Scale Invariance in Particle Physics and Cosmology

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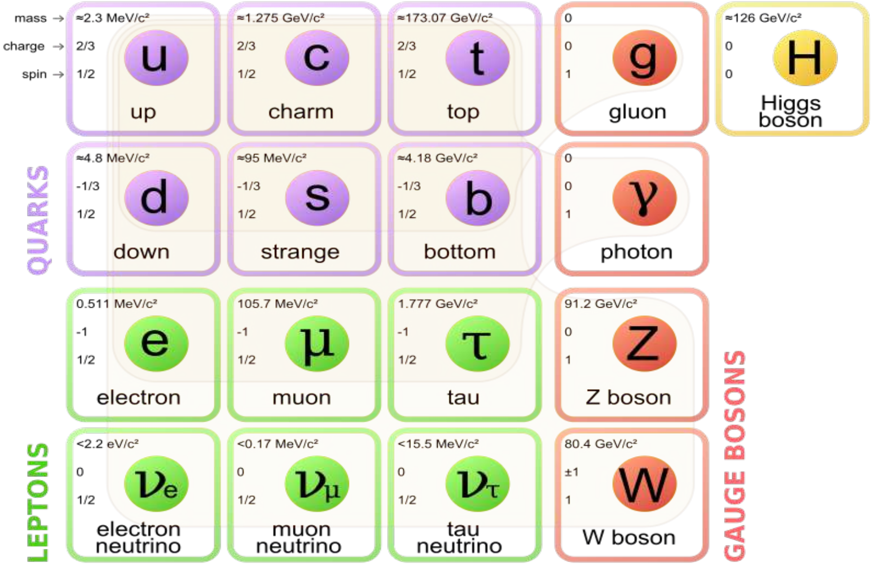
CoE DSoU | Centre of Excellence: Dark Side of the Universe

The Standard Model (SM)

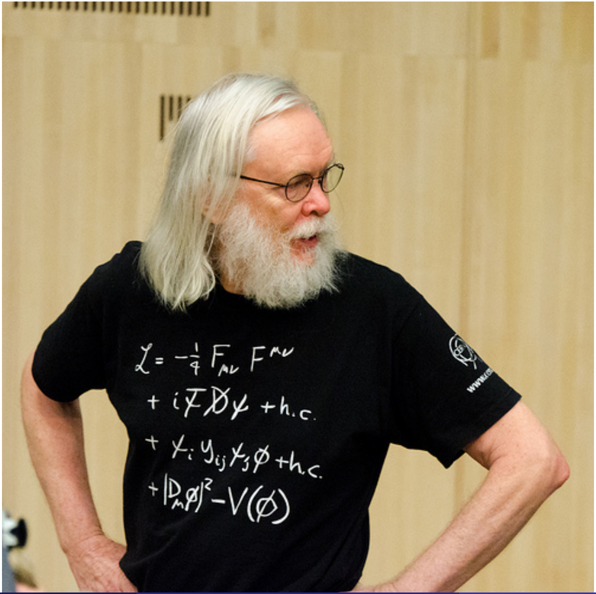


Beyond the Standard Model (BSM)

Shortcomings of the SM:

- **Dark Matter** - No particle(s) in the SM that can explain the measurements of extra non-baryonic mass at galactic and cosmic scales
- **Neutrino Masses** - How do neutrinos obtain their mass?
- **Vacuum Instability** - In the SM $\lambda_h \left(\mu \gtrsim 10^{10.5}\ \mathrm{GeV}\right) < 0$ which means the Higgs potential becomes unstable
- **Matter-Antimatter Asymmetry** - The SM does not explain why there is more matter than antimatter
- **Flavor Problem** - Why are there 6 flavours of quarks and leptons with different characteristics?
- **Strong CP Problem** - Why QCD does not break the CP symmetry?
- **Gauge Unification** - Do the 3 gauge forces of the SM become unified at a high energy scale?
- **Inflation** - Is there a Particle Physics description of inflation?
- ...

Standard Model Lagrangian



The Higgs field

$$\mathcal{L}_{\text{Higgs}} = (D^\mu H)^\dagger (D_\mu H) - y_f \overline{\psi}_L H \psi_R - V(H),$$

$$D_\mu H = \left(\partial_\mu + i\frac{g_2}{2}\tau^a A^a_\mu + i\frac{g_1}{2}B_\mu\right) H,$$

$$V(H) = -\frac{1}{2}\mu^2_{\text{SM}} H^\dagger H + \lambda_h (H^\dagger H)^2$$

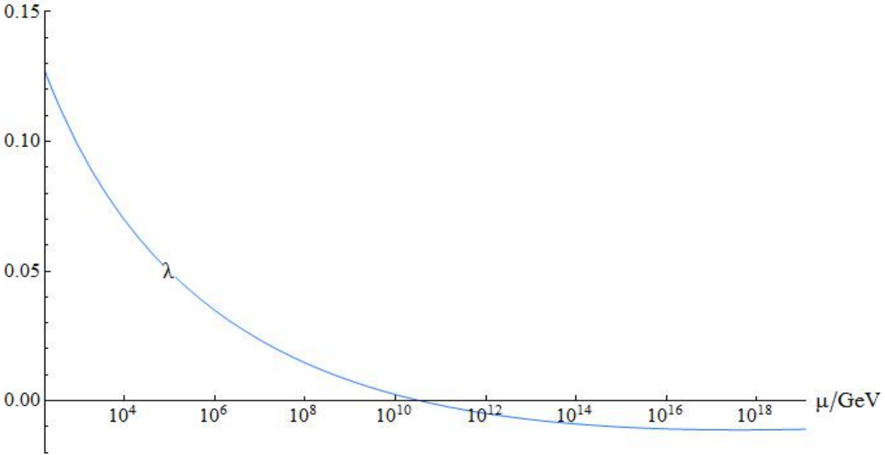
In the unitary gauge, $H^{\mathsf{T}}(x) = \frac{1}{\sqrt{2}}(0, h(x))$. The vacuum expectation value (**VEV**) v_h and the mass M_h of the physical Higgs field $h(x)$ are generated through the scale μ_{SM},

$$v_h = \frac{\mu_{\text{SM}}}{(2\lambda_h)^{1/2}} \simeq 246 \text{ GeV}, \quad M_h = \mu_{\text{SM}} \simeq 125 \text{ GeV}$$

Renormalization Group Equation (RGE) or β-function for λ_h:

$$(4\pi)^2 \frac{d\lambda_h}{d\ln\mu} = -6y_t^4 + 24\lambda_h^2 + \lambda_h\left(12y_t^2 - \frac{9}{5}g_1^2 - 9g_2^2\right) + \frac{27}{200}g_1^4 + \frac{9}{20}g_2^2 g_1^2 + \frac{9}{8}g_2^4$$

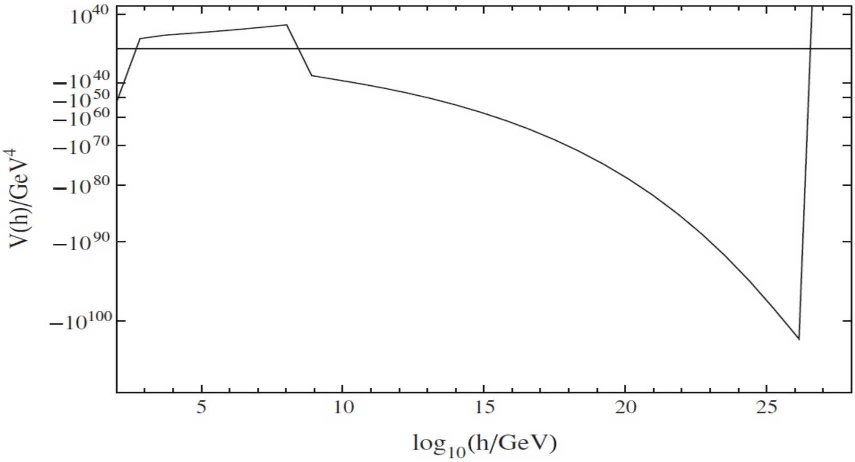
Running of λ_h at 3 loops



$\lambda_h(\mu) > 0$: $M_h > 129.40$ GeV or $M_t < 171.22$ GeV

Measured: $M_h = 125.09$ GeV and $M_t = 173.34$ GeV

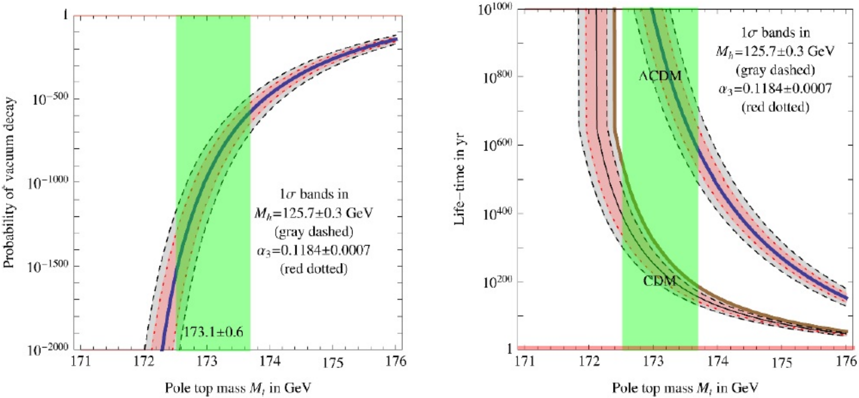
The SM Higgs effective potential as a function of the Higgs field strength



Gabrielli et al. PRD 89, 015017 (2014)

Past and Future of the Vacuum

Buttazzo et al. JHEP 1312 (2013) 089



Left: The probability that electroweak vacuum decay happened in our past light-cone, taking into account the expansion of the universe.

Right: The life-time of the electroweak vacuum, with two different assumptions for future cosmology: universes dominated by the cosmological constant (ΛCDM) or by dark matter (CDM).

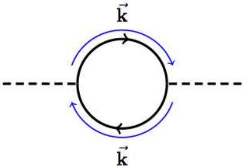
Conclusion 1

We should **add scalar fields** that couple to the Higgs in order to achieve vacuum stability.

The Hierarchy Problem

Solutions to the problems of the SM generally require new large scales $\Lambda_{\rm NP} \gg v_{\rm EW}$. These theories contain particles with mass $m \sim \Lambda_{\rm NP}$ that give loop corrections to the Higgs boson mass:

$$M_h^2 = M_0^2 + \Delta m_h^2, \quad \Delta m_h^2 \propto \Lambda_{\rm NP}^2$$



A huge amount of fine-tuning is needed between M_0^2 and $\Lambda_{\rm NP}^2$ in order to obtain $M_h = 125.09$ GeV.

A similar argument holds if we consider the SM as an effective theory, without any additional degrees of freedom, having a cut-off $\Lambda_{\rm UV}$.

The Hierarchy Problem

Popular Solution:

Supersymmetry

Alternate Solution:

Classical Scale Invariance

Classical Scale Invariance

A theory is said to have Classical Scale Symmetry if the Lagrangian is invariant under the transformations

$$x_\mu \to d x_\mu, \quad S(x) \to d S(dx), \quad V_\mu(x) \to d V_\mu(dx), \quad F(x) \to d^{3/2} F(dx)$$

There is only **one** scale in the SM, the parameter μ_{SM} in the scalar potential

$$V(H) = -\frac{1}{2}\mu_{\text{SM}}^2 H^\dagger H + \lambda_h (H^\dagger H)^2$$

With $\mu_{\text{SM}} = 0$ the SM is Classically Scale Invariant (**CSI**) $(T_\mu^\mu)_{\text{classical}} = 0$.
The quadratic sensitivity to the cut-off is now unphysical.
Quantum effects break the symmetry through the β-functions $(T_\mu^\mu = \beta_{\lambda_i} \mathcal{O}_i)$, but this does not reintroduce the quadratic divergences.
(Bardeen: On naturalness in the standard model).

