A string-inspired running-vacuum-model of cosmology and the current tensions in cosmological data

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EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

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Workshop on the Standard Model and Beyond

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E I S A

European Institute for Sciences and Their Applications


Corfu Summer Institute

Hellenic School and Workshops on Elementary Particle Physics and Gravity

Corfu, Greece

Dedicated to the memory of Prof. Costas Kounnas and Prof. Graham Ross

0. Outline

1. **Motivation**

2. **Topological invariants in Gravity theories**

3. **String-Inspired Gravitational Theory with Gravitational Anomalies & axions**

4. **Primordial Gravitational Waves (GW) induced Condensates of Anomalies, the role of supersymmetry**

5. **Running Vacuum Model (RVM) of Cosmology & inflation without external inflatons**

6. **(a) Post-inflationary Cosmic evolution**
   (b) Modern-era phenomenology: deviations from ΛCDM and alleviation of cosmological data tensions?

9. **Conclusions & Outlook**

This talk

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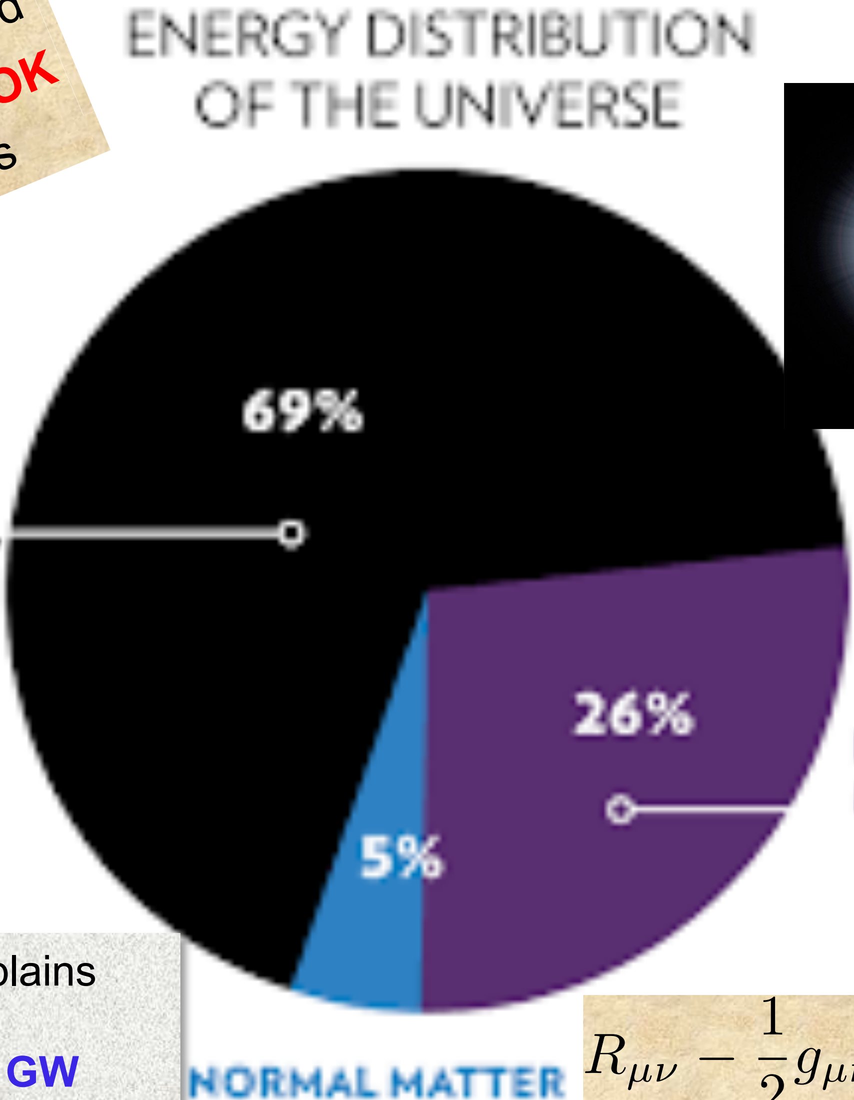
Bonus features

7. **Enhanced cosmic perturbations & densities of primordial black holes (PBH) & GW**
   → dark matter components: PBH, together with the torsion-induced axions
8. **Spontaneous Lorentz and CPT-Violation by axion backgrounds & Leptogenesis in radiation era → Baryogenesis ;**
9. **Conclusions & Outlook**

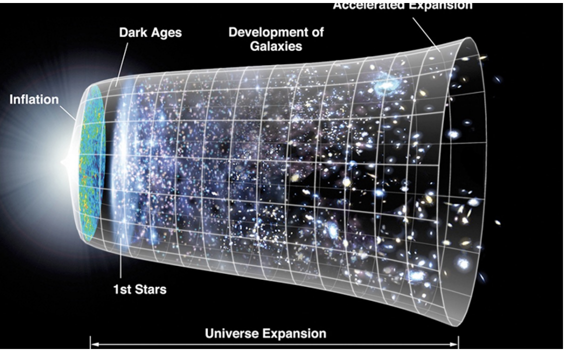
1. Motivation

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

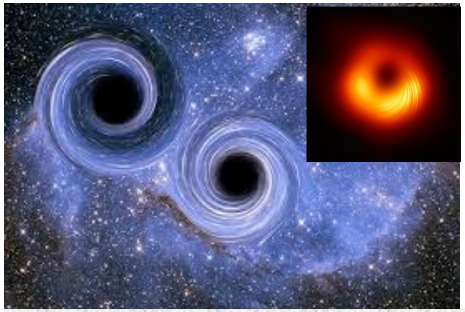
Simplest model based on **ΛCDM** works **OK** for **large** scales



Planck2018 data



+ SnIa, BaO, Lensing



Also **Einstein's GR** explains **sufficiently well**
Black-Hole Mergers + GW
(since 2015 **LIGO**),
Black-Hole `photographs' (EHT),...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, R - g_{\mu\nu}\, \Lambda = 8\pi \mathrm{G}\, T_{\mu\nu}$$

$$T_{\mu\nu} \ni \text{Cold Dark Matter}$$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

3 data

Sim

on

But....

Need to go Beyond....

What still we do not know/did not observe:

Nature of Dark Energy

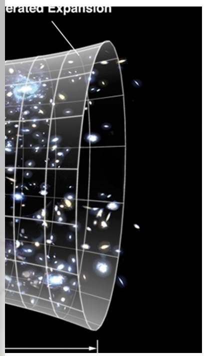
Nature of Dark matter

Primordial Gravitational Waves

(through detection of B-mode polarisation in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)



Lensing

Also

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Black

$8\pi G\, T_{\mu\nu}$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

Sim
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But....

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What still we
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Nature o
Nature o
Primordial G
(through de
polarisation
in CMB from very early Universe)

Microscopic models of Inflation
(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)

ΛCDM appears to be in tension with both, perturbative & non-perturbative string theory (& UV complete Theories of Quantum Gravity- swampland ?)

Also
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Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

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(through detection of B-mode polarisation
in CMB from very early Universe)

Microscopic models of Inflation
(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)

ΛCDM appears to be in tension with local measurements of present-era H_0 & also σ_8 galaxy-growth data ?

Also I
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(since
Black-

$8\pi G\, T_{\mu\nu}$

2. Topological Invariants in Gravity theories

Topological invariants (total derivatives) in (3+1)-dimensional (curved) space-times

- Gauss Bonnet (quadratic-curvature) combination

$$\boxed{\begin{aligned} R_{\rm GB} &\equiv R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\, R^{\mu\nu} + R^2 \\ &= \nabla_\mu \mathcal{F}^\mu_{\rm GB} \end{aligned}}$$

- Chern-Simons – Hirzenbruch signature (Chiral Anomaly terms)

$$\boxed{\begin{aligned} \mathrm{CS} &\equiv R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \\ &= \nabla_\mu \mathcal{K}^\mu_{\rm CS} \end{aligned}}$$

$$= 2\,\partial_\mu \Big[\epsilon^{\mu\nu\alpha\beta}\, \omega_\nu^{ab} \Big(\partial_\alpha\, \omega_{\beta ab} + \frac{2}{3}\, \omega_{\alpha a}^{\ \ \ c}\, \omega_{\beta cb} \Big) - 2\epsilon^{\mu\nu\alpha\beta} \Big(A_\nu^i\, \partial_\alpha A_\beta^i + \frac{2}{3}\, f^{ijk}\, A_\nu^i\, A_\alpha^j\, A_\beta^k \Big) \Big]$$

$$\widetilde{R}^{\mu\nu\rho\sigma} \equiv \frac{1}{2} \varepsilon^{\mu\nu\lambda\pi} R_{\lambda\pi}^{\ \ \ \ \rho\sigma}\,, \qquad \widetilde{F}^{\mu\nu} \equiv \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma}\, F_{\rho\sigma}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma}\,, \qquad \varepsilon^{\mu\nu\rho\sigma} = \frac{\mathrm{sgn}(g)}{\sqrt{-g}}\, \epsilon^{\mu\nu\rho\sigma} \qquad\qquad {}^*R^{\mu\nu\rho\sigma} \equiv \frac{1}{2} \epsilon^{\mu\nu\lambda\pi} R_{\lambda\pi}^{\ \ \ \ \rho\sigma}$$

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Spin connection **Gauge fields**

$$= 2\,\partial_\mu \Big[\epsilon^{\mu\nu\alpha\beta}\, \omega_\nu^{ab} \Big(\partial_\alpha\, \omega_{\beta ab} + \frac{2}{3}\, \omega_{\alpha a}^{\ \ \ c}\, \omega_{\beta cb} \Big) - 2\epsilon^{\mu\nu\alpha\beta} \Big(A^i_\nu\, \partial_\alpha A^i_\beta + \frac{2}{3}\, f^{ijk}\, A^i_\nu\, A^j_\alpha\, A^k_\beta \Big) \Big]$$

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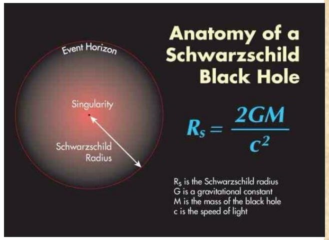
Flat Levi-Civita

Topological invariants in string-effective gravity theories

- **Gauss Bonnet coupling to dilatons** (spin-0 scalars of string massless gravitational multiplet)

$$\mathcal{S} \ni \int d^4x \sqrt{-g}\, \frac{1}{8g_s^2}\, e^{-2\Phi} \left(R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\, R^{\mu\nu} + R^2\right)$$

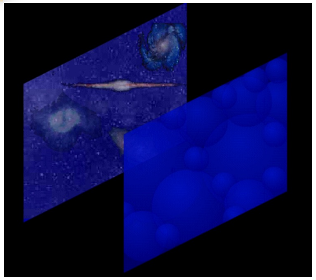
Kanti, NEM, Rizos, Tamvakis, Winstanley, … Bakopoulos….



Applications: (i) Dilaton scalar hair (secondary type) in Black Holes

(ii) String-Brane modified cosmologies

Binetruy, Charnousis, Odintsov,….



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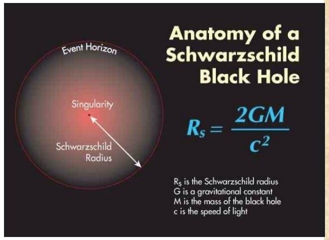

Applications: (i) Dilaton scalar hair (secondary type) in Black Holes

Dilaton sources of BHs

$$\Box\,\Phi \propto e^{-2\Phi}\, \frac{M_{\rm Pl}^{-2}}{g_s^2}\, R_{\rm GB}$$

Non trivial BH solutions

A Remark

- **Gauss Bonnet coupling to dilatons** (spin-0 scalars of string massless gravitational multiplet)

$e^{-2\Phi} \approx 1 - 2\Phi$, $|\Phi| << 1$

$$\mathcal{S} \ni \int d^4x\, \sqrt{-g}\, \frac{1}{8g_s^2}\, e^{-2\Phi} \Big(R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\, R^{\mu\nu} + R^2\Big)$$

Applications: (i) Dilaton scalar hair (secondary type) in Black Holes

NB: weak (linear dilaton coupling → belongs to shift-symmetric scalar-GB Horndeski Theories $\Phi \rightarrow \Phi + c$ (evasion of no hair theorem similar to string case)

Sotiriou, Zhou….

Topological invariants in string-effective gravity theories

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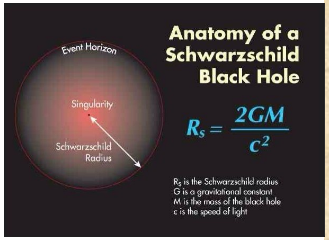
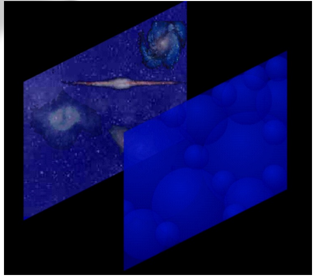


Applications: (i) Dilaton scalar hair (secondary type) in Black Holes

BH could be primordial

(ii) String-Brane modified cosmologies



Topological invariants in string-effective gravity theories

- **Chern-Simons coupling** to **axions** (pseudoscalars) **a(x)** (**string-model independent axions** (dual in (3+1)-dim to the field strength of Kalb-Ramond antisymmetric (spin-1) tensor field) as well as **axions from string compactification**)

$$\mathcal{S} \ni \int d^4x \sqrt{-g}\, \frac{1}{f_a}\, a(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - \mathbf{F}_{\mu\nu}\, \widetilde{\mathbf{F}}^{\mu\nu}\Big)$$

Jackiw, Pi, Yunes, Alexander, ….
Chatzifotis, Dorlis, NEM,
Papantonopoulos

Applications: (i) Pseudoscalar (axion) hair (secondary type) in Rotating (Kerr-like) Black Holes

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Applications: (i) Pseudoscalar (axion) hair (secondary type) in Rotating (Kerr-like) Black Holes

Axions source (via Chern-Simons coupling) rotating (spinning) black holes

$$\Box\, a(x) \propto \frac{1}{f_a} \Big(\underbrace{R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}}_{\text{Spinning BH solutions}} \Big)$$

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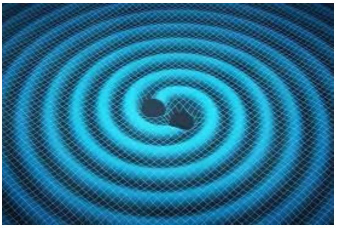
Alexander, Peskin, Sheikh-Jabbari
Lyth, Quimbay, Rodriguez



Applications: (ii) Non-trivial Chern-Simons gravity terms if chiral gravitational waves are present

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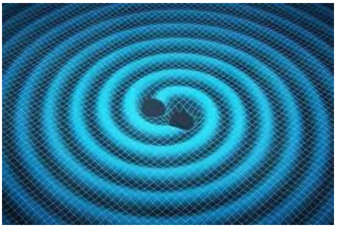
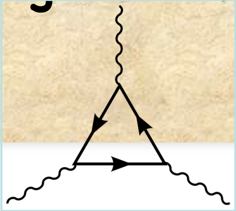
Non trivial if chiral gravitational Waves (GW) or spinning BH present (including primordial configurations.)

