Polarisation of the Gauge Field Theory Vacuum and Cosmological Inflation

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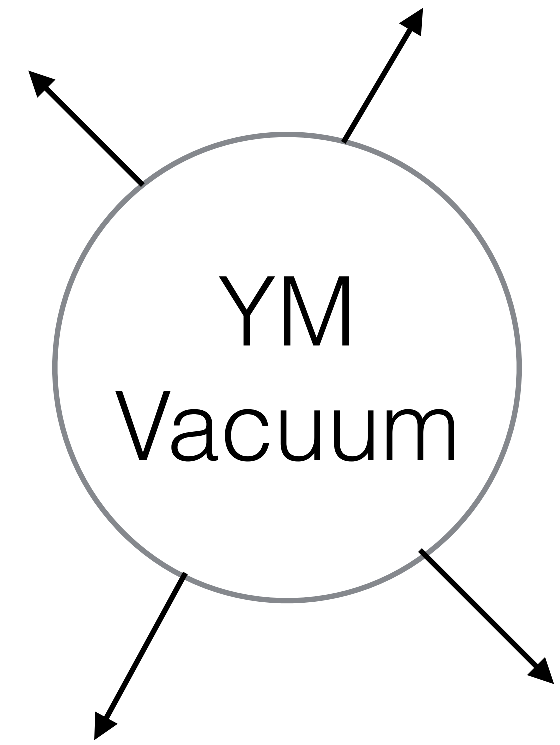
What is the Influence of the

Gauge Field Theory Vacuum

on the Cosmological Evolution?



Y. B. Zel'dovich, *The Cosmological constant and the theory of elementary particles,* Sov. Phys. Usp. **11** (1968) 381

S. Weinberg, *The Cosmological constant problem,* Rev. Mod. Phys. **61** (1989) 1-23

V. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, New York, 2005.

The vacuum energy density

$$E_0 = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\omega_p \sim \frac{1}{16\pi^2}\Lambda^4 \qquad \approx 1.44 \times 10^{110}\frac{g}{s^2 cm}$$

The contribution of zero-point energy exceed by many orders of magnitude the observational cosmological upper bound on the energy density of the universe

$$\epsilon_{crit} = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \approx 7.67 \times 10^{-9}\frac{g}{s^2 cm}$$

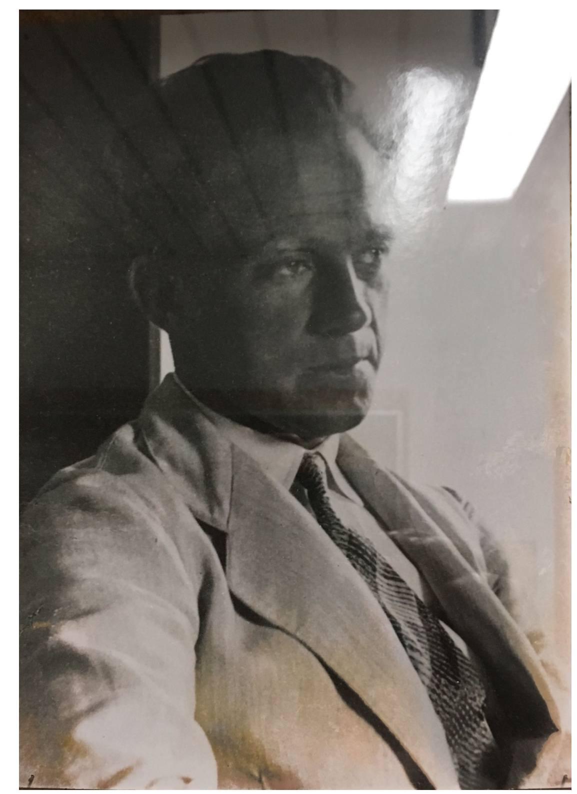
$$\epsilon_\Lambda = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda \approx 5.28 \times 10^{-9}\frac{g}{s^2 cm}$$

69%

Sauter 1931 LMU
Euler and Kockel 1935
Heisenberg and Euler 1936

The Effective Lagrangians

Hans Euler



Werner Heisenberg

Contribution of Vacuum Fluctuations to the Cosmological Constant

Only the difference between vacuum energy in the presence and in the absence of the external sources has a well defined physical meaning

Heisenberg and Euler - 1936

Heisenberg-Euler, 1936; Schwinger 1951;Coleman-Weinberg 1973; Vanyashin-Terentev 1965; Skalozub:1975; Brown-Duff,1975; Duff — Ramon-Medrano,1975; Nielsen and Olesen 1978; Skalozub 1978; Nielsen 1978; Ambjorn-Nielsen-Olesen1979; Nielsen and Ninomiya,1979; Nielsen and Olesen 1979; Nielsen-Ninomiya 1980; Nielsen-Olesen 1979; Ambjorn-Olesen 1980; Ambjorn-Olesen 1980; Skalozub1980; Leutwyler 1980; Leutwyler 1981; Duff 1977 ; Savvidy 1976, 1977, 2018, 2020, 2022

$$U^{\infty}_{\gamma} = \sum \frac{1}{2} \hbar \omega_k e^{-\gamma \omega_k}$$

$$\lim_{\gamma \to 0} [\ U^{\infty}_{\gamma}(J) - U^{\infty}_{\gamma}(0)\ ] = U_{phys}$$

Lamb shift - 1947 *Casimir effect 1948*

1. Effective Lagrangians in QED and YM theory

2. Quantum Energy Momentum Tensor

3. Vacuum Condensate in YM theories

4. Solution of Friedmann Equations in Gauge Field Theory Vacuum

5. Inflation

1. Annals of Phys. **436** (2022) 168681
2. PoS Corfu Meeting (2022)
3. Eur. Phys. J. **C 80** (2020) 165

Heisenberg-Euler Effective Lagrangian in QED

$$\mathcal{L}_{eff} = \frac{\mathcal{E}^2 - \mathcal{H}^2}{2} - \pi mc^2 (\frac{mc}{h})^3 \int_0^\infty \frac{ds}{s^3} e^{-s} \{ \frac{as\cos(as)}{\sin(as)} \, \frac{bs\cosh(bs)}{\sinh(bs)} - 1 + \frac{a^2 - b^2}{3} s^2 \}$$

where dimensionless fields are

$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$

$$mc^2 = 8.2 \cdot 10^{-7} \, \frac{g \; cm^2}{s^2} \qquad \lambda_c = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11} cm \qquad \frac{mc^2}{(\frac{\hbar}{mc})^3} = 1.43 \cdot 10^{25} \frac{g}{cm \; s^2}$$

$$\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \sim 10^{16} \; Volt/cm \qquad \mathcal{H}_c = \frac{m^2c^3}{e\hbar} \sim 4.4 \cdot 10^{13} \; Gauss$$

Contribution of Vacuum Fluctuations

Renormalisation of massless Heisenberg-Euler and Yang-Mills Effective Lagrangians

G.S. 1976

$$\frac{\partial \mathcal{L}}{\partial \mathcal{F}}\Big|_{t=\frac{1}{2}\ln(\frac{2e^2|\mathcal{F}|}{\mu^4})=\mathcal{G}=0} = -1, \qquad ($$

where $\mathcal{F} = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$ is the Lorentz and gauge invariant form of the YM field strength tensor

Heisenberg-Euler Effective Lagrangian

Massless limit of fermions G.S. 2020

$$\mathcal{L}_e = -\mathcal{F} + \frac{e^2 \mathcal{F}}{24\pi^2}\Big[\ln(\frac{2e^2\mathcal{F}}{\mu^4}) - 1\Big], \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}^2 - \vec{\mathcal{E}}^2}{2}, \quad \mathcal{G} = \vec{\mathcal{E}}\vec{\mathcal{H}} = 0,$$

the energy momentum tensor by using the formula derived by Schwinger in [5]:

$$T_{\mu\nu} = (F_{\mu\lambda}F_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}F^2_{\lambda\rho})\frac{\partial \mathcal{L}}{\partial \mathcal{F}} - g_{\mu\nu}(\mathcal{L} - \mathcal{F}\frac{\partial \mathcal{L}}{\partial \mathcal{F}} - \mathcal{G}\frac{\partial \mathcal{L}}{\partial \mathcal{G}}).$$

In massless QED using the one-loop expression (1.2) for $T_{\mu\nu}$ one can get

$$T_{\mu\nu} = T^M_{\mu\nu}\Big[1 - \frac{e^2}{24\pi^2}\ln\frac{2e^2\mathcal{F}}{\mu^4}\Big] + g_{\mu\nu}\frac{e^2}{24\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

Effective Lagrangian in Yang-Mills theory

The YM effective Lagrangian take the following form

$$\mathcal{L}^{(1)} = -\frac{1}{8\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} \frac{(gF_1 s)\ (gF_2 s)}{\sinh(gF_1 s)\ \sinh(gF_2 s)} - \\ - \frac{1}{4\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} (gF_1 s)\ (gF_2 s)[\frac{\sinh(gF_1 s)}{\sinh(gF_2 s)} + \frac{\sinh(gF_2 s)}{\sinh(gF_1 s)}]$$

$$F_1^2 = -\mathcal{F} - (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}, \qquad F_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$$

