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Cosmological Constant and Equation of State of the Quantum Vacuum

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Workshop on Standard Model and Beyond (Corfu August 28-September 8, 2022)

Interpretation of Einstein's eqs.

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}}$$

1915
↓
1917

Geometry ⟷ **Energy**

$$\nabla^\mu G_{\mu\nu} = 0, \text{ where } G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R$$

$$\nabla_\mu \Lambda = \partial_\mu \Lambda = 0 \quad \Rightarrow \quad \Lambda = \text{const}\,. \quad !!$$

if $\nabla^\mu \left(G_N T_{\mu\nu}\right) = 0\ldots$!!!

$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\, G_N}$$

Cosmological Constant

Dark Energy

➢ The old CC problem as a fine tuning problem

The CC problem stems from realizing that the effective or physical vacuum energy is the sum of two terms:

$$\boxed{\rho_{\Lambda\text{phys}} = \rho_{\Lambda\text{vac}} + \rho_{\Lambda\text{ind}}}$$

$$S_{EH} = \frac{1}{16\pi\,G_N}\int d^4x\sqrt{|g|}\,\left(R - 2\,\Lambda_{\text{vac}}\right) = \int d^4x\sqrt{|g|}\,\left(\frac{1}{16\pi\,G_N}R - \rho_{\Lambda\text{vac}}\right)$$

$$\boxed{\rho_{\Lambda\text{vac}} = \frac{\Lambda}{8\pi\,G_N}}$$

Vacuum bare term in Einstein eqs.

$$R_{ab} - \frac{1}{2}g_{ab}R = -8\pi\,G_N\left(\langle\tilde{T}^{\varphi}_{ab}\rangle + T_{ab}\right) = -8\pi\,G_N\,g_{ab}\,(\rho_{\Lambda\text{vac}} + \rho_{\Lambda\text{ind}} + T_{ab})$$

Quantum effects $\Rightarrow\ \rho_{\Lambda\text{ind}} = \langle V(\varphi)\rangle + \text{ZPE}$

Vacuum energy = bubbles+SSB

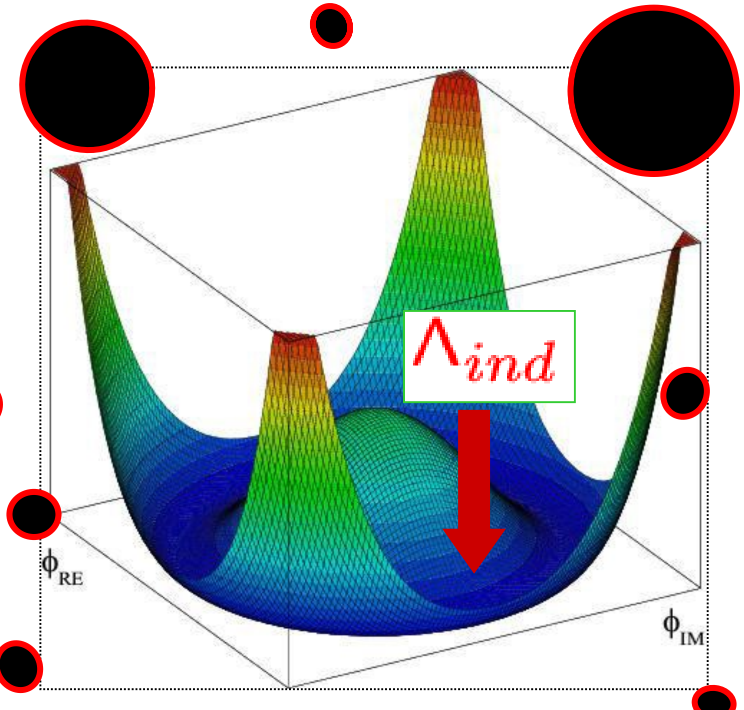
e+
e-

e+
e-

e+
e-

e+
e-

$$\frac{1}{2}\hbar\omega_k$$



Λ in the SM and beyond

Source	**Effect** (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

!!

$$\rho_\Lambda^0 = \Omega_\Lambda^0 \, \rho_c^0 \simeq 6\, h^2 \times 10^{-47}\,\mathrm{GeV}^4 \simeq 3 \times 10^{-47}\,\mathrm{GeV}^4$$

$$\boxed{m_\Lambda \equiv \sqrt[4]{\rho_\Lambda^0} \simeq 2 - 3 \ \mathrm{meV}}$$

RVM: inflation and cosmological expansion

Consider the class of time evolving vacuum models following a power series of the Hubble rate:

I. Shapiro and J. Solà (2000,2003,2009)

J. Solà and H. Stefancic (2005,2006)

J. Solà (2007) ...

$$\Lambda(H) = c_0 + c_1 H + c_2 H^2 + c_3 H^3 + c_4 H^4 + \ldots$$

C. Moreno-Pulido and JSP (2020-2022)

JS, A. Gómez-Valent, J. de Cruz Pérez (2015-2019)
+ C. Moreno-Pulido (2019-2021)

Reviews:

J. Solà (2011,2013,2014,,2016)

Joan Solà (2022)

Better fit than the ΛCDM and **alleviates H_0 and σ_8-tensions**

N. Mavromatos, J. Solà (2021)
("stringy-RVM" ...)

Basilakos-Mavromatos.Solà (2020)

Vacuum energy density: $\rho_\Lambda(H) = \Lambda(H)/(8\pi G)$

see talk by Nick!!

