String Excitation by Initial Singularity of Inflation

Based on JHEP10(2021)025

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Introduction (1/2)

・Inflation models are the most vigorously studied model of the early universe.

ex. Starobinsky model, hill-top model, etc...

・A singularity occurs at very early stage of inflation due to a component of Ricci tensor diverges.

→Does a singularity due to the divergence of a component of Ricci tensor cause problems?

Introduction (2/2)

・From what point of view do you find out “whether an initial singularity causes problems"?

・Effects of quantum gravity is important on the early stage of inflation.

→ Consider (bosonic) string theory to investigate the initial singularity of an inflationary universe.

・Focus on expectation values of mass of string.

・A singularity should be resolved by effects of quantum gravity if it causes a divergence of mass.

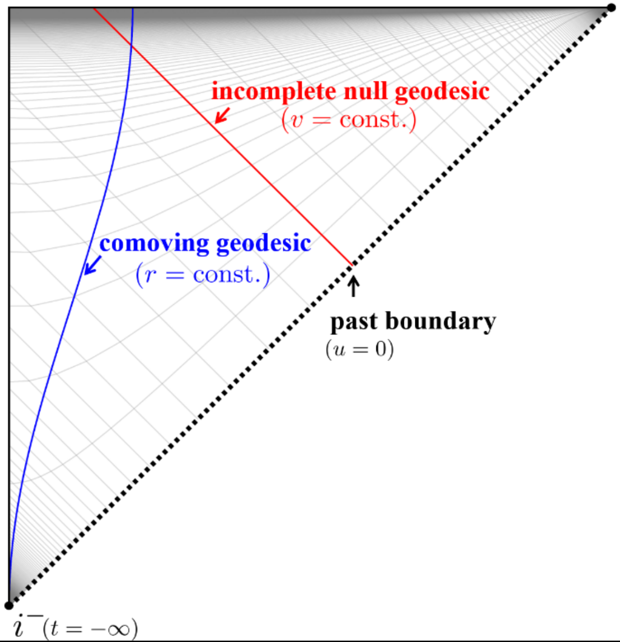
Contents

Initial singularity in FLRW space-time

・Flat FLRW space-time in light cone coordinates

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -2dudv + a^2(u)dv^2 + a^2(u)(v-\eta(u))^2 d\Omega^2$$

・A singularity cause at the past $u=0$ by divergence of uu component of Ricci tensor in inflationary universe.



$$R_{uu} = -2A(u)$$

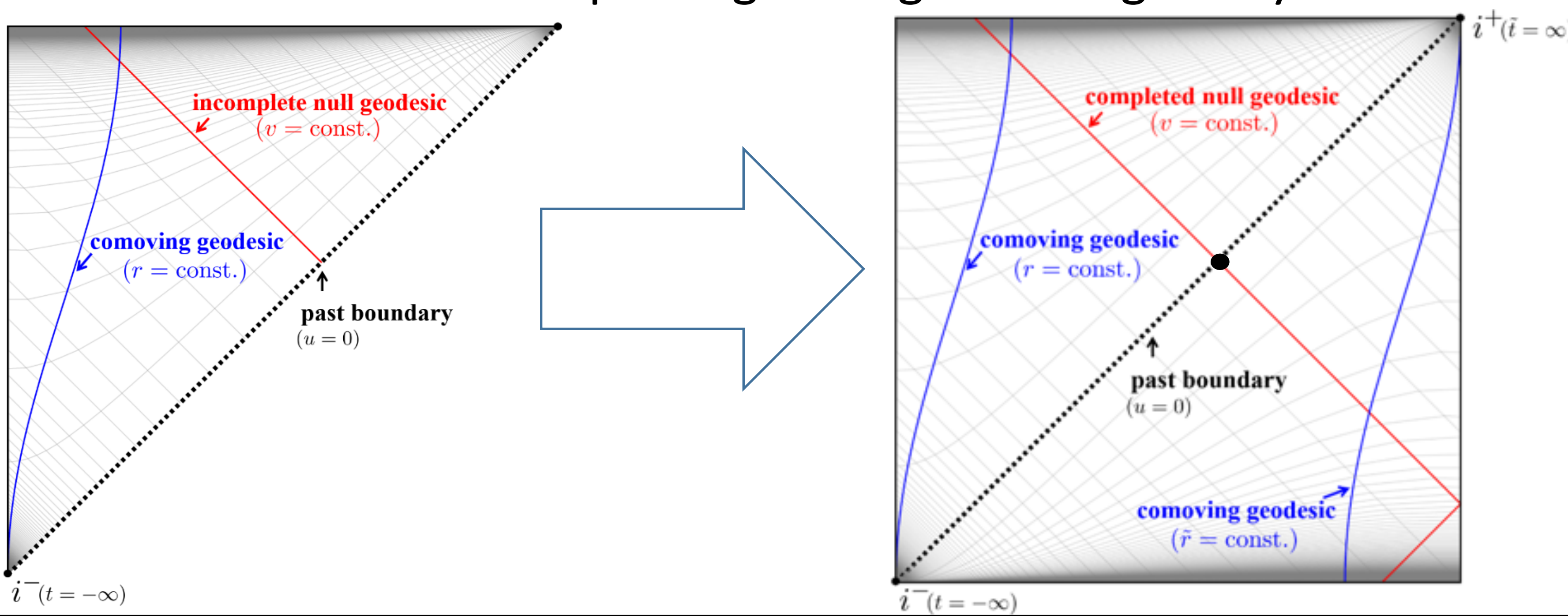
$$A(u) := \frac{\dot{H}}{a^2}, \quad a(u) \overset{u\to 0}{\sim} 0$$

