The Multipole Expansion of the Expansion Rate in the Local Universe

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Progress on Old and New Themes in cosmology (PONT) 04/05/2023

Introduction

The Standard Model of Cosmology

- The standard cosmological model relies on the theory of general relativity, with the cosmological principle (CP).

- In the standard model of cosmology, all the cosmological parameters (density parameters Ω, Hubble parameter H …) depend only on time.

- Hubble constant tension problem is one of the main problems in standard cosmology.

In the local universe:

$$H_0 = 73.2 \pm 1.3\, km/s/Mpc$$

(Riess et al. 2021)

By CMB:

$$H_0 = 67.4 \pm 0.5\, km/s/Mpc$$

(Planck Collaboration et al. 2020)

Introduction

- ❑ One of the possible solution for Hubble constant tension is that the local universe is not homogeneous and isotropic.

- ❑ Traditionally, deviations from the cosmological principle (CP) predictions are treated perturbatively by expanding the cosmological parameters into a smooth background component and a fluctuating part.

- ❑ We developed a completely non-perturbative and model-independent approach to investigate the local inhomogeneities.

The Expansion Rate Fluctuations

- An observable that captures deviations from the linear redshift-distance relation is the *expansion rate fluctuation* (natural units c = 1),

$$\eta \equiv \log \frac{z}{H_0 d_L}$$

- It is linear in the distance modulus

$$\hat{\eta} = \log\left[\frac{z}{H_0}\right] + 5 - \frac{\mu}{5}$$

- If we ignore the error in the redshift, then $\hat{\eta}$ is Gaussian and unbiased estimator.

The Expansion Rate Fluctuations

- If the observer is comoving with matter flow (≈z_{LG}), then $\hat{\eta}$ will converge to $\log(\mathfrak{H}_0/H_0)$ (for $z \longrightarrow 0$), where $\mathfrak{H}_0$ is the effective Hubble parameter (Kristian & Sachs 1966, Clarkson & Maartens 2010. Heinesen 2021):

$$\mathfrak{H}_0 = n^\mu n^\nu \nabla_\mu u_\nu|_0 = \frac{1}{3}\theta|_0 + n^\mu n^\nu \sigma_{\mu\nu}|_0$$

The line of sight vector

The four velocity of matter particles

monopole

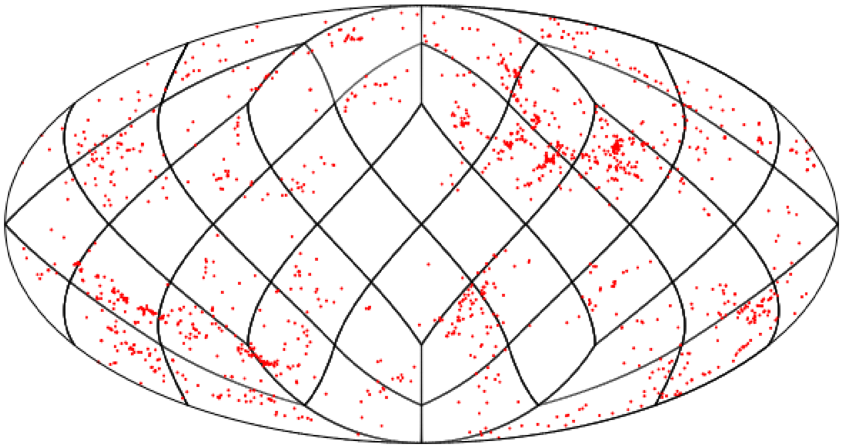
quadrupole

- However, if we choose an observer that does not see the CMB dipole (z_{CMB} in FRW background), then η provides insights into the radial component of peculiar velocities(if v<<1)

$$\eta \approx \frac{v(1+z)}{z \ln 10}$$

The Expansion Rate Fluctuations

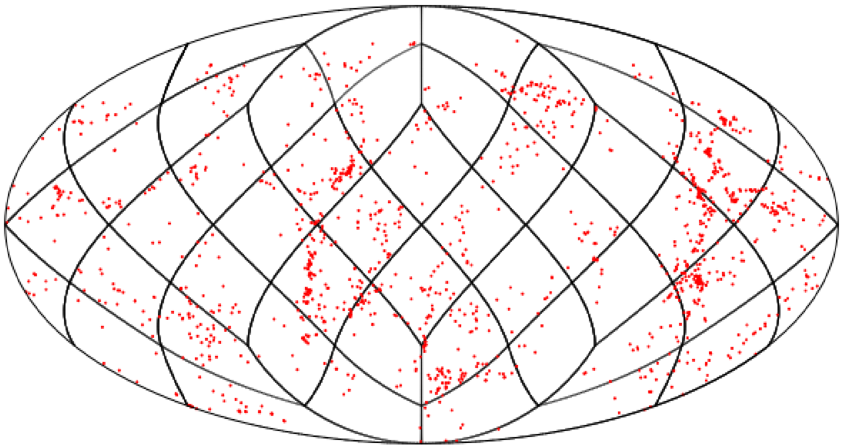
- By using **HEALPix** (Hierarchical Equal Area isoLatitude Pixelisation of a 2-sphere).



Number of pixels = 48

The Expansion Rate Fluctuations

- By using **HEALPix** (Hierarchical Equal Area isoLatitude Pixelisation of a 2-sphere).
- Rotation to handle angular incompleteness.



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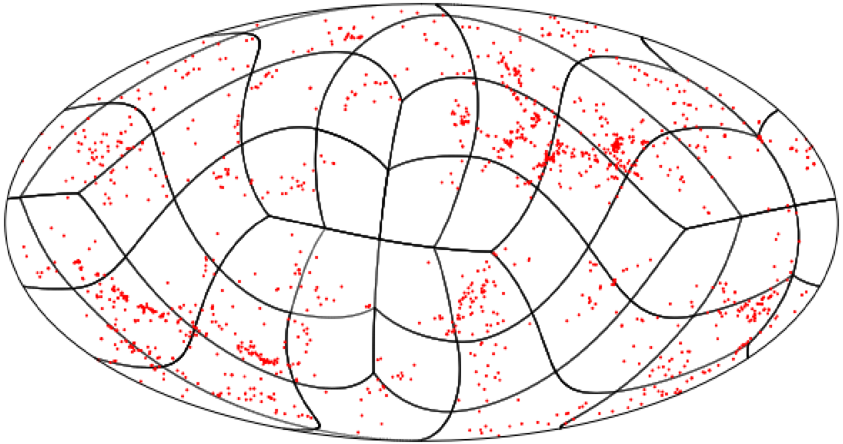