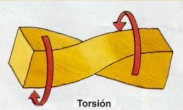


Applications: (ii) Non-trivial Chern-Simons gravity terms if chiral gravitational waves are present

(iii) String-inspired modified cosmologies with gravitational anomalies (& torsion)



NEM, Solà, Basilakos



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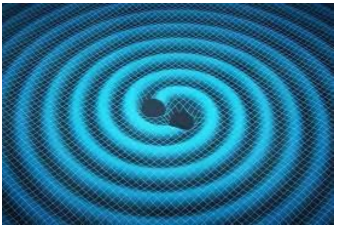

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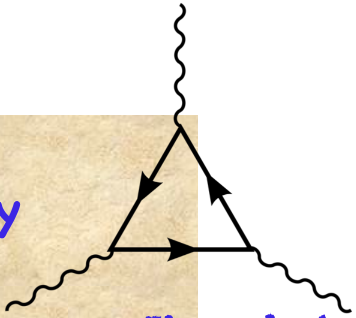
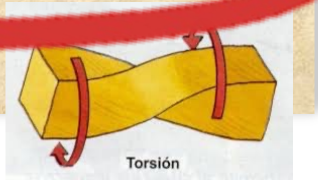
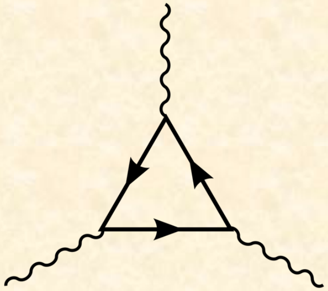


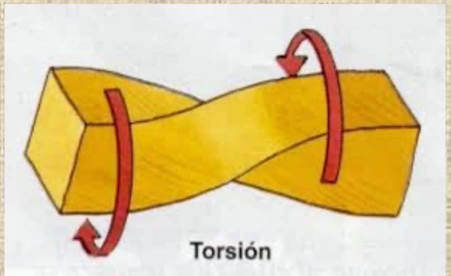
This talk

(iii) String-inspired modified cosmologies with gravitational anomalies (& torsion)

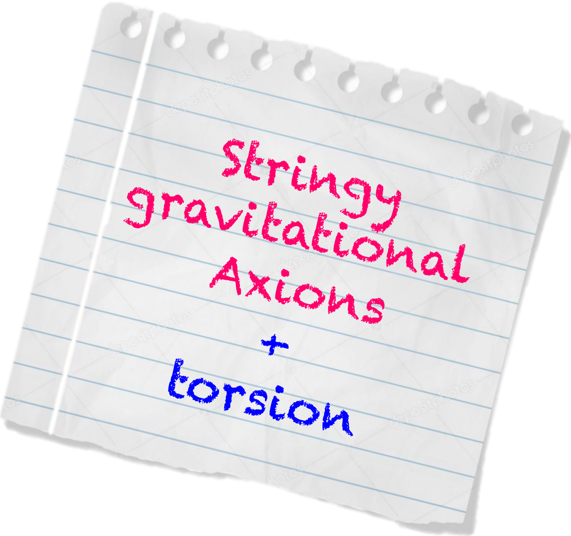




3. String-Inspired Gravitational Theory with Torsion & Grav. Anomalies, axions and torsion



Gross and Sloan, Metsaev and Tseytlin


Stringy gravitational Axions + torsion

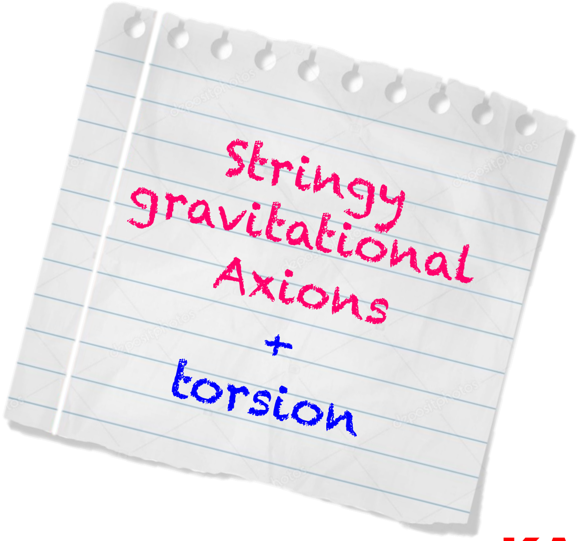
Massless Gravitational multiplet of (closed) strings:

spin 0 scalar (dilaton Φ)

spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)

spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $$B_{\mu\nu} = -B_{\nu\mu}$$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KALB-RAMOND FIELD

$$\mathrm{U(1)-symmetry}: B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

4-DIM action

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R+2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}e^{-4\Phi}H_{\lambda\mu\nu}H^{\lambda\mu\nu}+\dots\Big)$$

$\kappa^2 = 8\pi G$

Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} \Rightarrow \boxed{\boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)}$$

$$\Omega_{3\mathrm{L}} = \omega^a_{\ c}\wedge \mathbf{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c}\wedge\omega^c_{\ d}\wedge\omega^d_{\ a}, \quad \Omega_{3\mathrm{Y}} = \mathbf{A}\wedge\mathbf{dA} + \mathbf{A}\wedge\mathbf{A}\wedge\mathbf{A},$$

Gross and Sloan, Metsaev and Tseytlin

Stringy gravitational Axions
+
torsion

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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

Φ = constant throughout

No Gauss Bonnet combination

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

Gross and Sloan, Metsaev and Tseytlin

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$$\overline{R}(\overline{\Gamma})$$

quantum torsion →

gravitational axion b "dual" to H torsion

b(x) = Lagrange multiplier implementing **Bianchi identity constraint** for $H_{\mu\nu\rho}$:

$$d\star H \propto c_1 R\wedge\widetilde{R} - F\wedge\widetilde{F}$$

$H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$

ntorsion

Effective Actions & Anomaly Cancellation – Addition of Counterterms

Green, Schwarz

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R+2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}\,e^{-4\Phi}\,H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \dots\Big)$$

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$$\Omega_{3\mathrm{L}} = \omega^a_{\ c}\wedge \mathrm{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c}\wedge\omega^c_{\ d}\wedge\omega^d_{\ a}, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A}\wedge\mathrm{d}\mathbf{A} + \mathbf{A}\wedge\mathbf{A}\wedge\mathbf{A},$$

Modified Bianchi Constraint

$$\boxed{\varepsilon_{abc}{}^{\mu}\,\nabla_\mu\,H^{abc} = \frac{\alpha'}{32}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathcal{G}(\omega,\mathbf{A})} \;\neq\; 0$$

Implement in path-integral as a field theory δ(…) via
Lagrange multiplier b(x) pseudoscalar (axion-like) field
(**Kalb-Ramond (KR) Axion**) becomes dynamical after H-torsion integration

Effective Actions & Anomaly Cancellation – Addition of Counterterms

Green, Schwarz

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R+2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}\,e^{-4\Phi}\,H_{\lambda\mu\nu}H^{\lambda\mu\nu}+\ldots\Big)$$

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\,B_{\nu\rho]} \quad \Longrightarrow \quad \boxed{\mathcal{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)}$$

$$\Omega_{3\mathrm{L}} = \omega^a{}_c \wedge \mathrm{d}\omega^c{}_a + \frac{2}{3}\omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A}\wedge\mathrm{d}\mathbf{A} + \mathbf{A}\wedge\mathbf{A}\wedge\mathbf{A},$$

$$\Downarrow$$

Modified Bianchi Constraint

$$\boxed{\varepsilon_{abc}{}^{\mu}\,\nabla_\mu\,H^{abc} = \frac{\alpha'}{32}\sqrt{-g}\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) \equiv \sqrt{-g}\,\mathcal{G}(\omega,\mathbf{A})} \neq 0$$

Implement in path-integral as a field theory δ(...) via Lagrange multiplier b(x) pseudoscalar (axion-like) field (Kalb-Ramond (KR) Axion) becomes dynamical after H-torsion integration

$$\Pi_x\,\delta\Big(\varepsilon^{\mu\nu\rho\sigma}\,\mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega,\mathbf{A})\Big) = \int Db\ \exp\left[i\int d^4x\sqrt{-g}\,\frac{1}{\sqrt{3}}\,b(x)\Big(\mathcal{H}_{\nu\rho\sigma}(x)^{\varepsilon^{\mu\nu\rho\sigma}}_{;\mu} - \mathcal{G}(\omega,\mathbf{A})\Big)\right] =$$

$$\int Db\ \exp\left[-i\int d^4x\sqrt{-g}\,\Big(\partial^\mu b(x)\,\frac{1}{\sqrt{3}}\,\epsilon_{\mu\nu\rho\sigma}\,\mathcal{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}}\,\mathcal{G}(\omega,\mathbf{A})\Big)\right]$$

$$\mathcal{Z} = \int DH\,Db\,\exp(-H\wedge *H + c_1 b(dH-\mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

KR-axion anomalous CP-Violating interaction

Fermions and (generic) Torsion

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

$$\mathcal{L} = \sqrt{-g}\left(i\,\bar{\psi}\,\gamma^a D_a \psi - m\,\bar{\psi}\psi\right)$$

$$\gamma^a\gamma^b\gamma^c = \eta^{ab}\,\gamma^c + \eta^{bc}\,\gamma^a - \eta^{ac}\,\gamma^b - i\,\epsilon^{dabc}\,\gamma_d\,\gamma^5$$

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right),$$

$$g_{\mu\nu} = e^a_\mu\,\eta_{ab}\,e^b_\nu$$

$$\omega_{bca} = e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e^\gamma_c e^\mu_a\right).$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \tfrac{i}{2}\left[\gamma^a, \gamma^b\right]$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar{\psi}\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi,$$

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a\right)$$

If **torsion** then $\Gamma_{\mu\nu} \neq \Gamma_{\nu\mu}$ ***antisymmetric*** part is the contorsion tensor, contributes

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\overline{\mathcal{D}}_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\sigma^{bc}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

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$$\overline{\mathcal{D}}_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\sigma^{bc}$$

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contorsion

$$\sigma^{ab} = \tfrac{i}{2}\left[\gamma^a, \gamma^b\right]$$

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$\gamma^a\gamma^b\gamma^c = \\ \eta^{ab}\,\gamma^c + \eta^{bc}\,\gamma^a - \eta^{ac}\,\gamma^b - i\,\epsilon^{dabc}\,\gamma_d\,\gamma^5$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

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contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$H_{cab}$$

Non-trivial contributions to ***B^μ***

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\boxed{S_\psi \ni \int d^4x B_a\,\overline{\psi}\gamma^a\gamma^5\psi}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$B^d \sim \epsilon^{abcd}H_{bca}$$

$$H_{cab}$$

Non-trivial contributions to ***B***$^\mu$

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION

$$\boxed{S_\psi \ni \int d^4x B_a\,\overline{\psi}\gamma^a\gamma^5\psi}$$

$$B^d \sim \epsilon^{abcd}H_{bca}$$

$$\boxed{-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

***b(x)* = KR (gravitational) axion**

Non-trivial contributions to ***B^μ***

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^{\mu}_{\nu\rho} \;=\; \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION → **AXION-LIKE CP-VIOLATING INTERACTION**

$$\boxed{S_\psi \ni \int d^4x B_a\,\overline{\psi}\gamma^a\gamma^5\psi} \rightarrow \boxed{-\int d^4x\sqrt{-g}\,\partial_\alpha b\,\big(\overline{\psi}\,\gamma^\alpha\,\gamma^5\,\psi\big)}$$

Universal (gravitational) Coupling

$$B^d \sim \epsilon^{abcd}H_{bca} \rightarrow \boxed{-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

b(x) **= KR (gravitational) axion**

Non-trivial contributions to ***B^μ***

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} \;=\; \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\, \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}\, e_{b\lambda}\partial_a e^\lambda_c,$$ vielbeins

$$J^{5\,\mu} = \bar{\psi}_j\, \gamma^\mu\, \gamma^5 \psi_j$$

Axial Current

All fermion species

KR-axion anomalous CP-Violating interaction

torsion

cf. classically in 4 dim:
(duality relationship)

$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}\, H^{\mu\nu\rho}}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c,$$

vielbeins

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

All fermion species

KR-axion anomalous CP-Violating interaction

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cf. classically in 4 dim:
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$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}\,H^{\mu\nu\rho}}$$

NB: Anomalies: (CHIRAL)

Classically conserved current $J^{\mu 5}$
AXIAL FERMION CURRENT
CEASES to be conserved @ a quantum level

CHIRAL FERMIONS IN LOOP:

$$\boxed{\nabla_\mu J^{\mu 5} \propto c_1 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad c_1 \in \mathbb{R}}$$

$J^{\mu 5} \equiv \bar{\Psi}_j \gamma^\mu \gamma^5 \Psi_j$, $j = 1 \dots N$ SPECIES

chiral fermion $\gamma^5 \Psi_j = \mp \Psi_j$ (LEFT OR RIGHT HANDED)

$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$,

$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,F^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$\propto \alpha'$

or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c,$$

vielbeins

$\propto \alpha'$

Axial Current

All fermion species

KR-axion anomalous CP-Violating interaction

torsion

cf. classically in 4 dim:
(duality relationship)

$$\boxed{-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c,$$

vielbeins

4-fermion contact interaction characteristic of (integrating out) torsion

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

All fermion species

torsion

cf. classically in 4 dim:
(duality relationship)

$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}\,H^{\mu\nu\rho}}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S^{Free}_{Dirac} \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\,\partial_a\,e^\lambda_c;$$

vielbeins

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

torsion

cf. classically in 4 dim:
(duality relationship)

$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}\,H^{\mu\nu\rho}}$$

Inclusion of Fermions

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c;$$

vielbeins

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

Kalb-Ramond (KR) or string-model independent (``gravitational”) axion

torsion

cf. classically in 4 dim:
(duality relationship)

$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[- \frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\, \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots \Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\, \Big(\quad \cdot\, \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16} \int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots \Big] + \dots$$

or Majorana

$$J^{5\,\mu} = \bar{\psi}_j\, \gamma^\mu\, \gamma^5 \psi_j$$

All fermion species

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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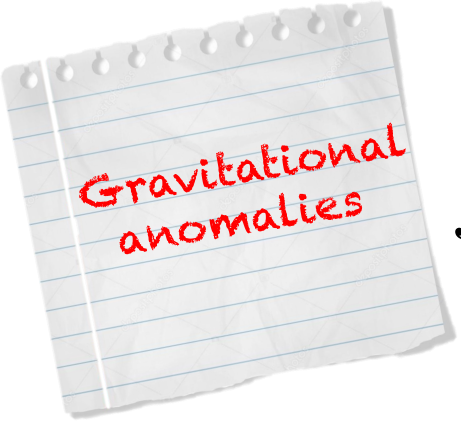
$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x\,\sqrt{-g}\,b(x)\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x\sqrt{-g}\,b\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\Big] = 4\int d^4x\sqrt{-g}\,\mathcal{C}^{\mu\nu}\,\delta g_{\mu\nu} = -4\int d^4x\sqrt{-g}\,\mathcal{C}_{\mu\nu}\,\delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma\left(\varepsilon^{\sigma\mu\alpha\beta}R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta}R^{\mu}{}_{\beta;\alpha}\right) + v_{\sigma\tau}\left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu}\right)\Big] = -\frac{1}{2}\Big[\left(v_\sigma\,\widetilde{R}^{\lambda\mu\sigma\nu}\right)_{;\lambda} + (\mu \leftrightarrow \nu)\Big]$$

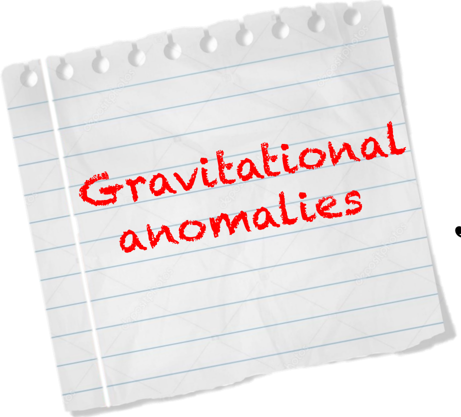
$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma},\;\; v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless

$$g_{\mu\nu}\,\mathcal{C}^{\mu\nu} = 0$$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x\,\sqrt{-g}\,b(x)\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big)$$

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Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma\Big(\varepsilon^{\sigma\mu\alpha\beta}R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta}R^{\mu}{}_{\beta;\alpha}\Big) + v_{\sigma\tau}\Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu}\Big)\Big] = -\frac{1}{2}\Big[\Big(v_\sigma\,\widetilde{R}^{\lambda\mu\sigma\nu}\Big)_{;\lambda} + (\mu\leftrightarrow\nu)\Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma},\quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

not necessarily positive contributions to vacuum energy

Traceless

$$g_{\mu\nu}\,\mathcal{C}^{\mu\nu} = 0$$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\boxed{\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\Big\downarrow$$

$$\boxed{\kappa^2 T^{\mu\nu}_{\rm matter\,;\mu} = - \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0}$$

Diffeomoprphism invariance breaking by gravitational anomalies?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\boxed{\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\Downarrow$$

$$\boxed{\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0}$$

No problem with diffeo

Conserved Modified stress-energy tensor

4. Primordial Gravitational Waves, Anomaly condensates – The role of Supersymmetry

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$\begin{aligned} S_B^{\rm eff} &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big] \\ &= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,, \end{aligned}$$

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

NB:

absent before formation of GW

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[\boxed{-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b} + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\cancel{R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\cancel{\mathcal{K}^\mu + \dots}\Big]\,,$$

No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM,
Solà (2019-20)

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$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[\boxed{-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b} + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\cancel{R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}} + \dots\Big]$$

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No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

c.f. Zeldovich
but for baryons
in his model;
cf. also Chavanis

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves
Potential Origins in pre-inflationary era?

