Progress in the Standard Model precision calculations

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Increasing precision allows us to change paradigms

- Tycho de Brahe ($\sim$ 1601) Mars orbits, Rudolphine tables;
  $\longrightarrow$ Johannes Kepler ($\sim$ 1609) - planets laws;
- O. Lummer et al. (1900) - blackbody radiation
  $\longrightarrow$ M. Planck (1900) - quanta; EXPERIMENT $\rightarrow$ THEORY
- Gravitational waves; THEORY $\rightarrow$ EXPERIMENT

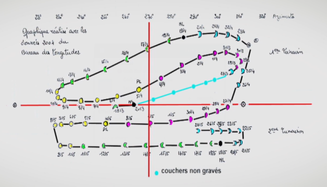
Particle Physics

- (i) muon discovery, J/Ψ
  (ii) $(g-2)_e$, $(g-2)_\mu$ EXPERIMENT $\rightarrow$ THEORY
  (iii) V-A, parity; Note the 100^{th} Birthday Anniversary of Prof. Chen Ning Yang, link
- (i) $\tau^\pm$ (tau lepton);
  (ii) Tevatron - top quark discovery; THEORY $\rightarrow$ EXPERIMENT
  (iii) H^0 (scalar Higgs-Englert boson)

  SM corrections matters! LEP, SLAC, LHC (see backup slides)
  $\longrightarrow$ M. Veltman (1977) ρ-parametr $\sim m_t^2, \ln(m_H^2)$;
- Neutrinos (masses, mixing angles, CP phase(s));

Future Colliders (FC); THEORY $\leftrightarrow$ EXPERIMENT

Precision, true inception



Abri Blanchard bone ($\sim$ 30 000 BC),

Alexander Marschack, ‘Cognitive Aspects of Upper Paleolithic Engraving’ , Current Anthropology (1972) Vol.13, 3/4

Interpretation: Chantal Jegues-Wolkiewicz
$\longrightarrow$ probably the first lunar calendar

Similarly Lascaux caves’ paintintings, $\sim$ 17 000 BC.

Discovery strategies in PP

Two ways for discoveries (in both cases precision is crucial):

1. within the known theory (anomalies[1])
2. new processes and (rare) phenomena;

[1]'I have always suspected that, one day, (...) they [JG: experimentalists] would like to see what would happen, just for the fun of it, if they falsely report that there exists a certain bump, or an oscillation in a certain curve, and see how the theorists predict it. I know these men so well that the moment I thought of that possibility I have honestly always been concerned that some day they will do just that. Then you can imagine how absurd the theoretical physicists would sound, making all these complicated calculations to demonstrate the existence of such a bump, while these fellows are laughing up their sleeves.' – R.P. Feynman)

I have chosen to discuss:

`(i) Z-pole precision physics;`
`(ii) Neutrino precision physics [2];`

$\longrightarrow$ K. Grzanka, continued

[2] Massive neutrinos (RHNs) are in the (non-minimal?) SM.

NOT COVERED

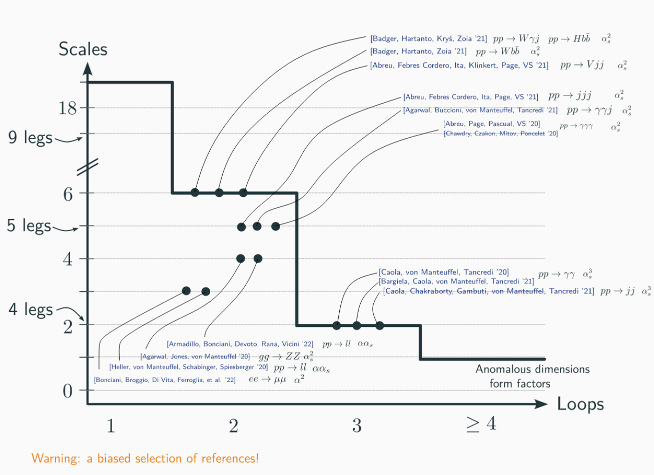
Future: W, t, H

- $e^+e^- \to W^+W^-$ at 161 GeV: $\delta m_W^{exp} = 0.5 \div 1$ MeV.
  Challenge to get the same TH error:
  $$\text{NNLO } e^+e^- \to 4f.$$
- $e^+e^- \to t\bar{t}$ at 350 GeV: $\delta {m_t}^{exp} =$ 17 MeV
  Big challenge for theory, today > 100 MeV, future projection $\leq$ 50 MeV:
  $\sim$ 10 MeV unc. from mass def.;
  $\sim$ 15 MeV from $\alpha_{\rm s}$ unc. to threshold mass def.;
  ~ 30 MeV - h. orders resummation
- $e^+e^- \to HZ$ at 240 GeV: Kinematic constraint fits with $Z \to ll$ and $H \to bb$, ...,
  $m_H = 125.35$ GeV ± 150 MeV [link CMS], $\Gamma_H = 4.1^{5.1}_{4.0}$ MeV, $\Gamma_H <$ 13 MeV at 95 % C.L., 1901.00174
  $\delta m_H^{exp} =$ 10 MeV; Theory errors subdominant.

Monte Carlo generators (**not discussed!**) 'QED challenges at FCC-ee precision measurements', S. Jadach and M. Skrzypek, Eur.Phys.J.C 79 (2019) 9, 756 1903.09895

Workshop: Precision calculations for future e+e colliders: targets and tools,

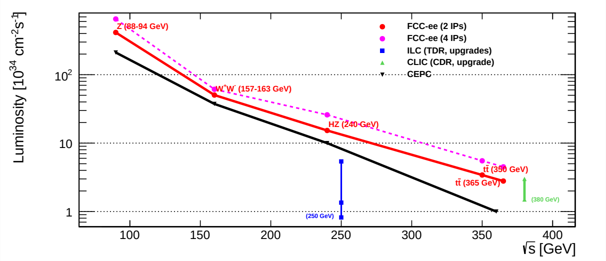
CERN 2022, talk by V. Sotnikov



LHC, HL-LHC: QCD, EW-QCD, ...

NEEDS FOR PRECISION: THE FUTURE

Motivation For Precision Studies: Z,W,H,t and flavour electroweak factories



https://arxiv.org/abs/2203.06520 [The Future Circular Collider: a Summary for the US 2021 Snowmass Process]

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})	Event Statistics
FCC-ee-Z	4	88-94	150	$5 \cdot 10^{12}$ **Z decays**
FCC-ee-W	2	157-163	10	10^8 WW events
FCC-ee-H	3	240	5	10^6 ZH events 25k WW $\rightarrow H$
FCC-ee-tt	5	340-365	0.2 $\div 1.5$	10^6 $t\bar{t}$ even ts 200k ZH 50k WW $\rightarrow H$

Jorgen D'Hondt, "Strategies and plans for particle physics in Europe",

Epiphany 2021, https://indico.cern.ch/event/934666

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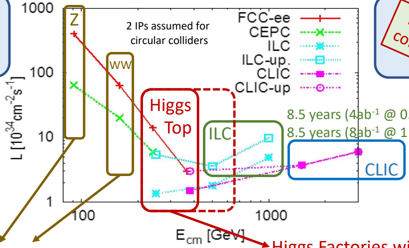
e^+e^- Higgs Factories (incl. B/c/τ/EW/top factories)

precision frontier — circular colliders

synchrotron radiation

for the same power, less luminosity at higher E_{cm}

(Energy Recovery Linac technology might mitigate this & allow to go to higher E_{cm})



linear colliders — energy frontier

8.5 years ($4ab^{-1}$ @ 0.5 TeV)

8.5 years ($8ab^{-1}$ @ 1 TeV)

7 years ($2.5ab^{-1}$ @ 1.5 TeV)

8 years ($5ab^{-1}$ @ 3 TeV)

B/c/τ/EW Factories

per detector in e^+e^-	# Z	# B	# τ	# charm	# WW
LEP	4×10^6	1×10^6	3×10^5	1×10^6	2×10^4
SuperKEKB	-	10^{11}	10^{11}	10^{11}	-
FCC-ee	2.5×10^{12}	7.5×10^{11}	2×10^{11}	6×10^{11}	1.5×10^8

Higgs Factories with complementarity

- g_{HZZ} (250GeV) versus g_{HWW} (380GeV)
- top quark physics
- beam polarization for EW precision tests

(transverse polarization in circular e^+e^- colliders only at lower E_{cm} while longitudinal polarization at linear colliders)

Tera-Z Physics:

BREATHTAKING

EXPERIMENTAL

PRECISION

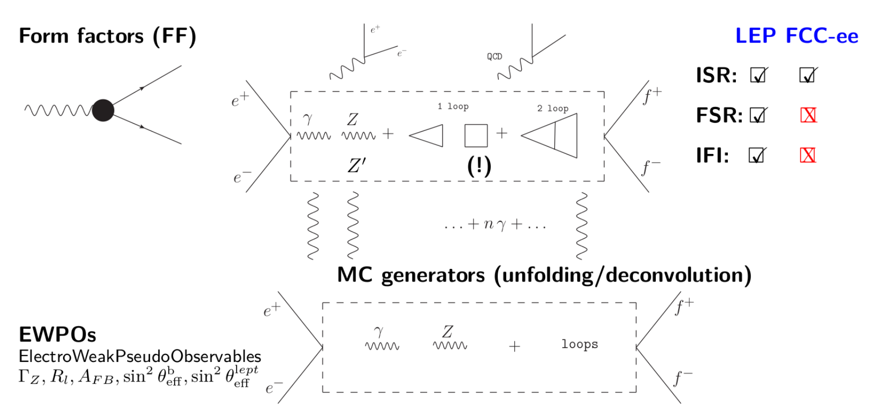
Table 3 Measurement of selected precision measurements at FCC-ee, compared with present precision. Statistical errors are indicated in boed phase. The systematic uncertainties are initial estimates, aim is to improve down to statistical errors. This set of measurements, together with those of the Higgs properties, achieves indirect sensitivity to new physics up to a scale Λ of 70 TeV in a description with dim 6 operators, and possibly much higher in specific new physics (non-decoupling) models