Vanyashin and Terentev 1965
Duff and Ramon-Medrano 1975
Skalozub 1976

Bartalin, Matinyan and Savvidy 1976
Savvidy 1977
Matinyan and Savvidy 1978

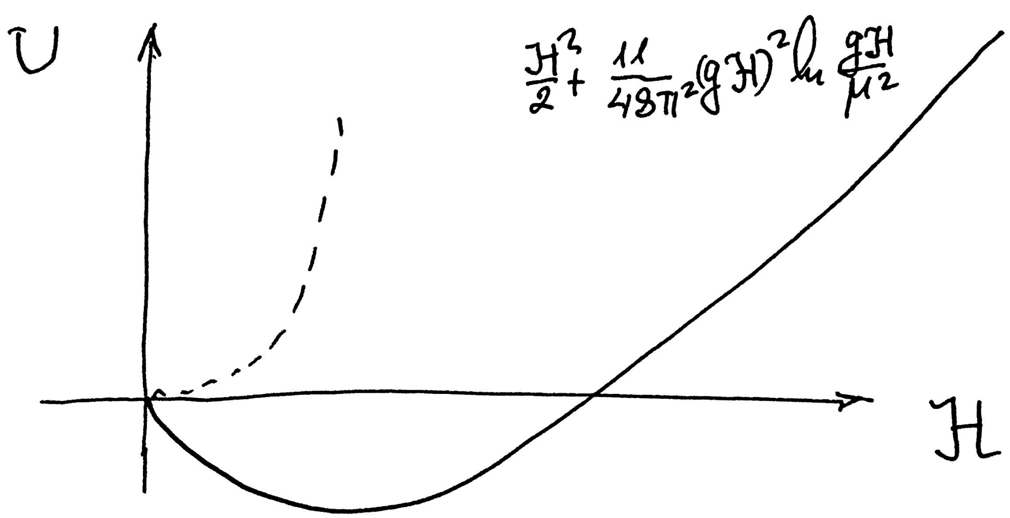
N.Nielsen and Olesen 1978
Ambjorn, N.Nielsen and Olesen 1979
H.Nielsen and Ninomia 1979
H.Nielsen and Olesen 1979
Ambjorn and Olesen1980

Dimensional Transmutation and Condensation

G.S. 1977, 2020

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big)\ , \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0\ .$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F}\Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1\Big]$$



$$2g^2\mathcal{F}_{vac} = \mu^4 \exp\big(-\frac{96\pi^2}{b\ g^2(\mu)}\big) = \Lambda^4_{YM},$$

where $b = 11N - 2N_f$.

$$T_{\mu\nu} = T^{YM}_{\mu\nu}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

Quantum Energy Momentum Tensor

$$T_{\mu\nu} = T^{YM}_{\mu\nu}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$T_{00} \equiv \epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big)$$

$$T_{ij} = \delta_{ij}\ \Big[\frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big)\Big] = \delta_{ij}\ p(\mathcal{F}).$$

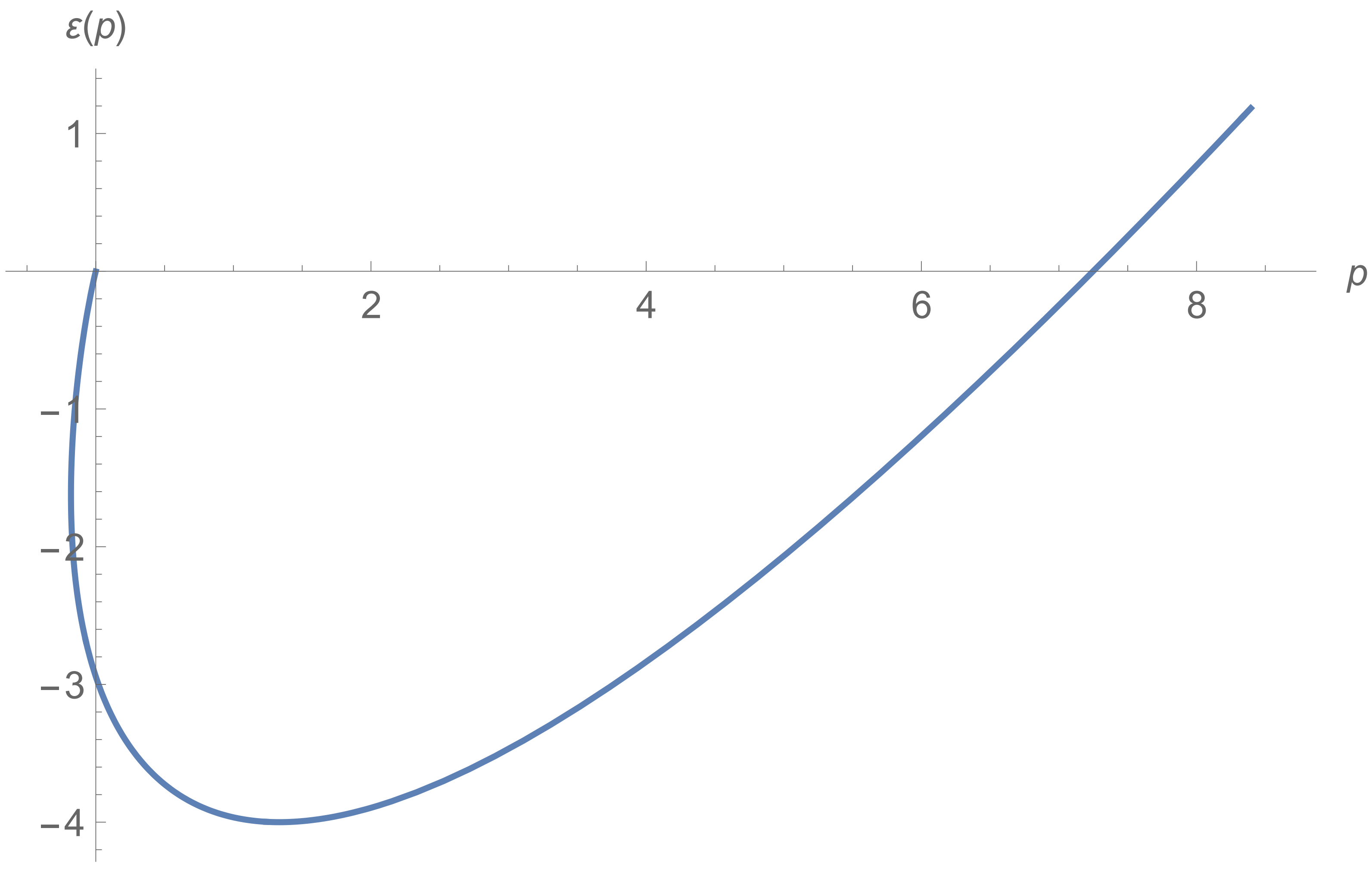
$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big),$$

$$p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

$$\mathcal{F} = \tfrac{1}{4}g^{\alpha\beta}g^{\gamma\delta}G^a_{\alpha\gamma}G_{\beta\delta} \geq 0$$

$$\mathcal{G} = G^*_{\mu\nu}G^{\mu\nu} = 0$$

Yang-Mills Quantum Equation of State



$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Isotropy of the Yang-Mills Energy Momentum Tensor

$$T_{00} = \frac{1}{2}(E^a_i)^2 + \frac{1}{2}(H^a_i)^2, \quad T_{0i} = \epsilon_{ijk} E^a_j H^a_k, \quad T_{ij} = \frac{1}{2}\delta_{ij}(E^a_i E^a_i + H^a_i H^a_i) - E^a_i E^a_j - H^a_i H^a_j.$$

The "white colour" solution

G. Baseyan, S. Matinyan and G. Savvidy, *Nonlinear plane waves in the massless Yang-Mills theory,* Pisma Zh. Eksp. Teor. Fiz. **29** (1979) 641-644

G. K. Savvidy, *Classical and Quantum Mechanics of Non-Abelian Gauge Fields,* Nucl. Phys. B **246** (1984) 302. doi:10.1016/0550-3213(84)90298-0

$$A^a_i = \delta^a_i f(t),$$

and the corresponding chromoelectric and chromomagnetic fields take the following form:

$$E^a_i = \delta^a_i \dot{f}(t), \qquad H^a_i = g\delta^a_i f^2(t).$$

The energy density therefore is:

$$\epsilon = T_{00} = \frac{3}{2}(\dot{f}^2 + g^2 f^4) = \mu^4,$$

[5] I would like to thank Prof. Viatcheslav Mukhanov for the discussion of this point.

