Scale invariant SM and inhomogeneous universe

Remembering Graham Ross Sept. 4th 2022

Photo: A. M. Kobos.
PLANCK 2012 Conference, Warsaw.
Paul Steinhardt, Subir Sarkar, Graham Ross, Zygmunt Lalak, Burt A. Ovrut.
R²/Higgs inflation and the hierarchy problem
e-Print: 2108.06095 [hep-ph]

Starobinsky inflation, gravitational contact terms, and the induced Brout-Englert-Higgs boson mass
Christopher T. Hill (Fermilab), Graham G. Ross (Oxford U.) (Mar 11, 2021)
Racetrack inflation and assisted moduli stabilisation

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Abstract

We present a model of inflation based on a racetrack model \textit{without} flux stabilization. The initial conditions are set automatically through topological inflation. This ensures that the dilaton is not swept to weak coupling through either thermal effects or fast roll. Including the effect of non-dilaton fields we find that moduli provide natural candidates for the inflaton. The resulting potential generates slow-roll inflation without the need to fine-tune parameters. The energy scale of inflation must be near the GUT scale and the scalar density perturbation generated has a spectrum consistent with WMAP data.

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Large scale structure from biased nonequilibrium phase transitions - percolation theory picture

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Abstract

We give an analytical description of the spatial distribution of domain walls produced during a biased nonequilibrium phase transition in the vacuum state of a light scalar field. We discuss in detail the spectrum of the associated cosmological energy density perturbations. It is shown that the contribution coming from domain walls can enhance the standard cold dark matter spectrum in such a way as to account for the whole range of IRAS data and for the COBE measurement of the microwave background anisotropy. We also demonstrate that in case of a biased phase transition which allows a percolative description, the number of large size domain walls is strongly suppressed. This offers a way of avoiding excessive microwave background distortions due to the gravitational field of domain walls present after decoupling.
Inflation in a scale invariant universe


No fifth force in a scale invariant universe


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Weyl Current, Scale-Invariant Inflation and Planck Scale Generation


SM + dilaton

with D. Ghilencea, P. Olszewski, P. Michalak
Quantum scale symmetric effective lagrangian

No scale anomaly in

\[ \mathcal{L}^{(0)}(\phi, \sigma) = \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} (\partial \sigma)^2 - \mu^2 \varepsilon(\sigma) \left[ V(\phi, \sigma) + \sum_{n=0} \lambda_n \frac{\phi^{4+2n}}{\sigma^{2n}} \right] \]

go to broken phase

\[ \mathcal{L}^{(0)}(\phi_0 + \phi', \sigma_0 + \phi') \]

compute loop corrections (in momentum expansion) & RGE functions \( \beta, \gamma \)

\[ \mathcal{L}_{\text{eff}}(\phi, \sigma) = -V_{\text{eff}}(\phi, \sigma) + \ldots \]

- Homogenous function (no mass-parameters, only vev's)
- \( \mathbb{Z}^2 \times \mathbb{Z}^2 \) sym.
- Satisfies Callan-Symanzik eq.
Quantum scale symmetric effective lagrangian

RG-improvement:

\[ \mu = e^t \mu_0 , \quad \lambda(t) \phi^4 + \frac{\lambda^2(t)}{64\pi^2} \log \left( \frac{\phi}{e^t \sigma} \right)^2 + \ldots \]

Choose \( t = t(\phi, \sigma) \sim \log \frac{\phi}{\sigma} \)

to avoid large logs.

Spontaneous scale-symmetry breaking:

\[
\begin{pmatrix} \phi \\ \sigma \end{pmatrix} = M \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix} , \quad V_{\text{eff}} = M^4 W(\theta) ,
\]

flat direction in \( V_{\text{eff}} \) \( \Rightarrow \nexists \theta = \theta_0 \ W(\theta_0) = W'(\theta_0) = 0 \)

renormalization condition, similar to choosing C.C.