Observable	Present value ± error	FCC-ee stat.	FCC-ee syst.	Comment and leading exp. error
m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan
				Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan
				Beam energy calibration
$\sin^2\theta_W^{eff}(\times 10^6)$	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak
				Beam energy calibration
$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	Small	From $A_{FB}^{\mu\mu}$ off peak
				QED&EW errors dominate
R_ℓ^Z $(\times 10^3)$	20767 ± 25	**0.06**	0.2–1	Ratio of hadrons to leptons
				Acceptance for leptons
$\alpha_s(m_Z^2)$ $(\times 10^4)$	1196 ± 30	**0.1**	0.4–1.6	From R_ℓ^Z above
σ_{had}^0 $(\times 10^3)$ (nb)	41541 ± 37	**0.1**	4	Peak hadronic cross section
				Luminosity measurement
$N_\nu(\times 10^3)$	2996 ± 7	**0.005**	1	Z peak cross sections
				Luminosity measurement
R_b $(\times 10^6)$	216290 ± 660	**0.3**	< 60	Ratio of $b\bar{b}$ to hadrons

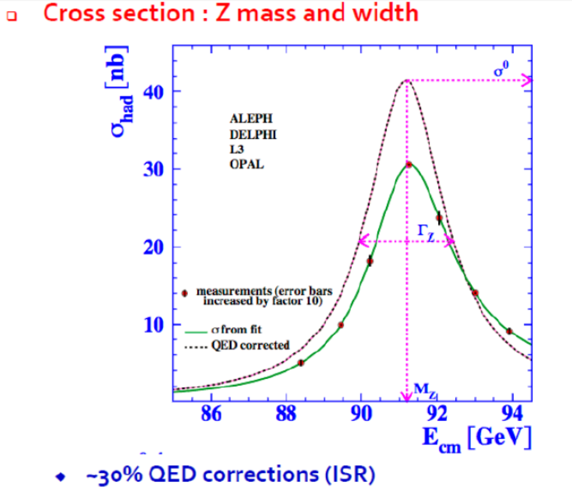
LEP/SLC HAS PAVED THE WAY

MC generators and theory (Z-pole)

Experimental measurements at Z-pole: after unfolding ($\longrightarrow$ Z. Was talk)



Altogether $17 \cdot 10^6$ Z-boson decays at LEP



Z-resonance: QED and EW, *Standard Model Theory for the FCC-ee Tera-Z stage,* https://e-publishing.cern.ch/index.php/CYRM/issue/view/89

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1. Z-resonance and $\gamma, Z', \ldots \longrightarrow$ Laurent series,

$$\mathcal{M} = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n \; B^{(n)}, \quad s_0 = \bar{M}_Z^2 + i\bar{M}_Z\bar{\Gamma}_Z.$$

2. We want to extract EW Z-vertex couplings and definitions like $\sin^2\theta^f_{\text{eff}}$, but in reality, we deal with complicated process

$$e^+e^- \to f^+f^- \quad + \text{invisible } (n\ \gamma + \mathrm{e^+e^-pairs} + \cdots)$$

$$\sigma^{e^+e^-\to f^+f^- + \cdots}(s) = \int dx \; \overbrace{f(x)} \;\; \underbrace{\sigma^{e^+e^-\to f^+f^-}(s')} \; \delta(x - s'/s)$$

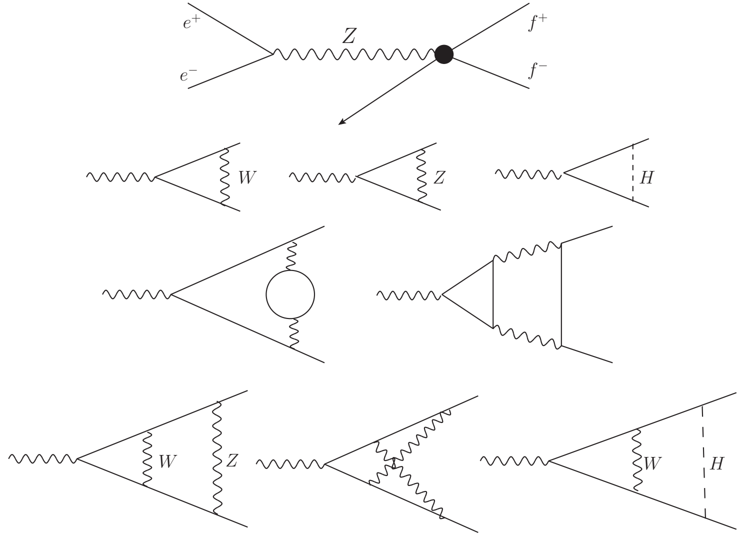
$\longrightarrow$ form factors, QED separation/deconvolution, non-factorizations (backup slides)

To determine the structure function/flux function kernels and hard scattering ansatz for data preparation or for unfolding is one of the challenges of the Tera-Z physics.

EWPOs and

$N^x LO$ SM CORRECTIONS

Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections



EWPOs, Z pole

$$\sigma^0_{\mathrm{had}} = \sigma[e^+e^- \to \text{hadrons}]_{s=M_Z^2},$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}],$$

$$R_\ell = \frac{\Gamma[Z \to \text{hadrons}]}{\Gamma[Z \to \ell^+\ell^-]}, \quad \ell = e, \mu, \tau,$$

$$R_q = \frac{\Gamma[Z \to q\bar{q}]}{\Gamma[Z \to \text{hadrons}]}, \quad q = u, d, s, c, b.$$

The remaining EWPOs are cross section asymmetries, measured at the Z pole, e.g., forward-backward asymmetry

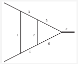
$$A^f_{\mathrm{FB}} = \frac{\sigma_f\left[\theta < \frac{\pi}{2}\right] - \sigma_f\left[\theta > \frac{\pi}{2}\right]}{\sigma_f\left[\theta < \frac{\pi}{2}\right] + \sigma_f\left[\theta > \frac{\pi}{2}\right]},$$

where θ is the scattering angle between the incoming e^- and the outgoing f.

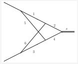
EWPOs and Form Factors

fermionic,bosonic

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) + a_b(s)\gamma_5] = \cdots +$$



$+$



$+ \cdots$

planar,non−planar

Note approximate factorization of weak couplings

$$A_{FB} = \frac{\left[\int_0^1 dcos\theta - \int_{-1}^0 dcos\theta\right] \frac{d\sigma}{dcos\theta}}{\sigma_T} \sim \overbrace{\frac{2a_e v_e}{a_e^2 + v_e^2}}^{A_e} \overbrace{\frac{2a_f v_f}{a_f^2 + v_f^2}}^{A_f} + \text{corrections}$$

$$A_f = \frac{2\Re e\frac{v_f}{a_f}}{1 + \left(\Re e\frac{v_f}{a_f}\right)^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^{\text{f}}}{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^{\text{f}} + 8(Q_f\sin^2\theta_{\text{eff}}^{\text{f}})^2},$$

$$\sin^2\theta_{\text{eff}}^{\text{f}} = F\left(\Re e\frac{v_f}{a_f}\right)$$

Input, theoretical and parametric errors,

A. Freitas et al., "Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee", https://arxiv.org/abs/1906.05379

Quantity	FCC-ee	Current **intrinsic** error		Projected intrinsic error (at start of FCC-ee)
M_{W} [MeV]	0.5–1 ‡	4	$(\alpha^3, \alpha^2\alpha_{\mathrm{s}})$	1
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	0.6	4.5	$(\alpha^3, \alpha^2\alpha_{\mathrm{s}})$	1.5
Γ_{Z} [MeV]	0.1	**0.4**	$(\alpha^3, \alpha^2\alpha_{\mathrm{s}}, \alpha\alpha_{\mathrm{s}}^2)$	**0.15**
R_b $[10^{-5}]$	6	11	$(\alpha^3, \alpha^2\alpha_{\mathrm{s}})$	5
R_l $[10^{-3}]$	1	6	$(\alpha^3, \alpha^2\alpha_{\mathrm{s}})$	1.5

‡ The pure experimental precision on M_{W} is $\sim 0.5\,\mathrm{MeV}$.

Quantity	FCC-ee	future **parametric** unc.	Main source
M_{W} [MeV]	$0.5-1$	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	0.6	2 (1)	$\delta(\Delta\alpha)$
Γ_{Z} [MeV]	0.1	0.1 (0.06)	$\delta\alpha_{\mathrm{s}}$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_{\mathrm{s}}$
R_ℓ $[10^{-3}]$	1	1.3 (0.7)	$\delta\alpha_{\mathrm{s}}$

Important input parameter errors are $\delta(\Delta\alpha) = 3\cdot 10^{-5}$, $\delta\alpha_{\mathrm{s}} = 0.00015$.

$M_H, m_t, \ldots \longrightarrow$ K. Grzanka's talk

Input and renormalization schemes

E.g. the bosonic 2-loop corrections shift the value of Γ_Z by 0.51 MeV when using M_W as input and 0.34 MeV when using G_μ as input.

Please recall: $\delta\Gamma_{Z,\mathrm{FCC-ee}} = 0.1$ MeV

Dubovyk et al, https://doi.org/10.1016/j.physletb.2018.06.037

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\mathrm{bos}})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

- 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV (see backup slides)
  2018, exact result: 0.505 MeV

* Fixed values of M_W

Complicated subject, theoretical, parametric errors.

1. *Standard Model Theory for the FCC-ee Tera-Z stage,*
https://e-publishing.cern.ch/index.php/CYRM/issue/view/89
2. *Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee,*
https://arxiv.org/abs/1906.05379

Errors, a simple observation:

1. Lack of knowledge about HO corrections is a real pain; estimates even in the perturbative regime can differ substantially from concrete results.
2. Estimations for each next piece of HO take into account **AMOUNT** of the correction
3. Real calculation gives a **CONCRETE** number, with an error which is at least 2 digits.