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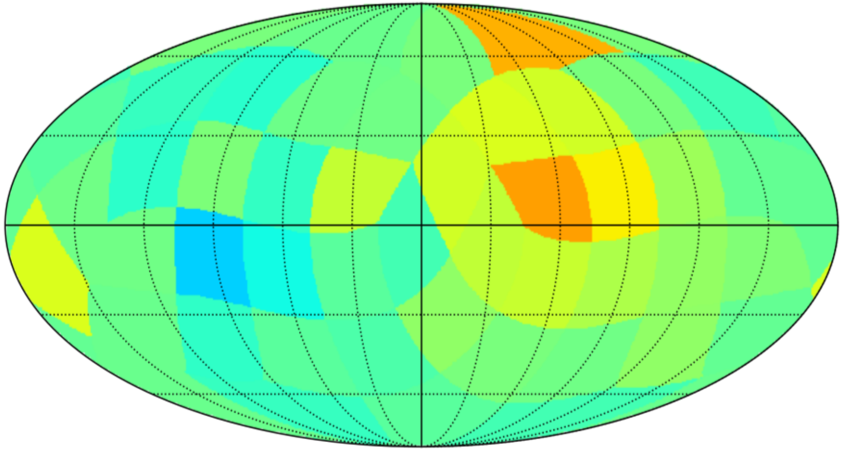


- Define a piece-wise function.

$$\eta = \left\langle \log \frac{z}{H_0 d_L} \right\rangle_\Omega$$

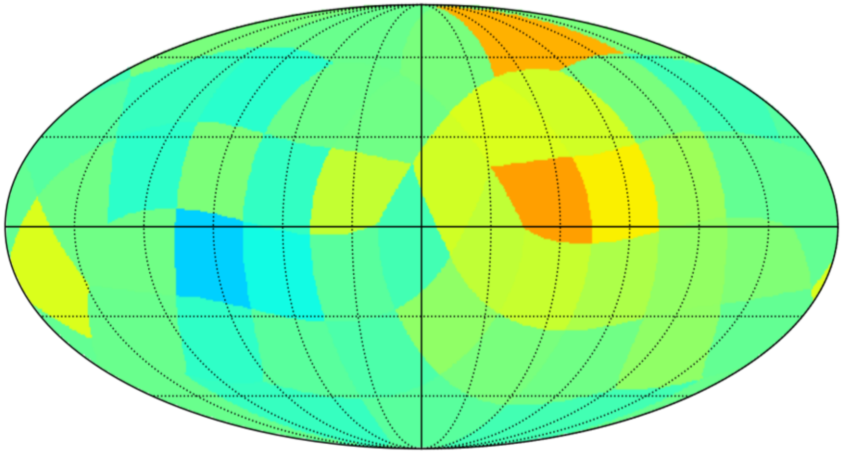
- It is then decomposed into the spherical harmonic basis

$$\eta = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

The Expansion Rate Fluctuations

- The spherical harmonic coefficients

$$a_{\ell m} \equiv \int_0^{2\pi} \int_0^{\pi} \eta(\theta, \phi) Y^*_{\ell m}(\theta, \phi) \sin\theta \, d\theta d\phi$$



- The zeroth order estimator is

$$\hat{a}^{(0)}_{\ell m} = \frac{4\pi}{N_{pix}} \sum_{p=1}^{N_{pix}} \eta(\Omega_p) Y^*_{\ell m}(\theta_p, \phi_p)$$

- The higher order estimator is

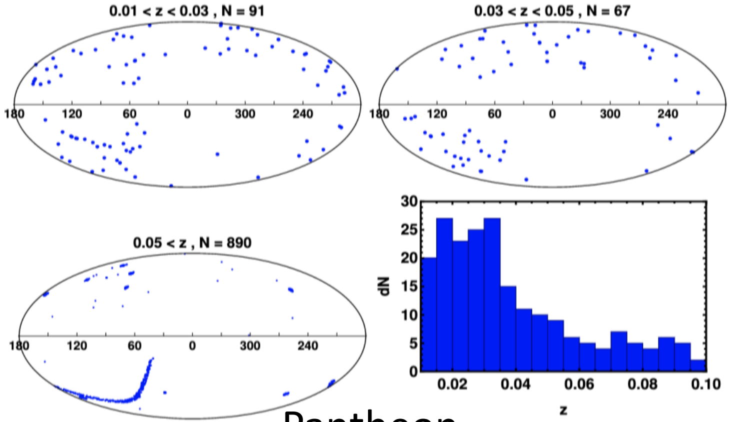
$$\hat{a}^{(k+1)}_{\ell m} = \hat{a}^{(k)}_{\ell m} + \frac{4\pi}{N_{pix}} \sum_{p=1}^{N_{pix}} \Big(\eta(\Omega_p) - \eta^{(k)}(\theta_p, \phi_p)\Big) Y^*_{\ell m}(\theta_p, \phi_p) \quad , \quad \eta^{(k)}(\theta_p, \phi_p) = \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} \hat{a}^{(k)}_{\ell m} Y_{\ell m}(\theta_p, \phi_p)$$

- The power spectrum $\hat{C}_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a^{(\infty)}_{\ell m}|^2$

Data

1) **Pantheon** SNIa sample (Scolnic et al. 2018)

- Almost isotropic for z < 0.05 with (158 objects).
- The typical error in the distance modulus ($\boldsymbol{\Delta\mu = 0.15}$).



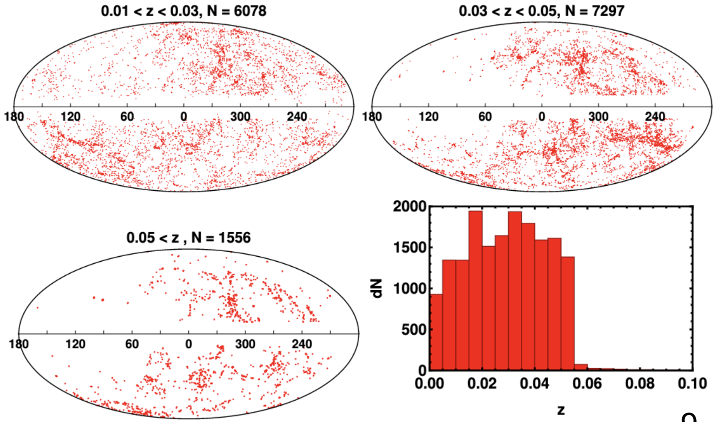
Pantheon

2) **Cosmic-Flows 3 (CF3)** galaxies (Tully et al. 2017)

- Almost isotropic for z < 0.05 with (13661 objects).
- We divided the sample into two independent subsamples:

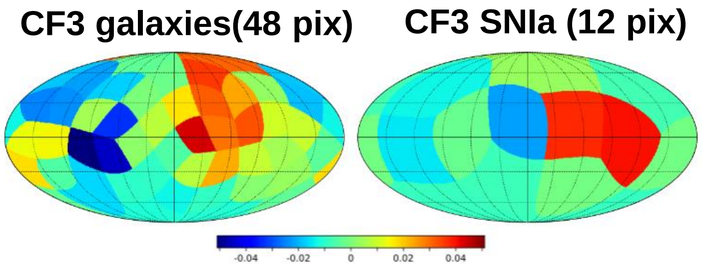
A) **CF3 SNIa**: 286 objects ($\boldsymbol{\Delta\mu = 0.18}$).

B) **CF3 galaxies**: 13375 objects ($\boldsymbol{\Delta\mu = 0.46}$).