・How do expectation values of mass behave when a string passes through the singularity?

Contracting universe

・Construct a contracting universe by using a continuity of the metric.

・The Bogoliubov transformation allows us to compare the masses before and after passing through the singularity.



EOM of string

・Consider a quantization of string and EOM in simplified FLRW space-time by taking the Penrose limit.

・$X_n^i(\tau)$ determine the motion of strings.

$$-\frac{d^2 X_n^i}{d\tau^2} + \mathcal{V}(\tau) X_n^i = E_n X_n^i, \quad \mathcal{V}(\tau) := \alpha'^2 p^2 A(\alpha' p \tau), \quad E_n := n^2$$

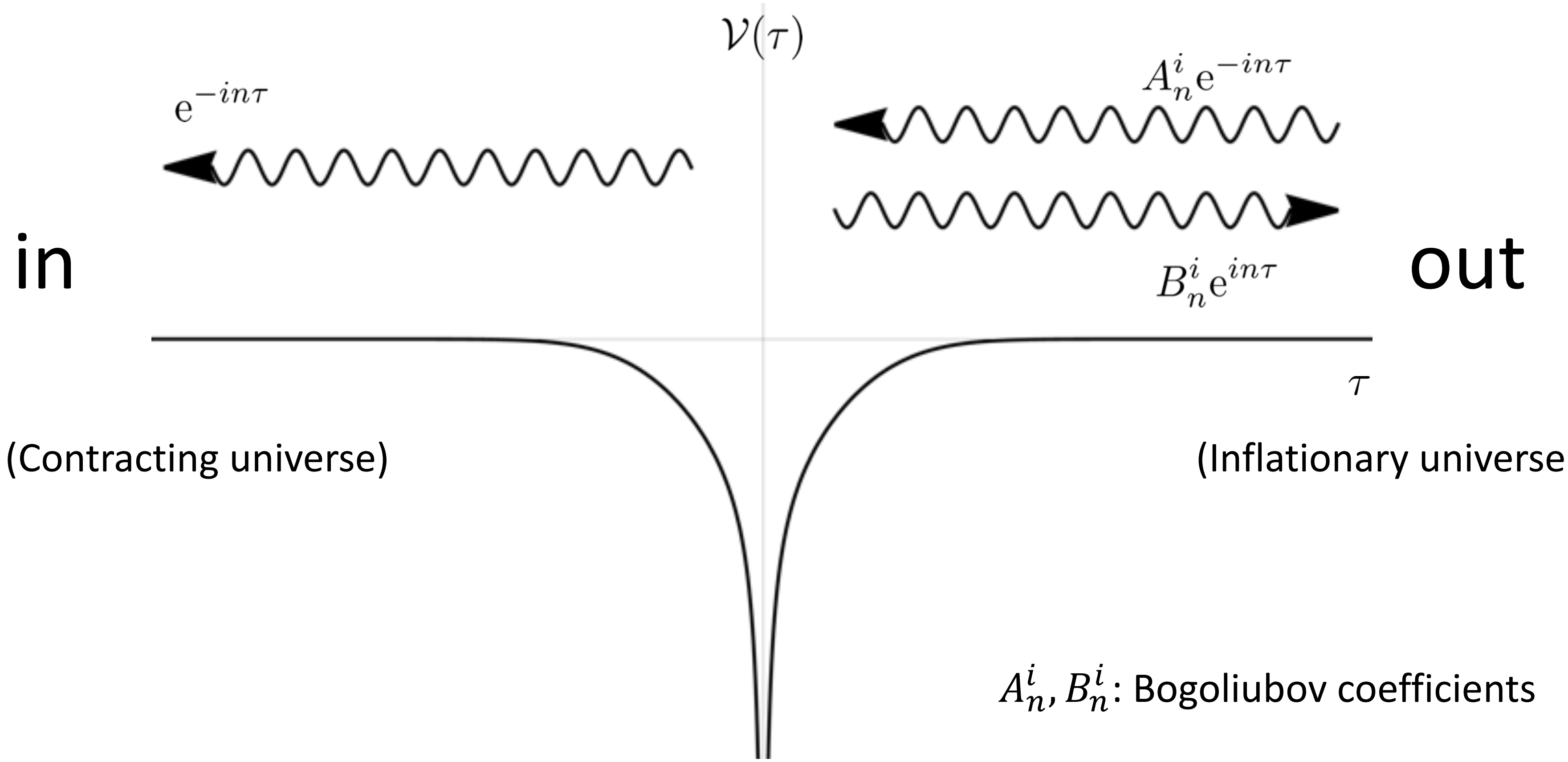
・This is Schrödinger-like equation has a potential determined by $A(u = \alpha' p \tau)$.

