A PROFILE OF A HIGGS MECHANISM UNDER A UV AND QUANTUM, 1ST ORDER PHASE TRANSITION

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HR EXCELLENCE IN RESEARCH

CONTENTS

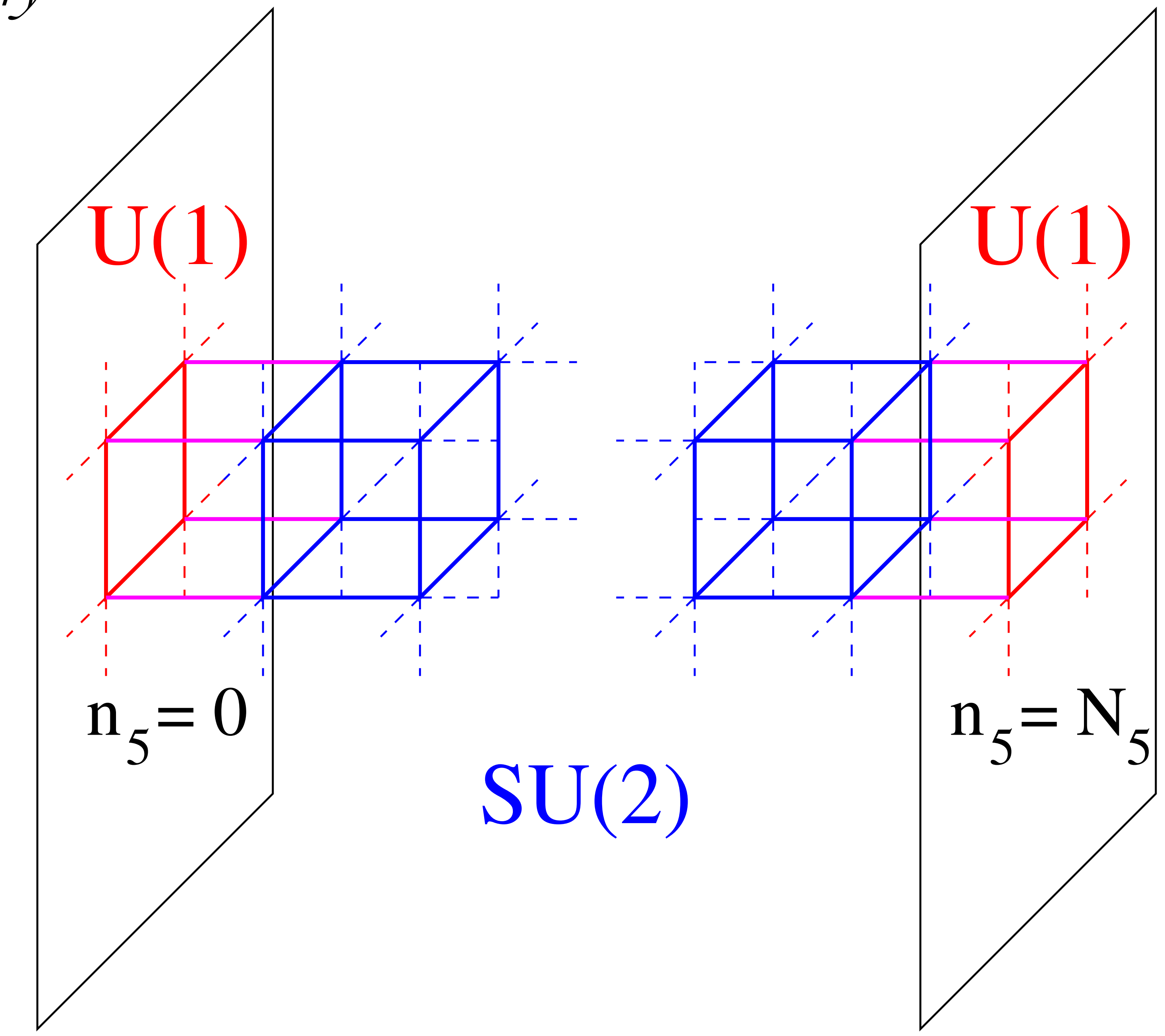
INTRO

- Ultimate goal is the proposal of a new approach to the Higgs-Hierarchy problem

- The Non-Perturbative Gauge-Higgs Unification (NPGHU) model:

1. *An anisotropic, in fifth dimension, lattice with orbifold boundary conditions generating a 4d boundary*

2. *A pure SU(2) gauge symmetry on the bulk, a U(1) gauge field coupled to a complex scalar survive on the boundary*



N. Irges and *F. Knechtli*, Nucl. Phys. B **719** (2005) 12
N. Irges and *F. Knechtli*, Nucl. Phys. B **775** (2007) 283
N. Irges, F. Knechtli and *K. Yoneyama*, Nucl. Phys. B **722** (2013) 378-383
M. Alberti, N. Irges, F. Knechtli and *G. Moir*, JHEP **09** (2015) 159
(After a lot of effort as you can see)

INTRO

- Construction of a 4d continuum effective action for a 5d model originated by the lattice model of NPGHU (What is the motivation?)

- A: The model exhibits, non-perturbatively, spontaneous breaking of its gauge symmetry in infinite fifth dimension (*Zero Temperature effect, dimensional reduction through localization*):

N. Irges and *F. Knechtli*, Nucl. Phys. B **719** (2005) 12
N. Irges and *F. Knechtli*, Nucl. Phys. B **775** (2007) 283
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- B: Three crucial characteristics:

1. *Even though extra dimensional, no finite-temperature type potential. No compactification, no Kaluza-Klein states*
2. *Pure bosonic nature of the Higgs mechanism. No need for fermions to trigger the mechanism*
3. *There are not any polynomial terms (not a Coleman-Weinberg (CW) like model) in the classical (nor in the (quantum) effective) potential*

↓

NPGHU model → Exhibits a pure quantum and bosonic spontaneous symmetry breaking

INTRO

- C: 1. *A non-perturbative (NP) new class of Higgs-type mechanisms*

  2. *The phase diagram of the lattice model exhibits a Higgs phase separated from two other phases by a1st order and "bulk" or "zero-temperature" or "quantum" phase transition:*

N. Irges and *F. Knechtli*, JHEP **06** (2014) 070; *M. Alberti, N. Irges, F. Knechtli* and *G. Moir*, JHEP **09** (2015) 159