Cosmic-Flows 3

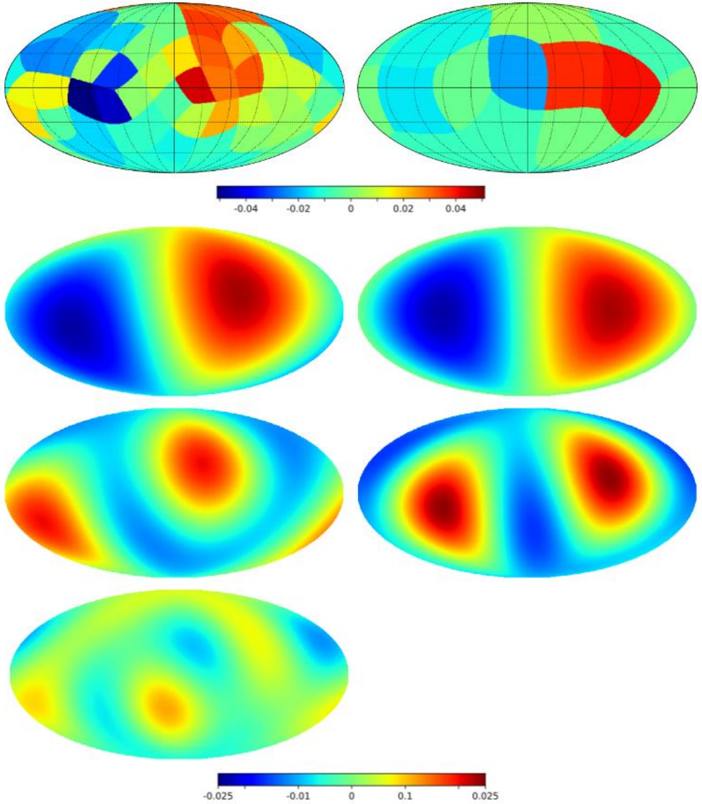
Results



η maps

Results

CF3 galaxies(48 pix) **CF3 SNIa (12 pix)**



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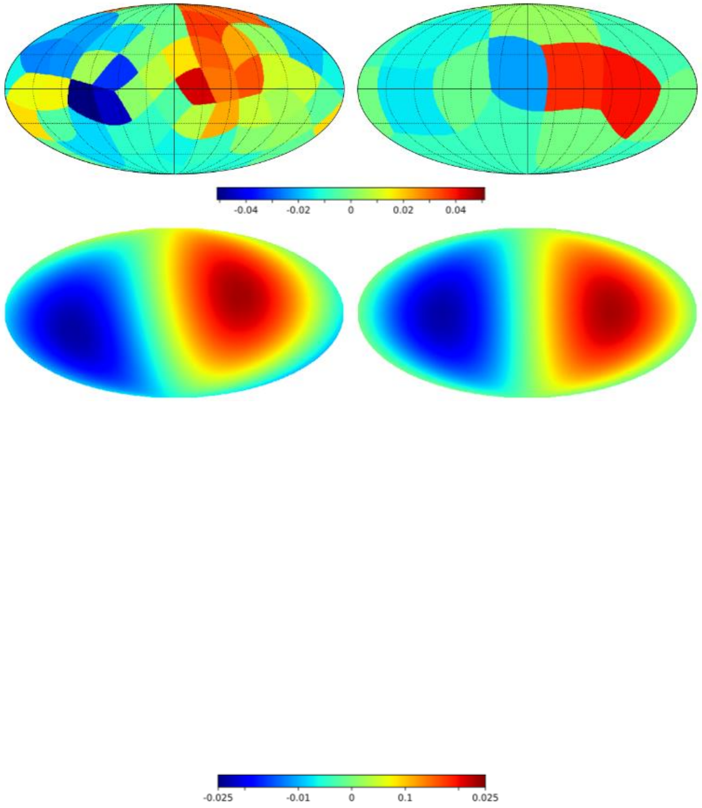
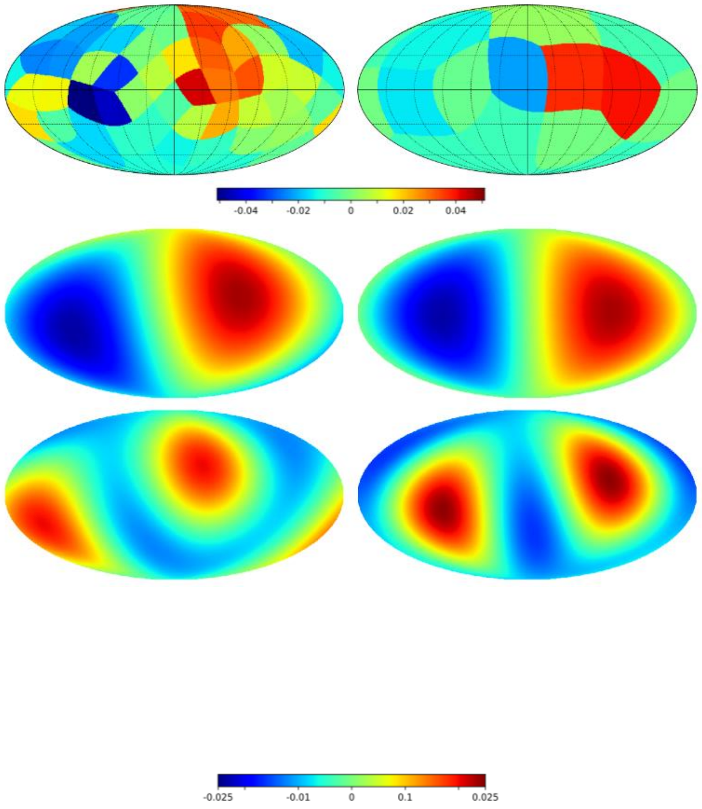


η maps

Same intensity and direction (CF3g, CF3sn and Pantheon).
Same direction as bulk component of local group motion.

Results

CF3 galaxies(48 pix) CF3 SNIa (12 pix)



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Quadrupole intensity is half that of the dipole
Alignement with the direction of the dipole
No quadrupole component in Pantheon data!

