

Leptogenesis and Charged Lepton Flavour Violation

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Includes:

P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352

Outline:

- **Matter–AntiMatter Asymmetry and Leptogenesis**
- **Resonant and Tri-Resonant Leptogenesis**
- **Flavour Covariant Transport Equations**
- **Charged Lepton Flavour and Number Violation**
- **Numerical Estimates**
- **Conclusions**

• Matter–AntiMatter Asymmetry and Leptogenesis

[N. Aghanim *et al.* [PLANCK Collaboration], *Astron. Astrophys.* 641 (2020) A6;
B.D. Fields, K.A. Olive, T.H. Yeh, C. Young, *JCAP*03 (2020) 010.]

$$\eta_B^{\text{CMB}} = \frac{n_B}{n_\gamma} = (6.104 \pm 0.058) \times 10^{-10}, \quad \eta_B^{\text{BBN+D}} = (6.148 \pm 0.161) \times 10^{-10}.$$

Sakharov's conditions for generating the **BAU**

(from an *initially B-symmetric Universe*): [A.D. Sakharov, *JETP Lett.* 5 (1967) 24.]

- **B**-violating interactions
- **C** and **CP** violation (assuming **CPT** invariance)
- Out-of-equilibrium dynamics

Popular Scenarios for Baryogenesis:

- **Baryogenesis through the decay of a heavy particle**

Out-of-equilibrium, B -violating decay of a heavy GUT particle, e.g. in $SO(10)$.

[M. Yoshimura, PRL41 (1978) 281; S. Dimopoulos, L. Susskind, PRD18 (1978) 4500;

D. Nanopoulos, S. Weinberg, PRD20 (1979) 2484; . . .

K. S. Babu, R. N. Mohapatra, PRL109 (2012) 091803.]

- **Baryogenesis at the electroweak phase transition**

BAU generated by $(B + L)$ -violating sphaleron interactions at $T \sim T_c \approx 140$ GeV, through a 1st order phase transition.

[V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, PLB155 (1985) 36;

MSSM: M. Carena, M. Quirós, C. Wagner '96; K. Rummukainen, M. Laine '98; . . .]

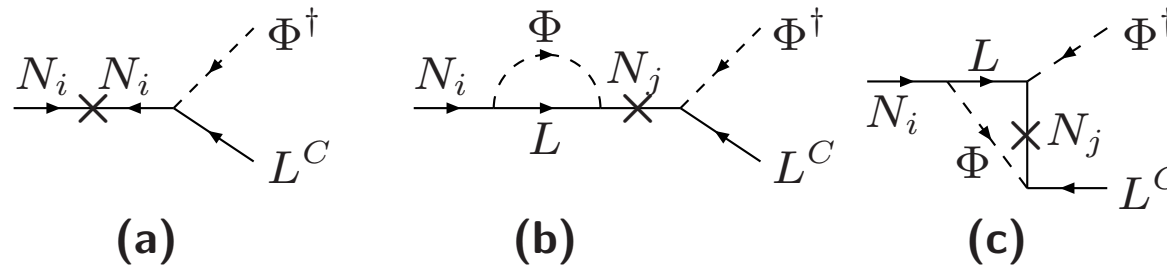
MSSM parameter space **squeezed** by **EDMs** and direct LHC searches

[see talk by M. Carena at SUSY2022.]

⇒ Baryogenesis through Leptogenesis

Out-of-equilibrium L -violating decays of heavy Majorana neutrinos produce a net lepton asymmetry, converted into the BAU through $(B + L)$ -violating sphaleron interactions.

[M. Fukugita, T. Yanagida, PLB174 (1986) 45.]



$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = i \frac{N_F}{8\pi} \left(-W_{\mu\nu} \tilde{W}^{\mu\nu} + B_{\mu\nu} \tilde{B}^{\mu\nu} \right) .$$

In-equilibrium sphaleron rates:

$$120 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}$$

– Models of **Leptogenesis**

- **Hierarchical Leptogenesis**

[M. Fukugita, T. Yanagida, PLB174 (1986) 45.]

Lower mass bound on $m_{N_1} \gtrsim 10^9$ GeV

[S. Davidson, A. Ibarra, PLB535 (2002) 25;

W. Buchmüller, P. Di Bari, M. Plümacher, Annals Phys. **315** (2005) 305.]

- **Resonant Leptogenesis**

[A.P. and T. Underwood, NPB692 (2004) 303,
and references therein]

- **Dirac Leptogenesis**

[K. Dick, M. Lindner, M. Ratz, D. Wright, PRL84 (2000) 4039.]

- **Other scenarios:**

Non-thermal leptogenesis

[G. Lazarides, Q. Shafi, PLB258 (1991) 305.]

Affleck–Dine, spontaneous leptogenesis [e.g., M. Dine, A. Kusenko, RMP76 (2004) 1.]

CPT-violating leptogenesis

[N. Mavromatos, S. Sarkar, EPJC73 (2013) 2359.]

● The Flavourdynamics of Leptogenesis

BAU can be generated from and protected in a single lepton flavour:

$$\frac{1}{3}B - L_{e,\mu,\tau}.$$

[e.g. J.A. Harvey, M.S. Turner, PRD42 (1990) 3344;
H. Dreiner, G.G. Ross, NPB410 (1993) 188;
J.M. Cline, K. Kainulainen, K.A. Olive, PRD49 (1994) 6394.]

Two sources of flavour effects:

- Charged-lepton Yukawa couplings $h_{e,\mu,\tau}$

[E. Nardi, Y. Nir, J. Racker, E. Roulet, JHEP0601 (2006) 068;
A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, A. Riotto, JCAP0604 (2006) 004.]

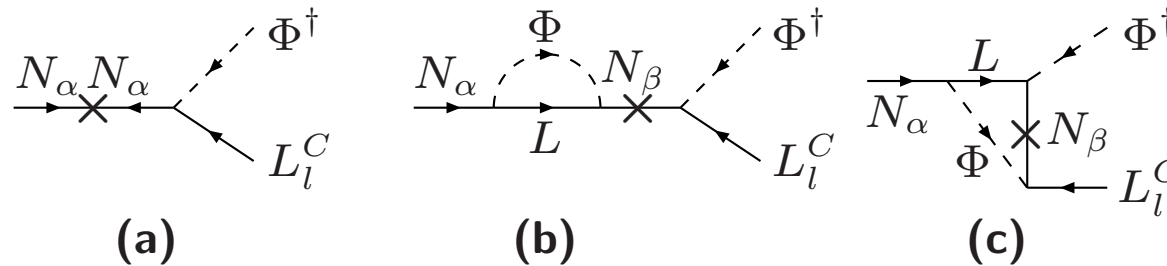
Modify BAU by up to 1-order of magnitude at $T \sim m_N \sim 10^9$ GeV.

- Heavy-neutrino Yukawa couplings $h_{l\alpha}^\nu$

[A.P., PRL95 (2005) 081602 [hep-ph/0408103];
T. Endoh, T. Morozumi and Z. h. Xiong, PTP111 (2004) 123;
A.P., T.E.J. Underwood, PRD72 (2005) 113001;
P. Di Bari, NPB727 (2005) 318; O. Vives, PRD73 (2006) 073006.]

Modify BAU by many orders of magnitude, e.g. $> 10^6$,
and render CLFV in RL models observable.

• Resonant and Tri-Resonant Leptogenesis



Importance of self-energy effects (when $|m_{N_1} - m_{N_2}| \ll m_{N_{1,2}}$)

[J. Liu, G. Segré, PRD48 (1993) 4609;
M. Flanz, E. Paschos, U. Sarkar, PLB345 (1995) 248;
L. Covi, E. Roulet, F. Vissani, PLB384 (1996) 169.]

Importance of the heavy-neutrino width effects: Γ_{N_α}

[A.P., PRD56 (1997) 5431; NPB504 (1997) 61;
A.P., T. Underwood, NPB692 (2004) 303;
inspired by A.P., ZPC47 (1990) 95]

Variants of Resonant Leptogenesis:

- **Soft RL** [Y. Grossman, T. Kashti, Y. Nir, E. Roulet, PRL91 (2003) 251801;
G. D'Ambrosio, G. F. Giudice, M. Raidal, PLB575 (2003) 75.]
- **Radiative RL** [R. Gonzalez Felipe, F. R. Joaquim and B. M. Nobre, PRD70 (2004) 085009;
G. C. Branco, A. J. Buras, S. Jager, S. Uhlig, A. Weiler, JHEP0709 (2007) 004;
G. C. Branco, R. Gonzalez Felipe, M. N. Rebelo, H. Serodio, PRD79 (2009) 093008.]
- **Coherent RL** (via sterile neutrino oscillations)
[E. K. Akhmedov, V. A. Rubakov, A. Y. Smirnov, PRL81 (1998) 1359;
T. Asaka, M. Shaposhnikov, PLB620 (2005) 17.]

- The Field-Theory of **Resonant Leptogenesis**:

[A.P., PRD56 (1997) 5431; NPB504 (1997) 61.]

LSZ-type formalism for **mixing** and **decay** of heavy Majorana neutrinos

$$\times [S_{\alpha\alpha}(\not{p})]^{-1} u_{N_\alpha}(p) \equiv \mathcal{T}^{\text{eff}}(N_\alpha \rightarrow L_l \Phi)$$

2- N Mixing Model:

$$S_{\alpha\beta}(\not{p}) = \left(\begin{array}{cc} \not{p} - m_{N_1} + \Sigma_{11}(\not{p}) & \Sigma_{12}(\not{p}) \\ \Sigma_{21}(\not{p}) & \not{p} - m_{N_2} + \Sigma_{22}(\not{p}) \end{array} \right)^{-1}$$

[For 3- N mixing, see, A.P., T. Underwood, NPB692 (2004) 303;

F. Deppisch, A.P., PRD83 (2011) 076007;

P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]

Effective (Resummed) Neutrino Yukawa Couplings:

$$\mathcal{T}^{\text{eff}}(N_\alpha \rightarrow L_l \Phi) = \mathbf{h}_{l\alpha} \bar{u}_l P_R u_{N_\alpha}$$

For 2- N mixing:

$$\mathbf{h}_{l\alpha} = h_{l\alpha}^\nu + iB_{l\alpha} - \frac{ih_{l\beta}^\nu m_{N_\alpha} (m_{N_\alpha} A_{\alpha\beta} + m_{N_\beta} A_{\beta\alpha})}{m_{N_\alpha}^2 - m_{N_\beta}^2 + 2i A_{\beta\beta} m_{N_\alpha}^2}$$

$$\mathbf{h}_{l\alpha}^c = h_{l\alpha}^{\nu*} + iB_{l\alpha}^* - \frac{ih_{l\beta}^{\nu*} m_{N_\alpha} (m_{N_\alpha} A_{\alpha\beta}^* + m_{N_\beta} A_{\beta\alpha}^*)}{m_{N_\alpha}^2 - m_{N_\beta}^2 + 2i A_{\beta\beta} m_{N_\alpha}^2}$$

Lepton Flavour Asymmetries

[A.P., T. Underwood, PRD72 (2005) 113001.]

$$\delta_{\alpha l} \equiv \frac{\Gamma_{\alpha l} - \Gamma_{\alpha l}^c}{\sum_{l=e,\mu,\tau} (\Gamma_{\alpha l} + \Gamma_{\alpha l}^c)} = \frac{|\mathbf{h}_{l\alpha}|^2 - |\mathbf{h}_{l\alpha}^c|^2}{(\mathbf{h}^\dagger \mathbf{h})_{\alpha\alpha} + (\mathbf{h}^{c\dagger} \mathbf{h}^c)_{\alpha\alpha}}$$

ϵ' -type CP violation :

$$\epsilon'_{N_\alpha} = \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{\alpha\beta}^2}{(h^{\nu\dagger} h^\nu)_{\alpha\alpha} (h^{\nu\dagger} h^\nu)_{\beta\beta}} \left(\frac{\Gamma_{N_\beta}}{m_{N_\beta}} \right) f \left(\frac{m_{N_\beta}^2}{m_{N_\alpha}^2} \right),$$

where

$$\Gamma_{N_\beta} = \frac{(h^{\nu\dagger} h^\nu)_{\beta\beta}}{8\pi} m_{N_\beta}$$

ϵ -type CP violation :

$$\epsilon_{N_\alpha} = \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{\alpha\beta}^2}{(h^{\nu\dagger} h^\nu)_{\alpha\alpha} (h^{\nu\dagger} h^\nu)_{\beta\beta}} \frac{(m_{N_\alpha}^2 - m_{N_\beta}^2) m_{N_\alpha} \Gamma_{N_\beta}}{(m_{N_\alpha}^2 - m_{N_\beta}^2)^2 + m_{N_\alpha}^2 \Gamma_{N_\beta}^2}$$

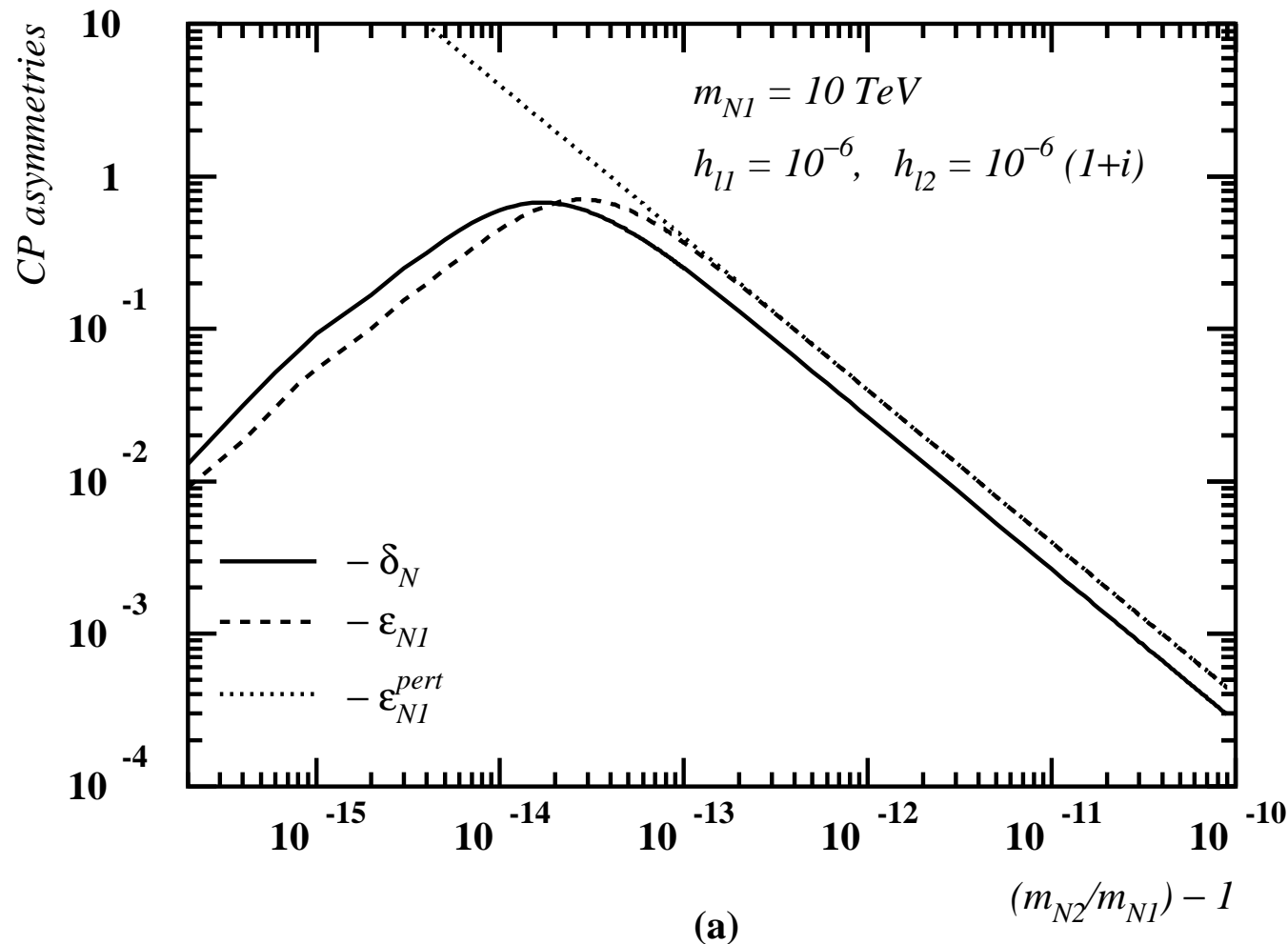
Note: $\epsilon_{N_{1,2}}$ have the same sign.

Resonant conditions for $O(1)$ leptonic asymmetries:

[A.P., PRD56 (1997) 5431.]

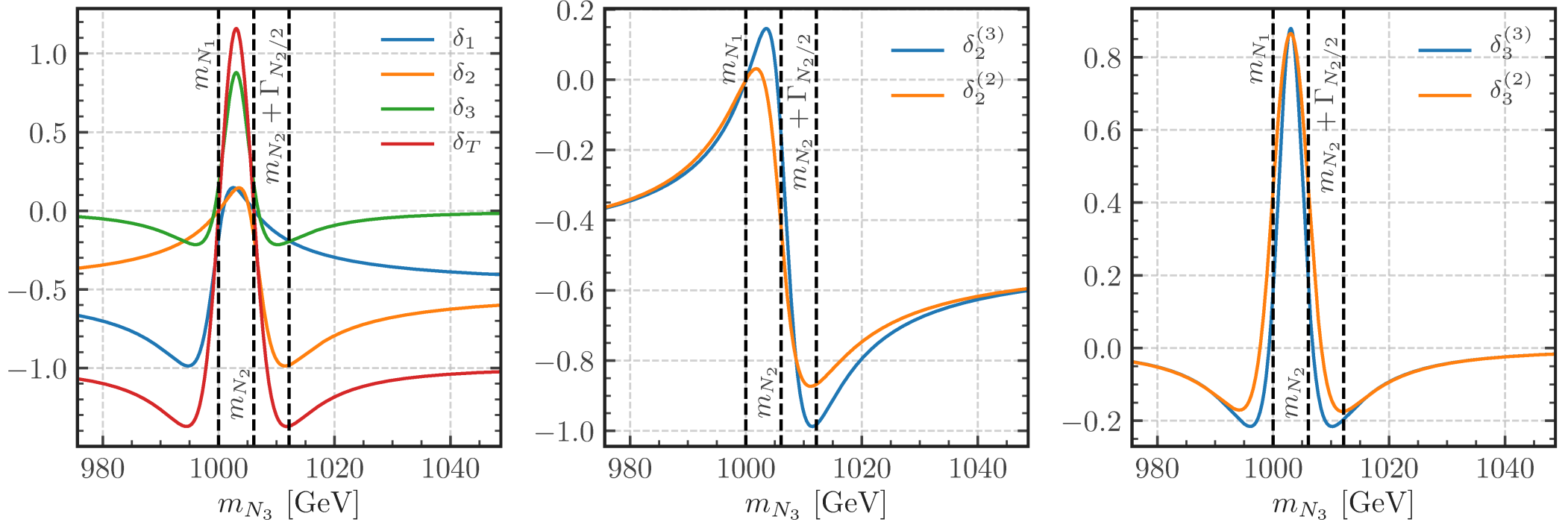
$$\Rightarrow m_{N_2} - m_{N_1} \sim \frac{1}{2} \Gamma_{N_{1,2}}$$

$$\Rightarrow \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{\alpha\beta}^2}{(h^{\nu\dagger} h^\nu)_{\alpha\alpha} (h^{\nu\dagger} h^\nu)_{\beta\beta}} \sim 1$$



– Tri-Resonant leptonic asymmetries

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



Mass and Yukawa parameters (with **approximate** Z_6 symmetry):

$$m_{N_2} = m_{N_1} + \frac{\Gamma_{N_1}}{2}, \quad m_{N_3} = m_{N_2} + \frac{\Gamma_{N_2}}{2}, \quad \mathbf{h}_0^\nu = \begin{pmatrix} a & a\omega & a\omega^2 \\ b & b\omega & b\omega^2 \\ c & c\omega & c\omega^2 \end{pmatrix},$$

with $\omega = \exp(i\pi/3)$, and $a, b, c \lesssim 10^{-3}$ for $m_\nu \lesssim 0.05$ eV.