NEM,Solà
EPJ-ST
(2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM,
Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\, \kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \dots \Big]$$

$$= \int d^4x\, \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\, \frac{\alpha'}{96\, \kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\, ,$$

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

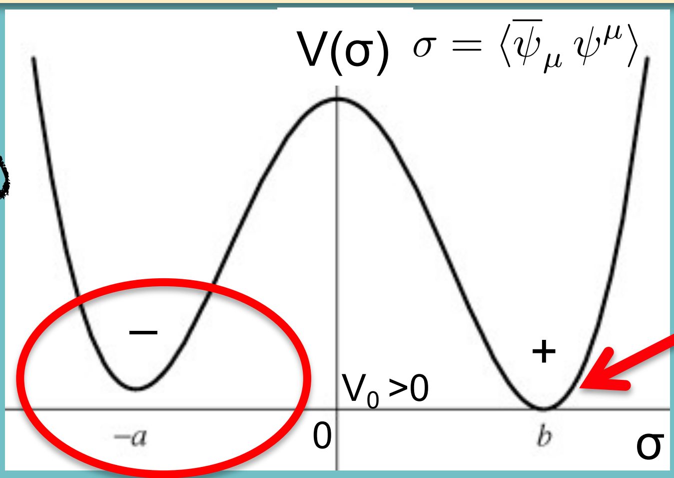
Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM,Solà
EPJ-ST
(2020)

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

Role of (Local) Supoersymmetry



SUGRA broken dynamically
gravitino
Condensate
stabilised →
RVM GW-induced Inflation

Statistical bias (percolation) in
occupation probabilities of the +,- vacua

Lalak, Ovrut,
Lola, G. Ross,
Thomas

Primordial Gravitational Waves
Potential Origins in pre-inflationary era?

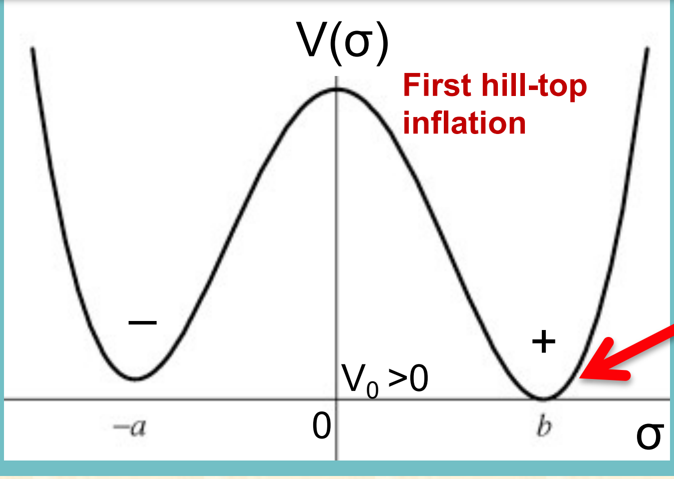
Collapse/collisions of Domain walls formed in theories with (approxmate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM,Solà
EPJ-ST
(2020)

Ellis, NEM,
Alexandre,
Houston

The Model in Early Universe:
only gravitational d.o.f. ($b, g_{\mu\nu}, \psi_\mu$)

Basilakos, NEM, Solà (2019-20)



SUGRA broken dynamically gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

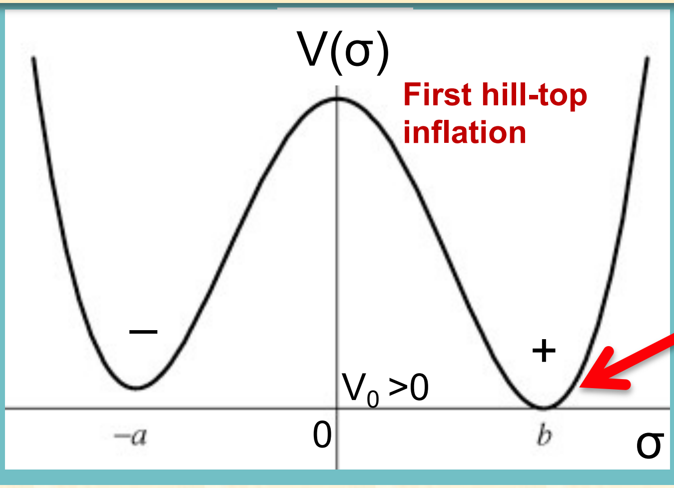
First Hill-top inflation = finite life –time → System **tunnels** to **RVM inflationary vacuum (GW condense)**

NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



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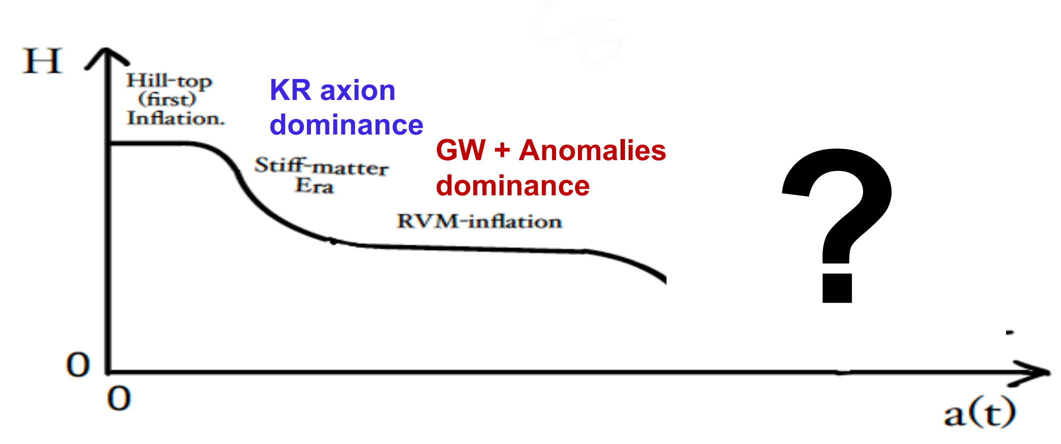
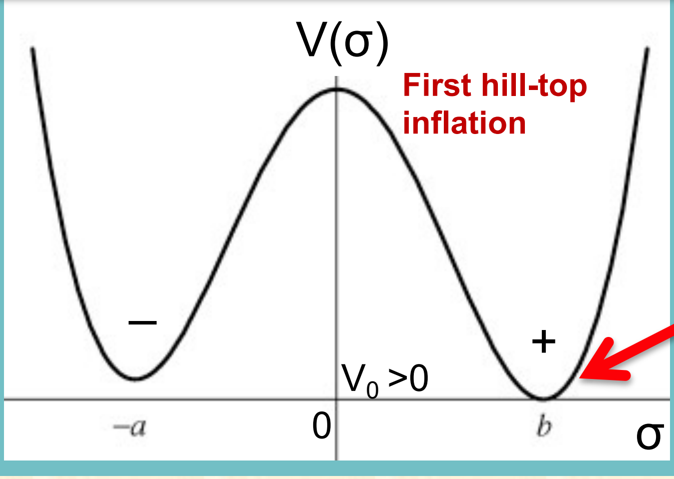


NEM,Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe:
only gravitational d.o.f. ($b, g_{\mu\nu}, \psi_\mu$)

Basilakos, NEM, Solà (2019-20)

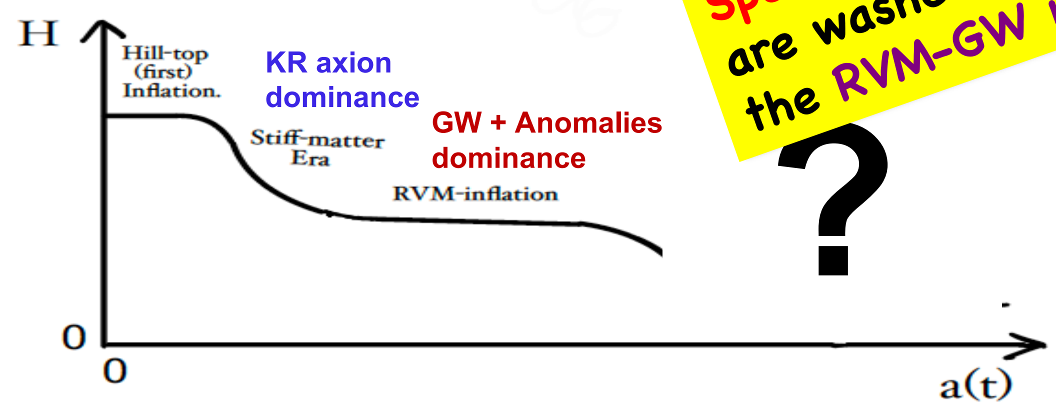


SUGRA broken dynamically gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

First Hill-top inflation = finite life –ti

System **tunnels** to **RVM inflationary va**



First inflation ensures any Spatial inhomogeneities are washed out before the RVM-GW inflation

NEM,Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

4b. Spontaneous Lorentz & CPT Violation by axion backgrounds and RVM Inflation

The Parts

Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz + CPT Violation from anomaly condensates

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Non-trivial if GW present

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves,

&

De Sitter space times &

Spontaneous Lorentz & CPT Violation

The Model in Early Universe:
only gravitational d.o.f. ($b, g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves →
Condensate < …> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe:
only gravitational d.o.f. ($b, g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational
Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

$$+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Condensate < …> of
Gravitational Anomalies

Cosmological-
Constant-like

Mild time
Dependence
through H(t)

$$gCS = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe: only gravitational d.o.f. ($b, g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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$$+\sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Condensate < ...> of Gravitational Anomalies

Cosmological-Constant-like

Mild time Dependence through H(t)

$$g\mathcal{CS} = -\sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle\partial_\mu b\,\mathcal{K}^\mu\rangle + \text{quantum flcts.}$$

Up to boundary terms

$$\sqrt{-g}\,\mathcal{K}^{\mu}(\omega)_{;\mu}$$

Effective action contains **CP violating axion-like coupling**

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad {}'\Big) + \dots\Big]$$

(i) Assume de Sitter era, first, to discuss anomaly condensate in the presence of GW perturbation

(ii) deduce RVM vacuum behaviour

and

(iii) Inflation is obtained self consistently from RVM evolution

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad {}'\Big) + \ldots\Big]$$

$$\begin{aligned} ds^2 = dt^2 - a^2(t)\Big[&(1 - h_+(t,z))\,dx^2 \\ &+ (1 + h_+(t,z))\,dy^2 \\ &+ 2h_\times(t,z)\,dx\,dy + dz^2\Big] \end{aligned}$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

b(x)=b(t)

Alexander, Peskin, Sheikh -Jabbari

n* = proper number density of sources of GW(assumed of O(1))

μ = UV k-momentum Cut-off

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \boxed{\langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2 n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)}$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

H ≈ const. (inflation)

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

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$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

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H ≈ const. (inflation)

Solutions (backgrounds) to the Eqs of Motion

$$\boxed{\alpha' = M_s^{-2}}$$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right]=0 \quad\Rightarrow\quad \dot{\bar{b}}=\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0$$

n* = proper number density of sources of GW(assumed of O(1))

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right)=\boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle=\frac{16}{a^4}\,\kappa^2 n^*\int^{\mu}\frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta+O(\Theta^3)}$$

$$\Theta=\sqrt{\frac{2}{3}\,\frac{\kappa^3}{12}}\,H\dot{b}\propto\mathcal{K}^0$$

time evolution of Anomaly

μ = UV k-momentum Cut-off

$$\boxed{\mathcal{K}^0(t)\simeq\mathcal{K}^0_{\rm begin}(0)\,\exp\left[-3Ht\left(1-0.73\times10^{-4}\,n^*\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]}$$

Solutions (backgrounds) to the Eqs of Motion

$$\boxed{\alpha' = M_s^{-2}}$$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^0$$

n* = proper number density of sources of GW(assumed of O(1))

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$$\boxed{\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3H\underbrace{\left(1 - 0.73\times 10^{-4}\,n^*\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)}_{\approx\, 0}\right]}$$

Solutions (backgrounds) to the Eqs of Motion

$$\boxed{\alpha' = M_s^{-2}}$$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0$$

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$$\boxed{\frac{\mu}{M_s} \simeq 15\,(n^\star)^{-1/4}\left(\frac{M_{\mathrm{Pl}}}{H}\right)^{1/2}} \Rightarrow \mathcal{K}^0 = \mathrm{const.}$$

Planck Data $H/M_{\mathrm{Pl}} < 10^{-4}$ ⇒

to ensure constant anomaly

$\mu = O(10^3\,(n^*)^{-1/4})\,M_s \leq M_{planck}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right]=0 \quad\Rightarrow\quad \boxed{\dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0\sim\text{constant}}$$

n* = proper number density of sources of GW(assumed of O(1))

$$\dot{\bar{b}}\propto\epsilon^{ijk}H_{ijk}=\text{constant}$$

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right)=\boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle=\frac{16}{a^4}\kappa^2 n^*\int^{\mu}\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta+O(\Theta^3)}$$

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$$\boxed{\mathcal{K}^0(t)\simeq\mathcal{K}^0_{\text{begin}}(0)\exp\left[-3Ht\left(1-0.73\times10^{-4}n^*\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]}$$

$$\boxed{\frac{\mu}{M_s}\simeq15\,(n^\star)^{-1/4}\left(\frac{M_{\rm Pl}}{H}\right)^{1/2}} \quad\Rightarrow\quad \mathcal{K}^0=\text{const.}$$

Spontaneous LV (+ CPTV) solution ⚠

Planck Data $H/M_{\rm Pl}<10^{-4}$ ⇒ to ensure constant anomaly

$\mu = O(10^3\,(n^*)^{-1/4})\,M_s \leq M_{planck}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right]=0 \quad\Rightarrow\quad \boxed{\dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0\sim\text{constant}}$$

n* = proper number density of sources of GW(assumed of O(1))

$$\dot{\bar{b}} \propto \epsilon^{ijk}H_{ijk} = \text{constant}$$

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \boxed{\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2 n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)}$$

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$$\boxed{\mathcal{K}^0(t)\simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3Ht\left(1-0.73\times 10^{-4}\, n^*\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]}$$

$$\boxed{\frac{\mu}{M_s}\simeq 15\,(n^\star)^{-1/4}\left(\frac{M_{\rm Pl}}{H}\right)^{1/2}} \Rightarrow \boxed{\mathcal{K}^0 = \text{const.}}$$

No transplanckian modes !

Planck Data $H/M_{\rm Pl} < 10^{-4}$ ⇒

to ensure constant anomaly

$\mu = O(10^3\ (n^*)^{-1/4})\, M_s \leq M_{planck}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk} H^{ijk}$$

≈ constant torsion

Using **slow-roll assumption** *b*

$$\varepsilon = \frac{1}{2}\,\frac{1}{(H M_{\rm Pl})^2}\,\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

$$\boxed{\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H}$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right]=0 \quad\Rightarrow\quad \boxed{\dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0\sim\text{constant}}$$

Using **slow-roll assumption** *b*

If n* of O(1)), otherwise M_s free parameter, μ = M_s

$$2.6 \times 10^{-5} M_{Pl} < M_s \leq 10^{-4} M_{Pl}$$

NEM + Solà (2021)

$$\boxed{\dot{\bar{b}}\sim\sqrt{2\varepsilon}\,M_{\rm Pl}\,H\sim0.14\,M_{\rm Pl}\,H}$$

$$H=H_{\rm infl}\simeq\text{const.}$$

Constant anomaly during inflation, no transplanckian modes !

