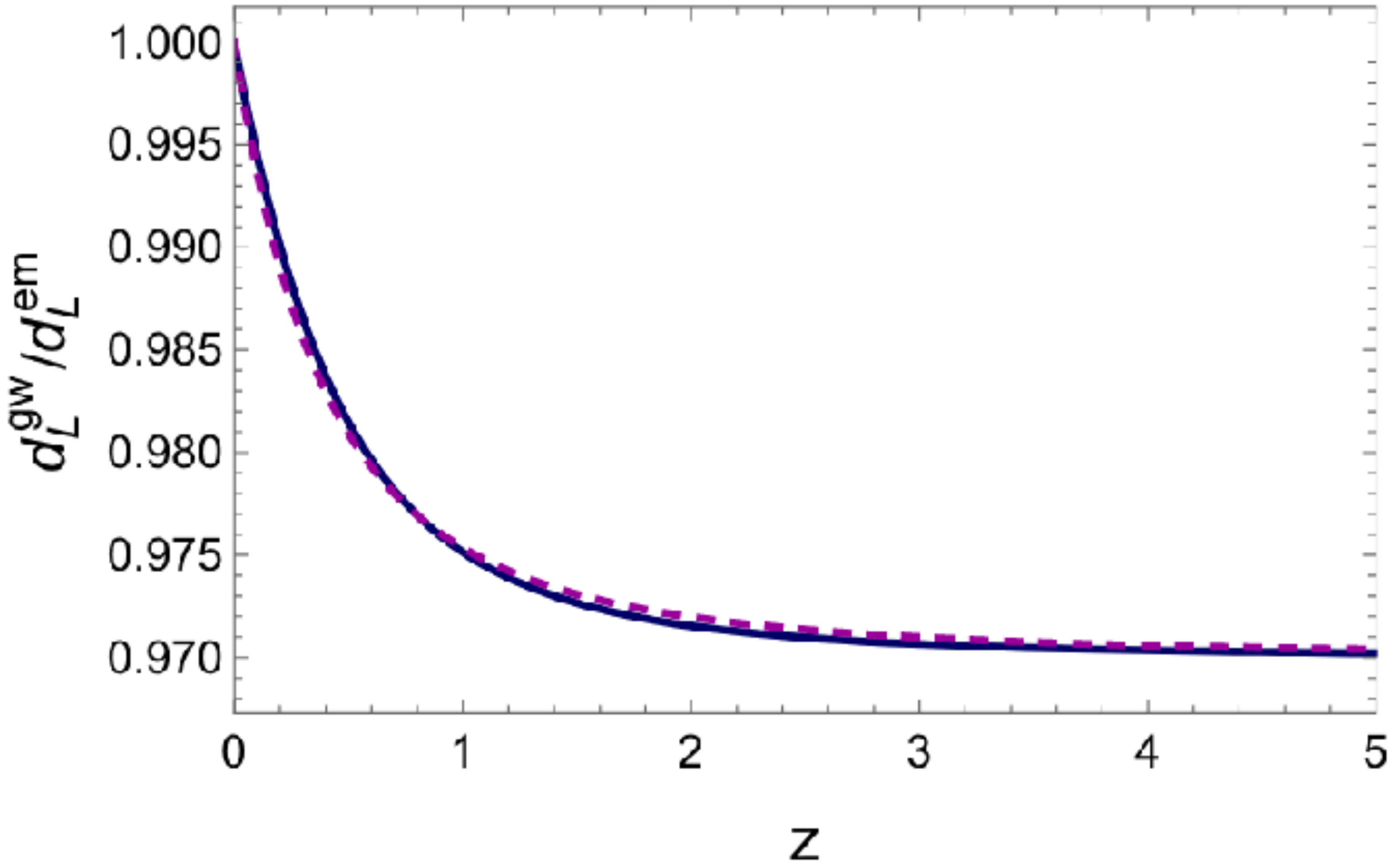
at lower redshift, expand the formula:

$$\frac{d_L^{\rm gw}(z)}{d_L^{\rm em}(z)} = 1 - z\delta(0) + \mathcal{O}(z^2)\,,$$

