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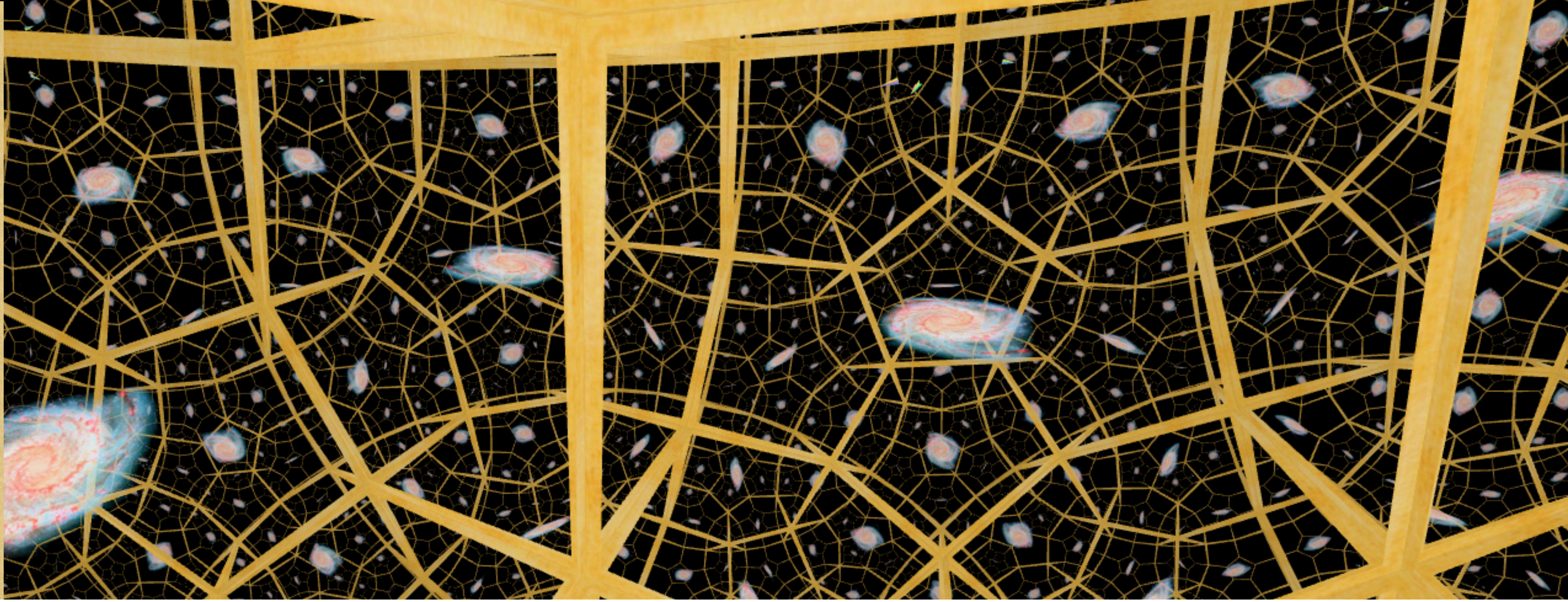

Progress on Old and New Themes in cosmology

*3 May 2023*

# COSMOLOGICAL MODEL WITH EXPANSION BLIND TO THE SPATIAL CURVATURE

Quentin Vigneron

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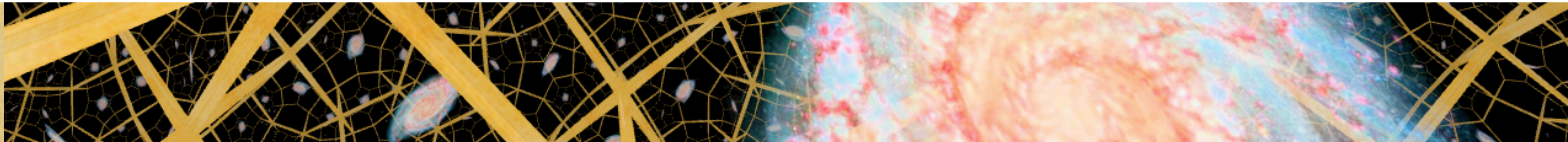

Image: *Curved Spaces*, Jeffrey Weeks

**Motivations:**

**The theory:**

**Cosmology:**

## Topological term in the Einstein equation

(arXiv: 2204.13980)

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = T_{\mu\nu} - T/2g_{\mu\nu} + \Lambda g_{\mu\nu}$$

Reference curvature related to
the spacetime topology

# TOPOLOGICAL TERM IN THE EINSTEIN EQUATION

**Rosen's bi-connection theory** [e.g. Rosen (1980), *General relativity with a background metric*]

The theory is composed of:

- The **physical** Lorentzian structure $(\boldsymbol{g}, {}^4\nabla)$ and its Riemann curvature tensor $R^{\mu}{}_{\alpha\beta\gamma}$
- A **reference, non-dynamical,** connection ${}^4\bar{\nabla}$ and its reference Riemann curvature tensor $\bar{R}^{\mu}{}_{\alpha\beta\gamma}$

Modified Einstein's equation:

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T^{*}_{\mu\nu} + \Lambda g_{\mu\nu} \quad ; \quad g^{\mu\nu}\nabla_{\mu}\bar{R}_{\nu\alpha} - \frac{1}{2}g^{\mu\nu}\nabla_{\alpha}\bar{R}_{\mu\nu} = 0$$

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My choice of reference metric: **the reference connection determines the spacetime topology**

**Vigneron Q.,** 2022c, arXiv: 2204.13980

$$\bar{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \bar{\mathscr{R}}_{ij}(x^k) \end{pmatrix}$$ where $\bar{\mathscr{R}}_{ij}$ = (maximally symmetric) curvature of the spatial covering space $\tilde{\Sigma}$.

| $\tilde{\mathcal{M}} = \mathbb{R} \times$ | $\mathbb{R}^3$ | $\mathbb{S}^3$ | $\mathbb{H}^3$ | others |
|---|---|---|---|---|
| $\bar{R}_{\mu\nu}$ | $\mathbf{0}$ | $\mathrm{diag}(0; 2\bar{h}_{ij})$ | $-\,\mathrm{diag}(0; 2\bar{h}_{ij})$ | $\neq 0$ |

## Motivations:

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Reference curvature related to
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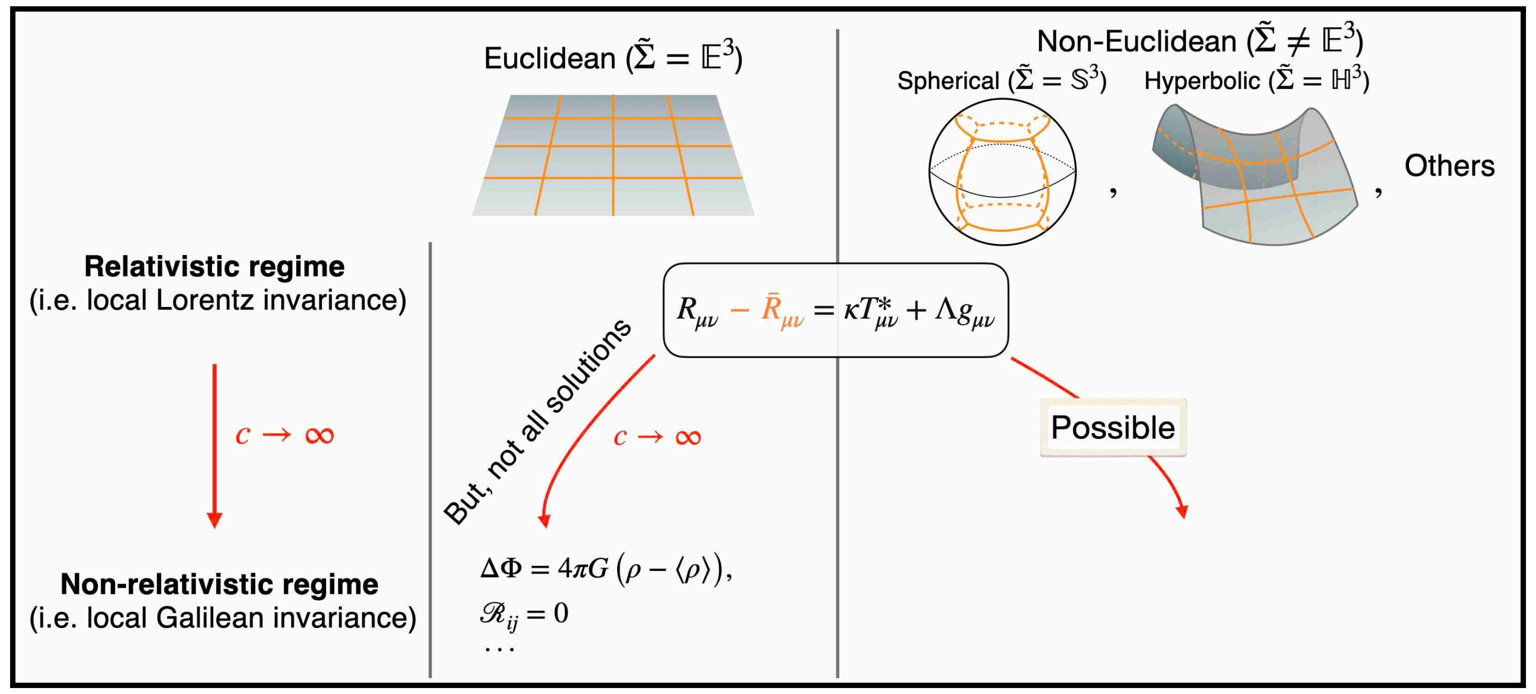
## Motivations:

### Non-relativistic limit



### First order covariant formalism

$$S_{\rm EH} = \int_{\mathscr{M}} \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu} \mathrm{d}x^4$$

## The theory:

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(arXiv: 2204.13980)

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$$\Delta\Phi = 4\pi G \left(\rho - \langle\rho\rangle\right),$$

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$$\dots$$

# MOTIVATION 1: EXISTENCE OF A NON-RELATIVISTIC LIMIT

$$\boxed{R_{\mu\nu} = \kappa T^{*}_{\mu\nu} + \Lambda g_{\mu\nu}}$$

But, not all solutions

$$\xrightarrow{c \to \infty}$$

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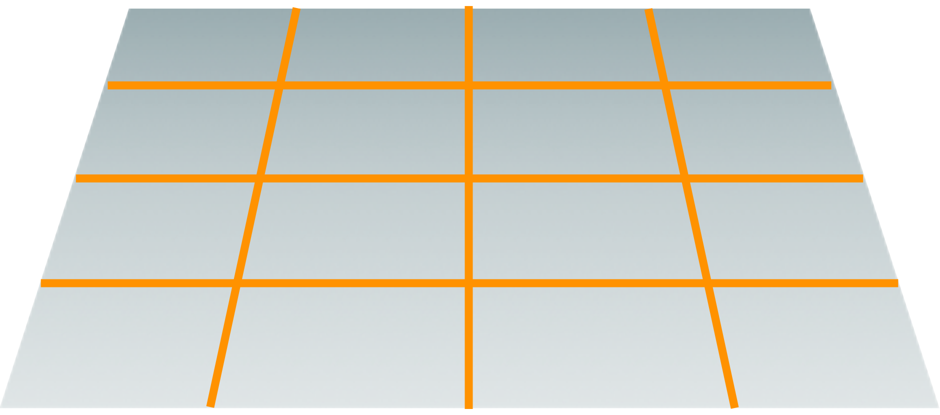
# MOTIVATION 1: EXISTENCE OF A NON-RELATIVISTIC LIMIT

Euclidean ($\tilde{\Sigma} = \mathbb{E}^3$)



Non-Euclidean ($\tilde{\Sigma} \neq \mathbb{E}^3$)

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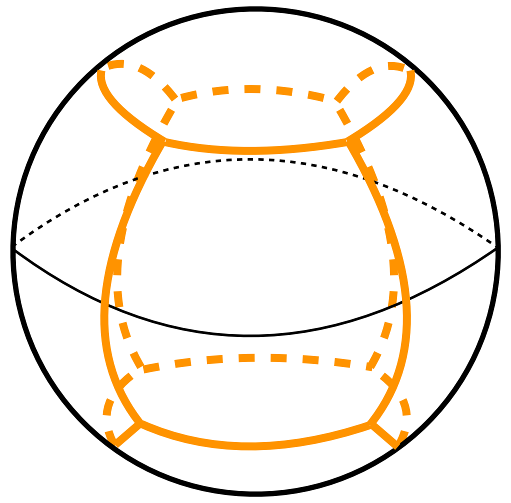

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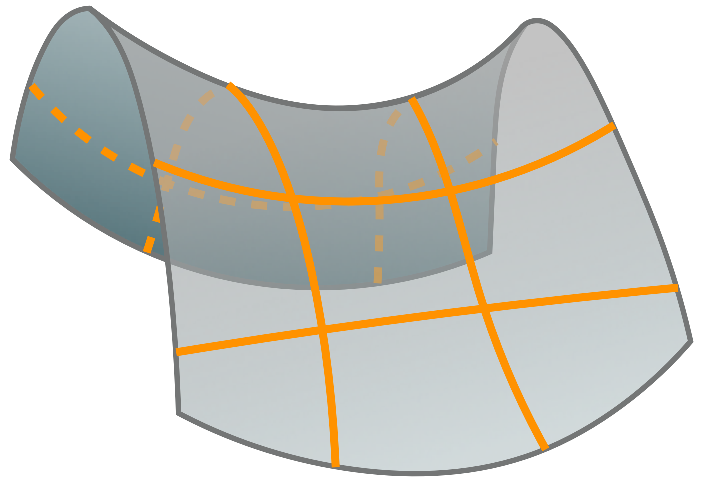
Hyperbolic ($\tilde{\Sigma} = \mathbb{H}^3$)



, Others

$$R_{\mu\nu} = \kappa T^{*}_{\mu\nu} + \Lambda g_{\mu\nu}$$

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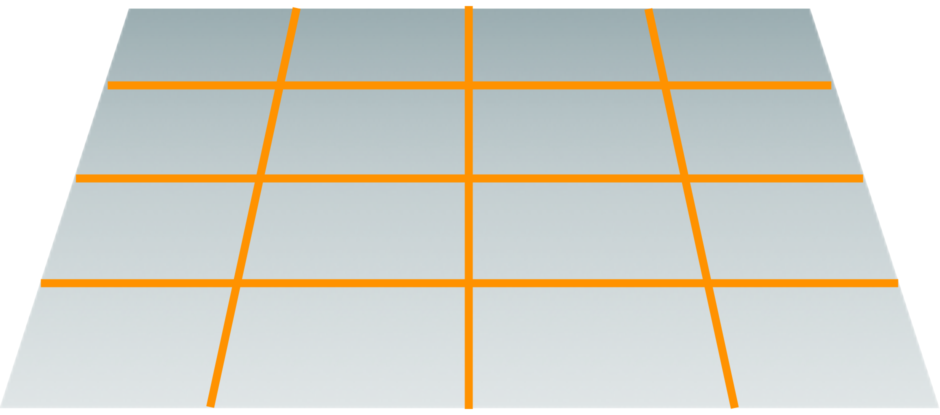

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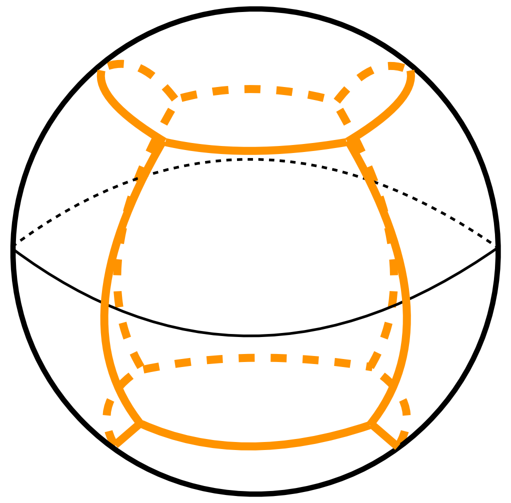

