Black Holes as Probes for Ultralight DM

a Tale of Boundary Conditions

SCUOLA
NORMALE
SUPERIORE

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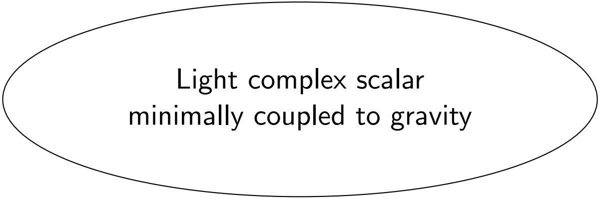
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minimally coupled to gravity

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Easy to get light scalars

$$\phi \to \phi + c$$

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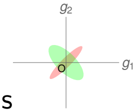

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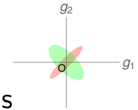
Interplay with black holes

No detected interactions



ϕ

What is the field profile?
Dependence on μ?
Relation $|\phi|_{r\to\infty}$ to $|\phi|_{r\to r_{BH}}$?

$$\Box\phi - \mu^2\phi = 0$$

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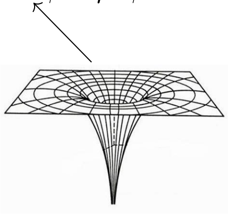

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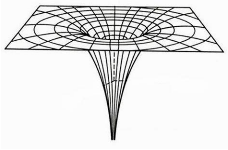

- BH domination first, no self gravity
- Nonrotating black hole

$\rightarrow$ Schwarzschild metric

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Dependence on μ?
Relation $|\phi|_{r\rightarrow\infty}$ to $|\phi|_{r\rightarrow r_{BH}}$?

$$\phi \to e^{-i\omega t} Y_{l,m} \phi(r), \qquad \omega = \mu, \quad V_l = r_{BH}\mu^2 r^3 - l(l+1) r(r - r_{BH})$$

$$r(r - r_{BH}) \frac{\mathrm{d}}{\mathrm{d}r} \left(r(r - r_{BH}) \frac{\mathrm{d}\phi}{\mathrm{d}r} \right) + V_l(r)\phi = 0$$

Hui '19 for $l = 0$

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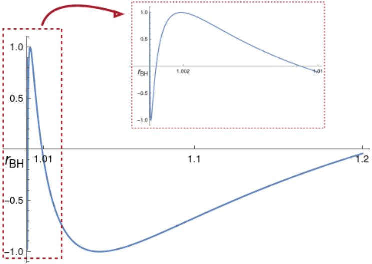
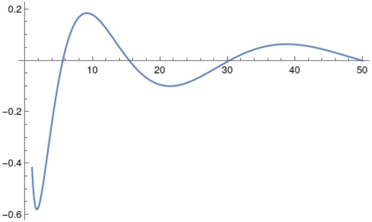


Near horizon limit
(numerically hard)



Far field

Local WKB

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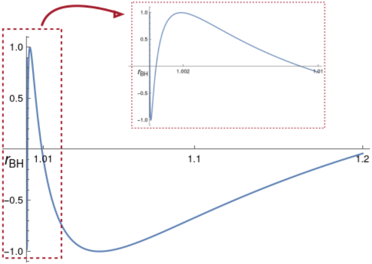

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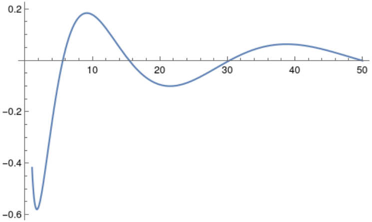


Far field

Local WKB

Can they be ‘glued’?

Hui ’19 for $l = 0$

Global WKB

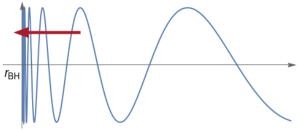
$r_{SgrA^*} \simeq 10^7\mathrm{km} \simeq 10^{17}\mathrm{eV}^{-1}$

$$(r - r_{BH})^{-i\mu r_{BH}} \longleftrightarrow \frac{1}{r^{3/4}} e^{-2i\mu\sqrt{r_{BH} r}}$$

$$|\phi_l| \propto V_l^{-1/4}(r)$$

l	0	1	2
$\mu r_{BH} \gtrsim$	0.3	0.7	1.2

Impose infalling b.c. at horizon (Nontrivial, see Love numbers, no hair thms.)