`These points are essential when we are at the level of accuracy which approaches experimental precision.`

Updates for error estimations

- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\text{had}}$, ...)

see, Ayres Freitas: https://arxiv.org/abs/1604.00406

E.g.: Intrinsic theory error estimation for Γ_Z, 1804.10236 [1604.00406]

1. Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV } [0.26 \text{ MeV}]$$

$$\delta_2 : \mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.21 \text{ MeV } [0.3 \text{ MeV}]$$

$$\delta_3 : \mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1} \text{ MeV } \underline{[\text{Now we know it!}]}$$

Total: $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5}\delta_i^2} \sim \mathbf{0.4}$ **MeV** [0.5 MeV]

Summary: estimations for higher order EW and QCD corrections

δ_1 :	δ_2 :	δ_3 :	δ_4 :	δ_5 :	$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha_{bos}^2)$	$=\sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from geometric series of perturbation)					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from geometric series of perturbation)					
0.2	0.21	0.23	0.035	$< 10^{-4}$	**0.4**

δ_1' :	δ_2' :	δ_3' :	δ_4 :		$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(N_f^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^2\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$		$\sqrt{\delta_1'^2+\delta_2'^2+\delta_2'^3+\delta_4^2}$
TH2 (extrapolation through prefactor scaling)					
0.04	0.1	0.1	0.035	10^{-4}	**0.15**

A. Blondel et al., "Theory for the FCC-ee : Report on the 11th FCC-ee Workshop Theory and Experiments",
https://e-publishing.cern.ch/index.php/CYRM/issue/view/110

Minimal precision of 3-loop EW calculations, an example.

1. Calculating N^3LO with 10% accuracy (two digits), we can replace intrinsic error estimation

$$\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim 0.4 \text{ MeV}$$

by

$$\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} (\delta_i/10)^2} \sim 0.04 \text{ MeV}.$$

2. The requirement FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV can be met.

METHODS AND TOOLS

Direct numerical approach[3]

- Sector decomposition (SD) method:
  - FIESTA [2016], [A.V.Smirnov]
  - pySecDec [2022], [Expansion by regions with pySecDec],
- The Mellin-Barnes (MB) method:
  - MB [M.Czakon, 2006]
  - MBnumerics [J.Usovitsch, I.Dubovyk, T.Riemann, 2015] – Minkowskian kinematics
- Differential equations (DEs) method:
  - `DiffExp` [F. Moriello, 2019; M. Hidding, 2021],
  - `AMFlow` [X. Liu, Y.-Q. Ma, 2022] `AMFlow`,
  - `SeaSyde` [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vi, 2022]

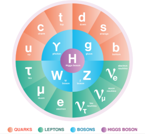
$M_H, m_t, \ldots \longrightarrow$ K. Grzanka's talk

[3] All programs are public

SM and BSM: NEUTRINOS*

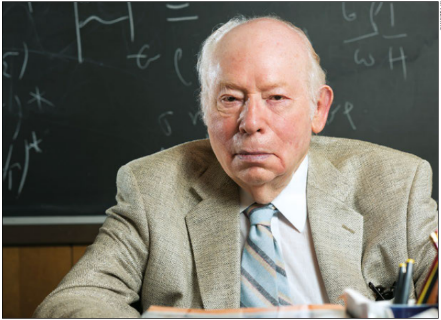
* Neutrino physics itself enters the precision era (mass ordering, C-nature, CP phases); more in backup slides.

LEP ($W^{\pm}, Z$), LHC (H^0) - shaping the Standard Model



STEVEN WEINBERG 1933–2021

A mind to rank with the greatest



M Valentine

Steven Weinberg radically changed the way we look at the universe.

Steven Weinberg, one of the greatest theoretical physicists of all time, passed away on 23 July, aged 88. He revolutionised particle physics, quantum field theory and cosmology with conceptual breakthroughs that still form the foundation of our understanding of physical reality.

Weinberg is well known for the unified theory of weak and electromagnetic forces, which earned him the Nobel Prize in Physics in 1979, jointly awarded with Sheldon Glashow and Abdus Salam, and led to the prediction of the Z and W vector bosons, later discovered at CERN in 1983. His breakthrough was the realisation that some new theoretical ideas, initially believed to play a role in the description of nuclear strong interactions, could instead explain the nature of the weak force. "Then it suddenly occurred to me that this was a perfectly good sort of theory, but I was applying it to the wrong kind of interaction. The right place to apply these ideas was not to the strong interactions, but to the weak and electromagnetic interactions," as he later recalled. With his work, Weinberg had made the next step in the unification of physical laws, after Newton understood that the motion of apples on Earth and planets in the sky are governed by the same gravitational force, and Maxwell understood that electric and magnetic phenomena are the expression of a single force.

In my life, I have built only *one* model

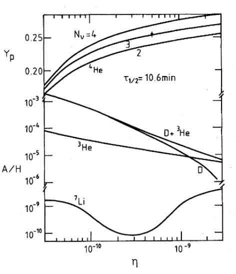
physicists, and will certainly continue to serve future generations.

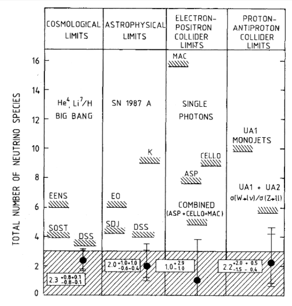
Steven Weinberg is among the very few individuals who, during the course of the history

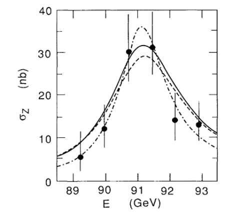
$N_{\rm eff}$: (Good) Things Come in 3s?

The Number of Neutrino Species,

D. Denegri, B. Sadoulet, M. Spiro, Rev.Mod.Phys. 62 (1990) 1







SLC,

$$N_\nu = 3.8 \pm 1.4$$

106 Z events

1989:

Initial measurements of Z-boson resonance parameters in e^+e annihilation, SLC Colaboration

Phys. Rev. Lett. 63, 724

$N_{\rm eff}$: LEP and Now

ALEPH, OPAL, L3, DELPHI, MARKII (SLC): $N_\nu = 3.12 \pm 0.19$
CERN, 13.10.1989, Video (∼12,000 Z decays)
[LEP, 2006] (∼17 mln Z decays)

$$N_\nu = 2.9840 \pm 0.0082$$

Update: [P. Janot and S. Jadach, 2019](only 1σ off from N=3)

$$N_\nu = 2.9963 \pm 0.0074$$

Theorem: [C. Jarlskog, 1990]
In the Standard Model with n left-handed lepton doublets and $N - n$ right-handed neutrinos, the effective number of neutrinos, N_ν, defined by

$$\Gamma(Z \to \nu' s) \equiv N_\nu \Gamma_0,$$

where Γ_0 is the standard width for one masseless neutrino, satisfies

$$N_\nu \leq n.$$

$$\nu_\alpha^{(f)} = \underbrace{(\mathrm{V}_{\rm osc})_{\alpha i}\, \nu_i^{(m)}}_{\text{SM part}} + \underbrace{(\mathrm{V}_{lh})_{\alpha j}\, \widetilde{\nu}_j^{(m)}}_{\text{BSM part}}$$

Cosmology: $N_{eff} = 3.044$. J. Froustey, C. Pitrou, M. Volpe, JCAP 12 (2020) 015, J. Bennett, G. Buldgen, M. Drewes, Y. Wong, JCAP 03 (2020) 003, JCAP 03 (2021) A01

PMNS data analysis (Nonunitarity), 3+1

New limits on neutrino non-unitary mixings based on prescribed singular values,
W. Flieger, JG, K. Porwit, JHEP 03 (2020) 169

- (I): $m > EW$.

  $$Ours: |U_{e4}| \in [0, 0.021], \quad |U_{\mu 4}| \in [0.00013, 0.021], \quad |U_{\tau 4}| \in [0.0115, 0.075].$$
  $$Others: |U_{e4}| \leq 0.041, \quad |U_{\mu 4}| \leq 0.030, \quad |U_{\tau 4}| \leq 0.087$$ [J. de Blas, 2013]

- (II): $\Delta m^2 \gtrsim 100 eV^2$.

  $$Ours: |U_{e4}| \in [0, 0.082], \quad |U_{\mu 4}| \in [0.00052, 0.099], \quad |U_{\tau 4}| \in [0.0365, 0.44].$$

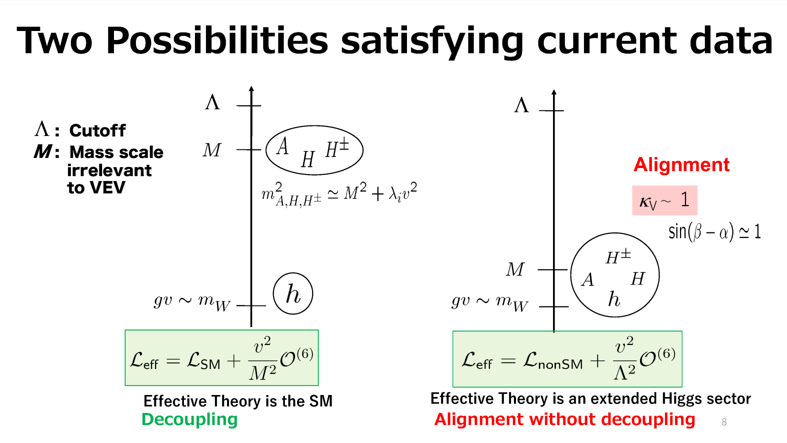
- (III): $\Delta m^2 \sim 0.1 - 1 eV^2$.

  $$Ours: |U_{e4}| \in [0, 0.130], \quad |U_{\mu 4}| \in [0.00052, 0.167], \quad |U_{\tau 4}| \in [0.0365, 0.436].$$
  $$Others: |U_{e4}| \in [0.114, 0.167], \quad |U_{\mu 4}| \in [0.0911, 0.148], \quad |U_{\tau 4}| \leq 0.361.$$
  [C. Giunti et al., 2017] [M. Dantler et al., 2018]

$\longrightarrow$ In some cases we improved (blue), in some not (red).

