# Gravitational wave production in higher derivative gravity


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Based on PLB 2211.02070 (A. Koshelev, A. Starobinsky, AT)
and ongoing work

# Higher derivative gravity – does it make sense?

$R^2_{\mu\nu\lambda\rho}$, $R_{\mu\nu\lambda\rho}\Box R^{\mu\nu\lambda\rho}$, $R_{\mu\nu}\Box R^{\mu\nu}$, ... – ghosts and instabilities

Two approaches:

1) low energy EFT – ghosts live beyond the cutoff scale

For cosmology: there's no renormalizable theory of inflation
$\Rightarrow$ higher derivatives must be there

For GWs: extra graviton production

2) Infinite derivative gravity can be ghost-free

$$S = \int d^4x \sqrt{-g}\left(-\frac{M_P^2}{2}R + R\,F(\Box)R + W_{\mu\nu\lambda\rho}F_W(\Box)W^{\mu\nu\lambda\rho}\right)$$

For cosmology: Starobinsky inflaton is an exact solution
$F, F_W \rightarrow$ modification of $r$, non-gaussianities

A. Koshelev, S. Kumar, A. Starobinsky, '22

For GWs: extra graviton production after inflation
only $W\Box W$ term is relevant

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

# Inflaton decay to gravitons

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) + \frac{\varphi}{\Lambda} W_{\mu\nu\rho\lambda} W^{\mu\nu\rho\lambda} + \dots$$

$\varphi \to hh$, $\Gamma_{GW} \sim \frac{m^7}{\Lambda^2 M_p^4}$ – decay rate after inflation

Observables:

GW signal

$$\frac{d\Omega_{GW}}{d\log k} = \frac{16 k^4}{m^4} \frac{\rho_{reh}}{\rho_0} \frac{\Gamma_{GW}}{\Gamma_{reh}} \frac{e^{-\delta(k)}}{\delta(k)}$$

$$\delta(k) = \left( \left(\frac{g_{reh}}{g_0}\right)^{1/3} \frac{T_{reh}}{T_0} \frac{2k}{m} \right)^{3/2}$$

$$\frac{d\Omega_{GW}}{d\log k} \sim k^{5/2}$$

GW contribution to dark radiation

$$\Delta N_{eff} = 2{,}85 \frac{\Gamma_{GW}}{\Gamma_{reh}} < 0{,}2 \text{ (Planck)}$$