• Flavour Covariant Transport Equations

[E. W. Kolb and S. Wolfram, NPB172 (1980) 224.]

– Flavour Diagonal Boltzmann Equations

$$\frac{dn_a}{dt} + 3Hn_a = \sum_{aX' \leftrightarrow Y} \left(-\frac{n_a n_{X'}^{\text{eq}}}{n_a^{\text{eq}} n_{X'}^{\text{eq}}} \gamma(aX' \rightarrow Y) + \frac{n_Y}{n_Y^{\text{eq}}} \gamma(Y \rightarrow aX') \right),$$

where n_a is the **number density**:

$$\begin{aligned} n_a(T) &= g_a \int \frac{d^3\mathbf{p}}{(2\pi)^3} \exp \left[- \left(\sqrt{\mathbf{p}^2 + m_a^2} - \mu_a(T) \right) / T \right] \\ &= \frac{g_a m_a^2 T e^{\mu_a(T)/T}}{2\pi^2} K_2 \left(\frac{m_a}{T} \right) \end{aligned}$$

and $\gamma(X \rightarrow Y)$ is the **collision term**:

$$\gamma(X \rightarrow Y) = \int d\pi_X d\pi_Y (2\pi)^4 \delta^{(4)}(p_X - p_Y) e^{-p_X^0/T} |\mathcal{M}(X \rightarrow Y)|^2.$$

– Flavour Diagonal BEs for Leptogenesis

[A.P., T.E. Underwood, NPB**692** (2004) 303; PRD**72** (2005) 113001.]

Define first

$$\eta^X \equiv n_X/n_\gamma, \quad z \equiv m_{N_1}/T, \quad H \equiv H(T = m_N) \approx 17 m_N^2/M_{\text{Planck}}$$

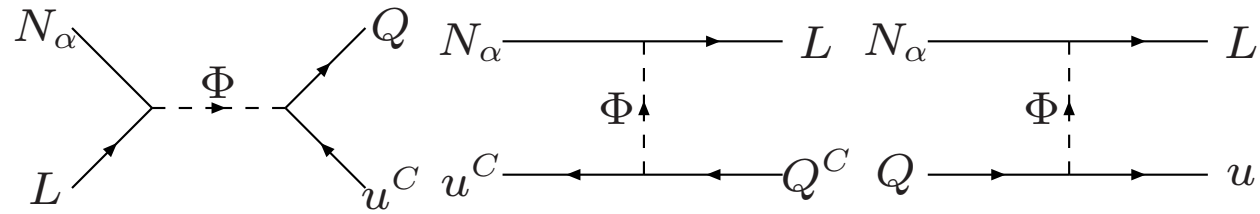
and the short-hands:

$$\begin{aligned} \gamma_Y^X &\equiv \gamma(X \rightarrow Y) + \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} \gamma_X^Y, \\ \delta\gamma_Y^X &\equiv \gamma(X \rightarrow Y) - \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} -\delta\gamma_X^Y. \end{aligned}$$

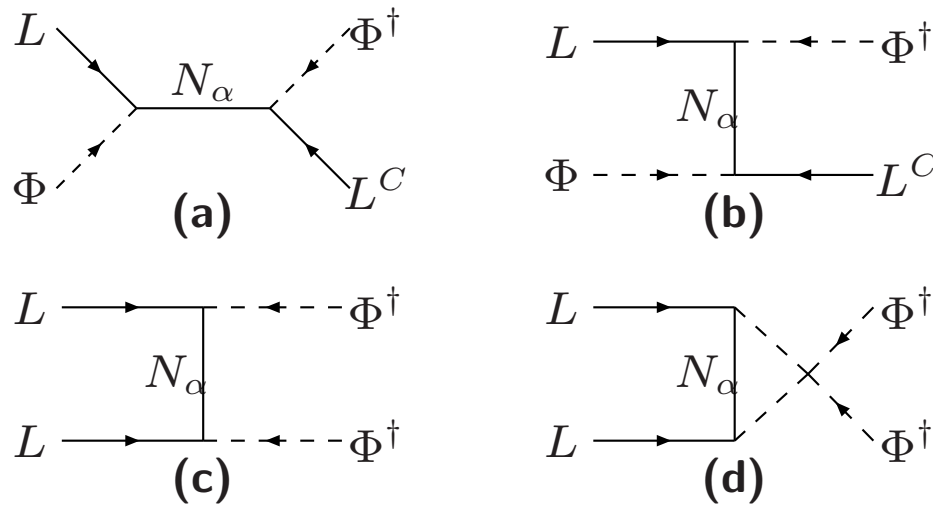
Write down the **Boltzmann equations**:

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d\eta_\alpha^N}{dz} &= \left(1 - \frac{\eta_\alpha^N}{\eta_{\text{eq}}^N}\right) \sum_k \gamma_{L_k\Phi}^{N_\alpha} + \dots \\ \frac{H n_\gamma}{z} \frac{d\delta\eta_l^L}{dz} &= \sum_\alpha \left(\frac{\eta_\alpha^N}{\eta_{\text{eq}}^N} - 1\right) \delta\gamma_{L_l\Phi}^{N_\alpha} - \frac{2}{3}\delta\eta_l^L \sum_k \left(\gamma_{L_k^c\Phi^c}^{L_l\Phi} + \gamma_{L_k\Phi}^{L_l\Phi}\right) \\ &\quad - \frac{2}{3} \sum_k \delta\eta_k^L \left(\gamma_{L_l^c\Phi^c}^{L_k\Phi} - \gamma_{L_l\Phi}^{L_k\Phi}\right) + \dots \end{aligned}$$

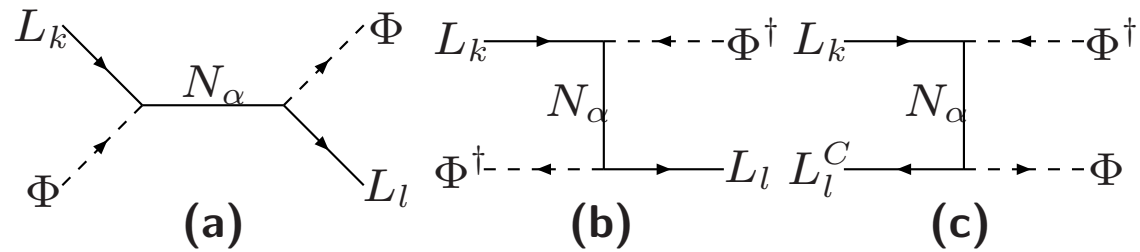
$\Delta L = 1$ scatterings involving L , N_α and quarks



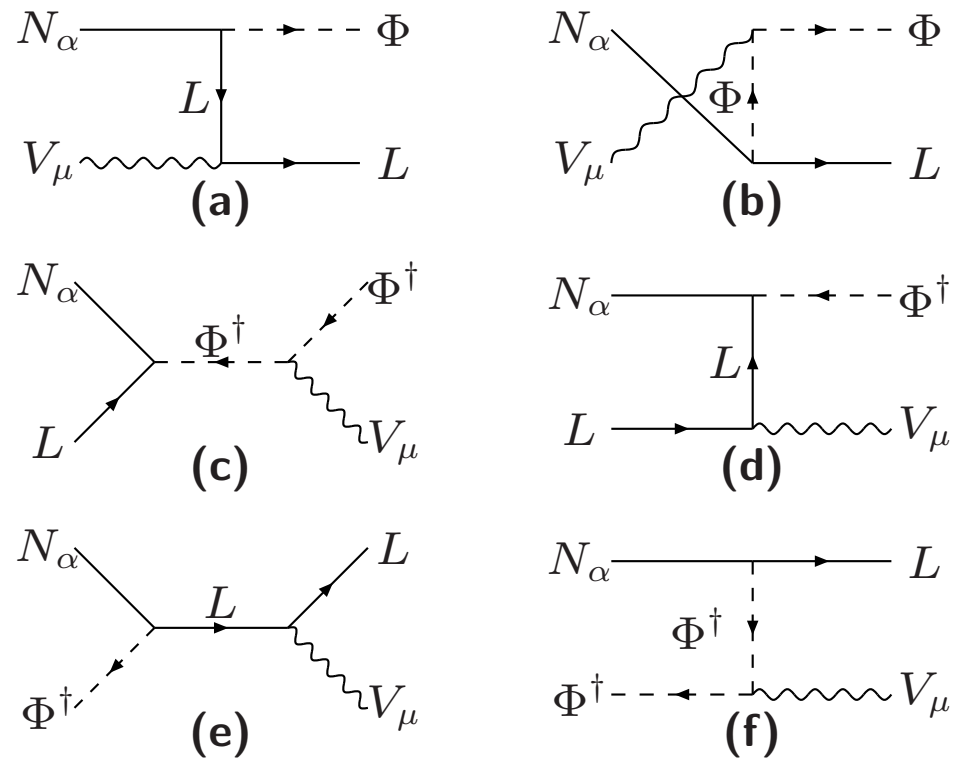
$\Delta L = 2$ scatterings involving L , Φ and N_α



$\Delta L = 0$ scatterings involving L , Φ and N_α



Gauge-mediated $\Delta L = 1$ scatterings



– Order-of-magnitude estimate of the **BAU**

Flavour-dependent decay width of heavy Majorana neutrino N_α :

$$\Gamma_{N_\alpha \rightarrow l} \equiv \Gamma(N_\alpha \rightarrow L_l \Phi) = (h^{\nu\dagger})_{\alpha l} h_{l\alpha}^\nu \frac{m_{N_\alpha}}{8\pi}$$

Define the effective wash-out K -factors:

$$K_l^{\text{eff}} \equiv \frac{\sum_{N_\alpha} \Gamma_{N_\alpha \rightarrow l}}{H}$$

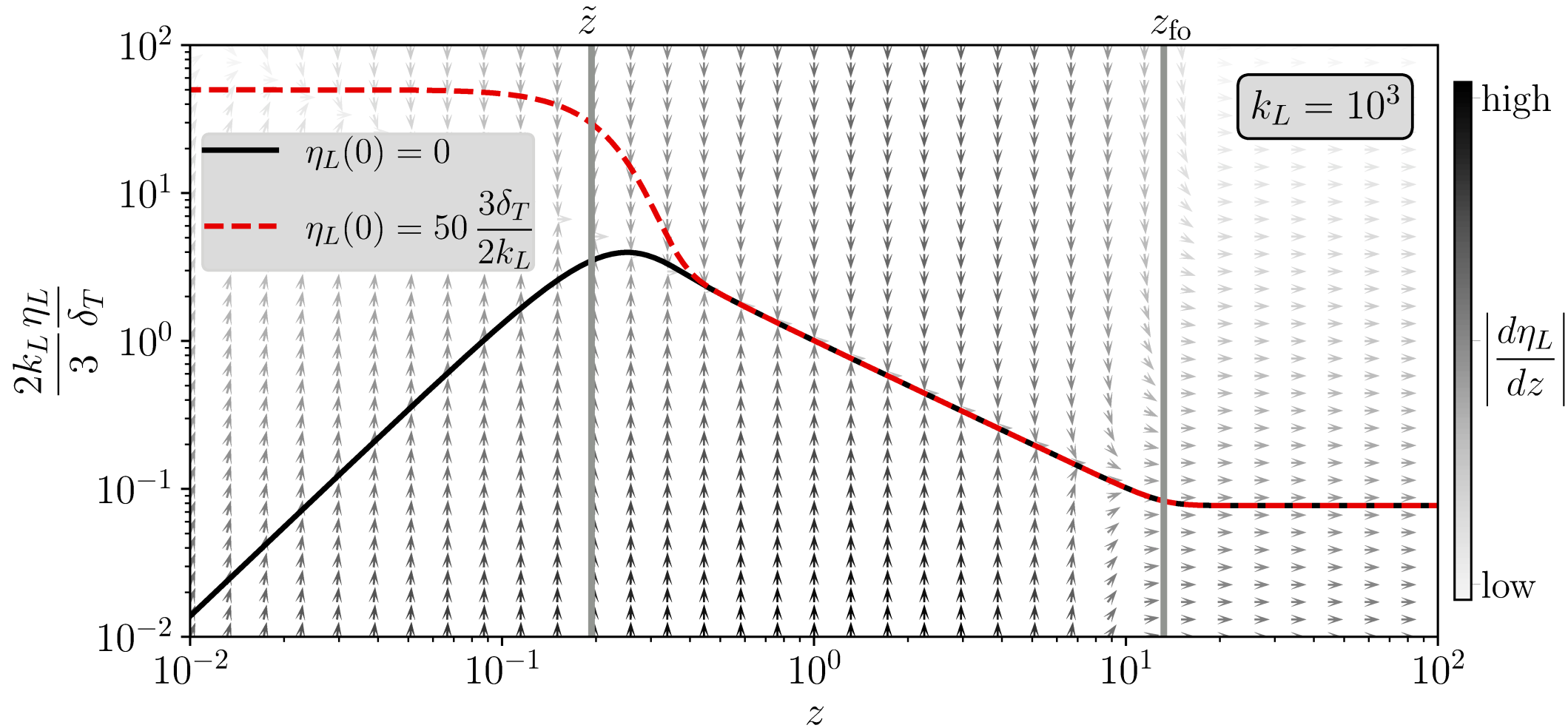
Estimate of the BAU (strong wash-out regime):

[F. Deppisch, A.P., PRD83 (2011) 076007.]

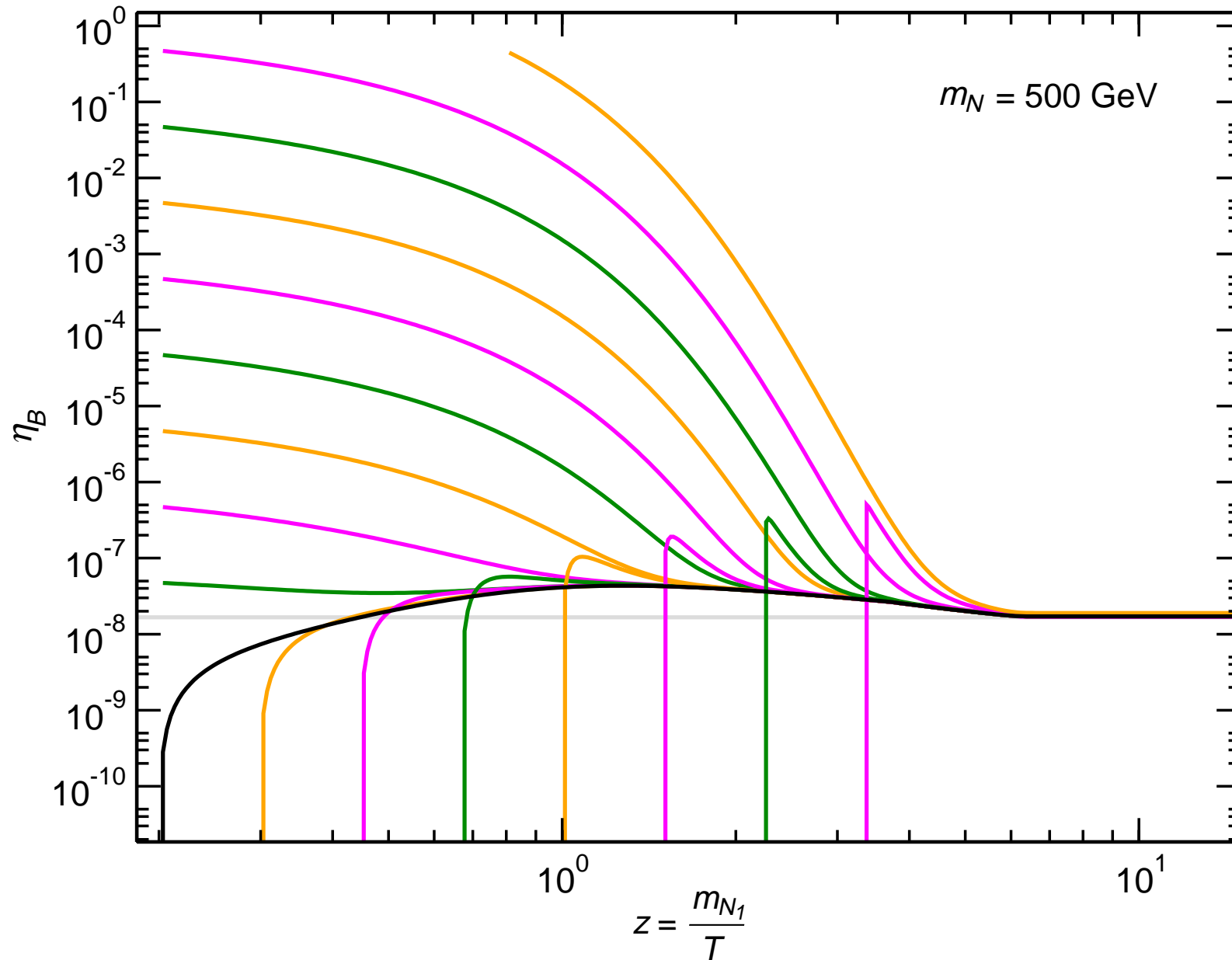
$$\eta_B^{\text{mix}} \sim -3 \cdot 10^{-2} \sum_{l=e,\mu,\tau} \frac{\delta_l^{\text{mix}}}{K_l^{\text{eff}} \min \left[m_N / T_c, 1.25 \ln(25 K_l^{\text{eff}}) \right]} .$$

Independence of **BAU** on Initial Conditions

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



with $z = m_{N_1}/T$.