BSM and new scales, S. Kanemura, FCC November Week 2020

https://indico.cern.ch/event/923801



Effective Theories vs Concrete Models, hep-ph/9909242.

$$\Delta r_{BSM} \stackrel{?}{=} \Delta r_{SM}(m_t, m_H, ...) + \text{''subleading effects from h.o.c.''}$$

$$(\Delta\rho)_{SM} \simeq \frac{m_t^2}{M_W^2};$$

$$(\Delta\rho)_{LRM} \simeq \frac{m_t^2}{\mathbf{M^2_{W_2}} - M^2_{W_1}}$$

$$G_F \sim \frac{g^2}{M_W^2}$$

Confronting electroweak precision measurements with New Physics models

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Abstract. Precision experiments, such as those performed at LEP and SLC, offer us an excellent opportunity to constrain extended gauge model parameters. To this end, it is often assumed that in order to obtain more reliable estimates, one should include the sizable one-loop standard model (SM) corrections, which modify the Z^0 couplings as well as other observables. This conviction is based on the belief that the higher order contributions from the "extension sector" will be numerically small. However, the structure of higher order corrections can be quite different when comparing the SM with its extension; thus one should avoid assumptions which do not take account of such facts. This is the case for all models with $\rho_{\mathrm{tree}} \equiv M_W^2/(M_Z^2 \cos^2\Theta_{\mathrm{W}}) \neq 1$. As an example, both the manifest left–right symmetric model and the $SU(2)_{\mathrm{L}} \otimes U(1)_Y \otimes \tilde{U}(1)$ model, with an additional Z' boson, are discussed, and special attention to the top contribution to $\Delta\rho$ is given. We conclude that the only sensible way to confront a model with the experimental data is to renormalize it self-consistently. If this is not done, parameters which depend strongly on quantum effects should be left free in fits, though essential physics is lost in this way. We should note that the arguments given here allow us to state that at the level of loop corrections (indirect effects) there is nothing like a "model-independent global analysis" of the data.

A theorist's commentary

Why are neutrino masses still Beyond the Standard Model Physics?

We do not know how to write neutrino masses:

- Are ν data described by left-handed Majorana masses?

$$\Delta\mathcal{L} = \frac{1}{2} m_L \overline{\nu_L^c} \nu_L \quad \text{(maybe!)}$$

- Are ν data described by right-handed Majorana masses?

$$\Delta\mathcal{L} = \frac{1}{2} m_R \overline{\nu_R^c} \nu_R \quad \text{(not by itself!)}$$

- Are ν data described by Dirac masses?

$$\Delta\mathcal{L} = m_D \overline{\nu_L} \nu_R + \text{H.c.} \quad \text{(maybe, but I hope not!)}$$

Experimentally establishing $1/2$ is probably worth a prize...

$Z \to l_1^{\pm} l_2^{\mp}$: $Z \to e\mu, \mu\tau, e\tau$

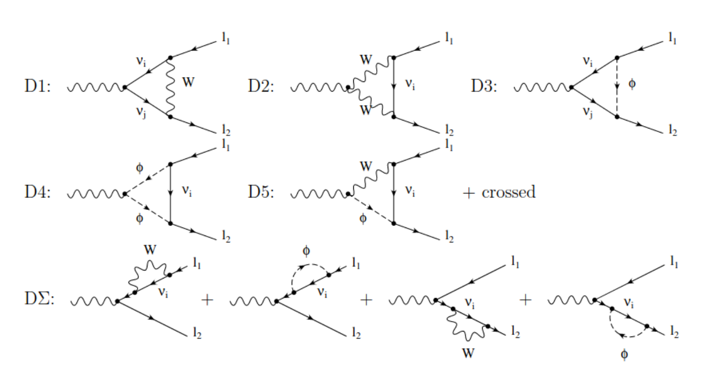


Fig: *Charged lepton flavor violation from massive neutrinos in Z decays*

J. Illana, T. Riemann, Phys.Rev.D 63 (2001) 053004, hep-ph/0010193

ν_i, ν_j - light, heavy, Dirac, Majorana, in general.

In addition: EWPOs and oblique corrections (i.e. S,T,U).

the list is not complete ...

Leptonic flavor changing Z0 decays in SU(2) x U(1) theories with right-handed neutrinos,
J. G. Korner, A. Pilaftsis and K. Schilcher, Phys. Lett. B **300** *(1993), 381, hep-ph/9301290*

Mixing renormalization in Majorana neutrino theories, B.A. Kniehl, A. Pilaftsis, Nucl.Phys.B 474 (1996) 286, hep-ph/9601390

Effects of heavy Majorana neutrinos on lepton flavor violating processes, G. Hernández-Tomé et al, Phys.Rev.D 101 (2020) 7, 075020 1912.13327

Improving Electro-Weak Fits with TeV-scale Sterile Neutrinos, E. Akhmedov et al, JHEP 05 (2013) 081, 1302.1872

Loop level constraints on Seesaw neutrino mixing, E. Fernandez-Martinez et al, JHEP 10 (2015) 130, 1508.03051

`For RHNs we can compare SM HO terms with BSM effects:`
$HO = HO_{\rm SM} + HO_{BSM}$ (work in progress, Dirac neutrinos).

Summary and Outlook

`Z-pole physics was essential in the past and will remain the butter and bread of precision particle studies` **if future e^+e^- colliders come to reality**.

1. Challenges at Z-pole:
   1.1 3-loop EW and mixed EW-QCD, leading 4-loop corrections for $Z \to 2f$ vertices
   1.2 QED interference effects, non-factorizable corrections
   1.3 Adjusting MC generators at NNLO and beyond (Bhabha (!), exclusive NNLO $e^+e^- \to f\bar{f}$).
2. Challenge to improve input parameters (α, α_s, physics at ZH, WW, tt)
3. Challenge to optimize/understand paths towards BSM discovery (RHNs, DM, CP effects,...)
4. Challenge: SM(BSM)EFT, precision physics for concrete BSM models
5. Challenge: Tools (MC generators, ~~multiloop numerical~~, analytical programs)

*'FCC-ee: the challenge for theory', talk at 4th FCC Physics and Experiments Workshop, link

BACKUP

Precision changed history: justice, law, crime, trade, economy, social, ...

Example: To be just was precisely to use balance.



Wisdom 11:20

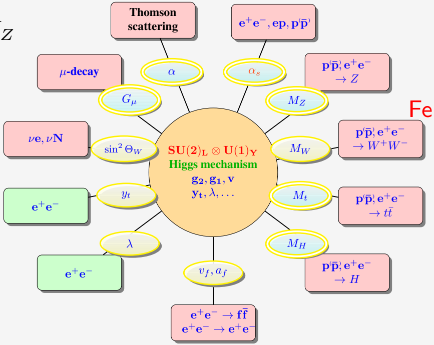
‘By weight, measure and number, God made all things’

Code of Hammurabi 1772 BC - any taverner using false weights could be served up with the death penalty

Input and calculated/measured parameters

Schemes: G_μ vs M_W,...

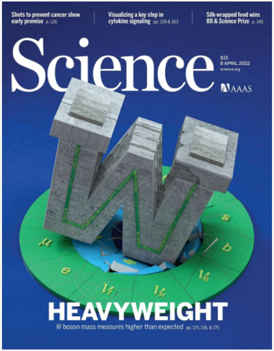
$$G_\mu,\ \sin^2\theta^\ell_{eff},\ M_Z$$

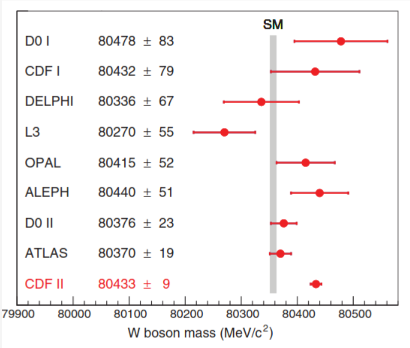


Fermilab!

Fig. from the FCC-ee report 'α_{QED}' by F. Jegerlehner in 1905.05078

Introduction to Precision Electroweak Analysis by J. Welss, 0512342





Science 376 (2022) 6589, 170-176

$$
\begin{aligned}
\mathrm{SM} : M_W &= 80.357 \pm 6 \text{ MeV}, \ (\mathrm{PDG2020}) \\
\mathrm{Global} : M_W &= 80.379 \pm 12 \text{ MeV}, \ (\mathrm{PDG2020}) \\
\mathrm{CDFII} : M_W &= 80433.5 \pm 9.4 \text{ MeV}
\end{aligned}
$$

$$
\texttt{FCC-ee} \text{ forecast} : M_W = X \pm \mathbf{0.4} \text{ MeV!}
$$

Input and calculated/measured parameters

Experimental values:

$$\begin{aligned}
\hat{\alpha} &= 1/137.0359895(61),\ \gamma^* \to e^+e^- \\
\hat{G}_F &= 1.16639(1)\times 10^{-5}\,\mathrm{GeV}^{-2}\ \text{muon decay} \\
\hat{m}_Z &= 91.1875 \pm 0.0021\,\mathrm{GeV} \\
\hat{m}_W &= 80.426 \pm 0.034\,\mathrm{GeV} \\
\hat{s}^2_{\rm eff} &= 0.23150 \pm 0.00016, \text{effective}\sin^2\theta_{\rm W}, A_{LR} \equiv \frac{(1/2-\hat{s}^2_{\rm eff})^2 - \hat{s}^4_{\rm eff}}{(1/2-\hat{s}^2_{\rm eff})^2 + \hat{s}^4_{\rm eff}} \\
\hat{\Gamma}_{l^+l^-} &= 83.984 \pm 0.086\,\mathrm{MeV}
\end{aligned}$$

$$\left\{\begin{array}{l}
\mathbf{g}(=e/s_W)\ SU(2) \\
\mathbf{g}'(e/c_W)\ U(1)_Y \\
\mathbf{v}\ \text{VEV},
\end{array}\right. \longrightarrow \left\{\begin{array}{l}
\hat{\alpha} = \frac{e^2}{4\pi} \\
\hat{G}_F = \frac{1}{\sqrt{2}v^2} \\
\hat{m}_Z^2 = \frac{e^2v^2}{4s^2c^2} \\
\hat{m}_W^2 = \frac{e^2v^2}{4s^2} \\
\hat{s}^2_{\rm eff} = s^2 \\
\hat{\Gamma}_{l^+l^-} = \frac{v}{96\pi}\frac{e^3}{s^3c^3}\left[\left(-\frac{1}{2}+2s^2\right)^2+\frac{1}{4}\right]
\end{array}\right.$$