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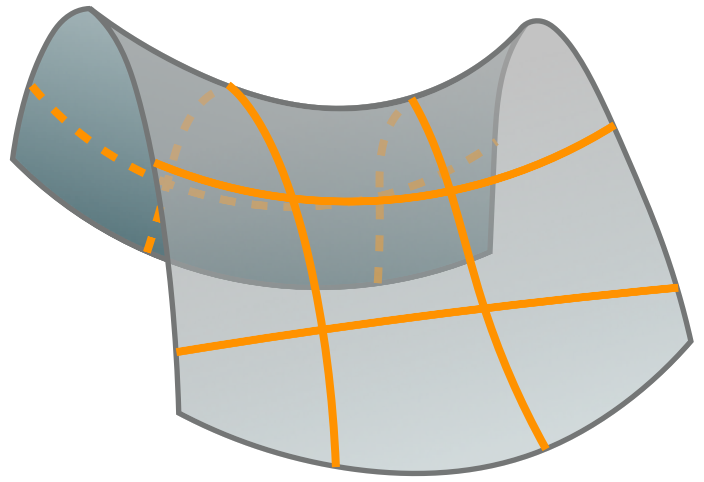

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?

# MOTIVATION 1: EXISTENCE OF A NON-RELATIVISTIC LIMIT

Limit defined for any topology

**Relativistic regime**
(i.e. local Lorentz invariance)

$c \to \infty$

**Non-relativistic regime**
(i.e. local Galilean invariance)

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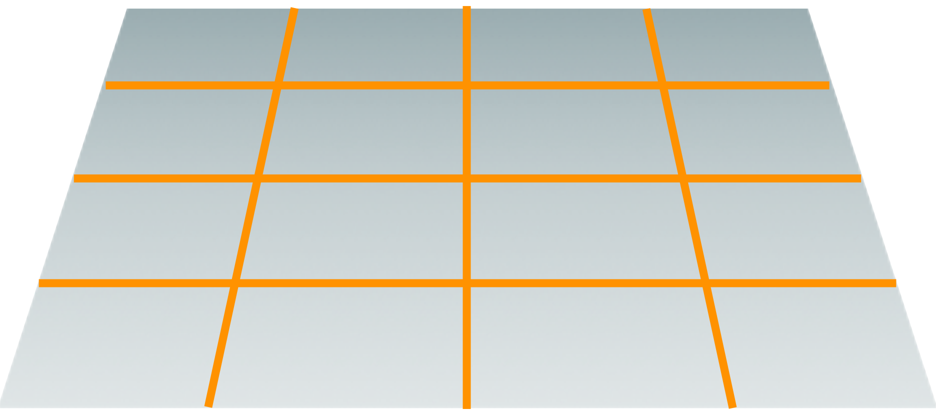

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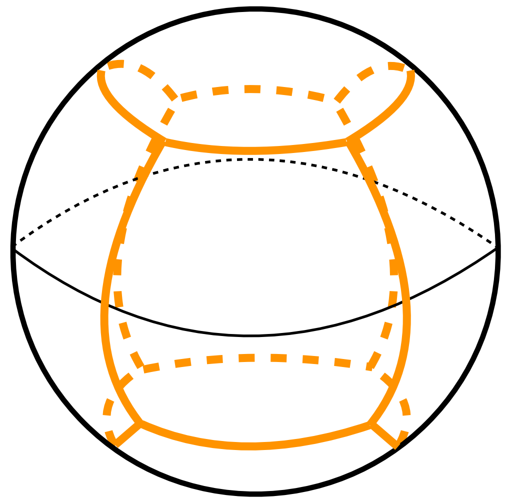
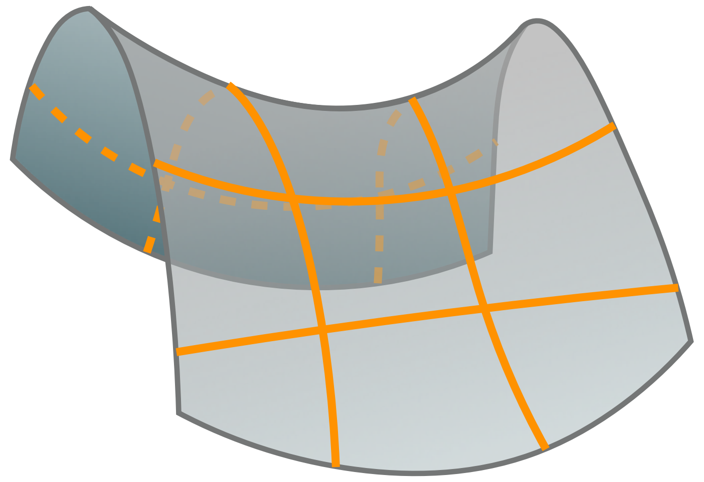
Spherical ($\tilde{\Sigma} = \mathbb{S}^3$)  ,

Hyperbolic ($\tilde{\Sigma} = \mathbb{H}^3$)  ,

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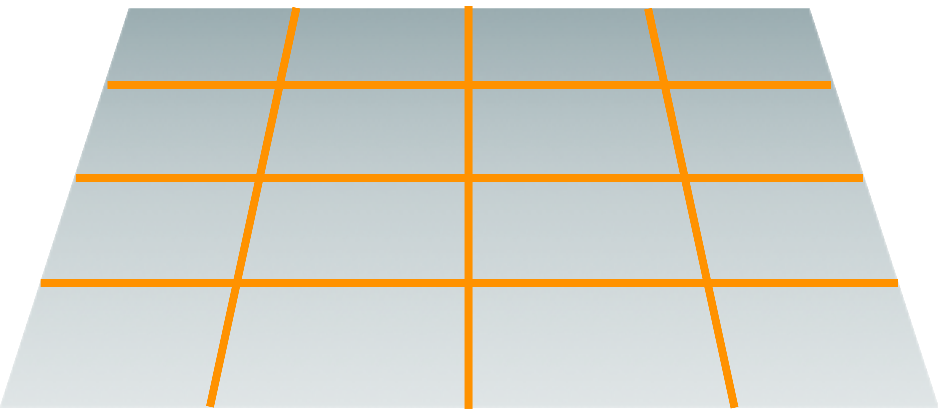

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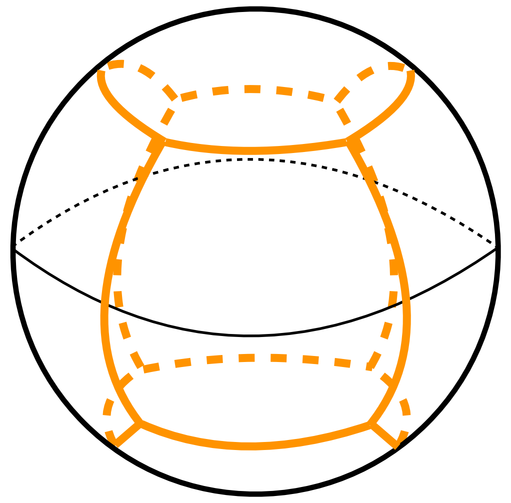

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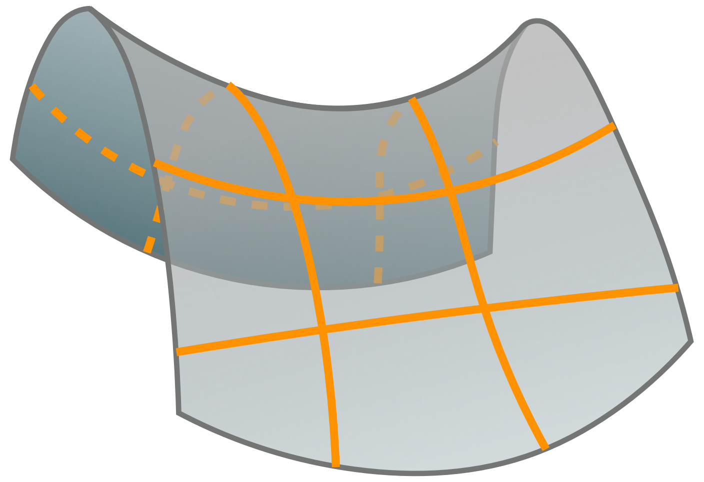