– Flavour Covariant Rate Equations (Markovian approximation)

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

$$\frac{H n_\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n_\gamma}{2} [\mathcal{E}_N, \delta\eta^N]_\alpha^\beta + [\text{Re}(\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^N]_\alpha^\beta}{dz} &= -2i n_\gamma [\mathcal{E}_N, \underline{\eta}^N]_\alpha^\beta + 2i [\text{Im}(\delta\gamma_{L\Phi}^N)]_\alpha^\beta \\ &\quad - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Im}(\delta\gamma_{L\Phi}^N) \right\}_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \delta\eta^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^L]_l^m}{dz} &= -[\delta\gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\tilde{\eta}_{\text{eq}}^N} [\delta\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta + \frac{[\delta\eta^N]_\beta^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta\eta^L, \gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta\eta^L]_k^n \left([\gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi}]_{n l}^k{}^m - [\gamma_{L\Phi}^{L\Phi}]_{n l}^k{}^m \right) \\ &\quad - \frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

– **Unified Description of 3 Physically Distinct Phenomena:**

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

- **Resonant Mixing between Heavy Neutrinos,**

through: $\mathbf{h}_{l\alpha}$ and $\mathbf{h}_{l\alpha}^c$ in $[\gamma_{L\Phi}^N]_l^m \alpha^\beta$ and $[\delta\gamma_{L\Phi}^N]_l^m \alpha^\beta$.

- **Coherent Oscillations between Heavy Neutrinos** ($\Delta m_N \ll m_N$),

from $[\mathcal{E}_N, \underline{\eta}^N]$ and the rank-4 tensor term $\frac{[\delta\eta^N]_\beta^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m \alpha^\beta$, yielding:

$$\delta\eta_{\text{osc}}^L \sim \frac{3}{2Kz} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11} (h^\dagger h)_{22}} \frac{2(m_{N_1}^2 - m_{N_2}^2) m_N \Gamma_N}{(m_{N_1}^2 - m_{N_2}^2)^2 + \left(\frac{2m_N \Gamma_N \text{Im}[h^\dagger h]_{12}}{|[h^\dagger h]_{12}|} \right)^2},$$

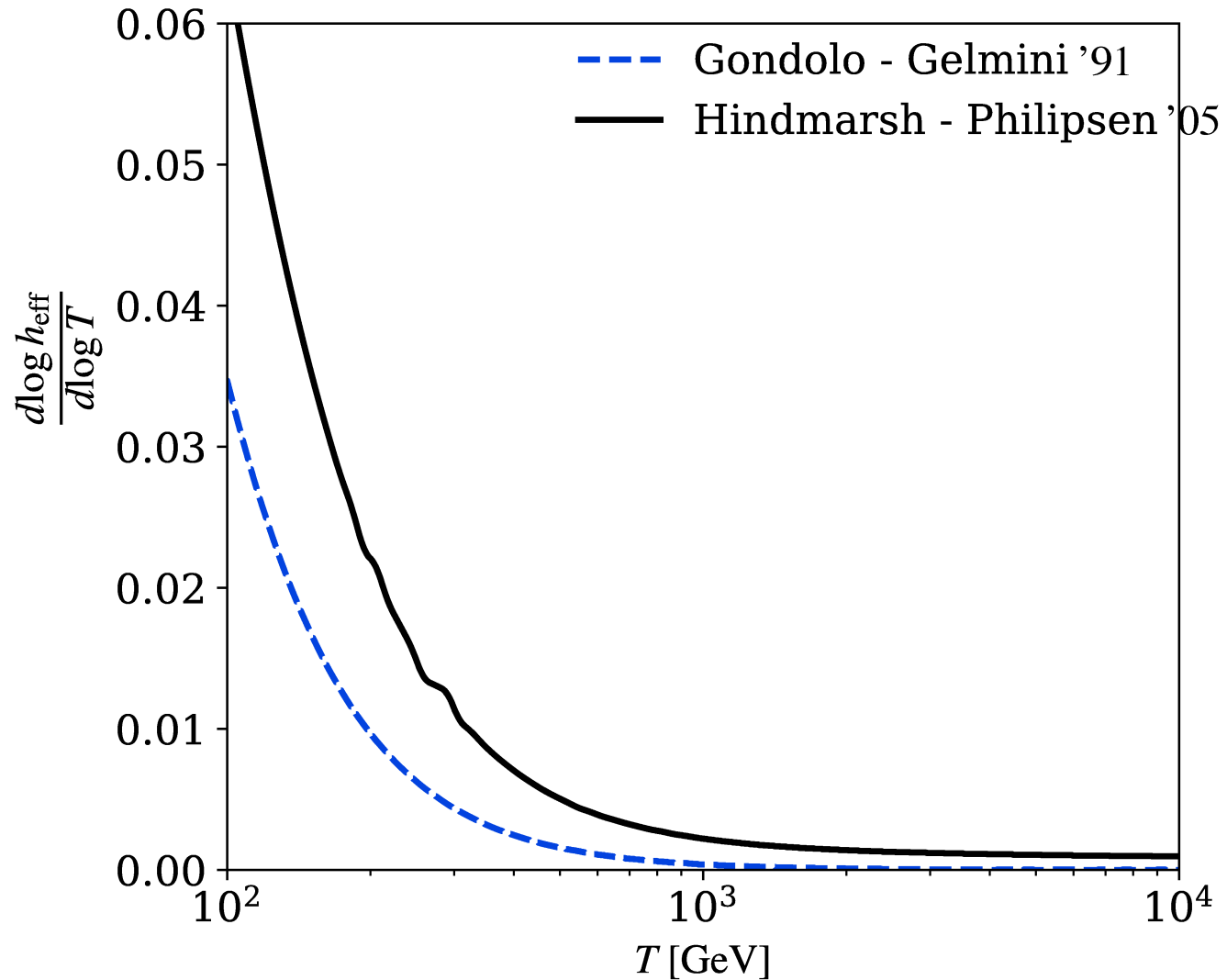
with $\Gamma_N = \frac{1}{2} \left(\Gamma_{N_1}^{(0)} + \Gamma_{N_2}^{(0)} \right)$. [**Note:** Different from the ARS mechanism.]

- **Decoherence Effects due to Charged Lepton Yukawa Couplings,**

from $-\frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m$

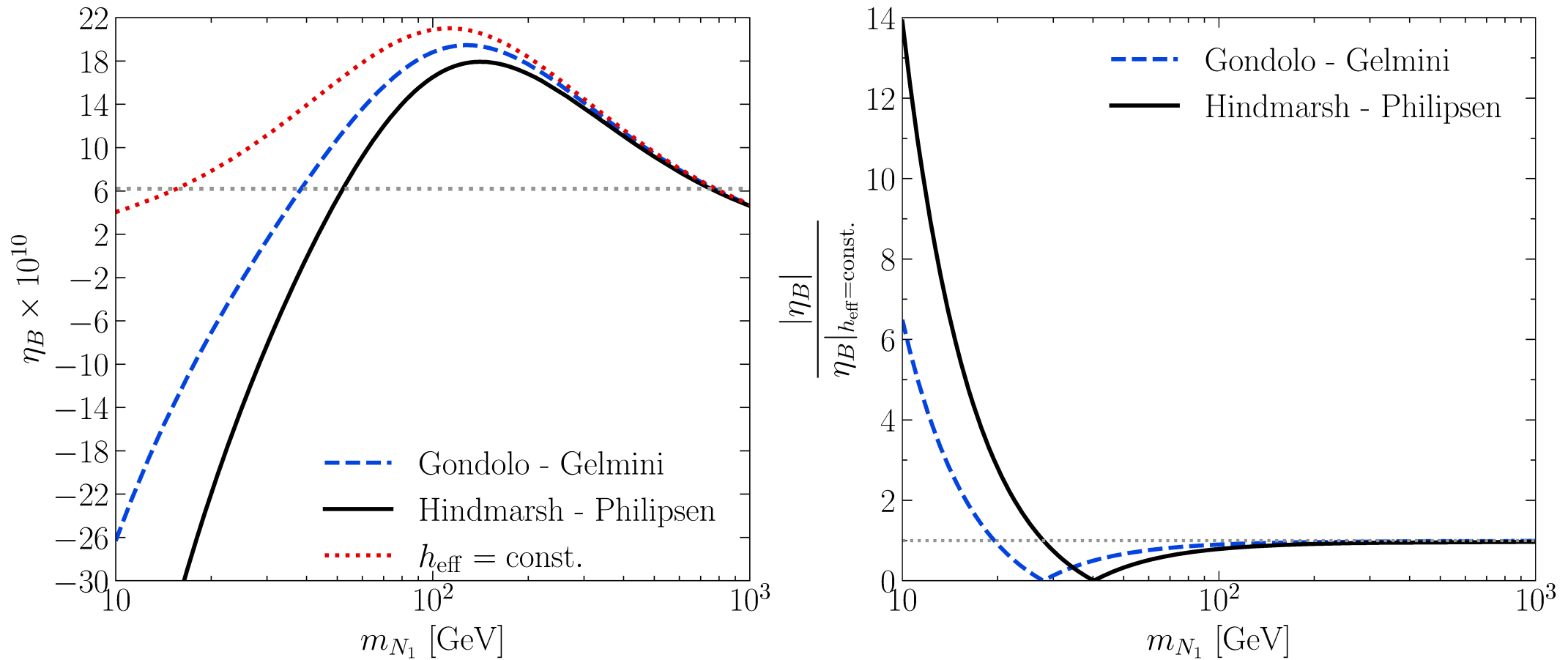
– The effect of **varying relativistic dofs** on **Transport Equations**

$$\rho(T) = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4, \quad s(T) = \frac{\pi^2}{45} h_{\text{eff}}(T) T^3$$



Modification of the **BAU** predictions due to **varying** $h_{\text{eff}}(T)$

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



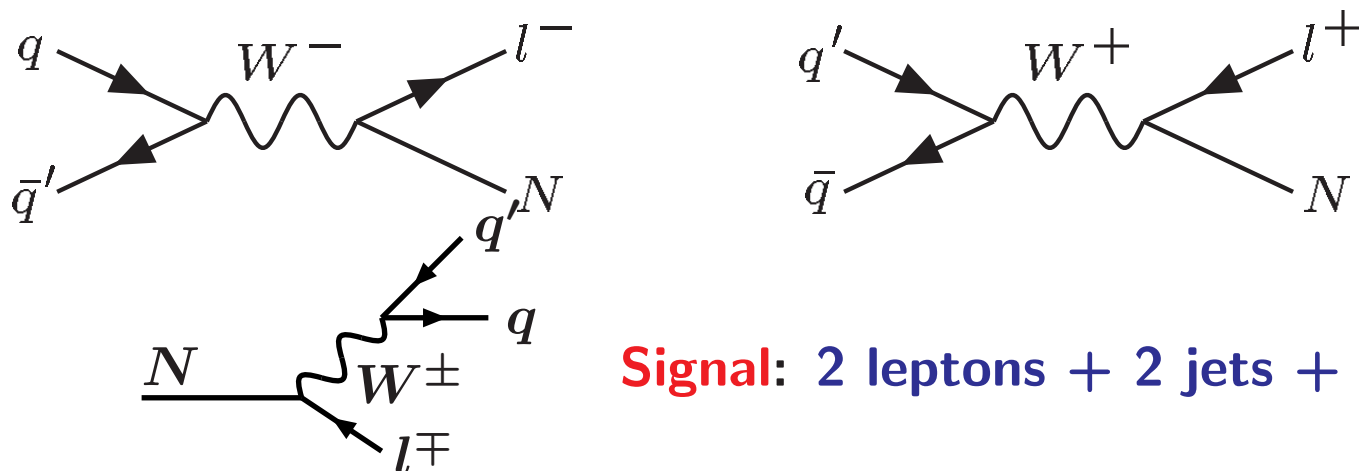
Isentropic FRW Universe ($ds/dt = -3Hs$):

$$\frac{d}{dt} = \frac{ds}{dt} \frac{dT}{ds} \frac{d}{dT} = H \delta_h^{-1} \frac{d}{dz}, \quad \text{with } \delta_h = 1 - \frac{1}{3} \frac{d \ln h_{\text{eff}}}{d \ln z}.$$

• Charged Lepton Flavour and Number Violation

– Heavy Majorana Neutrinos at the LHC

[A.P., ZPC55 (1992) 275; A. Datta, M. Guchait, A.P., PRD50 (1994) 3195;
 J. Kersten, A. Y. Smirnov, PRD76 (2007) 073005;
 F. del Aguila, J. A. Aguilar-Saavedra, R. Pittau, JHEP0710 (2007) 047.]



Signal: 2 leptons + 2 jets + no \cancel{p}_T

- **LNV** signatures: $pp \rightarrow e^+e^+, e^+\mu^+, e^-e^-, e^-\mu^-, e^-\tau^- \dots$

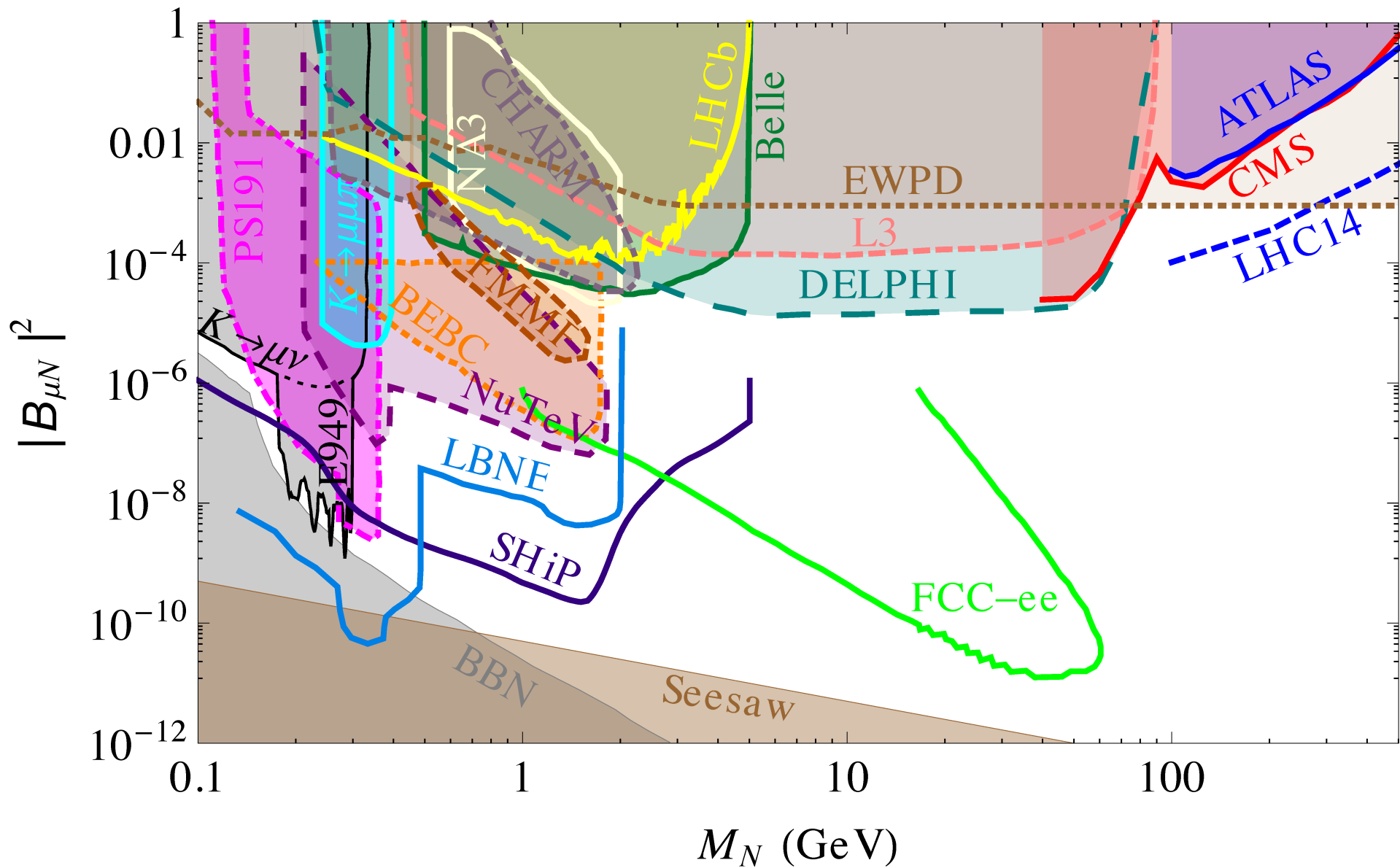
- **LFV** signatures: $pp \rightarrow e^+\mu^-, e^-\mu^+, e^-\tau^+ \dots$

- **CP Asymmetries**

[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]