Shaping the SM, tree level estimates

In terms of $\hat{\alpha}, \hat{G}_F$ and $\hat{m}_Z$

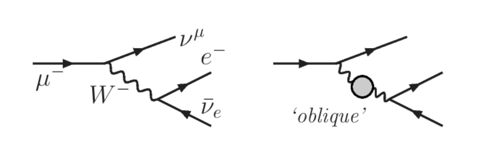
$$\hat{m}_W^2 = \pi\sqrt{2}\hat{G}_F^{-1}\hat{\alpha}\left(1-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^{-1}$$

$$\hat{s}^2_{\rm eff}\hat{c}^2_{\rm eff} = \frac{\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2} \quad \equiv \quad \hat{s}^2_{\rm eff} = \frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}$$

$$\hat{\Gamma}_{l^+l^-} = \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{12\pi}\left\{\left(\frac{1}{2}-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^2+\frac{1}{4}\right\}$$

$$\begin{aligned}
Prediction : \hat{m}_W &= 80.939 \pm 0.003\,\mathrm{GeV}\ 15\sigma \text{ away}\\
Prediction : \hat{s}^2_{\rm eff} &= 0.21215 \pm 0.00003\ 120\sigma \text{ away}\\
Prediction : \hat{\Gamma}_{l^+l^-} &= 84.843 \pm 0.012\,\mathrm{MeV}\ 10\sigma \text{ away}
\end{aligned}$$

Shaping SM, oblique corrections also not sufficient



$$\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} K(\alpha, m_e, m_\mu, m_W)$$

$$\begin{aligned} \frac{(\hat{G}_F)^{\text{th}}}{\sqrt{2}} &= \frac{g^2}{8m_W^2}\left[1 + i\Pi_{WW}(q^2)\left(\frac{-i}{q^2 - m_W^2}\right)\right]_{q\to 0} \\ &= \frac{1}{2v^2}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right]. \end{aligned}$$

Primary role of SM radiative corrections, F. Jegerlehner, in 1905.05078

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\,\frac{1}{1-\Delta r_i} \quad \Delta r_i = \Delta r_i(\alpha, G_\mu, M_Z, m_H, m_{f\neq t}, m_t)\,,$$

$$\Delta r_i = -\frac{c_W^2}{s_W^2}\,\Delta\rho + \Delta r_{i\,\mathrm{reminder}}\,,$$

$$\Delta\rho = \frac{3\,m_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}$$

$$\hat{\alpha}(m_Z) = \frac{\hat{\alpha}}{1-\Delta\alpha(m_Z)} = \frac{e^2}{4\pi}\left[1+\frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2}\right] \sim 128\ (137\ \text{at the Thomson limit})$$

Still, well visible disagreement between SM prediction and experiment for EWPOs without subleading SM corrections, and only with the leading corrections $\Delta\alpha(m_Z)$ and $\Delta\rho$.

$r_{i\,\mathrm{reminder}}$ **matters!** (see also backup slides)

F. Jegerlehner, in 1905.05078

Example: the W and Z mass from $\alpha(M_Z)$, G_μ and $\sin^2\Theta_{\ell\,\mathrm{eff}}$:
(i) $\sin^2\Theta_W = 1 - M_W^2/M_Z^2$,
$\sin^2\theta_{\ell,\mathrm{eff}}(M_Z) = \left(1 + \frac{\cos^2\Theta_W}{\sin^2\Theta_W}\,\Delta\rho\right)\sin^2\Theta_W$,

$$\Delta\rho = \frac{3\,M_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}\ ;\ \ M_t = 173 \pm 0.4\ GeV$$

The iterative solution with input $\sin^2\theta_{\ell,\mathrm{eff}}(M_Z) \equiv (1 - v_\ell/a_\ell)/4 = 0.23148$ (EXP!) is $\sin^2\Theta_W = 0.22426$.
(ii) $M_W^{\mathrm{exp}} = 80.379 \pm 0.012\ \ ;\ \ M_Z^{\mathrm{exp}} = 91.1876 \pm 0.0021$ GeV,
$\longrightarrow 1 - \mathrm{M_W^2/M_Z^2} = 0.22263.$
Predicting then the masses we have

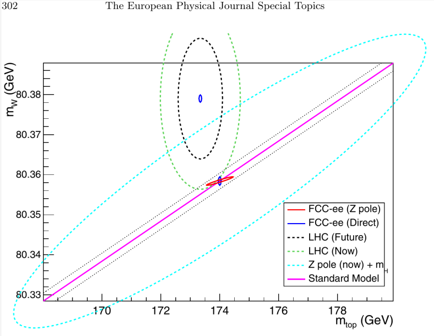
$$M_W = \frac{A_0}{\sin^2\Theta_W}\ ;\ \ A_0 = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_\mu}}\ ;\ \ M_Z = \frac{M_W}{\cos\Theta_W}$$

where, including photon VP correction $\alpha^{-1}(M_Z) = 128.953 \pm 0.016$. For the W, Z mass we then get

$$M_W^{\mathrm{the}} = 81.1636 \pm 0.0346\ \ ;\ \ M_Z^{\mathrm{the}} = 92.1484 \pm 0.0264\ \ .$$

Deviations (errors added in quadrature): $W:\ \ 23\,\sigma\ ;\ \ Z:\ \ 36\,\sigma$

FCC-ee fits, CDR link



Observable	present value ± error	FCC-ee **Stat.**	FCC-ee Syst.	Comment and leading exp. error
m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan Beam energy calibration
$\sin^2\theta_W^{eff}(\times 10^6)$	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	small	from $A_{FB}^{\mu\mu}$ off peak QED&EW errors dominate
R_ℓ^Z $(\times 10^3)$	20767 ± 25	**0.06**	0.2-1	ratio of hadrons to leptons **acceptance for leptons**
$\alpha_s(m_Z^2)$ $(\times 10^4)$	1196 ± 30	**0.1**	0.4-1.6	from R_ℓ^Z above
σ_{had}^0 $(\times 10^3)$ (nb)	41541 ± 37	**0.1**	4	peak hadronic cross section luminosity measurement
$N_\nu(\times 10^3)$	2996 ± 7	**0.005**	1	Z peak cross sections Luminosity measurement
R_b $(\times 10^6)$	216290 ± 660	**0.3**	< 60	ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD
$A_{FB}^b, 0$ $(\times 10^4)$	992 ± 16	**0.02**	1-3	b-quark asymmetry at Z pole from jet charge
$A_{FB}^{pol,\tau}$ $(\times 10^4)$	1498 ± 49	**0.15**	<2	τ polarization asymmetry τ decay physics
τ lifetime (fs)	290.3 ± 0.5	**0.001**	0.04	radial alignment
τ mass (MeV)	1776.86 ± 0.12	**0.004**	0.04	momentum scale
τ leptonic $(\mu\nu_\mu\nu_\tau)$ B.R. (%)	17.38 ± 0.04	**0.0001**	0.003	e/μ/hadron separation
m_W (MeV)	80350 ± 15	**0.25**	0.3	From WW threshold scan Beam energy calibration
Γ_W (MeV)	2085 ± 42	1.2	0.3	From WW threshold scan Beam energy calibration
$\alpha_s(m_W^2)(\times 10^4)$	1170 ± 420	**3**	small	from R_ℓ^W
$N_\nu(\times 10^3)$	2920 ± 50	**0.8**	small	ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV/c^2)	172740 ± 500	**17**	small	From $t\bar{t}$ threshold scan QCD errors dominate
Γ_{top} (MeV/c^2)	1410 ± 190	45	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	1.2 ± 0.3	**0.10**	small	From $t\bar{t}$ threshold scan QCD errors dominate
ttZ couplings	± 30%	0.5 – 1.5%	small	From $\sqrt{s} = 365\,\text{GeV}$ run

E.g. effective weak mixing angle

The weak mixing angle $s_W^2 \equiv \sin^2\theta_W$ has three potential different meanings or functions in the model-building:

(i) It describes the ratio of the two gauge couplings,

$$g'/g = c_W/s_W,$$

usually in the $\overline{\text{MS}}$ scheme.

(ii) It describes the ratio of two gauge boson (on-shell) masses,

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}.$$

(iii) It describes the ratio of the vector and axial-vector couplings of an (on-shell) Z boson to fermions,

$$\frac{v_f}{a_f} = 1 - 4|Q_f|s_W^2.$$

This definition is called the effective weak mixing angle, denoted as $\sin^2\theta_W^{f,\text{eff}}$.

How to unfold - rough scheme

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma),$$

S-matrix Ansatz in the complex energy plane

$$\mathcal{A}^{e^+e^-\to b\bar{b}} = \underbrace{\frac{R_Z}{s-s_Z} + \overbrace{\frac{R_\gamma}{s}}_{\gamma - Z\ interference} \overbrace{+ S + (s-s_Z)S' + \ldots}^{Background}},$$

$$s_Z = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z$$

- $R, S, S', \ldots$ are individually gauge-invariant and UV-finite - unitarity and analyticity of the S-matrix. IR-finite, when soft and collinear real photon emission is added. [Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994] [Passera, Sirlin, 1998] [Gambino, Grassi, 2000] [Awramik, Czakon, Freitas, 2006].