Isotropy of the Yang-Mills Energy Momentum Tensor

$$T_{0i} = S_i = \epsilon_{ijk} E^a_j H^a_k = 0. \tag{27}$$

Thus importantly the space components of $T_{\mu\nu}$ in (23) are diagonal:

$$T_{ij} = \frac{1}{2}\delta_{ij}\left(\dot{f}^2 + g^2 \dot{f}^4\right) = \delta_{ij} p. \tag{28}$$

The full energy momentum tensor has the form of a relativistic matter:

$$T_{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}. \tag{29}$$

It follows from relations (26), (27) and (28) that the *classical* Yang Mills equation of state is equivalent to a homogeneous relativistic matter

$$p = \frac{1}{3}\epsilon. \tag{30}$$

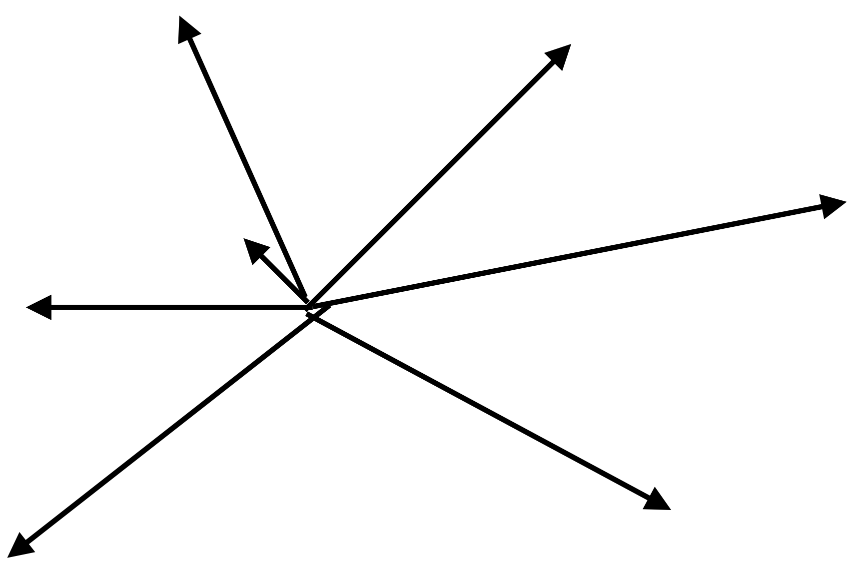
As we have seen above there are *quantum corrections to the classical equation of the state* (30) given by the first formula in (19)

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2 \mathcal{F}}{96\pi^2}\Lambda^4_{YM}. \tag{31}$$

Anisotropy of the Abelian Energy Momentum Tensor

A. Golovnev, V. Mukhanov and V. Vanchurin, *Vector Inflation*, JCAP **06** (2008), 009 doi:10.1088/1475-7516/2008/06/009 [arXiv:0802.2068 [astro-ph]].

A. Golovnev, V. Mukhanov and V. Vanchurin, *Gravitational waves in vector inflation*, JCAP **11** (2008), 018 doi:10.1088/1475-7516/2008/11/018 [arXiv:0810.4304 [astro-ph]].



N - randomly oriented vector fields

$$T_0^0 = \frac{1}{2}\left(\dot{B}_k^2 + m^2 B_k^2\right),$$

$$\begin{aligned} T_j^i \quad = \quad & \left[-\frac{5}{6}\left(\dot{B}_k^2 - m^2 B_k^2\right) - \frac{2}{3} H \dot{B}_k B_k - \frac{1}{3}\left(\dot{H} + 3H^2\right) B_k^2\right] \delta_j^i \\ & + \dot{B}_i \dot{B}_j + H\left(\dot{B}_i B_j + \dot{B}_j B_i\right) + \left(\dot{H} + 3H^2 - m^2\right) B_i B_j, \end{aligned}$$

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$

Equation of State

general parametrisation of the equation of state $p = w\epsilon$

when $w = -1$, $p = -\epsilon < 0$,

the acceleration is positive:

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3c^4}\epsilon > 0.$$

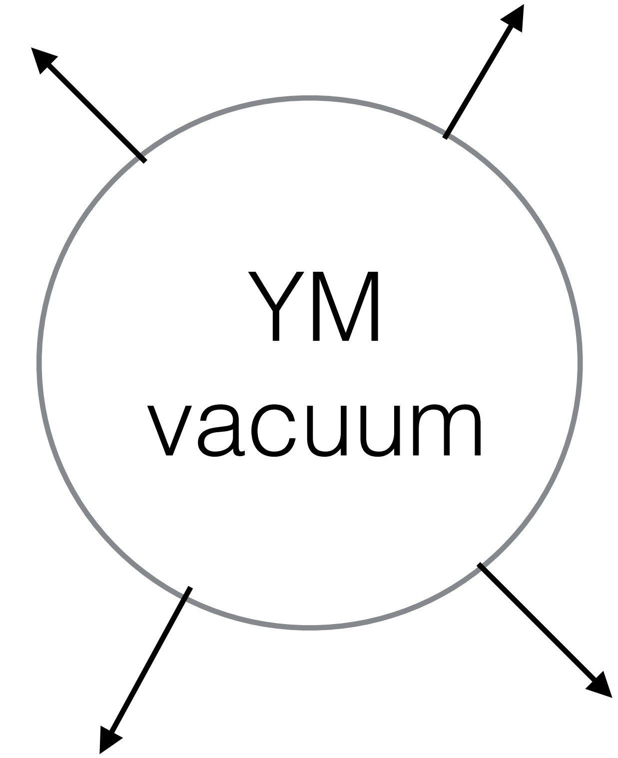
Yang-Mills Quantum Equation of State

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda_{YM}^4 \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3}{3\left(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\right)}$$

general parametrisation of the equation of state $p = w\epsilon$

GR Action

$$S=-\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x+\int(\mathcal{L}_q+\mathcal{L}_g)\,\sqrt{-g}d^4x.$$



$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi G}{c^4}\Big[T^{YM}_{\mu\nu}\Big(1+\frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big)-g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big].$$

$$\Lambda_{eff}=\frac{8\pi G}{c^4}\ \epsilon_{vac}=-\frac{8\pi G}{c^4}\frac{b}{192\pi^2}2g^2\mathcal{F}_{vac}=-\frac{8\pi G}{c^4}\frac{b}{192\pi^2}\Lambda^4_{YM}$$

The YM field strength $\mathcal{F}$ is not a constant function of time but evolve in time in accordance with the Feidmann equations, thus the cosmological term here is time dependent

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \longrightarrow \quad \epsilon + p = \frac{4\mathcal{A}}{3}\ (2g^2\mathcal{F})\ \log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \longrightarrow \quad \epsilon + 3p = 2\mathcal{A}\ (2g^2\mathcal{F})\ \left(\log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 1\right).$$

the first equation can be solved for the field strength

$$2g^2\dot{\mathcal{F}} + 4(2g^2\mathcal{F})\frac{\dot{a}}{a} = 0 \qquad\qquad 2g^2\mathcal{F}\ a^4 = const \equiv \Lambda_{YM}^4\ a_0^4,$$

Friedmann Evolution Equations

$$2g^2\mathcal{F}\ a^4 = const \equiv \Lambda^4_{YM}\ a_0^4,$$

the energy density and pressure are

$$\epsilon = \mathcal{A}\frac{a_0^4}{a^4}\Big(\log\frac{a_0^4}{a^4} - 1\Big)\Lambda^4_{YM}, \qquad p = \mathcal{A}\frac{a_0^4}{3a^4}\Big(\log\frac{a_0^4}{a^4} + 3\Big)\Lambda^4_{YM}.$$

the first Freidmann equation will take the form

$$\frac{k}{a^2} + \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3c^4}\epsilon,$$

$$\frac{da}{cdt} = \pm\sqrt{\frac{8\pi G}{3c^4}\ \mathcal{A}\ \Lambda^4_{YM}\ \frac{a_0^4}{a^2}\Big(\log\frac{a_0^4}{a^4} - 1\Big) - k}, \qquad k = 0, \pm 1.$$

Friedmann Evolution Equations

$$a(\tau) = a_0 \; \tilde{a}(\tau), \qquad ct = L \; \tau,$$

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4}\;\mathcal{A}\;\Lambda_{YM}^4 \equiv \Lambda_{eff}\;,$$

$$\mathcal{A} = \frac{b}{192\pi^2} = \frac{11N - 2N_f}{192\pi^2}.$$

Polarisation of the YM vacuum and the Effective Lagrangians

$$\epsilon_{YM} = 3 \frac{c^4}{8\pi G} \frac{1}{L^2}, \qquad \frac{1}{L^2} = \frac{8\pi G}{3c^4} \; \frac{11N - 2N_f}{196\pi^2} \; \Lambda^4_{YM}$$

Λ^4_{YM} *is the dimensional transmutation scale of YM theory*

$$\epsilon_{YM} = 3 \frac{c^4}{8\pi G} \frac{1}{L^2} = \begin{cases} 9.31 \times 10^{-3} & eV \\ 9.31 \times 10^{29} & QCD \\ 9.31 \times 10^{97} & GUT \\ 9.31 \times 10^{110} & Planck \end{cases} \qquad \frac{g}{s^2 cm}$$

the YM vacuum energy density is well defined, is finite and is time dependent quantity