At low energy:

$$\Lambda(H) = c_0 + c_2 H^2 = \Lambda_0 + 3\,\nu\,(H^2 - H_0^2)$$

proposed (RG) equation for the vacuum energy density of the expanding Universe |

$$\frac{d\rho_\Lambda(\mu)}{d\ln\mu^2} = \frac{1}{(4\pi)^2}\left[\sum_i B_i M_i^2\,\mu^2 + \sum_i C_i\,\mu^4 + \sum_i \frac{D_i}{M_i^2}\,\mu^6 \;+\ldots\right]$$

$$\Big\Downarrow \qquad \mu^2 = aH^2 + b\dot{H}$$

(J. Solà, 2013,2014)

(J. Solà, A. Gómez-Valent, 2015)

$$\rho_\Lambda(H,\dot{H}) = \boxed{a_0} + a_1\,\dot{H} + \boxed{a_2\,H^2} + a_3\,\dot{H}^2 + \boxed{a_4\,H^4} + a_5\,\dot{H}\,H^2$$

$$\Big\Downarrow \qquad \mu^2 = H^2$$

$$\rho_\Lambda(H) = \frac{3}{8\pi G_N}\left(c_0 + \boxed{\nu H^2} + \boxed{\frac{H^4}{H_I^2}}\right)$$

Distinctive from Starobinsky's inflation **!!**

Can this be substantiated in QFT or string theory?

Adiabatic renormalization of the VED in QFT in a FLRW background: absence of quartic mass terms

C. Moreno-Pulido and JSP arXiv:2005.03164 (EPJ-C)

- The gravitational field equations read

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{matter} ,$$

where Λ is the Cosmological constant, with energy density $\rho_\Lambda \equiv \Lambda/(8\pi G_N)$. (this is not yet the physical VED)

Consider a toy-model (but non-trivial) calculation of the VED.

- We will suppose that there is only one matter field contribution to the EMT in $T_{\mu\nu}^{matter}$ in the form of a real scalar field, ϕ.

(nonminimal coupling ξ)

$$S[\phi] = -\int d^4x \sqrt{-g} \left(\frac{1}{2} g_{\mu\nu} \partial_\nu \phi \partial_\mu \phi + \frac{1}{2} (m^2 + \xi R) \phi^2 \right)$$

(no SSB contribution!)

- The Energy-Momentum tensor (EMT) associated to the scalar field is

$$T_{\mu\nu}(\phi) = (1-2\xi)\,\partial_\mu\phi\partial_\nu\phi + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}\partial^\sigma\phi\partial_\sigma\phi$$

$$-\,2\xi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi + \xi G_{\mu\nu}\phi^2 - \frac{1}{2}m^2 g_{\mu\nu}\phi^2.$$

- We can take into account the quantum fluctuations of the field ϕ by considering the expansion of the field around its background (or classical mean field) value ϕ,

$$\phi(\tau,\mathbf{x}) = \phi_b(\tau) + \delta\phi(\tau,\mathbf{x}),$$

$$\boxed{\langle T_{\mu\nu}^{vac}\rangle \equiv -\rho_\Lambda g_{\mu\nu} + \langle T_{\mu\nu}^{\delta\phi}\rangle.}$$

Total
vacuum contribution

(**needs renormalization!!**)

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = a^2(\tau)\eta_{\mu\nu}\,dx^\mu dx^\nu \qquad \text{sign}(g_{\mu\nu}) = (-,+,+,+)$$

Fluctuations split in Fourier modes:

$$\delta\phi(\tau,\mathbf{x})=\frac{1}{(2\pi)^{3/2}a}\int d^3k\left[A_{\mathbf{k}}e^{i\mathbf{kx}}h_k(\tau)+A_{\mathbf{k}}^{\dagger}e^{-i\mathbf{kx}}h_k^*(\tau)\right]$$

$$(\Box-m^2-\xi R)\delta\phi(\tau,x)=0 \quad \Rightarrow \quad \boxed{h_k''+\Omega_k^2 h_k=0,} \quad \textbf{(mode equation)}$$

$$h_k'h_k^*-h_kh_k^{*\prime}=i$$

$$\Omega_k^2\equiv k^2+a^2m^2+\boxed{a^2(\xi-1/6)R} \quad \textbf{(non-trivial!)}$$

The solution is $h_k(\tau)\sim\dfrac{e^{i\int^{\tau}W_k(\tau_1)d\tau_1}}{\sqrt{W_k(\tau)}}$,

$$W_k^2=\Omega_k^2-\frac{1}{2}\frac{W_k''}{W_k}+\frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2$$