The term $R_\gamma(s)/s$ is part of the the background

- The poles of $\mathcal{A}$ have complex residua R_Z and R_γ.
- There is only ONE pole in mathematics, while in physics we observe two of them: photon exchange at $s = 0$, Z exchange at $s_0 = s_Z$. Mathematicaly, the appearance of the photon pole is result of summing of part of background around Z pole, $s_0 = s_Z$

[T. Riemann, APPB 2015]

$$
\begin{aligned}
\frac{R_\gamma(s)}{s} &= \frac{\sum_{n=0}^{\infty} R_n (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^{\infty} R_n (s-s_0)^n}{s_0 - (s_0 - s)} \\
&= \sum_{n=0}^{\infty} R_n (s-s_0)^n \; \frac{1}{s_0} \; \frac{1}{1 - \frac{s_0 - s}{s_0}} \\
&= \sum_{n=0}^{\infty} R_n (s-s_0)^n \; \frac{1}{s_0} \; \left[1 + \frac{s_0 - s}{s_0} + \left(\frac{s_0 - s}{s_0} \right)^2 \cdots \right];
\end{aligned}
$$

EWPOs - refers to $|M|^2$; EWPPs - refers to M

Beyond Born level, one can write

$$\begin{aligned}
\mathcal{M}_\gamma^{(0)}(e^-e^+ \to f^-f^+) &= \frac{4\pi i \alpha_{em}(s)}{s} Q_e Q_f \, \gamma_\alpha \otimes \gamma^\alpha, \\
\mathcal{M}_Z^{(0)}(e^-e^+ \to f^-f^+) &= 4ie^2 \frac{\chi_Z(s)}{s} \big[M_{vv}^{ef} \, \gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \\
& \quad - M_{va}^{ef} \, \gamma_\alpha \times \gamma^\alpha \gamma_5 + M_{aa}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \gamma_5 \big].
\end{aligned}$$

In the pole scheme, where $\bar{M}_Z$ is defined as the real part of the pole of the S matrix, one has

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2}\; 8\pi\alpha_{em}} K_Z(s) \simeq \frac{1}{1 + i\frac{\bar{\Gamma}_Z}{\bar{M}_Z}} \frac{s}{s - \bar{M}_Z^2 + i\bar{M}_Z \bar{\Gamma}_Z} \simeq \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z(s)},$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2} \; \Gamma_Z$$

EWPOs - refers to $|M|^2$; EWPPs - refers to M

Definitions are related:

$$\bar{M}_Z \approx M_Z - \frac{1}{2}\,\frac{\Gamma_Z^2}{M_Z} \;\approx\; M_Z - 34\ \mathrm{MeV},$$
$$\bar{\Gamma}_Z \approx \Gamma_Z - \frac{1}{2}\,\frac{\Gamma_Z^3}{M_Z^2} \;\approx\; \Gamma_Z - 0.9\ \mathrm{MeV}.$$

- Known from LEP. One of examples why changing frameworks/assumptions/simplifications of calculations matter (!).
- However, at FCC-ee $\delta\Gamma_Z \sim 0.1$ MeV. Non-facotrization effects must be added properly beyond 1-loop.
- Is it necessary for FCC-ee accuracy to implement MC with radiative corrections calculated at the amplitudes level?
- At this precision it is important which parameters are taken as input parameters in schemes.

EW SM theory at loops, an example ($\Delta_{ef} \neq 0$)

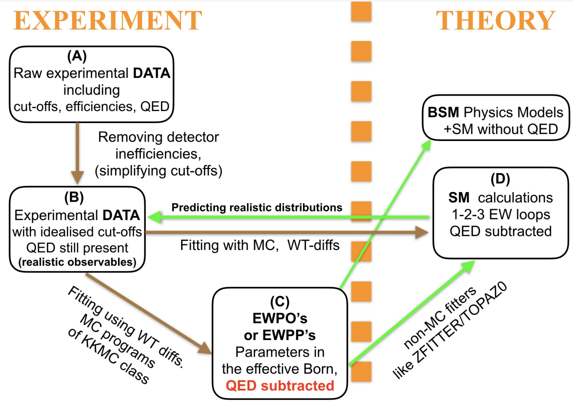
$$\left\{\begin{array}{l} \Gamma_Z, \Gamma_{partial} \\ \\ A^{eff.,Born}_{FB,peak}, A^{eff.,Born}_{LR,peak} \\ \\ R_b, R_\ell, ... \end{array}\right. \longrightarrow \left\{\begin{array}{l} v^{eff}_{\ell,\nu,u,d,b} \\ \\ a^{eff}_{\ell,\nu,u,d,b} \\ \\ \sin^2\theta^b_{\rm eff}, \sin^2\theta^{lept}_{\rm eff} \end{array}\right.$$

e.g. : improvements needed for subtle corrections $\Delta_{1,2}$ (e.g. boxes, **2L-boxes**)

$$\begin{aligned} A^{eff.,Born}_{FB,peak} &= \frac{2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\right] 2\Re e\left[\frac{v_f a_f^*}{|a_f|^2}\right]}{\left(1+\frac{|v_e|^2}{|a_e|^2}\right)\left(1+\frac{|v_f|^2}{|a_f|^2}\right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f, \\ \Delta_1 &= 2\Re e\left[\Delta_{ef}\right],\ \Delta_2 = |\Delta_{ef}|^2 + 2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\frac{v_f a_f^*}{|a_f|^2}\Delta^*_{ef}\right], \\ \Delta_{ef} &= 16|Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e\kappa_f) \end{aligned}$$

Scheme of construction and the use of EWPO/EWPP at FCC-ee

Jadach&Skrzypek **1903.09895**



Scheme DATA $\longrightarrow$ Born $\otimes$ QED still valid?

$\longrightarrow \mathcal{O}(\alpha^1)^{\mathrm{noQED}} \otimes \mathrm{QED}$

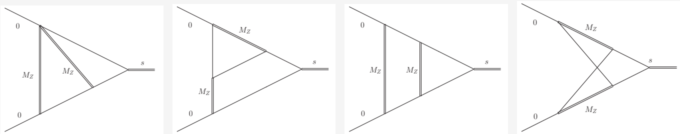
General remarks on usefulness of EWPOs

1. EWPOs encapsulate experimental data after extraction of well known and controllable QED and QCD effects, in a model-independent manner.
2. They provide a convenient bridge between real data and the predictions of the SM (or SM plus New Physics).
3. Contrary to raw experimental data (like differential crosssections), EWPOs are well suited for archiving and long term exploitation.
4. In particular archived EWPOscan be exploited over long periods of time for comparisons with steadily improving theoretical calculationsof the SM predictions, and for validations of the New Physics models beyond the SM.
5. They are also useful for comparison and combination of results from different experiments.

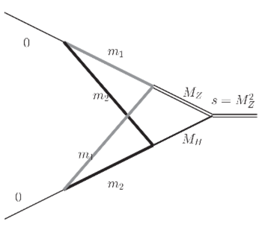
MB and SD methods are very much complementary!

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR);
  SD more useful for integrals with many internal masses

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods - in **Minkowskian region**.



2-loops $\longrightarrow$ 3-loops

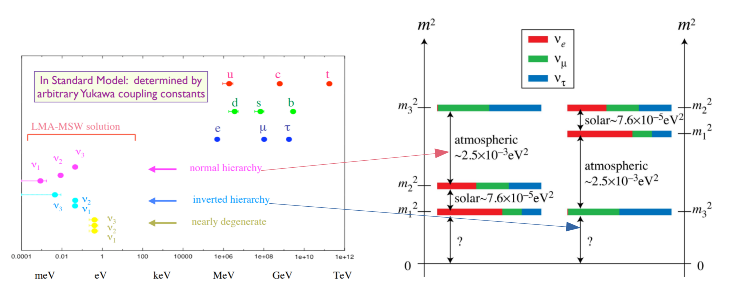


$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

Neutrino parameters, development



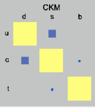
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{\mathrm{CP}}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

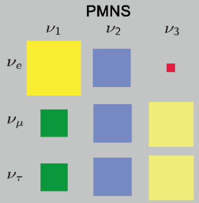
Neutrino Physics Enters Precisoin Era

Super-K, Hyper-K, T2K, NOvA, Antares, KM3NeT, Juno, Dune, SNO+, Daya Bay, Double Chooz, RENO, ...

$$U_{PMNS} = \begin{pmatrix} \{0.810, 0.829\} & \{0.539, 0.562\} & \{0.147, 0.169\} \\ \{-0.485, -0.479\} & \{0.467, 0.563\} & \{0.669, 0.743\} \\ \{0.278, 0.339\} & \{-0.683, -0.626\} & \{0.647, 0.728\} \end{pmatrix}$$

$$\theta_{12} = 33.9^o \pm 1.0^o$$
$$\theta_{23} = 36^o - 54^o$$
$$\theta_{13} = 9.12^o \pm 0.63^o$$
$$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ [eV^2]$$
$$\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\ [eV^2]$$

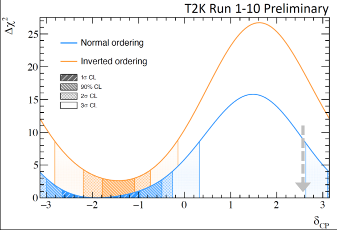


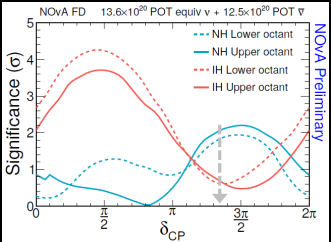


Conclusion: Neutrino Physics stepped in the precision era.

Till 2030: mass hierarchy, δ_{CP} (maybe), absolute masses, Majorana-Dirac, L. Wen, EPS2021.

The CP Phase



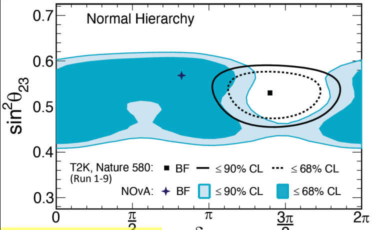


T2K
- $\delta = -\pi/2$ favored
- Large range of values of δ_{CP} around $+\pi/2$ are excluded at 99.7%

NOvA
- Best-fit $\delta = 0.82\,\pi$
- Exclude **IH** $\delta = \pi/2$ at $>3\sigma$
- Disfavor NH $\delta = 3\pi/2$ at ~2σ

Clear tension exists

NOvA + T2K joint analysis is underway



Alex Himmel @ Neutrino 2020

Dirac neutrinos: Cosmology,...