at very low redshift we are actually sensitive to $\delta(0) \equiv \delta(z=0)$

$$\delta(z) = \frac{n(1-\Xi_0)}{1-\Xi_0+\Xi_0(1+z)^n}$$

$$\delta(0) = n(1-\Xi_0)$$



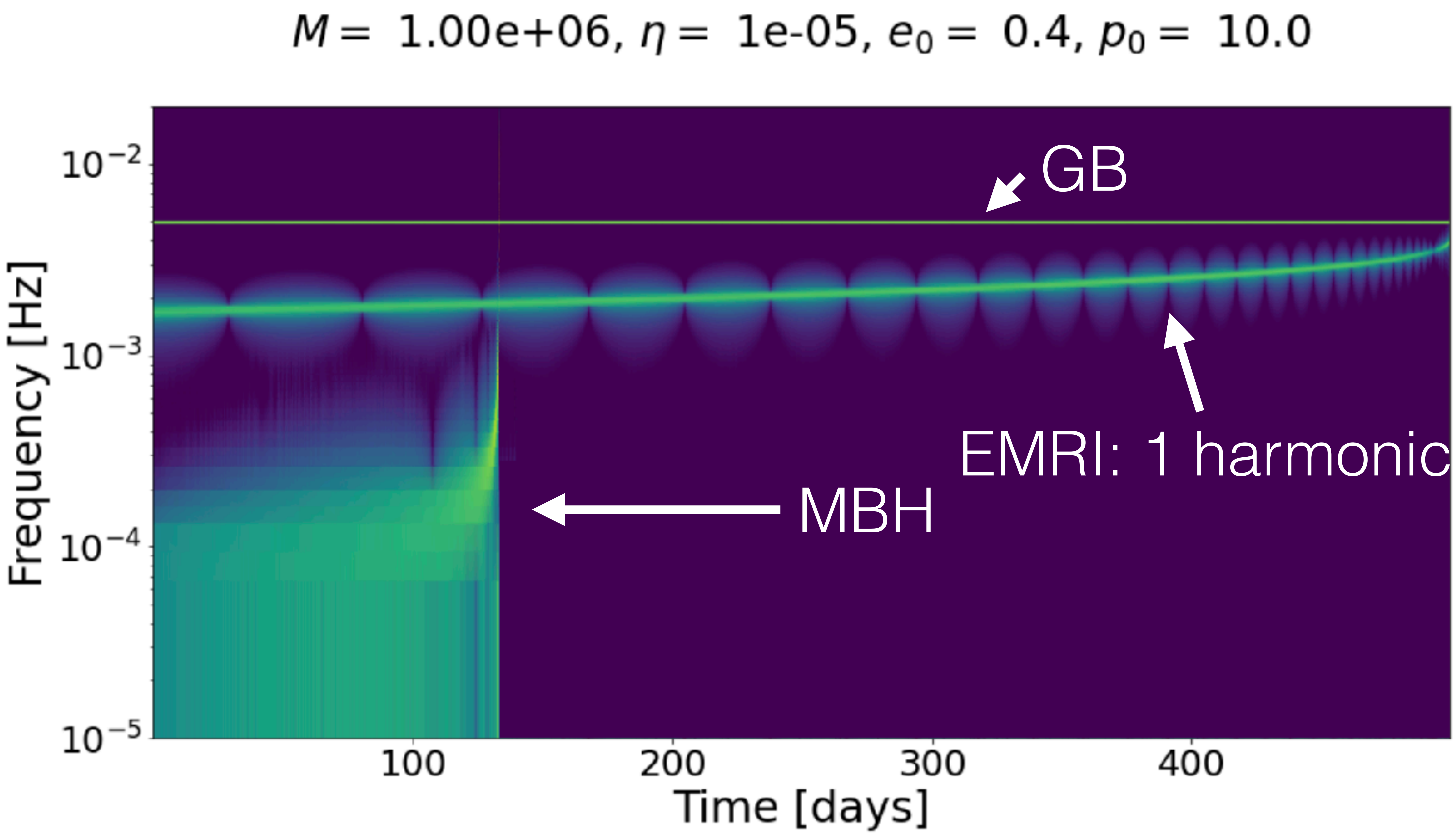
EMRI model

Model	Mass function	MBH spin	Cusp erosion	M–σ relation	$N_{\rm p}$	CO mass $[M_\odot]$
M1	Barausse12	a98	yes	Gultekin09	10	10
M2	Barausse12	a98	yes	KormendyHo13	10	10
M3	Barausse12	a98	yes	GrahamScott13	10	10
M4	Barausse12	a98	yes	Gultekin09	10	30
M5	Gair10	a98	no	Gultekin09	10	10
M6	Barausse12	a98	no	Gultekin09	10	10
M7	Barausse12	a98	yes	Gultekin09	0	10
M8	Barausse12	a98	yes	Gultekin09	100	10
M9	Barausse12	aflat	yes	Gultekin09	10	10
M10	Barausse12	a0	yes	Gultekin09	10	10
M11	Gair10	a0	no	Gultekin09	100	10
M12	Barausse12	a98	no	Gultekin09	0	10

fiducial

[Babak et al. 2017]