Results

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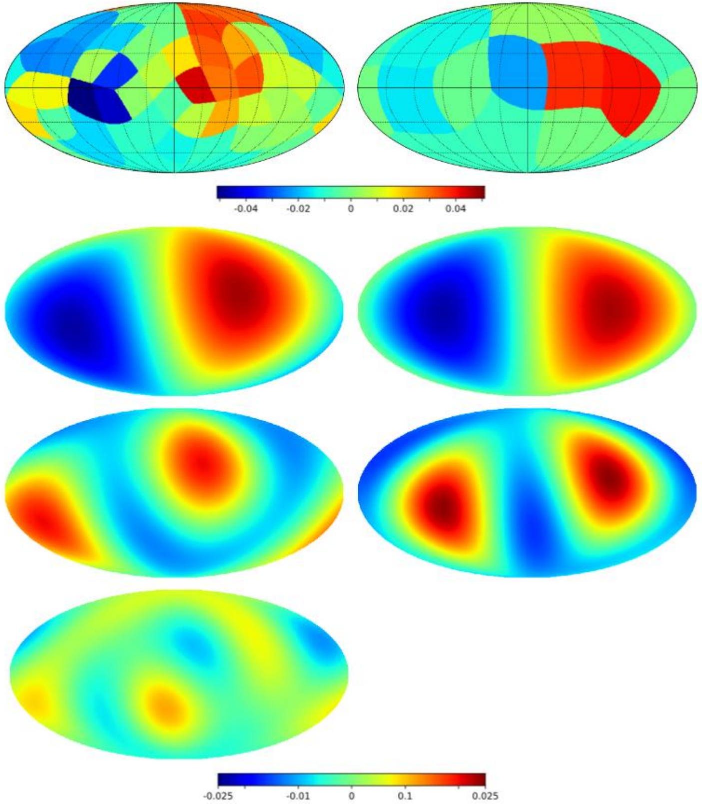

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Octupole: only for CF3 the S/N is acceptable (~3.1)

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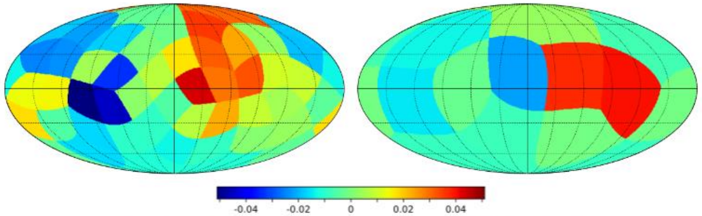
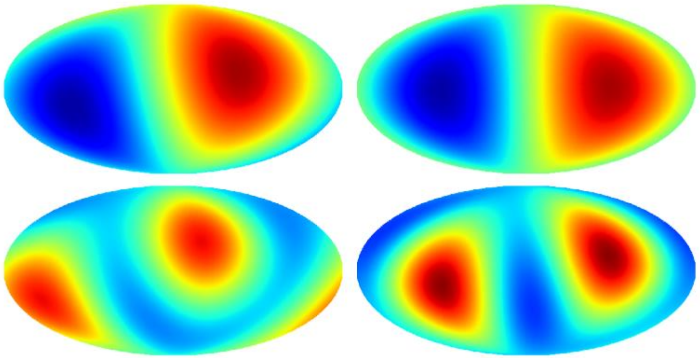


η maps



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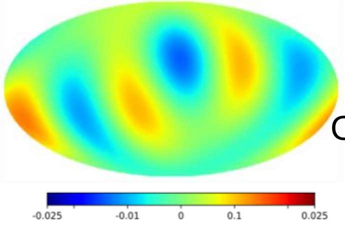
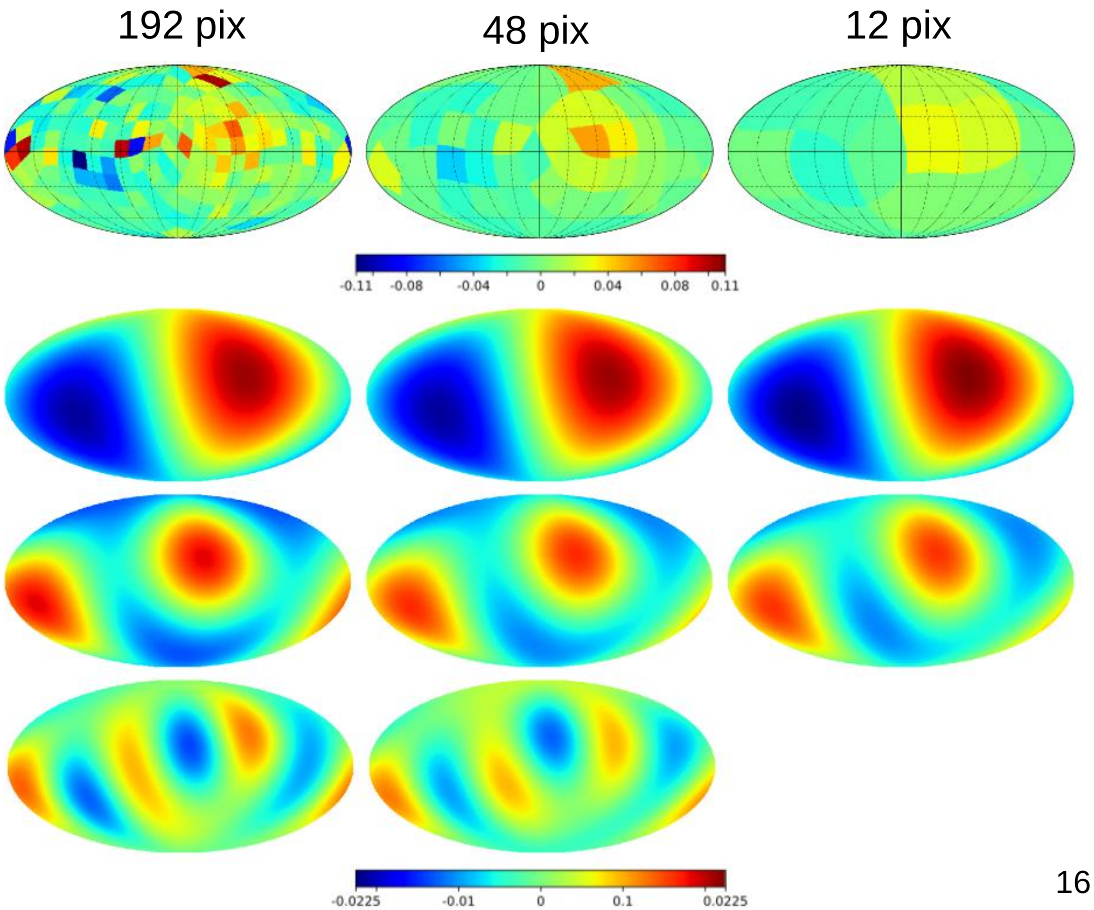


Octupole of full CF3 sample

Octopole: only for CF3 the S/N is acceptable (~3.1)

