Do the Small numbers in VCKM arise from New Physics?

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Work done in recent collaboration with J. Bastero and J.I. Silva Marcos
and older collaboration with F. Botella, M. N. Rebelo et al
Organization of the talk

• Identification of the small numbers in VCKM

• Conjecture: The small numbers in VCKM arise from New Physics

• Motivation for vector-like quarks (VLQs) and simple realization of the Conjecture with VLQs

• Phenomenological consequences

• Conclusions
Identification of the small numbers in VCKM:

$$|V_{ub}| \approx 3.6 \times 10^{-3}$$

$$|\text{Im } Q| \approx 3 \times 10^{-5}$$

Q → Replacing invariant quartet of VCKM

In the SM, |Im Q| has the same value for all quartets and gives the strength of CP violation in the SM
Details about Replacing invariant quantities

Example: \[ Q = V_{us} V_{cb} V_{es}^* V_{ub}^* \]

\[ \text{Im } Q = \lambda^5 \sin(\arg Q) \]

\[ |\text{Im } Q| \] has the same value for all quartets and measures the **strength of CP violation in the SM**. One can have restoring invariants of higher order in \( V_{ij} \), but they can be written in terms of \( Qs \) and moduli.
A surprising result: In the 3x3 up corner of a \( V^{CKM} \) matrix of arbitrary size one has:

\[ 9 - 5 = 4 \] rephasing invariant phase.

The following phase convention may be chosen, in general:

\[
\arg V^{3x3} = \begin{pmatrix}
0 & \beta & \delta \\
\pi & 0 & 0 \\
-\beta & \pi + \beta & 0
\end{pmatrix}
\]
The phases $\delta, \beta, \beta_3, \beta_k$ are arguments of the replacing invariant quartets:

\[ \gamma = \arg \left( -V_{ud} V_{cb} V_{ub}^* V_{cd}^* \right) \]
\[ \beta = \arg \left( -V_{cd} V_{tb} V_{cb}^* V_{td}^* \right) \]
\[ \beta_3 = \arg \left( -V_{cb} V_{ts} V_{cs}^* V_{tb}^* \right) \]
\[ \beta_k = \arg \left( -V_{us} V_{kd} V_{ud}^* V_{cs}^* \right) \]

Sometimes one also introduces $\chi = \arg \left( -V_{td} V_{ub} V_{ud}^* V_{tb}^* \right)$ which is unnecessary, because

\[ \chi \equiv \pi - \beta - \delta \quad \text{By definition!!!} \]
Within the SM, 3x3 unitarity implies some exact relations among rephasing invariant quantities:

\[
\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin \beta}{\sin \delta} \frac{|V_{tb}|}{|V_{ud}|}
\]

\[
\sin \beta_s = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin \beta = O(1^2)
\]

\[
\sin \beta_K = \frac{|V_{ub}|}{|V_{us}|} \frac{|V_{cb}|}{|V_{cs}|} \sin \delta = O(1^4)
\]
Conjecture: The small numbers in VCKM arise from New Physics.

The conjecture implies that within the SM,

\[ |V_{ub}| = 0 \]
\[ \text{Im } Q = 0 \]

A simple realization of the Conjecture can be constructed within SM + VLQs.
A crucial question: What can VLQs do for you?

(i) They provide a simple alternative solution to the Strong CP problem without axions. Barr and Nelson Bento, G.C.B and Parada

(ii) They provide the simplest extension of the SM with Spontaneous CP Violation in a model consistent with experiment.
Requirements to have a viable model of Spontaneous CP Violation:

- Lagrangian should be CP invariant but CP invariance should be broken by the vacuum.

One has to be careful. Often a geometrical vacuum phase does not violate CP.

- The vacuum phase should be able to generate a complex CKM matrix

Experimentally $\delta \neq 0, \pi$
(iii) Provide a simple framework where there are New Physics (NP) contributions to $B_d - \bar{B}_d$ mixing, $B_s - \bar{B}_s$ mixing and/or $D^0 - \bar{D}^0$ mixing; Also new contributions to $t \rightarrow c \bar{\tau} \nu$ may receive tree-level contributions in models with up-type VLQs.
IV VLQs may populate the desert between $V$ and some higher scale ($M_{GUT}$?) without worsening the hierarchy problem.

To my knowledge, this was first emphasized in a paper by Pierre Ramond.

"Fermions in the Desert" (talk given at Erice)

Appears in Spines.
V LQs may play an important role in providing an explanation for the

\textbf{VCKM} unitarity problem.

\[ |V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 < 1 \]

at the level of 2,3 standard deviations.