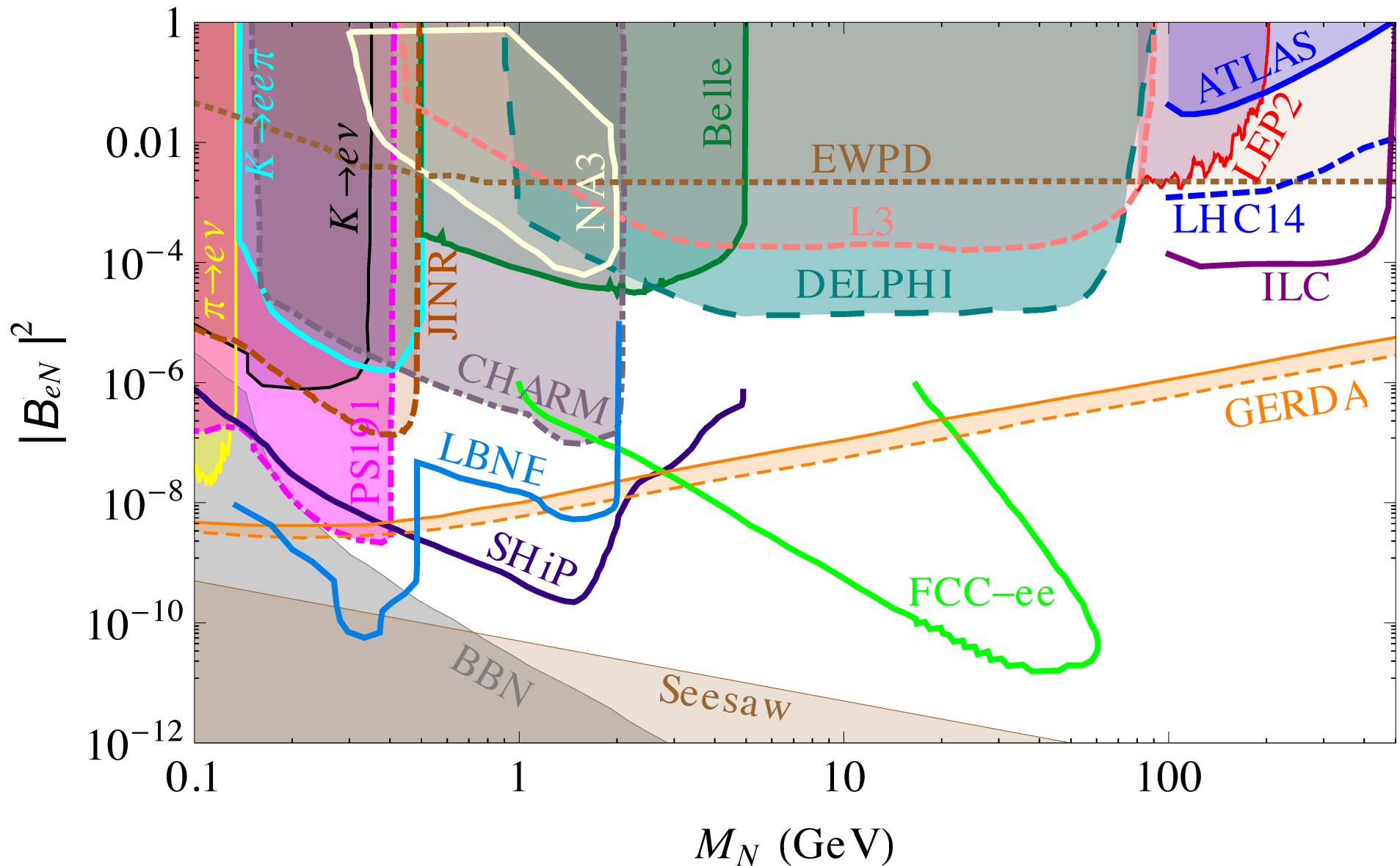
LHC and Other Constraints: μN -Sector

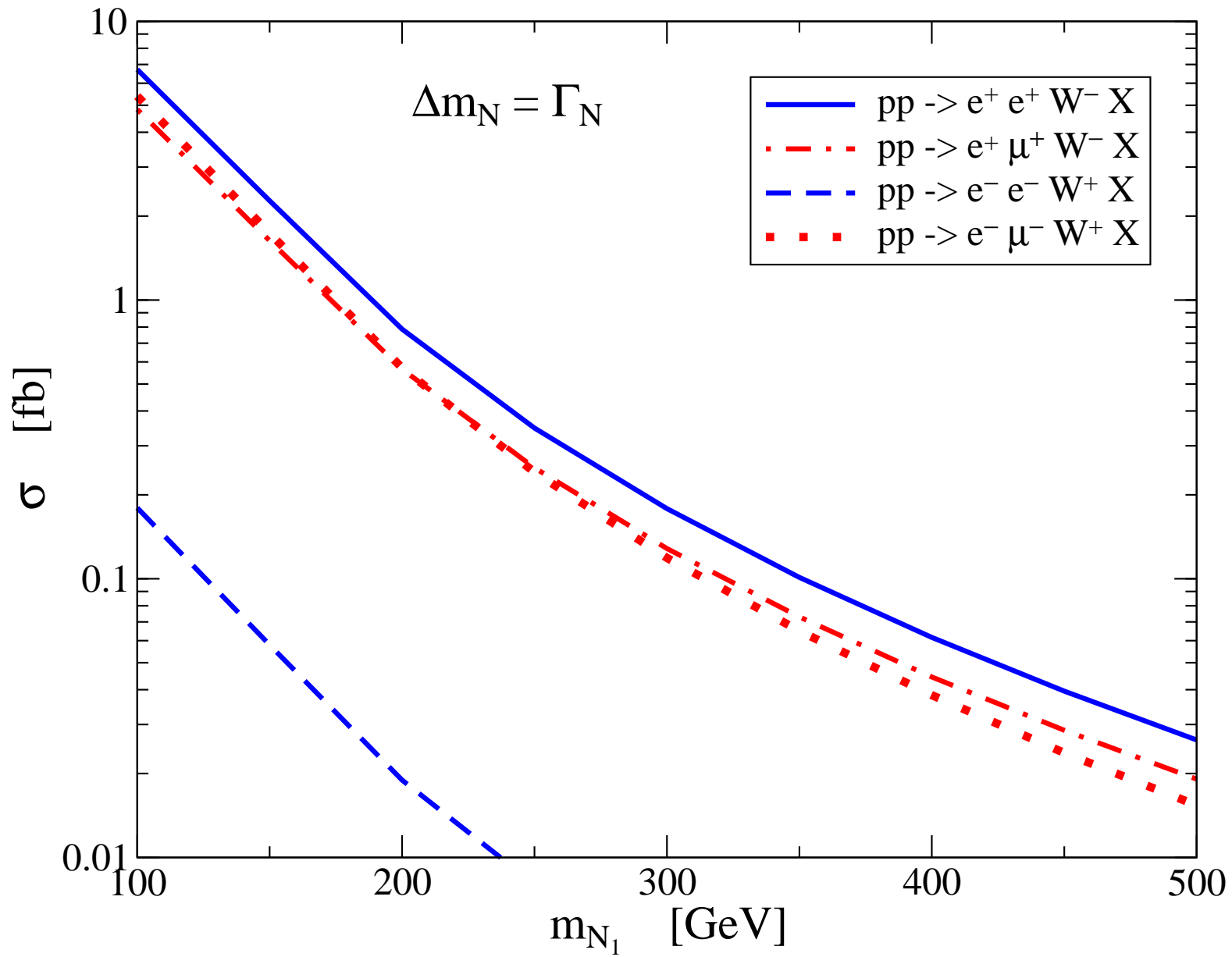
[F.F. Deppisch, P.S.B. Dev, A.P., NJP17 (2015) 7.]



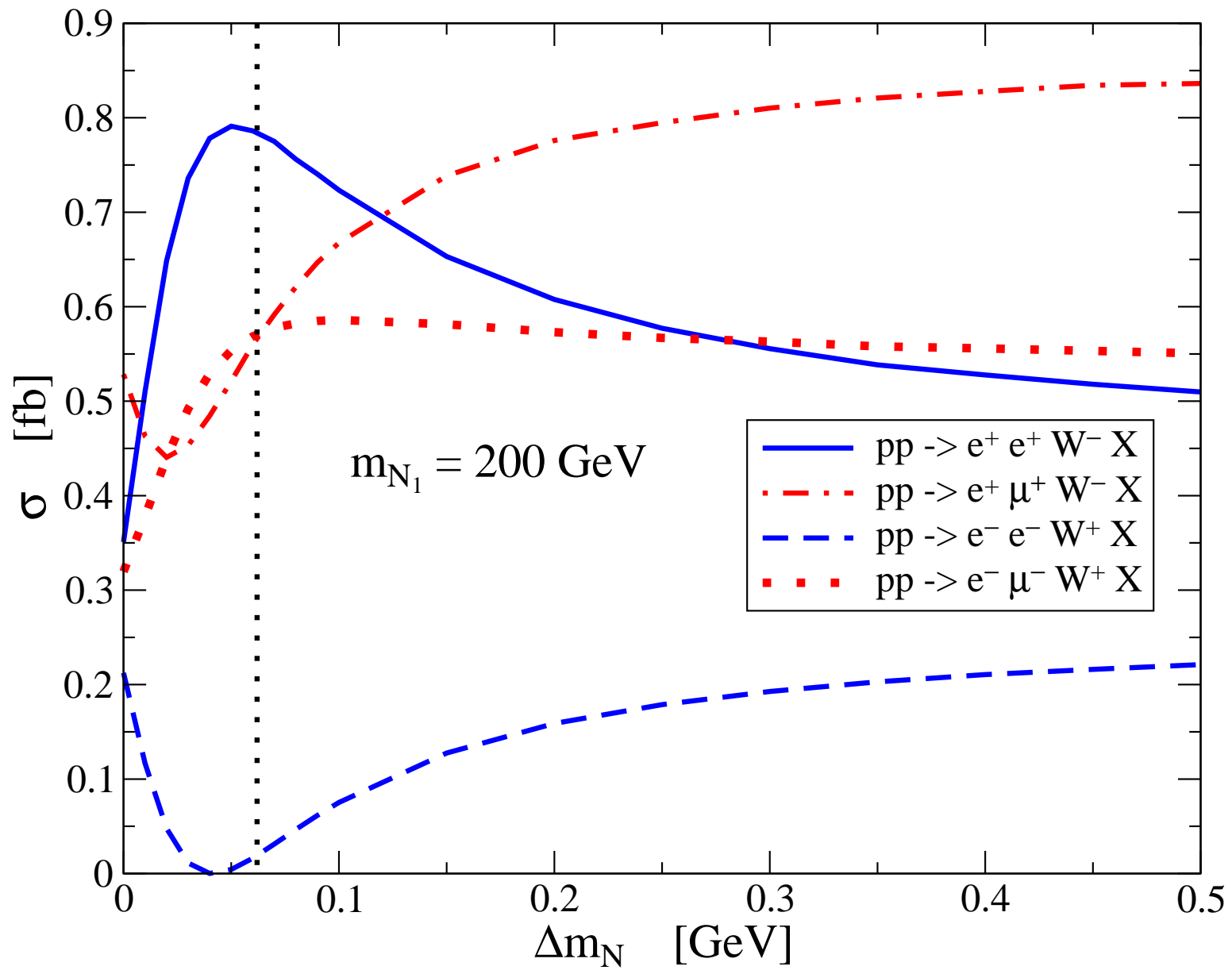
LHC and Other Constraints: eN -Sector

[F.F. Deppisch, P.S.B. Dev, A.P., NJP17 (2015) 7.]





[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]



[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]

- **Lepton Number Violation:**

$$A_{\text{CP}}(\text{LNV1}) = \frac{\sigma(pp \rightarrow e^+e^+W^-X) - K\sigma(pp \rightarrow e^-e^-W^+X)}{\sigma(pp \rightarrow e^+e^+W^-X) + K\sigma(pp \rightarrow e^-e^-W^+X)},$$

$$A_{\text{CP}}(\text{LNV2}) = \frac{\sigma(pp \rightarrow e^+\mu^+W^-X) - K\sigma(pp \rightarrow e^-\mu^-W^+X)}{\sigma(pp \rightarrow e^+\mu^+W^-X) + K\sigma(pp \rightarrow e^-\mu^-W^+X)},$$

$$R_{\text{CP}}(\text{LNV}) = \frac{\frac{\sigma(pp \rightarrow e^+e^+W^-X)}{\sigma(pp \rightarrow e^+\mu^+W^-X)} - \frac{\sigma(pp \rightarrow e^-e^-W^+X)}{\sigma(pp \rightarrow e^-\mu^-W^+X)}}{\frac{\sigma(pp \rightarrow e^+e^+W^-X)}{\sigma(pp \rightarrow e^+\mu^+W^-X)} + \frac{\sigma(pp \rightarrow e^-e^-W^+X)}{\sigma(pp \rightarrow e^-\mu^-W^+X)}}.$$

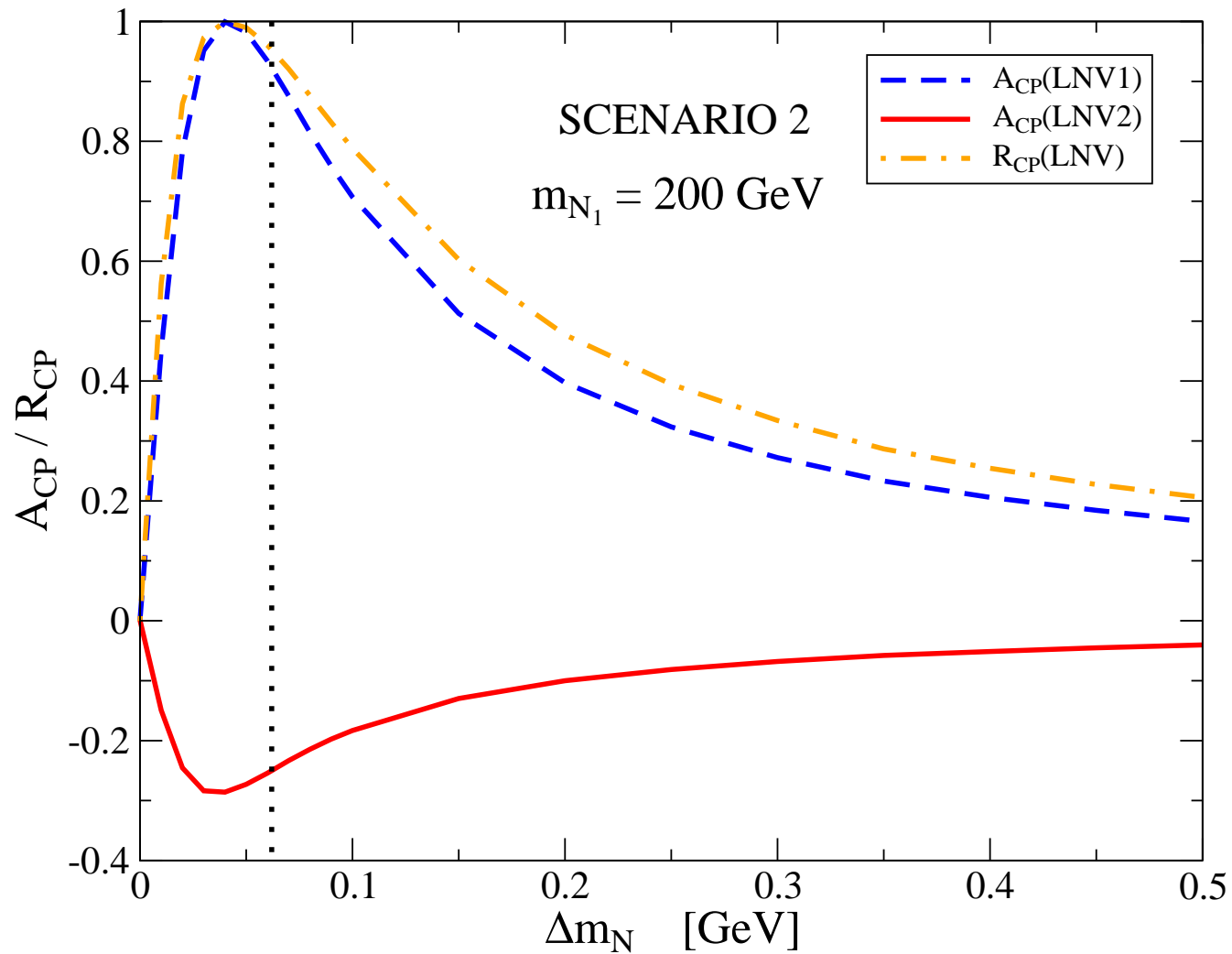
- **Lepton Flavour Violation:**

$$A_{\text{CP}}(\text{LNC}) = \frac{\sigma(pp \rightarrow e^+\mu^-W^\pm X) - \sigma(pp \rightarrow e^-\mu^+W^\pm X)}{\sigma(pp \rightarrow e^+\mu^-W^\pm X) + \sigma(pp \rightarrow e^-\mu^+W^\pm X)},$$

$$R_{\text{CP}}(\text{LNC}) = \frac{\frac{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}{\sigma(pp \rightarrow e^-\mu^+W^\pm X)} - \frac{\sigma(pp \rightarrow e^-\mu^+W^\pm X)}{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}}{\frac{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}{\sigma(pp \rightarrow e^-\mu^+W^\pm X)} + \frac{\sigma(pp \rightarrow e^-\mu^+W^\pm X)}{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}}.$$

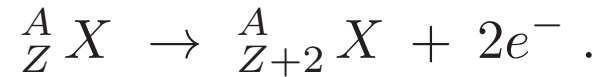
Resonant CP Violation via Heavy Neutrino Mixing

[AP, NPB504 (1997) 61;
S. Bray, J.S. Lee, AP, NPB786 (2007) 95.]



– **Charged LNV** and **LFV** at the **Intensity Frontier**

• **$0\nu\beta\beta$ Decay**



Half-life for $0\nu\beta\beta$ decay:

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{|\langle m \rangle|^2}{m_e^2} |\mathcal{M}_{0\nu\beta\beta}|^2 G_{01} .$$

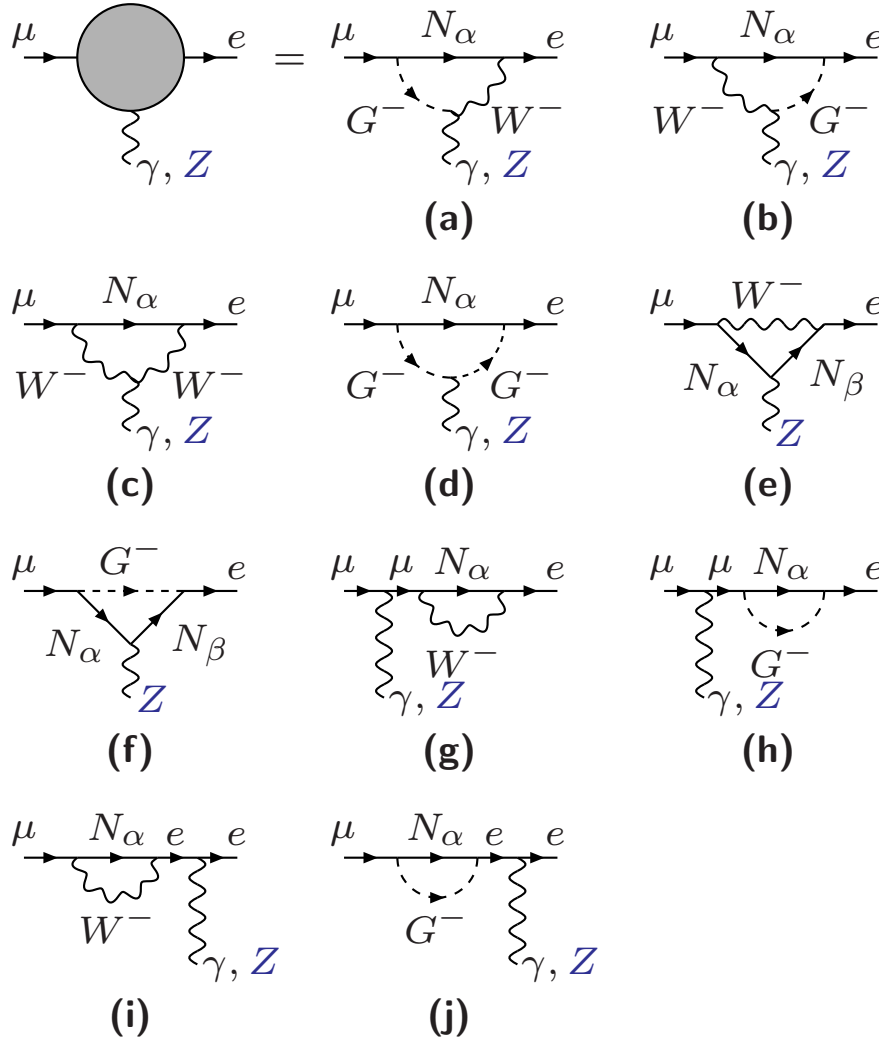
Weak limits on RL models:

$$|\langle m_{0\nu\beta\beta} \rangle| \simeq |(\mathbf{m}^\nu)_{ee}| \lesssim 0.015 \text{ eV} .$$

Future $0\nu\beta\beta$ experiments will be sensitive to $|\langle m \rangle| \sim 0.01\text{--}0.05$ eV, such as SuperNEMO . . .