The Robustness of the Results

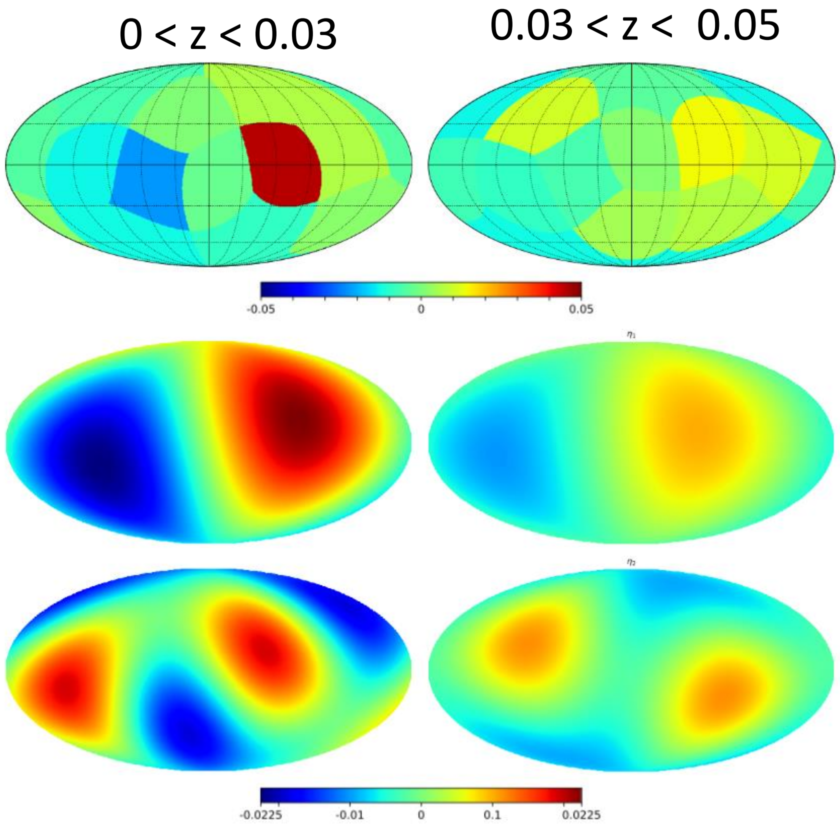
- Independent from pixelisation strategy.



The Robustness of the Results

- Independent from pixelisation strategy.

- Independent from the sample depth.



The Robustness of the Results

- Independent from pixelisation strategy.
- Independent from the sample depth.
- Consistent with results of fitting the data with the spherical harmonics.

	Fourier	Fit	Fourier	Fit	Fourier	Fit	Fourier	Fit
Sample	l_d		b_d		$C_1/10^4$		$C_2/10^4$	
CF3 [0.01, 0.05]	285 ± 5	287 ± 5	11 ± 4	10 ± 4	5.1 ± 0.8	4.3 ± 0.8	1 ± 0.3	1.2 ± 0.3
CF3g [0.01, 0.05]	296 ± 6	290 ± 5	18 ± 5	5 ± 4	4.0 ± 0.6	5.3 ± 0.9	1.3 ± 0.4	1.3 ± 0.3
CF3sn [0.01, 0.05]	322 ± 23	293 ± 14	−8 ± 18	0 ± 10	3.7 ± 1.5	5.5 ± 2.6	1.7 ± 1.4	1.8 ± 1.1
Pantheon [0.01, 0.05]	332 ± 39	312 ± 23	−9 ± 19	19 ± 26	3.9 ± 2.7	3.4 ± 3.2	0.5 ± 1.4	0.6 ± 1.0

Errors and Biases

- The errors and biases are computed by 1000 Monte Carlo simulations.

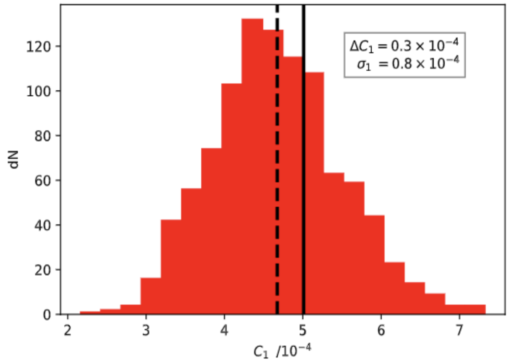
- Variance of the power spectrum coefficients

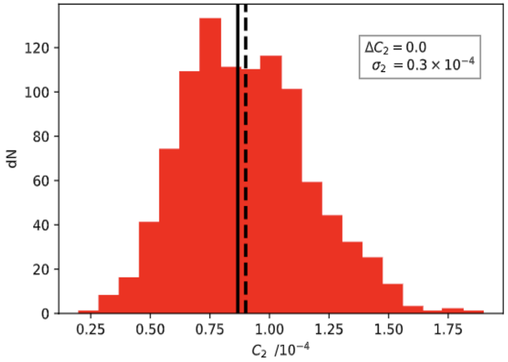
$$V\left[\hat{C}_\ell\right] = \left(\frac{1}{2\ell+1}\right)^2 \sum_{n=0}^{2\ell} V[w_n^{(\ell)}]$$

where

$$V[w_n^{(\ell)}] = \begin{cases} 2\sigma_{\ell 0}^4 + 4\sigma_{\ell 0}^2 a_{\ell 0}^2 & n = 0 \\ 8\sigma_{\ell n}^{(R)4} + 16\sigma_{\ell n}^{(R)2} \Re[a_{\ell n}]^2 & \ell \geq n > 0 \\ 8\sigma_{\ell(n-\ell)}^{(I)4} + 16\sigma_{\ell(n-\ell)}^{(I)2} \Im[a_{\ell(n-\ell)}]^2 & 2l \geq n > \ell \end{cases}$$

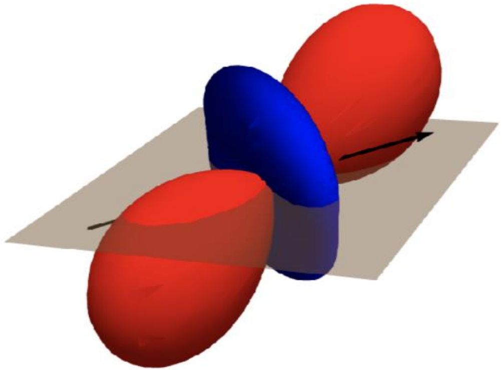
and $\sigma_{\ell m}^{(R)2} = V[\Re[\hat{a}_{\ell m}]]$, $\sigma_{\ell m}^{(I)2} = V[\Im[\hat{a}_{\ell m}]]$



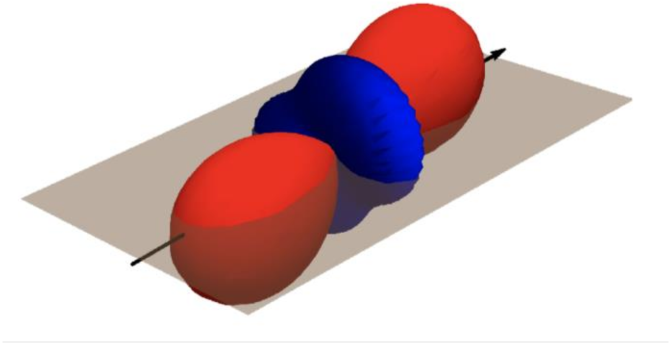


Axial Symmetry

- 3D structure of the quadrupole (Galactic plane shown for reference)



CF3 Galaxies



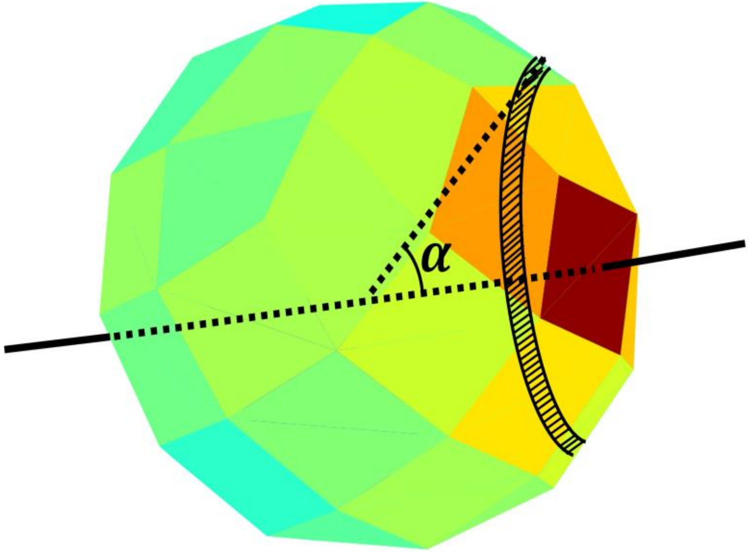
CF3 SNIa

- The quadrupole is symmetric around the axis of the maximum ($l \sim 285$, $b \sim 11$).