$\Longrightarrow$ Solution to the Hierarchy Problem

Coleman-Weinberg Mechanism

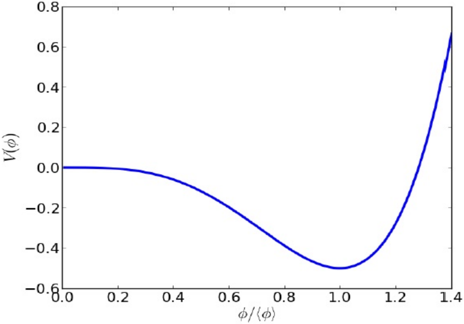
Massless scalar QED: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - \frac{\lambda_\phi}{4!}|\phi|^4$

1-loop effective potential:

$$V_{\text{eff}}(\phi) = \frac{\lambda_\phi}{24}|\phi|^4 + \frac{3g_\phi^4}{64\pi^2}|\phi|^4\left[\ln\left(\frac{|\phi|^2}{v_\phi^2}\right) - \frac{25}{6}\right]$$

Minimization at $\phi = v_\phi \neq 0$
gives $\lambda_\phi(v_\phi) = \frac{33}{8\pi^2}g_\phi^4(v_\phi)$

$$V_{\text{eff}}(\phi) \propto g_\phi^4|\phi|^4\left[\ln\left(\frac{|\phi|^2}{v_\phi^2}\right) - \frac{1}{2}\right]$$



Coleman-Weinberg Mechanism

Dimensional Transmutation: exchange a **dimensionless** parameter λ_ϕ for a **dimensionful** parameter v_ϕ.

$$\lambda_\phi(v_\phi) = \frac{33}{8\pi^2} g_\phi^4(v_\phi), \qquad v_\phi \sim \Lambda_{\mathsf{UV}} \times \exp\left[\frac{-24\pi^2}{g_\phi^2(v_\phi)}\right] \ll \Lambda_{\mathsf{UV}}$$

The scalar boson (**scalon**) obtains a 1-loop mass:

$$M_S^2 = \frac{3g_\phi^4}{8\pi^2} v_\phi^2 = \frac{3g_\phi^2}{8\pi^2} M_V^2$$

In the SM:

$$M_h^2 \approx \frac{1}{8\pi^2 v_h^2}\left[6M_W^4 + 3M_Z^4 - 12M_t^4\right] < 0$$

Conclusion 2

In a CSI extension of the SM, we should **add bosonic fields** in order to obtain a positive mass.

Types of CSI Models

- **Weakly-coupled** à la Coleman-Weinberg
  - λ corrections due to other scalar quartics
  - g corrections due to a gauge coupling

- **Strongly-coupled** where a gauge coupling g runs to non-perturbative values, inducing condensates

Classically Scale-Invariant Extension of the Standard Model 1

PHYSICAL REVIEW D **92,** 075010 (2015)

Dark matter and neutrino masses from a scale-invariant multi-Higgs portal

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We consider a classically scale invariant version of the Standard Model, extended by an extra dark $SU(2)_X$ gauge group. Apart from the dark gauge bosons and a dark scalar doublet which is coupled to the Standard Model Higgs through a portal coupling, we incorporate right-handed neutrinos and an additional real singlet scalar field. After symmetry breaking à la Coleman-Weinberg, we examine the multi-Higgs sector and impose theoretical and experimental constraints. In addition, by computing the dark matter relic abundance and the spin-independent scattering cross section off a nucleon we determine the viable dark matter mass range in accordance with present limits. The model can be tested in the near future by collider experiments and direct detection searches such as XENON 1T.

DOI: 10.1103/PhysRevD.92.075010 PACS numbers: 12.60.Cn, 14.60.St, 14.80.Ec, 95.35.+d

Classically Scale-Invariant Extension of the Standard Model 2

PHYSICAL REVIEW D **94,** 055004 (2016)

Dark matter from a classically scale-invariant $SU(3)_X$

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In this work we study a classically scale-invariant extension of the Standard Model in which the dark matter and electroweak scales are generated through the Coleman-Weinberg mechanism. The extra $SU(3)_X$ gauge factor gets completely broken by the vacuum expectation values of two scalar triplets. Out of the eight resulting massive vector bosons the three lightest are stable due to an intrinsic $Z_2 \times Z_2'$ discrete symmetry and can constitute dark matter candidates. We analyze the phenomenological viability of the predicted multi-Higgs sector imposing theoretical and experimental constraints. We perform a comprehensive analysis of the dark matter predictions of the model solving numerically the set of coupled Boltzmann equations involving all relevant dark matter processes and explore the direct detection prospects of the dark matter candidates.

DOI: 10.1103/PhysRevD.94.055004

The Model

CSI $\quad SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_X$

New fields:

- 1 complex scalar doublet Φ under $SU(2)_X$
- 3 gauge bosons X^a

Unitary gauge: $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix}, \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}$

$$V_0 = \frac{\lambda_h}{4} h^4 - \frac{\lambda_{h\phi}}{4} h^2 \phi^2 + \frac{\lambda_\phi}{4} \phi^4 \,, \qquad \lambda_h(\Lambda)\,\lambda_\phi(\Lambda) - 4\lambda_{h\phi}^2(\Lambda) = 0$$

The shifted scalar fields and VEVs are

$$h \ = \ (\varphi + v)\, n_1, \quad \phi \ = \ (\varphi + v)\, n_2$$

$$\langle h \rangle \ \equiv \ v_h = v\, n_1, \quad \langle \phi \rangle \ \equiv \ v_\phi = v\, n_2$$

Masses

From the shifted tree-level potential we can read off the scalar mass matrix

$$\mathcal{M}_0^2 = v^2 \begin{pmatrix} 2\lambda_h n_1^2 & -n_1 n_2 \lambda_{h\phi} \\ -n_1 n_2 \lambda_{h\phi} & 2\lambda_\phi n_2^2 \end{pmatrix} , \qquad v^2 = v_h^2 + v_\phi^2$$

in the (h, ϕ) basis. Introduce a general rotation

$$\mathcal{R}\,\mathcal{M}_0^2\,\mathcal{R}^{-1} \;=\; \mathcal{M}_d^2, \quad \mathcal{R}^{-1} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} , \quad \tan^2\alpha = \frac{v_h^2}{v_\phi^2} = \frac{\lambda_{h\phi}}{2\lambda_h}$$

The resulting mass eigenvalues are

$$M_{h_1}^2 \;=\; (2\lambda_h - \lambda_{h\phi})\, v_h \equiv M_{\rm Higgs}^2$$

$$\boxed{M_{h_2}^2 = 0}$$

Gauge Sector

$$\mathcal{L}_X = -\frac{1}{4} X^a_{\mu\nu} X^{a\,\mu\nu} + (D_\mu \Phi)^\dagger D^\mu \Phi, \quad a = 1, 2, 3$$

$$\boxed{M_{X^a} = \tfrac{1}{2} g_X v_\phi}$$

$$\mathcal{L}_{\mathrm{s-g}} = \frac{g_X^2}{4} v_\phi\, \phi\, X^a_\mu X^{a\,\mu} + \frac{g_X^2}{8}\, \phi^2\, X^a_\mu X^{a\,\mu}$$

$$\mathcal{L}_{\mathrm{g-g}} = -g_X \epsilon^{abc} (\partial_\mu X^{\,a}_\nu) X^{\mu\,b} X^{\nu\,c} - \frac{g_X^2}{4} \left[\left(X^{\,a}_\mu X^{\mu\,a} \right)^2 - X^{\,a}_\mu X^{\,a}_\nu X^{\mu\,a} X^{\nu\,a} \right]$$

The system possesses the $Z_2 \times Z_2'$ symmetry

$$\begin{aligned} Z_2 : &\qquad X^1_\mu \to -X^1_\mu, \quad X^2_\mu \to -X^2_\mu \quad \text{(gauge trans.)} \\ Z_2' : &\qquad X^1_\mu \to -X^1_\mu, \quad X^3_\mu \to -X^3_\mu \quad \text{(charge conj.)} \end{aligned}$$

1-loop Potential and Darkon Mass

The one-loop effective potential becomes

$$\left.\frac{\partial V_1(\mathbf{n}\varphi)}{\partial \varphi}\right|_{\varphi=v} = 0 \quad \Rightarrow \quad V_1(\mathbf{n}\varphi) = B\varphi^4 \left[\log\frac{\varphi^2}{v^2} - \frac{1}{2}\right],$$

$$B = \frac{1}{64\pi^2 v^4}\left(M_{h_1}^4 + 6M_W^4 + 3M_Z^4 + 9M_X^4 - 12M_t^4\right),$$

$$v^2 = v_h^2 + v_\phi^2$$

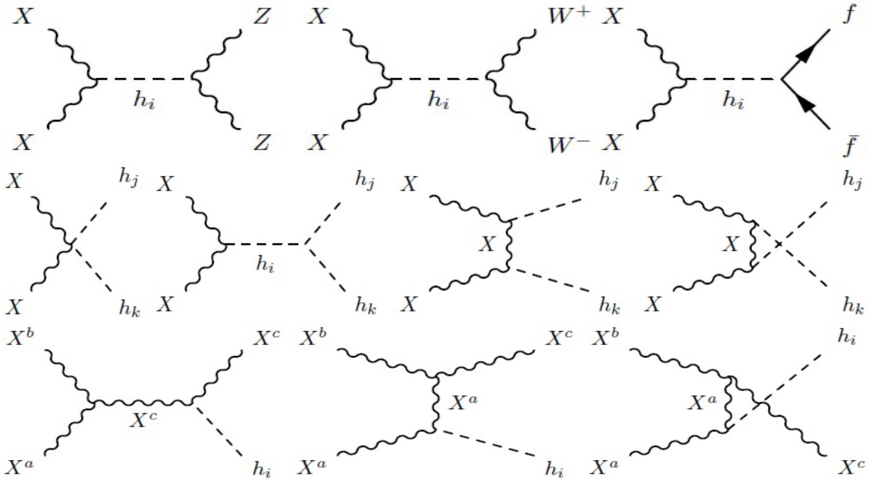
The pseudo-Goldstone boson (**darkon**) mass is now shifted from zero to

$$M_{h_2}^2 = 8Bv^2 = \frac{1}{8\pi^2 v^2}\left(M_{h_1}^4 + 6M_W^4 + 3M_Z^4 + 9M_X^4 - 12M_t^4\right)$$

Positivity condition $B > 0$ translates to

$$9M_X^4 > (317.26\ \text{GeV})^4$$

DM Annihilations and Semi-annihilations



Boltzmann Equation and Relic Abundance

The Boltzmann equation has the form

$$\frac{\mathrm{d}n}{\mathrm{d}t} + 3\,H\,n = -\frac{\langle\sigma v\rangle_{\rm ann}}{3}\left(n^2 - n_{eq}^2\right) - \frac{2\,\langle\sigma v\rangle_{\rm semi}}{3}\,n\,(n - n_{eq})$$

The annihilation cross section is dominated by the $XX \to h_2 h_2$ process

$$\langle\sigma v\rangle_{\rm ann} = \frac{11 g_X^4}{2304\pi M_X^2}$$

The semiannihilation cross section is dominated by the $XX \to X h_2$ process

$$\langle\sigma v\rangle_{\rm semi} = \frac{3 g_X^4}{128\pi M_X^2}$$

The semiannihilation processes dominate since $\langle\sigma v\rangle_{\rm semi} \sim 5\,\langle\sigma v\rangle_{\rm ann}$.
Solving the Boltzmann equation, we obtain the dark matter relic abundance

$$\Omega_X h^2 = \frac{1.04\times 10^9\ \mathrm{GeV}^{-1}}{\sqrt{g_*}\,M_P\,J(x_f)}, \qquad J(x_f) = \int_{x_f}^{\infty} dx\,\frac{\langle\sigma v\rangle_{\rm ann} + 2\,\langle\sigma v\rangle_{\rm semi}}{x^2},$$

The correct relic abundance ($\Omega_{\rm DM} h^2 = 0.120 \pm 0.001$) is reproduced if

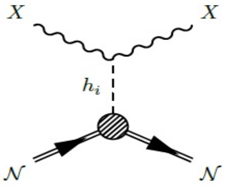
$$g_X \approx 0.9\times\sqrt{\frac{M_X}{1\ \mathrm{TeV}}}$$

Direct Detection

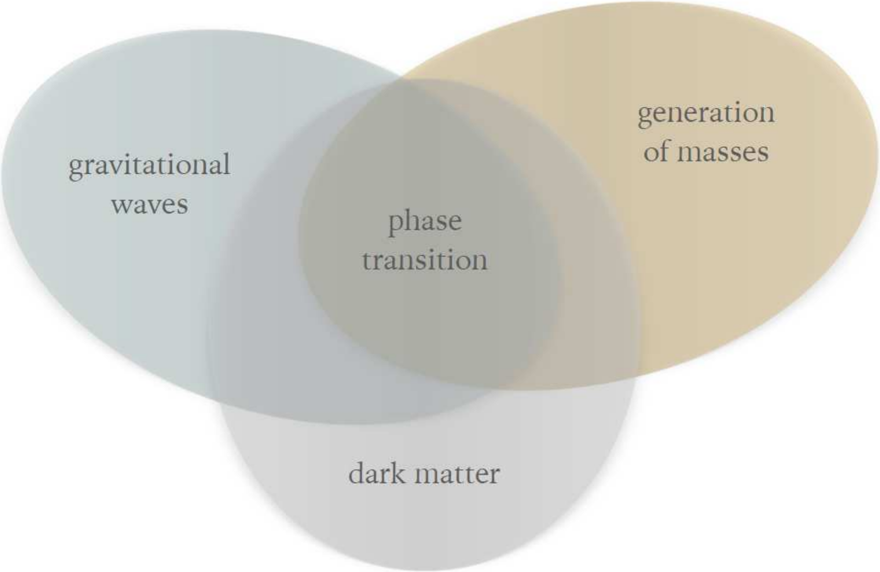
Scattering off of nucleons (spin-independent cross section)

$$\sigma_{\rm SI} = \frac{m_N^4 f^2}{16\pi v^2}\left(\frac{1}{M_{h_2}^2} - \frac{1}{M_{h_1}^2}\right)^2 g_X^2 \sin^2 2\alpha \simeq \frac{64\pi^3 f^2 m_N^4}{81 M_X^6}$$
$$\approx 0.6 \times 10^{-45}\ {\rm cm}^2 \left(\frac{\rm TeV}{\rm M_X}\right)^6$$

So $\sigma_{\rm SI} < 1.5 \times 10^{-45}\ {\rm cm}^2\ ({\rm M_X}/\,{\rm TeV})$ for $M_X > 0.88\,{\rm TeV}$



Phase transition



Finite Temperature Potential

The temperature-dependent effective potential

$$V_{\text{eff}}(h, \phi, T) = V^{(0)}(h, \phi) + V^{(1)}(h, \phi) + V^{T}(h, \phi, T) + V_{\text{daisy}}$$

The finite-temperature correction is

$$V^{T}(h, \phi, T) = \frac{T^4}{2\pi^2} \sum_a n_a J_a \left(\frac{M_a(h, \phi)^2}{T^2} \right),$$

where the sum runs over particle species. J_a denotes the thermal function, which is given by

$$J_{F,B}(y^2) = \int_0^\infty \mathrm{d}x x^2 \log(1 \pm e^{\sqrt{x^2+y^2}}),$$

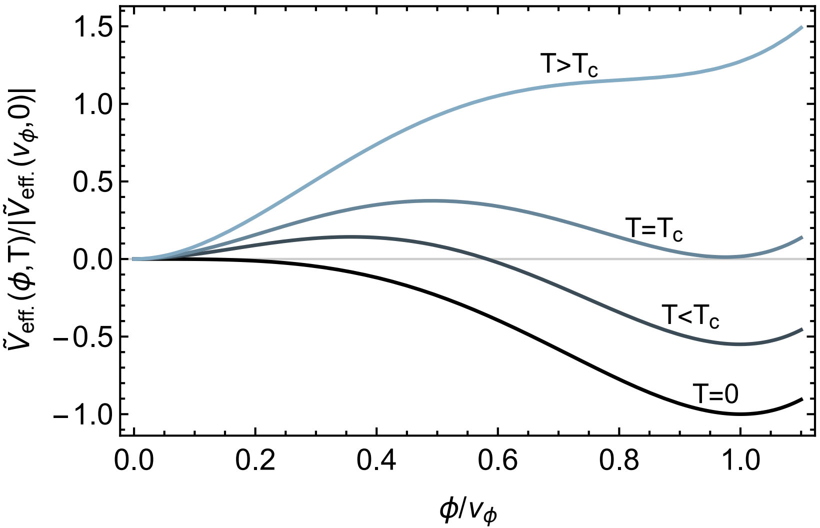
where ``$+$'' for fermions ($J_F$) and ``$-$'' for bosons (J_B). The correction from the daisy resummed diagrams its

$$V_{\text{daisy}} = -\frac{T}{12\pi} \sum_i n_i \left[(M_{i,\text{th}}^2)^{3/2} - (M_i^2)^{3/2} \right], \tag{1}$$

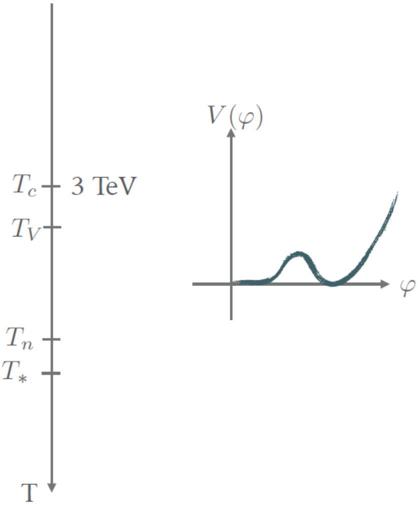
where n_i is the number of degrees of freedom, $M_{i,\text{th}}$ denotes thermally corrected mass, and M_i the usual field dependent mass.

Finite Temperature Potential

L. Marzola, A. Racioppi, V. Vaskonen: 1704.01034

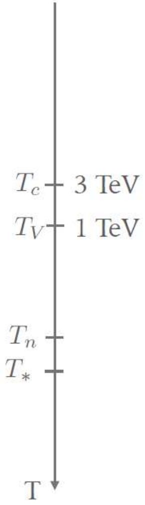


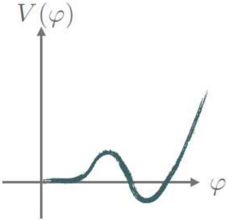
Temperature Evolution ($M_X = 9$ TeV, $g_X = 0.9$)



critical temperature: two degenerate minima

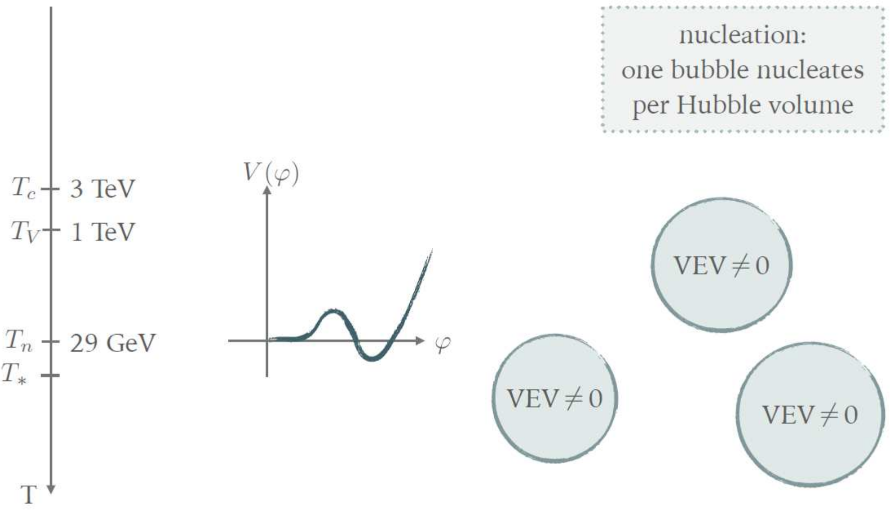
Temperature Evolution ($M_X = 9\ \text{TeV}$, $g_X = 0.9$)



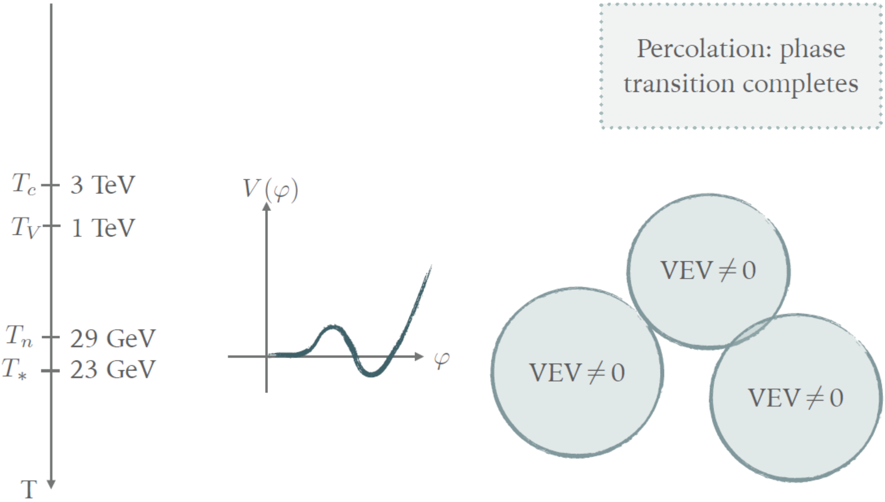


vacuum domination begins

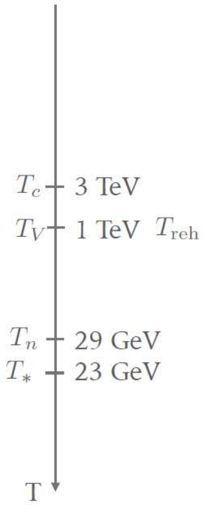
Temperature Evolution ($M_X = 9$ TeV, $g_X = 0.9$)



Temperature Evolution ($M_X = 9\ \text{TeV}$, $g_X = 0.9$)



Temperature Evolution ($M_X = 9\ \mathrm{TeV}$, $g_X = 0.9$)

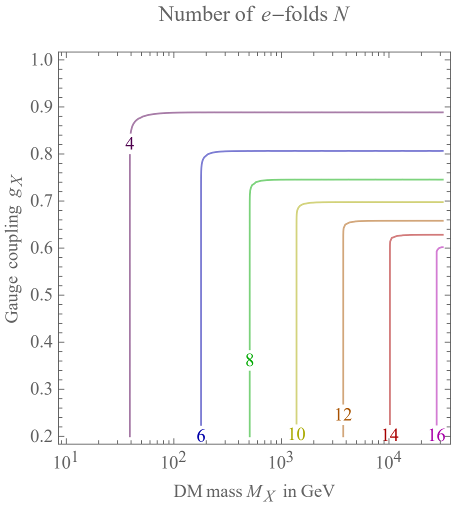


reheating

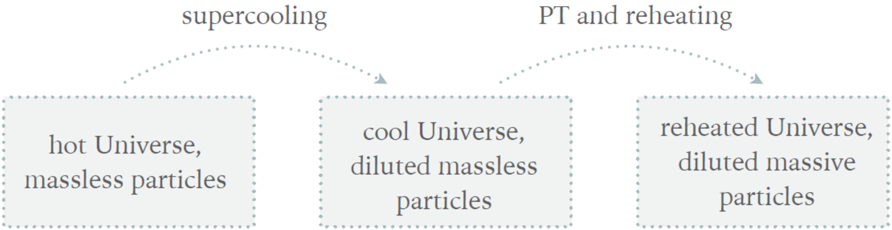
$$\alpha = \frac{\Delta V}{\text{energy of radiation}} \approx 4 \cdot 10^6$$

Thermal Inflation

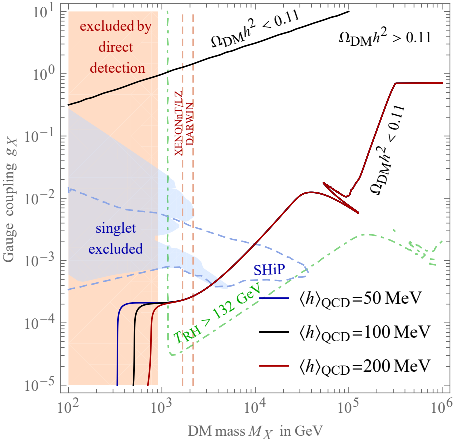
T. Hambye, A. Strumia, D. Teresi: 1805.01473



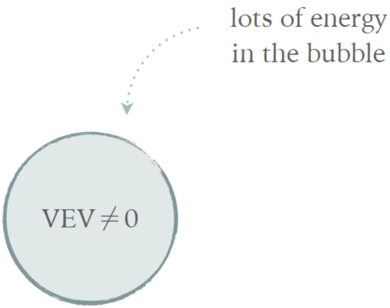
Supercooling



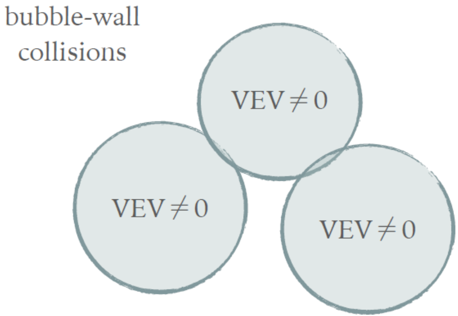
T. Hambye, A. Strumia, D. Teresi: 1805.01473



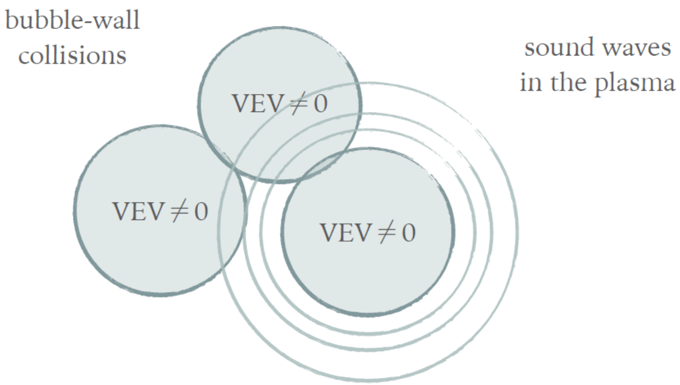
Bubbles



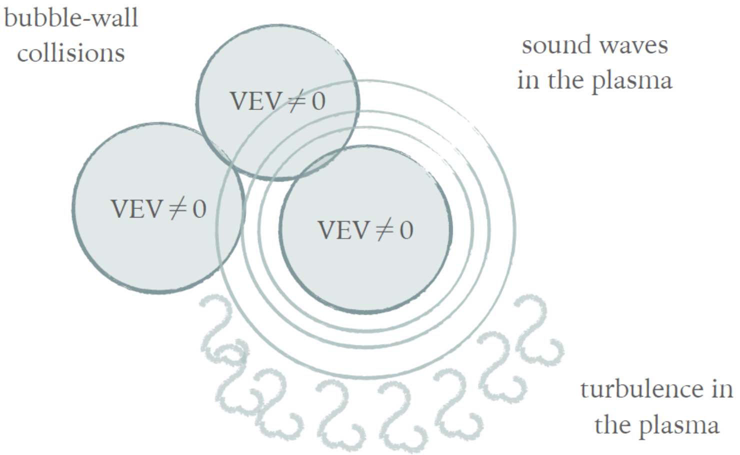
More Bubbles



Noisy Bubbles

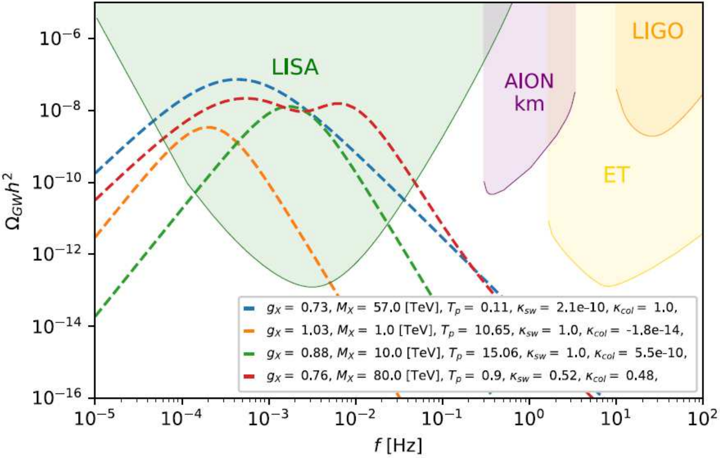


Turbulent Bubbles



Gravitational Waves Spectra

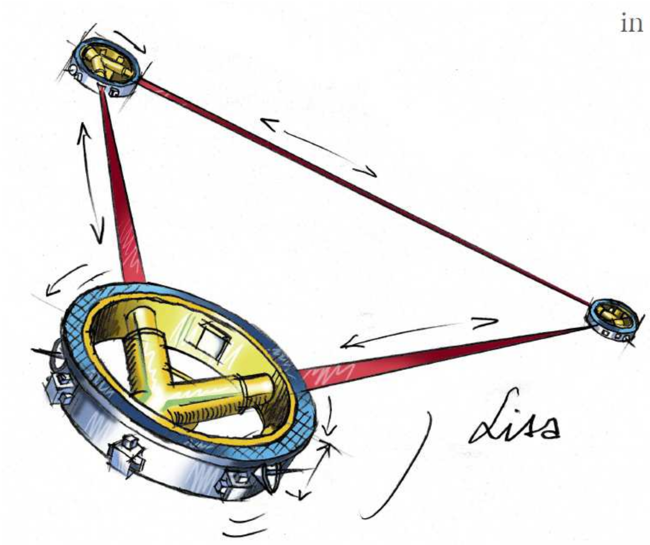
AK, M. Kierkla, B. Swiezewska: 2209.XXXXX



[sensitivity curves courtesy M. Lewicki]

LISA - Laser Interferometer Space Antenna

in the 2030's



[Image credit: ESA-C. Vijoux]

Minimal Inflation

- Inflation solves the **horizon** and **flatness** problems.
- When treated quantum-mechanically, it can also provide a mechanism for the generation of the perturbations that have resulted in the anisotropies observed in the CMB.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \qquad (M_{\mathrm{Pl}}^2 \equiv 1)$$

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{1}{3}\left[\frac{\dot{\phi}^2}{2} + V\right]$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2$$

Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Slow-roll Approximation (HSRPs)

Slow-roll approximation:

$$V(\phi) \gg \dot{\phi}^2 \,, \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|, \, |V'|$$

First **Hubble slow-roll parameter** (HSRP)

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V} \,, \quad \frac{\ddot{a}}{a} = H^2(1 - \epsilon_H)$$

Inflation ends **exactly** when $\epsilon_H = 1$.

Second HSRP

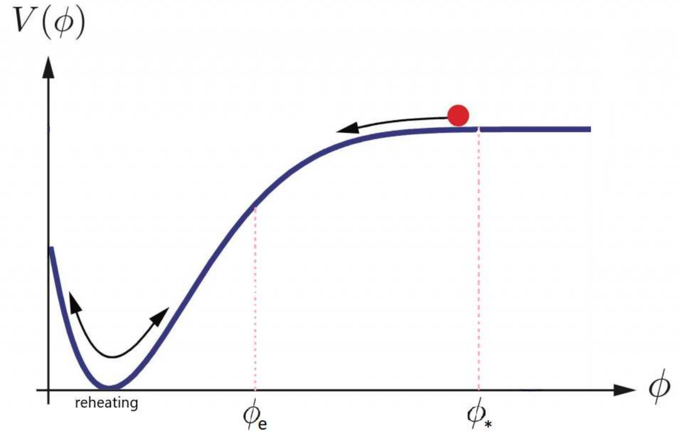
$$\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

Friedmann and Klein-Gordon equations become

$$H^2 \approx \frac{1}{3} V(\phi) \,, \quad \dot{\phi} \approx -\frac{V'}{3H} \,.$$

The shape of the potential is encoded in the **potential slow-roll parameters**

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta_V = \frac{V''}{V}$$



Number of e-folds and Inflationary Observables

The scalar curvature power spectrum is observed to have a power-law form

$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} , \quad A_s = \frac{1}{24\pi^2} \frac{V(\phi_*)}{\epsilon_V(\phi_*)} \simeq 2.1 \times 10^{-9} \;\; @ \, k_* = 0.05 \, \mathrm{Mpc}^{-1}$$

The spectral tilt is

$$n_s - 1 \equiv \frac{\mathrm{d} \ln \mathcal{P}_\zeta(k)}{\mathrm{d} \ln k} \simeq -6\epsilon_V + 2\eta_V \, ,$$

Tensor power spectrum

$$\mathcal{P}_T = 8 \left(\frac{H}{2\pi}\right)^2 \simeq \frac{2V}{3\pi^2}$$

The tensor-to-scalar ratio is

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \simeq 16\epsilon_V$$

Number of e-folds

$$N(\phi) = \int_t^{t_\mathrm{end}} H \mathrm{d}t = \int_{\phi_\mathrm{end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_H}} \approx \int_{\phi_\mathrm{end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V}} \sim 50 - 60$$

Inflationary Observables up to 3rd Order in Slow Roll

PHYSICAL REVIEW D **96**, 064036 (2017)

Frame-dependence of higher-order inflationary observables in scalar-tensor theories

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(Received 13 July 2017; published 21 September 2017)

In the context of scalar-tensor theories of gravity we compute the third-order corrected spectral indices in the slow-roll approximation. The calculation is carried out by employing the Green's function method for scalar and tensor perturbations in both the Einstein and Jordan frames. Then, using the interrelations between the Hubble slow-roll parameters in the two frames we find that the frames are equivalent up to third order. Since the Hubble slow-roll parameters are related to the potential slow-roll parameters, we express the observables in terms of the latter which are manifestly invariant. Nevertheless, the same inflaton excursion leads to different predictions in the two frames since the definition of the number of e-folds differs. To illustrate this effect we consider a nonminimal inflationary model and find that the difference in the predictions grows with the nonminimal coupling, and it can actually be larger than the difference between the first and third order results for the observables. Finally, we demonstrate the effect of various end-of-inflation conditions on the observables. These effects will become important for the analyses of inflationary models in view of the improved sensitivity of future experiments.

DOI: 10.1103/PhysRevD.96.064036

Inflationary Observables up to 3rd Order in the PSRPs

1707.00984: Scalar spectral index:

$$n_s = 1 - 6\epsilon_V + 2\eta_V + \left(24\alpha - \frac{10}{3}\right)\epsilon_V^2 - (16\alpha + 2)\,\epsilon_V\eta_V + \frac{2}{3}\eta_V^2 + \left(2\alpha + \frac{2}{3}\right)\zeta_V^2$$
$$- \left(90\alpha^2 - \frac{104}{3}\alpha + \frac{3734}{9} - \frac{87\pi^2}{2}\right)\epsilon_V^3 + \left(90\alpha^2 + \frac{4}{3}\alpha + \frac{1190}{3} - \frac{87\pi^2}{2}\right)\epsilon_V^2\eta_V$$
$$- \left(16\alpha^2 + 12\alpha + \frac{742}{9} - \frac{28\pi^2}{3}\right)\epsilon_V\eta_V^2 - \left(12\alpha^2 + 4\alpha + \frac{98}{3} - 4\pi^2\right)\epsilon_V\zeta_V^2$$
$$+ \left(\alpha^2 + \frac{8}{3}\alpha + \frac{28}{3} - \frac{13\pi^2}{2}\right)\eta_V\zeta_V^2 + \frac{4}{9}\eta_V^3 + \left(\alpha^2 + \frac{2}{3}\alpha + \frac{2}{9} - \frac{\pi^2}{12}\right)\rho_V^3$$

Tensor-to-scalar ratio:

$$r = 16\epsilon_V \left[1 - \left(4\alpha + \frac{4}{3}\right)\epsilon_V + \left(2\alpha + \frac{2}{3}\right)\eta_V + \left(16\alpha^2 + \frac{28}{3}\alpha + \frac{356}{9} - \frac{14\pi^2}{3}\right)\epsilon_V^2\right.$$
$$- \left(14\alpha^2 + 10\alpha + \frac{88}{3} - \frac{7\pi^2}{2}\right)\epsilon_V\eta_V + \left(2\alpha^2 + 2\alpha + \frac{41}{9} - \frac{\pi^2}{2}\right)\eta_V^2$$
$$\left. + \left(\alpha^2 + \frac{2}{3}\alpha + \frac{2}{9} - \frac{\pi^2}{12}\right)\zeta_V^2\right]$$

Chaotic Inflation

The potential is given by

$$V(\phi) = \lambda_n \phi^n \,.$$

The first two PSRPs are easily computed to be

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{n^2}{2}\frac{1}{\phi^2}\,, \qquad \eta_V = \frac{V''}{V} = n\,(n-1)\,\frac{1}{\phi^2}\,.$$

In the 1st order SR approximation, inflation ends when $\epsilon_H \simeq \epsilon_V = 1$, ergo $\phi_{\rm end} = n/\sqrt{2}$.
Number of e-folds

$$N(\phi_*) = \int_{\phi_{\rm end}}^{\phi_*} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V}} = \frac{\phi_*^2}{2n} - \frac{n}{4} \quad \rightarrow \quad \phi_*^2 = 2nN_* + \frac{n^2}{2}$$

Then

$$n_s = 1 - \frac{2n+4}{4N_*+1}\,, \qquad r = \frac{16n}{4N_*+1}$$

Let us now consider $N_* = 60$ and take the quadratic potential $V = \frac{1}{2}m^2\phi^2$. We find

$$n_s \simeq 0.97\,, \qquad r \simeq 0.13\,.$$

Similarly, for the quartic potential $V = \frac{1}{4}\lambda\phi^4$ we find

$$n_s \simeq 0.95\,, \qquad r \simeq 0.26\,.$$

Starobinsky Inflation

A simple extension of the Einstein-Hilbert action (Starobinsky, 1980):

$$S_{\text{Star.}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(R+\alpha R^2\right)\,, \qquad M_{\text{Pl}}^2 \equiv 1\,,$$

which belongs to the general class of $F(R)$ theories

$$S_F = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,F(R) \quad \rightarrow \quad S[g_{\mu\nu},\chi] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left[F'(\chi)(R-\chi)+F(\chi)\right]$$

After a Weyl rescaling of the metric $g_{\mu\nu}$ and a field redefinition

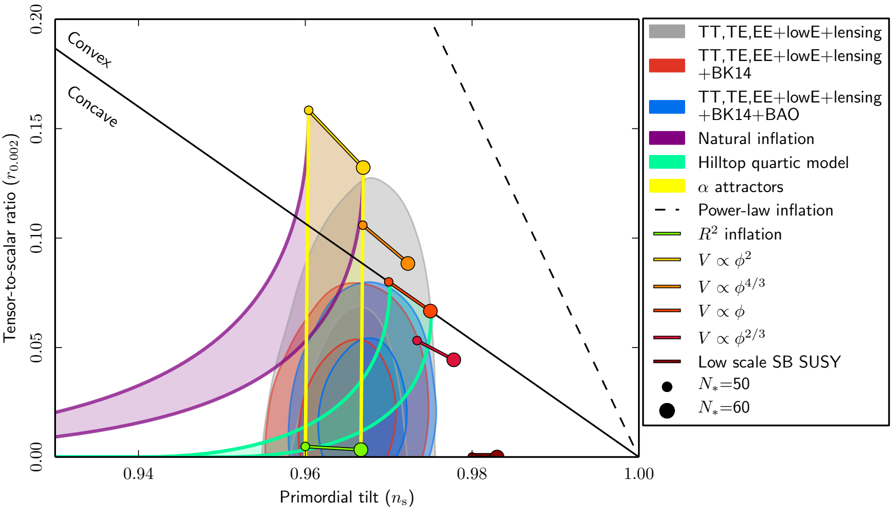
$$S[g_{\mu\nu},\varphi] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}R - \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right]\,,$$

where $V = \frac{1}{2}\frac{\chi F'(\chi)-F(\chi)}{F'(\chi)^2}$, $F'(\chi) = \exp\left(\sqrt{\frac{2}{3}}\varphi\right)$, $\varphi = \sqrt{\frac{3}{2}}\ln F'(\chi)$

For the $\left(R+\alpha R^2\right)$ model, $V(\varphi) = \frac{1}{8\alpha}\left[1-\exp\left(-\sqrt{\frac{2}{3}}\varphi\right)\right]^2$, we find for $N_* = 60$

$$n_s = 1-\frac{2}{N_*} = 0.9667\,, \qquad r = \frac{12}{N_*^2} = 0.0033$$

Planck 2018 Results



1807.06211, 2110.00483: $n_s = 0.9649 \pm 0.0042$ and $r < 0.036$

Dynamical generation of Planck scale and inflation

(Quasi-)scale-invariant potential 1502.01334, 1509.05423, 1710.04853, ..., 2006.09124

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\xi\phi^2 R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \qquad V(\phi) = \frac{1}{4}\lambda(\phi)\phi^4 + \Lambda^4$$

At the minimum

$$V(v) = \frac{1}{4}\lambda(v)v^4 + \Lambda^4 = 0\,, \quad v = \frac{M_P}{\sqrt{\xi}}$$

Minimization: $\beta(v) + 4\lambda(v) = 0$. This implies

a) $\beta(v) > 0,\ \lambda(v) < 0$

b) $\beta(v) \ = \ \lambda(v) \ = \ 0$

Taylor expansion around the VEV

$$\lambda(\phi) = \lambda(v) + \beta(v)\ln\frac{\phi}{v} + \frac{1}{2!}\beta'(v)\ln^2\frac{\phi}{v} + \frac{1}{3!}\beta''(v)\ln^3\frac{\phi}{v} + \cdots\,,$$

For the two cases we get

$$\lambda^a(\phi) \simeq \lambda(v) + \beta(v)\ln\frac{\phi}{v}\,, \qquad V(\phi) = \Lambda^4\left\{1 + \left[4\ln\left(\frac{\phi}{v}\right) - 1\right]\frac{\phi^4}{v^4}\right\}$$

$$\lambda^b(\phi) \simeq \frac{\beta'(v)}{2}\ln^2\frac{\phi}{v}\,, \qquad V(\phi) = \frac{1}{8}\beta'\phi^4\ln^2\left(\frac{\phi}{v}\right)$$

Non-minimal Coleman-Weinberg inflation with an R^2 term

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An IOP and SISSA journal

Nonminimal Coleman-Weinberg inflation with an R^2 term

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Abstract. We extend the Coleman-Weinberg inflationary model where a scalar field ϕ is non-minimally coupled to gravity with the addition of the R^2 term. We express the theory in terms of two scalar fields and going to the Einstein frame we employ the Gildener-Weinberg formalism, compute the one-loop effective potential and essentially reduce the problem to the case of single-field inflation. It turns out that there is only one free parameter, namely, the mixing angle between the scalars. For a wide range of this angle, we compute the inflationary observables which are in agreement with the latest experimental bounds. The effect of the R^2 term is that it lowers the value of the tensor-to-scalar ratio r.

Keywords: inflation, modified gravity, alternatives to inflation

JCAP02(2019)006

Non-minimal Coleman-Weinberg inflation with an R^2 term

A simple extension of the non-minimal Coleman-Weinberg inflationary model (1810.12884):

$$S^J = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left[\frac{\xi\phi^2}{2}\bar{R} + \frac{\alpha}{2}\bar{R}^2 - \frac{1}{2}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi - \frac{\lambda_\phi}{4}\phi^4\right],$$

Introducing an auxiliary scalar field χ:

$$S^J = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left[\frac{1}{2}\left(\xi\phi^2 + \alpha\chi^2\right)\bar{R} - \frac{\alpha}{8}\chi^4 - \frac{1}{2}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi - \frac{\lambda_\phi}{4}\phi^4\right].$$

After a Weyl rescaling of the metric

$$g_{\mu\nu} = \Omega^2\bar{g}_{\mu\nu}\,, \qquad \Omega^2 = \left(\alpha\chi^2 + \xi\phi^2\right)/M_{\mathrm{Pl}}^2 \equiv \frac{\zeta^2}{6M_{\mathrm{Pl}}^2}\,,$$

The action in the Einstein frame takes the form

$$S^E = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{6M_{\mathrm{Pl}}^2}{\zeta^2}\left(\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi + \frac{1}{2}\nabla^\mu\zeta\nabla_\mu\zeta\right) - V^{(0)E}(\phi,\zeta)\right],$$

where the tree-level potential becomes

$$V^{(0)E}(\phi,\zeta) = \frac{36M_{\mathrm{Pl}}^4}{\zeta^4}\left[\frac{\lambda_\phi}{4}\phi^4 + \frac{1}{8\alpha}\left(\frac{\zeta^2}{6} - \xi\phi^2\right)^2\right].$$

Gildener Weinberg approach

Due to the running of the couplings, the tree-level potential is flat at some renormalization scale $\Lambda_{\rm GW}$. Including the one-loop corrections, the effective potential obtains a radial shape along the flat direction and a non-zero VEV is dynamically generated (E. Gildener & S. Weinberg, 1976):

$$\frac{\mathrm{d}V^{(0)E}}{\mathrm{d}\phi}\bigg|_{\phi=v_\phi} = \frac{\mathrm{d}\,V^{(0)E}}{\mathrm{d}\zeta}\bigg|_{\zeta=v_\zeta} = 0\,, \qquad \Rightarrow \qquad v_\phi^2 = \frac{\xi}{6\left(\xi^2+2\alpha\lambda_\phi\right)} v_\zeta^2$$

Mass matrix diagonalization

$$\begin{pmatrix} \phi \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} s \\ \sigma \end{pmatrix}\,, \qquad \tan\omega = \frac{v_\zeta}{v_\phi}\,, \qquad v_s = \frac{v_\phi}{\cos\omega} = \frac{v_\zeta}{\sin\omega}$$

The mass eigenvalues are

$$m_s^2 = 0\,, \qquad m_\sigma^2 = \frac{\xi\left(\xi + 12\lambda_\phi\alpha + 6\xi^2\right)}{6\alpha\left(2\lambda_\phi\alpha+\xi^2\right)} M_{\rm Pl}^2$$

One-loop correction

$$V^{(1)} = \frac{m_\sigma^4}{64\pi^2 v_s^4} s^4 \left[\log\left(\frac{s^2}{v_s^2}\right) - \frac{1}{2}\right]\,, \qquad v_s^2 = v_\phi^2 + v_\zeta^2\,, \qquad v_\zeta^2 = 6M_{\rm Pl}^2$$

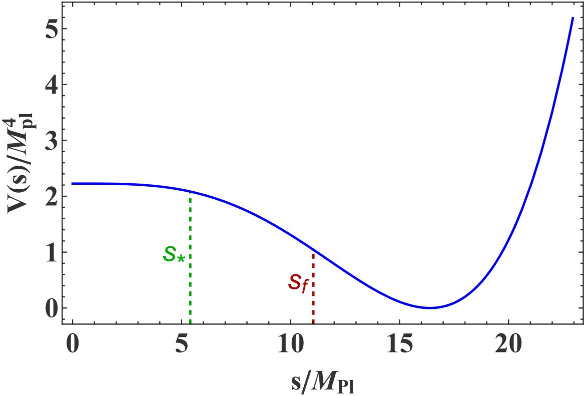
One-loop potential

We require the vanishing of the one-loop effective potential at the minimum

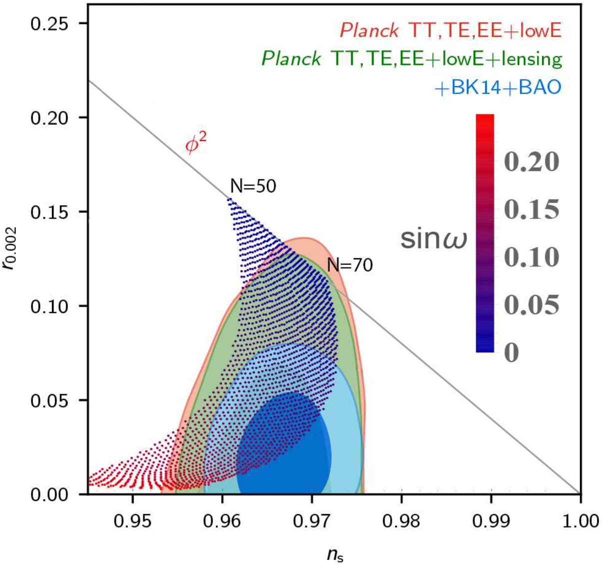
$$V(v_s) \equiv V^{(0)E}(v_s) + V^{(1)}(v_s) = 0$$

The total one-loop effective potential along the flat direction is given by

$$V(s) = \frac{m_\sigma^4}{128\pi^2}\left[\frac{\sin^2\omega}{36 M_{\rm Pl}^4} s^4 \left(2\ln\left[\frac{s^2\sin^2\omega}{6M_{\rm Pl}^2}\right] - 1\right) + 1\right], \qquad m_s^2 = \frac{\sin^2\omega}{48\pi^2}\frac{m_\sigma^4}{M_{\rm Pl}^2}$$



Inflationary predictions



Summary and Conclusions

- CSI models are minimal extensions of the SM that can solve many of its problems
- All mass scales are dynamically generated
- Scale invariance guarantees a strong first-order phase transition
- Leads to supercooling
- And potentially observable gravitational waves
- Viable inflation with the R^2 term

Thank you! ☺

Metric vs. Palatini

- In **metric formulation**, the metric is the only dynamical degree of freedom and the connection is always the Levi-Civita

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1+\xi\phi^2}{2} g^{\mu\nu} R_{\mu\nu}\left(g, \partial g, \partial^2 g\right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right)$$

- In **Palatini formulation**, both the metric and the connection are independent dynamical degrees of freedom

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1+\xi\phi^2}{2} g^{\mu\nu} R_{\mu\nu}\left(\Gamma, \partial \Gamma\right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right)$$

Scalar-Tensor Gravity: Metric vs. Palatini

$$S_J = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}A(\phi)g^{\mu\nu}R_{\mu\nu}(\Gamma) - \frac{1}{2}B(\phi)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right)$$

Variation with respect to Γ gives

$$\Gamma^\lambda_{\alpha\beta} = \left\{{\lambda \atop \alpha\beta}\right\} + (1-\kappa)\left[\delta^\lambda_\alpha\partial_\beta\omega(\phi) + \delta^\lambda_\beta\partial_\alpha\omega(\phi) - g_{\alpha\beta}\partial^\lambda\omega(\phi)\right], \quad \omega(\phi) = \ln\sqrt{A(\phi)}$$

where $\kappa = 1$ in metric and $\kappa = 0$ in Palatini. Performing a Weyl transformation

$$\bar{g}_{\mu\nu} \equiv A(\phi)g_{\mu\nu} \quad \to \quad \sqrt{-g} = A^{-2}\sqrt{-\bar{g}}, \quad R = A\left(\bar{R} - \kappa \times 6\,A^{1/2}\bar{\nabla}^\mu\bar{\nabla}_\mu A^{-1/2}\right),$$

the action becomes

$$S_E = \int \mathrm{d}^4x\sqrt{-\bar{g}}\left(\frac{1}{2}\bar{R} - \frac{1}{2}\left(\frac{B}{A} + \kappa \times \frac{3}{2}\frac{A_{,\phi}}{A^2}\right)\bar{\nabla}_\mu\phi\bar{\nabla}^\mu\phi - \frac{V(\phi)}{A^2}\right)$$

Field redefinition $\phi = \phi(\chi)$ to make it canonical

$$\frac{\mathrm{d}\phi}{\mathrm{d}\chi} = \sqrt{\frac{A^2}{AB + \kappa \times \frac{3}{2}A_{,\phi}}}.$$

Final action

$$S_{\mathrm{E}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\bar{R} - \frac{1}{2}\bar{\nabla}_\mu\chi\bar{\nabla}^\mu\chi - U(\chi)\right), \quad U(\chi) = \frac{V(\phi(\chi))}{A^2(\phi(\chi))}$$

Higgs Inflation: Metric vs. Palatini

We consider the Higgs-like inflationary potential

$$V(\phi) = \frac{\lambda}{4}\phi^4\,, \qquad A(\phi) = 1 + \xi\phi^2\,, \qquad B(\phi) = 1$$

Canonical field redefinition gives

$$\phi(\chi) \simeq \frac{1}{\sqrt{\xi}}\exp\left(\sqrt{\frac{1}{6}}\chi\right) \quad (\text{Metric})\,, \qquad \phi(\chi) = \frac{1}{\sqrt{\xi}}\sinh(\sqrt{\xi}\chi) \quad (\text{Palatini})$$

The Einstein-frame potential in terms of χ can be expressed as

$$U(\chi) \simeq \frac{\lambda}{4\xi^2}\left(1 - \exp\left(-\sqrt{\frac{2}{3}}\,\chi\right)\right)^2\,, \quad (\text{Metric})\,,$$

$$U(\chi) = \frac{\lambda}{4\xi^2}\tanh^4\left(\sqrt{\xi}\chi\right)\,, \qquad (\text{Palatini})$$

$$n_s \simeq 1 - \frac{2}{N_*} + \frac{3}{2N_*^2}\,, \qquad r \simeq \frac{12}{N_*^2}\,, \qquad A_s \simeq \frac{\lambda N_*^2}{72\pi^2\xi^2} \quad (\text{Metric})\,,$$

$$n_s \simeq 1 - \frac{2}{N_*} - \frac{3}{8\xi N_*^2}\,, \qquad r \simeq \frac{2}{\xi N_*^2}\,, \qquad A_s \simeq \frac{\lambda N_*^2}{12\pi^2\xi} \quad (\text{Palatini})\,.$$

When Metric and Palatini yield the same observables

PHYSICAL REVIEW D **102,** 044029 (2020)

Equivalence of inflationary models between the metric and Palatini formulation of scalar-tensor theories

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With a scalar field nonminimally coupled to curvature, the underlying geometry and variational principle of gravity—metric or Palatini—becomes important and makes a difference, as the field dynamics and observational predictions generally depend on this choice. In the present paper, we describe a classification principle which encompasses both metric and Palatini models of inflation, employing the fact that inflationary observables can be neatly expressed in terms of certain quantities which remain invariant under conformal transformations and scalar field redefinitions. This allows us to elucidate the specific conditions when a model yields equivalent phenomenology in the metric and Palatini formalisms and also to outline a method how to systematically construct different models in both formulations that produce the same observables.

DOI: 10.1103/PhysRevD.102.044029

n_s and r are the same if $A(\phi)B(\phi) \propto (A'(\phi))^2$ and $V(\phi) \propto A(\phi)^2 \left(\ln \frac{A(\phi)}{A_0}\right)^2$

Non-minimal Derivative Coupling in Palatini

PHYSICAL REVIEW D **102,** 063522 (2020)

Palatini-Higgs inflation with nonminimal derivative coupling

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The predictions of standard Higgs inflation in the framework of the metric formalism yield a tensor-to-scalar ratio $r \sim 10^{-3}$ which lies well within the expected accuracy of near-future experiments $\sim 10^{-4}$. When the Palatini formalism is employed, the predicted values of r get highly suppressed $r \sim 10^{-12}$ and consequently a possible nondetection of primordial tensor fluctuations will rule out only the metric variant of the model. On the other hand, the extremely small values predicted for r by the Palatini approach constitute contact with observations a hopeless task for the foreseeable future. In this work, we propose a way to remedy this issue by extending the action with the inclusion of a generalized nonminimal derivative coupling term between the inflaton and the Einstein tensor of the form $m^{-2}(\phi)G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$. We find that with such a modification, the Palatini predictions can become comparable with the ones obtained in the metric formalism, thus providing ample room for the model to be in contact with observations in the near future.

DOI: 10.1103/PhysRevD.102.063522

Non-minimal Derivative Coupling in Palatini

Consider the action

$$S = \int \mathrm{d}^4x \sqrt{-\widetilde{g}} \left(\frac{f(\phi)}{2} \widetilde{\mathcal{R}} - \frac{1}{2} \widetilde{\nabla}_\mu \phi \widetilde{\nabla}^\mu \phi + \frac{1}{2m^2(\phi)} \widetilde{G}^{\mu\nu} \widetilde{\nabla}_\mu \phi \widetilde{\nabla}_\nu \phi - V(\phi) \right) ,$$

where $\widetilde{G}^{\mu\nu} = \widetilde{\mathcal{R}}^{\mu\nu} - \frac{1}{2} \widetilde{g}^{\mu\nu} \widetilde{\mathcal{R}}$. Using a **disformal** transformation

$$\widetilde{g}_{\mu\nu} = \Omega^2(\phi, X) \left[g_{\mu\nu} + \beta^2(\phi, X) \nabla_\mu \phi \, \nabla_\nu \phi \right] , \qquad X = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$$

eventually we find

$$S_E \simeq \int \mathrm{d}^4x \sqrt{-g} \left(\frac{\mathcal{R}}{2} - K(\phi) \frac{(\nabla\phi)^2}{2} + L(\phi) \frac{(\nabla\phi)^4}{4} - U(\phi) \right) ,$$

where

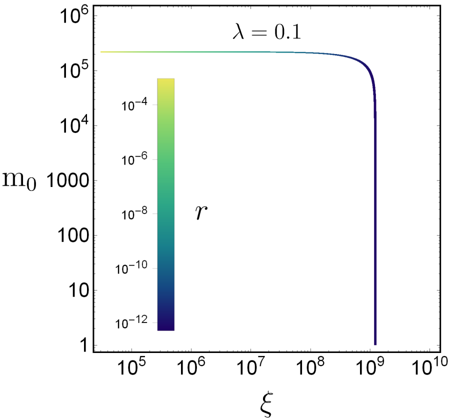
$$K(\phi) \equiv \frac{1}{f(\phi)} + \frac{U(\phi)}{m^2(\phi)} , \quad L(\phi) \equiv \frac{1}{m^2(\phi)} \left(\frac{1}{f(\phi)} + \frac{U(\phi)}{2m^2(\phi)} \right) , \quad U(\phi) \equiv \frac{V(\phi)}{f^2(\phi)} .$$

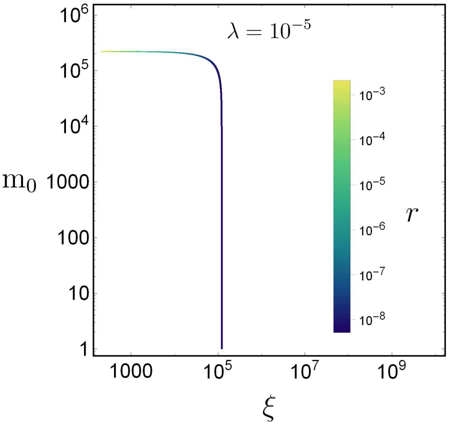
NMDC Palatini-Higgs Inflation

Field-dependent case: $m^2(\phi) = \phi^2/m_0^2$

The inflationary observables are obtained as

$$A_s \simeq \frac{\lambda N_*^2}{3\pi^2\,(m_0^2\lambda + 4\xi)}\,, \qquad n_s \simeq 1 - \frac{2}{N_*} - \frac{m_0^2\lambda + \xi}{8\xi^2 N_*^2}\,, \qquad r \simeq \frac{m_0^2\lambda + 4\xi}{2\xi^2 N_*^2}\,.$$





Palatini Inflation in Models with an R^2 Term

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Palatini inflation in models with an R^2 term

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Abstract. The Starobinsky model, considered in the framework of the Palatini formalism, in contrast to the metric formulation, does not provide us with a model for inflation, due to the absence of a propagating scalar degree of freedom that can play the role of the inflaton. In the present article we study the Palatini formulation of the Starobinsky model coupled, in general nonminimally, to scalar fields and analyze its inflationary behavior. We consider scalars, minimally or nonminimally coupled to the Starobinsky model, such as a quadratic model, the induced gravity model or the standard Higgs-like inflation model and analyze the corresponding modifications favorable to inflation. In addition we examine the case of a classically scale-invariant model driven by the Coleman-Weinberg mechanism. In the slow-roll approximation, we analyze the inflationary predictions of these models and compare them to the latest constraints from the Planck collaboration. In all cases, we find that the effect of the R^2 term is to lower the value of the tensor-to-scalar ratio.

Keywords: inflation, modified gravity, gravity, alternatives to inflation

ArXiv ePrint: 1810.10418

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Rescuing quartic and natural inflation in the Palatini formalism

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Abstract. When considered in the Palatini formalism, the Starobinsky model does not provide us with a mechanism for inflation due to the absence of a propagating scalar degree of freedom. By (non)-minimally coupling scalar fields to the Starobinsky model in the Palatini formalism we can in principle describe the inflationary epoch. In this article, we focus on the minimally coupled quartic and natural inflation models. Both theories are excluded in their simplest realization since they predict values for the inflationary observables that are outside the limits set by the Planck data. However, with the addition of the R^2 term and the use of the Palatini formalism, we show that these models can be rendered viable.

Keywords: inflation, modified gravity

ArXiv ePrint: 1812.00847

JCAP03(2019)005

Palatini inflation in models with an R^2 term

In 1810.10418, 1812.00847 we considered (see also Enckell et al.: 1810.05536)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\alpha}{2}R^2 + \frac{1}{2}A(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \quad R = g^{\mu\nu}R^\rho{}_{\mu\rho\nu}(\Gamma, \partial\Gamma)$$

Introducing an auxiliary scalar $\chi \equiv 2\alpha R$ and Weyl rescaling $g_{\mu\nu} \to \Omega^2 g_{\mu\nu} = [\chi + A(\phi)]g_{\mu\nu}$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\frac{1}{\chi + A(\phi)}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \hat{V}(\phi,\chi)\right],$$

with

$$\hat{V}(\phi,\chi) = \frac{1}{[\chi + A(\phi)]^2}\left[V(\phi) + \frac{\chi^2}{8\alpha}\right].$$

- No kinetic term has been generated for the field χ (scalaron in metric formalism)
- EOM of χ reduces to a constraint
- ϕ is the only propagating scalar DOF $\to$ inflaton

Palatini inflation in models with an R^2 term

Varying the action with respect to χ:

$$\delta_\chi S = 0 \to \chi = \frac{8\alpha V(\phi) + 2\alpha A(\phi)\,(\partial\phi)^2}{A(\phi) - 2\alpha\,(\partial\phi)^2}\,.$$

Substituting back

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}K(\phi)\,(\partial\phi)^2 + \frac{1}{4}L(\phi)\,(\partial\phi)^4 - \frac{\bar{U}}{1+8\alpha\bar{U}}\right]\,, \quad \bar{U}(\phi) \equiv \frac{V(\phi)}{[A(\phi)]^2}$$

with

$$K(\phi) \equiv \frac{1}{A(1+8\alpha\bar{U})}\,, \qquad L(\phi) = \frac{2\alpha}{A^2(1+8\alpha\bar{U})}\,.$$

Using $\left(\frac{\mathrm{d}\phi}{\mathrm{d}\zeta}\right)^2 = A(1+8\alpha\bar{U})$, we arrive at

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\,(\partial\zeta)^2 + \frac{\alpha}{2}\,\left(1+8\alpha\bar{U}(\zeta)\right)(\partial\zeta)^4 - U(\zeta)\right]\,,$$

$$\boxed{U \equiv \frac{\bar{U}}{1+8\alpha\bar{U}}}\,.$$

- Regardless of the shape of V, the αR^2 term decreases the height of the effective potential
- For large values of ϕ tends to a plateau $M_{\mathrm{P}}^4/8\alpha$
- The rate of change of the field is also modified

Slow-roll

In a flat FRW background

$$3H^2 = \frac{1}{2}[1 + 3\alpha(1 + 8\alpha\bar{U})\dot{\zeta}^2]\dot{\zeta}^2 + U$$

$$0 = [1 + 6\alpha(1 + 8\alpha\bar{U})\dot{\zeta}^2]\ddot{\zeta} + 3[1 + 2\alpha(1 + 8\alpha\bar{U})\dot{\zeta}^2]H\dot{\zeta} + 12\alpha^2\dot{\zeta}^4\bar{U}' + U'$$

Inflation takes place when

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\zeta}^2}{2H^2}\left[1 + 2\alpha\left(1 + 8\alpha\bar{U}\right)\dot{\zeta}^2\right] < 1\,.$$

First order expressions for observables:

$$24\pi^2 A_s = \frac{U}{\epsilon_U} = \frac{\bar{U}}{\epsilon_{\bar{U}}}\,, \qquad n_s = 1 - 6\epsilon_U + 2\eta_U = 1 - 6\epsilon_{\bar{U}} + 2\eta_{\bar{U}}\,,$$

Tensor-to-scalar ratio

$$r = 16\epsilon_U = \frac{\bar{r}}{1 + 8\alpha\bar{U}} = \frac{\bar{r}}{1 + 12\pi^2 A_s\bar{r}\alpha}$$

Sensitivity of future experiments will be approximately $\delta_r \approx 10^{-4}$.

Requiring near-future detectability means $\alpha < 4 \times 10^{10}$.

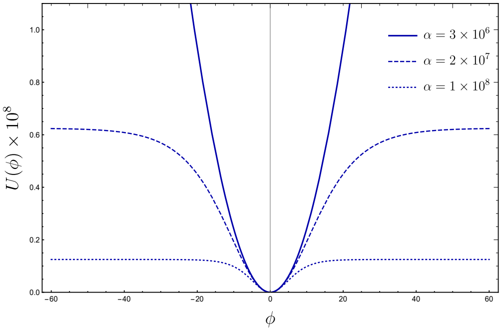
Example: Quadratic Inflation

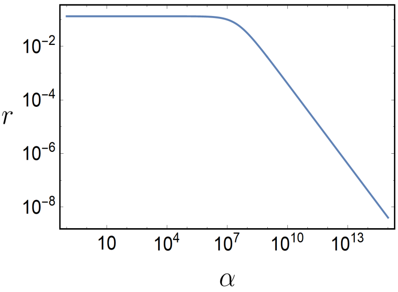
Consider the minimal quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ with $A(\phi) = 1$, (2102.02712)
The field redefinition and effective Einstein potential become

$$\chi = \frac{\sinh^{-1}(2m\sqrt{\alpha}\phi)}{2m\sqrt{\alpha}}, \qquad U = \frac{\tanh^2(2m\sqrt{\alpha}\chi)}{8\alpha}$$

and the observables

$$r = \frac{8}{N_* + 16\alpha m^2 N_*^2}, \qquad n_s \simeq 1 - \frac{2}{N_*}, \qquad A_s \simeq \frac{m^2 N_*^2}{6\pi^2}$$





Generalization to arbitrary $SU(N)$

SU(N) is broken completely by generic VEVs of $N-1$ fields Φ_i in the fundamental representation.

$$\Phi_1 = \begin{pmatrix} 0 \\ 0 \\ \ldots \\ 0 \\ \phi_1 \end{pmatrix} \ , \ \Phi_2 = \begin{pmatrix} 0 \\ 0 \\ \ldots \\ \phi_2^{(1)} \\ \phi_2^{(2)} e^{i\chi_2} \end{pmatrix} \ , \ \ldots \ , \ \Phi_{N-1} = \begin{pmatrix} 0 \\ \phi_{N-1}^{(1)} \\ \ldots \\ \phi_{N-1}^{(N-2)} e^{i\chi_{N-1}^{(N-3)}} \\ \phi_{N-1}^{(N-1)} e^{i\chi_{N-1}^{(N-2)}} \end{pmatrix}$$

The transformation properties of the gauge fields are identified with those of the corresponding SU(N) generators. The basis of the $N(N-1)$ off--diagonal generators $T^{ab}, \tilde{T}^{ab}$ can be chosen as

$$(T^{ab})_{ij} = \delta_{ia}\delta_{jb} + \delta_{ib}\delta_{ja} \ ,$$
$$(\tilde{T}^{ab})_{ij} = -i\delta_{ia}\delta_{jb} + i\delta_{ib}\delta_{ja} \ ,$$

where $a = 1, .., N-1$ and $b = 2, .., N$.