NB:

$$\Theta\equiv\sqrt{\frac{2}{3}}\frac{\alpha'\kappa}{12}H\dot{\bar{b}}\ll1.$$

$$\dot{\bar{b}}\ll H/\kappa$$

$H/M_s \ll 3.83$, $H\simeq(10^{-5}-10^{-4})M_{\rm Pl}$

$$\boxed{\frac{M_{\rm Pl}}{M_s}\ll3.83\times(10^4-10^5).}\qquad M_s\leq10^{-4}M_{\rm Pl}$$

The Parts

Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz + CPT Violation from anomaly condensates

Dynamical Inflation of Running Vacuum Model type without external inflatons

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right]=0 \quad\Rightarrow\quad \boxed{\dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0\sim\text{constant}}$$

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Planck Data

$$\boxed{\dot{\bar{b}}\sim\sqrt{2\varepsilon}\,M_{\rm Pl}\,H\sim 0.14\,M_{\rm Pl}\,H}$$

$$H=H_{\rm infl}\simeq\text{const.}$$

@ end of Inflationary era

$$b_{\rm end}\sim b_{\rm initial}+0.14\,M_{\rm Pl}\,H_{\rm infl}\,t_{\rm end},$$

$$t_{\rm end}H_{\rm infl}\sim\mathcal{N}=\text{e}-\text{foldings}$$

~ 55-70

Fix $b_{initial}$ *to arrange approx. constant condensate during appropriate time period (inflation)*

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

e–foldings

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

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Dark Energy
(“running
vacuum model
(RVM) type”)

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Dark Energy
("running
vacuum model
(RVM) type")

Equation of state :

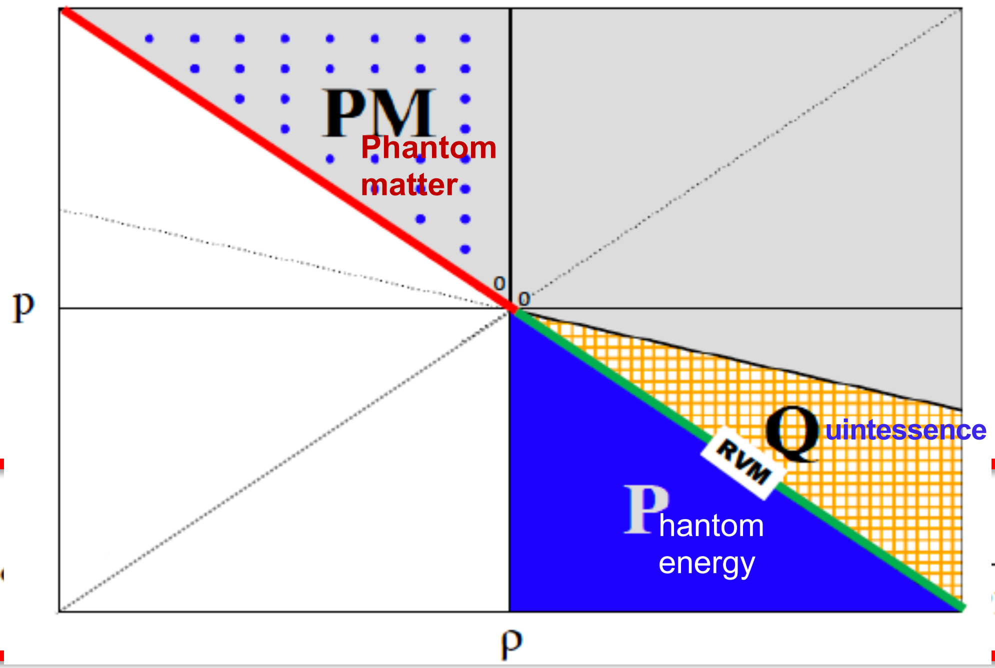
$0 > \rho_b + \rho_{gCS} = -\,(p_b + p_{gCS})$ cf. phantom ``matter"

$0 < \rho_\Lambda = -p_\Lambda$ → dominates →

$0 < \rho_b + \rho_{gCS} + \rho_\Lambda = -\,(p_b + p_{gCS} + p_\Lambda)$ true RVM vacuum

Gravitational Anomaly Condensates → Dynamical Inflation

NEM, Solà



$$10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive Cosmological Constant-like

ρ_t

$$\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7\, \big(\frac{H}{M_{\rm Pl}}\big)^4\Big] > 0$$

Dark Energy

("running vacuum model (RVM) type")

Equation of state :

$0 > \rho_b + \rho_{gCS} = -\,(p_b + p_{gCS}\,)$ cf. phantom "matter"

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Basilakos, NEM, Solà

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RVM-like terms
drive inflation
contain scalar d.o.f.
from the anomaly
condensate

Dark Energy
("running
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(RVM) type")

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{b}^2 \sim 10^{-2}$

5. Running-Vacuum Model Cosmology – Inflation without external inflatons

The Parts

Shapiro + Solà
Solà, ...

Dark Energy
("running
vacuum model
(RVM) type")

$$\boxed{\rho_\Lambda^{\rm RVM} = \kappa^{-2}\,\Lambda + c_1 H^2 + c_2 H^4 + \dots \equiv \kappa^{-2}\,\Lambda(t)}$$

$$\Lambda \equiv 3\,c_0 \qquad c_1 = 3\nu\kappa^{-2},\ c_2 = 3\alpha\kappa^{-2}\,H_I^{-2},$$

$H_I \sim 10^{-5}\kappa^{-1}$ (current pheno)

Vacuum energy density assumed de Sitter like but with time-dependent Cosmological parameter Λ(t) :

$$\rho^\Lambda_{\rm RVM}(t) = \Lambda(t)/\kappa^2 \qquad \kappa = \sqrt{8\pi \mathrm{G}} = M_{\rm Pl}^{-1}$$

$$\boxed{p(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)}$$

NB: Renormalization-Group-like equation for the evolution of **vacuum energy density**
Hubble parameter H(t) ←→ RG scale *μ*

$$\frac{d\,\rho_\Lambda^{\rm RVM}(t)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\left[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\left(\frac{H^6}{M_i^2}\right)\right]$$

general covariance →
even powers of *H*

The Parts

Shapiro + Solà
Solà, ...

Dark Energy
("running vacuum model (RVM) type")

$$\rho^{\rm RVM}_{\Lambda} = \kappa^{-2}\,\Lambda + c_1 H^2 + c_2 H^4 + \dots$$
$$\equiv \kappa^{-2}\,\Lambda(t)$$

$c_1 = 3\nu\kappa^{-2}$, $c_2 = 3\alpha\kappa^{-2}\, H_I^{-2}$,

$\sim 10^{-5}\kappa^{-1}$ (current pheno)

Vacuum energy de me-dependent Cosmological
parameter Λ(

$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$

$\rho(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)$

Any dH/dt ≈ −(1 + q) H², decel. parameter q ≈ const in each cosmic epoch

NB: Renor roup-like equation for the evolution of **vacuum energy density**
Hubble parameter H(t) ←→ RG scale *μ*

$$\frac{d\,\rho^{\rm RVM}_{\Lambda}(t)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\left[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\left(\frac{H^6}{M_i^2}\right)\right]$$

general covariance →
even powers of *H*

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \Rightarrow \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\boxed{\dot{H} + \frac{3}{2}(1+\omega) H^2 \left(1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2}\right) = 0}$$

Solution

$$\boxed{H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}}$$

$D > 0$

Early de Sitter (unstable) $D a^{4(1-\nu)} \ll 1 \Rightarrow H^2 = (1-\nu) H_I^2 / \alpha$

Radiation $D a^{4(1-\nu)} \gg 1 \Rightarrow H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $H^2(a) = H_0^2 \left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

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$$\nabla^\mu T_{\mu\nu} = 0 \Rightarrow \dot{\rho}_m + 3(1+\omega)H\rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

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Solution without fundamental inflatons

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable)

$$Da^{4(1-\nu)} \ll 1 \Rightarrow H^2 = (1-\nu)H_I^2/\alpha$$

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$$Da^{4(1-\nu)} \gg 1 \Rightarrow H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$$

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$\tilde{\Omega}_{\Lambda 0}$ dominant

Gravitational Anomaly Condensates → Dynamical Inflation

NEM, Solà

Cannot obtain such terms
in **ordinary Quantum Field Theories**
You need the **condensate of**
the gravitational anomalies
which have **CP-violating couplings**
with the **gravitational axions**

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + 1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4\Big] > 0$$

RVM-like terms
drive inflation
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But slow roll is due to the KR axion field $\epsilon :\sim \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\overline{b}}^2 \sim 10^{-2}$

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Another important
role of CP-violation
in Early Universe

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7\times 10^{-3}\Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big)\times 10^7\,(\frac{H}{M_{\rm Pl}})^4\Big] > 0$$

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$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon\, \mathcal{N}\, H^4 > 0$$

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

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Negative coefficient ν < 0
due to CS anomaly
in early Universe, unlike
late-era RVM

RVM-like terms
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Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

NB

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

approximately de Sitter provided during the duration of inflation ⚠

$$b(t) = \overline{b}(0) + 0.14 M_{\rm Pl}\, H\, t_{end} \simeq \overline{b}(0)$$

order of magnitude

< 0

N=e-folds

beginning of inflation

↓

$$|\overline{b}(0)| \gtrsim \mathcal{O}(10)\, M_{\rm Pl}$$

Distance-swampland conjectures?

Slow running of db/dt can be constrained by data

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

Undiluted KR axion background at the end of Inflation

@ end of Inflationary era

$$\boxed{\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H}$$

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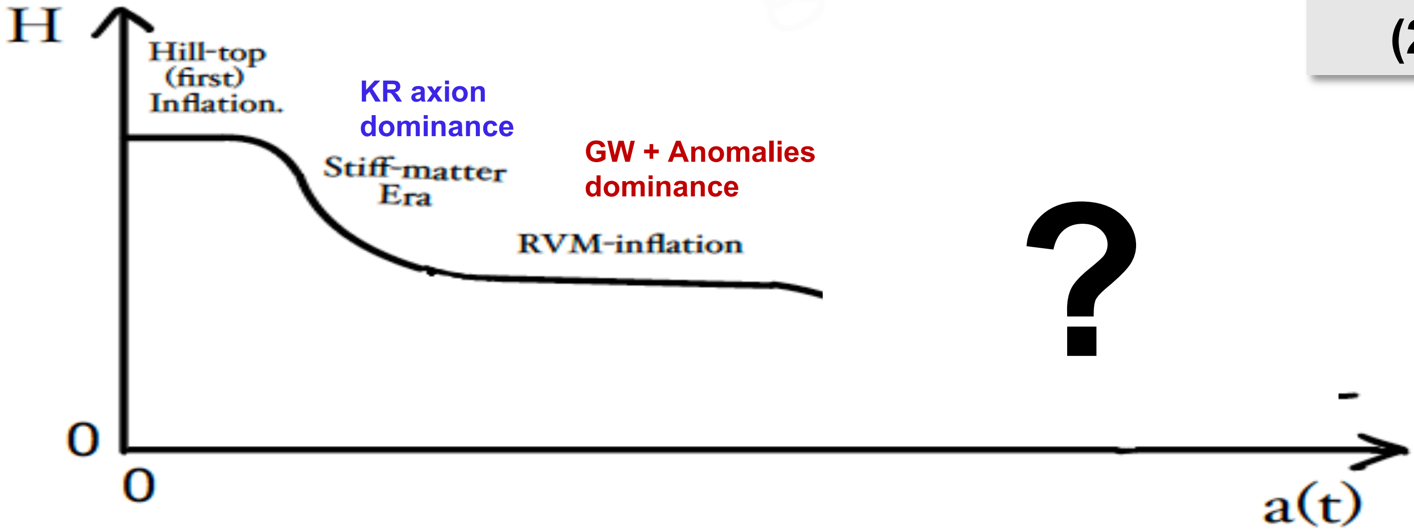
Important for Leptogenesis @ radiation era

6a. Post Inflationary Eras
&
Cosmic Evolution
of the stringy RVM

Post-RVM-Inflation Eras & Evolution

NEM,Solà
EPJ-ST
(2020)



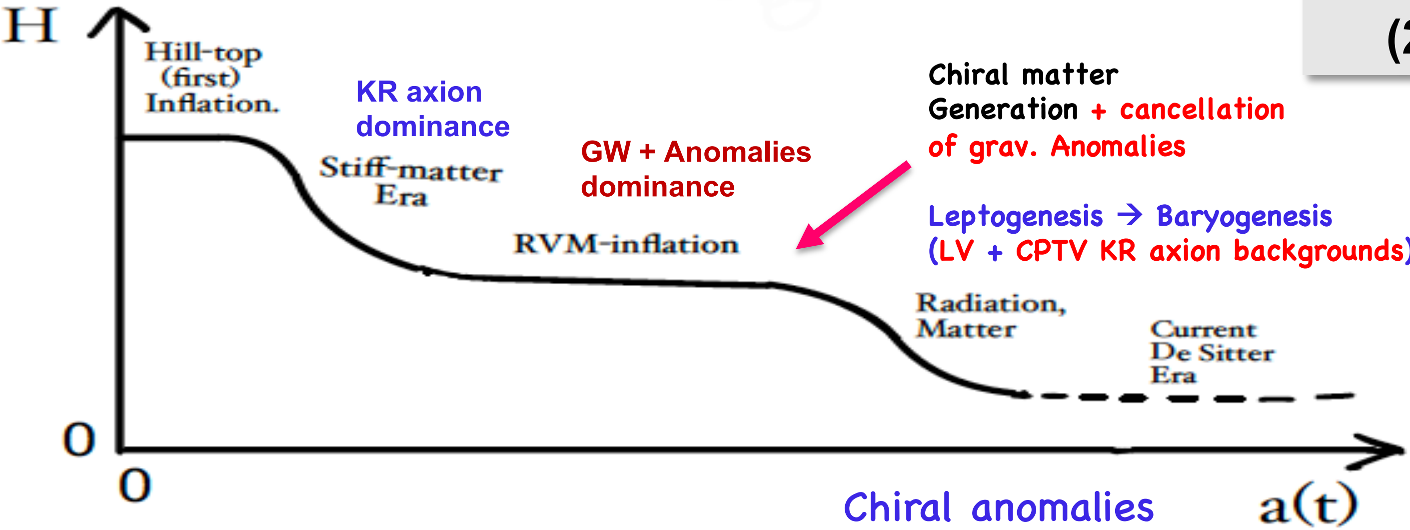
Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

NEM,Solà
EPJ-ST
(2020)



KR axion dominance

GW + Anomalies dominance

Chiral matter Generation + cancellation of grav. Anomalies

Leptogenesis → Baryogenesis
(LV + CPTV KR axion backgrounds)

NEM, Sarkar
+ De Cesare,
Bossingham

Chiral anomalies
Remain in matter era

KR axion mass generation through QCD instantons (Dark Matter)

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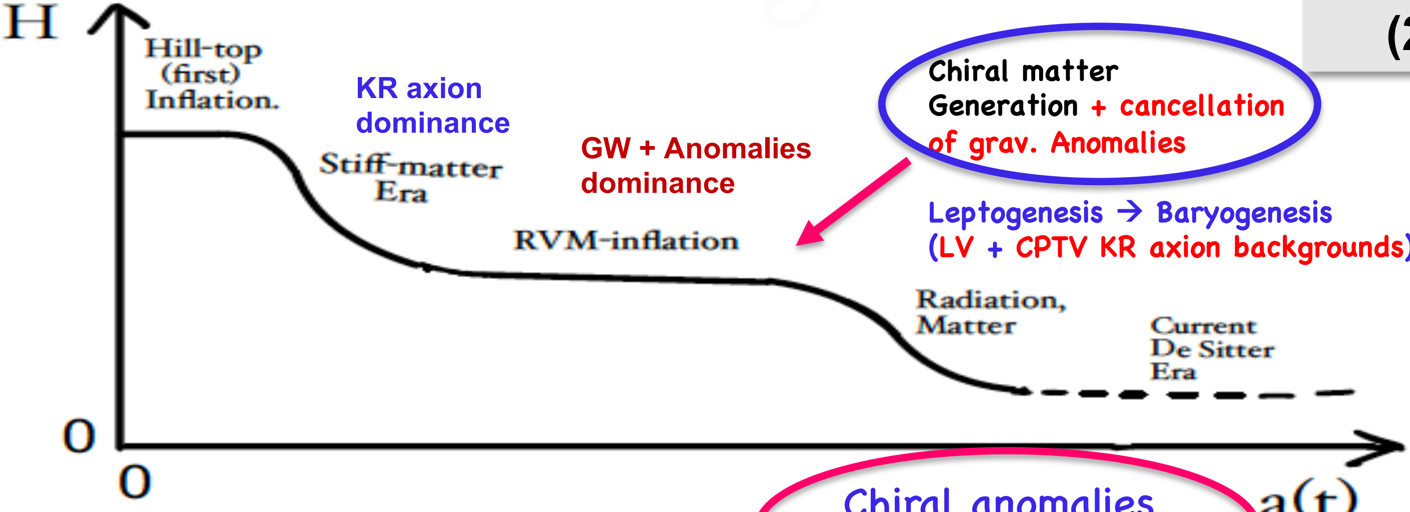

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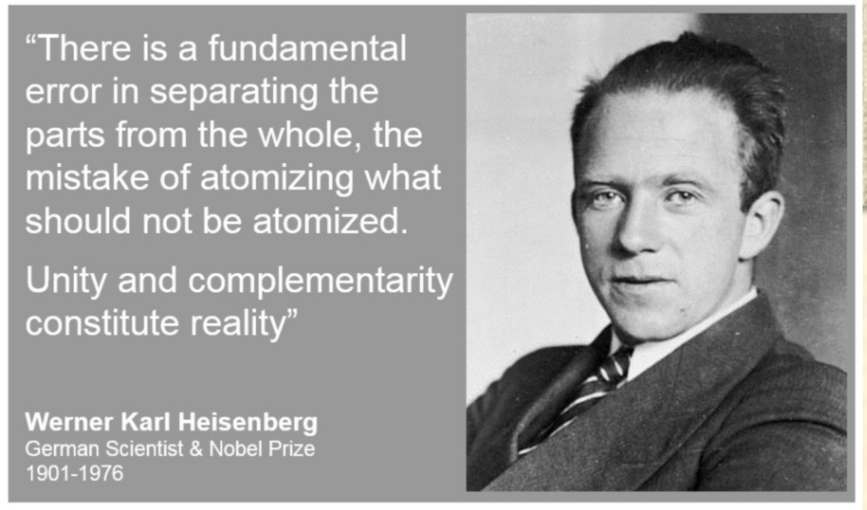
The Whole

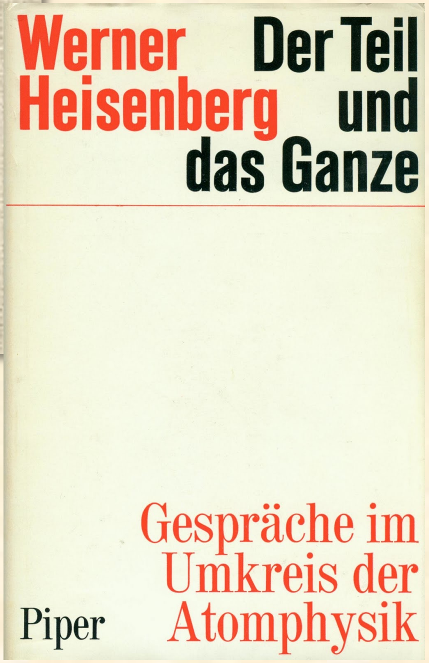
Stringy-RVM Cosmological Evolution



"There is a fundamental error in separating the parts from the whole, the mistake of atomizing what should not be atomized.