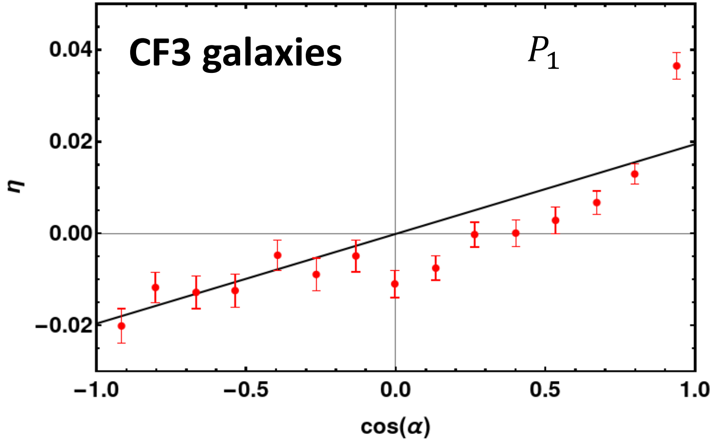
Axial Symmetry

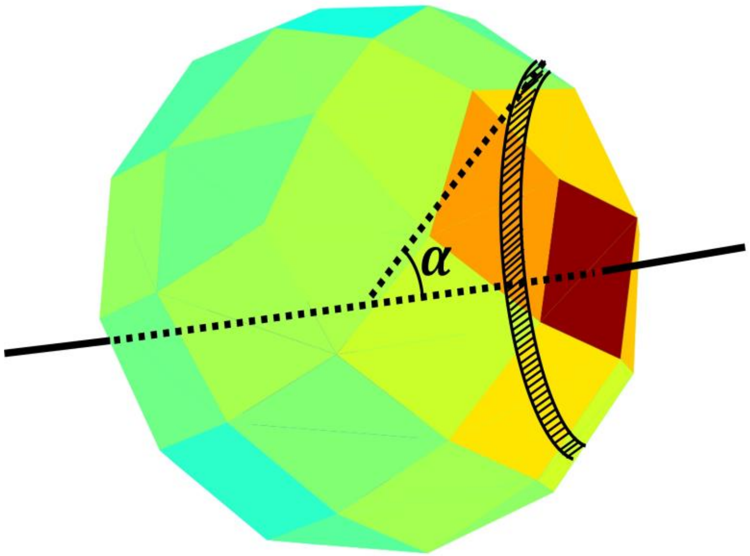
- Sufficient to expand using the Legendre basis about the axis of symmetry

$$\eta(\alpha) = \sum_{\ell=1}^{\infty} a_\ell P_\ell(\cos\alpha)$$



Axial Symmetry





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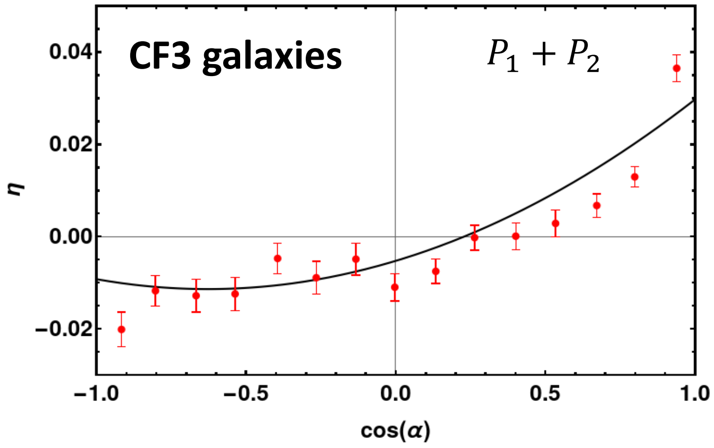

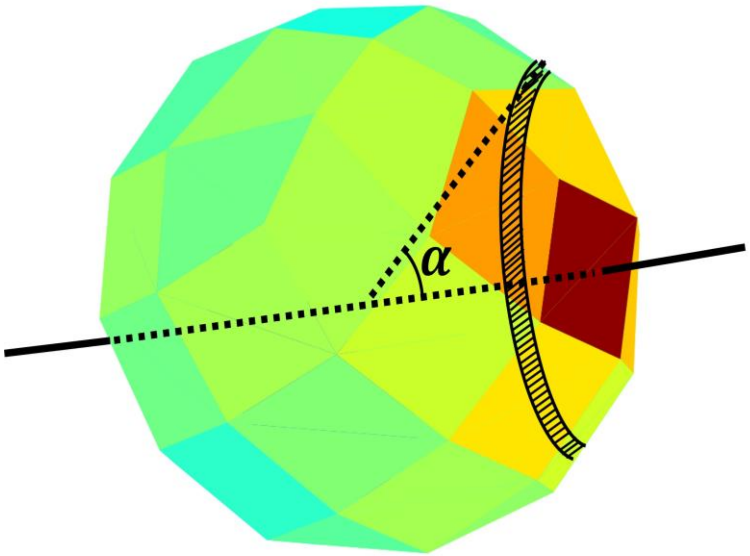

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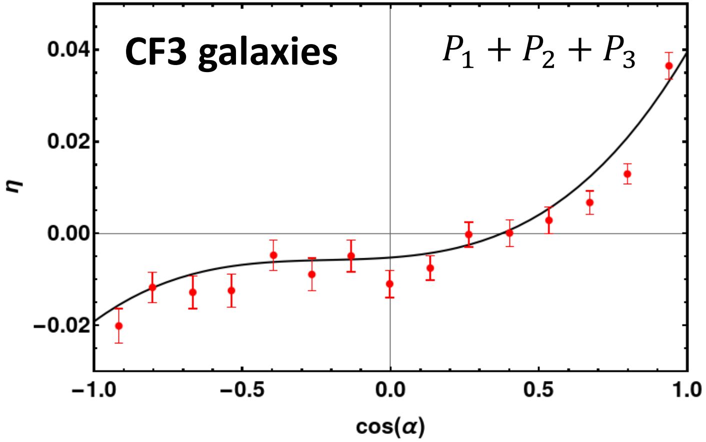