• $\mu \rightarrow e\gamma$

[T.P. Cheng, L.F. Li, PRL45 (1980) 1908;
 J.G. Körner, A.P., K. Schilcher, PLB300 (1993) 381;
 J. Bernabéu, J.G. Körner, A.P., K. Schilcher, PRL71 (1993) 2695.]



$$\Omega_{ll'} \equiv \sum_{\alpha} B_{l\alpha} B_{l'\alpha}^*$$

with $B_{l\alpha} \simeq (\mathcal{M}_D)_{l\alpha} m_{N\alpha}^{-1}$

$$B(\mu \rightarrow e\gamma) \approx 7 \cdot 10^{-4} |\Omega_{e\mu}|^2,$$

for $m_N \gtrsim 100$ GeV.

Compare with

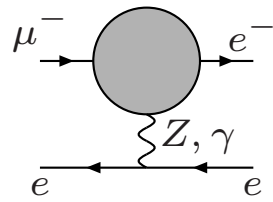
$$B^{\text{exp}}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$$

MEG sensitivity:

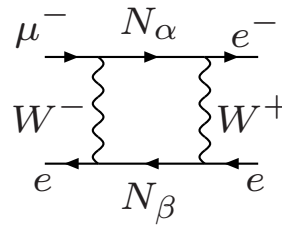
$$B(\mu \rightarrow e\gamma) \sim 10^{-13} - 10^{-14}$$

• $\mu \rightarrow eee$

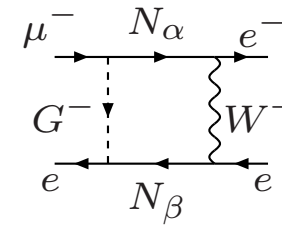
[A. Ilakovac, A.P., NPB437 (1995) 491.]



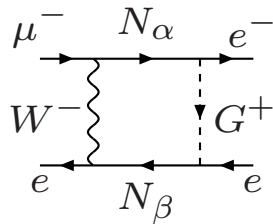
(a)



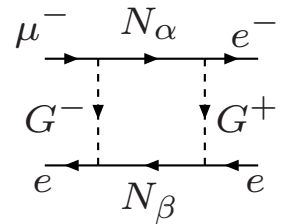
(b)



(c)



(d)



(e)

+ ($e \leftrightarrow e^-$)

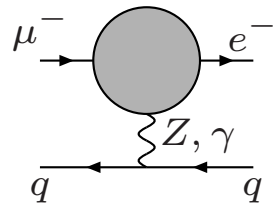
$m_N = 250 \text{ GeV}: \quad B(\mu \rightarrow eee) \approx 1.4 \cdot 10^{-2} \times B(\mu \rightarrow e\gamma) \sim 1.4 \times 10^{-14} .$

Current experimental limit: $B^{\text{exp}}(\mu \rightarrow eee) < 10^{-12}$ (SINDRUM)

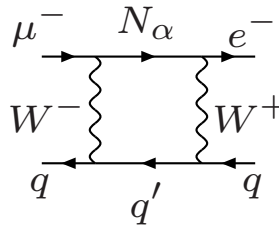
[Mu3E, A. Blondel et al, arXiv:1301.6113 [hep-ex].]

Future proposed sensitivity: $B^{\text{exp}}(\mu \rightarrow eee) \sim 10^{-16}$

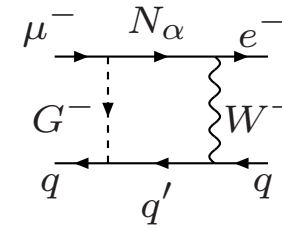
• **Coherent $\mu \rightarrow e$ Conversion in Nuclei (${}_{22}^{48}\text{Ti}$, ${}_{79}^{197}\text{Au}$)**



(a)



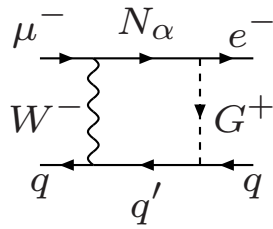
(b)



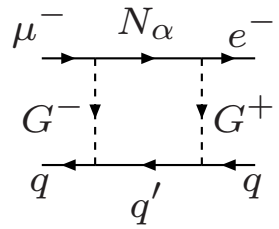
(c)

[A. Ilakovac, A.P., PRD80 (2009) 091902;

R. Alonso, M. Dhen, M. B. Gavela, T. Hambye, arXiv:1209.2679.]



(d)



(e)

$m_N = 250 \text{ GeV}: \quad B_{\text{Ti}}(\mu \rightarrow e) \approx 5 \times B(\mu \rightarrow e\gamma).$

$B_{\text{Ti}}^{\text{exp}}(\mu \rightarrow e) < 4.3 \times 10^{-12}, \quad B_{\text{Au}}^{\text{exp}}(\mu \rightarrow e) < 7 \times 10^{-13}.$

COMET/PRISM sensitivity: $B_{\text{Ti}}^{\text{exp}}(\mu \rightarrow e) \sim 10^{-13} - 10^{-18}.$

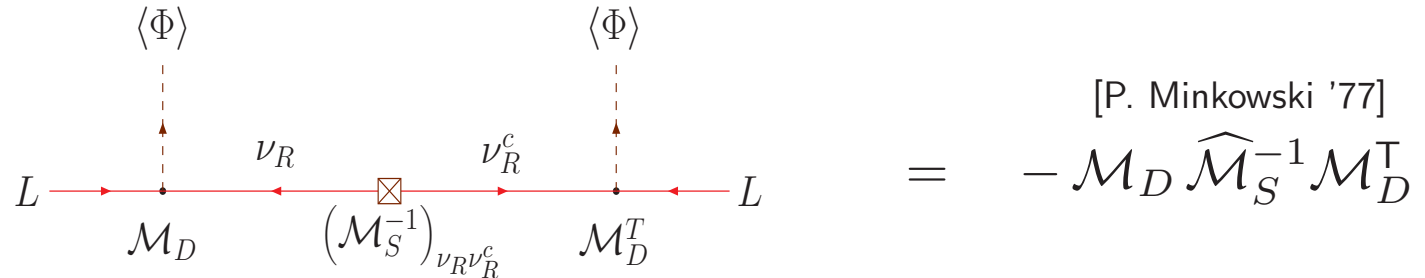
[M. Aoki et al, cLFV at FERMILAB, arXiv:2203.08278.]

• Numerical Estimates

– The Type I+II Radiative Seesaw Model

[A.P., ZPC55 (1992) 275;
P.S.B. Dev, A.P., PRD86 (2012) 113001.]

Type I:

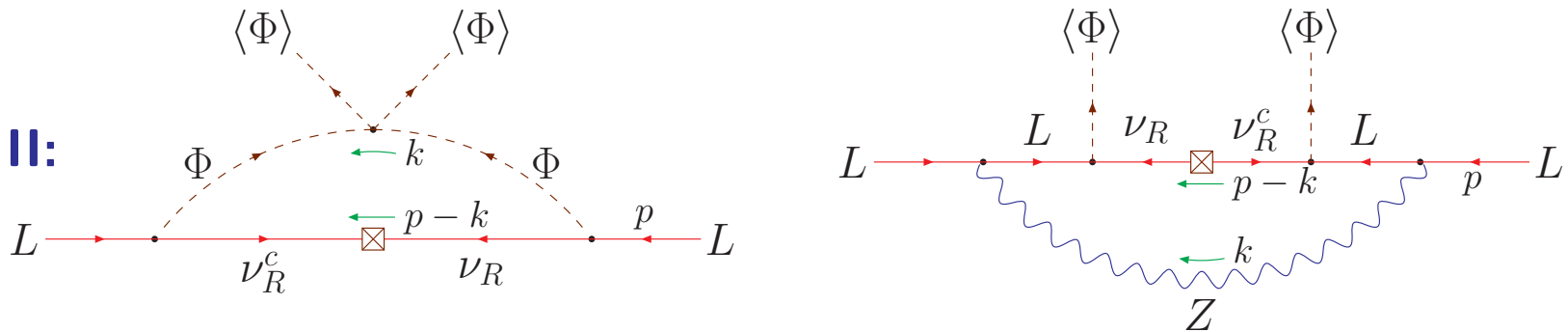


[P. Minkowski '77]

$$= -\mathcal{M}_D \widehat{\mathcal{M}}_S^{-1} \mathcal{M}_D^T$$

+

Type II:



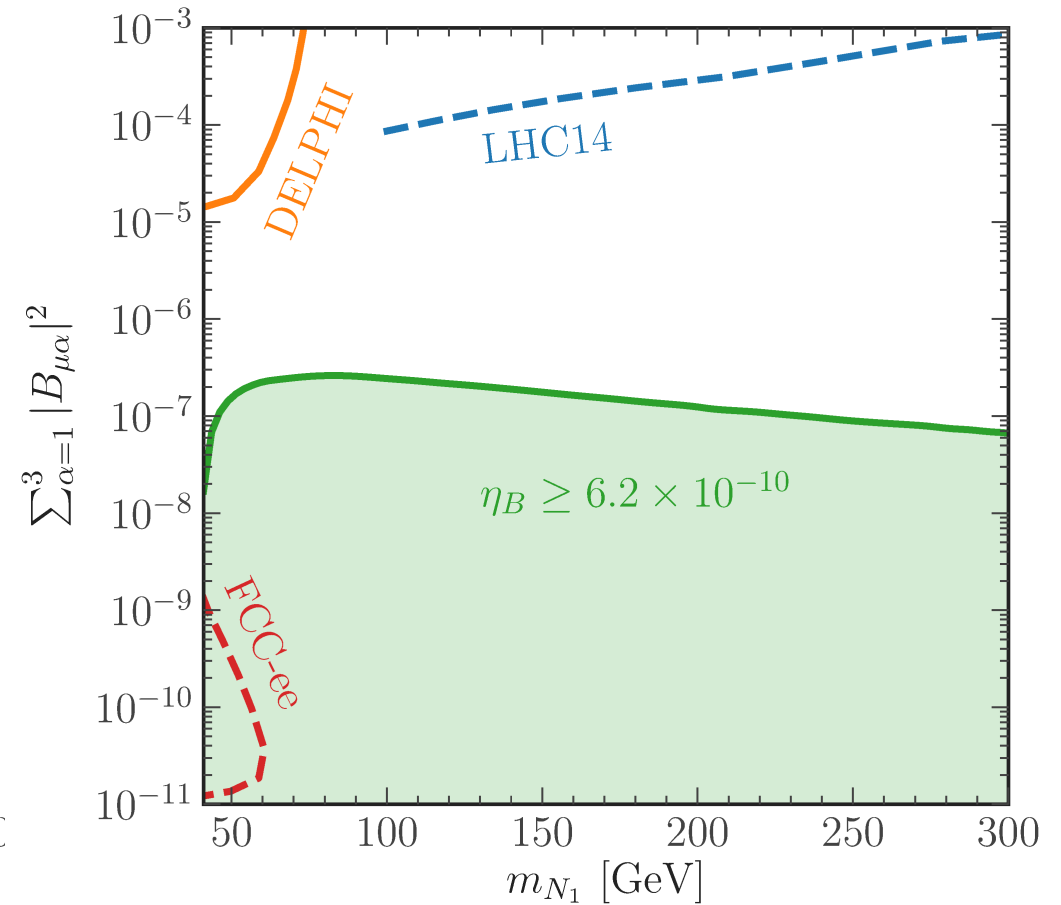
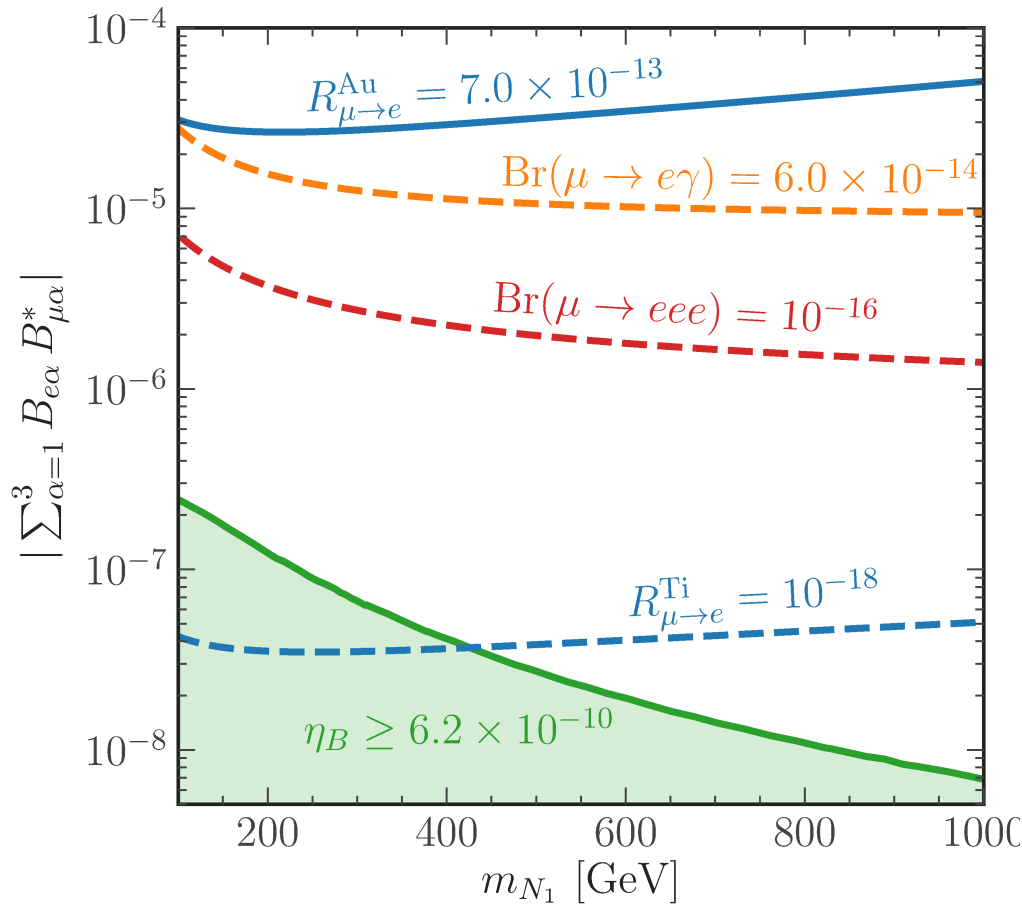
$$\mathbf{M}_{LL}^{1\text{-loop}} \simeq \frac{\alpha_w}{16\pi} \frac{M_H^2 + 3M_Z^2}{M_W^2} \mathcal{M}_D \ln \left(\frac{\widehat{\mathcal{M}}_S^2}{M_W^2} \right) \widehat{\mathcal{M}}_S^{-1} \mathcal{M}_D^T$$

Light-neutrino mass matrix:

$$\mathbf{m}_\nu = -\mathcal{M}_D \widehat{\mathcal{M}}_S^{-1} \mathcal{M}_D^T + \mathbf{M}_{LL}^{1\text{-loop}}$$

Allowed parameter space from cLFV processes and Colliders

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



$B_{l\alpha} \simeq (\mathcal{M}_D)_{l\alpha} m_{N_\alpha}^{-1}$: light-to-heavy neutrino mixings

Hypothesis: Democratic flavour models based on $\sim Z_3$ or $\sim Z_6$ symmetries.

• Conclusions

- **Matter–AntiMatter Asymmetry** through **Resonant Leptogenesis**

$$\implies \delta\eta_{\text{tot}}^L \sim \delta\eta_{\text{mix}}^L + \delta\eta_{\text{osc}}^L + \delta\eta_{\text{dec}}^L$$

independent of the *initial B-number* state of the early Universe.

Complete treatment may **enhance BAU** by 1-order of magnitude.

- **Tri– or Multi–Resonant Leptogenesis** offer **new possibilities** in **building models** with **observable cLFV** and **LNV**.

- **Varying relativistic dofs** modify significantly the **Transport Equations** and so the **BAU** predictions for $m_N \lesssim 100$ GeV.

- $B(\mu \rightarrow e\gamma) \sim 10^{-13}$ + **successful leptogenesis** [A.P. '04]
 $\implies \geq 3$ **RHNs** + **Non-trivial Flavour Effects**.

- **Low-Scale Heavy Majorana Neutrinos** can give rise to **observable signatures** of **LNV** and **CPV** at the **LHC**.

BACK-UP SLIDES

• Flavour Covariant Transport Equations

[E. W. Kolb and S. Wolfram, NPB172 (1980) 224.]

– Flavour Diagonal Boltzmann Equations

$$\frac{dn_a}{dt} + 3Hn_a = \sum_{aX' \leftrightarrow Y} \left(-\frac{n_a n_{X'}^{\text{eq}}}{n_a^{\text{eq}} n_{X'}^{\text{eq}}} \gamma(aX' \rightarrow Y) + \frac{n_Y}{n_Y^{\text{eq}}} \gamma(Y \rightarrow aX') \right),$$

where n_a is the **number density**:

$$\begin{aligned} n_a(T) &= g_a \int \frac{d^3\mathbf{p}}{(2\pi)^3} \exp \left[- \left(\sqrt{\mathbf{p}^2 + m_a^2} - \mu_a(T) \right) / T \right] \\ &= \frac{g_a m_a^2 T e^{\mu_a(T)/T}}{2\pi^2} K_2 \left(\frac{m_a}{T} \right) \end{aligned}$$

and $\gamma(X \rightarrow Y)$ is the **collision term**:

$$\gamma(X \rightarrow Y) = \int d\pi_X d\pi_Y (2\pi)^4 \delta^{(4)}(p_X - p_Y) e^{-p_X^0/T} |\mathcal{M}(X \rightarrow Y)|^2.$$

– Flavour Diagonal BEs for Leptogenesis

[A.P., T.E. Underwood, NPB**692** (2004) 303; PRD**72** (2005) 113001.]