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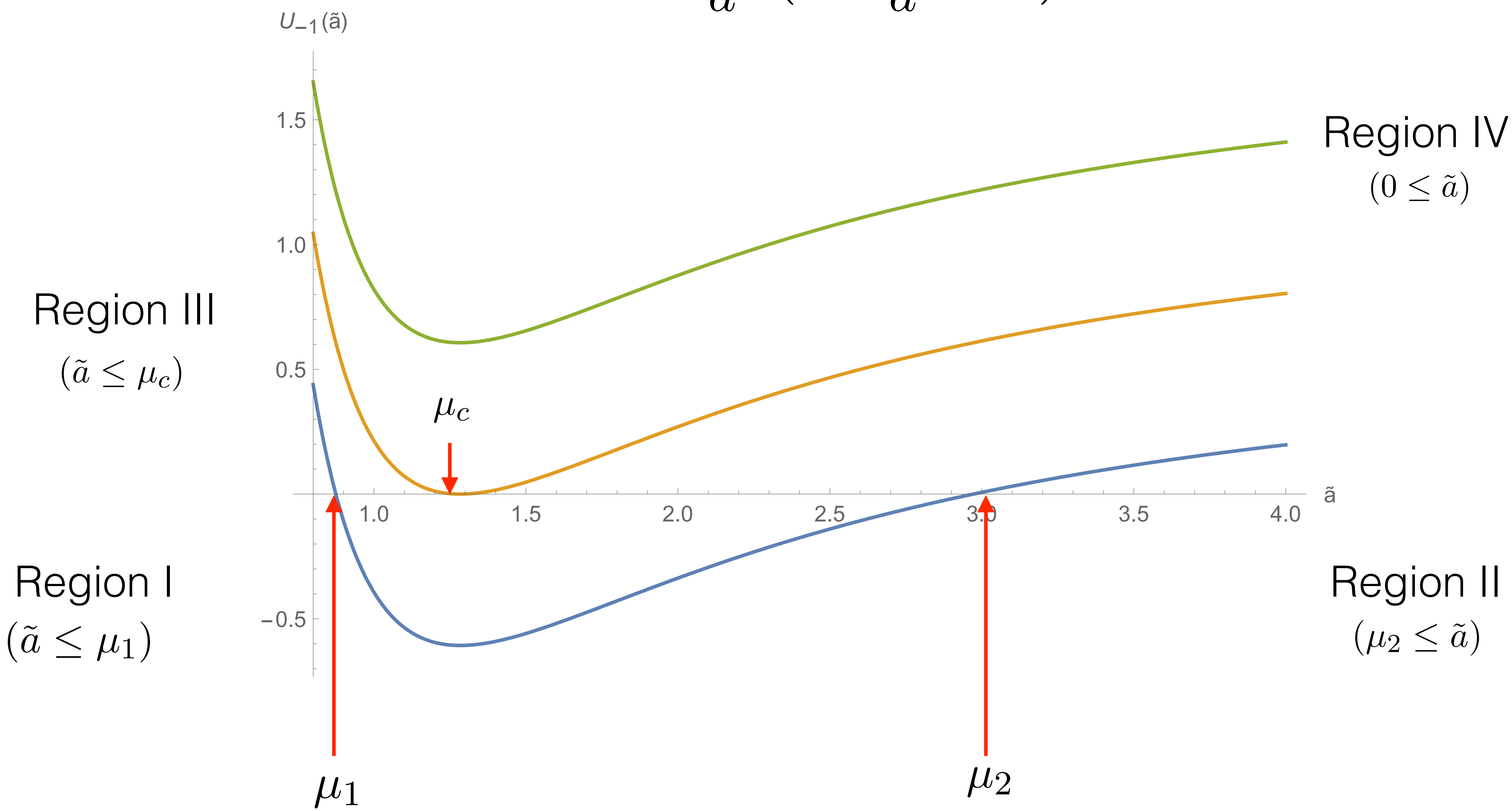
$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$



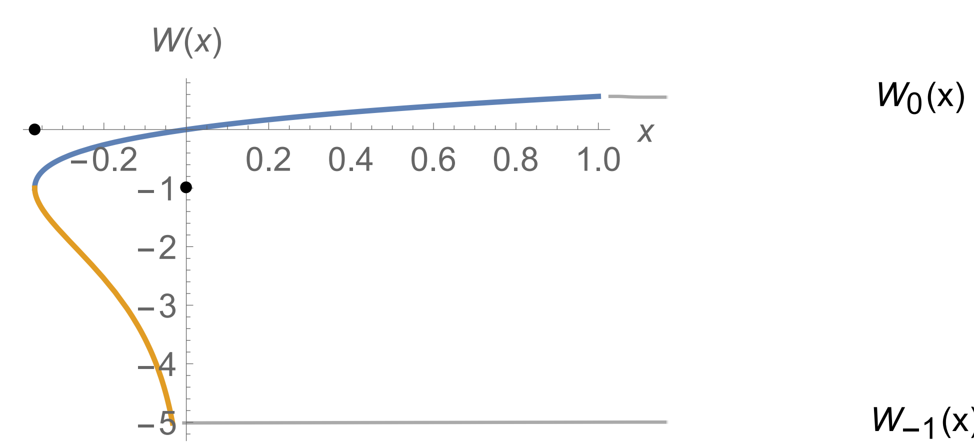
$$0 \leq \gamma^2 < \gamma_c^2$$

$$\gamma^2 = \gamma_c^2 = \frac{2}{\sqrt{e}}$$

$$\gamma_c^2 < \gamma^2$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) + \gamma^2.$$

$k = -1,$	$0 \leq \gamma^2 < \gamma_c^2$	Regions I ($\tilde{a} \leq \mu_1$) and II ($\mu_2 \leq \tilde{a}$)
$k = -1,$	$\gamma^2 = \gamma_c^2 = \dfrac{2}{\sqrt{e}}$	Region III (separatrix, $\tilde{a} \leq \mu_c$)
$k = -1,$	$\gamma_c^2 < \gamma^2$	Regions IV ($0 \leq \tilde{a}$)
$k = 0,$		
$k = 1,$	$0 \leq \gamma^2.$	



Lambert - Euler W function

Type II Solution — Initial Acceleration of Finite Duration

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

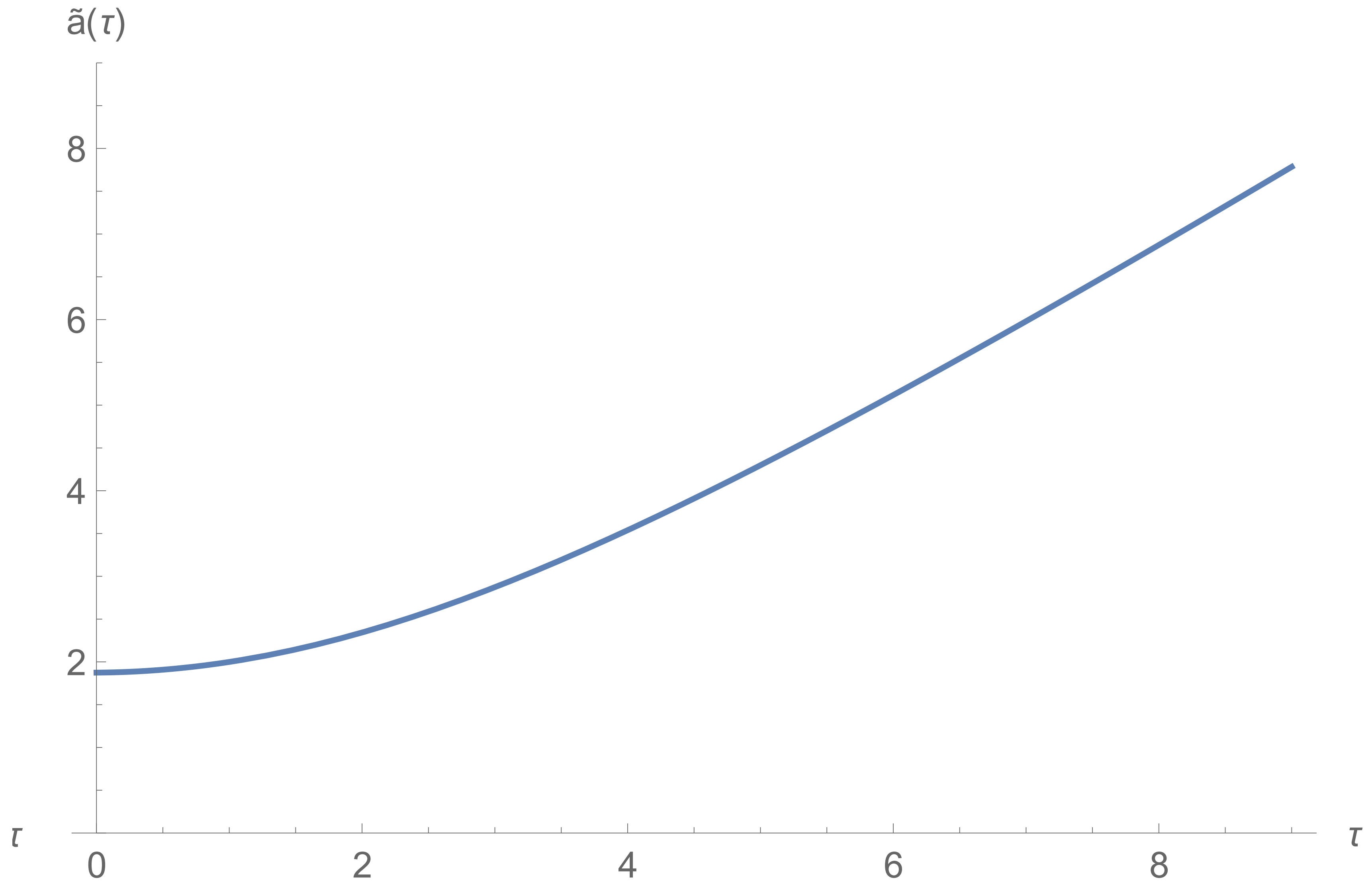
$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}} - 1) - 1\Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\ W_-\Big(-\frac{\gamma^2}{2\sqrt{e}}\Big), \qquad 0 \le \gamma^2 < \frac{2}{\sqrt{e}} \text{ and } \tilde{a} \ge \mu_2.$$