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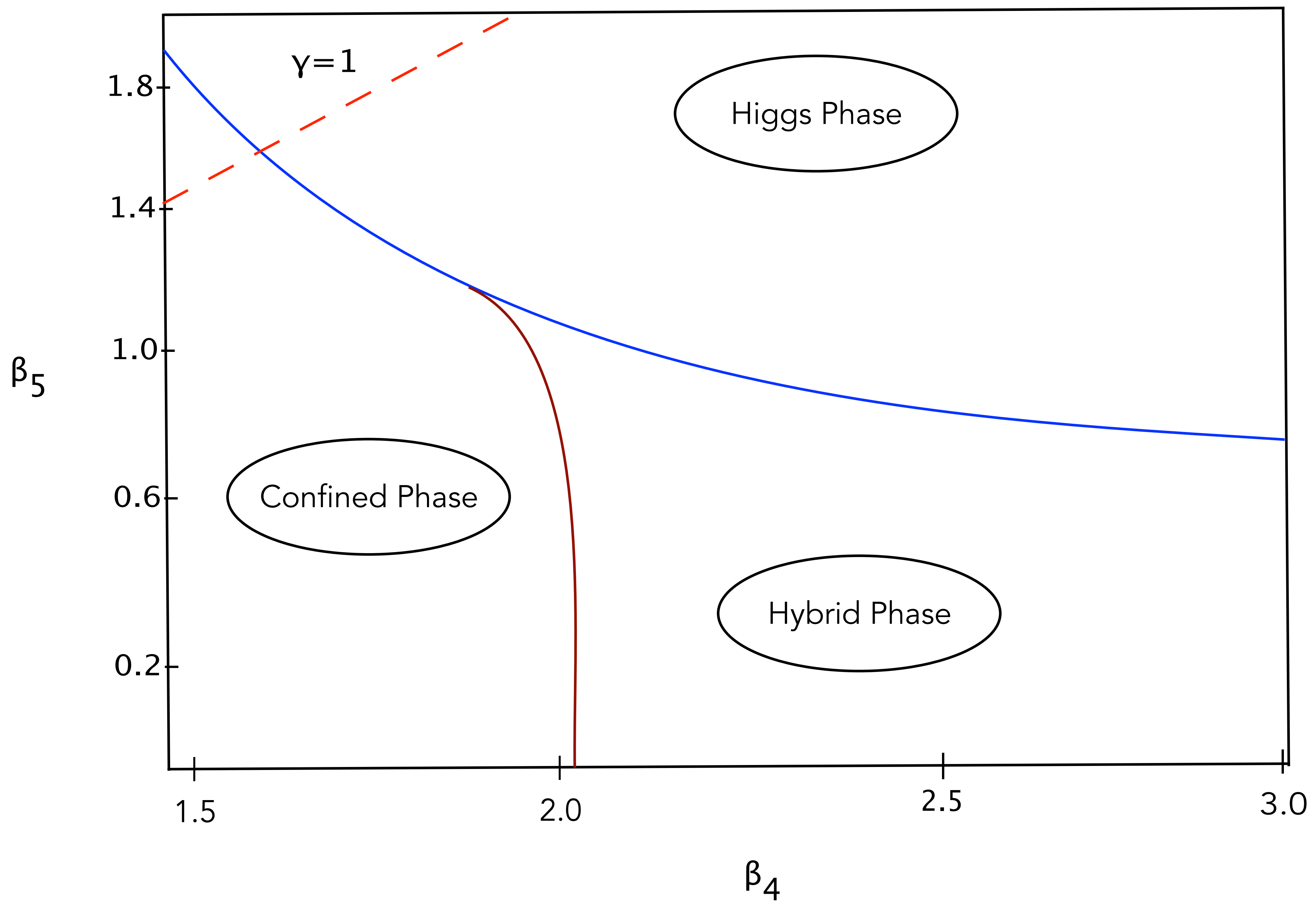


The Phase Diagram of the anisotropic orbifold lattice.

QUANTIZATION WITH HDO

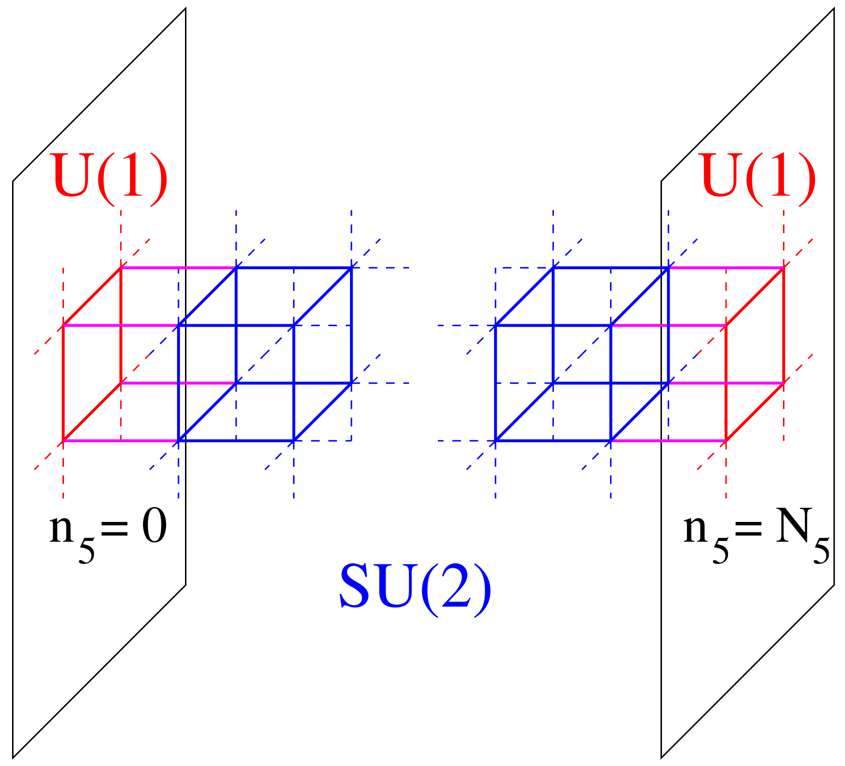
- What is the action to be quantized? Start from the lattice plaquette action $S^{\text{orb}} = S^{\text{b-h}} + S^{B}$

  The boundary action $S^{\text{b-h}}$

$$S^{\text{b-h}} = \frac{1}{2N} \sum_{n_\mu} \left[\frac{\beta_4}{2} \sum_{\mu<\nu} \text{tr} \left\{ 1 - U^b_{\mu\nu}(n_\mu, 0) \right\} + \beta_5 \sum_{\mu} \text{tr} \left\{ 1 - U^h_{\mu 5}(n_\mu, 0) \right\} \right]$$

  The bulk action S^B

$$S^B = \frac{1}{2N} \sum_{n_\mu, n_5} \left[\beta_4 \sum_{\mu<\nu} \text{tr} \left\{ 1 - U_{\mu\nu}(n_\mu, n_5) \right\} + \beta_5 \sum_{\mu} \text{tr} \left\{ 1 - U_{\mu 5}(n_\mu, n_5) \right\} \right]$$



- The parameters of the model $\beta_4 = \dfrac{4a_5}{g_5^2} = \dfrac{4}{g_4^2}$, $\beta_5 = \dfrac{4a_4^2}{a_5 g_5^2} = \dfrac{4a_4^2}{a_5^2 g_4^2}$, $\gamma = \dfrac{a_4}{a_5}$, $g_4^2 = \dfrac{g_5^2}{a_5} = \dfrac{g_5^2}{a_4}\gamma$