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Limit does not exist (arXiv: 2204.13980)

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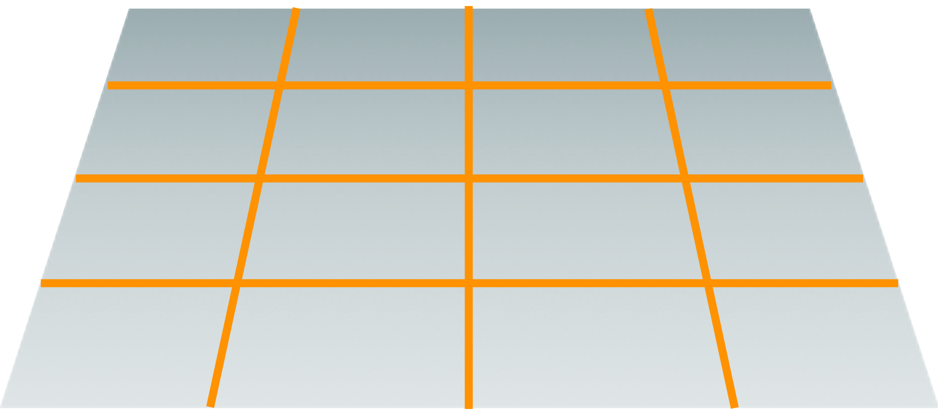

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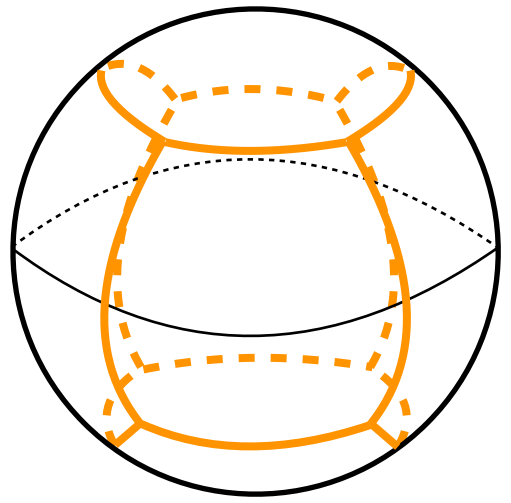

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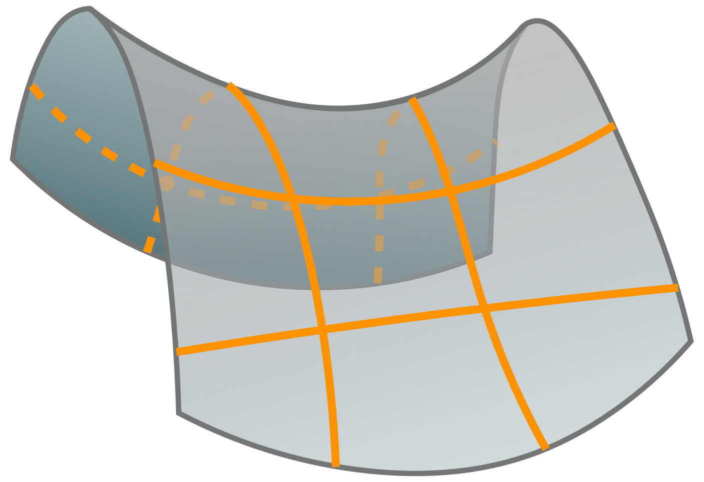

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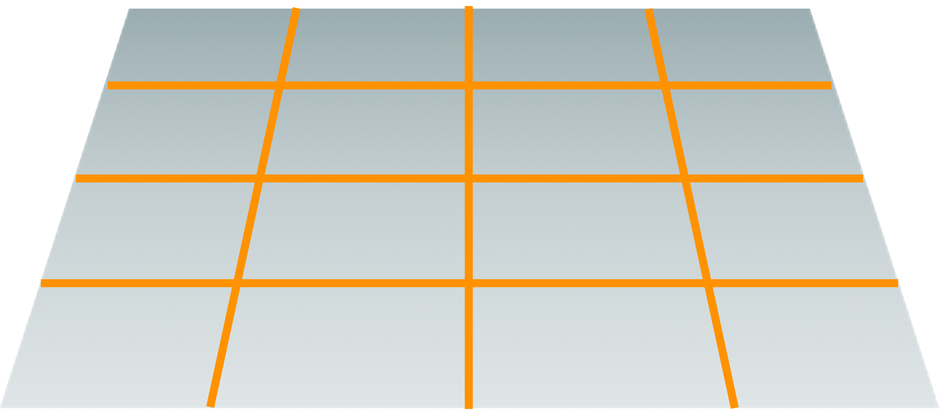

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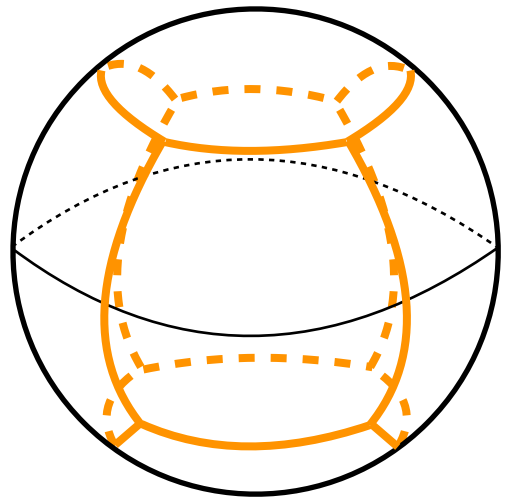

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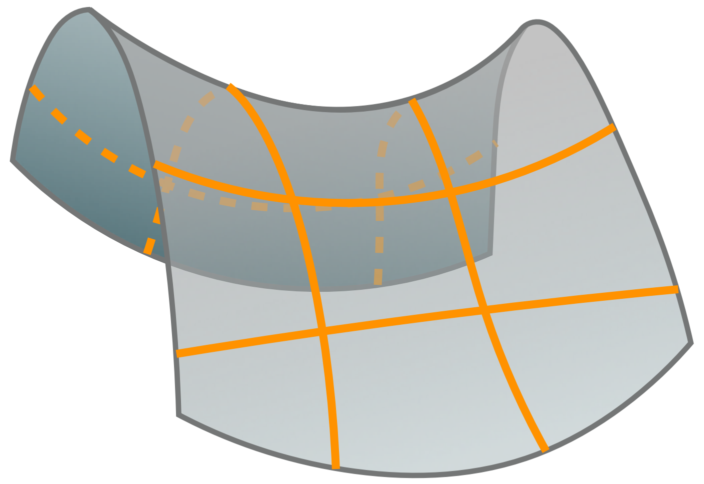

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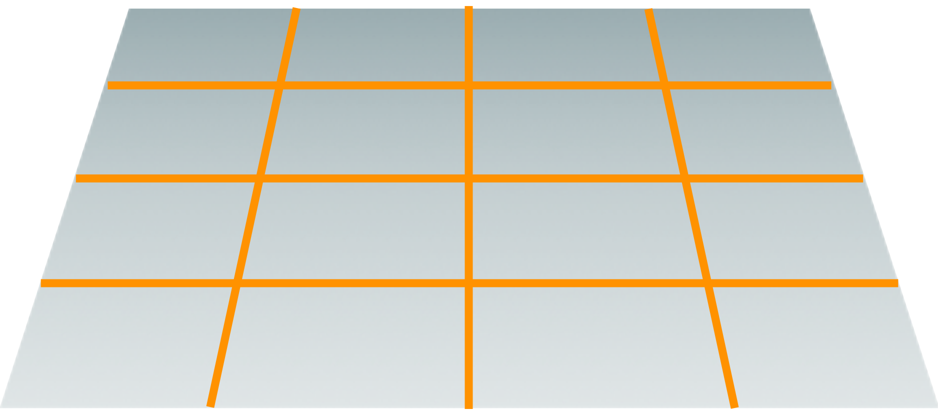