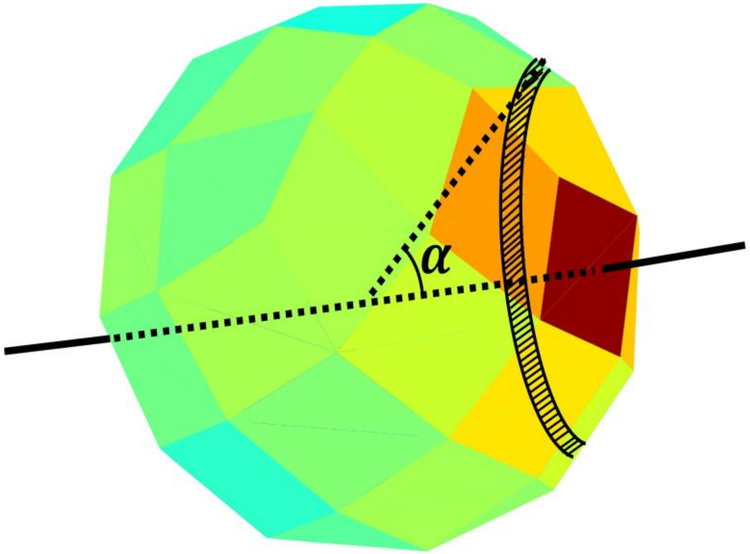

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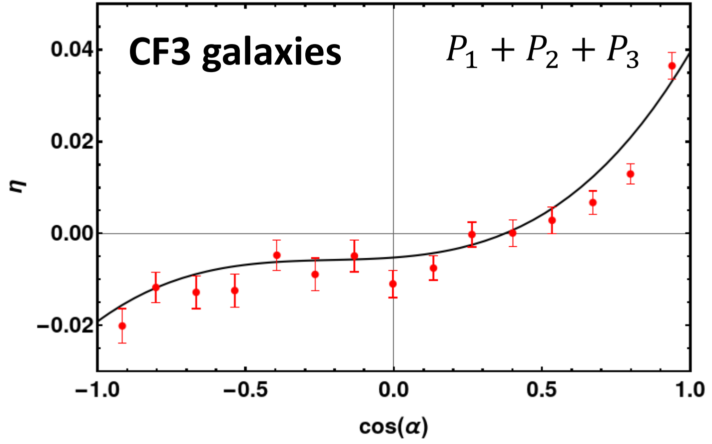
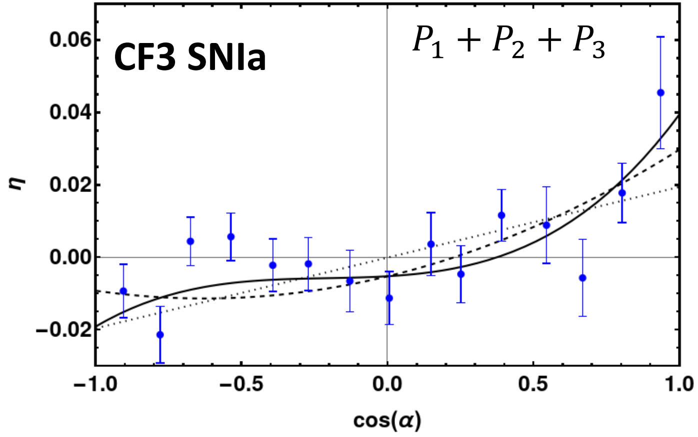
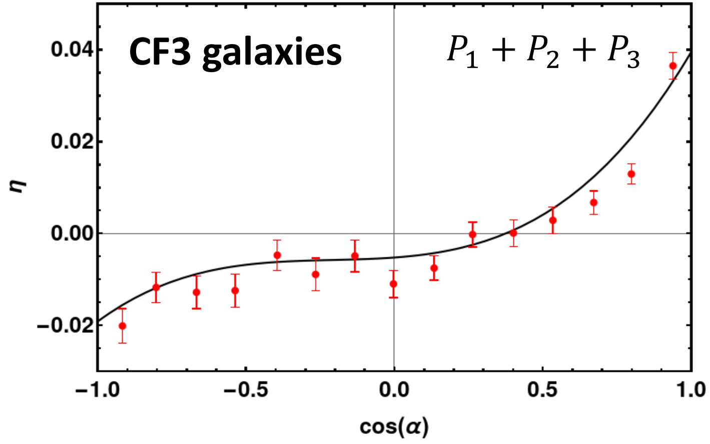




- The same 3 Legendre coefficients can describe the measurements for both samples.

$$a_1 = (1.9 \pm 0.1) \times 10^{-2}$$
$$a_2 = (1.1 \pm 0.1) \times 10^{-2}$$
$$a_3 = (1.1 \pm 0.1) \times 10^{-2}$$

Axial Symmetry



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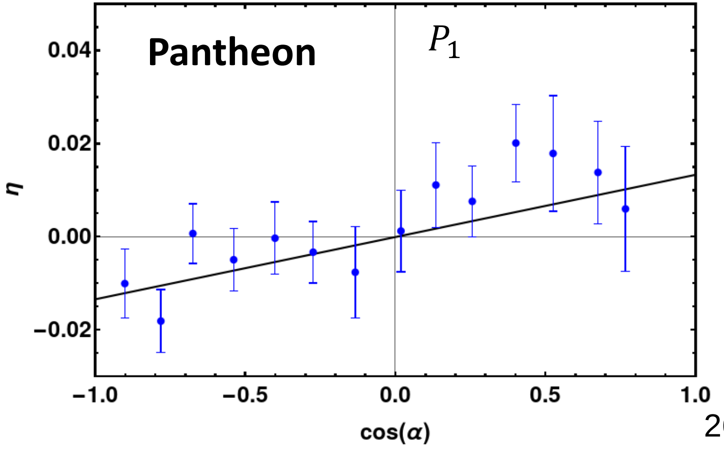
- For Pantheon sample, the dipole is enough, since the error is larger (due to the small number of objects), and there is no objects in the interesting direction.



Bulk Motion

- The bulk velocity is related to the dipole.

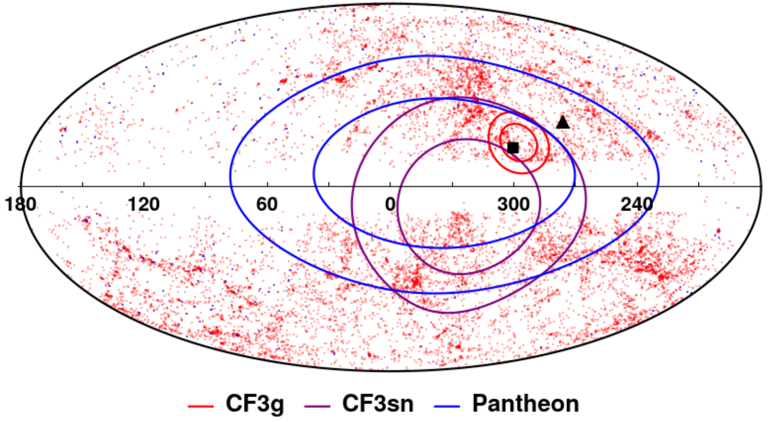
$$v_b = \frac{a_1 \ln 10}{\langle (1+z)/z \rangle} \quad = 318 \pm 22\ \mathrm{km/s}$$