Detection of a single EMRI



Credit: Ollie Burke

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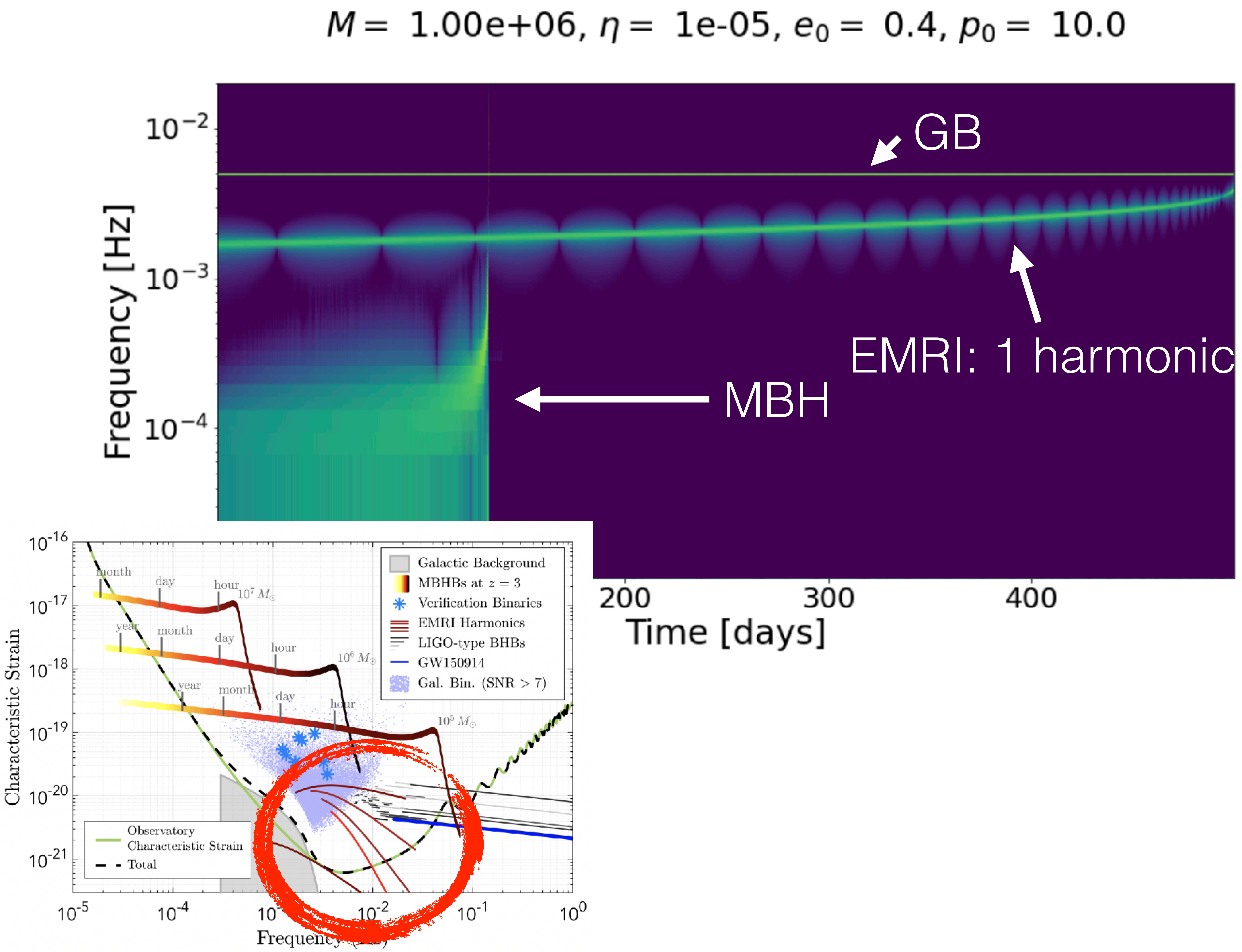

