Amplification and oscillations in the power spectrum from features in the inflaton potential

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Motivation

- We are interested in scales of the power spectrum of cosmological perturbations much smaller than the CMB scale.
- Observational data for such scales are related to:
  1. Possible production of primordial black holes (PBHs) (B. J. Carr and S. W. Hawking (1974))
  2. Gravitational waves induced during the collapse of scalar density perturbations at 2nd order in perturbation theory (S. Matarrese, O. Pantano and D. Saez (1993), S. Matarrese, O. Pantano and D. Saez (1994), S. Matarrese, S. Mollerach and M. Bruni (1998), ...)
- In order for such phenomena to be detectable, the amplitude of the primordial power spectrum must be larger by several orders of magnitude than the value favored by the CMB.
- Features of the inflaton potential that can generate such an enhancement of the curvature power spectrum are:
  1. inflection points
  2. steps
  3. turns in field space (multi-field phenomenon)

Relevant Publications

- K. Kefala, G. P. Kodaxis, I. D. Stamou and N. Tetradis, "Features of the inflaton potential and the power spectrum of cosmological perturbations," [arXiv:2106.02467 [astro-ph.CO]] Phys. Rev. D **104**, no.2, 023506 (2021)
- I. Dalianis, G. P. Kodaxis, I. D. Stamou, N. Tetradis and A. Tsigkas-Kouvelis, "Spectrum oscillations from features in the potential of single-field inflation," [arXiv:2010.12483 [astro-ph.CO]] Phys. Rev. D **104**, no.10, 103510 (2021)
- K. Boutivas, I. Dalianis, G. P. Kodaxis and N. Tetradis, "The effect of multiple features on the power spectrum in two-field inflation," [arXiv:2203.15605 [astro-ph.CO]] JCAP **08** (2022) no.08, 021

Contents

The Mukhanov-Sasaki equation

The most general scalar metric perturbation around the FRW background takes the form

$$ds^2 = a^2(\tau)\left\{(1+2\phi)d\tau^2 - 2B_{,i}\,dx^i d\tau - ((1-2\psi)\delta_{ij} + 2E_{,ij})\,dx^i dx^j\right\}$$

with $B_{,i} = \partial_i B$, $E_{,ij} = \partial_i \partial_j E$.
If we write the inflaton field as $\varphi(\tau) + \delta\varphi(\tau, x)$, we can define a gauge-invariant perturbation as

$$v = a\left(\delta\varphi + \frac{\varphi'}{\mathcal{H}}\psi\right),$$

which satisfies the Mukhanov-Sasaki equation (in Fourier space):

$$v_k''(\tau) + \left(k^2 - \frac{z''}{z}\right) v_k(\tau) = 0.$$

where $z = a\varphi'/\mathcal{H}$.
The comoving curvature perturbation $R = -v/z$ satisfies

$$R_k'' + 2\frac{z'}{z}R_k' + k^2 R_k = 0$$

It is more convenient to use the number of efoldings N as the independent variable. Then the Hubble parameter and the slow-roll parameters take the form:

$$\begin{aligned} H^2 &= \frac{V(\varphi)}{3M_{\rm Pl}^2 - \frac{1}{2}\varphi_{,N}^2} \\ \varepsilon_H &= -\frac{d\ln H}{dN} = \frac{\varphi_{,N}^2}{2M_{\rm Pl}^2} \\ \eta_H &= \varepsilon_H - \frac{1}{2}\frac{d\ln\varepsilon_H}{dN} = \frac{\varphi_{,N}^2}{2M_{\rm Pl}^2} - \frac{\varphi_{,NN}}{\varphi_{,N}}. \end{aligned}$$

The evolution of the background field is governed by the equation

$$\varphi_{,NN} + 3\varphi_{,N} - \frac{1}{2M_{\rm Pl}^2}\varphi_{,N}^3 + \left(3M_{\rm Pl}^2 - \frac{1}{2}\varphi_{,N}^2\right)\frac{V_{,\varphi}}{V} = 0,$$

while the equation for the curvature perturbation becomes:

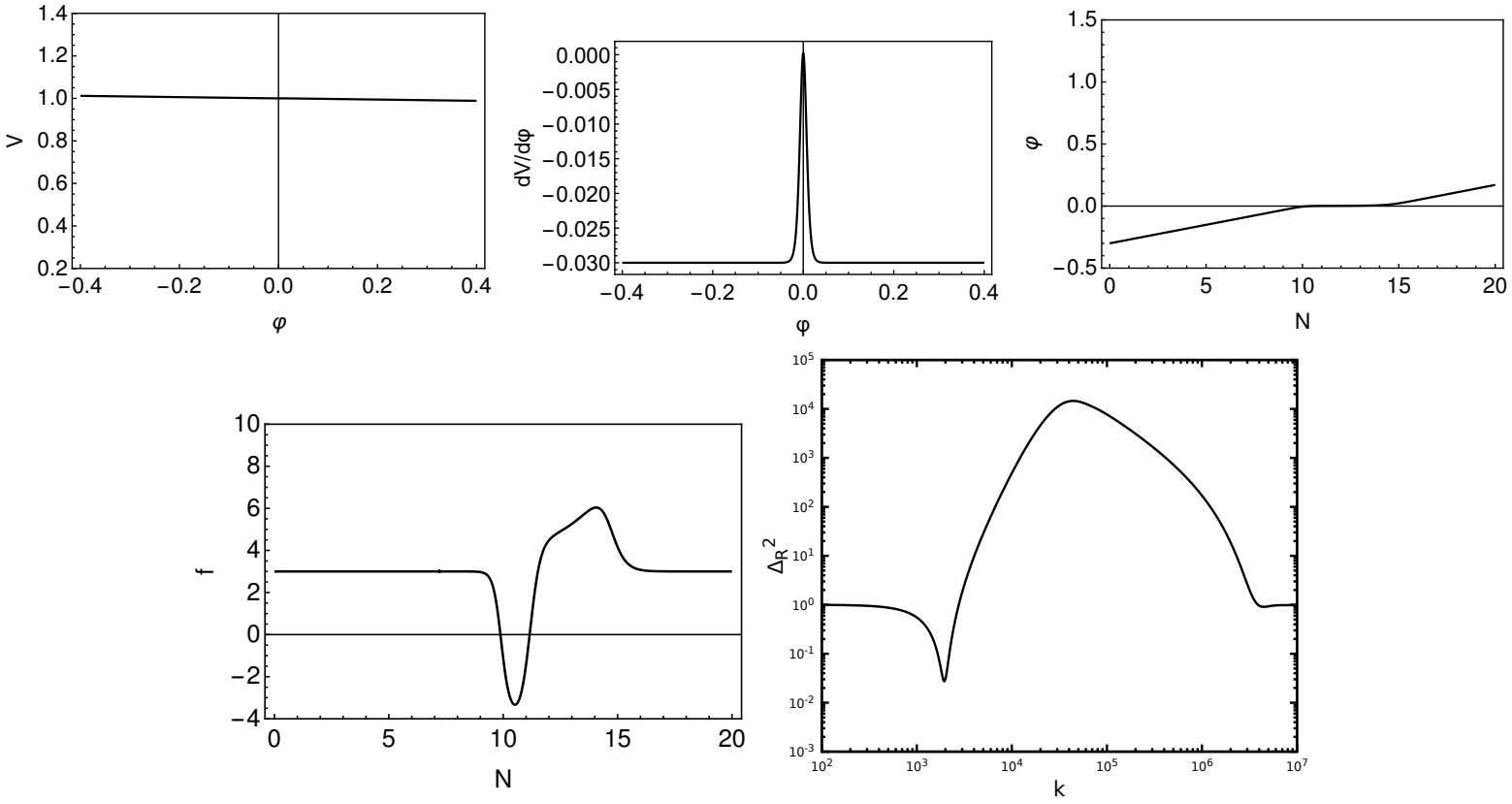
$$R_{k,NN} + f(N)\,R_{k,N} + \frac{k^2}{e^{2N}H^2}R_k = 0,$$