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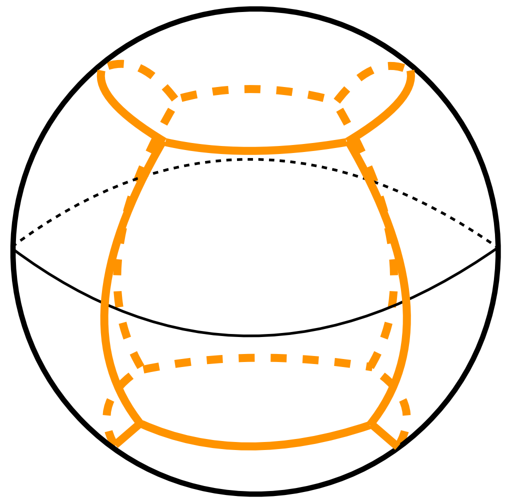

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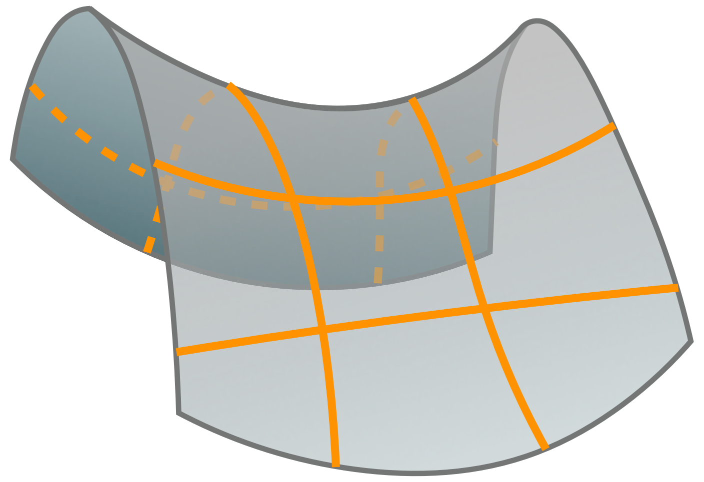

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**Summary: Requiring compatibility between the relativistic and the non-relativistic regime in any topology.**

# MOTIVATION 2: FIRST ORDER COVARIANT FORMULATION

Action of general relativity:

- For a mathematically **well-posed** action
- For a **covariant first order** Lagrangian

$$S_{\mathrm{EH+GHY}} = \int_{\mathscr{M}} \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu} \mathrm{d}x^4 + \ 2\int_{\partial\mathscr{M}} \sqrt{|h|}\, n^{\mu} h^{\alpha\beta} \left( \bar{\nabla}_{\mu} g_{\alpha\beta} - \bar{\nabla}_{\alpha} g_{\beta\mu} \right) \mathrm{d}x^3$$

$$= \int_{\mathscr{M}} \sqrt{|g|}\, \mathscr{C}^{\alpha}_{\mu[\beta} \mathscr{C}^{\beta}_{\nu]\alpha} g^{\mu\nu} \mathrm{d}x^4$$

where $\mathscr{C}^{\alpha}_{\mu\nu} := \Gamma^{\alpha}_{\mu\nu} - \bar{\Gamma}^{\alpha}_{\mu\nu}$ and $\bar{R}^{\alpha}{}_{\beta\mu\nu} = 0$.

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Only possible
in Euclidean topologies

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**Hypothesis:** $\bar{\Gamma}^{\alpha}_{\mu\nu}$ should be adapted as function of the spacetime topology, i.e. $\bar{R}_{\mu\nu} \neq 0$ for non-Euclidean topologies.

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$$= \underset{\text{First order Lagrangian}}{\boxed{\int_{\mathscr{M}} \sqrt{|g|}\, \mathscr{C}^{\alpha}_{\mu[\beta} \mathscr{C}^{\beta}_{\nu]\alpha} g^{\mu\nu} \mathrm{d}x^4}}$$

where $\mathscr{C}^{\alpha}_{\mu\nu} := \Gamma^{\alpha}_{\mu\nu} - \bar{\Gamma}^{\alpha}_{\mu\nu}$ and $\boxed{\bar{R}^{\alpha}{}_{\beta\mu\nu} = 0.}$

Only possible in Euclidean topologies

**Hypothesis:** $\bar{\Gamma}^{\alpha}_{\mu\nu}$ should be adapted as function of the spacetime topology, i.e. $\bar{R}_{\mu\nu} \neq 0$ for non-Euclidean topologies.

$$\delta S_{\mathrm{EH+GHY}} \left( \bar{R}_{\mu\nu} \neq 0 \right) = \int_{\mathscr{M}} \sqrt{-g} \left[ R_{\mu\nu} - \bar{R}_{\mu\nu} - \frac{1}{2} \left( R_{\alpha\beta} - \bar{R}_{\alpha\beta} \right) g^{\alpha\beta} g_{\mu\nu} \right] \delta g^{\mu\nu} \mathrm{d}x^4$$

**Bi-connection equation :** $R_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T^{*}_{\mu\nu} + \Lambda g_{\mu\nu}$

# MOTIVATION 2: FIRST ORDER COVARIANT FORMULATION

Action of general relativity:

- For a mathematically **well-posed** action
- For a **covariant first order** Lagrangian

—> A reference connection $\bar{\Gamma}^{\alpha}_{\mu\nu}$ is needed

$$S_{\mathrm{EH+GHY}} = \overset{\text{Einstein-Hilbert}}{\boxed{\int_{\mathscr{M}} \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu} \mathrm{d}x^4}} + \overset{\text{Gibbons-Hawking-York term}}{\boxed{2\int_{\partial\mathscr{M}} \sqrt{|h|}\, n^{\mu} h^{\alpha\beta} \left( \bar{\nabla}_{\mu} g_{\alpha\beta} - \bar{\nabla}_{\alpha} g_{\beta\mu} \right) \mathrm{d}x^3}}$$

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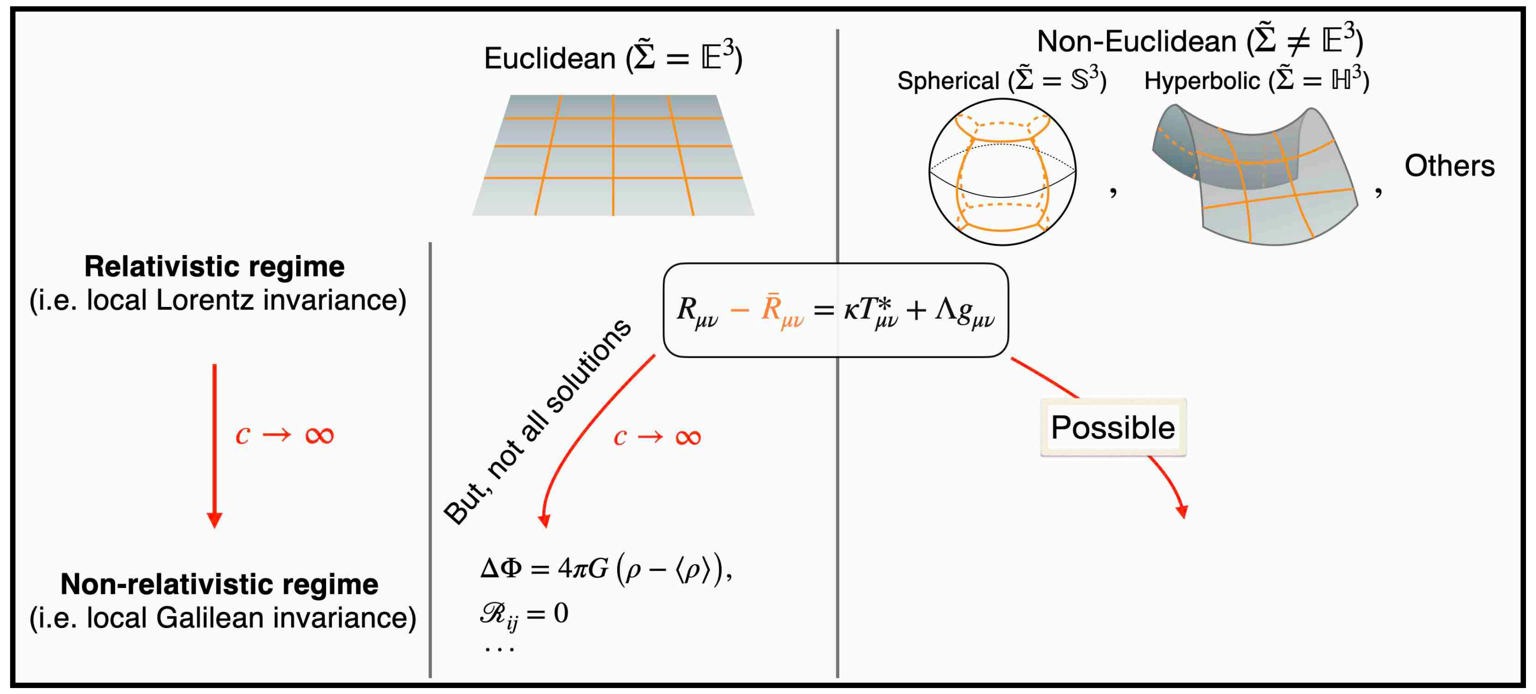
**Summary: Having a well-posed and first order action of gravitation, for any topology.**