- Expanding w.r.t the lattice spacings and truncate at NLO in the expansion

$$S^{\text{b-h}} = \sum_{n_\mu} a_4^4 \sum_{\mu} \left[\sum_{\nu} \left(\frac{1}{4} F^3_{\mu\nu} F^3_{\mu\nu} + \frac{1}{16} a_4^2 (\hat{\Delta}_\mu F^3_{\mu\nu})(\hat{\Delta}_\mu F^3_{\mu\nu}) \right) + |\hat{D}_\mu \phi|^2 + \frac{a_4^2}{4} |\hat{D}_\mu \hat{D}_\mu \phi|^2 \right]$$

- Consider the naive continuum limit and go to Minkowski space with metric $\eta_{\mu\nu} = (+,-,-,-)$ to get the boundary effective action

$$S^{\text{b-h}} = \int d^4x \left[-\frac{1}{4} F^3_{\mu\nu} F^{3,\mu\nu} + |D_\mu \phi|^2 + \frac{c_\alpha^{(6)}}{2\mu^2} (\partial^\mu F^3_{\mu\nu})(\partial_\mu F^{3,\mu\nu}) - \frac{c_2^{(6)}}{\mu^2} |D^\mu D_\mu \phi|^2 \right]$$

QUANTIZATION WITH HDO

- $A_M^{\mathbf{A}}$ is the bulk gauge field. $\mathbf{A} = 1{,}2{,}3$ denotes the adjoint index and $M = \mu{,}5$ the 5d Minkowski index

- $F^3_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu$ with A^3_μ the gauge field and $\phi = \dfrac{A^1_5 + iA^2_5}{\sqrt{2}}$ the scalar field. $\mu, \nu \ldots$ denote the 4d Minkowski index

- $c_\alpha^{(6)}$ and $c_2^{(6)}$ are introduced for the HDO of the gauge and scalar field respectively absorbing the function $F(\beta_4, \beta_5)$ of $\mu = F(\beta_4, \beta_5)/a_4$

- Why NLO truncation?

  Truncation at LO in lattice spacing expansion is not enough. It generates a 2nd order phase transition → *N. Irges and F.K.*, Nucl. Phys. B **937** (2018) 135-195

- Set $\Lambda^2 \equiv \Lambda^2(\mu)$ as a cut-off for the Effective Field Theory (EFT)

  In this case Λ is not an external scale that must be introduced by hand. It is rather an internal scale, given by the value of the regulating scale at the phase transition, μ_*, where it assumes its maximum value. HDO are of quantum origin

  N. Irges and F.K., Nucl. Phys. B **950** (2020) 114833

- One more step, before renormalize diagrammatically at 1-loop order the boundary action and obtain its quantum effective version, is to deal with the extra pole instability

QUANTIZATION WITH HDO

- Expanding the gauge fixed action

$$S^{\mathrm{b-h}} = \int d^4x \Big[-\frac{1}{4} F^3_{\mu\nu} F^{3,\mu\nu} - \frac{1}{2\xi} (\partial^\mu A^3_\mu)^2 + |D_\mu \phi|^2 + \frac{c^{(6)}_\alpha}{2\Lambda^2} (\partial^\mu F^3_{\mu\nu})(\partial_\mu F^{3,\mu\nu}) - \frac{c^{(6)}_2}{\Lambda^2} |D^\mu D_\mu \phi|^2 + \partial^\mu \bar{c}^3 \partial_\mu c^3 \Big]$$

- To deal with the Ostrogradsky ghosts (O-ghosts) perform the most general field redefinition

$$\phi_0 \to \hat{\phi}_0 = \phi_0 + \frac{x}{\Lambda^2} D^2 \phi_0 + \frac{y}{\Lambda^2} (\bar{\phi}_0 \phi_0) \phi_0$$
$$A^3_{\mu,0} \to \hat{A}^3_{\mu,0} = A^3_{\mu,0} + \frac{x_\alpha}{\Lambda^2} (\eta_{\mu\rho} \Box - \partial_\mu \partial_\rho) A^{3,\rho}_0$$

- This introduces the Reparameterization ghosts (R-ghosts) which cancel the O-ghosts pole by pole at classical and quantum level

N. Irges and F.K., Phys. Rev. D 100, 065004 (2019)
N. Irges and F.K., Nucl. Phys. B **950** (2020) 114833

QUANTIZATION WITH HDO

- Expanding the gauge fixed action

$$
\begin{aligned}
S_0^{\mathrm{b-h}} &= \int d^4x \Bigg[-\frac{1}{4} F^3_{\mu\nu,0} F_0^{3,\mu\nu} + \frac{1}{2\xi} A^3_{\mu,0} \partial^\mu \partial_\nu A_0^{3,\nu} - \bar\phi_0 \Box \phi_0 - \frac{c^{(6)}_{\alpha,0}}{2\Lambda^2} F^3_{\mu\nu,0} \Box F_0^{3,\mu\nu} - \frac{c^{(6)}_{2,0}}{\Lambda^2} \bar\phi_0 \Box^2 \phi_0 - \bar c_0^3 \Box c_0^3 \\
&+ i g_0 \sqrt{\gamma_0} A^3_{\mu,0} \left(\left\{ \bar\phi_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \bar\phi_0 \right\} + \frac{c^{(6)}_{2,0}}{\Lambda^2} \left\{ \Box \bar\phi_0 \partial^\mu \phi_0 - \Box \phi_0 \partial^\mu \bar\phi_0 \right\} \right) + g_0^2 \gamma_0 (A^3_{\mu,0})^2 \bar\phi_0 \phi_0 \\
&+ g_0^2 \gamma_0 \left(\frac{c^{(6)}_{2,0}}{\Lambda^2} \left\{ \phi_0 \Box^2 \bar\phi_0 + \bar\phi_0 \Box^2 \phi_0 \right\} - \frac{c^{(6)}_{2,0}}{\Lambda^2} \partial^\mu (A^3_{\mu,0} \bar\phi_0) \partial_\mu (A_0^{3,\mu} \phi_0) \right) \\
&- \frac{i g_0^3 \gamma_0^{3/2} c^{(6)}_{2,0}}{\Lambda^2} (A^3_{\rho,0})^2 A^3_{\mu,0} \left(\bar\phi_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \bar\phi_0 \right) - \frac{g_0^4 \gamma_0^2 c^{(6)}_{2,0}}{\Lambda^2} (A^3_{\rho,0})^4 \bar\phi_0 \phi_0 \Bigg] \quad , \; g^2 = \frac{g_5^2}{a_4}
\end{aligned}
$$