where

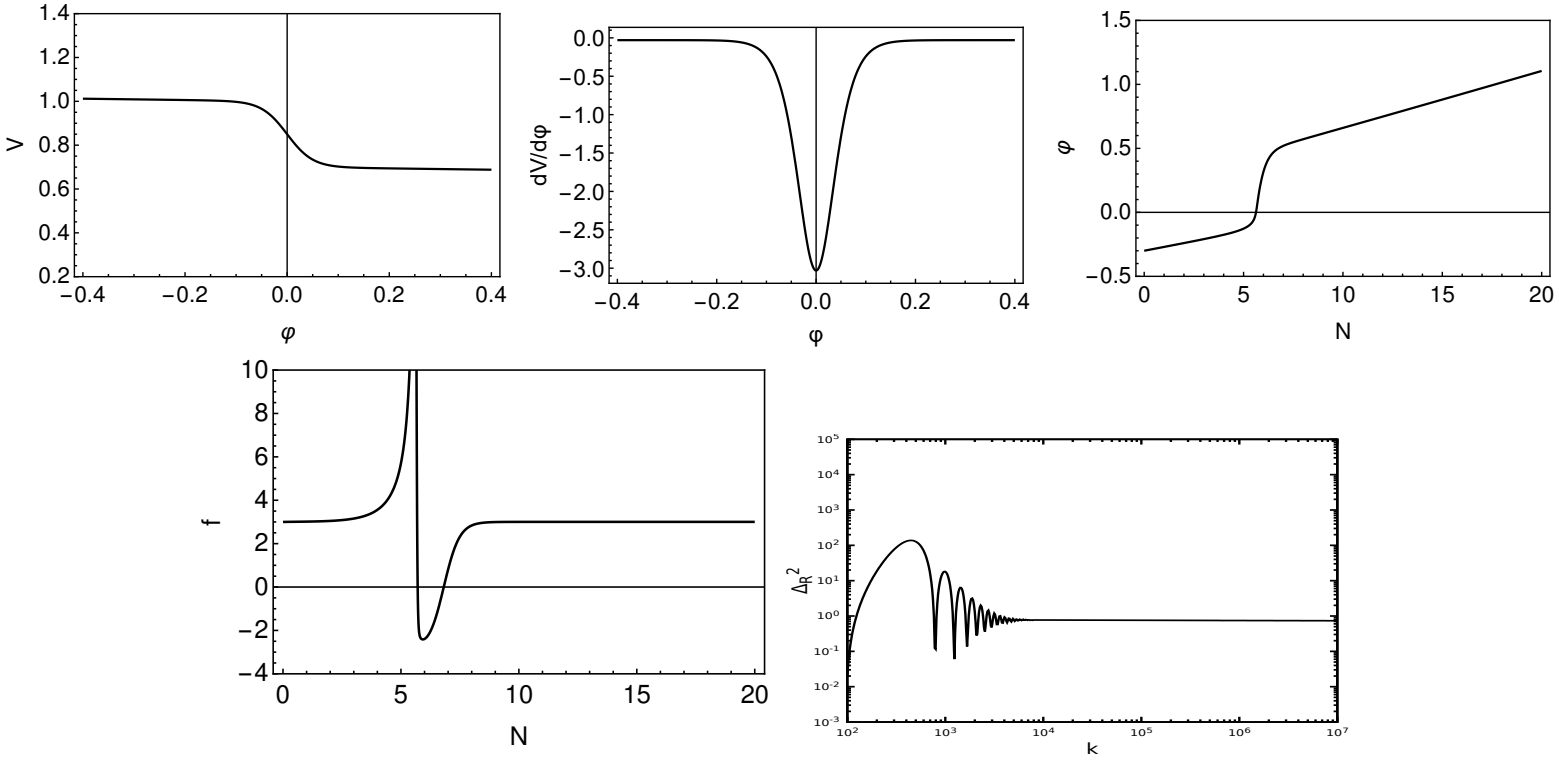
$$f(N) = 3 + \frac{2\varphi_{,NN}}{\varphi_{,N}} - \frac{\varphi_{,N}^2}{2M_{\rm Pl}^2} = 3 + \varepsilon_H - 2\eta_H$$

Features of the inflaton potential

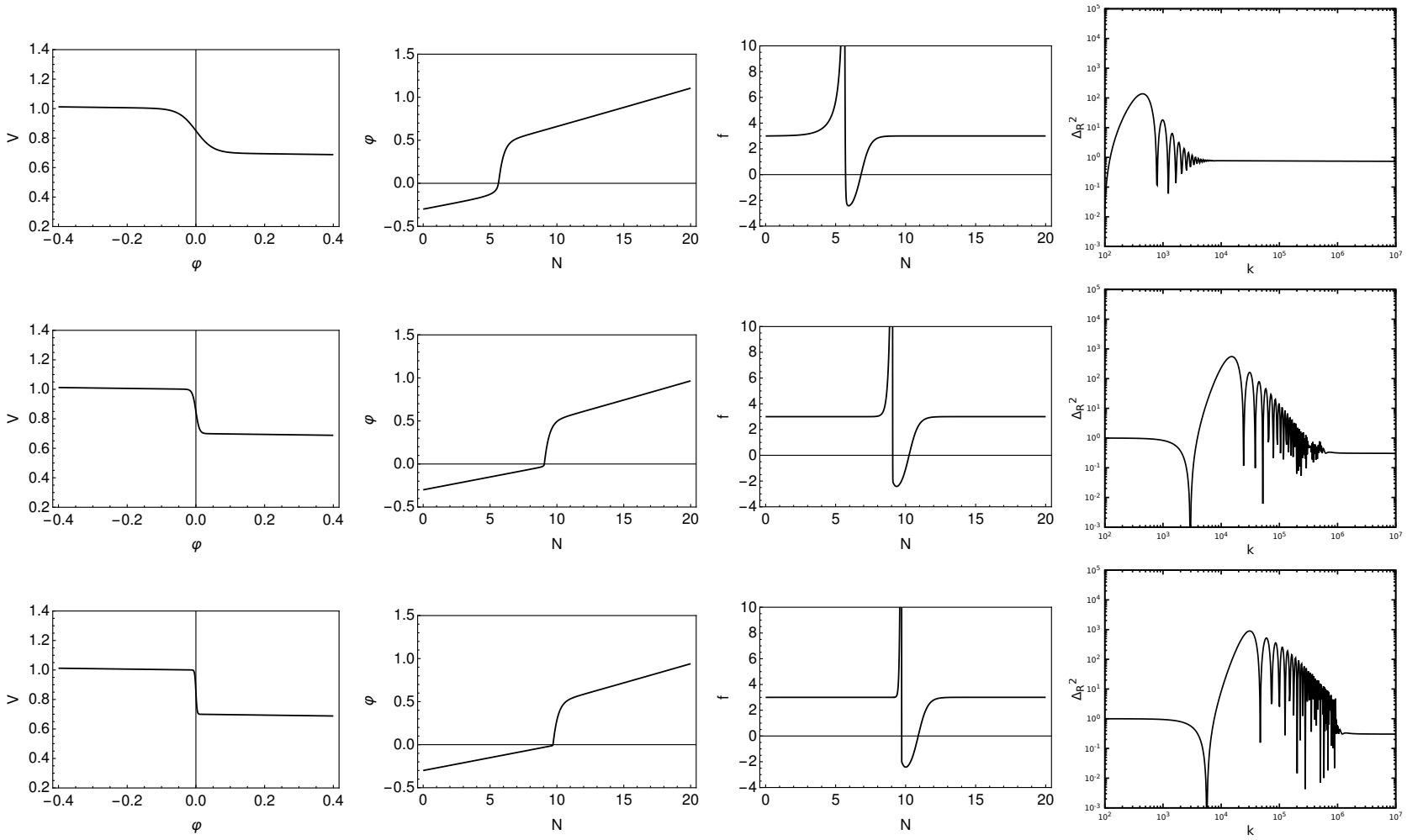
- We want to induce an amplification of the spectrum of curvature perturbations by several orders of magnitude.
- We keep only the minimal number of elements required for addressing the problem.
- In single-field inflation, the features of interest are:
  1. An inflection point, at which the first and second derivatives of the effective potential vanish. This requires the fine tuning of the parameters of the potential.
  2. One or more field values at which the potential decreases sharply (steps). Such steps can occur at transition points at which the vacuum energy jumps from one constant value to another.