Credit: Ollie Burke

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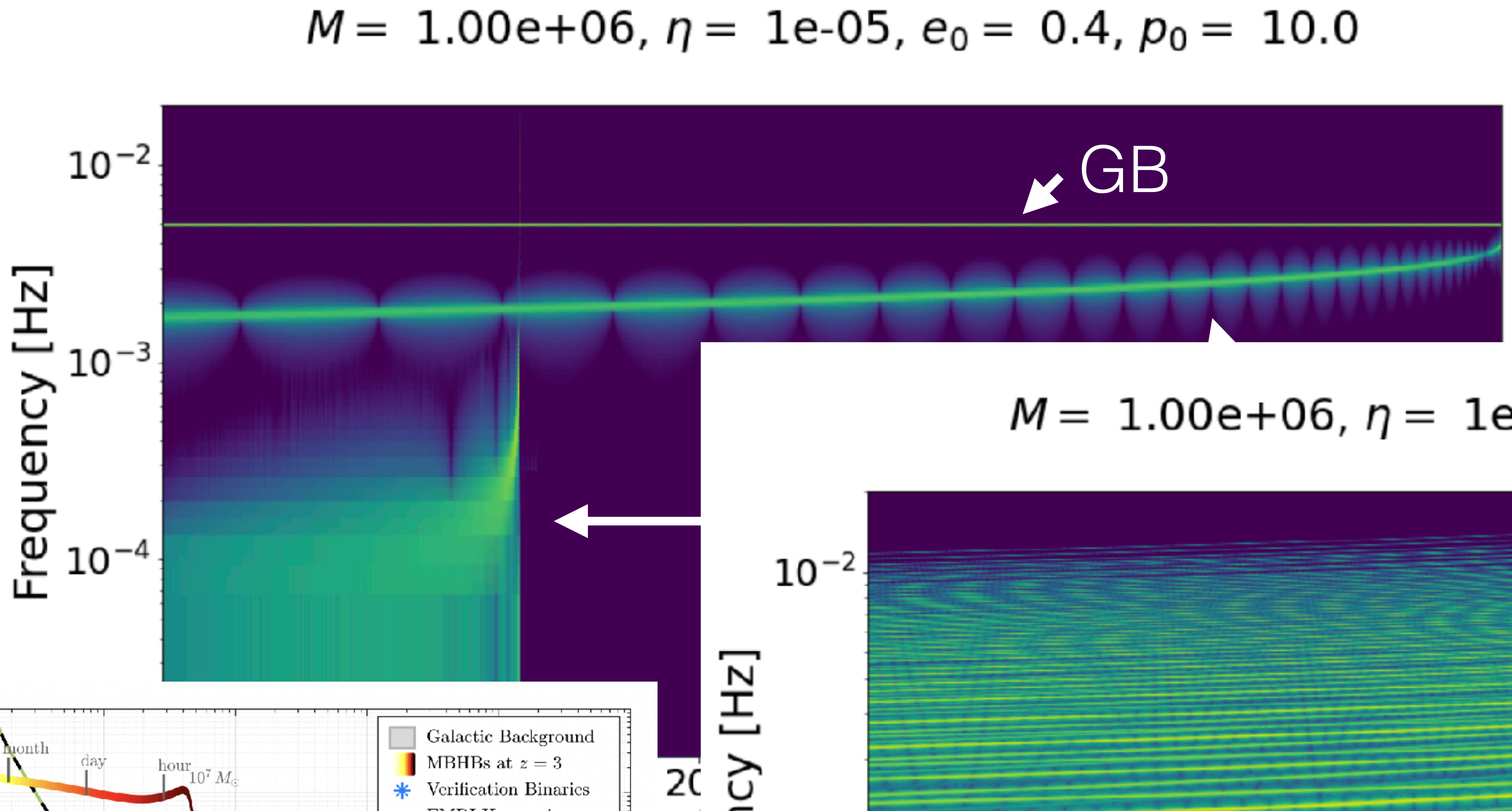