※ $R_{uu} = -2A(u)$

$$A(u) := \frac{\dot{H}}{a^2}, \quad a(u) \overset{u \to 0}{\sim} 0$$

String excitation by singularity (1/3)

・The Bogoliubov transformation from in-state (contracting universe) to out-state (inflationary universe)



A_n^i, B_n^i: Bogoliubov coefficients

String excitation by singularity (2/3)

・Calculate an expectation value of mass of out-state looked from in-state by using the Bogoliubov coefficients.

$$\langle M_{\rm out}^2 \rangle = \frac{\langle 0_{\rm in}|\, M_{\rm out}^2 \,|0_{\rm in}\rangle}{\langle 0_{\rm in}|0_{\rm in}\rangle} \sim \frac{2}{\alpha'} \sum_{n=1}^{\infty} \sum_{i=1}^{2} n |B_n^i|^2$$

・The Bogoliubov coefficient B_n^i is the following as,

$$B_n^i \simeq \frac{p^2 \alpha'^2}{2in} \int_{-\infty}^{\infty} d\tau \; e^{-2in\tau} A(\alpha' p \tau)$$

so we can calculate the mass when we obtain uu components of Ricci tensor $A(u = \alpha' p \tau)$.

String excitation by singularity (3/3)

・In general, $A(u)$ takes the following form in an inflation models. (The values of β are different for each models)

$$A(u) = -\frac{\kappa}{|u|^{\beta}}, \quad \kappa > 0 : \mathrm{const}, \quad 0 < \beta < 2$$

・The expectation value of mass converges for $0 < \beta < 1$ and diverges for $1 \leq \beta < 2$.

$$\langle M_{\mathrm{out}}^2 \rangle = \begin{cases} \text{converge} & 0 < \beta < 1, \\ \text{diverge} & 1 \leq \beta < 2 \end{cases}$$

→We can distinguish models whether break down or not by calculating the value of β.

Starobinsky model

- The potential of Starobinsky model is

$$V(\phi) = 3M_{pl}^2 \bar{H}^2 \left(1 - \mathrm{e}^{-\frac{\phi}{\mu}} + \cdots \right), \quad \mu : \mathrm{const}$$

- We can calculate $A(u)$ easily as

$$A(u) \sim -\frac{\kappa}{u^2 (\log u)^2}$$

This corresponds to $1 \leq \beta < 2$, so the mass diverges.

→The initial singularity in Starobinsky model should be removed by effects of quantum gravity.

Cosine type hill-top model

・The potential of Cosine type hill-top model is

$$V(\phi) = \frac{3M_{pl}^2 \bar{H}^2}{2}\left(1 + \cos\frac{\phi}{f}\right) = 3M_{pl}^2 \bar{H}^2 \left(1 - \frac{1}{2}\frac{M_{pl}^2}{2f^2}\frac{\phi^2}{M_{pl}^2} + \cdots\right)$$

Then, we obtain

$$A(u) = \frac{-\kappa}{u^{2(1-\gamma)}}, \quad \gamma = \frac{-3 + \sqrt{9 + \frac{6M_{pl}^2}{f^2}}}{2}$$

・If we ignore the slow-roll condition,

$$\langle M_{\rm out}^2 \rangle = \begin{cases} \text{converge} & f < \sqrt{\frac{6}{7}} M_{pl}, \\ \text{diverge} & f \geq \sqrt{\frac{6}{7}} M_{pl} \end{cases}$$

Summery

・The initial singularity causes at early stage of inflation.

→Do inflation models break down due to the singularity as EFT of string theory?

	Is it OK?	Value of β
Starobinsky model	×	$1 \leq \beta < 2$
Cosine type hill-top model	△	$0 < \beta < 1$ or $1 \leq \beta < 2$

・We suggested the method to distinguish inflation models whether they break down or not due to the initial singularity.

・Stringy corrections should resolve the initial singularity even though the value of the Hubble parameter remains smaller than the string scale.

Outlook

・ Calculate the time-dependence of the expectation value of mass (string excitation) without the contracting universe.

・ Take into account a back reaction of the excitation.

Calculations

- $A(u)$ is calculated by potential $V(\phi)$.

$$V(\phi) = 3M_{pl}^2 \bar{H}^2 \left(1 + \delta V(\phi)\right)$$

$$A(u) = \frac{1}{a^2}\frac{dH}{dt} \simeq -\frac{M_{pl}^2}{2u^2} \times \delta V'(\phi(u))^2$$

- EOMs of inflaton ϕ are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \qquad H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$