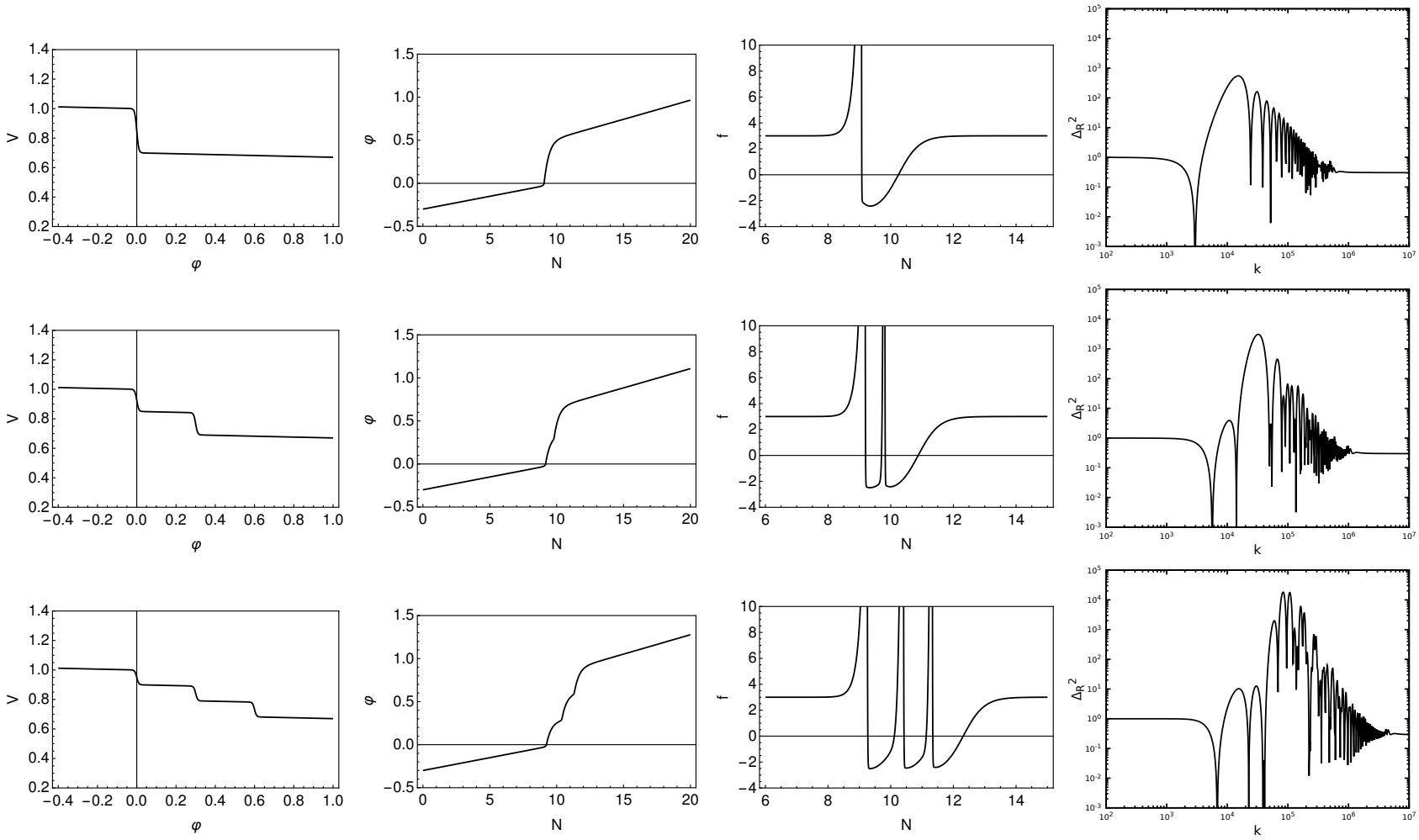
Potential with inflection point.