Define first

$$\eta^X \equiv n_X/n_\gamma, \quad z \equiv m_{N_1}/T, \quad H \equiv H(T = m_N) \approx 17 m_N^2/M_{\text{Planck}}$$

and the short-hands:

$$\begin{aligned} \gamma_Y^X &\equiv \gamma(X \rightarrow Y) + \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} \gamma_X^Y, \\ \delta\gamma_Y^X &\equiv \gamma(X \rightarrow Y) - \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} -\delta\gamma_X^Y. \end{aligned}$$

Write down the **Boltzmann equations**:

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d\eta_\alpha^N}{dz} &= \left(1 - \frac{\eta_\alpha^N}{\eta_{\text{eq}}^N}\right) \sum_k \gamma_{L_k\Phi}^{N_\alpha} + \dots \\ \frac{H n_\gamma}{z} \frac{d\delta\eta_l^L}{dz} &= \sum_\alpha \left(\frac{\eta_\alpha^N}{\eta_{\text{eq}}^N} - 1\right) \delta\gamma_{L_l\Phi}^{N_\alpha} - \frac{2}{3}\delta\eta_l^L \sum_k \left(\gamma_{L_k^c\Phi^c}^{L_l\Phi} + \gamma_{L_k\Phi}^{L_l\Phi}\right) \\ &\quad - \frac{2}{3}\sum_k \delta\eta_k^L \left(\gamma_{L_l^c\Phi^c}^{L_k\Phi} - \gamma_{L_l\Phi}^{L_k\Phi}\right) + \dots \end{aligned}$$

– Order-of-magnitude estimate of the **BAU**

Flavour-dependent decay width of heavy Majorana neutrino N_α :

$$\Gamma_{N_\alpha \rightarrow l} \equiv \Gamma(N_\alpha \rightarrow L_l \Phi) = (h^{\nu\dagger})_{\alpha l} h_{l\alpha}^\nu \frac{m_{N_\alpha}}{8\pi}$$

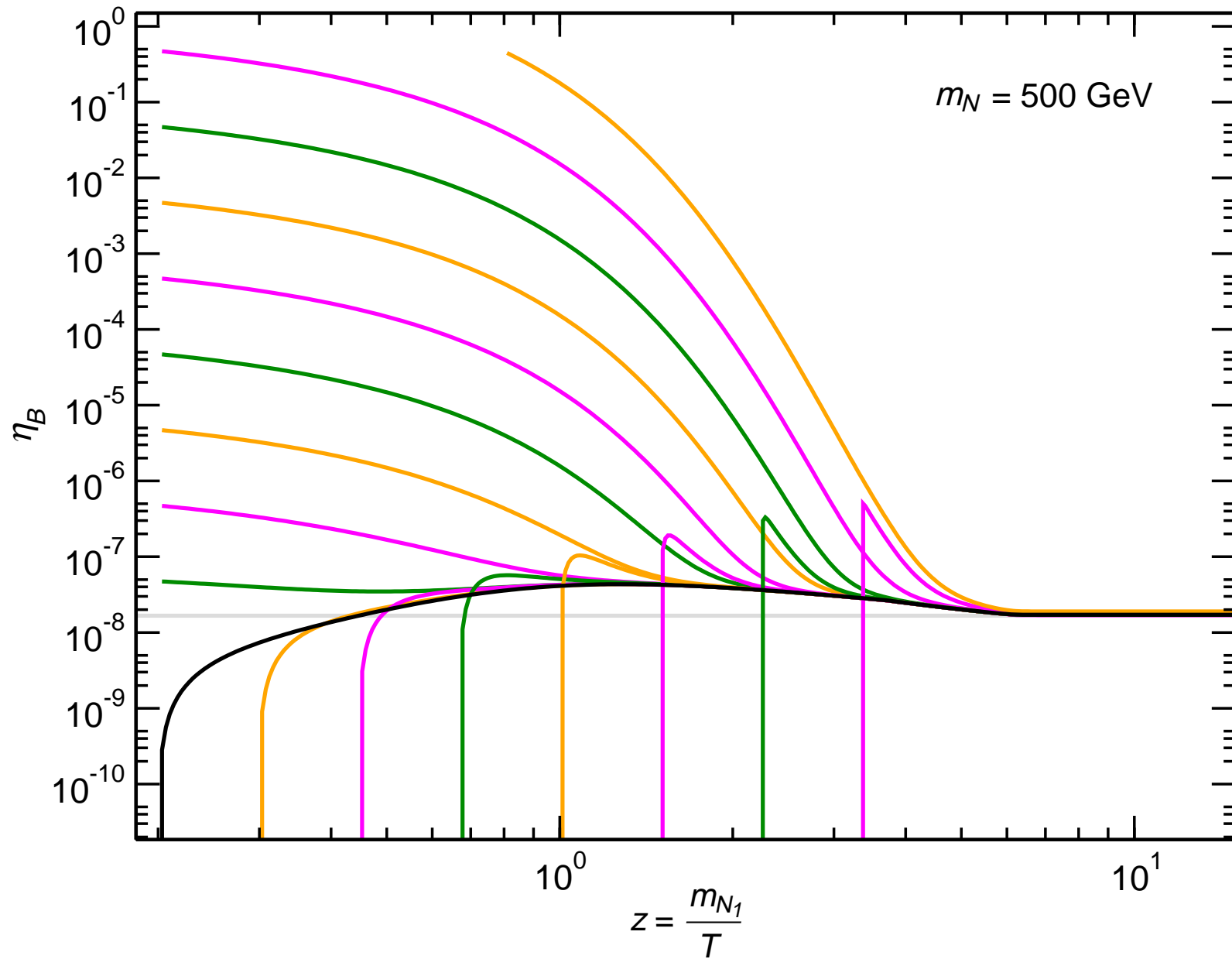
Define the effective wash-out K -factors:

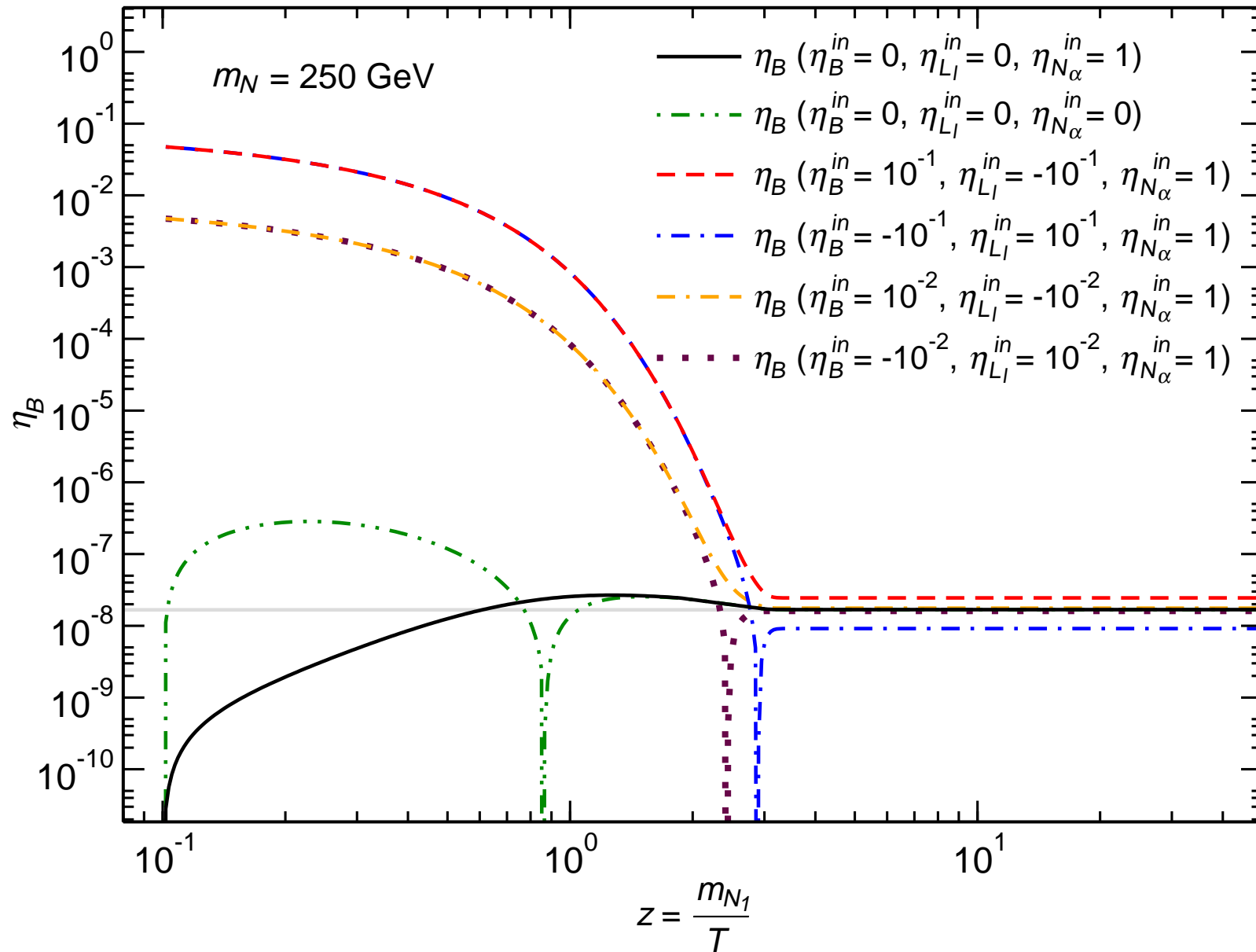
$$K_l^{\text{eff}} \equiv \frac{\sum_{N_\alpha} \Gamma_{N_\alpha \rightarrow l}}{H}$$

Estimate of the BAU (strong wash-out regime):

[F. Deppisch, A.P., PRD83 (2011) 076007.]

$$\eta_B^{\text{mix}} \sim -3 \cdot 10^{-2} \sum_{l=e,\mu,\tau} \frac{\delta_l^{\text{mix}}}{K_l^{\text{eff}} \min \left[m_N / T_c, 1.25 \ln(25 K_l^{\text{eff}}) \right]} .$$





– Flavour Covariant Quantization

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

$U(\mathcal{N}_L) \otimes U(\mathcal{N}_N)$ flavour-invariant Lagrangian:

$$-\mathcal{L}_N = h_l^\alpha \bar{L}^l \tilde{\Phi} N_{R,\alpha} + \bar{N}_{R,\alpha}^C [M_N]^{\alpha\beta} N_{R,\beta} + \text{H.c.}$$

Under $U(\mathcal{N}_L) \otimes U(\mathcal{N}_N)$ flavour transformations:

$$\begin{aligned} L_l &\rightarrow L'_l = V_l^m L_m, & L^l &\equiv (L_l)^\dagger \rightarrow L'^l = V_m^l L^m, \\ N_{R,\alpha} &\rightarrow N'_{R,\alpha} = U_\alpha^\beta N_{R,\beta}, & N_R^\alpha &\equiv (N_{R,\alpha})^\dagger \rightarrow N'^\alpha = U^\alpha_\beta N_R^\beta, \end{aligned}$$

\mathcal{L}_N is invariant provided:

$$h_l^\alpha \rightarrow h'_l{}^\alpha = V_l^m U^\alpha_\beta h_m^\beta, \quad [M_N]^{\alpha\beta} \rightarrow [M'_N]^{\alpha\beta} = U^\alpha_\gamma U^\beta_\delta [M_N]^{\gamma\delta}.$$

Quantization rules:

$$\begin{aligned} \{b_l(\mathbf{p}, s), b^m(\mathbf{p}', s')\} &= \{d^{\dagger,m}(\mathbf{p}, s), d_l^\dagger(\mathbf{p}', s')\} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{ss'} \delta_l^m \\ \{a_\alpha(\mathbf{k}, r), a^\beta(\mathbf{k}', r')\} &= (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{rr'} \delta_\alpha^\beta \end{aligned}$$

– Flavour Covariant Number Densities and Collision Rates

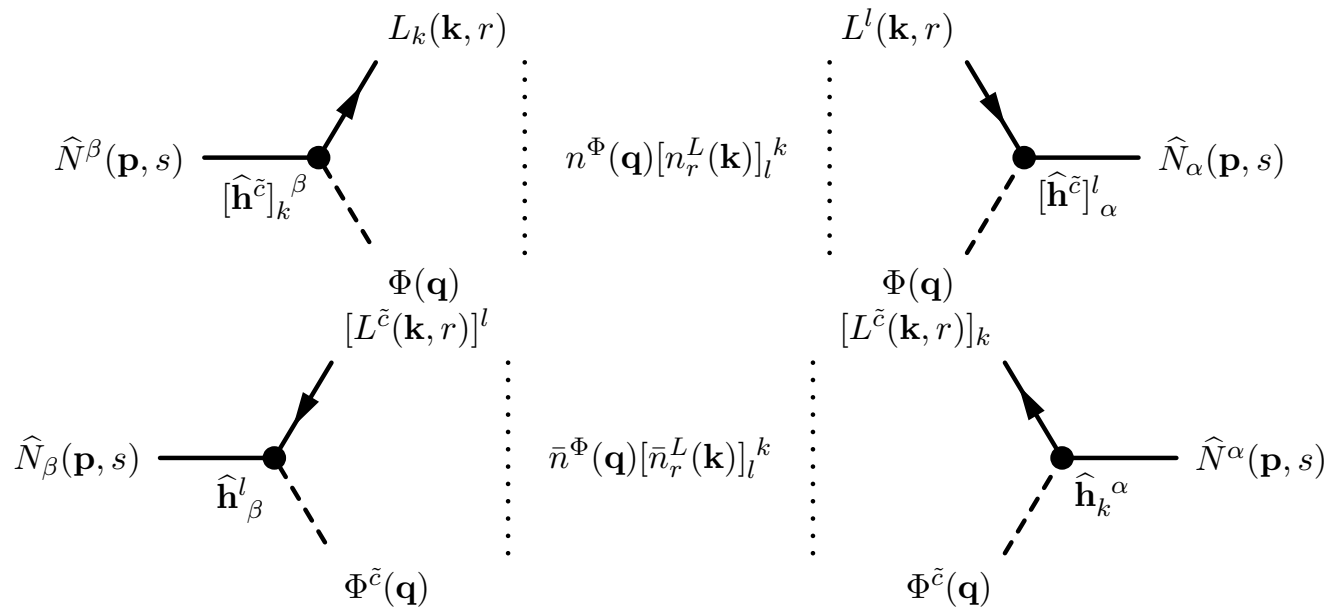
Flavour-covariant number densities:

$$\begin{aligned}
 [n_{s_1 s_2}^L(\mathbf{p})]_l^m &\equiv \frac{1}{\mathcal{V}} \langle b^m(\mathbf{p}, s_2) b_l(\mathbf{p}, s_1) \rangle , \\
 [\bar{n}_{s_1 s_2}^L(\mathbf{p})]_l^m &\equiv \frac{1}{\mathcal{V}} \langle d_l^\dagger(\mathbf{p}, s_1) d^{\dagger, m}(\mathbf{p}, s_2) \rangle , \\
 [n_{r_1 r_2}^N(\mathbf{k})]_\alpha^\beta &\equiv \frac{1}{\mathcal{V}} \langle a^\beta(\mathbf{k}, r_2) a_\alpha(\mathbf{k}, r_1) \rangle .
 \end{aligned}$$

Flavour-covariant generalization of the collision rates:

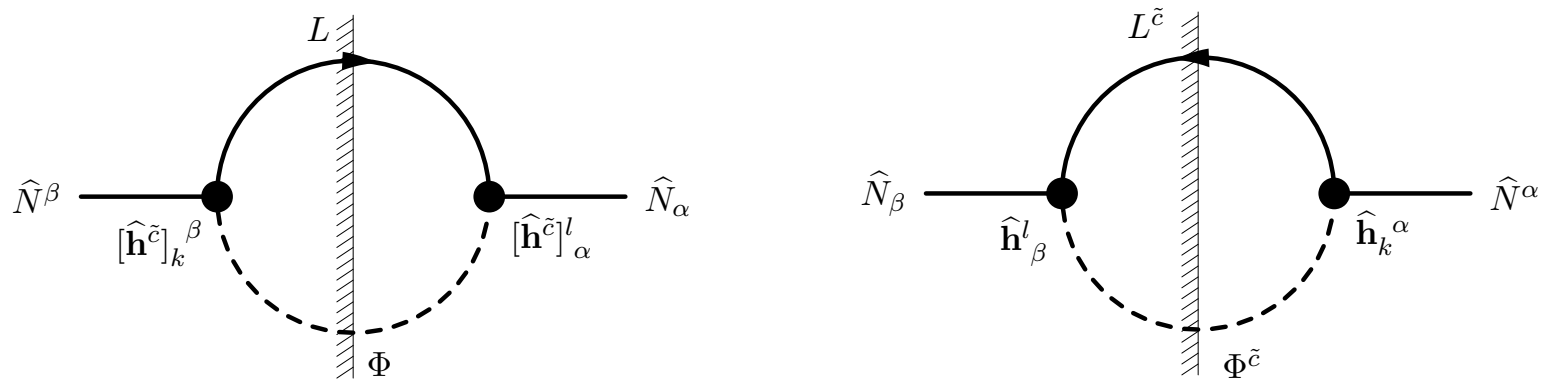
$$\begin{aligned}
 [\gamma(N \rightarrow L\Phi)]_l^m \alpha^\beta &= \frac{m_N^4}{\pi^2 z} \frac{K_1(z)}{16\pi} \mathbf{h}_\alpha^m \mathbf{h}_l^\beta , \\
 [\gamma(N \rightarrow L\tilde{\Phi}\tilde{\Phi}^c)]_l^m \alpha^\beta &= \frac{m_N^4}{\pi^2 z} \frac{K_1(z)}{16\pi} [\mathbf{h}^{\tilde{c}}]_\alpha^m [\mathbf{h}^{\tilde{c}}]_l^\beta , \\
 [\gamma_{L\Phi}^N]_l^m \alpha^\beta &\equiv [\gamma(N \rightarrow L\Phi)]_l^m \alpha^\beta + [\gamma(N \rightarrow L\tilde{\Phi}\tilde{\Phi}^c)]_l^m \alpha^\beta , \\
 [\delta\gamma_{L\Phi}^N]_l^m \alpha^\beta &\equiv [\gamma(N \rightarrow L\Phi)]_l^m \alpha^\beta - [\gamma(N \rightarrow L\tilde{\Phi}\tilde{\Phi}^c)]_l^m \alpha^\beta .
 \end{aligned}$$

– Feynman Diagrammatic Representation



– Closed Time Path (CTP) Representation

[e.g., P. Millington and A.P., PRD88 (2013) 085009.]



– Flavour Covariant Rate Equations (Markovian approximation)

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

$$\frac{H n_\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n_\gamma}{2} [\mathcal{E}_N, \delta\eta^N]_\alpha^\beta + [\text{Re}(\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^N]_\alpha^\beta}{dz} &= -2i n_\gamma [\mathcal{E}_N, \underline{\eta}^N]_\alpha^\beta + 2i [\text{Im}(\delta\gamma_{L\Phi}^N)]_\alpha^\beta \\ &\quad - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Im}(\delta\gamma_{L\Phi}^N) \right\}_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \delta\eta^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^L]_l^m}{dz} &= -[\delta\gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\tilde{\eta}_{\text{eq}}^N} [\delta\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta + \frac{[\delta\eta^N]_\beta^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta\eta^L, \gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta\eta^L]_k^n \left([\gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi}]_{n l}^k{}^m - [\gamma_{L\Phi}^{L\Phi}]_{n l}^k{}^m \right) \\ &\quad - \frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

– **Unified Description of 3 Physically Distinct Phenomena:**

- **Resonant Mixing between Heavy Neutrinos,**

through: $\mathbf{h}_{l\alpha}$ and $\mathbf{h}_{l\alpha}^c$ in $[\gamma_{L\Phi}^N]_l^{m\ \beta}$ and $[\delta\gamma_{L\Phi}^N]_l^{m\ \beta}$.