See J.T. Penedo, Pedro Pereira, M.N. Rebelo published in JHEP

See also nice work by Belfatto and Berezhiani
Question: Should we take this "deviation of unitarity" seriously?

My approach was: "When you are not sure, ask a friend who is a specialist."

In this case we asked Bill Marciano.

His answer: Yes, it should be taken seriously!!
The generation of $|V_{ub}|$ and $Im\,Q_{13}$ from New Physics

We propose that $V_{CKM}^{\text{eff}}$ is generated from three different contributions:

\[
V_{CKM}^{\text{eff}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\phi} \\ 0 & i\delta & 0 \\ -s_{13}e^{-i\phi} & 0 & c_{13} \end{pmatrix}_{\text{NP}} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{down}}
\]
In order to obtain the proposed CKM structure, we assume that there is a weak-basis where $M_d, M_u$ have the following structure:

$$
M_d = \begin{pmatrix}
&m_d^d & m_d^t & 0 \\
&m_{21}^d & m_{22}^d & 0 \\
&0 & 0 & m_b
\end{pmatrix} ; \\
M_u = \begin{pmatrix}
&m_u^u & 0 & 0 \\
&0 & m_{22}^u & m_{23}^u \\
&0 & m_{32}^u & m_{33}^u
\end{pmatrix}
$$

It can be shown that one can obtain these structures through the introduction of a $\mathbb{Z}_4$ symmetry at the Lagrangian level.
Without New Physics one has:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
-s_{23}s_{12} - s_{23}c_{12} & c_{23}
\end{bmatrix}
\]

At this level, one has \(|V_{31}| = |V_{12}V_{23}|; V_{13} = 0\).

Our conjecture offers an explanation why \(|V_{31}| > |V_{13}|\) !!!
Introduce an up-type VLQ and assume the $4 \times 4$ up-type quark mass matrix:

$$M_u = \begin{bmatrix}
0 & 0 & 0 & m_{14} \\
0 & m_{22} & m_{23} & 0 \\
0 & m_{32} & m_{33} & 0 \\
m_{41} & 0 & m_{43} & M
\end{bmatrix}$$

Then one can generate

$$|v_{\text{CKM}}|_{13} \neq 0 \quad |\text{Im} Q|_{44} \neq 0$$
Numerical Example

Mass matrices in GeV at Ms scale

\[
M_d = \begin{bmatrix}
0.0029 & -1.35 \times 10^2 & 0 \\
6.73 \times 10^4 & 0.058 & 0 \\
0 & 0 & 2.9
\end{bmatrix}
\]

\[m_d = 0.003 \quad m_\gamma = 0.05 \quad m_b = 2.9\]

\[
M_u = \begin{bmatrix}
0 & 0 & 0 & 53.73 \\
0.59 & -6.91 & 1.25 e & -0.285 i \\
0 & -0.024 i & 172.8 & 0 \\
0.06 & 0 & 14.88 e^{-189 i} & 1250
\end{bmatrix}
\]

\[m_u = 0.02 \quad m_c = 0.60 \quad m_t = 173 \quad m_\tau = 1251\]
The CKM matrix is the 4x3 left submatrix of the following 4x4 unitary matrix:

\[ \mathbf{U} = \begin{pmatrix}
0.9735 & 0.2244 & 0.0037 & 0.0423 \\
0.2244 & 0.9736 & 0.0399 & 0.00099 \\
0.00834 & 0.0393 & 0.999 & 0.00151 \\
0.04163 & 0.0105 & 0.001674 & 0.999
\end{pmatrix} \]
These mass matrices lead to:

\[ \delta \approx 68^\circ \]

\[ \sin 2\beta \approx 0.796 \]

\[ \beta_3 \approx 0.02 \]

\[ I_{CP} \equiv |Im Q| \approx 3 \times 10^{-5} \]
Conclusions

- VLQs are one of the simplest extensions of the SM, with a large number of phenomenological implications.
- VLQs are "cousins" of LR which provide through seesaw the most plausible explanation of the smallness of neutrino masses.
- The effects of VLQs may have been seen already in deviation of unitarity in the first line of VCKM.
• Weak point: No firm prediction for the scale of VLQs.

This is a **universal weak point** in all (so far) proposed New Physics proposals!!

The SM was an notable exception.

Before gauge interactions the Segregation was IVB with \( \approx 2 \) GeV!

Intermediate vector boson...