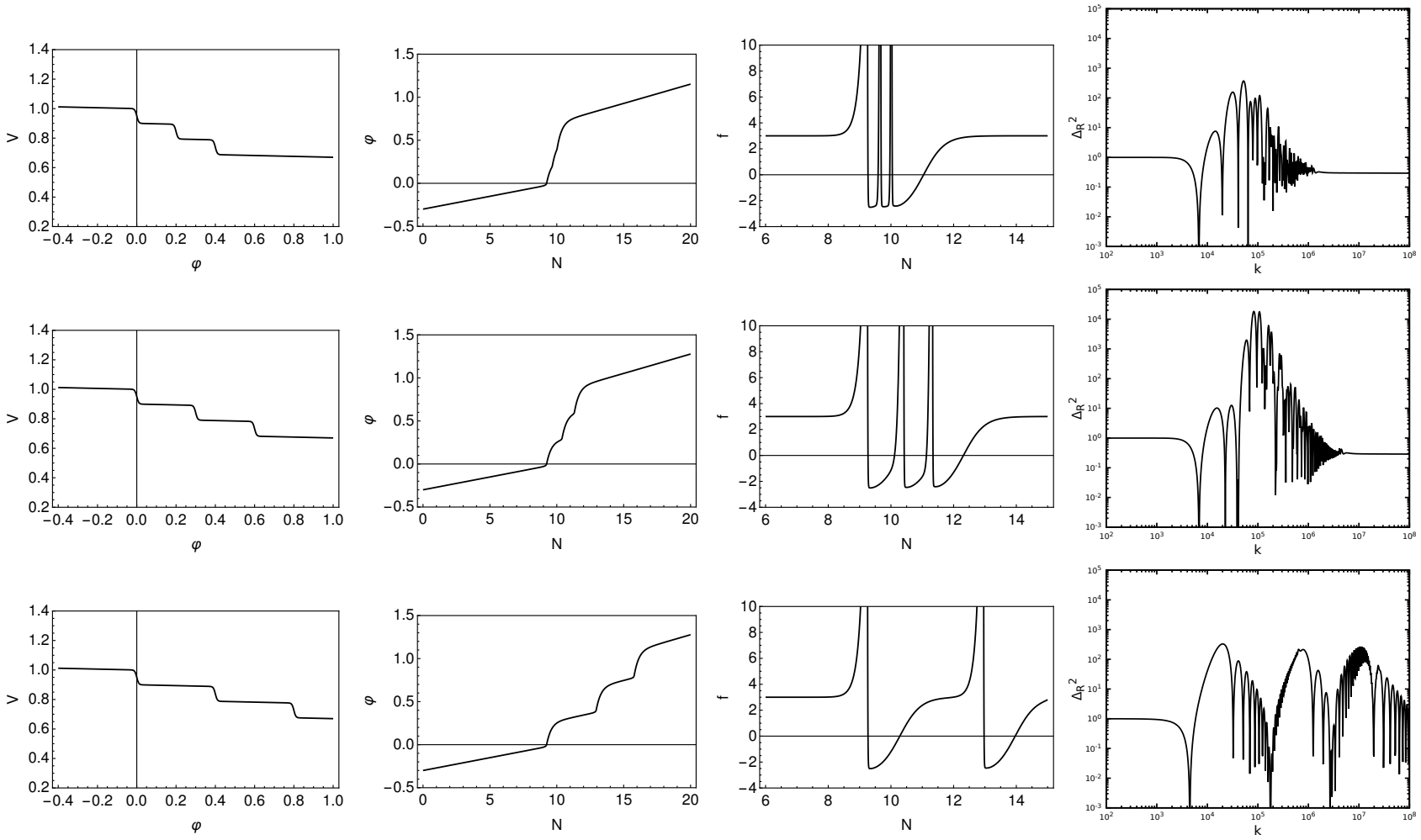
Potential with step.



Steps of different steepness.



Multiple steps.



Different distance between steps.