Type II Solution   Initial Acceleration of Finite Duration

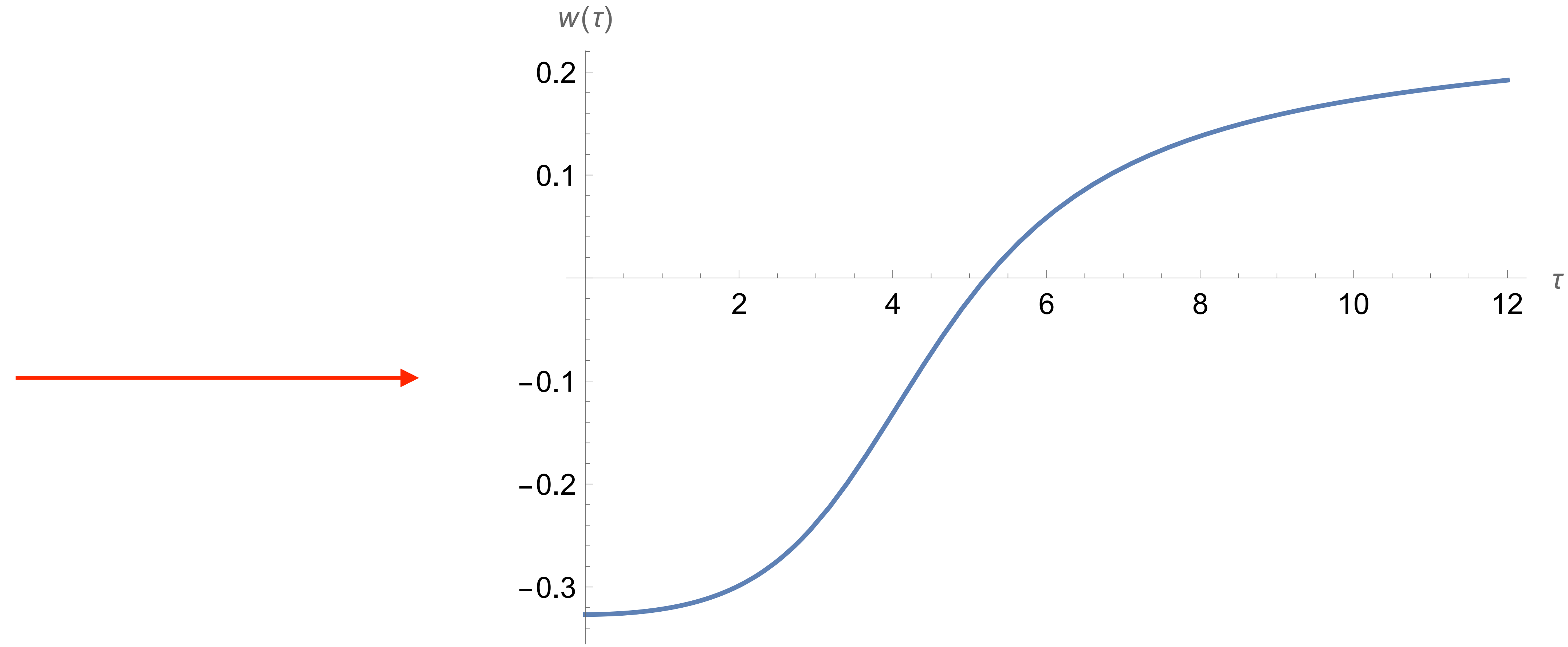
$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}}-1)-1\Big)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b\in[0,\infty],$$



The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq\ ct, \qquad a(\eta)\simeq a_0 e^{\eta}. \tag{5.87}$$

Type II Solution — Effective Parameter w



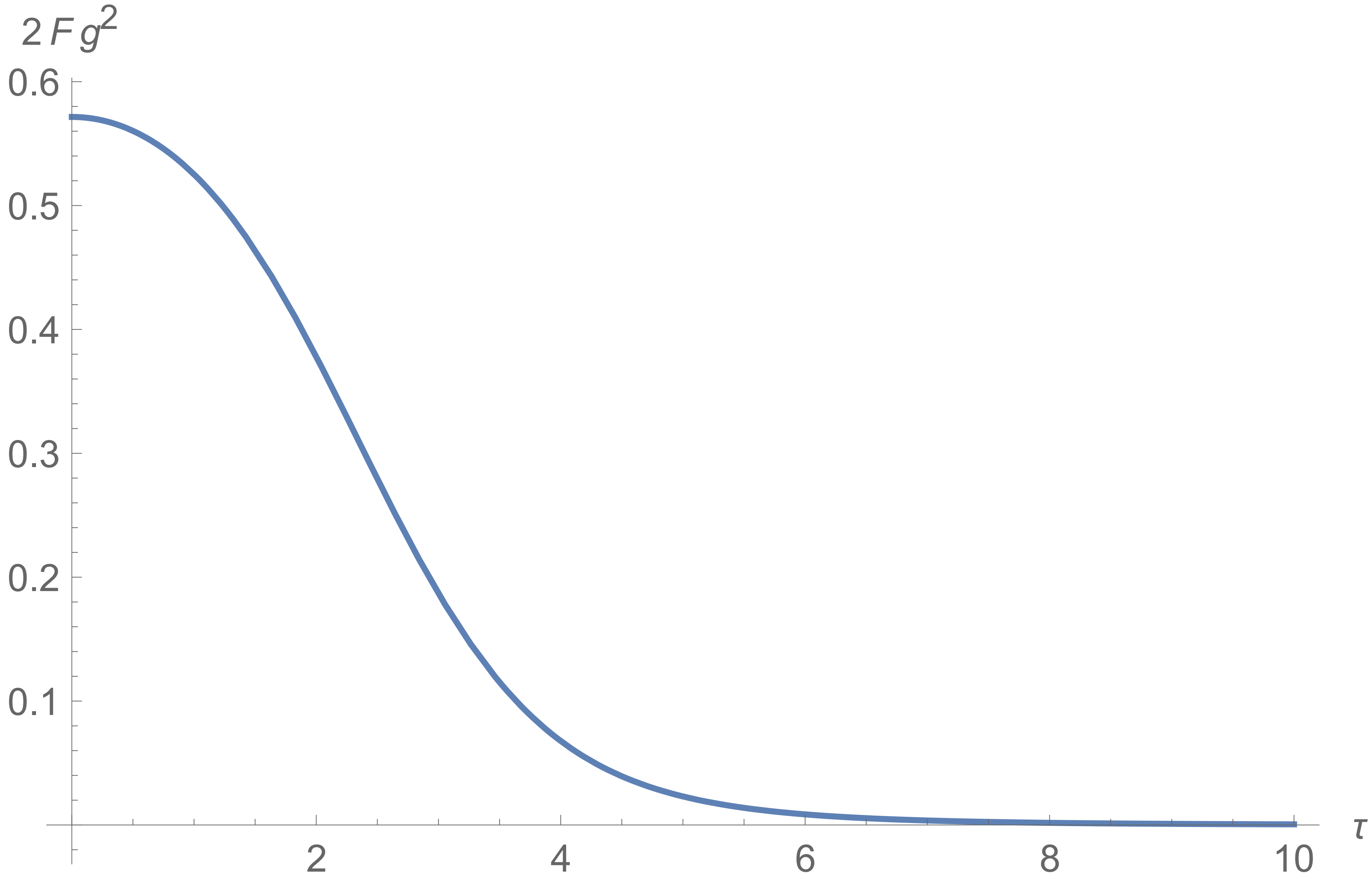
For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

$$w_{II} = \frac{b^2(\tau) + \gamma^2\mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2\mu_2^2\Big)}, \qquad -1 \leq w_{II},$$

$$w = \frac{p}{\epsilon} = \frac{\log\frac{1}{\tilde{a}^4(\tau)} + 3}{3\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)}.$$