- **Coherent Oscillations between Heavy Neutrinos** ($\Delta m_N \ll m_N$),

from $[\mathcal{E}_N, \underline{\eta}^N]$ and the rank-4 tensor term $\frac{[\delta\eta^N]_\beta^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^{m\ \beta}$, yielding:

$$\delta\eta_{\text{osc}}^L \sim \frac{3}{2Kz} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{2(m_{N_1}^2 - m_{N_2}^2)m_N\Gamma_N}{(m_{N_1}^2 - m_{N_2}^2)^2 + \left(\frac{2m_N\Gamma_N\text{Im}[h^\dagger h]_{12}}{|[h^\dagger h]_{12}|}\right)^2},$$

with $\Gamma_N = \frac{1}{2}(\Gamma_{N_1}^{(0)} + \Gamma_{N_2}^{(0)})$. [**NB:** Different from the ARS mechanism.]

- **Decoherence Effects due to Charged Lepton Yukawa Couplings,**

from $-\frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m$

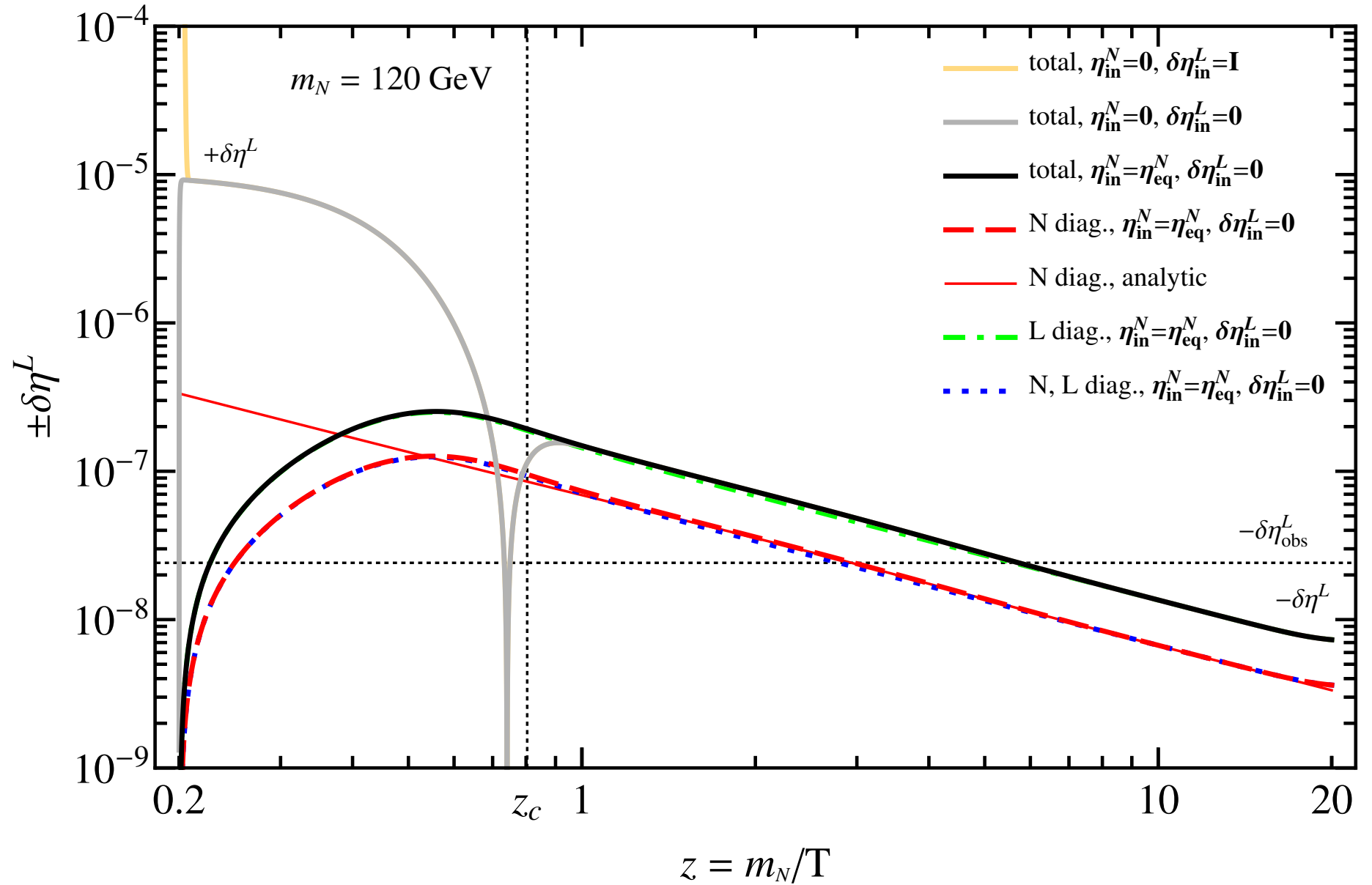
– Minimal Resonant Leptogenesis Model

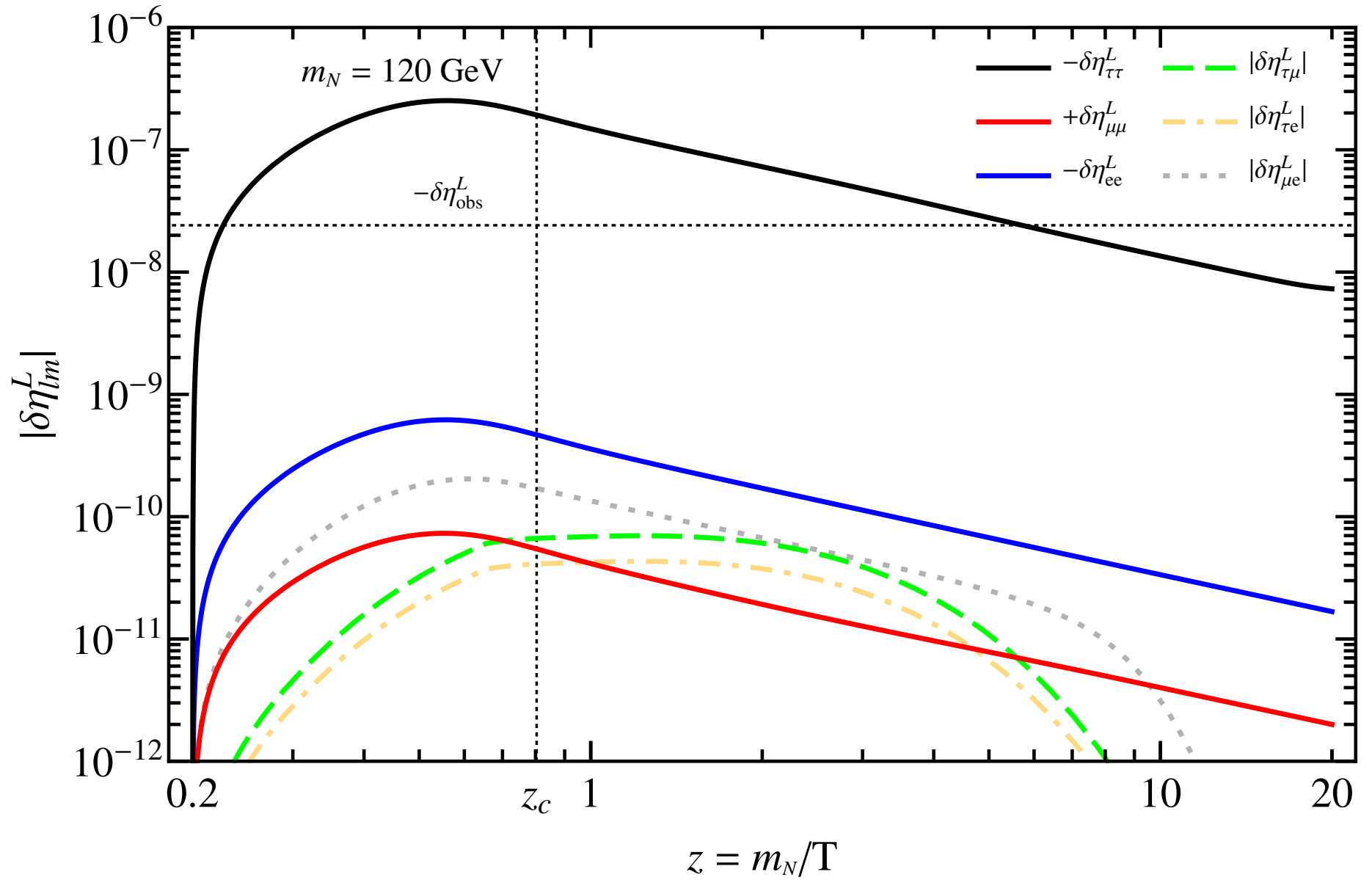
[F. Deppisch, A.P., PRD83 (2011) 076007.]

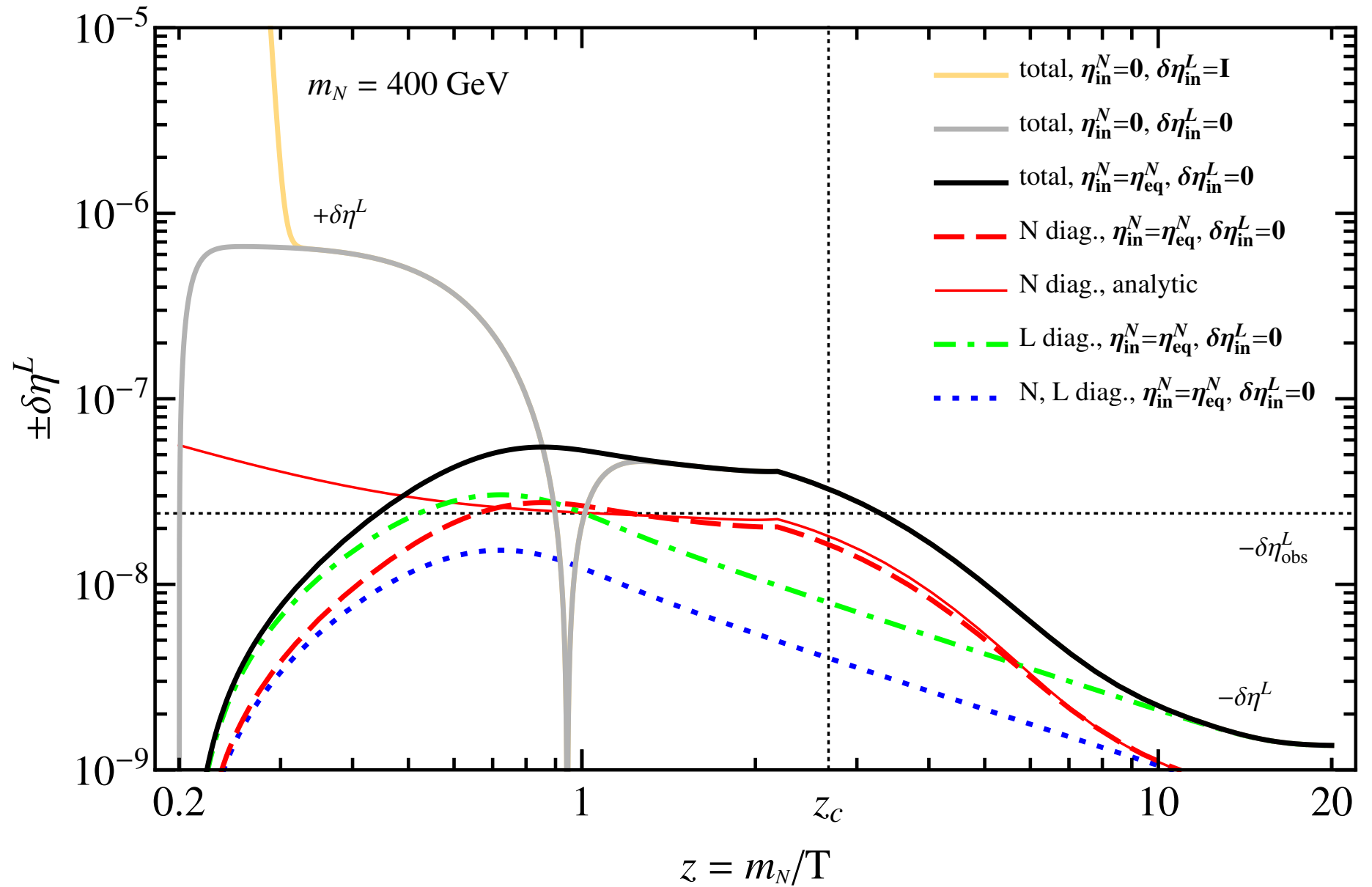
$$\mathbf{m}_M = m_N \mathbf{1}_3 \quad \text{and} \quad \mathbf{m}_D = \frac{v_{\text{SM}}}{\sqrt{2}} \begin{pmatrix} \epsilon_e & a e^{-i\pi/4} & a e^{i\pi/4} \\ \epsilon_\mu & b e^{-i\pi/4} & b e^{i\pi/4} \\ \epsilon_\tau & \kappa_1 e^{-i(\pi/4-\gamma_1)} & \kappa_2 e^{i(\pi/4-\gamma_2)} \end{pmatrix}$$

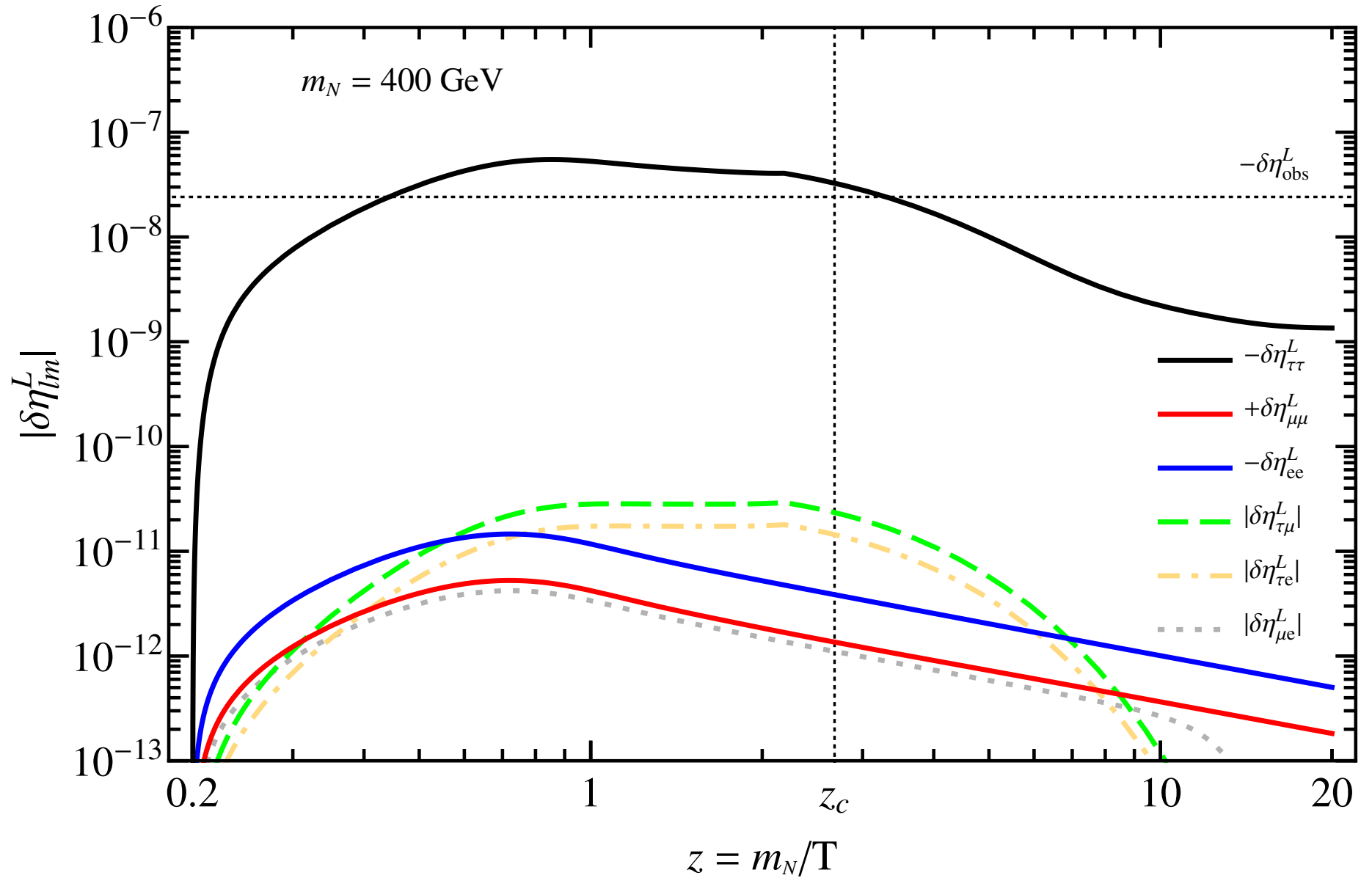
Parameters	BP1	BP2	BP3
m_N (GeV)	120	400	5000
γ_1	$\pi/4$	$\pi/3$	$3\pi/8$
γ_2	0	0	$\pi/2$
κ_1	4×10^{-5}	2.4×10^{-5}	2×10^{-4}
κ_2	2×10^{-4}	6×10^{-5}	2×10^{-5}
a	$(7.41 - 5.54 i) \times 10^{-4}$	$(4.93 - 2.32 i) \times 10^{-3}$	$(4.67 + 4.33 i) \times 10^{-3}$
b	$(1.19 - 0.89 i) \times 10^{-3}$	$(8.04 - 3.79 i) \times 10^{-3}$	$(7.53 + 6.97 i) \times 10^{-3}$
ϵ_e	3.31×10^{-8}	5.73×10^{-8}	2.14×10^{-7}
ϵ_μ	2.33×10^{-7}	4.3×10^{-7}	1.5×10^{-6}
ϵ_τ	3.5×10^{-7}	6.39×10^{-7}	2.26×10^{-6}