Unity and complementarity constitute reality"

Werner Karl Heisenberg
German Scientist & Nobel Prize
1901-1976



Werner Heisenberg
Der Teil und das Ganze

Gespräche im Umkreis der Atomphysik

Piper

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

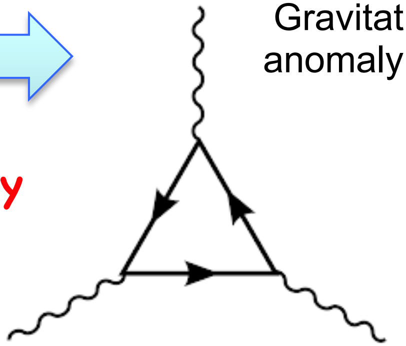
forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

NEM, Sarkar + De Cesare, Bossingham

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Basilakos, NEM, Solà

Cosmic Time

forward direction

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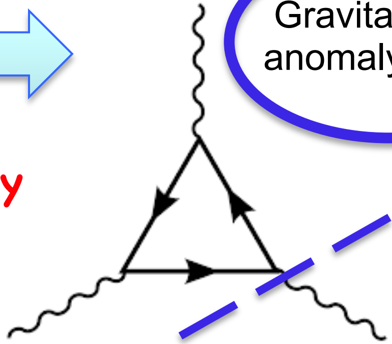

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Undiluted constant KR axial backround

$B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$

$\dot{b} \sim \sqrt{2\varepsilon}\, M_{\rm Pl} H \sim 0.14\, M_{\rm Pl} H$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

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Basilakos, NEM, Solà

Cosmic Time

forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

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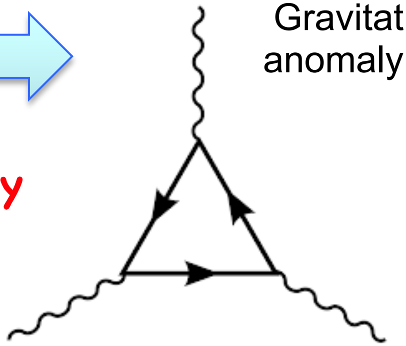


Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$ ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Chiral anomalies @ QCD era (instantons)

Summary of (stringy-RVM) Cosmological Evolution

Cosmic
Time

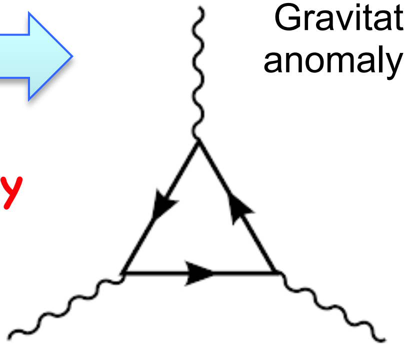
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial
Gravitational
Waves



Gravitational
anomaly (GA)

Undiluted constant
KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter
generation
@ inflation exit

From a pre-inflationary
era after Big-Bang

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by
RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathscr{O}(10^{-2})$$ **Phenomenology**

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

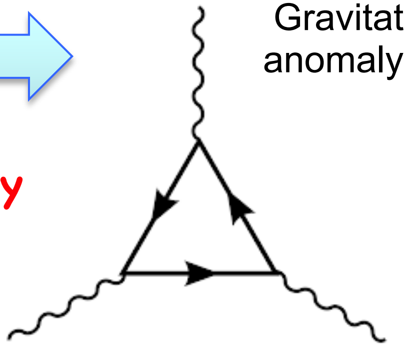
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\to\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era

Possible potential (mass) generation f b – → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Phenomenology

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

Summary of (stringy-RVM) Cosmological Evolution

Cosmic
Time

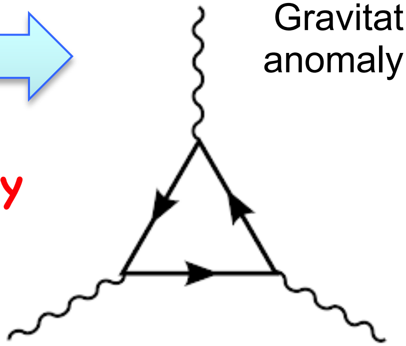
Basilakos, NEM, Solà

Big-Bang, pre-inflationary phase (broken Sugra)

forward direction

RVM Inflationary (de Sitter) Phase

Primordial
Gravitational
Waves



Gravitational
anomaly (GA)

Undiluted constant
KR axial backround

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$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter
generation
@ inflation exit

From a pre-inflationary
era after Big-Bang

Radiation Era

$$B_0 \propto \boxed{\dot{\bar{b}} \propto T^3 + \text{subleading}\ (\sim T^2)\ \text{chiral U}(1)\ \text{anomaly terms}}$$

$$\boxed{B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\,{\rm eV}}$$

Consistent with current
bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era

Possible potential (mass) generation f b – → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$H_0 \sim 10^{-42}$ GeV
$\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

$\varepsilon' \sim \varepsilon = \mathscr{O}(10^{-2})$ **Phenomenology**

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

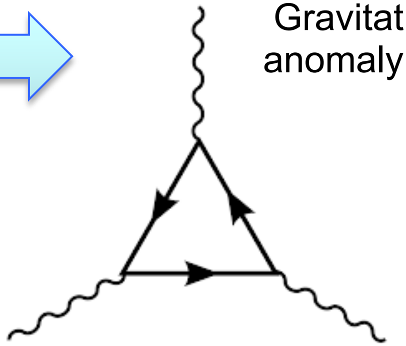
forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\;\phi\,\overline{\ell}$$

B-L conserving sphel

Consistent with current bounds on LV & CPTV

$B_0 < 10^{-2}$ eV,

$B_i < 10^{-22}$ eV

Need to understand Modern Era better

Matter Era

Fo

Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Phenomenology

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

6b. Modern Era & Cosmological data Tension(s) potential alleviation by the stringy RVM

Recall:

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\boxed{\dot{H} + \frac{3}{2}(1+\omega) H^2 \left(1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2}\right) = 0}$$

Solution

$$\boxed{H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}}$$

$D > 0$

Early de Sitter (unstable) $Da^{4(1-\nu)} \ll 1 \Rightarrow H^2 = (1-\nu) H_I^2/\alpha$

Radiation $Da^{4(1-\nu)} \gg 1 \Rightarrow H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $H^2(a) = H_0^2 \left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

Gómez-Valent, Solà

Fit
Cosmological
Data

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2} c_0 \simeq 10^{-122} M_{\rm Pl}^4$$

$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2 + \beta\frac{H_0^4}{M^4_{\rm Pl}}\right), \quad \beta > 0.$$

Not dominant today

Running RVM
Dark Energy

Gómez-Valent, Solà

Fit Cosmological Data

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

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$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2 \cancel{+\beta\frac{H_0^4}{M^4_{\rm Pl}}}\right), \quad \beta > 0.$$

Not dominant today

Running RVM Dark Energy

Gómez-Valent, Solà

Could **Alleviate** Tensions in Data, e.g.
H_0 , σ_8 tensions

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$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

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Running RVM
Dark Energy

Gómez-Valent, Solà

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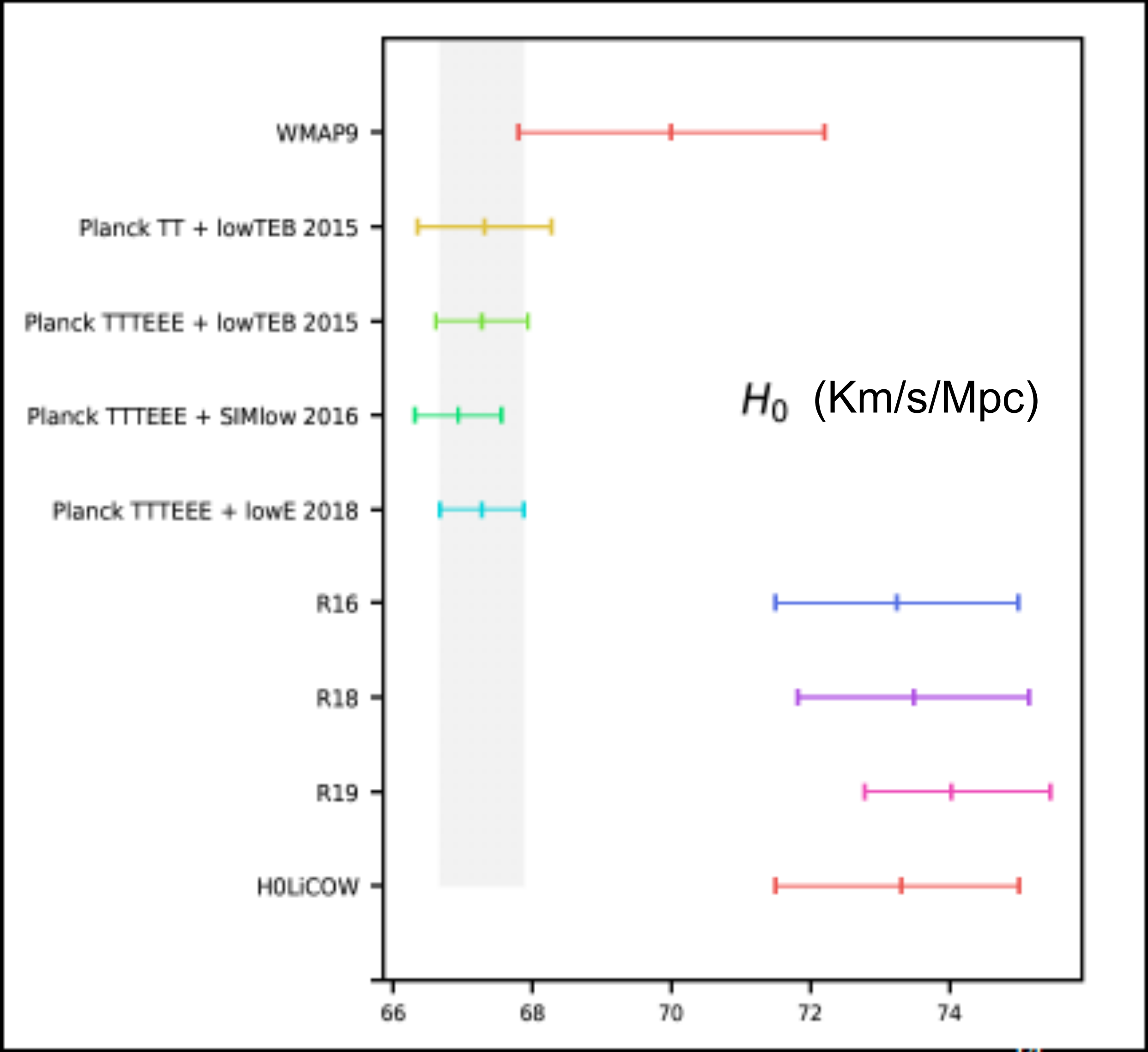
Not dominant today

Running RVM
Dark Energy

The H0 tension

We have two different blocks giving estimates of the Hubble constant in tension with each other:

- CMB (WMAP, Planck, ground based telescopes), BAO, BBN, Pantheon;
- Direct local distance ladder measurements (HST, SH0ES) and Strong lensing (H0LiCOW).

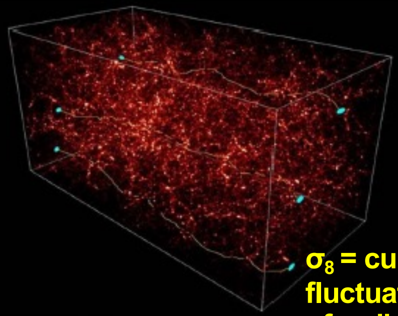


E. Di Valentino

E. Di Valentino

September 10th, 2019
VII Meeting on Fundamental Cosmology
Madrid

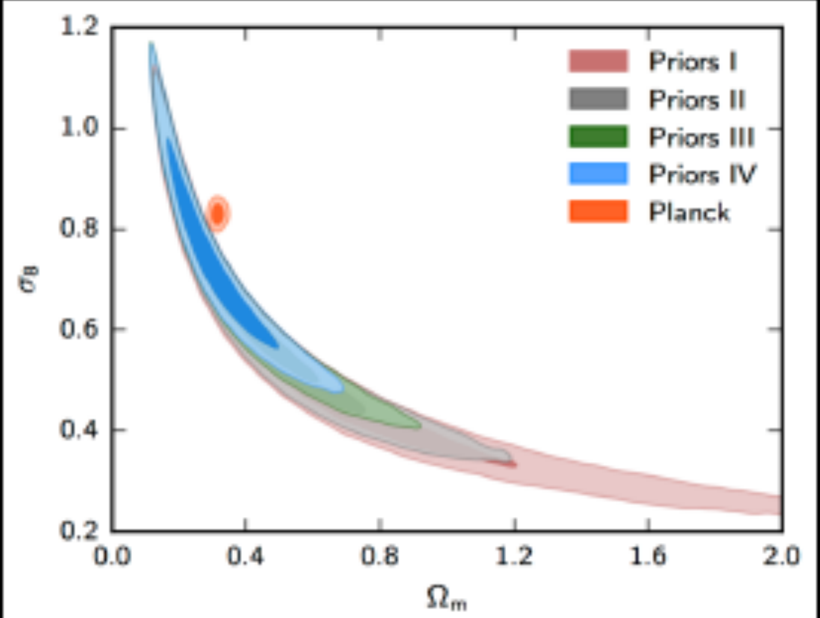
S8 tension



σ_8 = current matter density rms fluctuations within spheres of radius $8h^{-1}$ ($h= H_0/100$ = reduced Hubble constant)

$$S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$$

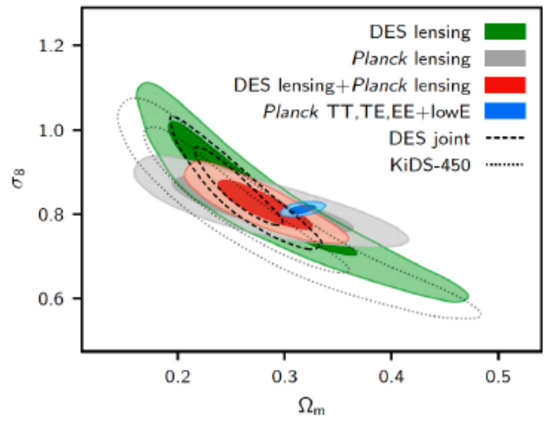
A tension on S8 is present between the Planck data in the ΛCDM scenario and the cosmic shear data.



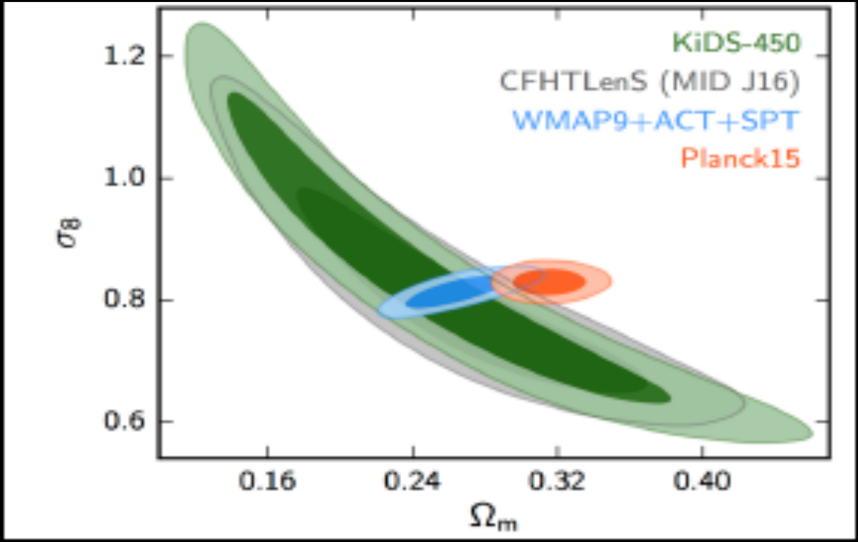
Joudaki et al, arXiv:1601.05786

S8 tension

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



While there is no tension with DES galaxy lensing, a tension at about 2.5 sigma level is present for the DES results that include galaxy clustering.