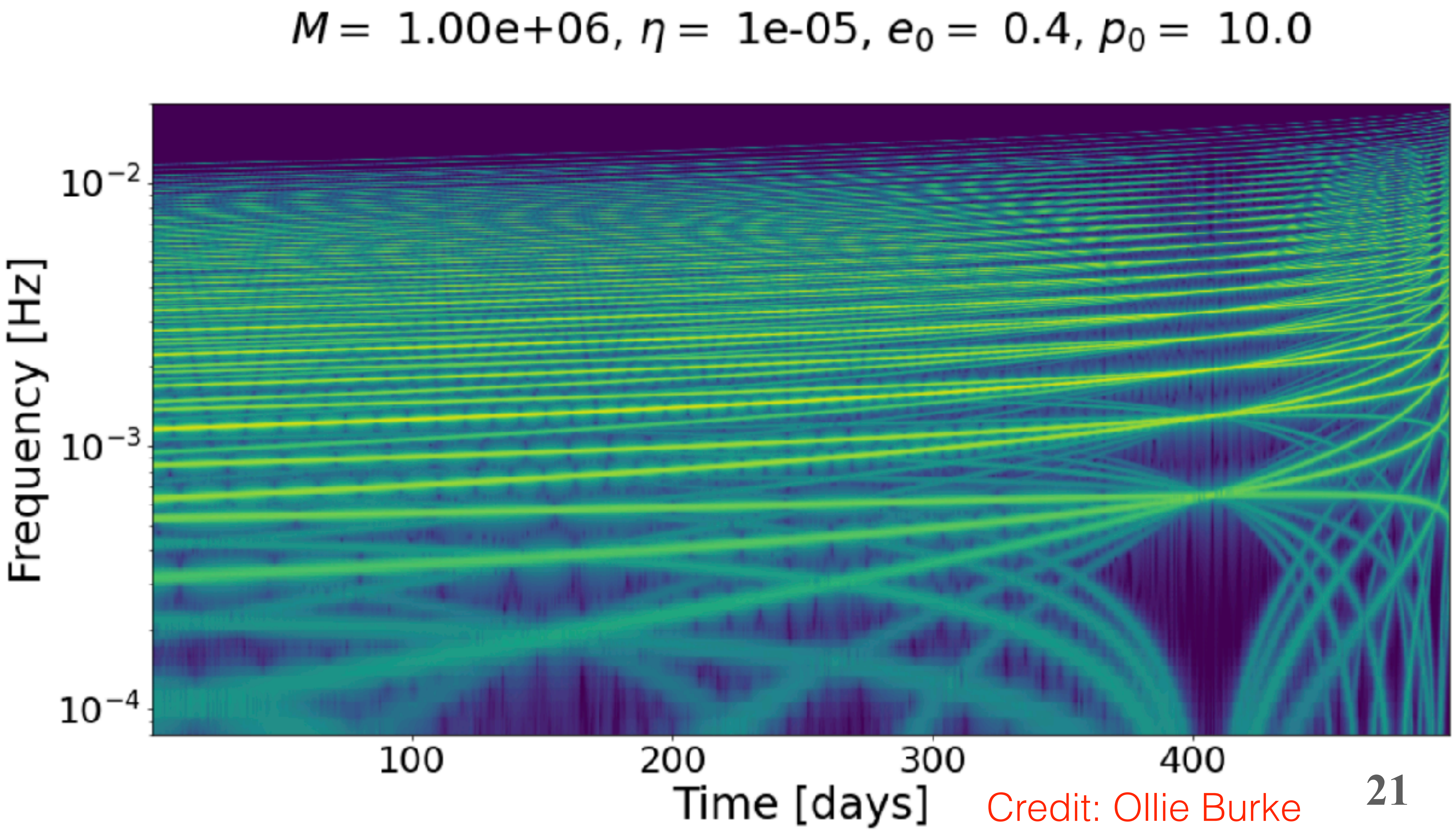

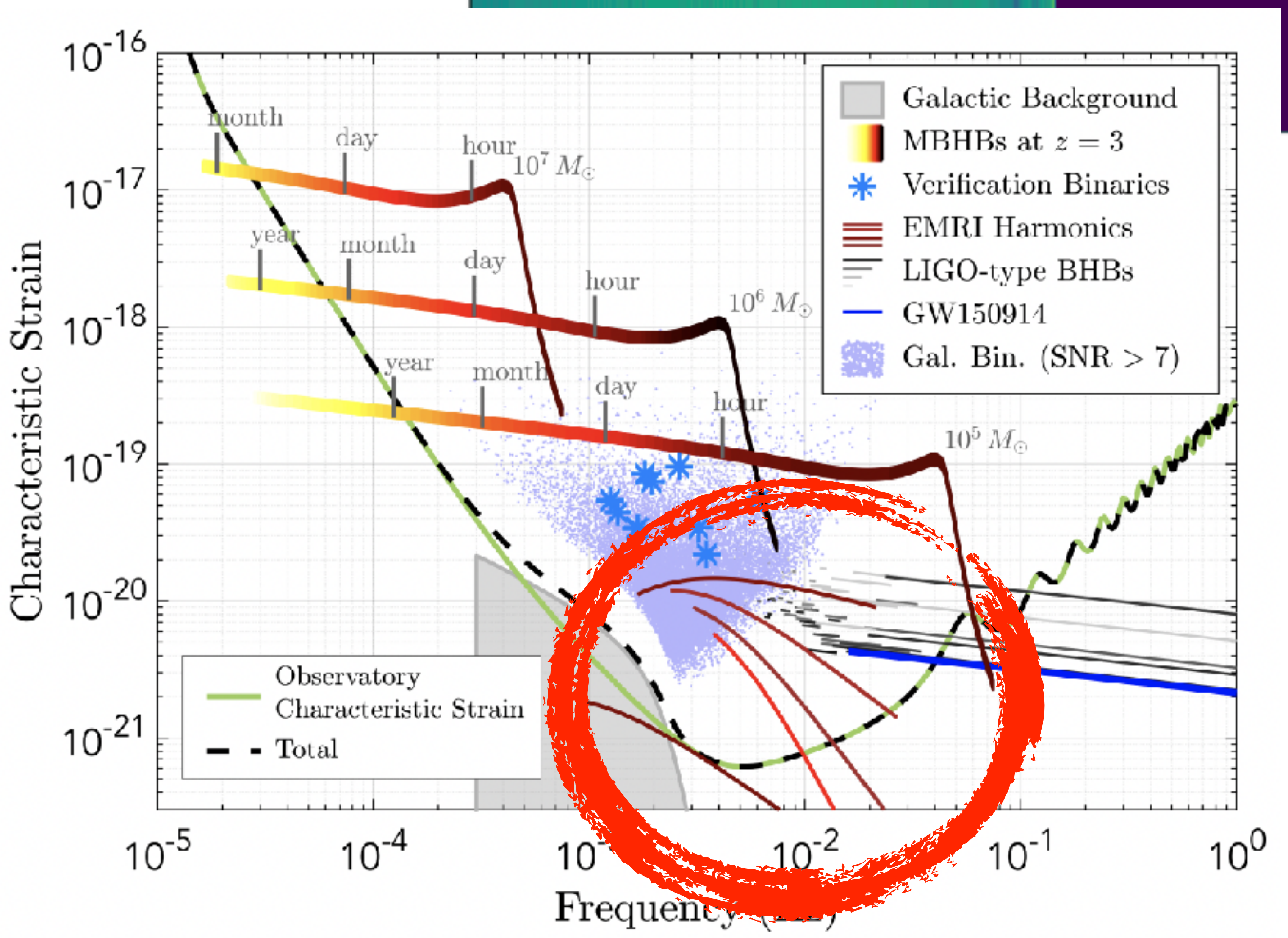