In order to solve this equation we should use the **WKB approximation** or **adiabatic regularization**. (slowly varying) Ω_k **!!**

$$W_k = \omega_k^{(0)} + \omega_k^{(2)} + \omega_k^{(4)} \ldots ,$$ (**Adiabatic expansion**)$^{(*)}$

$$\begin{cases} \omega_k^{(2)} = \dfrac{a^2\Delta^2}{2\omega_k} + \dfrac{a^2 R}{2\omega_k}(\xi - 1/6) - \dfrac{\omega_k''}{4\omega_k^2} + \dfrac{3\omega_k'^2}{8\omega_k^3} , \\ \omega_k^{(4)} = -\dfrac{1}{2\omega_k}\left(\omega_k^{(2)}\right)^2 + \dfrac{\omega_k^{(2)}\omega_k''}{4\omega_k^3} - \dfrac{\omega_k^{(2)\prime\prime}}{4\omega_k^2} - \dfrac{3\omega_k^{(2)}\omega_k'^2}{4\omega_k^4} + \dfrac{3\omega_k'\omega_k^{(2)\prime}}{4\omega_k^3} . \end{cases}$$

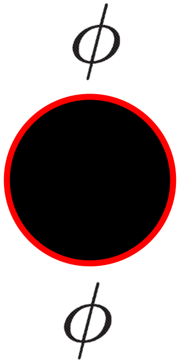
$$\begin{cases} \omega_k^{(0)} \equiv \omega_k = \sqrt{k^2 + a^2 M^2}, \\ \omega_k' = a^2\mathcal{H}\dfrac{M^2}{\omega_k}, \quad \omega_k'' = 2a^2\mathcal{H}^2\dfrac{M^2}{\omega_k} + a^2\mathcal{H}'\dfrac{M^2}{\omega_k} - a^4\mathcal{H}^2\dfrac{M^4}{\omega_k^3} . \end{cases}$$

The non-appearance of the odd adiabatic orders is justified by means of general covariance.

Explains why only even powers of H:

$$\boxed{\Lambda(H) = c_0 + c_1 \cancel{H} + c_2 H^2 + \cancel{c_3 H^3} + c_4 H^4 + \ldots}$$

(*) Adiabatic methods: cf. Bunch, Parker, Fulling (70's), Birrell&Davies (80's) etc
Recent improv.: Ferreiro & Navarro-Salas etc (2019)



one-loop

$T_{00}^{\delta\phi}$ up to 4th adiabatic order:

$$\left\langle T_{00}^{\delta\phi} \right\rangle = \int dk k^2 \Big[|h_k'|^2 + \left(\omega_k^2 + a^2\Delta^2\right)|h_k|^2$$
$$\left(\xi - \frac{1}{6}\right)\left(-6\mathcal{H}^2|h_k|^2 + 6\mathcal{H}(h_k' h_k^* + h_k^{*\prime} h_k)\right) \Big]$$

unrenormalized

ZPE

UV-divergent !!

$$\begin{aligned}
\left\langle T_{00}^{\delta\phi} \right\rangle = \frac{1}{8\pi^2 a^2}\int dk k^2 \bigg[& 2\omega_k + \frac{a^4M^4\mathcal{H}^2}{4\omega_k^5} - \frac{a^4M^4}{16\omega_k^7}(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 + 8\mathcal{H}'\mathcal{H}^2 + 4\mathcal{H}^4) \\
& + \frac{7a^6M^6}{8\omega_k^9}(\mathcal{H}'\mathcal{H}^2 + 2\mathcal{H}^4) - \frac{105a^8M^8\mathcal{H}^4}{64\omega_k^{11}} \\
& + \left(\xi - \frac{1}{6}\right)\bigg(-\frac{6\mathcal{H}^2}{\omega_k} - \frac{6a^2M^2\mathcal{H}^2}{\omega_k^3} + \frac{a^2M^2}{2\omega_k^5}(6\mathcal{H}''\mathcal{H} - 3\mathcal{H}'^2 + 12\mathcal{H}'\mathcal{H}^2) \\
& - \frac{a^4M^4}{8\omega_k^7}(120\mathcal{H}'\mathcal{H}^2 + 210\mathcal{H}^4) + \frac{105a^6M^6\mathcal{H}^4}{4\omega_k^9}\bigg) \\
& + \left(\xi - \frac{1}{6}\right)^2\bigg(-\frac{1}{4\omega_k^3}(72\mathcal{H}''\mathcal{H} - 36\mathcal{H}'^2 - 108\mathcal{H}^4) + \frac{54a^2M^2}{\omega_k^5}(\mathcal{H}'\mathcal{H}^2 + \mathcal{H}^4)\bigg)\bigg] \\
& + \frac{1}{8\pi^2 a^2}\int dk k^2 \bigg[\frac{a^2\Delta^2}{\omega_k} - \frac{a^4\Delta^4}{4\omega_k^3} + \frac{a^4\mathcal{H}^2M^2\Delta^2}{2\omega_k^5} - \frac{5}{8}\frac{a^6\mathcal{H}^2M^4\Delta^2}{\omega_k^7} \\
& + \left(\xi - \frac{1}{6}\right)\bigg(-\frac{3a^2\Delta^2\mathcal{H}^2}{\omega_k^3} + \frac{9a^4M^2\Delta^2\mathcal{H}^2}{\omega_k^5}\bigg)\bigg] + \dots,
\end{aligned}$$

- We compute terms up to 4th order because the divergences are only present up to this adiabatic order.
- We define the renormalized ZPE in curved space-time at the scale M as follows:

$$\boxed{\langle T_{00}^{\delta\phi}\rangle_{Ren}(M) \equiv \langle T_{00}^{\delta\phi}\rangle(m) - \langle T_{00}^{\delta\phi}\rangle^{(0-4)}(M)}$$

$$\langle T_{00}^{\delta\phi}\rangle_{\rm Ren}(M) = \frac{a^2}{128\pi^2}\left(-M^4+4m^2M^2-3m^4+2m^4\ln\frac{m^2}{M^2}\right)$$
$$-\left(\xi-\frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2}\left(m^2-M^2-m^2\ln\frac{m^2}{M^2}\right)+\left(\xi-\frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H}-\mathcal{H}'^2-3\mathcal{H}^4\right)}{16\pi^2a^2}\ln\frac{m^2}{M^2}+\dots$$

$$\Downarrow$$

$$\boxed{\mathcal{M}_{\rm Pl}^2(M)G_{\mu\nu}+\rho_\Lambda(M)g_{\mu\nu}+\alpha(M)\;{}^{(1)}H_{\mu\nu}=\langle T_{\mu\nu}^{\delta\phi}\rangle_{\rm ren}(M)\,.}$$

$\mathcal{M}_{\rm Pl}^2(M)=\dfrac{G^{-1}(M)}{8\pi}$ **Off-shell subtraction:** ⟹ **Exploring different scales**

➤ **VED =** $\rho_\Lambda + \mathrm{ZPE}$

$$\langle T_{\mu\nu}^{vac}\rangle \equiv -\rho_\Lambda g_{\mu\nu} + \langle T_{\mu\nu}^{\delta\phi}\rangle \quad\Longrightarrow\quad \rho_{vac}(M) = \rho_\Lambda(M) + \frac{\left\langle T_{00}^{\delta\phi}\right\rangle_{Ren}(M)}{a^2}$$

$$\rho_{\rm vac}(M) = \rho_\Lambda(M) + \frac{1}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\ln\frac{m^2}{M^2}\right)$$

$$+\left(\xi - \frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2a^2}\left(M^2 - m^2 + m^2\ln\frac{m^2}{M^2}\right) + \left(\xi - \frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2a^4}\ln\frac{m^2}{M^2} + \cdots$$

in Minkowski space ($H = 0$)

$\rho_{\rm vac}(M)$ must be RG invariant

$$\beta_{\rho_\Lambda}(M) = M\frac{\partial\rho_\Lambda(M)}{\partial M} = \frac{1}{2(4\pi)^2}(M^2 - m^2)^2$$

➢ Beta Function of the VED

C. Moreno-Pulido and JSP (2020-2022)

2201.05827 [gr-qc]

$$\beta_{\rho_{\rm vac}} = M\frac{\partial\rho_{\rm vac}(M)}{\partial M}$$

$$\boxed{= \left(\xi-\frac{1}{6}\right)\frac{3H^2}{8\pi^2}\left(M^2-m^2\right)}$$

$$+\left(\xi-\frac{1}{6}\right)^2\frac{9\left(\dot{H}^2-2H\ddot{H}-6H^2\dot{H}\right)}{8\pi^2}$$

$$\beta_{\rho_{\rm vac}} \propto \xcancel{m^4} \quad !!$$

➢ VED evolution

$$\rho_{\rm vac}(M,H)-\rho_{\rm vac}(M_0,H_0)=\frac{3\left(\xi-\frac{1}{6}\right)}{16\pi^2}\left[H^2\left(M^2-m^2+m^2\ln\frac{m^2}{M^2}\right)\right.$$
$$\left.-H_0^2\left(M_0^2-m^2+m^2\ln\frac{m^2}{M_0^2}\right)\right]+\cdots,$$

$M=H$ and $M_0=H_0$

for the current universe

$$\boxed{\rho_{\rm vac}(H)=\rho_{\rm vac}^0+\frac{3\nu_{\rm eff}(H)}{8\pi}m_{\rm Pl}^2\left(H^2-H_0^2\right)+\mathcal{O}(H^4)}$$

$$\boxed{\nu_{\rm eff}(H)\equiv\frac{1}{2\pi}\left(\xi-\frac{1}{6}\right)\frac{m^2}{m_{\rm Pl}^2}\left(-1+\ln\frac{m^2}{H^2}-\frac{H_0^2}{H^2-H_0^2}\ln\frac{H^2}{H_0^2}\right)}$$

naturally small parameter

RVM structure !!

$$\nu_{\rm eff}\simeq\epsilon\ln\frac{m^2}{H_0^2}\qquad\epsilon\equiv\frac{1}{2\pi}\left(\xi-\frac{1}{6}\right)\frac{m^2}{m_{\rm Pl}^2}$$

J. Solà (2011,2013,2014,,2016)

from action: 0710.4151

(J.Phys.A **41** (2008) 164066)

➢ QFT-driven INFLATION

2201.05827 [gr-qc]

C. Moreno-Pulido and JSP (2020-2022)

$$\boxed{\rho_{\rm vac}^{\rm inf} = \frac{\langle T_{00}^{\delta\phi}\rangle_{\rm Ren}^{\rm 6th}(m)}{a^2} = \frac{\tilde{\xi}}{80\pi^2 m^2} H^6 + f(\dot{H}, \ddot{H}, \dddot{H}\ldots)}$$

$$\tilde{\xi} = \left(\xi - \tfrac{1}{6}\right) - \tfrac{2}{63} - 360\left(\xi - \tfrac{1}{6}\right)^3$$

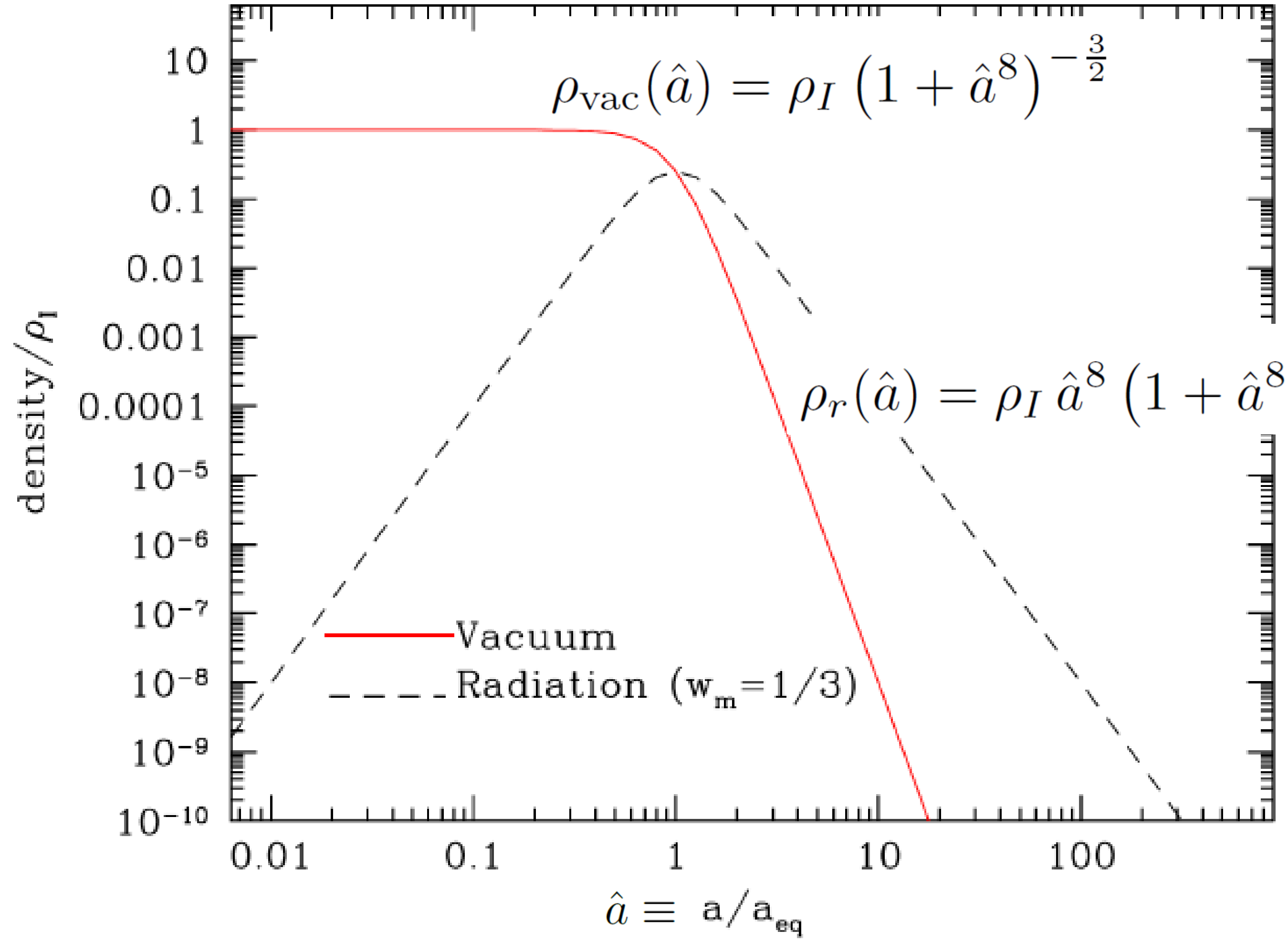
$$\boxed{P_{\rm vac}(M) = -\rho_{\rm vac}(M)}$$

$$+ f_2(M, \dot{H}) + f_4(M, H, \dot{H}, ..., \dddot{H}) + f_6(\dot{H}, ..., \overset{\cdot\cdot\cdot\cdot\cdot}{H})$$

e.g. $$f_2(M, \dot{H}) = \frac{\left(\xi - \frac{1}{6}\right)}{8\pi^2}\dot{H}\left(m^2 - M^2 - m^2 \ln\frac{m^2}{M^2}\right)$$

RVM-inflation

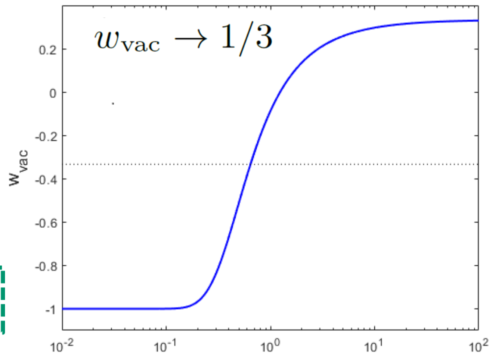
2201.05827 [gr-qc]



$$\rho_{\rm vac}(\hat{a}) = \rho_I \left(1 + \hat{a}^8\right)^{-\frac{3}{2}}$$

$$\rho_r(\hat{a}) = \rho_I \, \hat{a}^8 \left(1 + \hat{a}^8\right)^{-\frac{3}{2}}$$

(EoS)



$$\boxed{w_{\rm vac} \simeq -1}$$

Early Universe

In the early universe, before and during inflation, it is assumed that only fields from the gravitational multiplet of the string exist, which implies that the relevant bosonic part of the effective action pertinent to the dynamics of the inflationary period is given by

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]\,.$$

$\alpha' = M_s^{-2}$ $\qquad$ $\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$ $\qquad$ $M_{\rm Pl} \neq M_s$ in general

It involves the usual Hilbert-Einstein term and the Kalb-Ramond axion field, $b(x)$, which is coupled to the **gravitational Chern-Simons topological density** through the string tension α'. Such topological term when averaged over the de Sitter spacetime produces an effective contribution to the vacuum energy density of the form $\sim H^4$.

During inflation $R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}$ ➡ triggers H^4 contributions to ρ_Λ ⚠

$\sim\; H^4$ terms are back **!!**

See talk by Nick Mavromatos !

Detailed review:

(N.E. Mavromatos and JSP)

"Stringy RVM"

arXiv:2012.07971 (EPJ-ST 2021)

Minimal unified model at high energy (early universe):

S. Basilakos,J.A.S Lima, and JS arXiv:1509.00163, arXiv:1307.6251
JS and A. Gómez-Valent arXiv:1501.03832
JS arXiv:1505.05863
JS and H. Yu arXiv:1910.01638

$$\boxed{\Lambda(t) = c_0 + 3\nu H^2 + 3\alpha \frac{H^4}{H_I^2}}$$

$$\Big\Downarrow$$

$$\dot{H} + \frac{3}{2}(1+\omega_m)H^2\left[1 - \nu - \frac{c_0}{3H^2} - \alpha\frac{H^2}{H_I^2}\right] = 0$$

Inflationary solution: $H^2 = (1-\nu)H_I^2/\alpha$!!

$a(t) \propto e^{H_I t}$.

Running vacuum energy at the expense of matter non-conservation

$$\boxed{\rho_\Lambda = C_1 + C_2\, H^2\,.} \qquad \rho_\Lambda(H) = \frac{3}{8\pi G}\left(c_0 + \nu H^2\right)$$

Bianchi identity

$$\dot{\rho}_\Lambda + \dot{\rho}_m + 3\, H(\rho_m + p_m) = 0$$

(matter non-conservation!!)

$$C_2 \propto \nu = \frac{M^2}{12\pi\, M_P^2}$$

$$\rho_m(z) = \rho_m^0\,(1+z)^{3(1-\nu)}$$

and “running“ vacuum energy: (**RVM**)

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu\,\rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu)} - 1\right]$$

Equation of State of the QUANTUM VACUUM (EoS)

C. Moreno-Pulido and JSP (2020-2022)

2201.05827 [gr-qc]
2207.07111 [gr-qc]

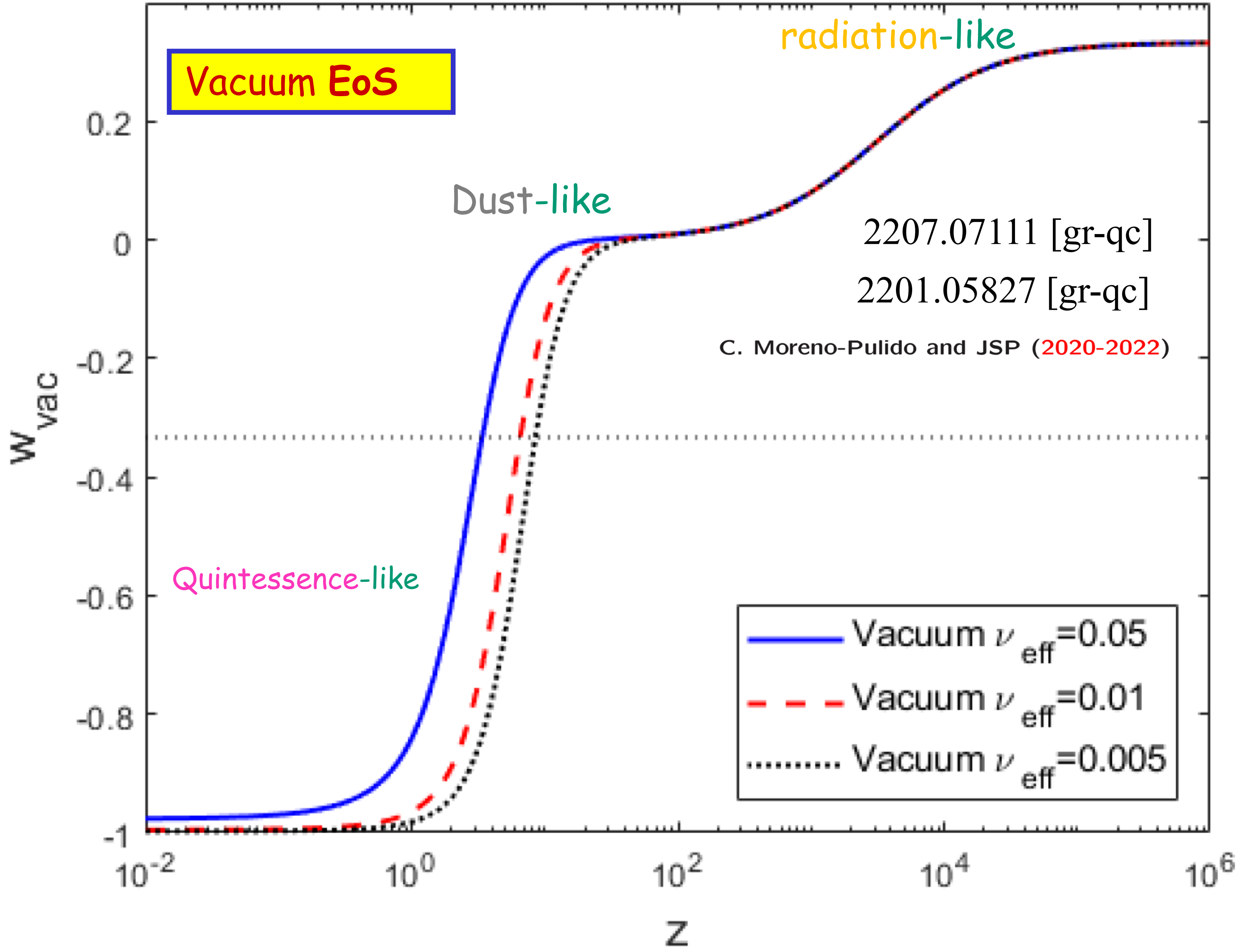
$$w_{\rm vac}(H) \equiv \frac{P_{\rm vac}(H)}{\rho_{\rm vac}(H)} \simeq -1 + \frac{f_2(\dot{H})}{\rho_{\rm vac}(H)}$$

$$\simeq -1 + \left(\xi - \frac{1}{6}\right)\frac{\dot{H}m^2}{8\pi^2\rho_{\rm vac}(H)}\left(1 - \ln\frac{m^2}{H^2}\right)$$

$$= -1 + \frac{\nu_{\rm eff}\left(\Omega_m^0(1+z)^3 + \frac{4}{3}\Omega_r^0(1+z)^4\right)}{\Omega_{\rm vac}^0 + \nu_{\rm eff}\left[-1 + \Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + \Omega_v^0\right]}$$

$$= \begin{cases} \frac{1}{3} & \text{for } \; z \gg z_{\rm eq} \;\text{ with }\; \Omega_r^0(1+z) \gg \Omega_m^0, \quad \text{radiation behavior } (\nu_{\rm eff} \neq 0), \\ 0 & \text{for } \; \mathcal{O}(1) < z \ll z_{\rm eq} \;\text{ with }\; \Omega_m^0 \gg \Omega_r^0(1+z), \quad \text{dust behavior } (\nu_{\rm eff} \neq 0), \\ \boxed{-1 + \nu_{\rm eff}\frac{\Omega_m^0}{\Omega_{\rm vac}^0}(1+z)^3} & \text{for } \; -1 < z < \mathcal{O}(1)\,, \quad \text{quintessence behavior } (\nu_{\rm eff} > 0) \end{cases}$$

Vacuum EoS



radiation-like

Dust-like

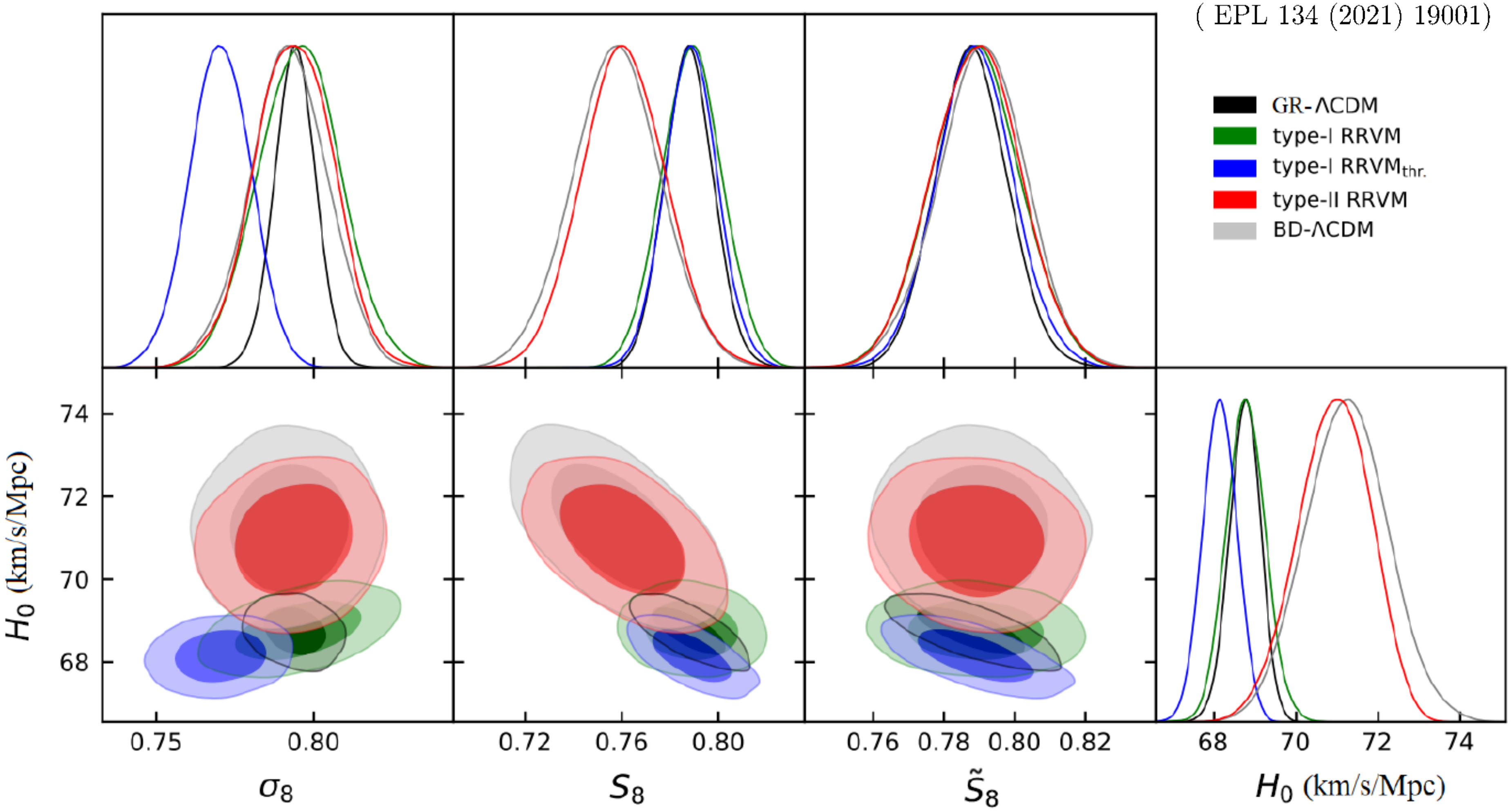
Quintessence-like

2207.07111 [gr-qc]

2201.05827 [gr-qc]

C. Moreno-Pulido and JSP (2020-2022)

(EPL 134 (2021) 19001)



Type II RVM can alleviate **both tensions** at a time **!!**

Joan Solà (Corfu 2022)

Summarized conclusions

- **Dynamical DE**: natural proposal for an **expanding Universe**
- The **RVM** based on a **running Λ term** in interaction with matter or **G** is theoretically **well motivated**
- **Running vacuum models** seem to describe **better** the observations SNIa+BAO+$H(z)$+LSS+CMB **than the ΛCDM**
- Provide a **consistent solution** to the main **tensions**
- These ideas may signal a **connection** between the the **LSS** of the Universe and the **quantum phenomena** in the **microcosmos**