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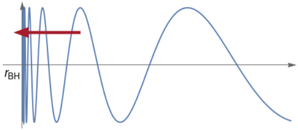
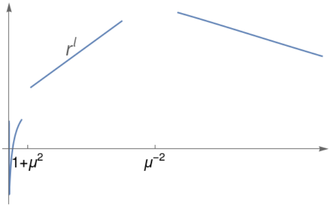


Small μr_{BH}?

$\mu r_{BH} \ll 1$ accurate unless $\frac{r}{r_{BH}} \lesssim 1 + (\mu r_{BH})^2$ or $\frac{r}{r_{BH}} \gtrsim 1/(\mu r_{BH})^2$



$l = 1$

$$\mathcal{O}(1) \quad \sim r^l \quad \sim \frac{\mu^{-\frac{3}{2}-2l}}{r^{3/4}} \cos(2\mu\sqrt{r_{BH} r})$$

- ✓ Uniform approximation
- ✓ Analytic control (causality)
- ✓ μ dependence
- ✓ $|\phi_\infty/\phi_{hor.}|$
- ☹ Little phenomenology

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$\omega < \mu$ almost allowed... needs $Im[\omega] < 0$

$\rightarrow$ SOLITON, bound state

$$M_s \simeq 10^9 M_\odot \propto M_{halo}^{1/3}$$

easier → BH domination

harder → Soliton domination (self-gravity)

Aim to

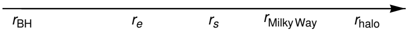
- Soliton domination
- Uniform solution
- Causal boundary conditions
- GR gravity close to the horizon
- No non-relativistic approximation

$$l = 0^1, \qquad \nabla^2 \Phi_N = -4\pi G \rho$$

$$V \to V - 2\mu^2 r^4 \Phi_N + (\omega^2 - \mu^2) r^4$$

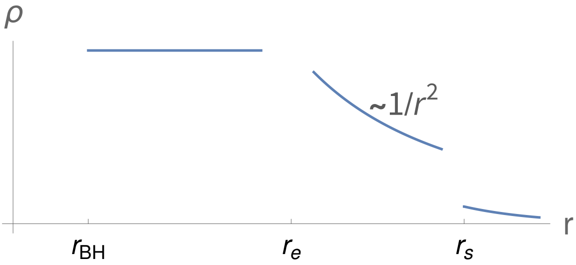
[1] Instabilities at $l > 0$, Dmitriev '21

Sketch: given Φ_N, compute ϕ in WKB $\to$ compute ρ, Φ_N



$$r_{BH} \simeq 10^7 \mathrm{km}, \;\; r_e \simeq 10^{13} \mathrm{km} \simeq 1 \mathrm{pc}$$
$$r_s \simeq 10^{15} \mathrm{km}, \;\; r_{M.W.} \simeq 10^{17} \mathrm{km}$$

$$r_s = \frac{1}{\mu^2 G M_s}, \qquad r_e = r_s \frac{M_{BH}}{M_s}, \qquad \Delta M_s / M_s \simeq 10^{-4} \text{ in } 10^{10} \mathrm{y}$$



Stability is crucial! It selects small μr_{BH}

$$GM_s \ll r_s \to GM_s \ll \frac{1}{\mu^2 GM_s} \to \mu GM_s \ll 1$$
$$\longrightarrow \mu r_{BH} \ll \frac{M_{BH}}{M_s} \ll 1$$

The hierarchy of scales makes boundary conditions unimportant at large distances

$$r_s \gg GM_s \to r_e \gg GM_s \frac{M_{BH}}{M_s} = r_{BH}$$

For $M_{BH} > M_s$ the known result $GM_{BH} r_s \mu^2 \simeq \mathcal{O}(1)$ gives

$$\mu r_{BH} \ll \sqrt{\frac{M_{BH}}{M_s}}$$

Conclusions

No soliton case:

✓ know $\rho(r)$ for spinning DM

⚠ boundary conditions have teeth when $\mu r_{BH} \gtrsim \mathcal{O}(1)$

Soliton case:

✓ understand $\rho(r)$, even with self-gravity

✓ b.c unimportant in soliton domination

✓ b.c unimportant in BH domination, small μr_{BH}

⚠ b.c. have teeth when $M_{BH} > M_s$ and $\mu r_{BH} \gtrsim \mathcal{O}(1)$
For $SgrA^*$: $M_s \lesssim 10^7 M_\odot, \quad \mu \gtrsim 10^{-17}\mathrm{eV}$