## Motivations:

Non-relativistic limit



First order covariant formalism

$$S_{\mathrm{EH+GHY}} = \int_{\mathscr{M}} \sqrt{|g|}\,\mathscr{C}^{\alpha}_{\mu[\beta}\mathscr{C}^{\beta}_{\nu]\alpha}g^{\mu\nu}$$

## The theory:

### Topological term in the Einstein equation

(arXiv: 2204.13980)

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = T_{\mu\nu} - T/2g_{\mu\nu} + \Lambda g_{\mu\nu}$$

Reference curvature related to the spacetime topology

## Cosmology:

Expansion blind to the curvature (arXiv: 2212.00675)

$$\Omega_{\neq K} = 1, \quad \forall\ \Omega_K$$

# CONSEQUENCE FOR THE ROLE OF SPATIAL CURVATURE

**1. Exact homogeneous and isotropic solution:**

$\Lambda$**CDM:**
$$\begin{cases} \Omega_{\neq K} + \Omega_K = 1, \\ q = \Omega_{\rm m}/2 + \Omega_{\rm rad} - \Omega_\Lambda \end{cases}$$

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**2. Weak field limit: for** $\Pi = 0 = \Pi_i$

$\Lambda$**CDM:**
$$(\Delta + 3K)\,\Psi = 4\pi G a^2 \delta\rho_\Delta,$$
$$(\Delta + 2K)\,Q_i = -\,16\pi G a^2 q_i,$$
$$f_{ij}'' + 2\mathscr{H} f_{ij}' + (2K - \Delta)\, f_{ij} = 8\pi G a^2 \Pi_{ij},$$
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—> Dynamical and geometrical effects of spatial curvature

—> Mainly geometrical effects of spatial curvature.

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**Strengths of this cosmological model:**

1. Origin of the modifications not related to cosmology or tensions of $\Lambda$CDM.
2. Has the same number of free-parameters than the $\Lambda$CDM model.

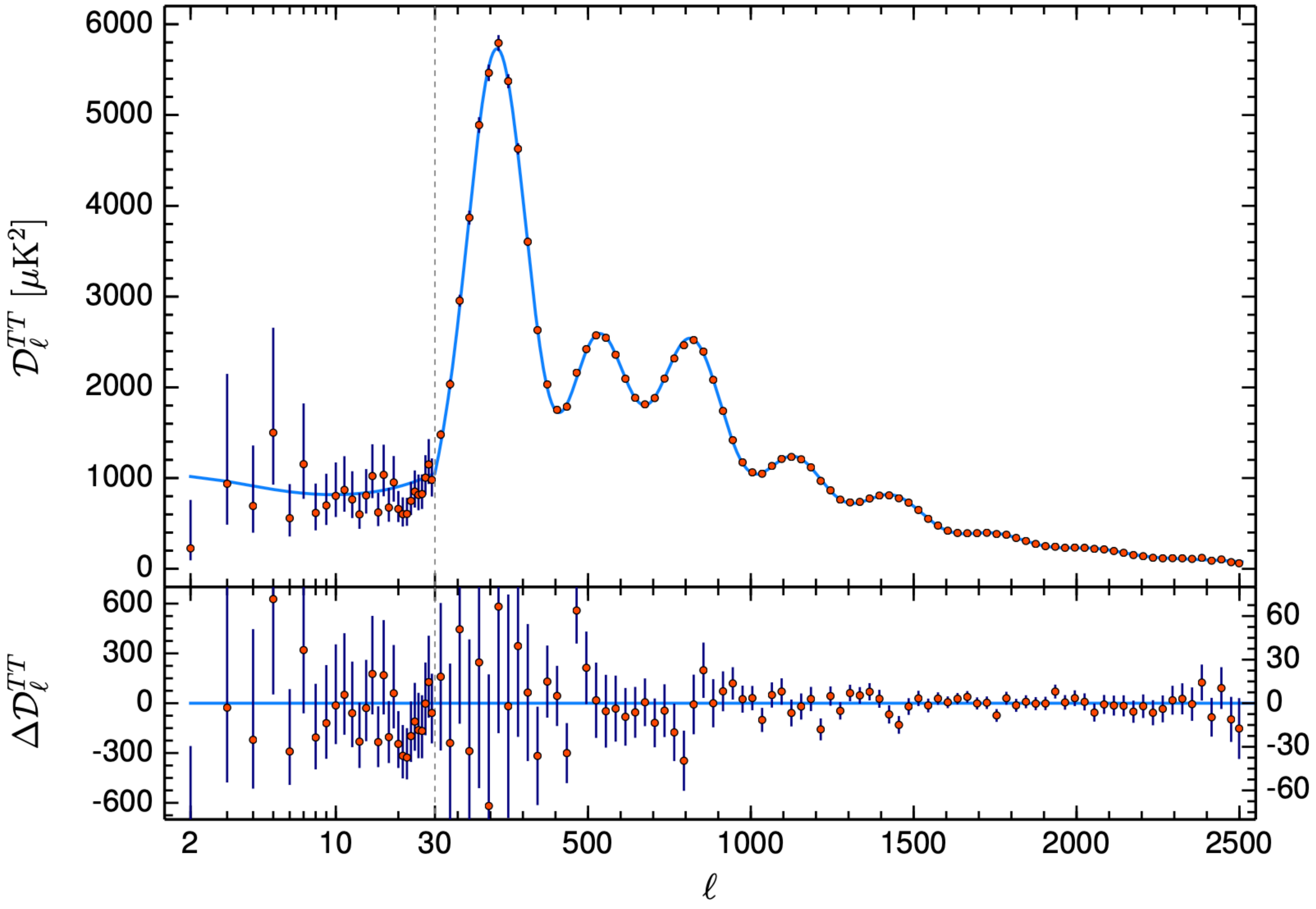
# CONSEQUENCE FOR THE ROLE OF SPATIAL CURVATURE

## A possibility:

- A positive curvature ($\Omega_K \sim -0.05$) gives a better fit of the CMB power spectrum alone.

  [e.g. Di-Valentino et al. (2020): 1911.02087]

- But, it increases the Hubble tension: $\Delta H_0$ from $67 \leftrightarrow 73$ to $55 \leftrightarrow 73$ km/s/Mpc: "**Curvature tension**"



Planck VI (2018)

$$S_\chi(x) = \begin{cases} \sin x, & \Omega_K < 0 \\ x, & \Omega_K = 0 \\ \sinh x, & \Omega_K > 0 \end{cases}$$

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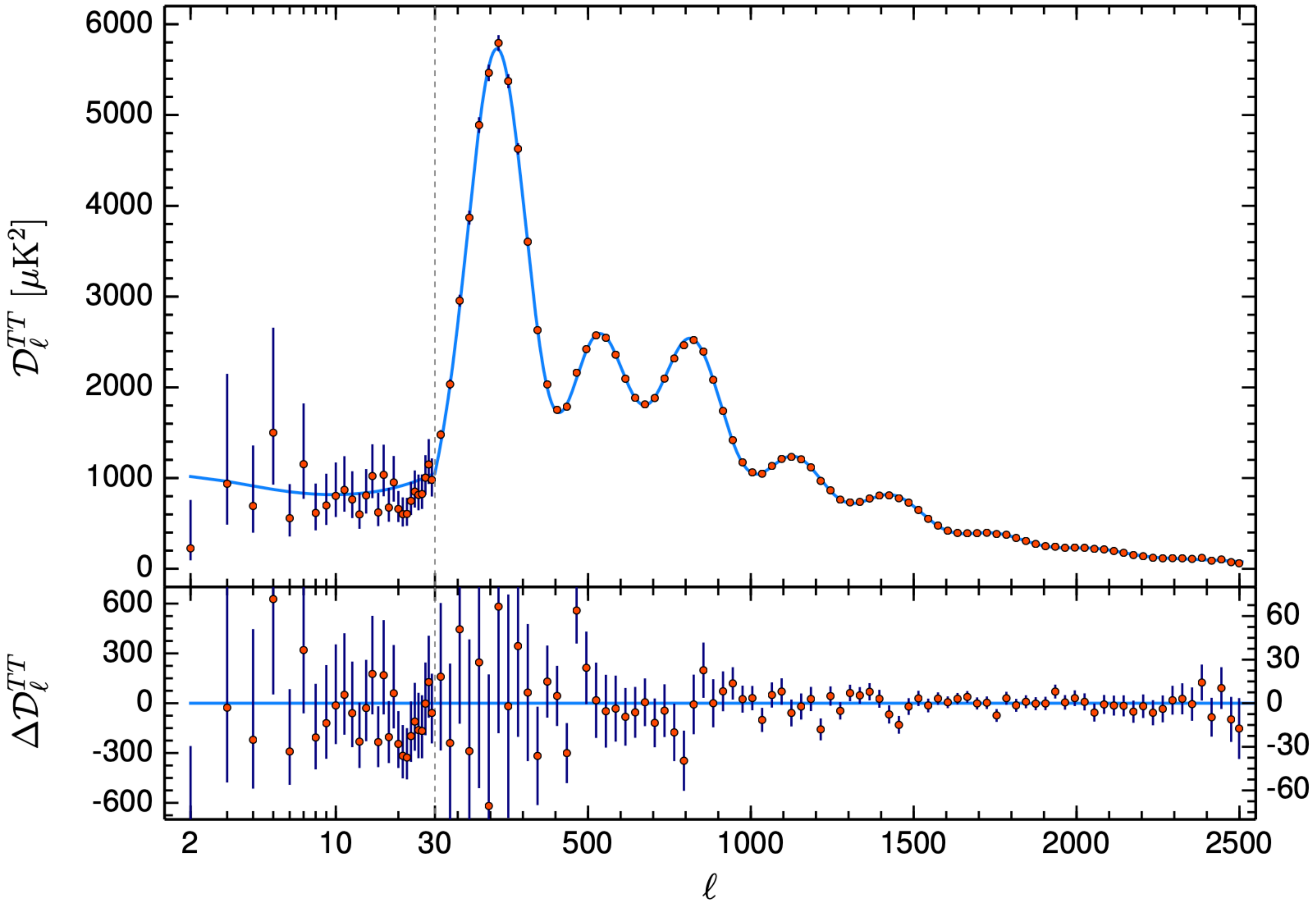

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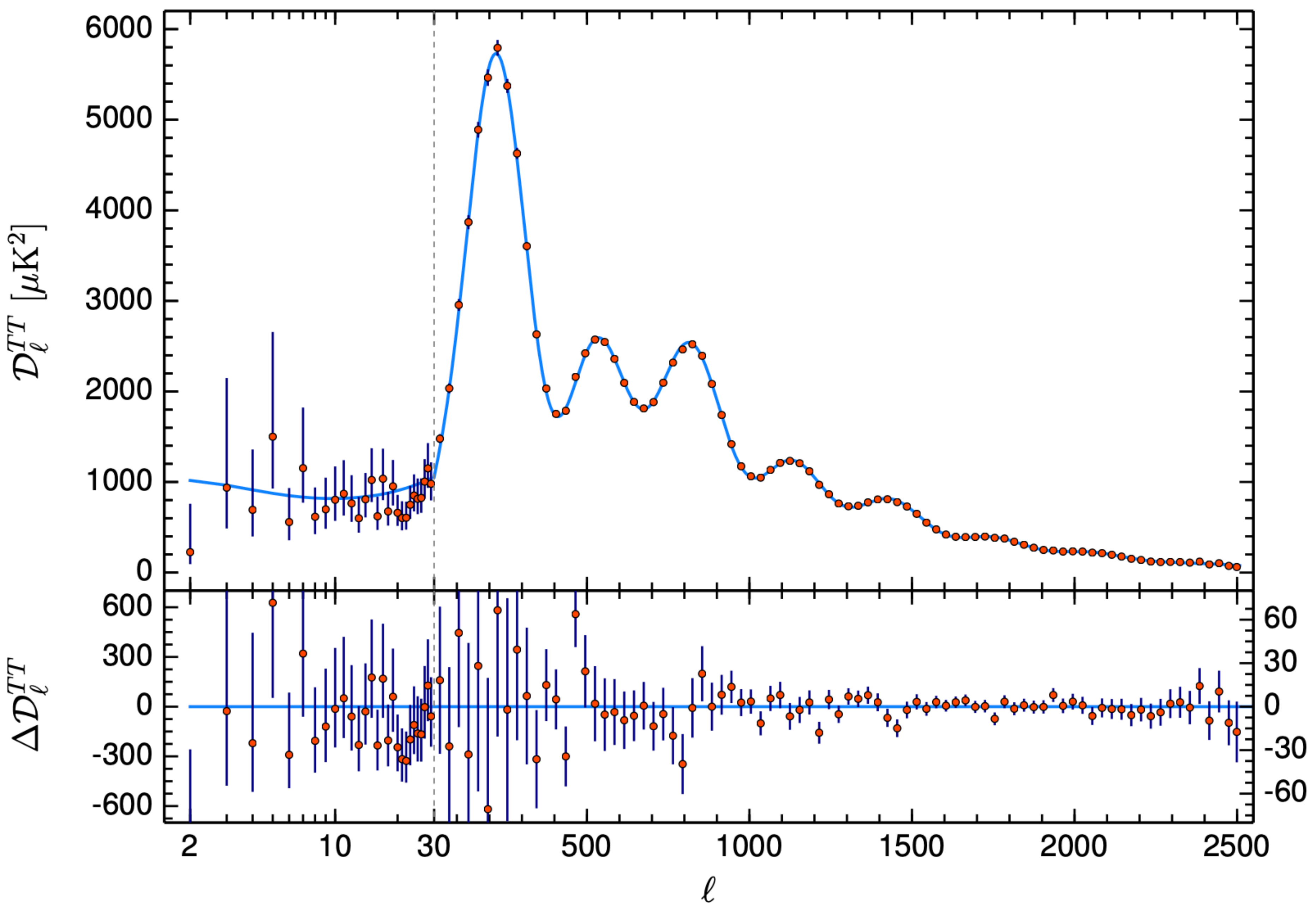

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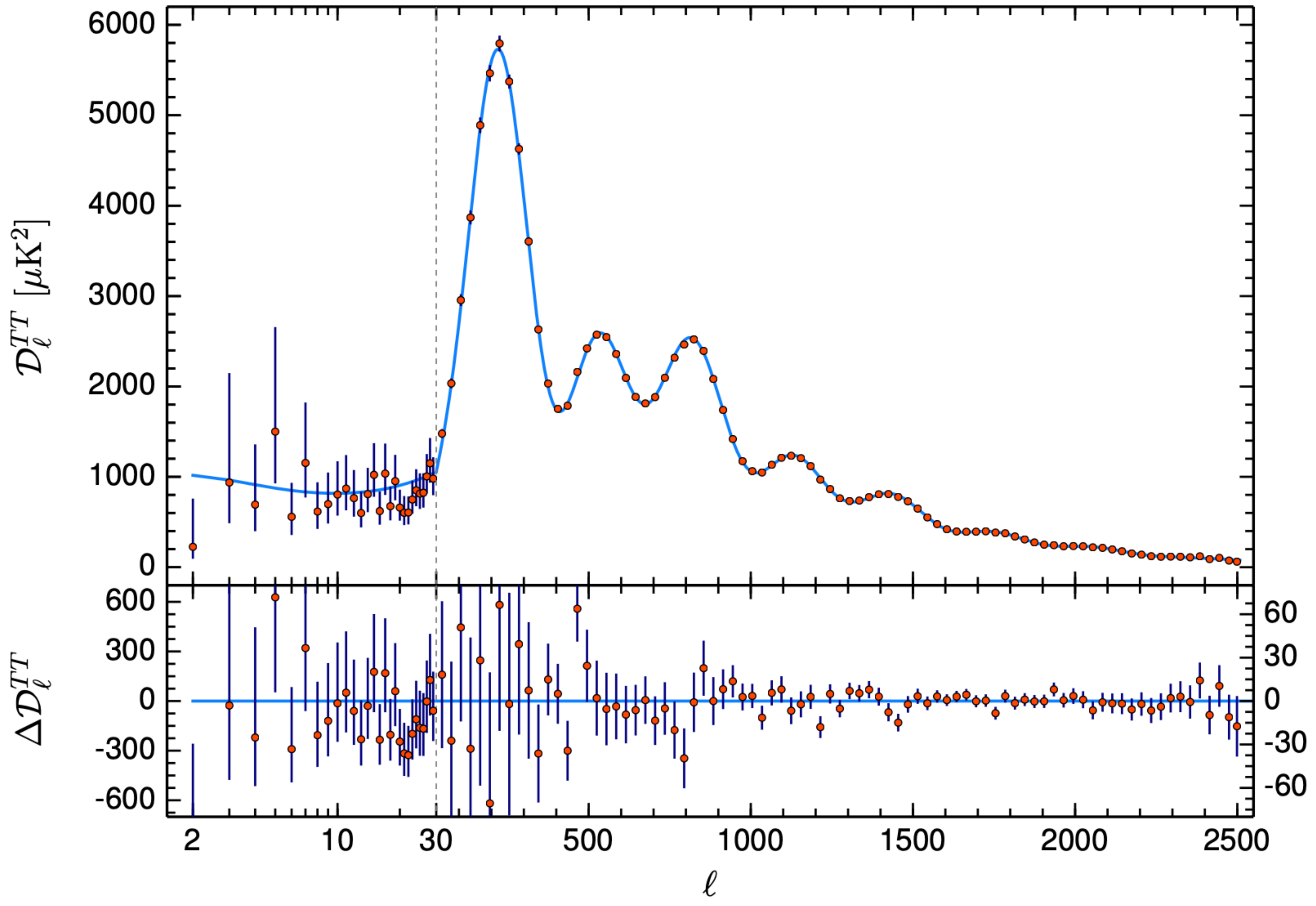

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<u>Question:</u> **Could the tension be solved by my cosmological model?**

<u>My guess:</u> A priori NO, because the curvature tension is mainly due to geometrical effects of curvature.

# CONSEQUENCE FOR EARLY UNIVERSE

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T^{*}_{\mu\nu} + \Lambda g_{\mu\nu}$$

Two additional gauge invariant variables:

- "Spatial velocity" scalar mode: $\mathscr{C}$
- "Spatial velocity" vector mode: $\mathscr{C}^i$

**Wave equation sourced by anisotropic stress**:

$$\mathscr{C}'' + 2\mathscr{H}\mathscr{C}' - \Delta\mathscr{C} = -a^2\kappa\Pi,$$
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<u>Ex:</u> scalar mode equations

$$\Delta\tilde{\Psi} = a^2\frac{\kappa}{2}\rho D_\rho + \mathscr{H}\left(\tilde{\Psi}' - \tilde{\Phi}'\right)$$
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$$\ldots$$

▸ Reference Bardeen potentials:

- Acceleration: $\tilde{\Phi} := \Phi - \mathscr{C}'' - \mathscr{H}\mathscr{C}'$
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—–> **Early Universe analysis required.**

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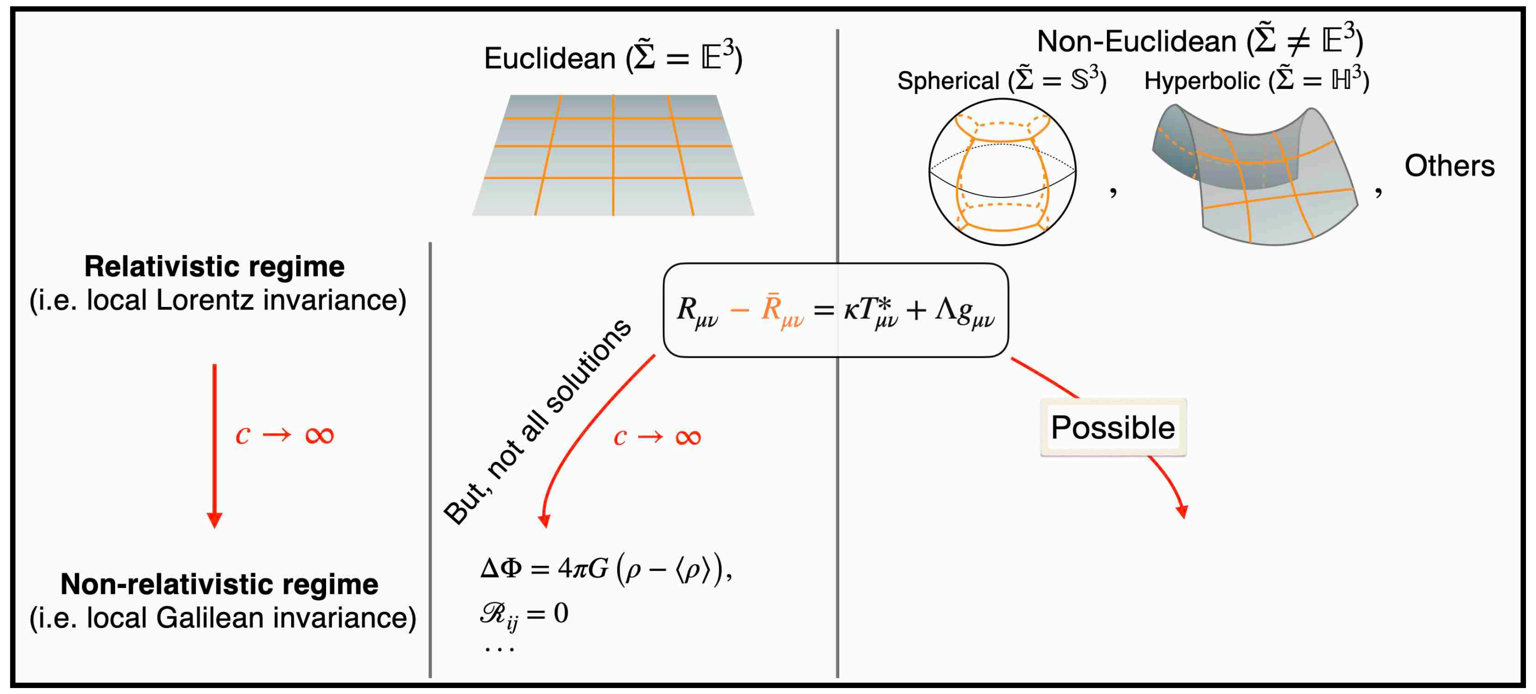

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