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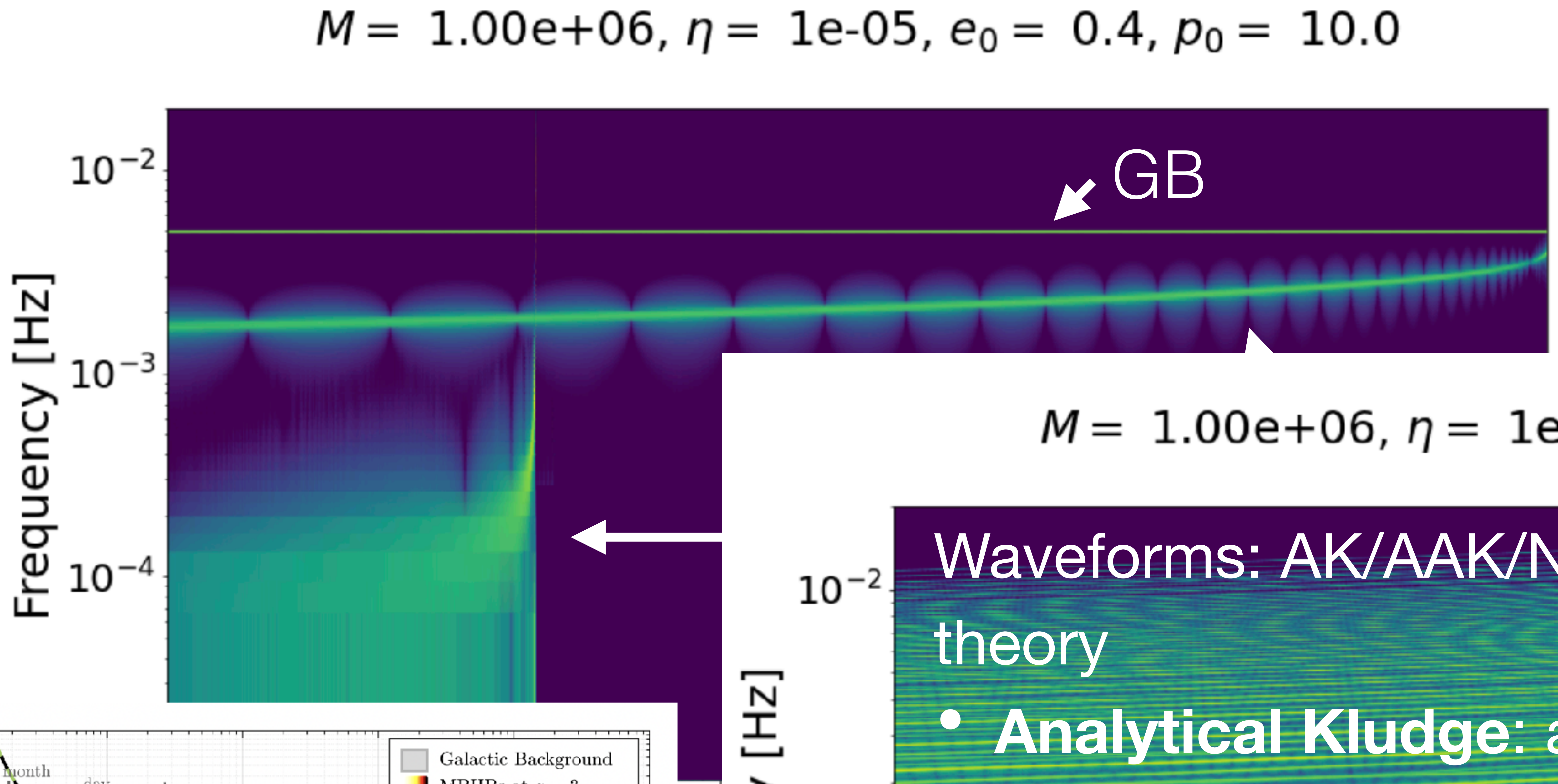