significant effect for low reheating temperatures

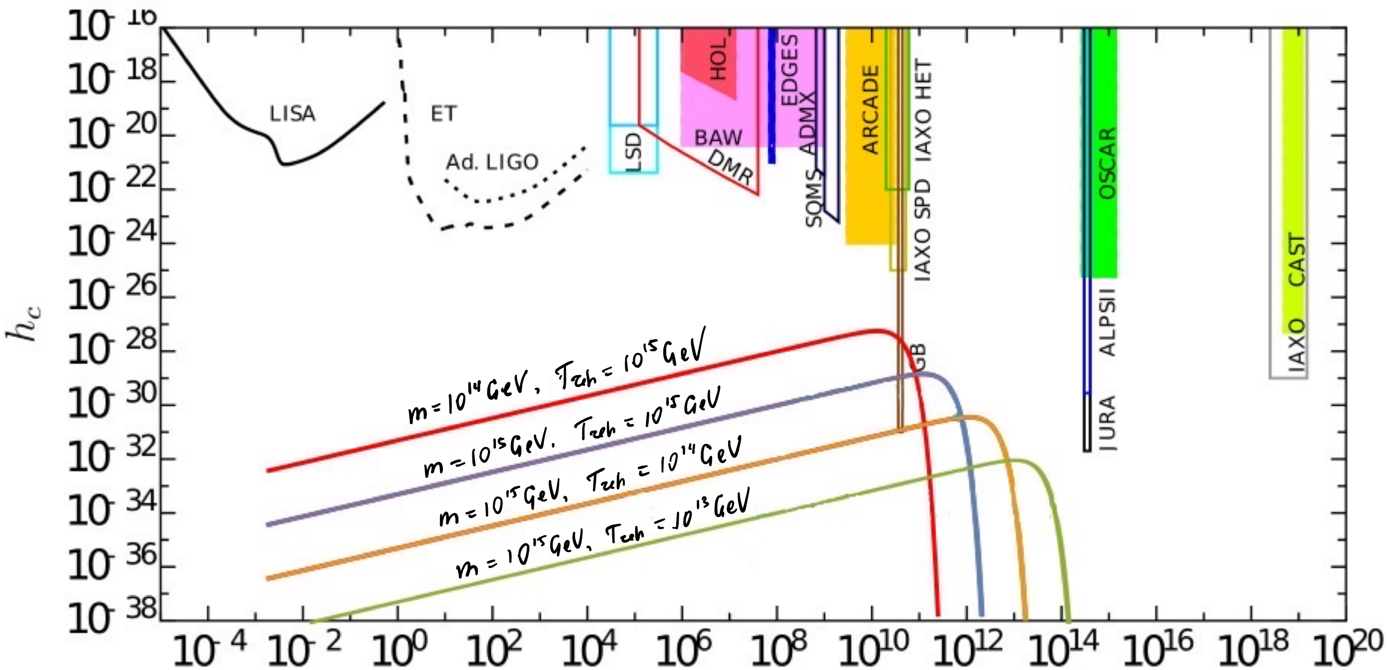
# Some results

* Planck-suppressed operators **do matter** if $T_{reh}$ is low

If $T_{reh} \lesssim \frac{m^{7/2}}{M_p^{3/2}\Lambda} \to$ overproduction $(\Delta N_{eff} > 0{,}2)$

$$\boxed{\begin{array}{l} m = 10^{13}\,\mathrm{GeV} \to T_{reh} = 1\,\mathrm{GeV} \\ m = 10^{16}\,\mathrm{GeV} \to T_{reh} = 10^{10}\,\mathrm{GeV} \end{array}}$$

* High frequency GW signal for $\Delta N_{eff} < 0{,}2$ and $\Lambda > 10^8\,\mathrm{GeV}$




G. Franciolini, A. Maharana, F. Muia, arxiv:2205.02153

# UV completion to the Starobinsky model

$$S = -\frac{M_p^2}{2}\int\sqrt{-g}\,d^4x\left(R - \frac{R^2}{6M^2}\right) + \boxed{\gamma\, W_{\mu\nu\lambda\rho}W^{\mu\nu\lambda\rho}}$$

makes the theo renormalizable

K. S. Stelle, Phys. Rev. D **16** (1977), 953-969

**Problem:** spin 2 ghost in the spectrum

**Possible solution:** infinite derivative gravity

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_p^2}{2}R + R\,F(\Box)\,R + W_{\mu\nu\lambda\rho}\,F_W(\Box)\,W^{\mu\nu\lambda\rho}\right)$$

Possible relation to asymptotic safety proposal

M.Reuter, F. Saueressig, '19
A. Platania, 2206.04072
B.Knorr, C. Ripken, F. Saueressig, 2210.16072