Generalization to arbitrary $SU(N)$[1]

With the Cartan generators denoted by H^α, the Z_2 associated with complex conjugation of the group elements acts as

$$T^{ab} \to -T^{ab} \;\;,\;\; \tilde{T}^{ab} \to \tilde{T}^{ab} \;\;,\;\; H^\alpha \to -H^\alpha$$

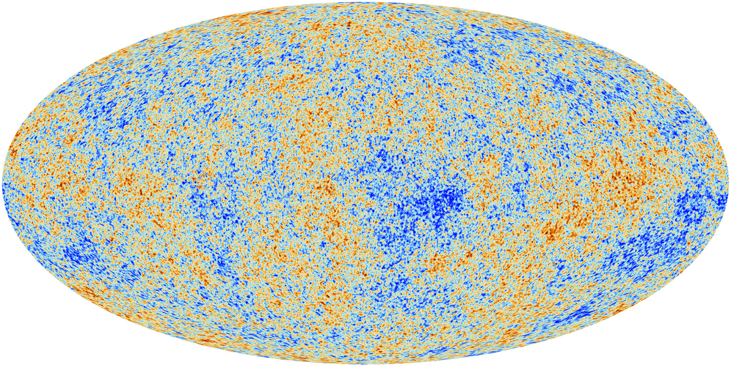
This is an **outer automorphism** of SU(N) which entails the corresponding symmetry of the Yang--Mills Lagrangian.
Another Z_2 can be defined by reflecting the off--diagonal generators containing nonzero elements in the first row:

$$\begin{aligned}
&T^{1a} \to -T^{1a} \;\;,\;\; \tilde{T}^{1a} \to -\tilde{T}^{1a} \;\;, \\
&T^{bc} \to T^{bc} \quad\;\;,\;\; \tilde{T}^{bc} \to \tilde{T}^{bc} \quad (b, c \geq 2), \\
&H^\alpha \to H^\alpha
\end{aligned}$$

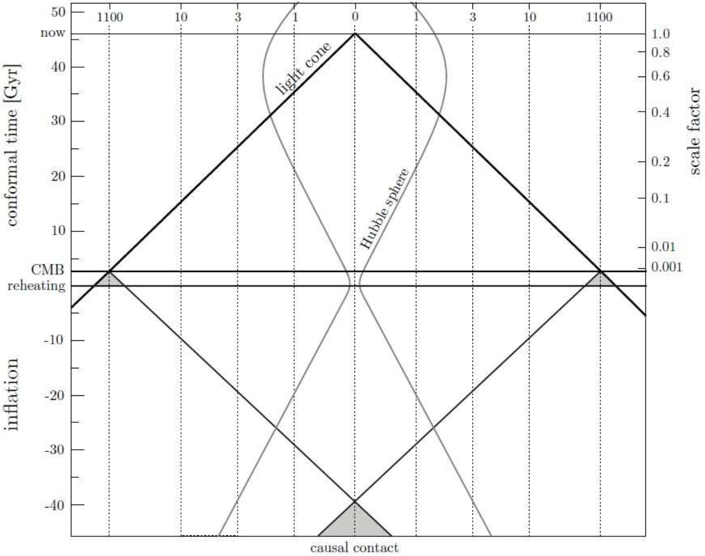
This Z_2 is an **inner automorphism**.

[1] Gross et al.: JHEP 1508 (2015) 158

Cosmic Microwave Background Radiation



Horizon Problem



Flatness Problem

The Friedmann equation (with zero cosmological constant),

$$H^2 = \frac{1}{3}\rho - \frac{\mathcal{K}}{a^2}\,, \qquad \Rightarrow \quad 1 - \Omega(t) = \frac{-\mathcal{K}}{(aH)^2}$$

For some earlier time t_i we can write

$$1 - \Omega(t_i) = [1 - \Omega(t_0)] \left(\frac{\dot{a}(t_0)}{\dot{a}(t_i)} \right)^2 .$$

If we go back to the Planck time, $t_{\rm Pl} \sim 5 \times 10^{-44}{\rm s}$, we find

$$1 - \Omega(t_{\rm Pl}) < 10^{-64} .$$

During inflation, the Hubble parameter H_I is almost constant and the scale factor grows as

$$a(t) \simeq a_{\rm end} \exp\left[H_I\left(t - t_{\rm end}\right)\right] = \exp\left[-N(t)\right]\,, \quad 1 - \Omega \propto e^{-2N} \to 0$$

Mukhanov-Sasaki Equation

The evolution of linear (tensor and scalar) curvature cosmological perturbations in a flat FLRW background and in the presence of a scalar inflaton field is governed by the *Mukhanov-Sasaki equation* (MSE) which reads

$$\frac{\mathrm{d}^2\nu}{\mathrm{d}\tau^2} + \left(k^2 - \frac{1}{z}\frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2}\right)\nu = 0, \qquad \nu \equiv z\mathcal{R}_k$$

For tensor and scalar perturbations:

$$z = \frac{\bar{a}}{\sqrt{\mathcal{I}_m}} = \hat{a}, \qquad z = \sqrt{\frac{2}{\mathcal{I}_m}}\frac{\bar{a}}{\bar{H}\left(1-\bar{\lambda}_0\right)}\frac{\mathrm{d}\mathcal{I}_\phi}{\mathrm{d}\bar{t}} = \sqrt{2}\frac{\hat{a}}{\hat{H}}\frac{\mathrm{d}\mathcal{I}_\phi}{\mathrm{d}\hat{t}}.$$

Asymptotic solutions for ν (subhorizon and superhorizon limit)

$$\nu \to \begin{cases} \frac{1}{\sqrt{2k}}e^{-ik\tau} & \text{as} \quad -k\tau \to \infty, \\ A_k z & \text{as} \quad -k\tau \to 0. \end{cases}$$

Green's Function Method

By introducing the dimensionless variable $x \equiv -k\tau$ and redefining the field as $y \equiv \sqrt{2k}\nu$, the asymptotic solutions become

$$y \to \begin{cases} e^{-ix} & \text{as} \quad x \to \infty, \\ \sqrt{2k}A_k z & \text{as} \quad x \to 0. \end{cases} .$$

Also, by assuming the following ansatz for z:

$$z = \frac{1}{x} f(\ln x),$$

we can recast the MSE in the form

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{1}{x^2} g(\ln x) y,$$

where the function g is defined through

$$g(\ln x) = \frac{1}{f(\ln x)} \left[-3 \frac{\mathrm{d} f(\ln x)}{\mathrm{d} \ln x} + \frac{\mathrm{d}^2 f(\ln x)}{\mathrm{d}(\ln x)^2}\right].$$

Green's Function Method

The homogeneous solution with the appropriate asymptotic behavior at $x \to \infty$ is

$$y_0(x) = \left(1 + \frac{i}{x}\right) e^{ix}.$$

We can rewrite the MSE as an integral equation

$$y(x) = y_0(x) + \frac{i}{2} \int_x^{\infty} \mathrm{d}u \frac{1}{u^2} g(\ln u) y(u) \left[y_0^*(u) y_0(x) - y_0^*(x) y_0(u) \right]$$

Taylor-expanding xz around $x = 1$ in the following way:

$$xz = f(\ln x) = \sum_{n=0}^{\infty} \frac{f_n}{n!} (\ln x)^n, \qquad f_n = \frac{\mathrm{d}^n (xz)}{\mathrm{d}(\ln x)^n}$$

The second-order power spectrum is then given in terms of the coefficients f_0, f_1 and f_2 as

$$P(k) = \frac{k^2}{(2\pi)^2} \frac{1}{f_0^2} \left[1 - 2\alpha \frac{f_1}{f_0} + \left(3\alpha^2 - 4 + \frac{5\pi^2}{12} \right) \left(\frac{f_1}{f_0} \right)^2 + \left(-\alpha^2 + \frac{\pi^2}{12} \right) \frac{f_2}{f_0} \right]$$

Disformal transformation

After using the disformal transformation, the action can be written as

$$S_D = \int \mathrm{d}^4x\,\sqrt{-g}\Bigg[F_1(\phi,X)\frac{\mathcal{R}}{2} - F_2(\phi,X)\frac{(\nabla\phi)^2}{2} + F_3(\phi,X)\mathcal{R}_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi + F_4(\phi,X)\frac{(\nabla\phi)^4}{4} - F_5(\phi,X)\,V(\phi)\Bigg] .$$

The functions F_i are given by

$$\begin{aligned}
F_1(\phi,X) &= f(\phi)\Omega^2\sqrt{1+\varepsilon u^2} - \frac{1}{2m^2(\phi)}\frac{\varepsilon u^2/\beta^2}{\sqrt{1+\varepsilon u^2}}\,,\\
F_2(\phi,X) &= \Omega^2\sqrt{1+\varepsilon u^2}\,,\\
F_3(\phi,X) &= -\frac{f(\phi)}{2}\frac{\Omega^2\beta^2}{\sqrt{1+\varepsilon u^2}} + \frac{1}{4m^2(\phi)}\frac{2+\varepsilon u^2}{(1+\varepsilon u^2)^{3/2}}\,,\\
F_4(\phi,X) &= \frac{2\Omega^2\beta^2}{\sqrt{1+\varepsilon u^2}}\,,\\
F_5(\phi,X) &= \Omega^4\sqrt{1+\varepsilon u^2}\,,
\end{aligned}$$

where $\varepsilon u^2 = u_\mu u^\mu = \beta^2(\nabla\phi)^2$.

Solving the system

In order to obtain the action in the Einstein frame, we essentially have to solve the system

$$F_1(\phi, X) = 1 \qquad \text{and} \qquad F_3(\phi, X) = 0\,,$$

which results in obtaining the solutions for the transformation functions Ω^2 and β^2 as functions of the field and its velocity. The solution of the system is easily obtained and reads

$$\Omega^2 = \frac{2+\varepsilon u^2}{2f(\phi)\sqrt{1+\varepsilon u^2}} \qquad \text{and} \qquad \beta^2 = \frac{1}{m^2(\phi)\sqrt{1+\varepsilon u^2}}\,.$$

Then we obtain the Einstein-frame action

$$S_E = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{\mathcal{R}}{2} - \hat{F}_2(\phi, X)\frac{(\nabla\phi)^2}{2} + \hat{F}_4(\phi, X)\frac{(\nabla\phi)^4}{4} - \hat{F}_5(\phi, X)\,V(\phi)\right)\,,$$

with

$$\begin{aligned}
\hat{F}_2(\phi, X) &= \frac{2+\varepsilon u^2}{2f(\phi)}\,,\\
\hat{F}_4(\phi, X) &= \frac{2+\varepsilon u^2}{f(\phi)m^2(\phi)(1+\varepsilon u^2)^{3/2}}\,,\\
\hat{F}_5(\phi, X) &= \frac{(2+\varepsilon u^2)^2}{4f^2(\phi)\sqrt{1+\varepsilon u^2}}\,.
\end{aligned}$$

Expanding in X

Using the canonical kinetic term X and substituting $u^2 = 2\beta^2 X$ in Ω^2, we find that

$$u^2(1-u^2)^{1/2} = \frac{2X}{m^2(\phi)}\,,$$

where we have used that $\varepsilon = -1$. Expanding in terms of X , we find that

$$u^2 \simeq \frac{2X}{m^2(\phi)}\left(1 + \frac{X}{m^2(\phi)}\right)\,.$$

Substituting above and keeping terms up to $\mathcal{O}\left(X^2\right)$ we obtain

$$\begin{aligned}
\hat{F}_2(\phi, X) &\simeq \frac{1}{f(\phi)}\left(1 - \frac{X}{m^2(\phi)} - \frac{X^2}{m^4(\phi)}\right)\,,\\
\hat{F}_4(\phi, X) &\simeq \frac{2}{f(\phi)m^2(\phi)}\left(1 + \frac{2X}{m^2(\phi)} + \frac{13X^2}{2m^4(\phi)}\right)\,,\\
\hat{F}_5(\phi, X) &\simeq \frac{1}{f^2(\phi)}\left(1 - \frac{X}{m^2(\phi)} - \frac{X^2}{2m^4(\phi)}\right)\,.
\end{aligned}$$