- **Hierarchy** of scales via aligning the flat direction \( \downarrow \phi \rightarrow \theta_0 \approx \frac{\phi_0}{\sigma_0} \ll 1 \)

- New perspective on **naturalness**: is this alignment stable wrt. embedding in a UV completion?
Symmetry breaking at finite temperature
Scale symmetric Lagrangian - minimal vn

\[
\frac{\mathcal{L}}{\sqrt{g}} = -\frac{1}{12} (\xi_0 \phi_0^2 + \xi_1 \phi_1^2) R + \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 + \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - V(\phi_0, \phi_1),
\]

\[
V(\phi_0, \phi_1) = \lambda_0 \phi_0^4 + \lambda_1 \phi_0^2 \phi_1^2 + \lambda_2 \phi_1^4.
\]

hierarchy of small couplings

\[
\lambda_2 \gg |\lambda_1| \gg \lambda_0
\]

leads to the hierarchy of vets

\[
\langle \phi_1^2 \rangle = -\frac{\lambda_1}{2 \lambda_2} \langle \phi_0^2 \rangle, \quad \lambda_0 = \frac{\lambda_1^2}{4 \lambda_2}, \quad \langle R \rangle = 0,
\]
Scale symmetric Lagrangian - minimal \( vn \)

\[
m_H^2 = -4\lambda_1 \left( 1 - \frac{\lambda_1}{2\lambda_2} \right) \langle \phi_0^2 \rangle, \quad v^2 = \langle \phi_1^2 \rangle = -\frac{\lambda_1}{2\lambda_2} \langle \phi_0^2 \rangle.
\]

\[
\frac{1}{6} \left( \xi_0 - \frac{\lambda_1}{2\lambda_2} \xi_1 \right) \langle \phi_0^2 \rangle = M_{Planck}^2.
\]
Scale symmetric Lagrangian - non-minimal

\[ V = \lambda_0 \phi_0^4 + \sum_{n=0}^{N} \left( \lambda_{2n+1} \phi_n^2 \phi_{n+1}^2 + \lambda_{2n+1} \phi_{n+1}^4 \right) \]

Take \( N=2 \)

\[ \phi_1^2 = -\frac{2\lambda_0}{\lambda_1} \phi_0^2, \quad \phi_2^2 = -\frac{\lambda_3}{2\lambda_4} \phi_1^2 \]

with tuning of couplings

\[ \lambda_0 = \frac{\lambda_1^2}{4\lambda_2 - \lambda_3/\lambda_4} \]

assuming \( \lambda_2 \sim \lambda_4 \sim 1 \)

hierarchy of vevs

\[ \phi_1^2 = -\lambda_1 \phi_0^2, \quad \phi_2^2 = -\lambda_3 \phi_1^2 \]
Scale symmetric Lagrangian - minimal vn thermal corrections

To obtain temperature corrections one adds to potential temperature dependent parts:

\[ V(\phi_0, \phi_1) \to V(\phi_0, \phi_1) + \delta V_T(\phi_0, \phi_1, T) + \delta V_{ring}(\phi_0, \phi_1, T). \]

\( \delta V_T \) stands for standard temperature corrections of first order:

\[ \delta V_T(\phi_0, \phi_1, T) = \frac{T^4}{2\pi^2} \left[ \sum_{i=\text{bosons}} n_i \cdot J_B(\frac{m_i^2(\phi_k)}{T^2}) + \sum_{j=\text{fermions}} n_j \cdot J_F(\frac{m_j^2(\phi_k)}{T^2}) \right], \]

\[ \delta V_{ring} = -\frac{T}{12\pi} \left( m_{eff}(\phi_i, T)^3 - m_i(\phi_i)^3 \right), \]

\[ V_{eff} = V_{T=0} + \frac{1}{2} \phi_1^2 \cdot \left( \lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_i^2}{4} \right) T^2 + \frac{1}{2} \phi_0^2 \cdot \frac{\lambda_1}{6} T^2 = V_{T=0} + \frac{\gamma T^2}{2} \phi_1^2 + \frac{\lambda_1 T^2}{12} \phi_0^2. \]
\[ V_{\text{eff}} = V_{T=0} + \frac{1}{2} \phi_1^2 \cdot \left( \lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_i^2}{4} \right) T^2 + \frac{1}{2} \phi_0^2 \cdot \frac{\lambda_1}{6} T^2 = V_{T=0} + \frac{\gamma T^2}{2} \phi_1^2 + \frac{\lambda_1 T^2}{12} \phi_0^2. \]

mismatch between eoms gives tree-level tuning

\[ \left( \frac{\lambda_2}{12} - \gamma \right) T^2 \neq 0. \] (41)