**How does it work?**

# Non-local scalar field

$$S = \int d^4x \sqrt{-g} \left( \varphi F(\Box) \varphi - V(\varphi) \right)$$

$F(\Box) = (\Box - m^2) e^{\sigma(\Box)}$, $\sigma(\Box)$ - entire function

no new degrees of freedom except the scalar with mass $m$

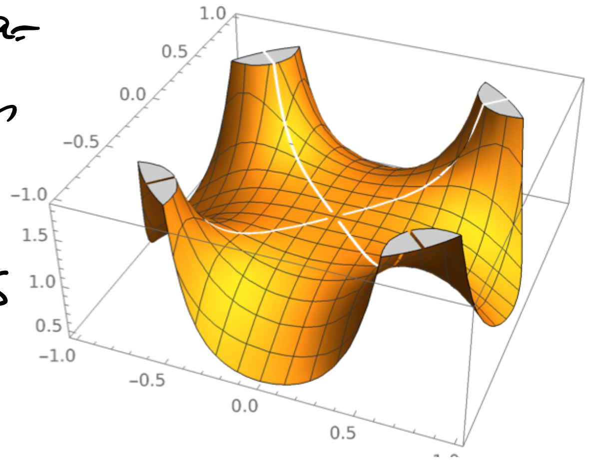
The propagator: $P(k^2) = \dfrac{i e^{-\sigma(k^2)}}{k^2 - m^2}$

If $P(k^2)$ is falling for large $k^2$ loops are convergent for any $V(\varphi)$

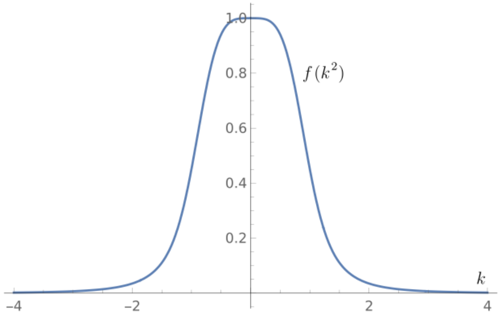
model is UV finite

# Can gravity be renormalizable?

$\boxed{\text{Renormalizable + ghost-free = non-local infinite-derivative}}$




* Power-counting arguments imply that if the propagator falls fast enough than all diagrams from 2 loops are convergent
* One-loop divergencies are local and the same as in the Einstein theory
* The propagator can be decaying as a power of momentum for both real and euclidian $k^2$ – but there are growth directions on the complex plane
* Amplitudes have the same analytic properties as in local theories except essential singularity at infinity





E. T. Tomboulis, 'Superrenormalizable gauge and gravitational theories," [arXiv:hep-th/9702146 [hep-th]]

A. S. Koshelev, L. Modesto, L. Rachwal and A. A. Starobinsky, 'Occurrence of exact $R^2$ inflation in non-local UV-complete gravity," JHEP **11** (2016), 067 [arXiv:1604.03127 [hep-th]]

A. S. Koshelev, S.Kumar, A. A. Starobinsky '$R^2$ inflation to probe non-perturbative quantum gravity", JHEP 03 (2018) 071

# Starobinsky solution in non-local gravity

The solution satisfying $\Box R = M^2 R$ appears to be an exact solution for

$$L = R F(\Box) R + W_{\mu\nu\lambda\sigma} F_W(\Box) W^{\mu\nu\lambda\sigma}$$

if $F'(M^2) = 0$

A. S. Koshelev, L. Modesto, L. Rachwal and A. A. Starobinsky, JHEP **11** (2016), 067 [arXiv:1604.03127 [hep-th]]

Degrees of freedom:

- massless graviton
- scalar with mass $M$

No ghosts $\to$ connection between $F$ and $F_W$

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

# Ghost-free conditions

$$\boxed{\begin{aligned} F(\Box) &= \frac{M_p^2}{6M^2\Box}\left((\Box - M^2)e^{\sigma(\Box)} + M^2\right) \\ F_W(\Box) &= M_p^2\,\frac{e^{\sigma(\Box)}-1}{2\Box} \end{aligned}}$$

The theory is parametrized by one function $\sigma(\Box)$

$\sigma(\Box) = \sigma\left(\frac{\Box}{\Lambda^2}\right)$, $\Lambda$ - non-locality scale

$M < \Lambda < M_p$

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

# Graviton production from higher derivatives

Main source: $\varphi \to hh$ (perturbative decay)

$W \Box W$, $RW^2$ – the only terms which contribute to the graviton production

$\Box h^{TT}_{ij} = 0 \longrightarrow$ leads to cancellations

$$\mathcal{L} = A\, W \Box W + B\, R W^2$$

$$\boxed{\Gamma = \frac{6}{\pi} \frac{M^{11}}{M_p^6} (A + 2B)^2}$$

$$\Gamma = \frac{3}{2\pi} \alpha_1^2 \frac{M^3}{M_p^2} \left(\frac{M}{\Lambda}\right)^8, \quad \alpha_1 \sim 1$$