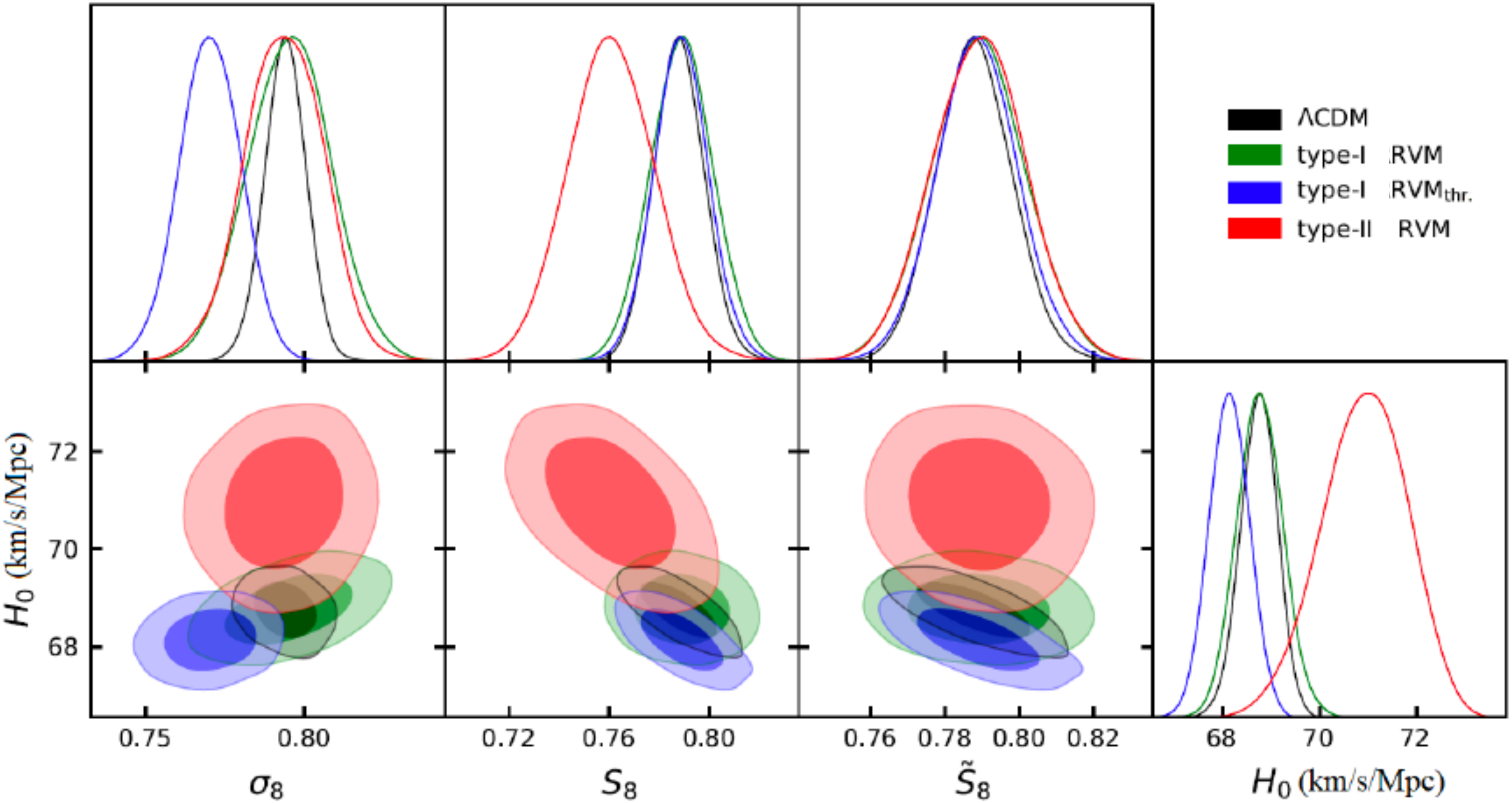
Hildebrandt et al., arXiv:1606.05338.

The S8 tension is at about 2.6 sigma level between the Planck data in the ΛCDM scenario and CFHTLenS survey and KiDS-450.

If tensions
are not due
to statistics

Solà, Gómez-Valent,
De Cruz Perez, Moreno-Pulido,
(Planck 2018 data)

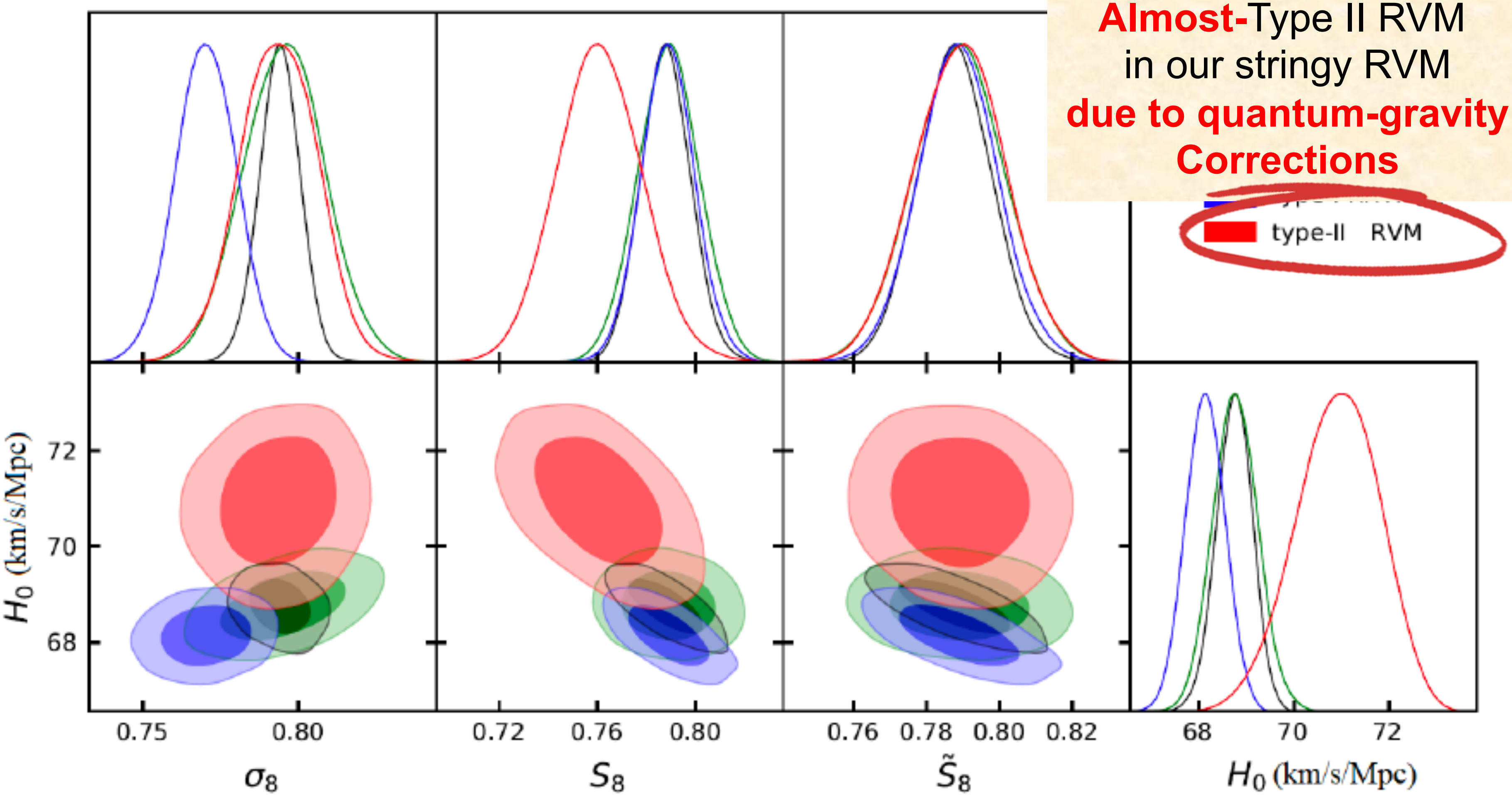
Alleviation of the H_0 , σ_8 tension by RVM model



NEM, Solà (2021)

Integrating out graviton flcts

$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (c_3 + c_4 \ln H)\, H^4 + \Lambda$$



Almost-Type II RVM
in our stringy RVM
due to quantum-gravity Corrections

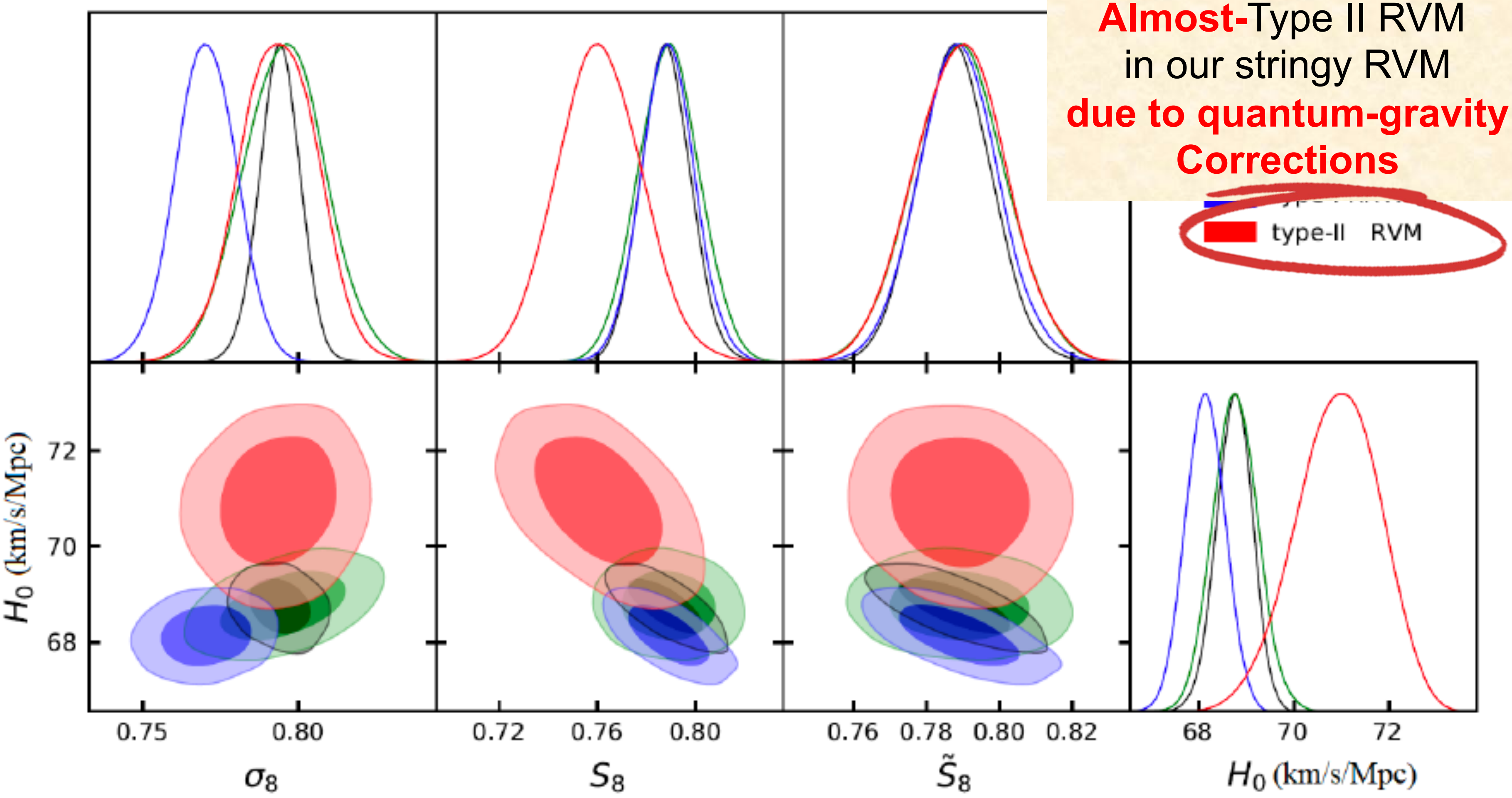
NEM, Solà (2021)

Integrating out graviton flcts

Not Dominant today

Metastable in strings

$$\rho \propto \left(c_1 + c_2 \ln H\right) H^2 + \left(c_3 + c_4 \ln H\right) H^4 + \Lambda$$

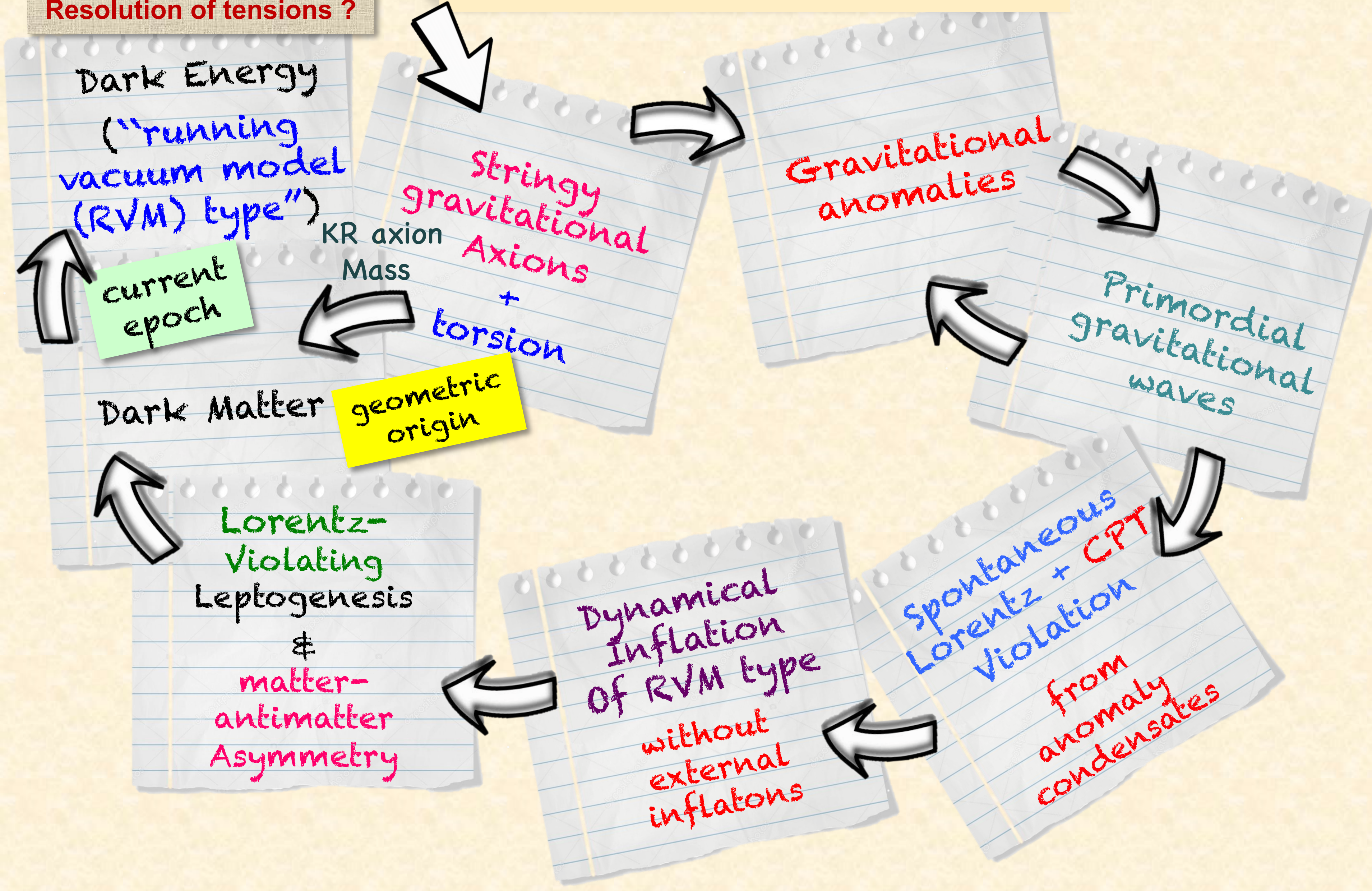


Almost-Type II RVM
in our stringy RVM
**due to quantum-gravity
Corrections**

9. Conclusions & Outlook

The Basic ``Cosmic Cycle’’

Deviations from ΛCDM
Resolution of tensions ?



Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz + CPT Violation
from anomaly condensates

Dynamical Inflation of RVM type
without external inflatons

Lorentz-Violating Leptogenesis
&
matter-antimatter Asymmetry

Dark Matter

geometric origin

KR axion Mass

Dark Energy
(“running vacuum model (RVM) type”)

current epoch

Deviations from ΛCDM
Resolution of tensions ?

The Parts/the Whole

Stringy gravitational Axions + torsion

Gravitational anomalies

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Spontaneous Lorentz + CPT Violation

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Dark Matter

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Dark Energy

("running vacuum model (RVM) type")

current epoch

STRINGY RVM

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic
Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial
Gravitational
Waves

Gravitational
anomaly (GA)

Undiluted constant
KR axial backround

We exist because
of Anomalies !