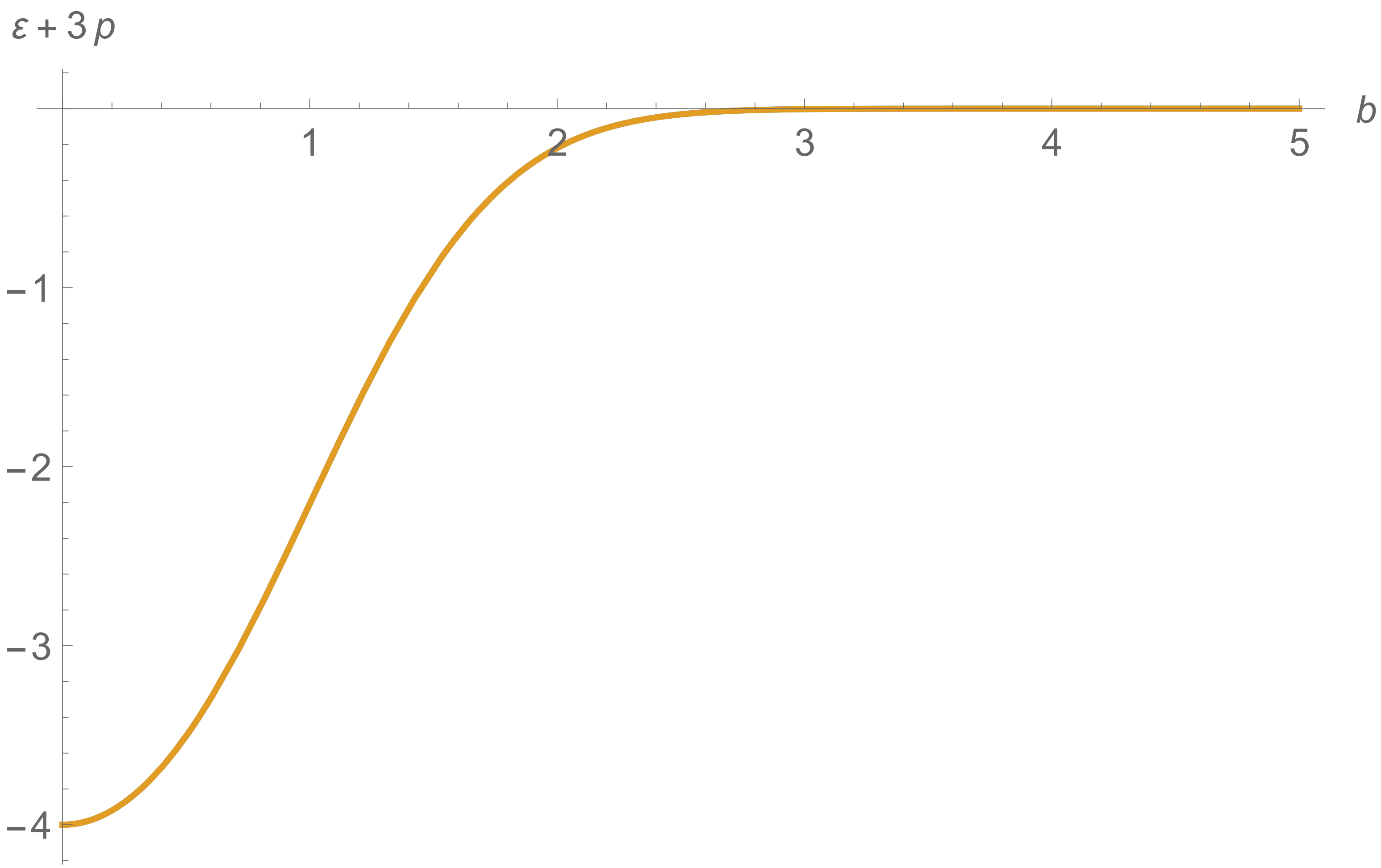
Evolution of the Field Strength

$$2g^2\mathcal{F} = \frac{\Lambda_{YM}^4}{\tilde{a}^4(\tau)}$$



Type II Solution — Initial Acceleration of Finite Duration

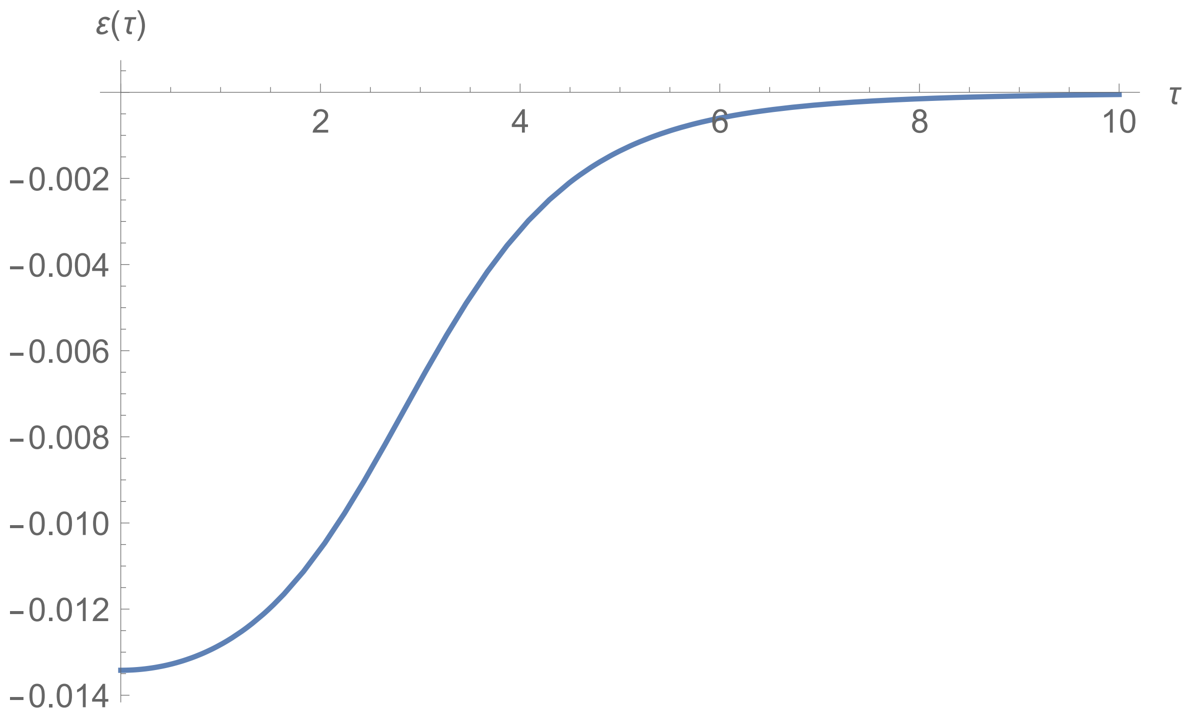
$$\epsilon + 3p = -\frac{2\mathcal{A}}{\mu_2^4}\ e^{-b^2(\tau)}(b^2(\tau) + \gamma^2\mu_2^2 - 2)\Lambda_{YM}^4, \qquad b \in [0, +\infty],$$

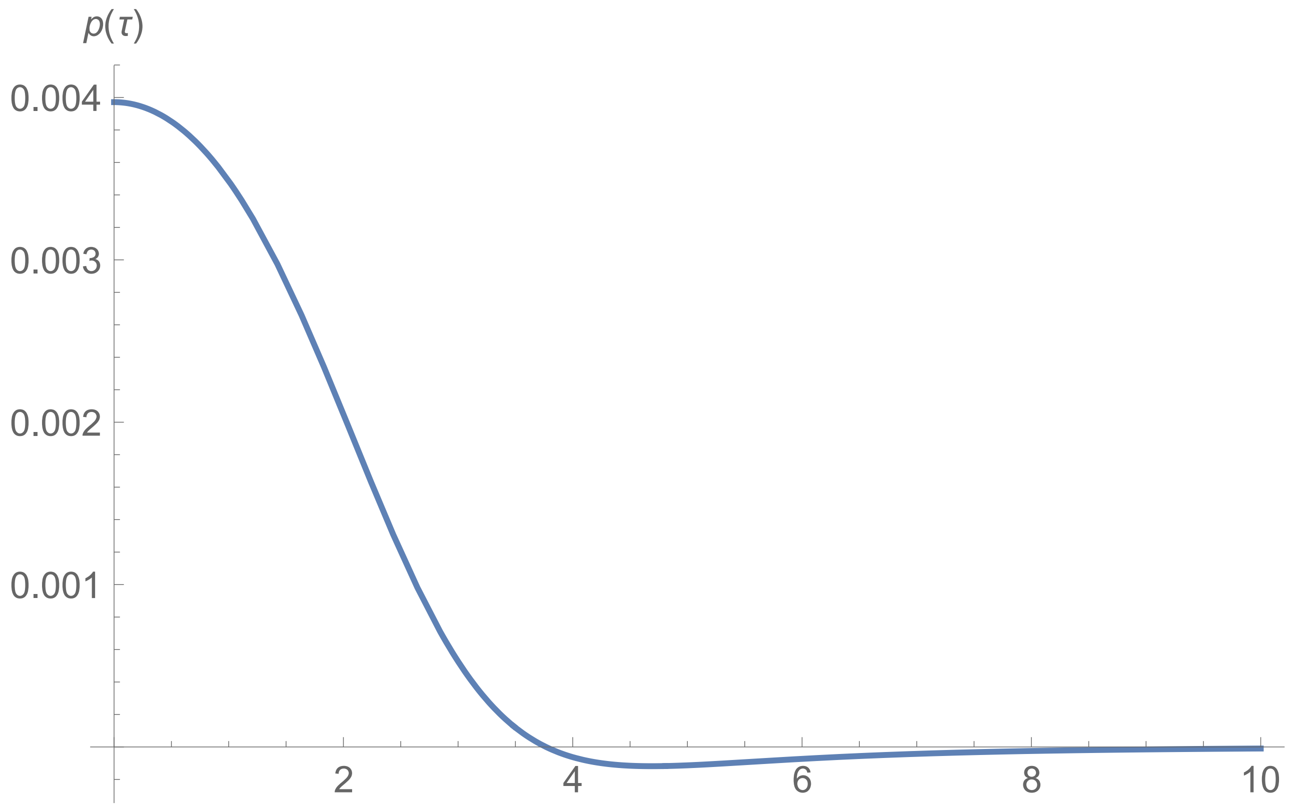


The r.h.s $\epsilon + 3p$ of the Friedmann acceleration equation (1.4) always negative