The framework of α-attractors

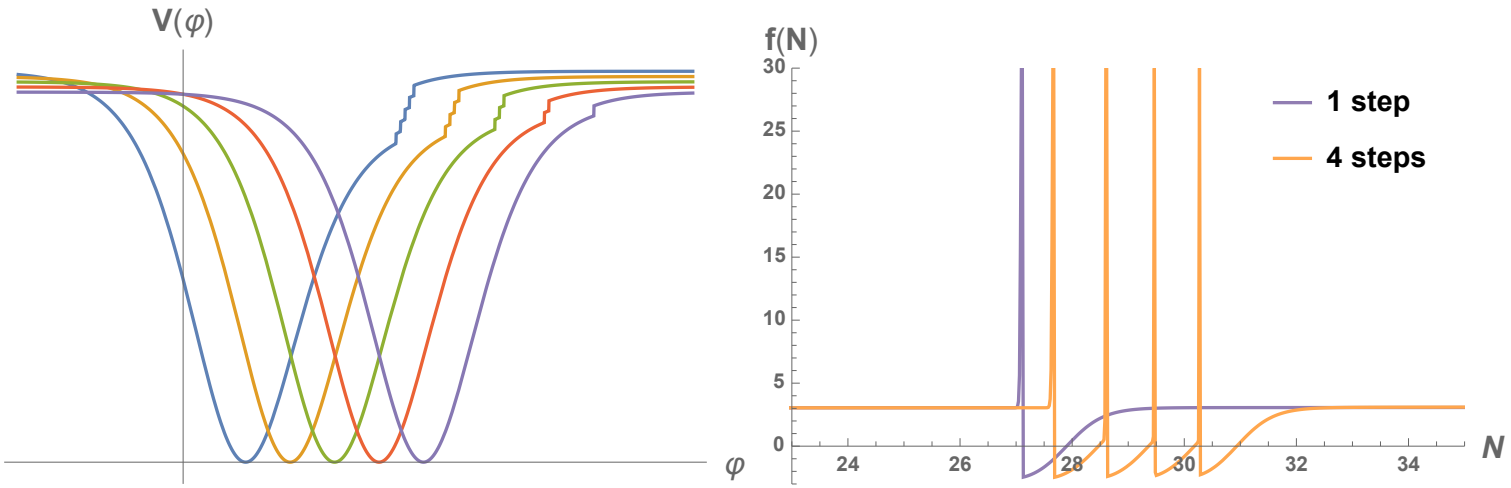
In the framework of α-attractors (Kallosh, Linde, Roest 2014), the Lagrangian has the typical form

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R(g) - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - F^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)\right].$$

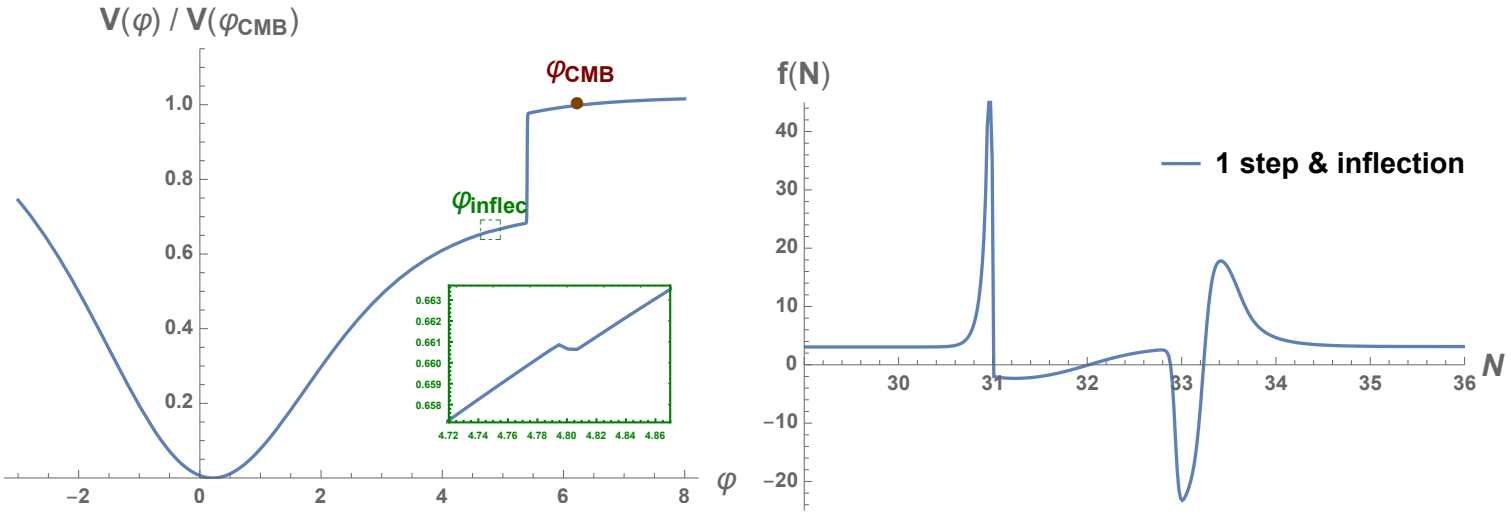
A constant value of F results in a cosmological constant.
However, transitions between two constant values are possible, if F takes the typical form

$$F(x) = x^n + \sum_i A_i\,\Theta(x - x_i),$$

for which the potential features n step-like transitions.

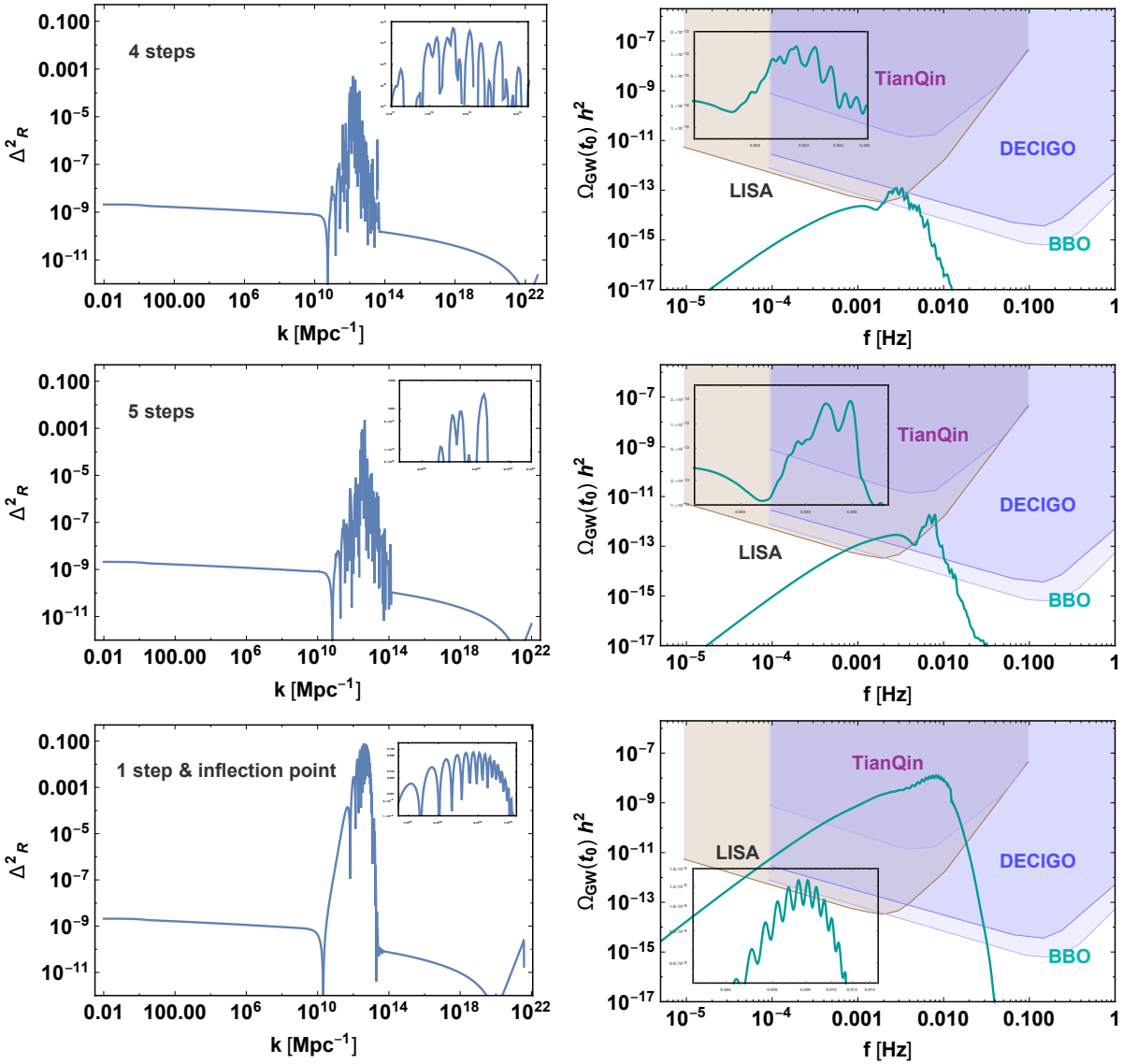


Potentials that include one-five steps, arbitrarily placed on the φ axis, and the function $f(N)$ for potentials with one and four steps.

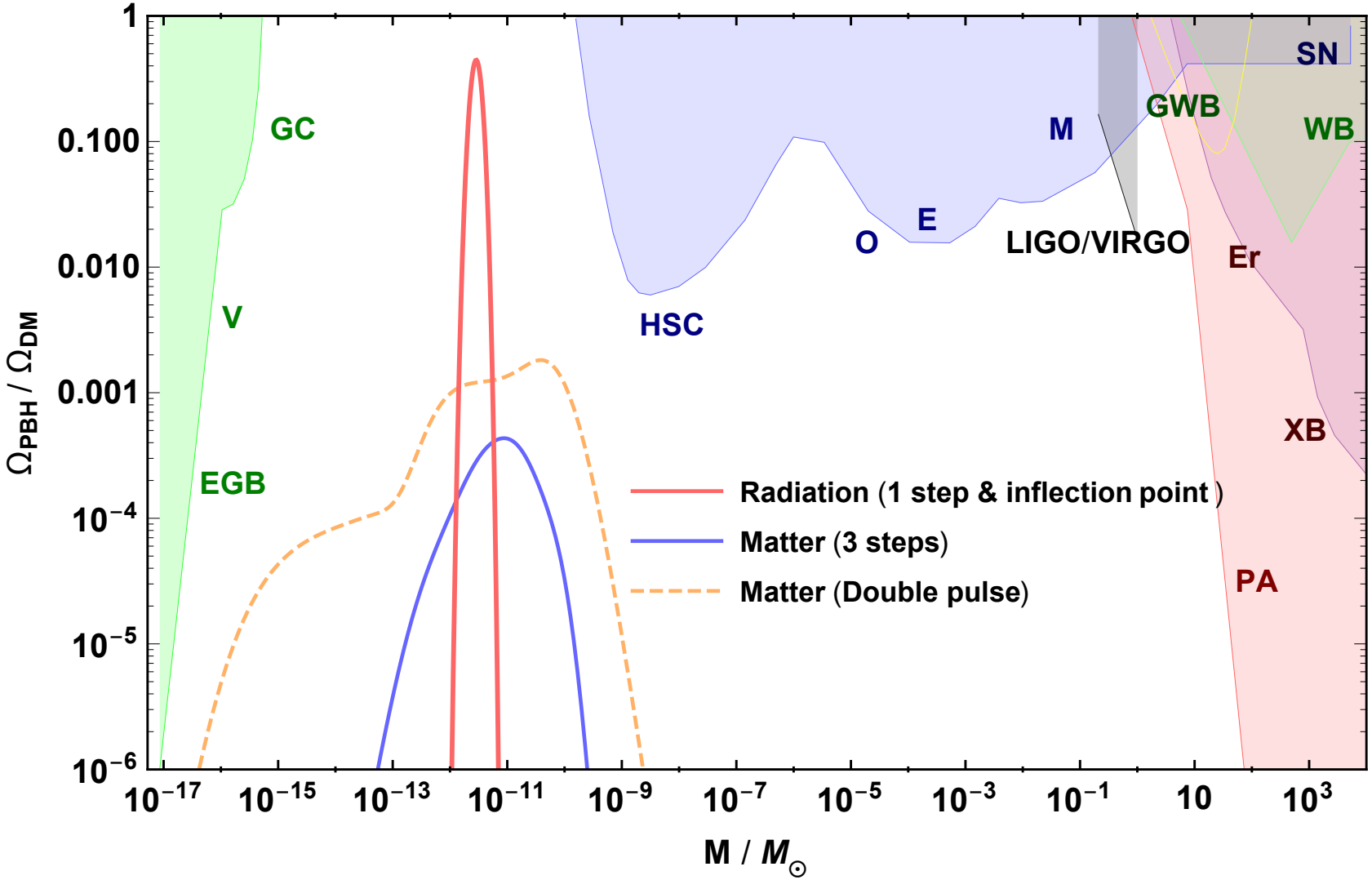


Potential with one step and an inflection point and the corresponding function $f(N)$.

Induced gravitational waves



Primordial black holes



Conclusions

- The amplification or suppression of the spectrum is determined by the function $f(N)$.
- Apart from inflection points, a significant amplification is also caused by steep steps in the inflaton potential.
- The number of steps, the distance from each other and their steepness affect the total amplification of the spectrum.
- A distinctive feature is the appearance of oscillations in the spectrum.
- Inflationary models that include such features can be constructed in the framework of α-attractors.
- The form of the curvature spectrum is reflected on the abundance of PBHs and the spectrum of induced gravitational waves.
- Similar phenomena occur in two-field models with sharp turns in field space during the field evolution.

Thank you for your attention!

The origin of Oscillations

- The origin of the oscillations can be understood if one considers the evolution of the curvature perturbation for constant $f(N) = \kappa$. Its solution has the form

$$R_k(N) = A e^{-\frac{1}{2}\kappa N} \left(J_{\kappa/2}\left(e^{-N}\frac{k}{H}\right) + c\, J_{-\kappa/2}\left(e^{-N}\frac{k}{H}\right)\right).$$

- The evolution of the perturbation during the initial slow-roll regime corresponds to $\kappa = 3$ and $c = i$. The amplitude of $R_k(N)$ does not have oscillatory behaviour.
- If κ varies strongly, the relative coefficient of the Bessel functions varies as well. The phase difference between the real and imaginary part changes, which leads to oscillations of the amplitude of $R_k(N)$.
- The freezing of R_k at horizon exit can occur at any stage of the oscillatory cycle, depending on the value of k. Eventually, this is reflected in the strong oscillatory behaviour of the spectrum as a function of k.