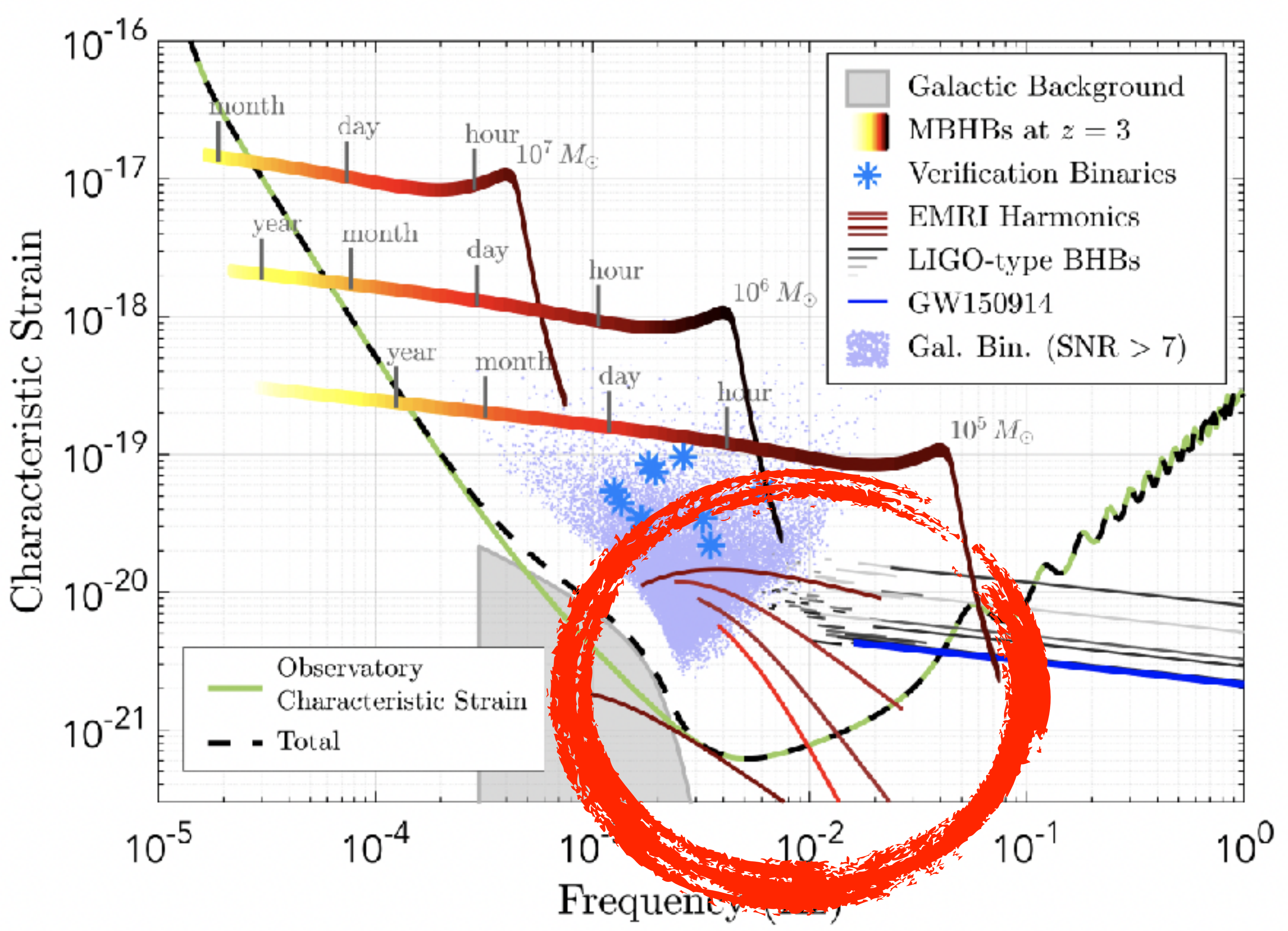

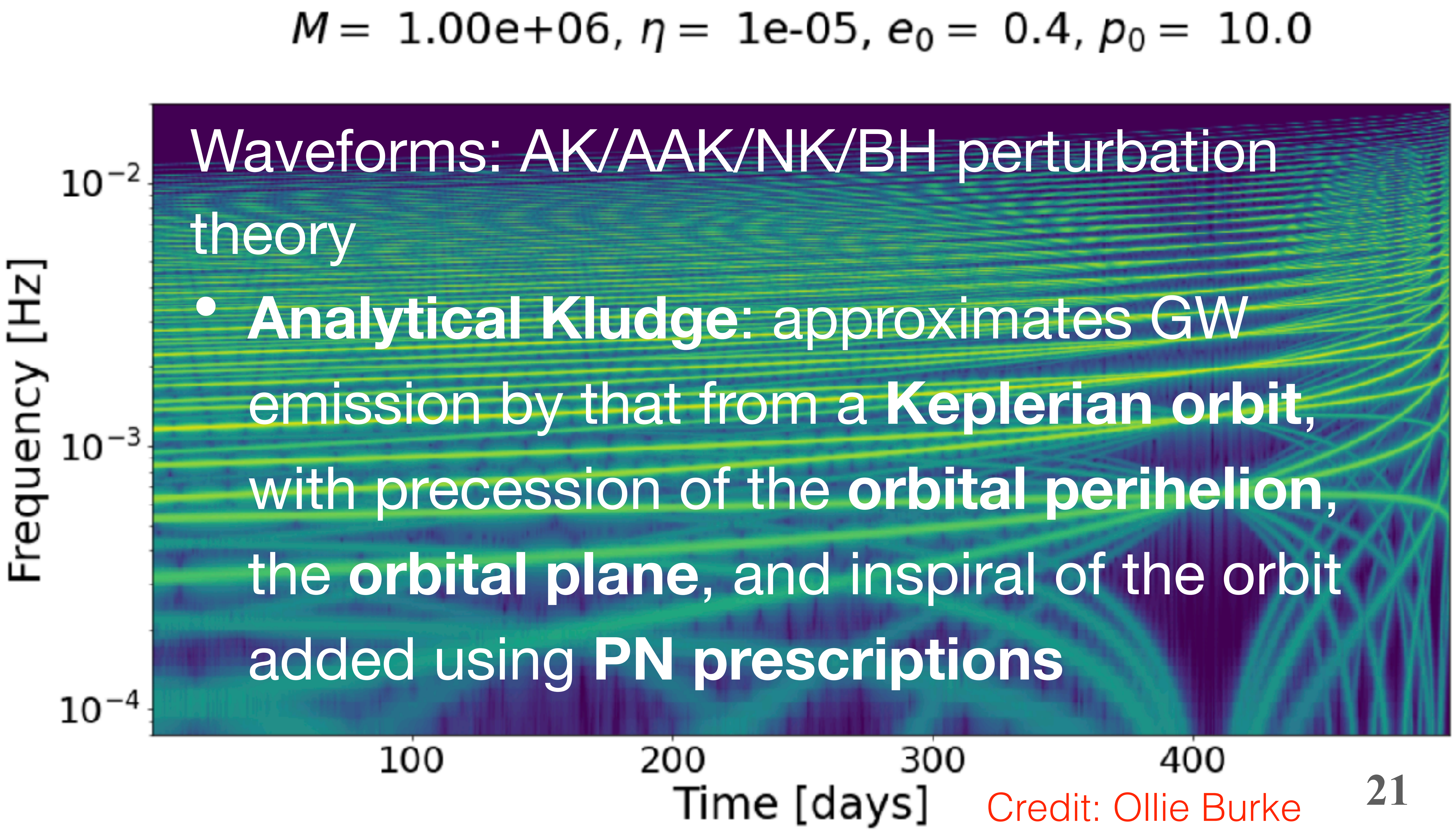

Waveforms: AK/AAK/NK/BH perturbation theory

- **Analytical Kludge**: approximates GW emission by that from a **Keplerian orbit**, with precession of the **orbital perihelion**, the **orbital plane**, and inspiral of the orbit added using **PN prescriptions**

Credit: Ollie Burke