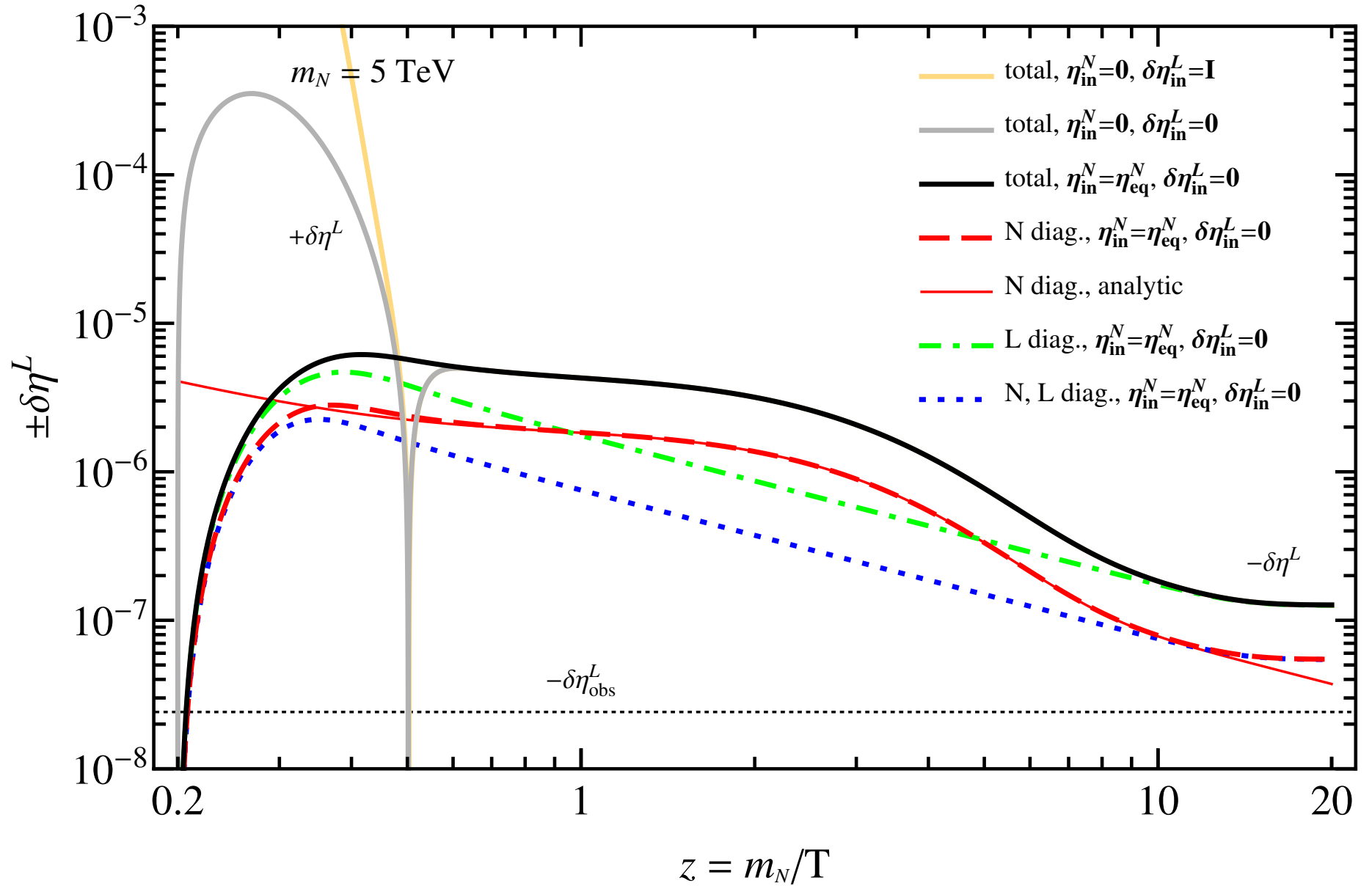
\implies **normal hierarchy for light neutrinos:** $\Delta m_{\text{sol}}^2 = 7.54 \times 10^{-5} \text{ eV}^2$, $\Delta m_{\text{atm}}^2 = 2.44 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.308$, $\sin^2 \theta_{23} = 0.425$, $\sin^2 \theta_{13} = 0.0234$.

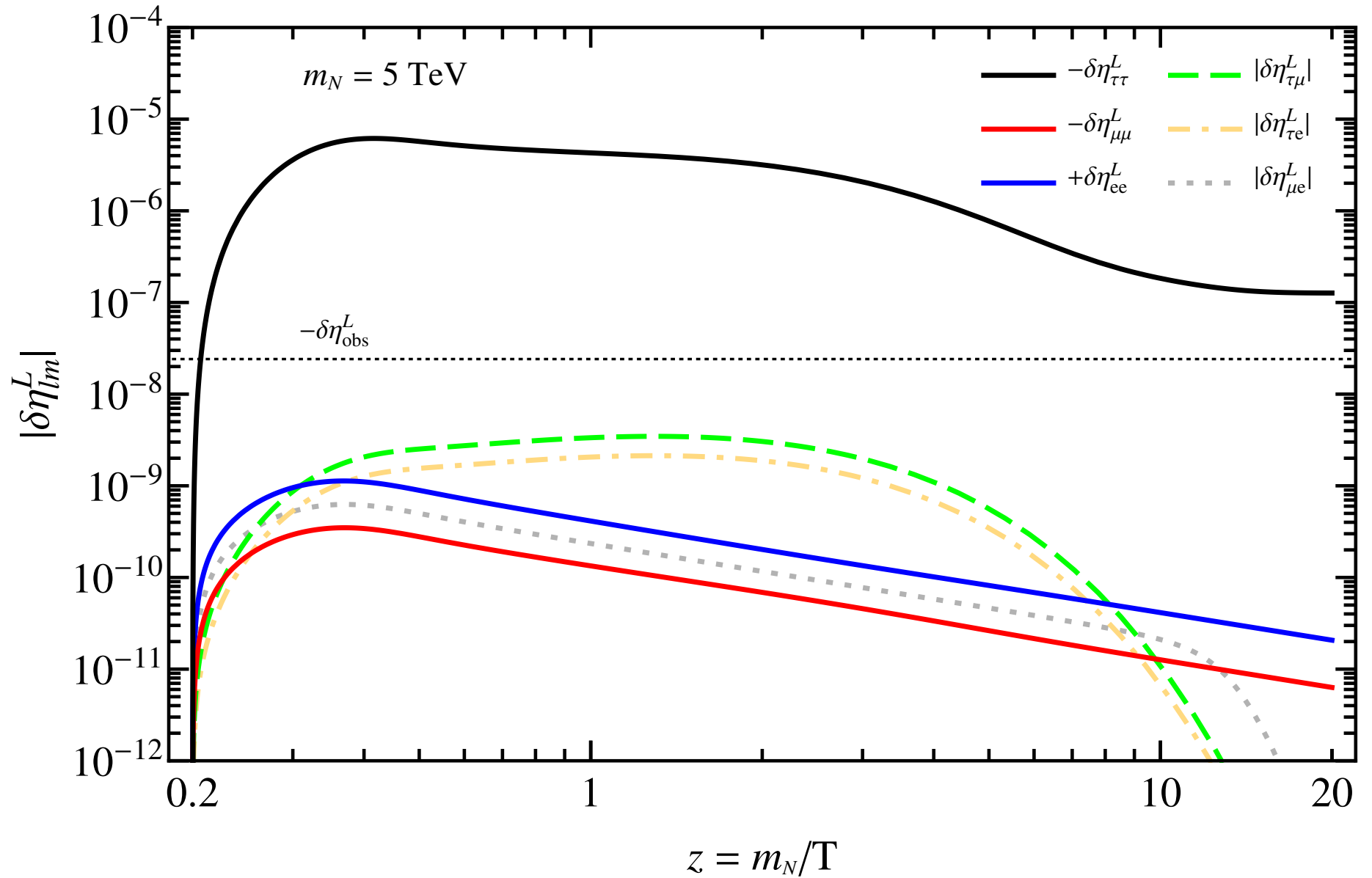


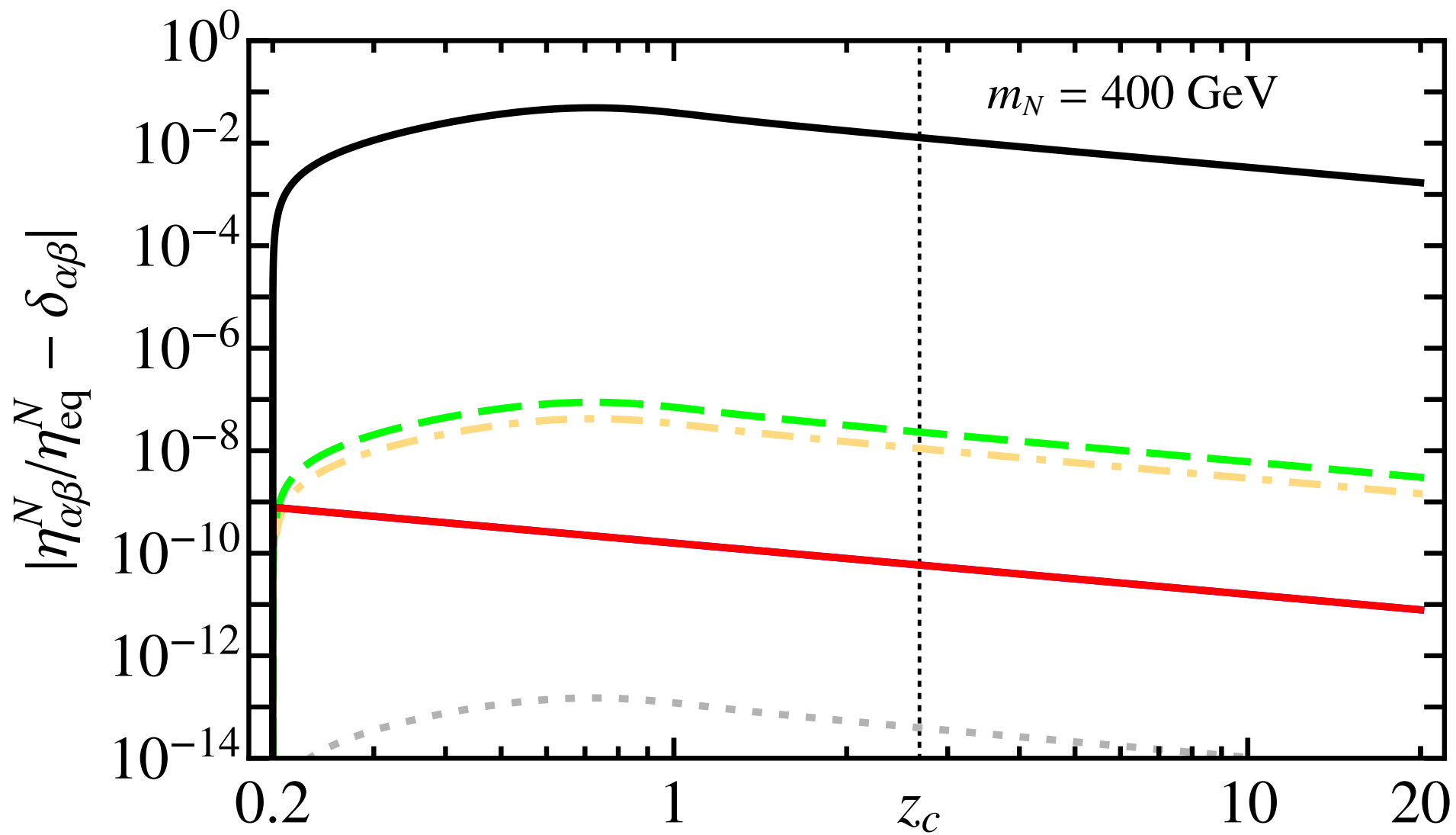












– Phenomenological Implications

[P.S.B. Dev, P. Millington, A.P., D. Teresi, arXiv:1404.1003.]

Low-energy observables	BP1 m_N 120 GeV	BP2 m_N 400 GeV	BP3 m_N 5 TeV	Experimental Limit
BR($\mu \rightarrow e\gamma$)	4.5×10^{-15}	1.9×10^{-13}	2.3×10^{-17}	$< 5.7 \times 10^{-13}$
BR($\tau \rightarrow \mu\gamma$)	1.2×10^{-17}	1.6×10^{-18}	8.1×10^{-22}	$< 4.4 \times 10^{-8}$
BR($\tau \rightarrow e\gamma$)	4.6×10^{-18}	5.9×10^{-19}	3.1×10^{-22}	$< 3.3 \times 10^{-8}$
BR($\mu \rightarrow 3e$)	1.5×10^{-16}	9.3×10^{-15}	4.9×10^{-18}	$< 1.0 \times 10^{-12}$
$R_{\mu \rightarrow e}^{\text{Ti}}$	2.4×10^{-14}	2.9×10^{-13}	2.3×10^{-20}	$< 6.1 \times 10^{-13}$
$R_{\mu \rightarrow e}^{\text{Au}}$	3.1×10^{-14}	3.2×10^{-13}	5.0×10^{-18}	$< 7.0 \times 10^{-13}$
$R_{\mu \rightarrow e}^{\text{Pb}}$	2.3×10^{-14}	2.2×10^{-13}	4.3×10^{-18}	$< 4.6 \times 10^{-11}$
$\langle m \rangle$ (eV)	3.8×10^{-3}	3.8×10^{-3}	3.8×10^{-3}	$< 0.11 - 0.25$

- **Comparison with Other Methods:** ϵ_{N_1} [F. Deppisch, A.P., PRD83 (2011) 076007.]

Consider a simple Inverse Seesaw-like Model ($1L + 2\nu_R$):

[R. N. Mohapatra, PRL56 (1986) 561;
R. N. Mohapatra, J. W. F. Valle, PRD34 (1986) 1642;
P. S. B. Dev and A.P., PRD86 (2012) 113001.]

$$M_\nu = \begin{pmatrix} 0 & vy & 0 \\ vy & \mu_1 & M \\ 0 & M & \mu_2 e^{i\alpha} \end{pmatrix}$$

Heavy neutrino masses:

$$M_{1,2} \approx M \mp \frac{\mu}{2}, \quad \text{with } \mu = |\mu_1 + \mu_2 e^{i\alpha}|$$

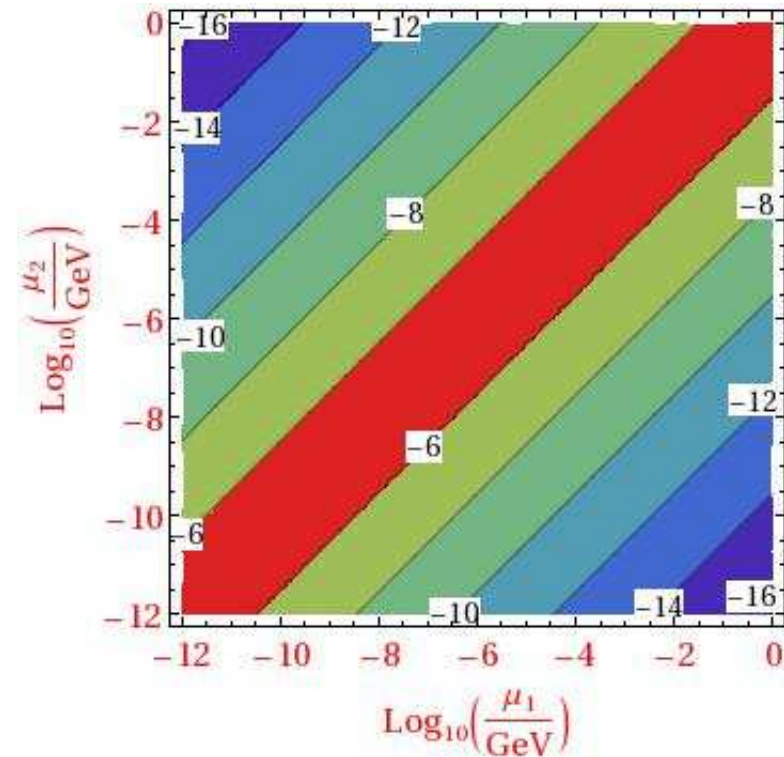
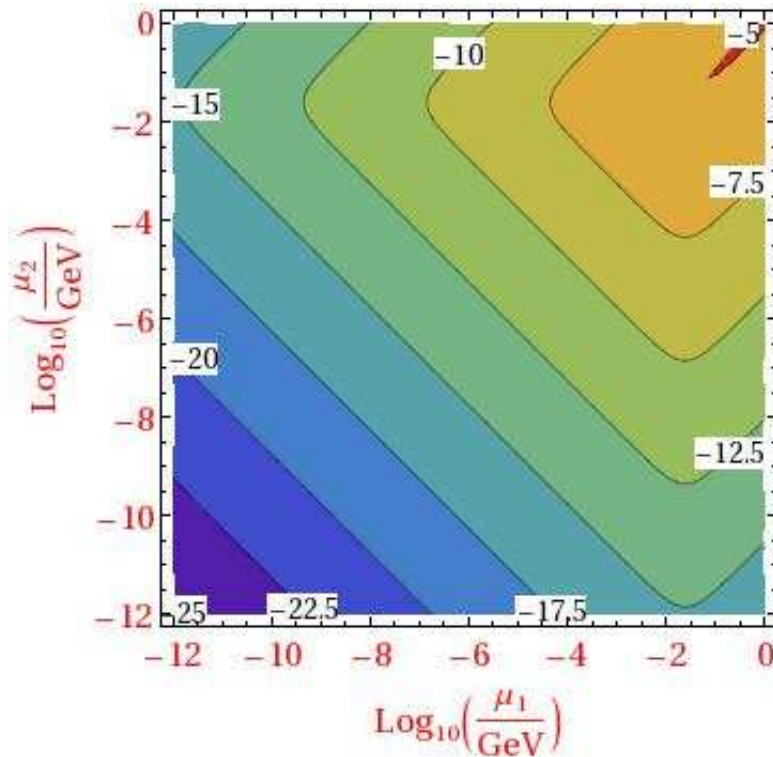
Lepton asymmetry ϵ_{N_1} :

$$\epsilon_{N_1} = \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{12}^2}{(h^{\nu\dagger} h^\nu)_{11} (h^{\nu\dagger} h^\nu)_{22}} f_{\text{reg}}$$

$\implies \epsilon_{N_1} \rightarrow 0$, when $\mu_{1,2} \rightarrow 0$.

L -conserving limits of ϵ_{N_1}

[P.S.B. Dev, P. Millington, A.P., D. Teresi, arXiv:1404.1003.]



Singular regulator:

[W. Buchmüller and M. Plümacher, PLB431 (1998) 354.]

$$f_{\text{reg}}^{\text{BP}} = \frac{|m_{N_1}^2 - m_{N_2}^2| m_{N_1} \Gamma_{N_2}}{(m_{N_1}^2 - m_{N_2}^2)^2 + (m_{N_1} \Gamma_{N_1} - m_{N_2} \Gamma_{N_2})^2}$$

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