- To deal with the Ostrogradsky ghosts (O-ghosts) perform the most general field redefinition

$$
\phi_0 \to \hat\phi_0 = \phi_0 + \frac{x}{\Lambda^2} D^2 \phi_0 + \frac{y}{\Lambda^2} (\bar\phi_0 \phi_0) \phi_0
$$
$$
A^3_{\mu,0} \to \hat A^3_{\mu,0} = A^3_{\mu,0} + \frac{x_\alpha}{\Lambda^2} (\eta_{\mu\rho} \Box - \partial_\mu \partial_\rho) A_0^{3,\rho}
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- This introduces the Reparameterization ghosts (R-ghosts) which cancel the O-ghosts pole by pole at classical and quantum level

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QUANTIZATION WITH HDO

- Fixing $x_\alpha = -c^{(6)}_{\alpha,0}$, $x = -\frac{c^{(6)}_{2,0}}{2}$ and $y = \frac{c^{(6)}_{1,0}}{8}$ gives the bare and redefined boundary action

$$\begin{aligned}
S_0^{\rm b-h} &= \int d^4x \Bigg[-\frac{1}{4} F^3_{\mu\nu,0} F_0^{3,\mu\nu} + \frac{1}{2\xi} A^3_{\mu,0} \partial^\mu \partial_\nu A_0^{3,\nu} - \bar\phi_0 \Box \phi_0 - \frac{c^{(6)}_{1,0}}{4\Lambda^2} (\bar\phi_0\phi_0) \bar\phi_0 \Box \phi_0 - \bar c_0^3 \Box c_0^3 \\
&+ i g_{4,0} \Big\{ \eta_{\mu\rho} - \frac{\eta_{\mu\rho}\Box - \partial_\mu \partial_\rho}{\Lambda^2} \Big\} A_0^{3,\rho} \left(\bar\phi_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \bar\phi_0 \right) + g^2_{4,0} (A^3_{\mu,0})^2 \bar\phi_0 \phi_0 \\
&+ \frac{g^2_{4,0}}{2\Lambda^2} \left(A^3_{\mu,0} A^3_{\rho,0} \partial^\rho \bar\phi_0 \partial^\mu \phi_0 + A^3_{\mu,0} \partial^\rho A^3_{\rho,0} \partial^\mu (\bar\phi_0 \phi_0) \right) - 2 g^2_{4,0} \frac{A_0^{3,\mu} (\eta_{\mu\rho}\Box - \partial_\mu \partial_\rho) A_0^{3,\rho}}{\Lambda^2} \bar\phi_0 \phi_0 \\
&+ i \frac{g_{4,0}\, c^{(6)}_{1,0}}{4\Lambda^2} A^3_{\mu,0} \bar\phi_0 \phi_0 \left(\bar\phi_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \bar\phi_0 \right) + \frac{g^2_{4,0}\, c^{(6)}_{1,0}}{4\Lambda^2} (A^3_{\mu,0})^2 (\bar\phi_0 \phi_0)^2 \Bigg]
\end{aligned}$$

- Now the boundary action is ghost-free and has developed a scalar quartic term $\bar\phi\phi\,\Box\,\bar\phi\phi$ (Recall that these HDO are of quantum nature)

- One coupling in the beginning and two couplings, g_4 and the "quartic coupling" $c_1^{(6)}$ at the end. However is expected to be connected (`a la CW)

- The Feynman rules are straightforward but non-trivial due to the HDO. Ready for the 1-loop level, diagrammatic, renormalization

QUANTIZATION WITH HDO

- The renormalization procedure suggests

$$g_{4,0} = (1+\delta_{g_4})g_4 \text{ or } \alpha_{4,0} = (1+\delta_{\alpha_4})\alpha_4 \text{ with } \alpha_4 = \frac{g_4^2}{16\pi^2} \qquad c_{1,0}^{(6)} = (1+\delta_{c_1^{(6)}})c_1^{(6)} \quad \phi_0 = \sqrt{1+\delta_\phi}\phi \qquad A^3_{\mu,0} = \sqrt{1+\delta_A}A^3_\mu$$

$$\downarrow$$

- The countqerterms and the associated β-functions of the boundary action are fixed (the off-shell scheme $p_i^2 = \Lambda^2$ is used)

$$\delta_{g_4} = -\frac{1}{2}\delta A,\ \delta g_4 = \frac{1}{16\pi^2}\frac{g_4^3}{\varepsilon} \text{ or } \delta\alpha_4 = 2\frac{\alpha_4^2}{\varepsilon} \qquad \delta c_1^{(6)} = \frac{1}{16\pi^2}\frac{4(c_1^{(6)})^2 + 34g_4^4}{\varepsilon} \qquad \delta\phi = 0$$

$$\beta_{g_4} = \frac{g_4^3}{16\pi^2} \text{ or } \beta_{\alpha_4} = 2\alpha_4^2 \qquad \beta_{c_1^{(6)}} = \frac{4(c_1^{(6)})^2 + 34g_4^4}{16\pi^2}$$

- For completeness apply all the previous steps in the bulk lattice action to get its continuum version (5d Lee-Wick version)

$$\mathcal{L}^B = -\frac{1}{4}F^A_{\mu\nu}F^{A,\mu\nu} + \frac{1}{16\Lambda^2}(D^\mu F^A_{\mu\nu})(D_\mu F^{A,\mu\nu}) - \frac{g_5}{24\Lambda^2}f_{ABC}F^A_{\mu\nu}F^B_{\nu\rho}F^C_{\rho\mu} + (\overline{D_\mu\Phi^A})(D^\mu\Phi^A) - \frac{1}{4\Lambda^2}(\overline{D^2\Phi^A})(D^2\Phi^A)$$

$$\beta_{g_5\mu^{-\varepsilon/2}} = -\frac{\varepsilon}{2}g_5\mu^{-\varepsilon/2} - \frac{125}{6}\frac{g_5^3\mu^{-3\varepsilon/2}}{16\pi^2} \text{ or } \beta_{\alpha_5} = -\varepsilon\alpha_5 - \frac{125}{12}\alpha_5^2$$

THE HIGGS PHASE

- The desired Higgs phase is revealed when a CW procedure is followed
- The algorithm:

1. *Consider the 4d bare potential in momentum space*
2. *Construct the renormalized and improved effective potential using the scalar field as the running parameter and minimize it to find the non-trivial minimum*
3. *Find the relation between the couplings* g_4, $c_1^{(6)}$ *and then determine the scalar and gauge field masses and from those the scalar-to-gauge mass ratio*

- The improved 1-loop effective potential is of a CW type

$$V_{\rm imp.}(\bar\phi,\phi) = \frac{c_1^{(6)}}{4}(\bar\phi\phi)^2 + \left\{2(c_1^{(6)})^2 + 17g_4^4\right\}\frac{(\bar\phi\phi)^2}{64\pi^2}\left(\ln\frac{\bar\phi\phi}{v^2} - 3\right)$$

- The minimization suggests

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or using $\bar{\phi}\phi = \frac{(A_5^1)^2 + (A_5^2)^2}{2} \equiv \phi_r^2$ $\quad V_{\text{imp.}}(\phi_r) = \frac{c_1^{(6)}}{4}\phi_r^4 + \left\{2(c_1^{(6)})^2 + 17g_4^4\right\}\frac{\phi_r^4}{64\pi^2}\left(\ln\frac{\phi_r^2}{v^2} - 3\right)$