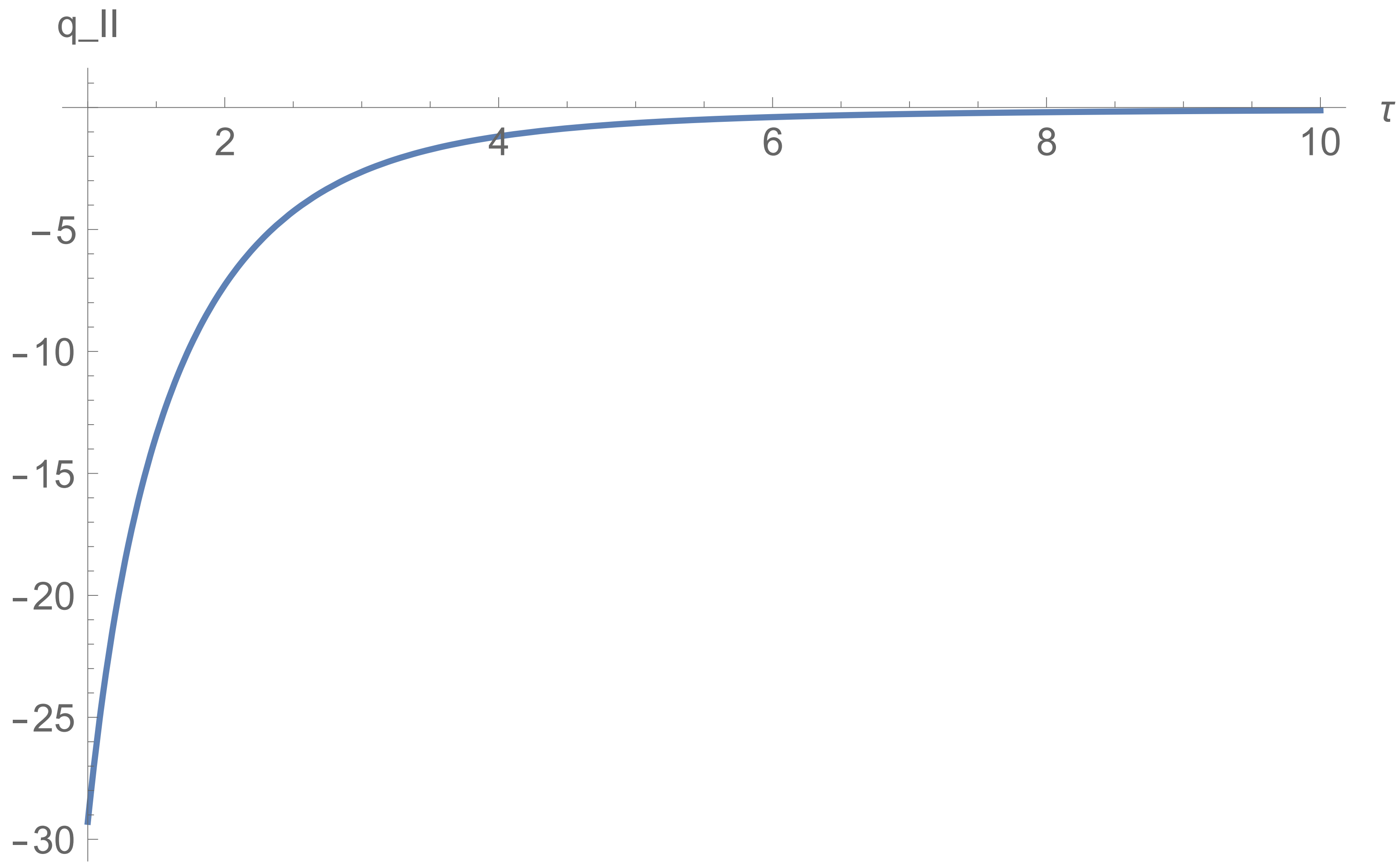
Evolution of Energy Density and Pressure

$$\epsilon = \frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)\Lambda^4_{YM}, \qquad p = \frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} + 3\Big)\Lambda^4_{YM}.$$





Type II Solution   *Deceleration of finite duration*



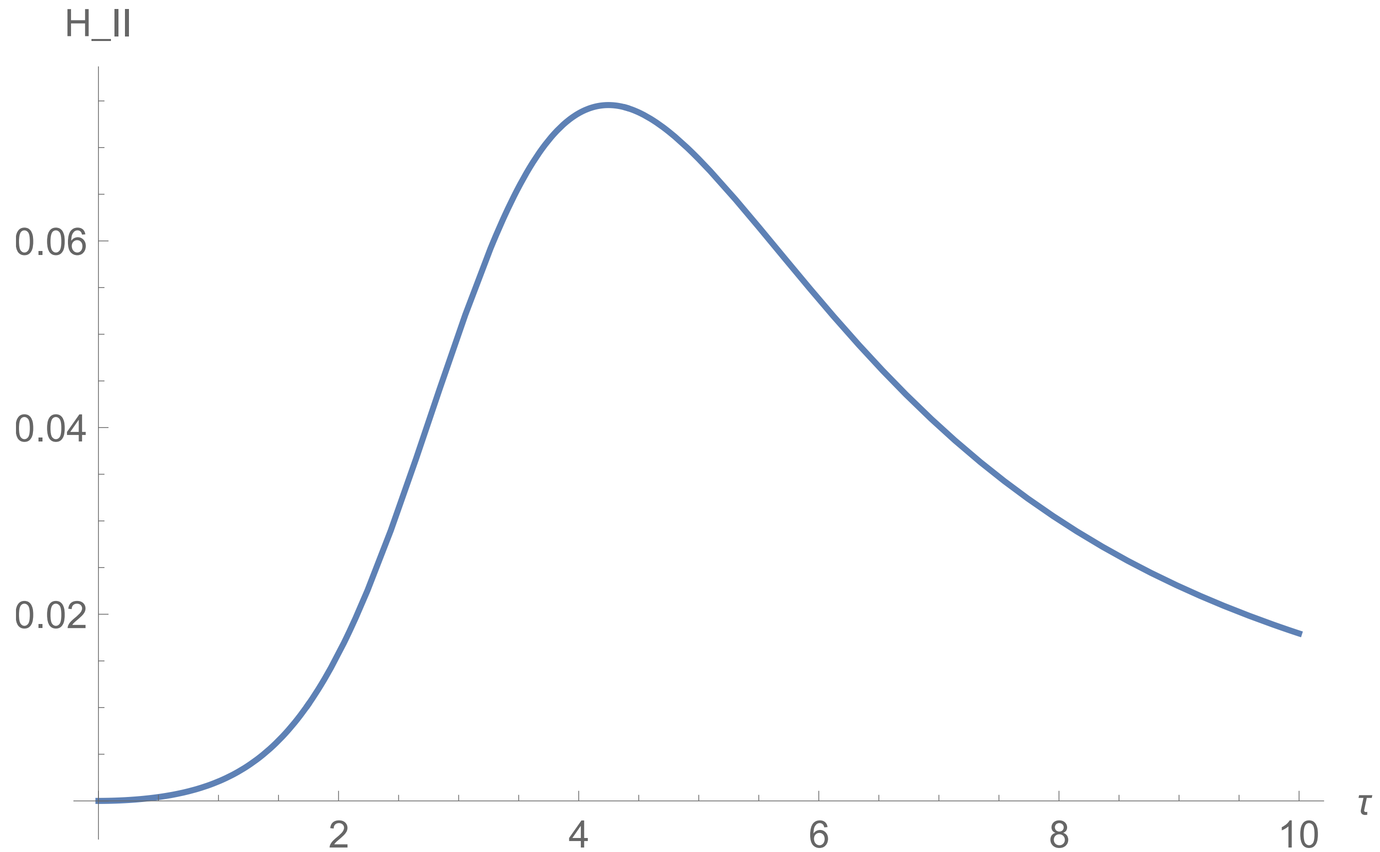
The deceleration parameter of the Type II solution is always negative:

$$q_{II} = \frac{b^2 + \gamma^2 \mu_2^2 - 2}{b^2 + \gamma^2 \mu_2^2 (1 - e^{b^2/2})} < 0 \qquad q_{II} \propto -\frac{2}{b^2} \qquad q_{II} \propto -\frac{b^2}{\gamma^2 \mu_2^2} e^{-b^2/2} \to 0.$$

Hubble Parameter

$$L^2H^2 = L^2\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{1}{\tilde{a}^2}\Big(\frac{d\tilde{a}}{d\tau}\Big)^2 = \frac{1}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big) - \frac{k\gamma^2}{\tilde{a}^2(\tau)}$$

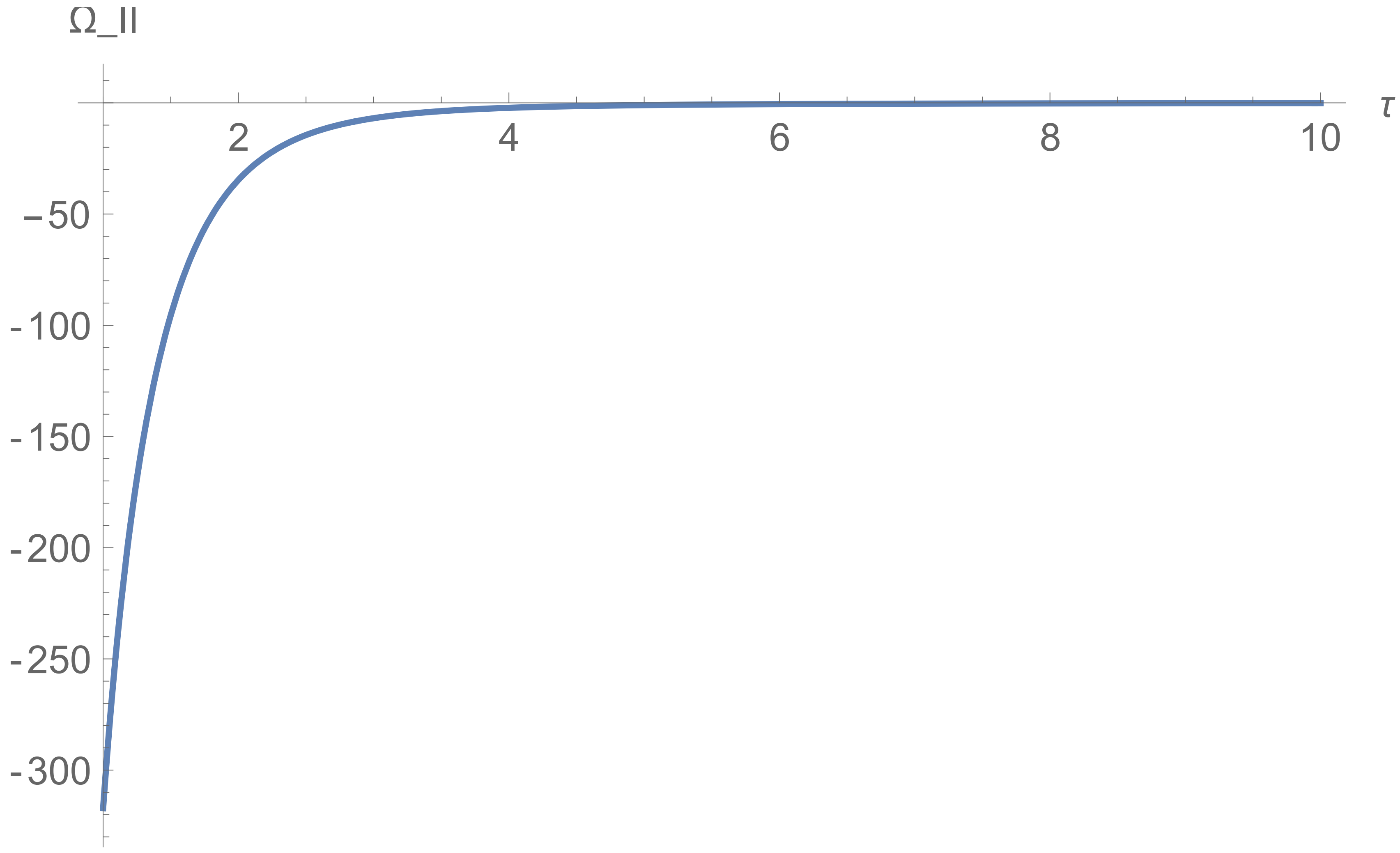
$$L^2H^2 = \frac{e^{-b^2}}{\mu_2^4}\Big(\gamma^2\mu_2^2(e^{b^2/2} - 1) - b^2\Big).$$