This is the amount of the scale symmetry breaking by the finite temperature effects. As the result, the only consistent solution to the corrected equations of motion becomes at this order

\[ \phi_1 = 0, \quad \phi_0 = -\frac{\lambda_2}{24\lambda_1} T^2. \] (42)
Thermal fluctuations force the dilation vev away from the origin

Thermal fluctuations in Higgs direction

$$\langle \phi_1^2 \rangle_{T,p} = T \frac{p^3}{\omega_p^2}$$

result in the negative mass squared term for the dilation

$$\delta_m V = \lambda_1 \phi_0^2 \phi_1^2 \rightarrow \lambda_1 T^2 \phi_0^2,$$

and in the repulsive force

$$-\frac{\partial \delta_m V}{\partial \phi_0} = -2\lambda_1 T^2 \phi_0,$$
Figure 1: Plots of $V_{full}(\phi_0, \phi_1, T)$ for different temperatures and $\lambda_1 = -10^{-6}$ value. Orange dashed line marks flat direction $\phi_1^2 = -\frac{\Delta_1}{2\lambda_2}$. It is easy to see that as the temperature increase, the flat direction no longer exists and the scale symmetry is broken.
Figure 5: Ratio $f(T) = (T^3\langle \sigma v \rangle)/H$ as a function of temperature for different $\lambda_1$. $\phi_0$ field can reach thermal equilibrium for sufficiently large $|\lambda_1|$ value.
Figure 7: Evolution of $\phi_i$ fields and $H$ with time for coupling constants values fulfilling requirements from section 3.1: $\lambda_2 = 0.03125$, $\lambda_1 = -4.37 \cdot 10^{-26}$, $\xi_0 = 10^{10}$, $\xi_1 = 0.1$. Initial conditions: $\phi_0(0) = 8 \cdot 10^{13}$, $\dot{\phi}_0(0) = 5 \cdot 10^{13}$, $\phi_1(0) = 0$, $\dot{\phi}_1(0) = 10$, and two different $H(0) = H_0$ values. The bigger the initial $H_0$, the faster $\phi_i$ fields loose their velocity and settles in lower values. Two plots for $\phi'_0(t)$ are shown, one for the same time range as in the evolution of $\phi_1$ and $H$, one for later times, to show that $\phi_0$ indeed loose its velocity and settles in desired value.
Figure 8: Evolution of $\phi_i$ fields and $H$ with time for non-zero temperature and coupling constants values $\lambda_2 = 0.03125$, $\lambda_1 = -10^{-6}$, $\xi_0 = 10^3$, $\xi_1 = 0.1$. Initial conditions: $\phi_0(0) = 3 \cdot 10^4$, $\dot{\phi}_0(0) = 5 \cdot 10^3$, $\phi_1(0) = 0$, $\dot{\phi}_1(0) = 10$, and two different $H(0) = H_0$ values. Initial temperature $T_0 = 10^4$ GeV. After fields $\phi_i$ land in flat direction, they start to roll along that flat valley to higher values and they don’t stop.
Conclusions I

- Scale symmetry as the underlying symmetry offers a way to understand the origin of scales as expected.
- Scale symmetry is broken at finite T with thermal dilaton vev proportional to T.
- Cosmological evolution can easily lead to large dilaton vev needed to model hierarchy.
Quantum scale symmetric SM + \( \sigma \)

\[
H = \begin{pmatrix}
0 \\
\frac{\phi}{\sqrt{2}}
\end{pmatrix}
\]
(electroweak vacuum \(\rightarrow\) electroweak flat direction)

\[
\mathcal{L}_{SM} \bigg|_{m^2 = 0, \mu = \mu(\sigma)} + \frac{1}{2} (\partial \sigma)^2 - \lambda_m |H|^2 \sigma^2 - \frac{\lambda_\sigma}{4} \sigma^4 + \sum_{n=0} \lambda_n \frac{|H|^{4+2n}}{\sigma^{2n}}
\]

\[
V_{\text{eff}}^{\text{SM}}(\phi, \sigma) \approx \frac{1}{4} \lambda_\text{eff} \left( \log \frac{\phi}{\sigma} \right) \phi^4 = M^4 \lambda_\text{eff} (\log \tan \theta) \frac{\tan^4 \theta}{(1 + \tan^2 \theta)^2}
\]