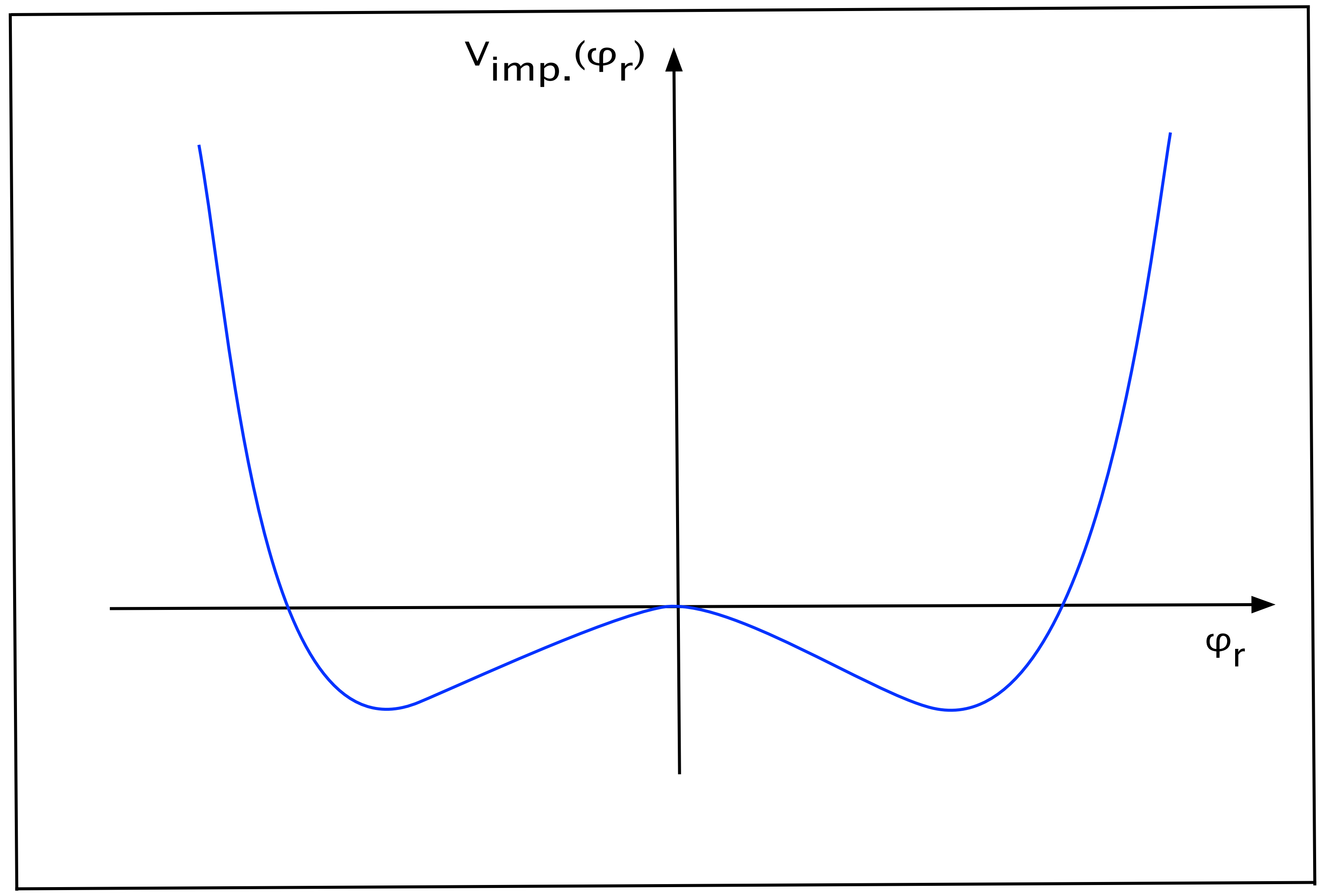
- The minimization suggests

$$\left.\frac{\partial V_{\text{imp.}}(\phi_r)}{\partial \phi_r}\right|_{\phi_r=v} = \frac{-(10(c_1^{(6)})^2 + 85g_4^4 - 32\pi^2 c_1^{(6)})v^3}{32\pi^2} = 0 \Rightarrow$$

$$c_1^{(6)} = \frac{85}{32\pi^2}g_4^4$$

THE HIGGS PHASE

- The expected connection between the couplings is achieved $\rightarrow$ $V_{\text{imp.}}(\phi_r) = \frac{17 g_4^4 \phi_r^4}{128\pi^2}\left(2\ln\frac{\phi_r^2}{v^2} - 1\right) + \mathcal{O}(g_4^8)$



The mexican hat potential $V_{\text{imp.}}(\phi_r)$

- The non-trivial vev ($<\phi_r> = v$) triggers the spontaneous breaking of the gauge symmetry $\phi_r = h + v$

$$m_h^2 \equiv \frac{\partial^2 V(h)}{\partial h^2}\Big|_{h=0} = \frac{210}{8\pi^2} g_4^4 v^2 \qquad m_{A_\mu^3}^2 = g_4^2 v^2 \equiv m_Z^2 \qquad \frac{m_h^2}{m_Z^2} \equiv \rho_{\text{bh}}^2 = \frac{210}{8\pi^2} g_4^2 \Rightarrow \rho_{\text{bh}} = \sqrt{\frac{210}{8\pi^2}}\, g_4 \simeq 1.64\, g_4$$

THE HIGGS PHASE

- Comparison with the CW case in the classical level

$$\rho_{\rm CW} = \sqrt{\frac{3}{8\pi^2}}e \simeq 0.19\, e \quad \text{and} \quad \lambda = \frac{33}{8\pi^2}e^4$$

- The numerical difference originates from the higher derivative nature of the quartic coupling $c_1^{(6)}$, a crucial point for the model

- At quantum level the solution of the RG equations $\mu\frac{d\, g_4(\mu)}{d\mu} = \beta_{g_4}$ and $\mu\frac{d\, e(\mu)}{d\mu} = \beta_e$ with $\beta_e = \frac{e^3}{48\pi^2}$

$$g_4(\mu) = \frac{g_{4,R}}{\sqrt{1 - \frac{g_{4,R}^2}{16\pi^2}\ln\frac{\mu^2}{m_R^2}}} \quad \text{or} \quad \alpha_4(\mu) = \frac{\alpha_{4,R}}{1 - \alpha_{4,R}\ln\frac{\mu^2}{m_R^2}} \quad \text{and} \quad e(\mu) = \frac{e_R}{\sqrt{1 - \frac{e_R^2}{48\pi^2}\ln\frac{\mu^2}{M_R^2}}}$$

- The IR boundary conditions are m_R, M_R, $g_4(m_R) = g_{4,R}$ $(\alpha_4(m_R) = \alpha_{4,R})$ and $e(M_R) = e_R$.