- Neutrino oscillation experiments are insensitive to the nature of neutrinos.
- Experiments (such as $0\nu\beta\beta$ decay) looking for lepton number violating signatures can probe the Majorana nature of neutrinos.
- Plethora of $0\nu\beta\beta$ Experiments :
  Past - Heidelberg-Moscow, GERDA-I, NEMO-3;
  Present- EXO-200, KamLAND-Zen;
  Future- GERDA-II, NEXT, nEXO, PandaX-III, SpuerNEMO..
- So far **NOT** so good : absence of any positive result.
- Caution : Null result in $0\nu\beta\beta$ expts. do not necessarily prove neutrinos are Dirac particle.
- Nevertheless, its worth to come up with ideas with implication of Dirac neutrinos in Cosmology too

• **Dirac Leptogenesis:** [H. Murayama et al., 2002] [T. Dick et al., 1999]

→ Traditional Leptogenesis: small neutrio mass and LNV required to produce lepton asymmetry is provided by heavy right-handed Majorana neutrinos.
→ Strategy: (a) additional symmetry introduced to forbid Dirac Yukawa coupling and Majorana mass term, (b) A set of heavy, vector-like pairs of fields introduced whose couplings to the standard model fields contain nontrivial, CP-violating phases which plays the role of heavy right-handed Majorana neutrinos.

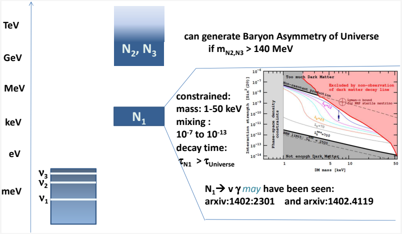
• **Dirac Neutrinos and freeze-in dark matter:** [E. Ma, 2021] [D. Borah et al., 2018]

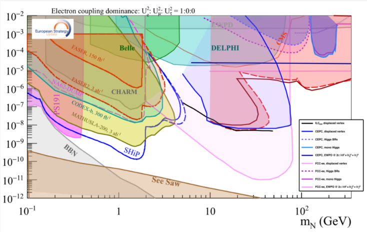
→ Neutrinos to have only Dirac mass and dark matter relic abdunce generated through freeze-in mechanism, both requires coupling $\mathcal{O}(10^{-11})$. Such a small coupling may have some unified origin.

BSM and RHNs, FCC-ee CDR vol.1

LFV Z-decays: $(10^{-6} \div 10^{-5})$. FCC-ee $\longrightarrow \sim 10^{-9}$ branching fractions.

A. Blondel et al. 1411.5230 ESPPU Briefieng Book 1910.11775





Low-scale leptogenesis with flavour and CP symmetries, M. Drewes et al, 2203.08538

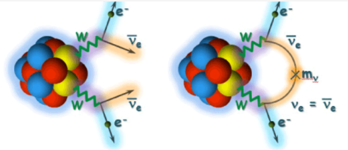
Discrete Flavor Symmetries and Lepton Masses and Mixings, G. Chauhan, et al, 2203.08538 (Snowmass contribution)

Resonant Leptogenesis, Collider Signals and Neutrinoless Double Beta Decay from Flavor and CP Symmetries, G. Chauhan, B. Dev, 2203.08538

LNV, Majorana neutrinos, $H^{\pm\pm}$, ...

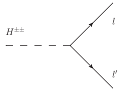
Proces	Obecne ograniczenie	Oczekiwany limit	Eksperyment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	5×10^{-14}	MEG II
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	10^{-16}	Mu3e
$\mu^-\mathrm{Al} \to e^-\mathrm{Al}$	$< 6.1 \times 10^{-13}$	10^{-17}	Mu2e, COMET
$\mu^-\mathrm{Si/C} \to e^-\mathrm{Si/C}$	$-$	5×10^{-14}	DeeMe
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	5×10^{-9}	Be lle II, FC
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	10^{-9}	Belle II , FC
$\tau \to eee$	$< 2.7 \times 10^{-8}$	5×10^{-10}	Belle I I, FC
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	5×10^{-10}	Belle II, FC
$\tau \to e\ \mathrm{had}$	$< 1.8 \times 10^{-8}$	3×10^{-10}	Belle II, FC
$\mathrm{had} \to \mu e$	$< 4.7 \times 10^{-12}$	10^{-12}	NA 62
$h \to e\mu$	$< 3.5 \times 10^{-4}$	3×10^{-5}	HL-LHC , FC
$h \to \tau\mu$	$< 2.5 \times 10^{-3}$	3×10^{-4}	HL-LH C, FC
$h \to \tau e$	$< 6.1 \times 10^{-3}$	3×10^{-4}	HL-LH C, FC

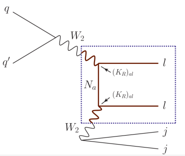
Matter-antimatter asymmetry! $\eta = \frac{n_B - n_{\bar{B}}}{n_m a} \simeq \frac{n_B}{n_\gamma} \simeq 10^{-10}$



Double beta decay
which emits anti-neutrinos

Neutrinoless
double beta decay





$Z \to l_1^{\pm} l_2^{\mp}$: $Z \to e\mu, \mu\tau, e\tau$

$$\Gamma(Z \to \bar{\ell}\ell') = \frac{\alpha}{3} M_Z |F_L^Z(M_Z^2)|^2$$

$$F_L^Z(q^2) = \frac{\alpha_W}{8\pi s_W c_W} \sum_{i,j}^{5} \mathbf{B}^*_{\ell \mathbf{i}} \mathbf{B}_{\ell' \mathbf{j}} \big[\delta_{ij} F(x_i; q^2)$$

$$+ \quad C^*_{ij} G(x_i, x_j; q^2) + C_{ij} \sqrt{x_i x_j} H(x_i, x_j; q^2)\big],$$

F,G,H - combination of the Passarino-Veltman functions,

G. Hernández-Tomé et al, Phys.Rev.D 101 (2020) 7, 075020 1912.13327

$$\mathcal{L}_{W^\pm} = -\frac{g}{\sqrt{2}} W_\mu^- \sum_{i=1}^{3} \sum_{j=1}^{5} B_{ij}\, \bar{\ell}_i \gamma^\mu P_L \chi_j + \mathrm{h.c.},$$

$$\mathcal{L}_Z = -\frac{g}{4c_W} Z_\mu \sum_{i,j=1}^{5} \bar{\chi}_i \gamma^u \left(C_{ij} P_L - C^*_{ij} P_R\right) \chi_j,$$

$Z \to l_1^{\pm} l_2^{\mp}$: $Z \to e\mu, \mu\tau, e\tau$

Important for renormalization

$$\sum_{k=1}^{5} B_{ik} B_{jk}^* = \delta_{ij}, \quad \sum_{k=1}^{3} B_{ki}^* B_{kj} = \sum_{k=1}^{5} C_{ik} C_{jk}^* = C_{ij},$$
$$\sum_{k=1}^{5} m_{\chi_k} C_{ik} C_{jk} = \sum_{k=1}^{5} m_{\chi_k} B_{ik} C_{kj}^* = \sum_{k=1}^{5} m_{\chi_k} B_{ik} B_{jk} = 0.$$

Simple model. 3 active neutrino masses irrelevant: BR$(Z \to ll') \sim 10^{-55}$

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & 0 & m_2 \\ 0 & 0 & 0 & 0 & m_3 \\ 0 & 0 & 0 & 0 & M \\ m_1 & m_2 & m_3 & M & \mu \end{pmatrix}, \quad \begin{matrix} m_{\chi_{1,2,3}} = 0, \\ m_{\chi_{4,5}} = \frac{1}{2}\left(\sqrt{4M'^2 + \mu^2} \mp \mu\right), \\ M'^2 = m_1^2 + m_2^2 + m_3^2 + M^2, \end{matrix}$$

$\mu = 0 \longrightarrow$ Dirac neutrino.

$$\begin{pmatrix} -\frac{m_2}{\sqrt{m_1^2+m_2^2}} & -\frac{m_1 m_3}{m\sqrt{m_1^2+m_2^2}} & -\frac{m_1 M}{m M'} & -i\frac{m_1 m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_1}{\sqrt{m_{\chi_5}^2+M'^2}} \\ \frac{m_1}{\sqrt{m_1^2+m_2^2}} & -\frac{m_2 m_3}{m\sqrt{m_1^2+m_2^2}} & -\frac{m_2 M}{m M'} & -i\frac{m_2 m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_2}{\sqrt{m_{\chi_5}^2+M'^2}} \\ 0 & \frac{\sqrt{m_1^2+m_2^2}}{m} & -\frac{m_3 M}{m M'} & -i\frac{m_3 m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_3}{\sqrt{m_{\chi_5}^2+M'^2}} \\ 0 & 0 & \frac{m}{M'} & -i\frac{M m_{\chi_5}}{M'\sqrt{m_{\chi_5}^2+M'^2}} & \frac{M'}{\sqrt{m_{\chi_5}^2+M'^2}} \\ 0 & 0 & 0 & i\frac{M'}{\sqrt{m_{\chi_5}^2+M'^2}} & \frac{m_{\chi_5}}{\sqrt{m_{\chi_5}^2+M'^2}} \end{pmatrix} \longrightarrow \mu = 0$$

$$m_{\chi_4} = m_{\chi_4} = M'$$

$$\begin{pmatrix} -\frac{m_2}{\sqrt{m_1^2+m_2^2}} & -\frac{m_1 m_3}{m\sqrt{m_1^2+m_2^2}} & -\frac{m_1 M}{m M'} & -\frac{i}{\sqrt{2}}\frac{m_1}{M'} & \frac{1}{\sqrt{2}}\frac{m_1}{M'} \\ \frac{m_1}{\sqrt{m_1^2+m_2^2}} & -\frac{m_2 m_3}{m\sqrt{m_1^2+m_2^2}} & -\frac{m_2 M}{m M'} & -\frac{i}{\sqrt{2}}\frac{m_2}{M'} & \frac{1}{\sqrt{2}}\frac{m_2}{M'} \\ 0 & \frac{\sqrt{m_1^2+m_2^2}}{m} & -\frac{m_3 M}{m M'} & -\frac{i}{\sqrt{2}}\frac{m_3}{M'} & \frac{1}{\sqrt{2}}\frac{m_3}{M'} \\ 0 & 0 & \frac{m}{M'} & -\frac{i}{\sqrt{2}}\frac{M}{M'} & \frac{1}{\sqrt{2}}\frac{M}{M'} \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The simplest 3+1 model (toy model)

$$\mathcal{L}_\nu^M = -\frac{1}{2}\bar{N}\begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ a_1 & a_2 & a_3 & M \end{pmatrix} + H.c.$$