Paraphrasing
C. Sagan:
we are
anomalously
made of star
stuff !

Leptogenesis induced by
RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

axion Dark matter

Matter Era

Modern de-Sitter Era

RVM-type
Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

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KR axial backround

Primordial
Gravitational
Waves

Gravitational
anomaly (GA)

We exist because
of Anomalies !

OUTLOOK: (i) Incorporate other model-dependent stringy axions → Axiverse
Interesting Cosmology
(*eg* Marsh 2015)
could be ultralight → AION etc

Leptogenesis induced by
RHN (tree-level) decays

Spontaneous

axion Dark matter

Matter Era

Modern de-Sitter Era

RVM-type
Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

ist because
malies !

OUTLOOK: (ii) Look for imprints of the LV & CPTV KR axial background in CMB in early eras.

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type
Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

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RVM Inflationary (de Sitter) Phase

Primordial
Gravitational
Waves

Gravitational
anomaly (GA)

Undiluted constant
KR axial backround

We cause
o !

OUTLOOK: (iii) Can we also get evidence of v < 0 coefficient of H^2 during RVM inflation?

$$\rho_{\rm RVM}^{\rm string} \simeq 3\, M_{\rm Pl}^4 \Big[-\,1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \mathcal{O}(10^7) \Big(\frac{H}{M_{\rm Pl}}\Big)^4 \Big]$$

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type
Running Dark Energy

References:

Thank you!

a microscopic
(string-
inspired)
model for
RVM Universe....

Basilakos, NEM, Solà
(i)JCAP 12 (2019) 025
(ii) IJMD28 (2019) 1944002
(iii) Phys.Rev.D 101 (2020) 045001
(iv) Phys.Lett.B 803 (2020) 135342
(v) Universe 2020, 6(11), 218
NEM, Solà
(vi) EPJST 230 (2020), 2077
(vii) EPJPlus 136 (2021), 1152
NEM
(viii) arXiv:2205.07044
(ix) Universe 7 (2021), 480
(x) Phil. Trans. A380 (2022) 2222
NEM, Spanos, Stamou,
(xi) hep-th:2206.07963

Links with :
spontaneous Lorentz violation
(via (gravitational axion)
backgrounds)
and
Matter-Antimatter Asymmetry
in theories with
Right-Handed Neutrinos

(i) NEM & Sarben Sarkar, EPJC 73 (2013), 2359
(ii) John Ellis, NEM & Sarkar, PLB 725 (2013), 407
(iii) De Cesare, NEM & Sarkar, EPJC 75 (2015), 514
(iv) Bossingham, NEM & Sarkar, EPJC 78 (2018), 113; 79 (2019), 50
(v) NEM & Sarben Sarkar, EPJC 80 (2020), 558

SPARES:

Bonus Features

7. Enhanced cosmic perturbations and densities of primordial black holes and Gravitational Waves

7. Enhanced cosmic perturbations

and

densities of primordial black holes

and Gravitational Waves

⚠

Assume RVM models with almost Instantaneous reheating;

Prolonged reheating in some RVM models leads to even more enhanced primordial BH densities

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480,
e-Print: 2111.05675 [hep-th]

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

approximately de Sitter provided during the duration of inflation

$$b(t) = \overline{b}(0) + 0.14 M_{\rm Pl}\, H\, t_{end} \simeq \overline{b}(0)$$

order of magnitude

< 0

N=e-folds

beginning
of inflation

$$|\overline{b}(0)| \gtrsim \mathcal{O}(10)\, M_{\rm Pl}$$

Distance-swampland conjectures?

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

**NEM, Universe 7 (2021) 12, 480,
e-Print: 2111.05675 [hep-th]**

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

$$V(b) \simeq b\,\widetilde{\Lambda}_0^4 \sqrt{\frac{2}{3}}\,\frac{M_{\rm Pl}}{96\,M_s^2} \equiv b\,\frac{\widetilde{\Lambda}_0^4}{f_b} \equiv b\,\Lambda_0^3$$

Such a potential can also arise in appropriate brane compactifications
(eg type IIB strings)

**L. McAllister, E. Silverstein and A. Westphal,
Phys. Rev. D 82 (2010), 046003
[arXiv:0808.0706 [hep-th]].**

We may extend the model to include other stringy axions arising from compactification

$$V^{\rm lin}_{a_I} = a_I(x)\,\frac{f_b}{f_a}\,\Lambda_0^3$$

$$\Lambda_0 = 8.4 \times 10^{-4}\, M_{\rm Pl}.$$

f_a = axion coupling

**canonical kinetic
terms for a-axions**

$$f_b \equiv \left(\sqrt{\frac{2}{3}}\,\frac{M_{\rm Pl}}{96\,M_s^2}\right)^{-1} \overset{Eq.(9)}{\simeq} 5.3 \times 10^{-6}\, M_{\rm Pl}$$

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480,
e-Print: 2111.05675 [hep-th]

NEM, Stamou, Spanos, gr-qc...

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V^{b}_{\rm wsinst} \simeq \Lambda_b^4 \cos\Big(\frac{b}{f_b}\Big) \qquad \Lambda_b^4 \sim M_s^4\, e^{-S_{\rm wsinst}} \quad \to \quad \Lambda_b \ll \Lambda_0\,.$$

$$V^{a_I}_{\rm wsinst} \simeq \Lambda_I^4 \cos\Big(\frac{a_I}{f_{a_I}}\Big) \qquad \Lambda_0 \gg \Lambda_I \neq \Lambda_b,$$

Restrict to $I = 1:\ a_1 \equiv a$

$$V_{brane-compact.-effects}(a) \ni \Lambda_2^4\, \frac{1}{f_a}\, a + \Lambda_I^4 \Big(1 + \xi_a\, \frac{a}{f_a}\Big) \cos\Big(\frac{a}{f_a}\Big)$$

warp factor

$$\frac{\Lambda_2^4}{f_a} \sim \frac{\epsilon}{L} \sqrt{\frac{3}{(2\pi)^3}}\, M_s^3$$

L. McAllister, E. Silverstein and A. Westphal,
Phys. Rev. D 82 (2010), 046003
[arXiv:0808.0706 [hep-th]].

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$\boxed{V(a,\,b) = {\Lambda_1}^4 \Big(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\Big)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\, {\Lambda_0}^3 + \Lambda_2^4\Big)\, a(x) + {\Lambda_0}^3\, b(x)}$$

Case I

$$\Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

NEM, Sola + Basilakos
Stamou, Spanos, gr-qc…

Case II

$$\Lambda_0 \ll \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1$$

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$\boxed{V(a,\, b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\right)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)}$$

Case I $$\left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

NEM, Sola + Basilakos
Stamou, Spanos, gr-qc...

Enhancement of cosmic perturbations

Case $$\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$$

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle \, b(x)$

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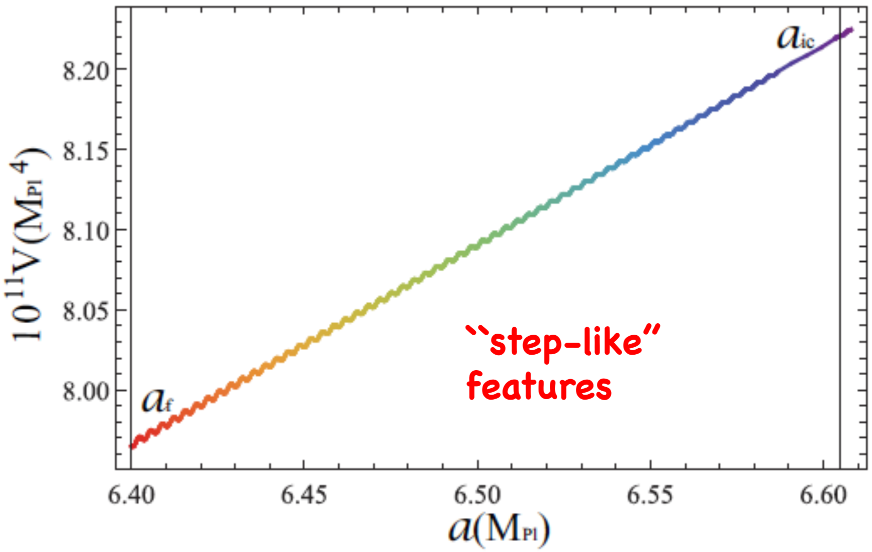
Case I $$\left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

NEM, Stamou, Spanos, gr-qc...

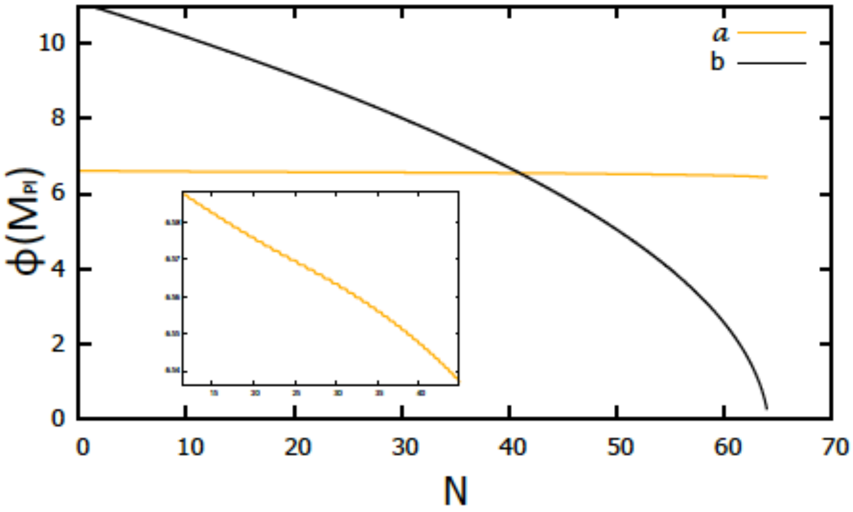
The enhancement of cosmic perturbations mechanism

NEM, Stamou, Spanos, gr-qc...

b-field + condensate drive inflation, **a-axion ends inflation**



"step-like" features



$$V(a,\,b) = \Lambda_1{}^4\left(1 + f_a^{-1}\,\tilde{\xi}_1\,a(x)\right)\,\cos(f_a{}^{-1}a(x)) + \frac{1}{f_a}\left(f_b\,\Lambda_0{}^3 + \Lambda_2^4\right)a(x) + \Lambda_0{}^3\,b(x)$$

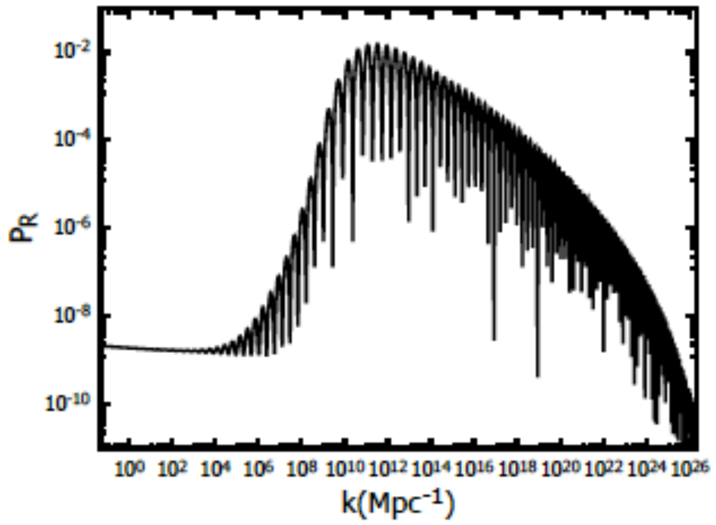
$$n_s = 1 + \frac{d\ln P_R}{d\ln k} \qquad r = \frac{P_T}{P_R} \qquad P_T = \frac{2}{\pi^2}H^2$$

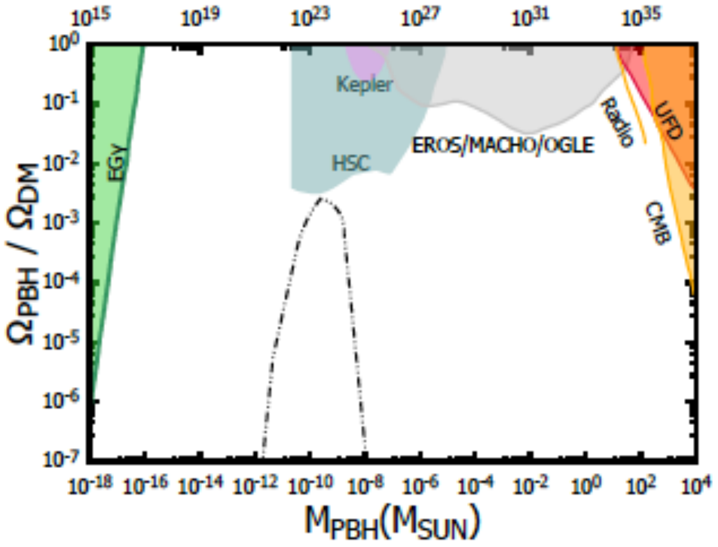
SET	g_1	g_2	ξ	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	-0.15	2.5×10^{-4}	8.4×10^{-4}	8.19×10^{-4}	2.32×10^{-4}
2	0.026	0.774	-0.20	2.5×10^{-4}	8.4×10^{-4}	7.89×10^{-4}	2.49×10^{-4}

SET	a_{ic}	b_{ic}	n_s	r
1	6.605	11.1	0.9638	0.062
2	4.932	11.4	0.9619	0.060

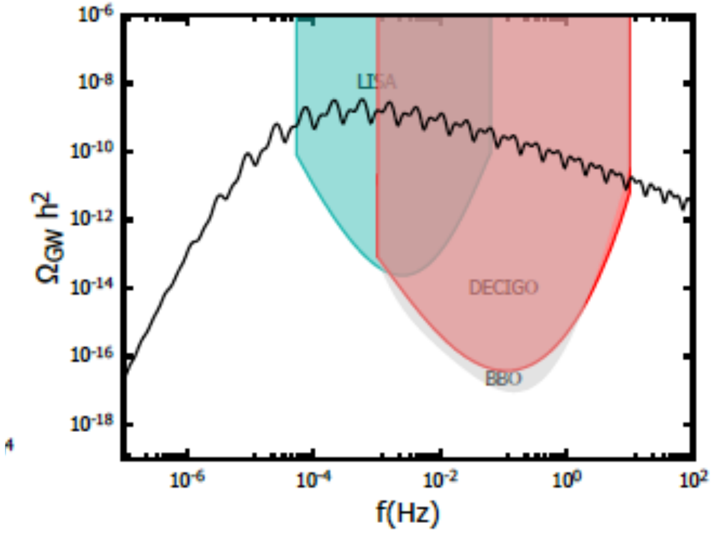
Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Stamou, Spanos, gr-qc...





SET 1



fractional PBH abundance

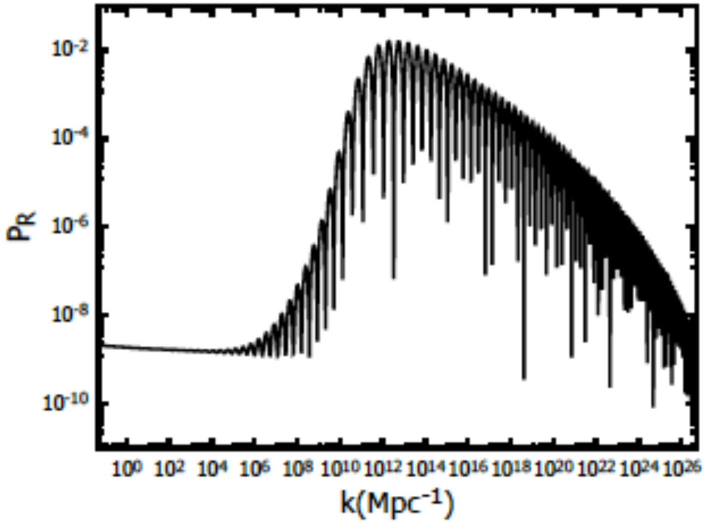
$$f_{PBH} = \int_k dM_{\rm PBH}(k) \frac{1}{M_{\rm PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

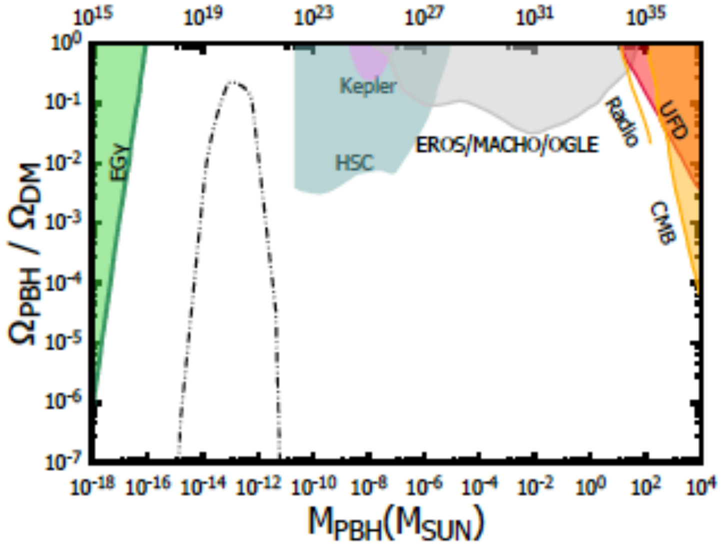
$$f_{PBH} = 0.01$$

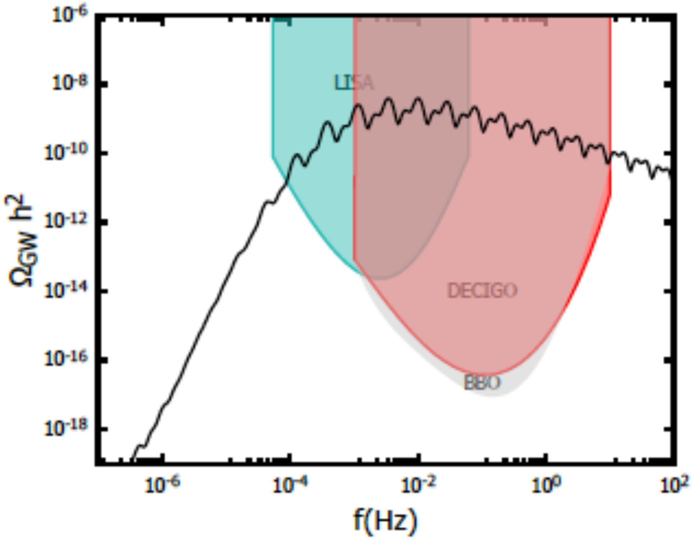
Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Stamou, Spanos, gr-qc...