Tunneling via **2-dim instanton** (Coleman’s bounce), in the presence of nonrerm. terms. (Even stronger) motivation to stabilise the \(V_{\text{eff}}\) completely: \(\lambda_\text{eff} > 0\)
FIG. 2. Contour plots of the effective potentials $-\mathcal{V}_{\text{SM}}(\phi, \sigma)$ for various choices of $\langle \sigma \rangle$. Lower green dashed line marks the electroweak vacuum-direction, higher green dashed line marks the direction of greatest instability. Red continuous line is a plot of the bounce configuration $(\phi_B, \sigma_B)$. (Note that, mainly due to varying contribution of the nonrenormalizable interaction from one plot to another, the plots present differing potentials and it would be misleading to plot the bounce configurations in a single frame.)
Summary

1) You may use a field as the scale $\mu$ in Dim-Reg to preserve scale symmetry at the quantum level.

2) The price to pay: infinitely many nonpolynomial $\phi/\sigma$ operators and corresponding couplings: nonrenormalizability.

3) Minimal subtraction scheme involves evanescent interactions.

4) Presence of a flat direction $\leftarrow$ tuning.

5) Naturalness: aligning the flat direction perpendicular to Higgs

6) Instability = unboundedness below
Domain walls

T. Krajewski, M. Lewicki
Phys. Rev. D 104, 123522
Network of walls prefers the true vacuum!
Models of interest

- Radiatively generated minima (e.g., SM at large field strength)

- Run-away potentials (moduli of stringy models), Quantum Scale Symmetric SM

- Models of strong first-order phase transitions - colliding bubbles (thermal effects play a role)
Models of interest

• Monodromy axion models, relaxion

\[ V_{\text{monodromy}}(\phi) = m^2 \phi^2 + \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] \]

\[ V_{\text{relaxion}}(\phi) = g\phi + \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] \]
Generic potential

\[ V_{AS}(\phi) = \frac{V_0}{60} \phi \left( 15\phi^3 \left( e^2 (2d(a + b + c) + ab + ac + bc + d^2) + 1 \right) - 60abc (d^2 e^2 + 1) 
\right.

\left. - 20\phi^2 \left( e^2 (d^2(a + b + c) + 2d(a(b + c) + bc) + abc) + a + b + c \right) - 12e^2 \phi^4 (a + b + c + 2d)
\right.

\left. + 30\phi \left( de^2 (a(b + c) + 2abc + bcd) + ab + ac + bc \right) + 10e^2 \phi^5 \right) . \tag{3.6} \]

\[ \frac{\partial^3 V_{AS}}{\partial \phi^3}(\phi) = 2V_0 \left( e^2(a - \phi)(\phi - b)(c + 2d - 3\phi)
\right.

\left. + (-a - b + 2\phi) \left( e^2(d - \phi)(2c + d - 3\phi) + 1 \right) + (\phi - c) \left( e^2(d - \phi)^2 + 1 \right) \right) \tag{3.7} \]

\[ a, b - \text{positions of minima, } c - \text{position of maximum} \]

\[ \delta V = V_{AS}(b) - V_{AS}(a), \]

\[ d3V = \frac{\partial^3 V}{\partial \phi^3}(c), \]

\[ 5 = w, \]
Quantities of interest: energy density and peak frequency

\[ \Omega_{GW}(\eta) := \frac{1}{\rho_c(\eta)} \frac{d\rho_{GW}}{d \log |k|}(\eta, k). \]

\[ \Omega_{GW}(\eta_{dec})\big|_{peak} = \frac{\tilde{\epsilon}_{GW} A^2 \sigma_{wall}^2}{24 \pi H_{dec}^2 M_{Pl}^4}, \]

\[ \Omega_{GW}(\eta_0) = \left( \frac{a(\eta_{dec})}{a(\eta_0)} \right)^4 \left( \frac{H(\eta_{dec})}{H(\eta_0)} \right)^4 \Omega_{GW}(\eta_{dec}) \]
Quantities of interest: energy density and peak frequency

\( \tilde{\varepsilon}_{GW} \) efficiency parameter between 0.7 and 1

\( \sigma_{walls}, \eta_{dec} \) - taken from simulations

\[
\frac{A}{V} = \frac{a(t)S_{wall}}{H^{-3}} \propto \frac{a(t)}{t}.
\]

\[
\frac{A}{V} = A\eta^{-1},
\]

stable DW: \( A \) in the range 0.8 ± 0.1
Quantities of interest: energy density and peak frequency

more generally

\[ \log \left( \frac{A}{V} \right) = -\nu \log \eta + \log A \]

scaling regime: obtained \( \nu \) ranges from 0.81 to 1.0

meta-stable DW: \( A \) in the range 0.08 – 0.34
**Figure 9:** The evolution of conformal surface area of domain walls per unit volume $\frac{A}{V}$ in function of conformal time $\eta$ (blue) and the fitted scaling behavior defined by eq. (5.8) (orange) for the best (left panel) and the worst (right panel) fits obtained by procedure described in the main text. Vertical dashed lines correspond to the estimated beginning and end of the scaling regime.
Quantities of interest: energy density and peak frequency

\[ \Omega_{GW}(\eta_0)_{\text{peak}} = 4.6 \times 10^{-81} A^2 \left( \frac{\text{GeV}}{H_{\text{dec}}} \right)^2 \left( \frac{\sigma_{\text{wall}}}{\text{GeV}^3} \right)^2 h^{-2} \left( \frac{100}{g_* (\eta_{\text{dec}})} \right)^{1/3} . \]

\[ f_0 |_{\text{peak}} = \frac{a(\eta_{\text{dec}})}{a(\eta_0)} H_{\text{dec}} = 1.63 \times 10^2 \left( \frac{H_{\text{dec}}}{\text{GeV}} \right)^{1/2} \text{ Hz}. \]

\[ \Omega_{GW}(\eta_0)_{\text{peak}} = 0.29 \times 10^{-77} A^2 \left( \frac{\eta_{\text{dec}}}{w} \right)^4 \left( \frac{\sigma_{\text{wall}}}{w^{-3}} \right)^2 \left( \frac{\text{GeV}^{-1}}{w} \right)^4 , \]

\[ f_0 |_{\text{peak}} = 3.3 \times 10^1 \left( \frac{w}{\eta_{\text{dec}}} \right) \left( \frac{\text{GeV}^{-1}}{w} \right)^{1/2} \text{ Hz}, \]
Quantities of interest: energy density and peak frequency

We have estimated overall factors present in eqs. (6.7) and (6.7) basing on values of $\mathcal{A}$, $\eta_{dec}$ obtained in simulations in which networks entered scaling regime and previously computed $\sigma_{wall}$. The maximal value of the prefactor in eq. (6.7) obtained in this way is equal to:

$$
\Omega_{GW}^{\text{max}}(\eta_0)|_{\text{peak}} = 0.1 \times 10^{-66} \left( \frac{\hbar c}{\text{GeV}} \right)^4, \quad f_0^{\text{max}}|_{\text{peak}} = 0.7 \left( \frac{\hbar c}{\text{GeV}} \right)^{\frac{1}{2}} \text{Hz}, \quad (6.9)
$$

where the frequency of the peak for this network is denoted as $f_0^{\text{max}}$. On the other hand, the minimal prefactor computed from data from simulations is equal to:

$$
\Omega_{GW}^{\text{min}}(\eta_0)|_{\text{peak}} = 0.6 \times 10^{-68} \left( \frac{\hbar c}{\text{GeV}} \right)^4, \quad f_0^{\text{min}}|_{\text{peak}} = 1.3 \left( \frac{\hbar c}{\text{GeV}} \right)^{\frac{1}{2}} \text{Hz}. \quad (6.10)
$$
Figure 10: Hypothetical peak amplitudes of GWs emitted from cosmological domain walls as a function of the peak frequency $f$ compared to predicted sensitivities of current and planned detectors LIGO [59–62], LISA [63, 64], AEDGE [65], AION-1km [66], ET [67, 68] as well as upper bound induced by the CMB/BBN [69, 70].
Summary II

• For a strong signal and a low frequency peak a period of stable evolution is needed
• Bias of the initial distribution easily destabilises the network
• Asymmetry of the potential destabilises the network for symmetric distributions
• Short living networks may give a strong signal if the energy scale is very large - but this produces a high frequency peak, beyond current sensitivity
• Decaying networks of domain walls produce a signal in the form of gravitational waves - too weak to be detected anytime soon - if a signal is detected then either fine-tuning or non-standard cosmology have occurred
Thanks Graham!