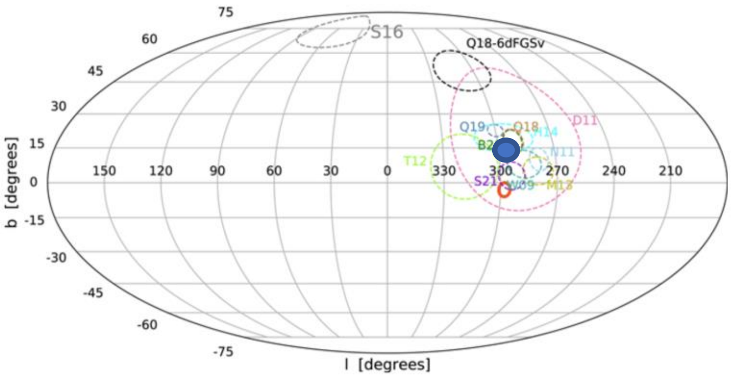
$$v_b = 252 \pm 11\ \mathrm{km/s}$$

(S. S. Boruah, M. J. Hudson, and G. Lavaux. 2020)

$$v_b = 292 \pm 28\ \mathrm{km/s}$$

(T. Hong et al. 2014)





(Qin et al. 2021)

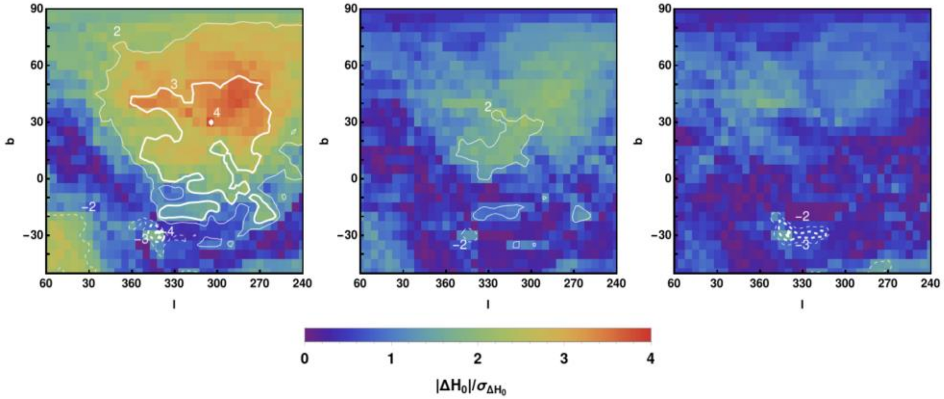
Bulk flow direction

Example of Practical Use of η : subtract peculiar velocity fields

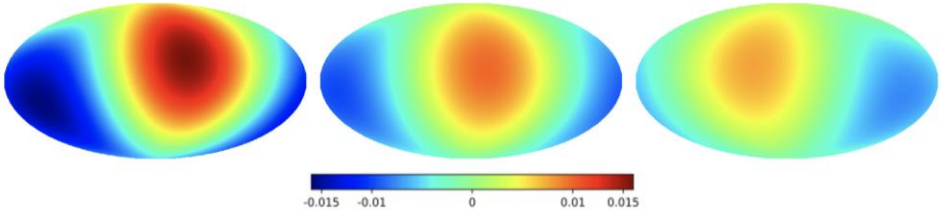
- Since (η) is related to the radial peculiar velocity.

$$\mu = 5\log\left(\frac{z}{H_0}\right) + 25 - 5\eta$$

The map of the signal-to-noise ratio for ΔH_0, between each direction and its opposite direction



The dipole of Pantheon sample



60%

45%

Conclusion

- Designed a new observable measuring the *expansion rate fluctuations* and determined its multipolar structure.
- A simple dipole term is a poor representation of the angular fluctuations in the local expansion rate.
- Find a signal for the quadrupolar component for both galaxies and SNIa of the CF3.
- The maximum of the quadrupole is aligned with dipole, and it is axially symmetric.
- The octupole (from galaxy sample only) has a maximum in the direction of the dipole.
- Need to build on the current work with updated and enlarged datasets: CosmicFlows-4, Pantheon+.

Spherical Harmonic analysis : parameters

Sample	N_{pix}	l_d	b_d	$\hat{C}_1$ (10^{-4})	$\frac{\eta 1_{max}-\lvert\eta 1_{min}\rvert}{2}$ (10^{-2})	p-value (%)	l_q	b_q	$\hat{C}_2$ (10^{-4})	$\frac{\eta 2_{max}-\lvert\eta 2_{min}\rvert}{2}$ (10^{-2})	p-value (%)	l_t	b_t	$\hat{C}_3$ (10^{-4})	$\frac{\eta 3_{max}-\lvert\eta 3_{min}\rvert}{2}$ (10^{-2})	p-value (%)
CF3 [0.01, 0.05]	192	291 ± 15	12 ± 7	3.0 ± 1.6	1.6	0.03	323 ± 16	9 ± 4	2.4 ± 0.9	1.9	0.12	289 ± 24	16 ± 15	0.1 ± 0.4	1.2	30.44
CF3 [0.01, 0.05]	48	283 ± 6	12 ± 5	5.3 ± 0.8	1.9	< 0.01	310 ± 11	4 ± 8	0.9 ± 0.3	1.1	< 0.01	284 ± 7	12 ± 5	0.5 ± 0.2	1.3	0.01
CF3g [0.01, 0.05]	48	286 ± 7	4 ± 6	7.0 ± 1.0	2.0	< 0.01	338 ± 8	22 ± 5	1.1 ± 0.4	1.3	< 0.01	255 ± 9	11 ± 5	0.7 ± 0.2	1.5	0.01
CF3 [0.01, 0.05]	12	285 ± 5	11 ± 4	5.1 ± 0.8	1.9	< 0.01	308 ± 7	1 ± 7	1 ± 0.3	1.1	< 0.01	-	-	-	-	-
CF3g [0.01, 0.05]	12	296 ± 6	18 ± 5	4.0 ± 0.6	1.7	< 0.01	323 ± 34	2 ± 17	1.3 ± 0.4	1.4	< 0.01	-	-	-	-	-
CF3sn [0.01, 0.05]	12	322 ± 23	−8 ± 18	3.7 ± 1.5	1.5	0.27	343 ± 15	−8 ± 10	1.7 ± 1.4	1.7	2.90	-	-	-	-	-
Pantheon [0.01, 0.05]	12	334 ± 42	6 ± 20	3.5 ± 2.7	1.6	4.37	337	−5	0.6 ± 1.9	1.6	33.33	-	-	-	-	-
CF3 [0.01, 0.03]	12	279 ± 5	12 ± 5	7.8 ± 1.0	2.3	< 0.01	310 ± 8	11 ± 6	2.9 ± 0.6	1.9	< 0.01	-	-	-	-	-
CF3 [0.03, 0.05]	12	301 ± 15	10 ± 14	1.1 ± 0.7	1.0	0.03	277 ± 28	−12 ± 11	0.9 ± 0.3	1.0	2.04	-	-	-	-	-