SET 2







fractional PBH abundance

$$f_{PBH} = \int_k dM_{\rm PBH}(k)\,\frac{1}{M_{\rm PBH}(k)}\,\frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.80.$$

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$\boxed{V(a,\, b) = {\Lambda_1}^4 \Big(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\Big) \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\, {\Lambda_0}^3 + \Lambda_2^4\Big)\, a(x) + {\Lambda_0}^3\, b(x)}$$

Case II $\Lambda_0 \ll \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1$

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

Stamou, Spanos, gr-qc...

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

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Case II $\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$

Stamou, Spanos, gr-qc...

specific set of parameters enhancement due to **inflection points** in the potential → different enhancement mechanism than in

Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\, b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\right) \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)$$

$$\Lambda_0 = 8.4 \times 10^{-4} M_{\rm Pl}, \quad g_1 = 110, \quad g_2 = 1.779 \times 10^4, \quad \xi = -0.09, \quad f = 0.09\, M_{\rm Pl}\, .$$

SET 3 $(a_{ic}, b_{ic}) = 7.5622, 0.522$

Stamou, Spanos, gr-qc...

specific set of parameters enhancement due to **inflection points** in the potential → different enhancement mechanism than in

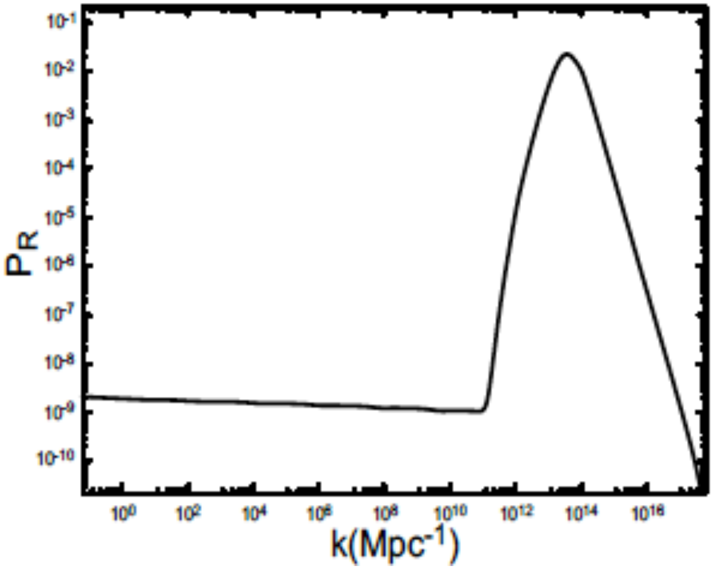
Zhou, Jiang, Cai, Sasaki, Pi,
Phys. Rev. D 102 (2020) no.10, 103527

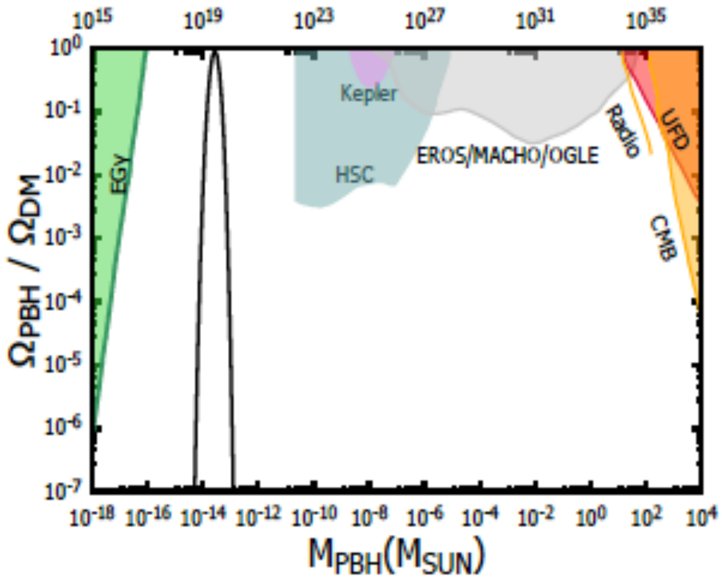
Case II $\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$

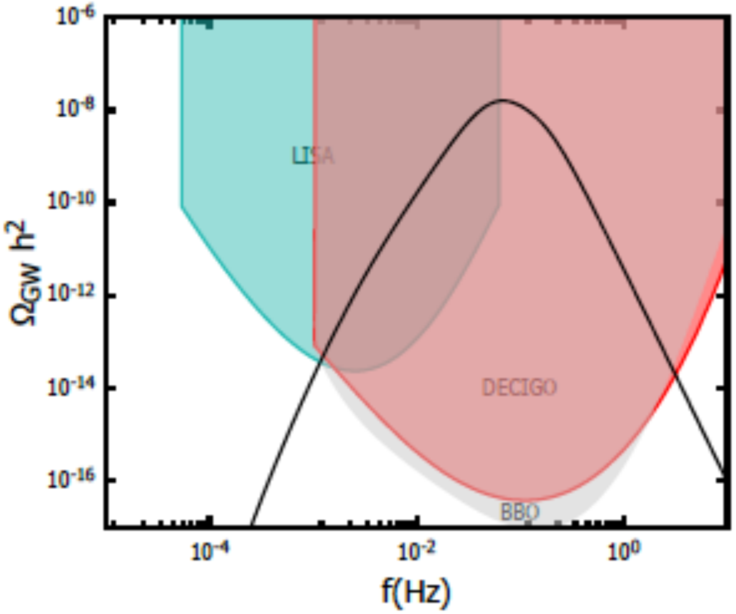
Primordial Black Hole (PBH) and GW enhanced production during inflation in Case 2

NEM, Stamou, Spanos, gr-qc...

SET 3







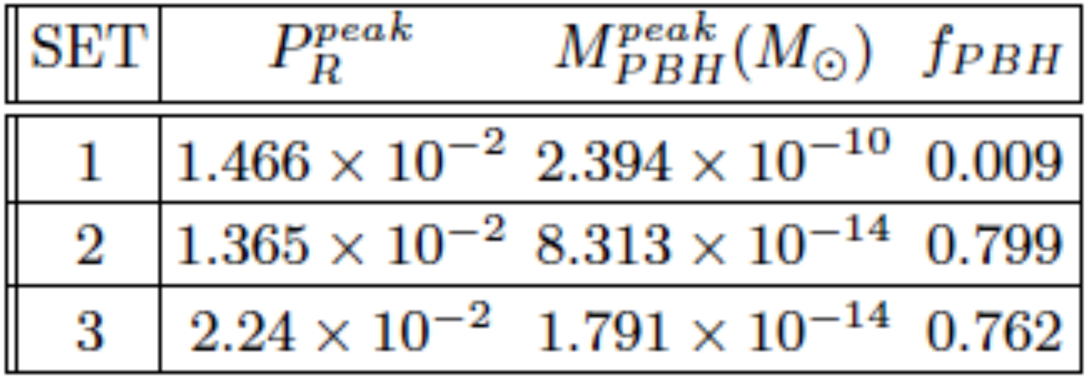
fractional PBH abundance

$$f_{PBH} = \int_k dM_{\rm PBH}(k) \frac{1}{M_{\rm PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.762$$

SUMMARY: Primordial Black Hole (PBH) and GW enhanced production during inflation in Cases 1 + 2

NEM, Stamou, Spanos, gr-qc...



SET	P_R^{peak}	$M_{PBH}^{peak}(M_\odot)$	f_{PBH}
1	1.466×10^{-2}	2.394×10^{-10}	0.009
2	1.365×10^{-2}	8.313×10^{-14}	0.799
3	2.24×10^{-2}	1.791×10^{-14}	0.762

Hence in both hierarchies of scales :

1: $\left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$, 2: $\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$

one may get **significant enhancement** of cosmic perturbations, ands PBH production, and thus a **significant portion** of PBH could play **the role of DM**, also, as a result, **profiles of GW** could **change** during radiation, in principle **falsifiable predictions** at **interferometers.**

8a. Post-RVM-Inflationary Era

Cancellation of Gravitational Anomalies

Cancellation of Gravitational Anomalies in Radiation Era
by:
Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R+\frac{1}{2}\partial_\mu b\,\partial^\mu b+\frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\,\frac{1}{96}\,\mathcal{K}^\mu-\sqrt{\frac{3}{8}}\,J^{5\mu}\Big)\Big]+\dots,$$

$$J^{5\mu}=\sum_j \bar{\psi}_j\,\gamma^\mu\,\gamma^5\,\psi_j$$

Chiral current, including RHN

Cancellation of Gravitational Anomalies in Radiation Era by:

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$$J^{5\mu}=\sum_j \bar{\Psi}_j\,\gamma^\mu\,\gamma^5\,\psi_j$$

Chiral current, including RHN

$$\boxed{\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu}-\frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}K^\mu\Big)\Big]=\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\Big(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu}+\frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)}$$

chiral U(1)

Gluon QCD

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

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$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots$$

$$J^{5\mu} = \sum_j \bar{\Psi}_j\,\gamma^\mu\,\gamma^5\,\psi_j;$$

Chiral current, including RHN

$$\boxed{\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\Big(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)}$$

chiral U(1)

Gluon QCD

instanton generated potential for KR axion b-field during matter dominance → axion Dark Matter

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Scale factor ***a(t) ~ T^{-1}***

Possibly also QCD

$$\boxed{\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U}(1) \text{ anomaly terms}}$$

sufficiently slowly varying during leptogenesis (brief) epoch → qualitatively similar to approximately const. background

Bossingham, NEM, Sarkar

8b. Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive Right-handed Neutrinos

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

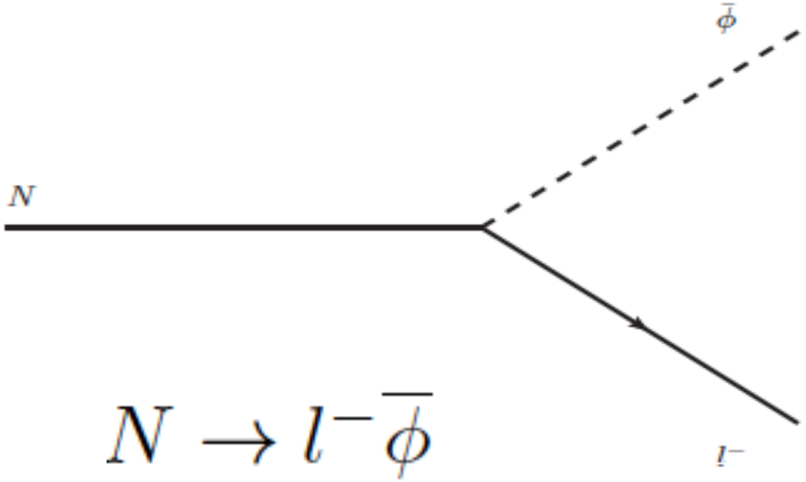
Early Universe
$T >> T_{EW}$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Heavy Right-Handed-Neutrinos (N) interact with **axial (approx.) constant background** with only temporal component $B_0 \propto \dot{b} \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background



$$N \to l^+\phi \qquad\qquad N \to l^-\overline{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega + B_0}{\Omega - B_0} \underset{B_0 \neq 0}{\neq} \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega - B_0}{\Omega + B_0}$$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

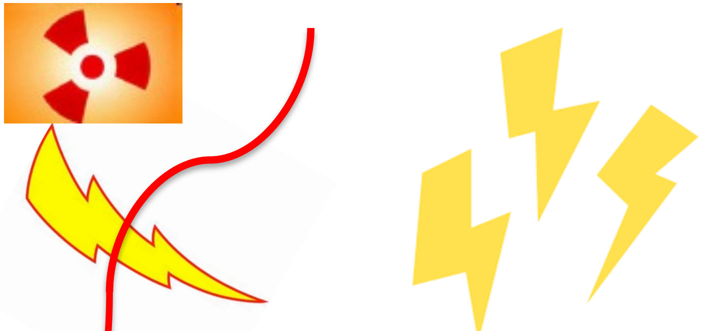
$$\mathcal{L} = i\overline{N}\not\partial N - \frac{m}{2}(\overline{N^c}N \;{}^{m}\overline{N}N^c) - \overline{N}\not B\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

(approx.) Constant B_0 Background

CPT Violation

Early Universe

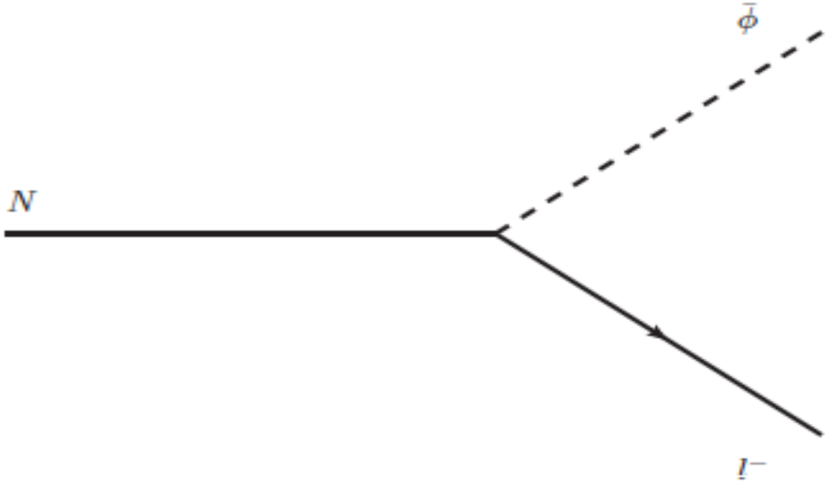
$T >> T_{EW}$



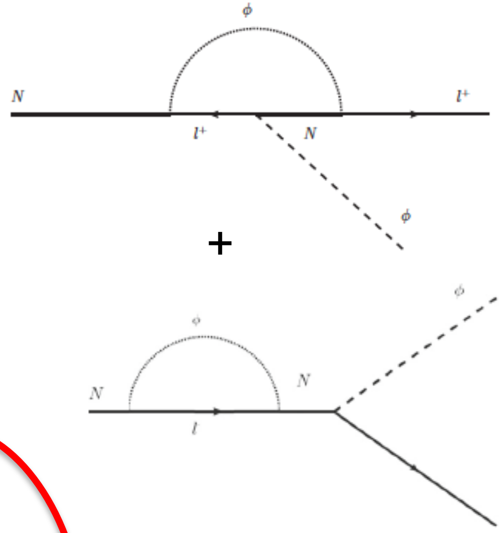
Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\;\phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial} N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

$$Y_k \sim 10^{-5}$$
$$m \geq 100\text{TeV} \to$$
$$B^0 \sim 1\text{MeV}$$
$$T_D \simeq m \sim 100\ \text{TeV}$$

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

(approx.) Constant $B^0 \neq 0$
background

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Equilibrated electroweak
B+L violating sphaleron interactions

B-L conserved

Environmental
Conditions Dependent

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Observed Baryon Asymmetry
In the Universe (BAU)

Fukugita, Yanagida,

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV