Attractor solutions and features in the power spectrum from turns in field space

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Why consider more fields?

Observables

Attractor solutions

Multiple turns

Summary

- Inflation is the leading paradigm for the generation of anisotropies of CMB
- Scalar fields suffice to derive models that fit the Planck data
- Single-field inflation phenomenological viable
- Inflation is a cosmological collider
- Why examine multi-field models? High-energy embedding of inflation remains an open problem

The **botom-up** approach

- Write down all higher-order operators

$$\sum \frac{c_n \phi^n}{\Lambda^{n-4}} \,, \qquad \sum \frac{d_n (\partial_\mu \phi \partial^\mu \phi)^n}{\Lambda^{4n-4}} \,. \tag{1}$$

- Specifically for the former, sensitivity to dimension-six operators

$$\frac{\mathcal{O}_6}{M_{\rm pl}^2} = \frac{\mathcal{O}_4}{M_{\rm pl}^2} \phi^2 \,, \tag{2}$$

with $\langle \mathcal{O}_4 \rangle \sim V$. For a given energy scale

$$\tilde{\eta}_v \approx \eta_v + \mathcal{O}(1) \,, \tag{3}$$

violation of slow-roll condition [Copeland, Liddle, Lyth, Stewart, Wands 94]

The **top-down** approach

- High-energy theories include at least two scalar fields: e.g. α-attractors [Kalosh, Linde, Roest 13] derived from supergravity. Stringy models predict a plethora of scalar fields at high energies
- Higgs inflation is inherently multi-field [Bezrukov, Shaposhnikov 08][Kaiser, Sfakianakis 13]
- Usually extra fields are stabilized by giving them very large masses ($\sim \mathcal{O}(H)$)
- The reduction to one field not always possible

- Multiple scalar fields with minimal derivative couplings.

$$\mathcal{L}_{\rm m} = \sqrt{-g}\left(\frac{1}{2}G_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V\right) \tag{4}$$

where G_{ij} behaves as a **metric**

- **Non-minimal** models with $L_{\rm gr} = \sqrt{-g}f(\phi)R_{\rm ein}$ can be brought in previous form via a conformal transformation $g \to \Omega(\phi)g$ Jordan frame $\to$ Einstein frame [Kaiser 10] (Higgs inflation)

- Supergravity models typically yield flat or hyperbolic-like metrics [Kalosh, Linde, Roest...]

- Non-trivial G_{ij} quite generic

- Fields start with random initial conditions
- The heuristic picture is the following: for a 'suitable' multi-variable potential
  1. Hubble friction dissipates excess kinetic energy (slow-roll evolution)
  2. Fields move towards the minimum, where 'heavy' degrees decay first
  3. The lightest field drives evolution
- Last phase is the **attractor** solution. If it is reached fast we avoid the initial conditions dependence of multi-field models. Important for self-consistency of inflation
- During attractor phase are fields at minimum of potential or away from it?

- Heavy fields stabilized at minima of an **effective potential** [Tolley, Wyman 09], [**PC**, Roest, Sfakianakis 20]

$$V_{\text{eff}}^{,i} = V^{,i} + \Gamma^{i}_{LL}\dot{\phi}_L^2 \tag{5}$$

  Centrifugal forces tend to drift fields away from their minima

- When fields are not in the minimum of potential, single-field inflation becomes incompatible; problem fully multifield
- Example: α-attractors for certain parameter values (e.g. for $\alpha \ll 1$) never follow traditional slow-roll and deviate from universal predictions [**PC**, Roest, Sfakianakis 19]
- Negatively curved field spaces can lead to geometrical destabilization of usual slow-roll inflation [Turzinsky, Renaux-Petel 15]

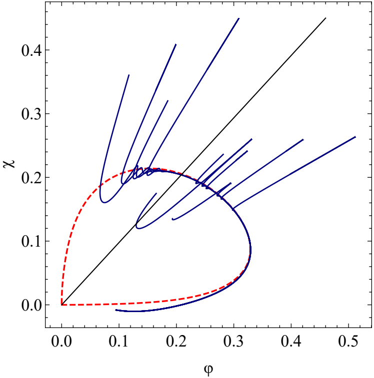

Figure: The attractor solution of two-field alpha attractors with $V = \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} m_\phi^2 \phi^2$ and $G_{ij} = \frac{\alpha}{(1-\chi^2-\phi^2)^2}\delta_{ij}$. *[PC, Roest, Sfakianakis]*

- For $\mathcal{N}$ fields: 1 curvature and $\mathcal{N}-1$ isocurvature (entropic). The latter source the curvature perturbation
- If 'heavy' then can be integrated out. However, they leave imprints on observables [Achucarro, Gong, Hardeman, Palma, Patil 10]

$$S=\int d\tau d^3x a^2 \epsilon M_{\rm pl}^2 \left[\frac{1}{c_s^2}\left(\frac{d\zeta}{d\tau}\right)^2-\nabla^2\zeta^2\right] \tag{6}$$

where the speed of sound of fluctuations c_s^2 depend on quantities that quantify the strength of multi-field effects

$$c_s^2 \equiv 1+\frac{4\Omega^2}{\mathcal{M}_{nn}-\Omega^2}$$

- When $c_s^2 < 0$ prior to horizon crossing transient instability in the power spectrum [Cremonini, Lalak, Turzynski 10].
- Power spectrum grows exponentially and can account for **PBH** production [Palma, Sypsas, Zenteno 20] [Fumagalli, Renaux-Petel, Ronayne, Witkowski 20] [Braglia, Hazra, Finelli, Smoot, Sriramkumar, Starobinsky 20] [many more...]
- PBH can be **dark matter** candidates
- An imaginary sound speed can be achieved with large turns

- Equations of motion:

$$D_N^2\phi^i + (3-\epsilon)\left(D_N\phi^I + (\ln V)^{,i}\right) = 0 \qquad (7)$$

- Rate of change of tangent vector $t^i \equiv D_N\phi^i/|D_N\phi^i|$

$$D_N t^i \equiv \mathbf{\Omega} n^i$$

  Quantifies the deviation from following a geodesic

- When small ($\Omega \ll 1$): $D_N t^i \equiv D_N^2\phi^i \approx 0$. These models follow the gradient flow $D_N\phi^i \approx -(\ln V)^i$. **Slow-turn** models have been extensively studied in the last 20 years

- For slow-turn solutions heavy fields are stabilized at the minima of their potential.
- However, specific potentials and field geometries support long-lasting solutions with heavy fields away from their minima: hyperinflation, angular, sidetracked, orbital,... In all cases $\Omega \gg 1$
- For slow-turn models solution is already in a coordinate invariant way: $D_N^2 \phi^i \approx 0$. Can the attractor solution be found for any type of G_{ij} and V?

- The solution can be found in a generic way using a particular orthonormal basis [Bjorkmo 19] or a special coordinate system [**PC**, Roest, Sfakianakis 20]
- Construct coordinate invariants from potential $w = \ln V$ and its covariant derivatives [**PC**, Rosati in progress] :

$$c_n \equiv w^{,i} \underbrace{w_{;i}{}^{j} \cdots w_{;m}{}^{l}}_{\text{n times}} w^{,m} \tag{8}$$

$$d_n \equiv \mathrm{Tr}\left(\underbrace{w_{;i}{}^{j} \cdots w_{;m}{}^{l}}_{\text{n times}} \right) \tag{9}$$

- Evaluate the previous invariants in the kinematic frame (adiabatic/entropic decomposition)
- Form linear combinations of c_n, d_n and solve for $\epsilon, \Omega, \cdots$

- For two fields only c_1, c_2 and d_1 are linearly independent. With c_1 and c_2

$$\epsilon = \epsilon_V - \frac{1}{2}\frac{c_1^2}{c_2} \tag{10}$$

or using c_1 and d_1

$$\epsilon_\pm = -\frac{1}{2}A \pm \frac{1}{2}\sqrt{A^2 - \frac{12\epsilon_V^2}{d_1 + \epsilon_V}}\,, \tag{11}$$

where

$$A = \frac{c_1 - 6\epsilon_V}{2(d_1 + \epsilon_V)} - \epsilon_V\,, \tag{12}$$

- For more fields need extra curvature invariants

- For three fields c_3, c_4, d_2, d_3 are also linearly independent. Now, ϵ is given as a solution of algebraic fourth and sixth order equations
- To find explicit expression need to make assumptions. For instance with zero torsion
- This method can not be applied to $\mathcal{N} > 3$ fields without simplifications.
- Attractor solution can be found in some restricted cases (e.g. diagonal metrics with isometries)
- With the attractor solution knwon can investigate whether large turns are allowed

- For two fields the rapid-turn regime is realized for large field-space curvatures.
- For simple $\mathcal{N}$-field extensions rapid-turn seems to require even larger curvatures.
- Has been observed in certain supergravity constructions with many fields [Aragam, Chiovoloni, Paban, Rosati, Zavala 22]
- Possible explanation: on average dynamical fields traverse shorter distances (away from minimum) during the last 55-60 e-folds. Compensate milder gradients with stronger curvatures

- **Holographic** extensions: cosmological evolution has similarities with RG flows in the bulk

- For cosmology FLRW metric

$$ds^2 = -dt^2 + e^{2\ln a(t)} ds_E^2 \tag{13}$$

Important quantity $\epsilon \equiv -\dot{H}/H^2$

- Consider other spacetimes

$$ds^2 = dz^2 + e^{2A(z)} ds_M^2 \tag{14}$$

Solutions with scalar fields are known as domain walls

- RG flow $\Leftrightarrow \epsilon$

- These solutions can describe some ‘exotic’ RG flows

- Multiple turns can have resonant effect [Boutivas, Dalianis, Kodaxis, Tetradis 22] [**PC**, Gong in progress]

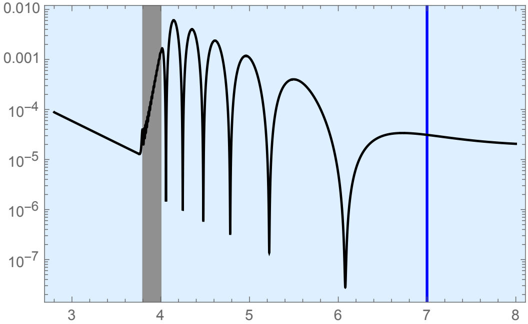

Figure: Evolution of a k-mode for a sharp turn

- During turn P_R increases exponentially and after the turn oscillates with frequency $\kappa \equiv k/(aH)$ (both positive and negative frequencies)

- If a subsequent turn is placed at a distance $1/\kappa$ resonance can happen
- Successive smaller turns achieve the same (or larger) effect. Perturbative control requires $\Omega^n_{\mathrm{max}} P_R \ll 1$. With multiple turns the system remains under control
- For more fields one can have the same effect without extremely large turns by increasing the magnitude of higher order bending parameters (e.g. the torsion of the curve)

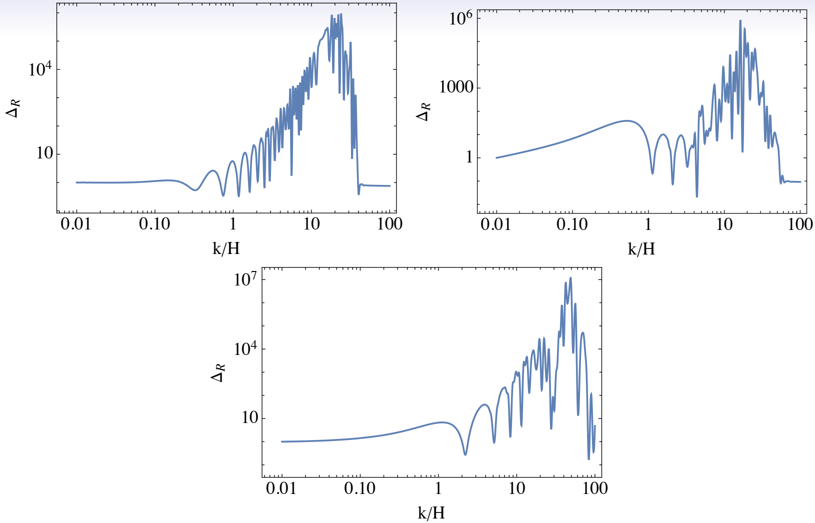

Figure: One vs two vs three sharp turns of equal angle $\theta = 10\pi$. Maximum value for each turn satisfies $\Omega_1 > \Omega_2 > \Omega_3$

- Scalar fields are useful in the early universe
- Multi-field models are motivated by high-energy theories
- They introduce novel behaviour at the background and perturbations level
- Have applications in early (inflation) and late universe (dark matter and dark energy)
- Connections with other fields: dark matter and holography

Thank you for your attention!