- The vev is fixed $v \equiv v_*$

REMARKS BEFORE THE PHASE DIAGRAM

- The effective boundary action is not completely decoupled from the 5d bulk

- The RG flow in the Higgs phase is constrained from the one in the Hybrid phase

- The Hybrid phase contains 4d slices with SU(2) gauge group in the bulk. 4d coupling g_s or $\alpha_s = \dfrac{g_s^2}{16\pi^2}$ connected with $\beta_{4,s}$ and $\beta_{5,s}$

↓

1. *Dimensional reduction through localization when the Higgs-Hybrid phase transition (1st order, quantum) is approached. The entire Hybrid phase is layered in the fifth dimension*

↓

2. *The bulk driven Higgs-Hybrid phase transition is approached simultaneously from either side when the system is driven towards the UV (proven NP) due to common μ*

↓

3. *A matching of all physical observables is possible at the scale μ_* where the running of $g_s(\mu)$ and $g_4(\mu)$ stops and it never reaches the continuum limit so the model inherits a finite cut-off*

THE CONTINUUM PHASE DIAGRAM

- The needed ingredients:

3. The value of the parameters on the phase transition denoted by *

1. The RG evolution of the couplings in both phases

2. The connection of β_4 and β_5 with the running gauge couplings of Higgs and Hybrid phase

The parameters

Higgs Phase	Hybrid Phase
$\alpha_4(\mu) = \dfrac{\alpha_{4,R}}{1 - \alpha_{4,R} \ln \frac{\mu^2}{m_R^2}}$	$\alpha_s(\mu) = \dfrac{c'_s}{\ln \frac{\mu}{\Lambda_s}}$ with $\Lambda_s = e^{-\frac{c'_s}{\alpha_{s,R}}} m_R$
$\mu = \exp\left[\dfrac{\alpha_4(\mu) - \alpha_{4,R}}{2\alpha_4(\mu)\, \alpha_{4,R}}\right] m_R$	$\mu = e^{\frac{c'_s}{\alpha_s(\mu)}} \Lambda_s$
$\beta_4(\mu) = \dfrac{1}{4\pi^2 \alpha_4(\mu)}$	$\beta_{4,s}(\mu) = \dfrac{1}{4\pi^2 \alpha_s(\mu)}$
$\beta_5(\mu) = \gamma^2(\mu) \beta_4(\mu)$	$\beta_{5,s}(\mu) = \gamma_s^2(\mu) \beta_{4,s}(\mu)$

THE CONTINUUM PHASE DIAGRAM

- On the Higgs-Hybrid phase transition $\mu = \mu_*$:

$$\alpha_4(\mu_*) = \alpha_s(\mu_*) = \alpha_* \qquad \mu_* = e^{\frac{c'_s}{1+2c'_s}\left[\frac{1}{\alpha_{4,R}} + \frac{2c'_s}{\alpha_{s,R}}\right]}\Lambda_s$$

$$\alpha_* = \frac{\alpha_{4,R}\,\alpha_{s,R}(1+2c'_s)}{\alpha_{s,R} + 2c'_s\alpha_{4,R}} \qquad m_{h*} = \sqrt{\frac{210}{8\pi^2}}16\pi^2 v_*\alpha_*$$

- The above are controlled by four variables: $\alpha_{4,R}$, $\alpha_{s,R}$, v_*, and Λ_s ($c'_s = 3/125$)

- $\alpha_{s,R} = 0.014$ (SM's strong gauge coupling) and $\Lambda_s = m_p = 1000$ MeV (proton mass), fixed by physical motivation

- The first necessary condition for the validity of the effective action is the hierarchy of the scales

$$m_R < m_{h*} < \mu_*$$

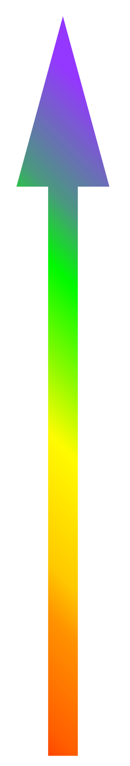
- The second necessary condition is to generate a SM-like spectrum

$$m_{h*} \simeq 125\,\text{GeV} \quad \text{and} \quad \rho_{\text{bh}} > 1$$

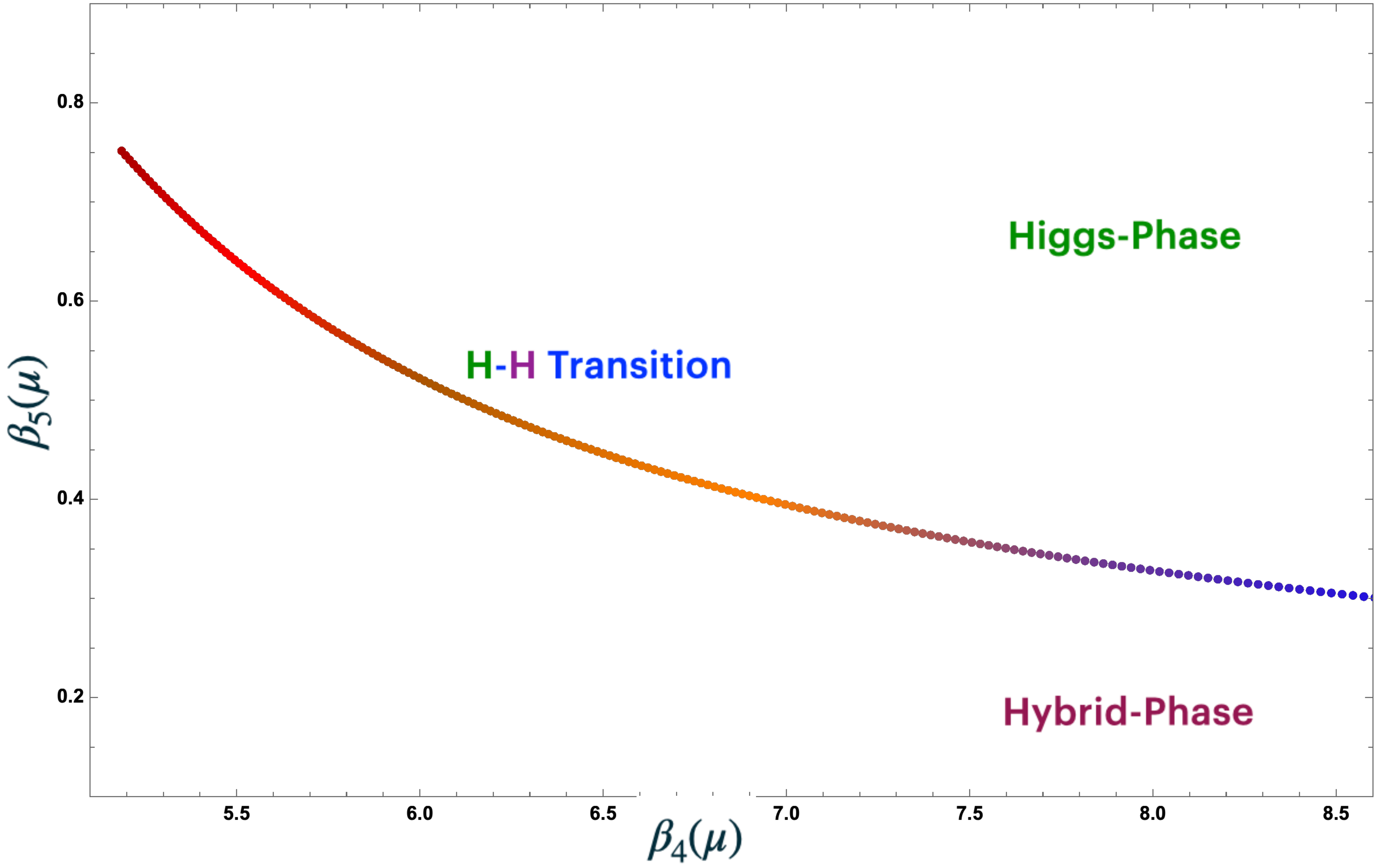
- Standard Model spectrum for $\alpha_{4,R} = 0.00435$ and $v_* = 108.2$ GeV → $m_R = 5.55\,\text{GeV}$, $m_{h*} \approx 125.1\,\text{GeV}$, $\mu_* \approx 209\,\text{GeV}$ and $\rho_{\text{bh}} \approx 1.373$