Four physical neutrino $\nu_1, \nu_2, \nu_3, \nu_4$, with ν_1, ν_2 massless and

$$m_3 < m_4$$

This mass matrix is diagonalized by the following mixing matrix

$$\begin{pmatrix} c_\beta & -s_\beta s_\gamma & -s_\beta c_\gamma & 0 \\ 0 & c_\gamma & -s_\gamma & 0 \\ c_\alpha s_\beta & c_\alpha c_\beta s_\gamma & c_\alpha c_\beta c_\gamma & -s_\alpha \\ s_\alpha s_\beta & s_\alpha c_\beta s_\gamma & s_\alpha c_\beta c_\gamma & c_\alpha \end{pmatrix} \qquad s_\alpha \sim \sqrt{m_3/(m_3+m_4)}$$

[C. Jarlskog, 1990] [C.O. Escobar et al., 1993]

Isolating non-standard contribution, we can write this as

$$N_\nu - 2 = \frac{1}{(x+y)^2}\left[x^2F(y) + y^2F(x) + 2xyG(x,y)\right],$$

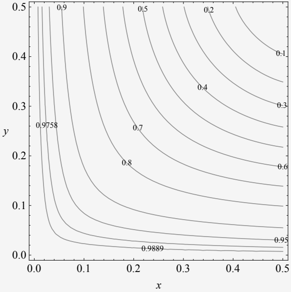
where

$$F(z) = (1-4z^2)^{\frac{3}{2}},$$
$$G(x,y) = \sqrt{1+(x^2+y^2)^2 - 2(x^2-y^2)}\left[1 - \frac{x^2+y^2}{2} - 3xy - \frac{(x^2-y^2)^2}{4}\right],$$

and

$$x = m_4/M_Z,\ y = m_3/M_Z, m_{3,4} \leq M_Z/2.$$

Nonstandard Mixing Estimation



We are looking for the line $y = x\tan^2\alpha$ lying below experimental limits

$$\mathrm{LEP}: N_\nu = 2.9840 \pm 0.0082 \rightarrow \sum_i |U_{i4}| \equiv \sin\alpha < 0.174 \quad i = e, \mu, \tau$$

$$\mathrm{NEW}: N_\nu = 2.9963 \pm 0.0074 \rightarrow \sum_i |U_{i4}| \equiv \sin\alpha < 0.121 \quad i = e, \mu, \tau$$

New constraints on heavy neutral leptons coming from oscillation data analysis and precision e^+e physics, W. Flieger, K. Grzanka, PoS ICHEP2020 (2021) 129

"Personal Remarks on SMEFT for Snowmass",
M. E. Peskin, EF1-EF4 meeting, Sept. 2020, pdf

1. "... the interpretation depends on the connection to explicit models of BSM physics".
   How $\frac{C_i}{\Lambda^2}$, O_H, O_{WW}, ..., are releated to the BSM param eters?
2. Linear dependence on SMEFT parameters?
3. SMEFT at high Q^2 vs specific models?
4. h.o. SMEFT corrections and cancellations with SMEFT tree level
5. SMEFT contribution to SM background.

Higgs boson decays: theoretical status, M. Spira, pdf

Estimated theoretical uncertainties from missing higher orders and the perturbative orders (QCD/elw.) of the results included in the analysis.

Partial Width	**QCD**	**Electroweak**	**Total**	**on-shell Higgs**
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$	$\sim 0.5\%$	$\mathrm{N^4LO}$ / NLO
$H \to \tau^+\tau^-/\mu^+\mu^-$	—	$\sim 0.5\%$	$\sim 0.5\%$	— / NLO
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$	$\mathrm{N^3LO}$ / NLO
$H \to \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$	NLO / NLO
$H \to Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$	LO / LO
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$\sim 0.5\%$	$\sim 0.5\%$	NLO/NLO

Higgs boson decays: theoretical status

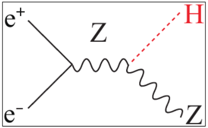
Projected intrinsic and parametric uncertainties for the partial and total Higgs-boson decay width predictions. The last column: the target of FCC-ee precisions.

decay	intrinsic	para. m_q	para. α_s	para. M_H	FCC-ee prec. on g^2_{HXX}
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	$\sim 0.8\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	$\sim 1.4\%$
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	$\sim 1.1\%$
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	$\sim 12\%$
$H \to gg$	$\sim 1\%$		0.5% (0.3%)	–	$\sim 1.6\%$
$H \to \gamma\gamma$	$< 1\%$	–	–	–	$\sim 3.0\%$
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \to WW$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
$H \to ZZ$	$\lesssim 0.3\%^\dagger$	–	–	$\sim 0.1\%$	$\sim 0.3\%$
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

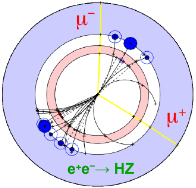
† From $e^+e^- \to HZ$ production

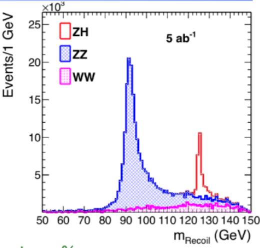
Absolute coupling and width measurement

- **Higgs tagged by a Z, Higgs mass from Z recoil**



$$m_H^2 = s + m_Z^2 - 2\sqrt{s}(E_+ + E_-)$$





- Total rate $\propto g_{HZZ}^2$ → measure g_{HZZ} to 0.2%
- ZH → ZZZ final state $\propto g_{HZZ}^4 / \Gamma_H$ → measure Γ_H to a couple %
- ZH → ZXX final state $\propto g_{HXX}^2\, g_{HZZ}^2 / \Gamma_H$ → measure g_{HXX} to a few per-mil / per-cent
- Empty recoil = invisible Higgs width; Funny recoil = exotic Higgs decays

- **Note: The HL-LHC is a great Higgs factory (10^9 Higgs produced) but …**
  - $\sigma_{i\to f}^{(\mathrm{observed})} \propto \sigma_{prod}\, (g_{Hi})^2\, (g_{Hf})^2 / \Gamma_H$
    - Difficult to extract the couplings : σ_{prod} is uncertain and Γ_H is largely unknown
      - Must do physics with ratios or with additional assumptions.

SM precision parameters determination: $\alpha(M_Z^2)$, F. Jegerlechner, pdf

1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics: besides top Yukawa y_t and Higgs self-coupling λ

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions**

50% non-perturbative $\qquad\qquad\qquad\qquad$ $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$
\begin{array}{lccccl}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} & \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} & \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} & \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 0.9 \div 1.6 & \times & 10^{-4} & (\text{present : lost } 10^5 \text{ in precision!}) \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} & (\mathtt{FCC-ee/ILC\ requirement})
\end{array}
$$

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - v_l/a_l)/4 = 0.23148 \pm 0.00017$

$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} = 0.00007$; $\delta M_W/M_W \sim 4.3 \times 10^{-5}$

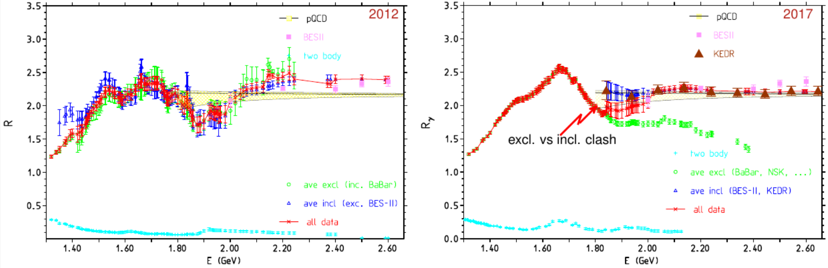
affects most precision tests and new physics searches!!!

$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}\,,\quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}\,,\quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ **Lattice-QCD**

❐ Still an issue in HVP

❐ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty



● illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Three approaches should be further explored for better error estimate

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in α:	**present**	**direct**	1.7×10^{-4}
		Adler	1.2×10^{-4}
	future	**Adler QCD 0.2%**	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
	future	**via $A_{\rm FB}^{\mu\mu}$ off Z**	3×10^{-5}

- **Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$**

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3\,a^2}{4\,v^2}\frac{\mathcal{I}}{\mathcal{Z}+\mathcal{G}}$$

where

$\gamma - Z$ **interference term**	$\mathcal{I} \propto \alpha(s)\,G_\mu$
Z **alone**	$\mathcal{Z} \propto G_\mu^2$
γ **only**	$\mathcal{G} \propto \alpha^2(s)$
v **vector Z coupling**	**also depends on $\alpha(s \sim M_Z^2)$ and $\sin^2\Theta_f(s \sim M_Z^2)$**
a **axial Z coupling**	**sensitive to ρ-parameter (strong M_t dependence)**

❑ **using v, a as measured at Z-peak**

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$

$\sigma_{\mu\mu}$:

1. the photon-exchange term, $\mathcal{G}$, proportional to $\alpha^2(s)$;
2. the Z-exchange term, $\mathcal{Z}$, proportional to G_F^2 (where G_F is the Fermi constant);
3. the Z-photon interference term, $\mathcal{I}$, proportional to $\alpha(s) \times G_F$

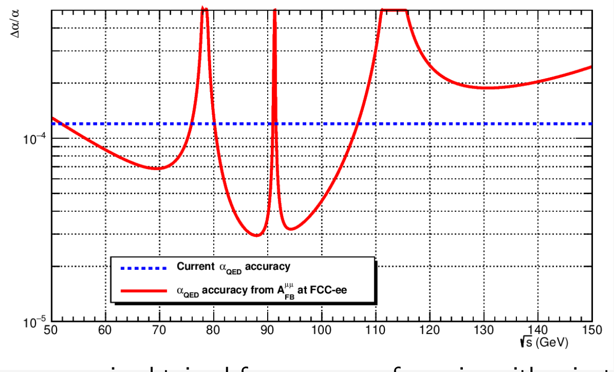
The muon forward-backward asymmetry, $A_{\rm FB}^{\mu\mu}$, is maximally dependent on the interference term

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{2}{2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}},$$

varies with $\alpha_{\rm QED}(s)$ as follows:

$$\Delta A_{\rm FB}^{\mu\mu} = \left(A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}\right) \times \frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.$$

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$



The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.