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

# Gravitational wave signal

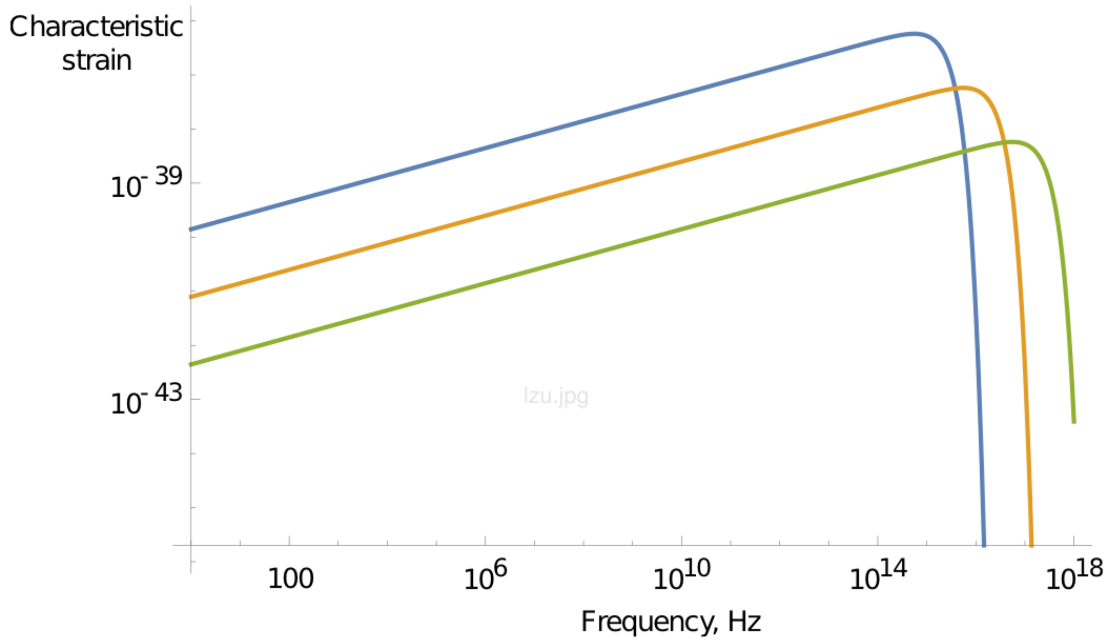



Figure 1: The blue curve shows the gravitational wave signal for $T_{reh} = 10^{10}$ GeV. The orange curve is for $T_{reh} = 10^{9}$ GeV and the green curve is plotted for $T_{reh} = 10^{8}$ GeV. In all cases we assumed $\Gamma_{GW}/\Gamma_{SM} = 10^{-3}$. The lowest characteristic strain available for future gravitational wave detectors is $10^{-24}$ for the frequencies $1 - 10^{6}$ Hz.

GW dark radiation:

$$\Delta N_{eff} = 2{,}85 \frac{\rho_{GW}}{\rho_H} = 2{,}85 \frac{\Gamma_{GW}}{\Gamma_H} = 821\, \alpha_1^2 \frac{M^8}{\Lambda^8}$$

Bound on $\Lambda$: $\Lambda \gtrsim 3M$

A. Koshelev, A. Starobinsky, AT, arXiv: 2211.02070

# Conclusions

* High frequency GWs can be sensitive to the quantum gravity effects
* Perturbative decay of inflaton to gravitons can be non-negligible for low reheating temperatures → high frequency GWs & $\Delta N_{eff}$
* Non-local gravity can be renormalizable and ghost-free UV-completion of Starobinsky $R^2$ inflation → non-perturbative GW production – for future work
* Constraints on quantum gravity scale!

Thank you

for your attention!

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