THE CONTINUUM PHASE DIAGRAM

- Keep $\alpha_{s,R} = 0.014$, $\Lambda_s = 1000\,\text{MeV}$ and $v_* = 108.2\,\text{GeV}$. Vary $\alpha_{4,R} \rightarrow$ Varies $\mu_* \rightarrow$ different pair (β_{4*}, β_{5*})



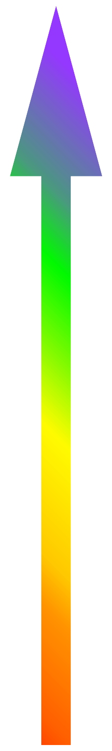
Increasing μ and μ_*

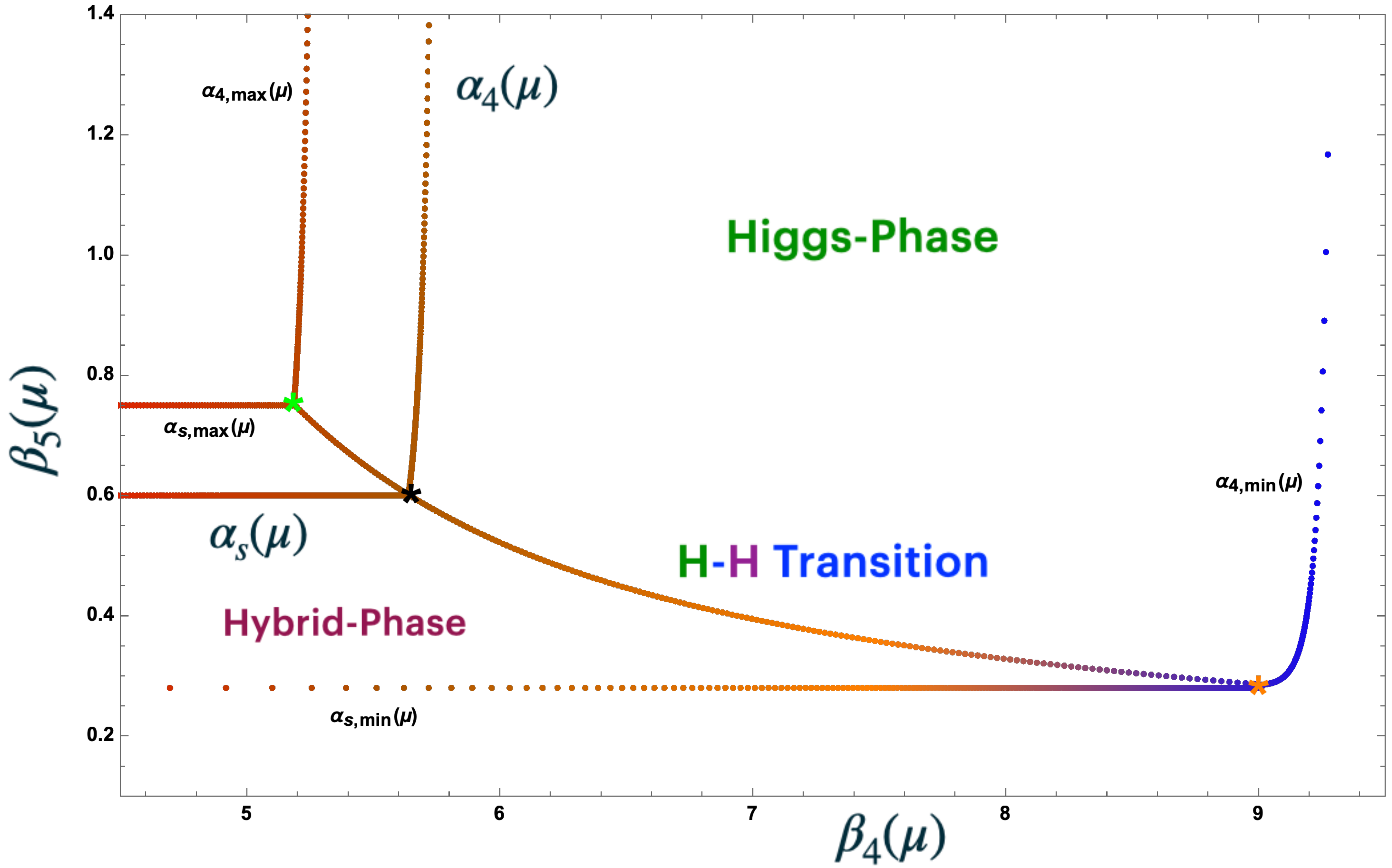


THE CONTINUUM PHASE DIAGRAM

- The phase diagram and three RG flows: $\alpha_{4,R} \rightarrow$ $(\alpha_{4,R}^{\min} = 0.0027)$, $(\alpha_{4,R} = 0.00435)$, $(\alpha_{4,R}^{\max} = 0.00473)$

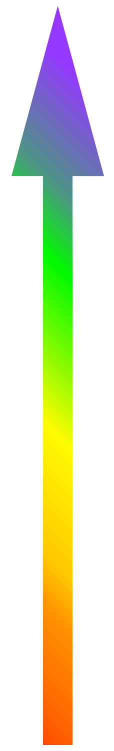
$(m_{h^*} \approx 78\,\text{GeV}, \mu_* \approx 5123\ \text{GeV})$ $(m_{h^*} \approx 125.1\,\text{GeV}, \mu_* \approx 209\ \text{GeV})$ $(m_{h^*} = \mu_* \approx 136.1\,\text{GeV})$