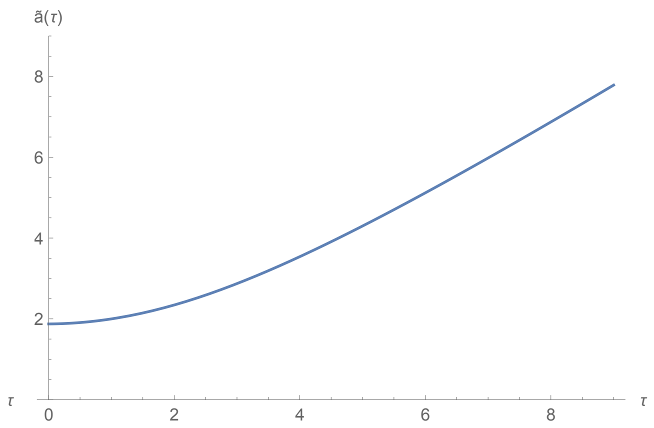
Type II Solution *Density Parameter*

$$\Omega_{vac} \equiv \frac{8\pi G}{3c^4}\frac{\epsilon}{H^2}$$

$$\Omega_{vac} - 1 = -\frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = -\frac{\gamma^2\mu_2^2 e^{b^2/2}}{\gamma^2\mu_2^2(e^{b^2/2}-1) - b^2}$$



Type II Solution   Initial Acceleration of Finite Duration



The number of e-foldings

typical parameters around $\gamma^2 = 1.211$, $\mu_2^2 \simeq 1.75$ we get $\tau_s = 10^{23}$ and $\mathcal{N} \simeq 53$. $\quad \mathcal{N} = \ln \frac{a(\tau_s)}{a(0)}$.

$$t_s^{GUM} = \frac{L_{GUM}}{c}\tau_s \simeq 4.2 \times 10^{-13}\ sec, \qquad \text{where } L_{GUM} \simeq 1.25 \times 10^{-25} cm$$

$$a(0) = L_{GUM}\frac{\mu_2}{\gamma} \simeq 1.5 \times 10^{-25} cm, \qquad a(t_s) = L_{GUM}\frac{\mu_2}{\gamma}e^{\mathcal{N}} \simeq 1.25 \times 10^{-2} cm,$$

The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \qquad (5.87)$$

The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor[‡]

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At the late stages of the inflation the asymptotic behaviour of the scale factor became linear in time $a(t) \approx ct$ and corresponds to a flat geometry. The metric

$$ds^2 = c^2 dt^2 - c^2 t^2 (d\chi^2 + \sin^2 \chi d\Omega^2)$$

transformation $r = ct \sinh \chi$, $\tau = t \cosh \chi$ reduce metric to flat metric $ds^2 = c^2 d\tau^2 - (dr^2 + r^2 d\Omega^2)$.

Type IV Solution - Late time Acceleration

The type IV solution is defined in the region $\gamma^2 > \gamma_c^2$ where the equation

$$U_{-1}(\mu) = \frac{1}{\mu^2}\Big(\log\frac{1}{\mu^4} - 1\Big) + \gamma^2 = 0$$

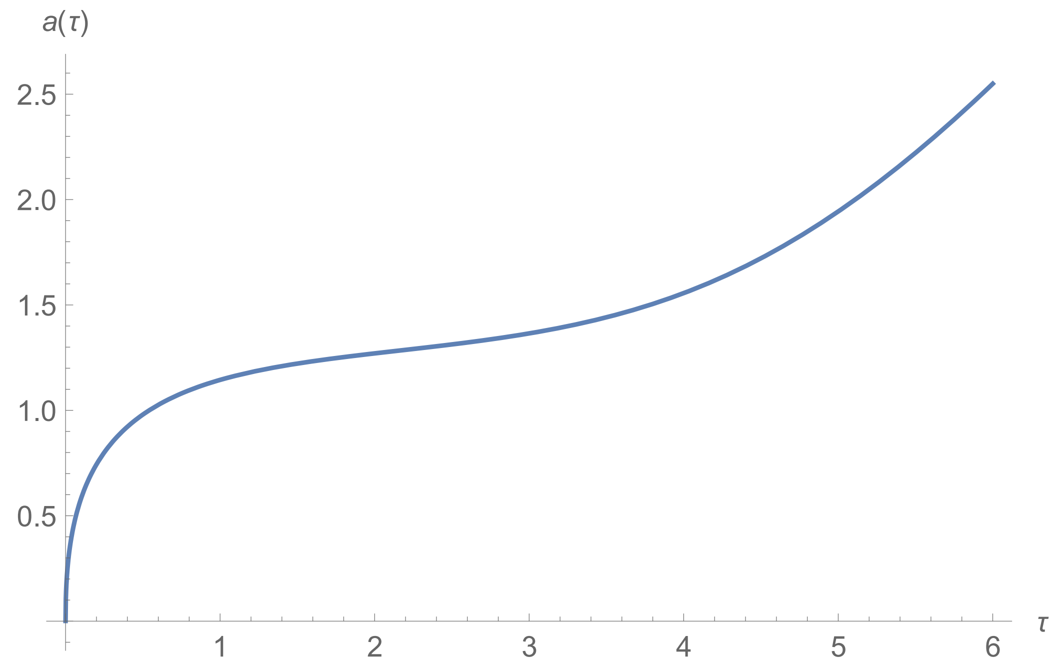
$$\tilde{a} = \mu_c e^b, \qquad b \in [-\infty, \infty], \qquad 2 < \gamma^2\mu_c^2, \qquad \gamma_c^2 = \frac{2}{\sqrt{e}},$$

$$\frac{db}{d\tau} = \sqrt{\frac{2}{e}}\; e^{-2b}\Big(\frac{\gamma^2}{\gamma_c^2}e^{2b} - 1 - 2b\Big)^{1/2}.$$

$$2g^2\mathcal{F} = e^{-4b(\tau)-1}\Lambda_{YM}^4,$$

$$\epsilon = 2\mathcal{A}e^{-4b(\tau)-1}\Big(-2b(\tau) - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{2\mathcal{A}}{3}e^{-4b(\tau)-1}\Big(-2b(\tau) + 1\Big)\Lambda_{YM}^4.$$

Type IV Solution - Late time Acceleration



$$q_{IV} \simeq -\frac{2}{\gamma^2 \mu_c^2} b e^{-2b} \to 0.$$

$$H = \sqrt{\frac{2}{e}} \frac{e^{-2b}}{L} \Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \Big)^{1/2} \simeq \frac{1}{ct}.$$

$$\Omega_{vac} = 1 - \frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = 1 - \frac{\gamma^2 e^{2b}}{\gamma_c^2 \Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \Big)} \to 0.$$

Primordial Gravitational Waves

The Freidmann equation in the gauge field theory vacuum

$$\Big(\frac{\tilde{a}^{'}}{\tilde{a}}\Big)^2 = \frac{1}{\gamma^2}\frac{1}{\tilde{a}^2}\Big(\ln\frac{1}{\tilde{a}^4} - 1\Big) - k$$

together with the acceleration equation

$$\frac{\tilde{a}^{''}}{\tilde{a}} - \Big(\frac{\tilde{a}^{'}}{\tilde{a}}\Big)^2 = -\frac{1}{\gamma^2}\frac{1}{\tilde{a}^2}\Big(\ln\frac{1}{\tilde{a}^4} + 1\Big).$$

gives

$$\tilde{a}^{''} = -\frac{2}{\gamma^2}\frac{1}{\tilde{a}} - k\tilde{a}$$

and the linear perturbation equation will take the form

$$\theta^{''} + \theta\Big(n^2 + \frac{2}{\gamma^2}\frac{1}{\tilde{a}^2} + k\Big) = 0.$$

In case of Type II solution with $\tilde{a}(0) = \mu_2$ the system avoids a singular behaviour in vicinity $\eta = 0$.

The amplification of the primordial gravitational waves is due to the second term when $n^2 < 2/\gamma^2\mu_2^2$.

Thank You !