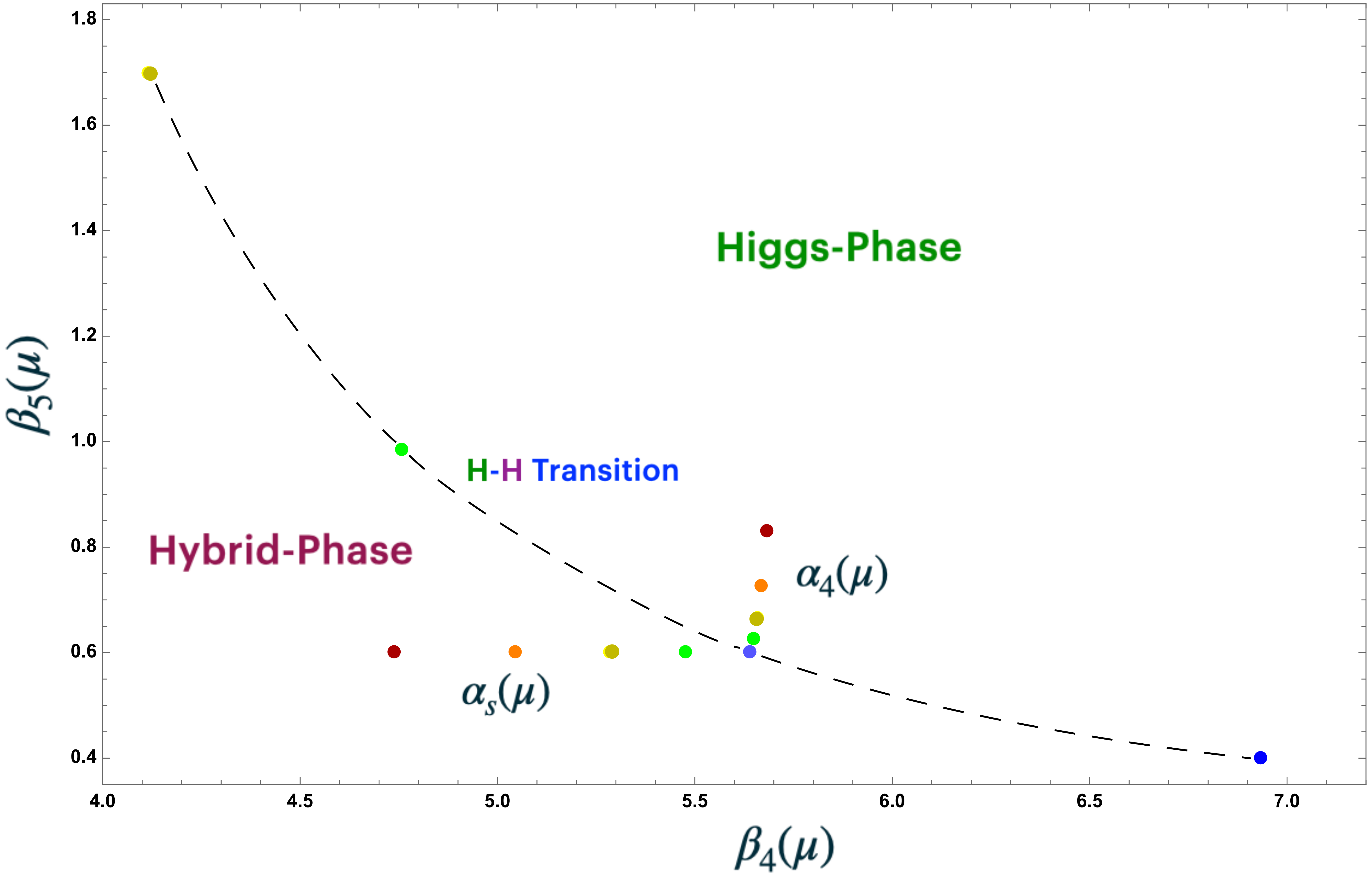


THE CONTINUUM PHASE DIAGRAM

- A zoomed version of the phase diagram



Increasing μ and μ_*



THE CONTINUUM PHASE DIAGRAM

- Numerical analysis shows that the fine tuning of an RG flow that respects the physical constraints is equal or less than $\mathcal{O}(10^2)$

Case 1: $\alpha_{s,R} \geq \mathcal{O}(10^{-1})$ only for $\alpha_{4,R} = 0.00435$ a realistic spectrum, however the 1st order phase transition is below the 2nd order phase transition

Case 2: $\alpha_{s,R} = \mathcal{O}(10^{-2})$ $(0.010 \leq \alpha_{s,R} \leq 0.098)$, only for $\alpha_{4,R} = 0.00435$ a realistic spectrum

Case 3: $\alpha_{s,R} \leq \mathcal{O}(10^{-3})$ a realistic spectrum for $\alpha_{4,R} \neq 0.00435$, however the the hierarchy condition is not respected

- The relation $\alpha_4(\mu_*) < \alpha_5(\mu_\star)$ is true for Case 2. The system reaches the 1st order phase transition before the 2nd order one

- Same arguments keeping $\alpha_{s,R} = 0.014$ and $\alpha_{4,R} = 0.00435$ fixed and varying Λ_s and v_*

  Viable conditions for $0.6\,\mathrm{GeV} \leq \Lambda_s \leq 16\,\mathrm{GeV}$ and $v_* = 108.2\,\mathrm{GeV}$

- Then the fine tuning in the Higgs mass is very small. The dynamics do not allow a high cut-off for the effective action

CONCLUSIONS

- The 1-loop effective action of an SU(2) gauge theory in five dimensions with boundary conditions that leave a U(1)-complex scalar theory on the boundary, located at the origin of a semi-infinite fifth dimension was constructed

- At perturbative level, the boundary theory is a version of the Coleman-Weinberg model where the quartic term is replaced by a dimension-6 derivative operator. A qualitatively similar to the CW model Higgs mechanism is at work but with different coefficients in the scalar mass and the β-functions that change things towards a more realistic direction

- Imposing on the effective action non-perturbative features known from the lattice, the system becomes highly constrained. The picture is that the model possesses a non-trivial phase diagram where the phases are separated by 1st order, quantum phase transitions located in the UV

- In order to use the model as a cartoon of a possible origin of the Standard Model Higgs sector, then it turns out that we have to sit on, or near the interface of the phase transition that separates the Higgs phase and a layered-type of phase, the Hybrid phase. There, dimensional reduction happens via localization in both phases and the effective action must be constructed with a dynamically generated finite cut-off but also with RG flows that are correlated below and above the phase transition

- Alternative resolution to the Higgs mass hierarchy problem: The fine tuning involved is about one part in a hundred and it is related to the choice of a ”physical RG flow” on the phase diagram while the dynamics do not allow a high cut-off for the effective action. Once such a physical RG flow is picked, there is very little fine tuning that takes place along it

- Several features of the model could be tested at Higgs-factories and future colliders

THANK YOU