FIELD INHOMOGENEITIES
and
HOW TO SPECIFY MEASUREMENT REQUIREMENTS

- **What is the effect of field inhomogeneities and errors on emittance?**
- **How well do fields have to be measured?**
- **Hmmm…**
  - ***Often asked; no quantitative answer so far***
  - ***Difficult to study with Monte Carlo***
    - **Requires**
      - **A model with correlated errors**
      - **Extensive, time-consuming simulations**
      - **Someone with nothing better to do**

- **This is an *attempt* to find a general answer**

- ***Field quality criteria:***
  1. Emittance growth < 0.1% of 6mm
  2. Displacement of reference muon < something
  3. p_t acquired by reference muon < something
- 1, 2 and 3 are related via beam optics

- Define some measurable property of field that ensures 1 / 2 / 3

- If field good enough, usable without correction (map in S/W)

- May be different criteria in Tracker regions
  – Software people to define

- RMS errors not sufficient
  – There will be correlations over some distance

GENERALITIES

- **Assume $B = B_{desired} + B_o$**

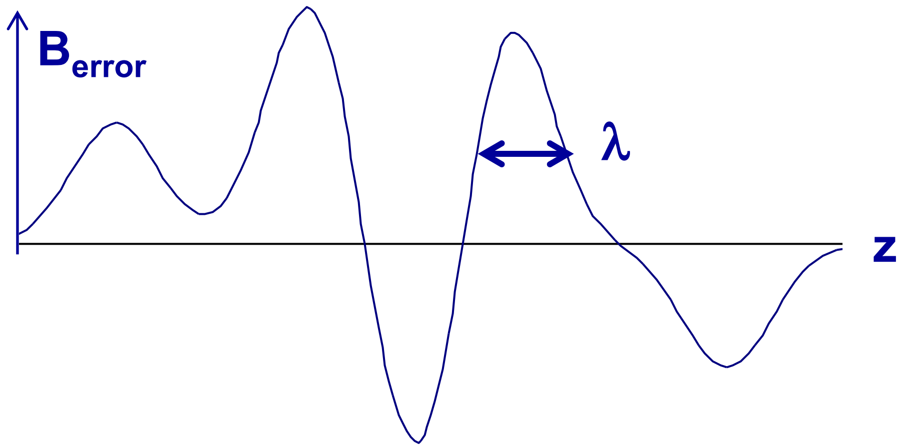
- **The error field components, $B_{o,}$ could give emittance growth**

- **How to characterise & quantify?**
  - **The *integrals* B_o *dz* matter rather than B_o *per se***
  - ***e.g.* 5 Gauss x 1/5 metre == 1 Gauss x 1 metre & so on**
  - **rms field errors are not very useful – need to account for correlations**

- **Imagine a Worst Worst-Case**
  - **Each muon experiences a different set of inhomogeneities, characterised by a transverse (x,y) error field $B_o$and a length λ**

  - **Each muon will experience a p_t kick at each inhomogeneity**

  - **Treat the problem like multiple scattering from the inhomogs.**



- Characterise transverse field inhomogeneities by impulses:

$$I_i = \int B_i dz = B_0 \lambda$$

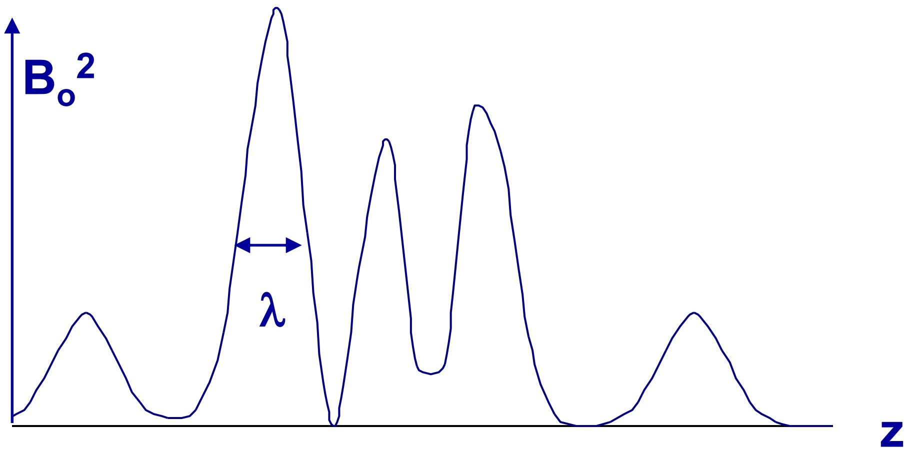
where λ is *correlation length* ($\sim$ 'width' of inhomogeneity)

- Each inhomogeneity gives $p_\perp$ kick of

$$p_i = qB_0\lambda$$

- At each kick the angle changes by

$$\theta_i = \frac{p_i}{p_z} = \frac{qB_0\lambda}{p_z}$$



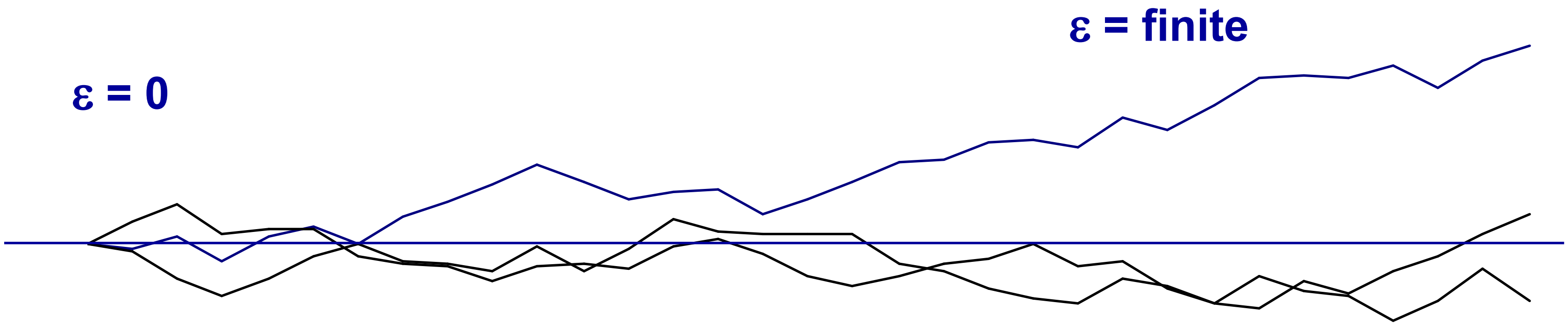
- Number of inhomogeneities per unit length $= \frac{1}{\lambda}$
- Assume $\langle B_0 \rangle = 0$ then $p_\perp$ kicks add in quadrature and

x c^2 !!!

$$\frac{d\theta^2}{dz} = \left(\frac{qB_0\lambda}{p_z}\right)^2 \frac{dz}{\lambda} = \frac{q^2}{p_z^2} B_0^2 \lambda dz$$

- *The $p_\perp$ kicks will give emittance growth, just like multiple scattering*
- The problem is described by two parameters: B_0^2 and λ

B_o^2 is the *mean square transverse error field,*
λ is a *correlation length*



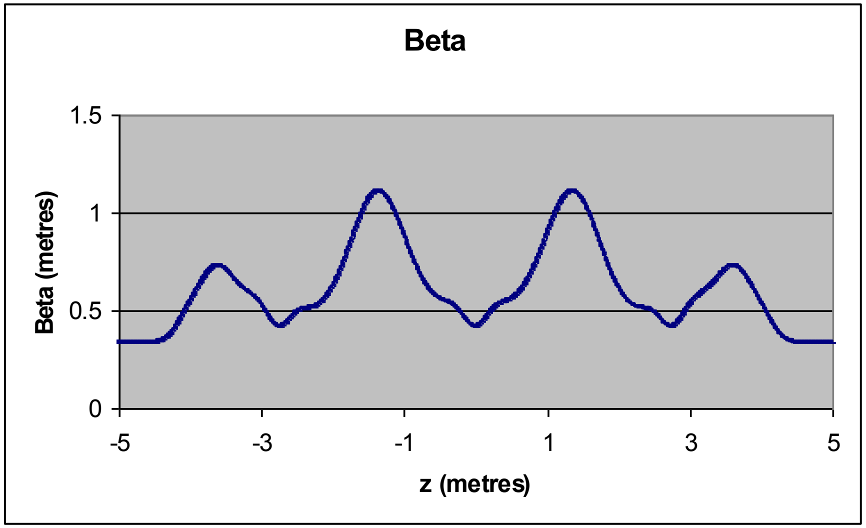
- The moments of the beam distribution can be evolved to find the trace-space emittance growth:

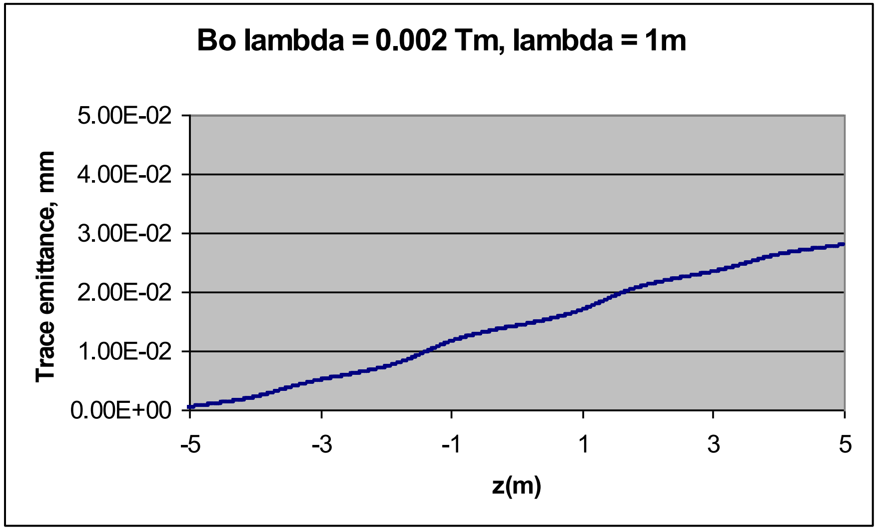
$$\frac{d\epsilon}{dz} = \frac{\beta_t}{2}\frac{d\theta^2}{dz} = \frac{\beta_t}{2}\frac{q^2}{p_z^2}B_0^2\lambda$$

- The overall emittance growth is

$$\Delta\epsilon = \int_0^L \frac{\beta_t}{2}\frac{q^2}{p_z^2}B_0^2\lambda dz = \frac{q^2}{p_z^2}B_0^2\lambda\int_0^L \frac{\beta_t}{2}dz$$

- Errors are characterised by B_0^2 and λ
- *What happens with some (guessed) values?*





RH plot shows emittance growth of 0.03 mm for

$\langle B_o^2 \rangle = (0.002\ T)^2 = (20\ Gauss)^2$ and λ = 1 metre

Trace emittance growth of 0.03mm
= Normalised emittance growth of 0.015mm (for 200 MeV/*c* muons)

➔ 0.25% emittance growth of 6mm beam for $\langle B_o^2 \rangle \lambda$ = 400 Gauss^2-m

➔ Require $\langle B_o^2 \rangle \lambda$ < 160 Gauss^2-m for <0.1% of 6mm ←

to keep emittance growth acceptably low

But with artificial assumption that all muons see independent errors

So think about it slightly differently…

- **Have considered what happens if all muons see independent sets of inhomogeneities**

- **That is artificial**
  - **Most of the beam will see the same (similar) field errors**
  - **Instead of increasing the emittance, the errors will wobble the beam around**
  - **COG of beam will be displaced from the axis**
  - **i.e. reference muon moves off axis**

- **Use previous result to predict the rms uncertainty in *x* or *y* due to same field error (i.e. 160 Gauss^2-m)**
  - **i.e. if we know fields are good to this value, but have no more detailed information than that**

- Imagine building are large ensemble of MICEs (*how long would that take?)*

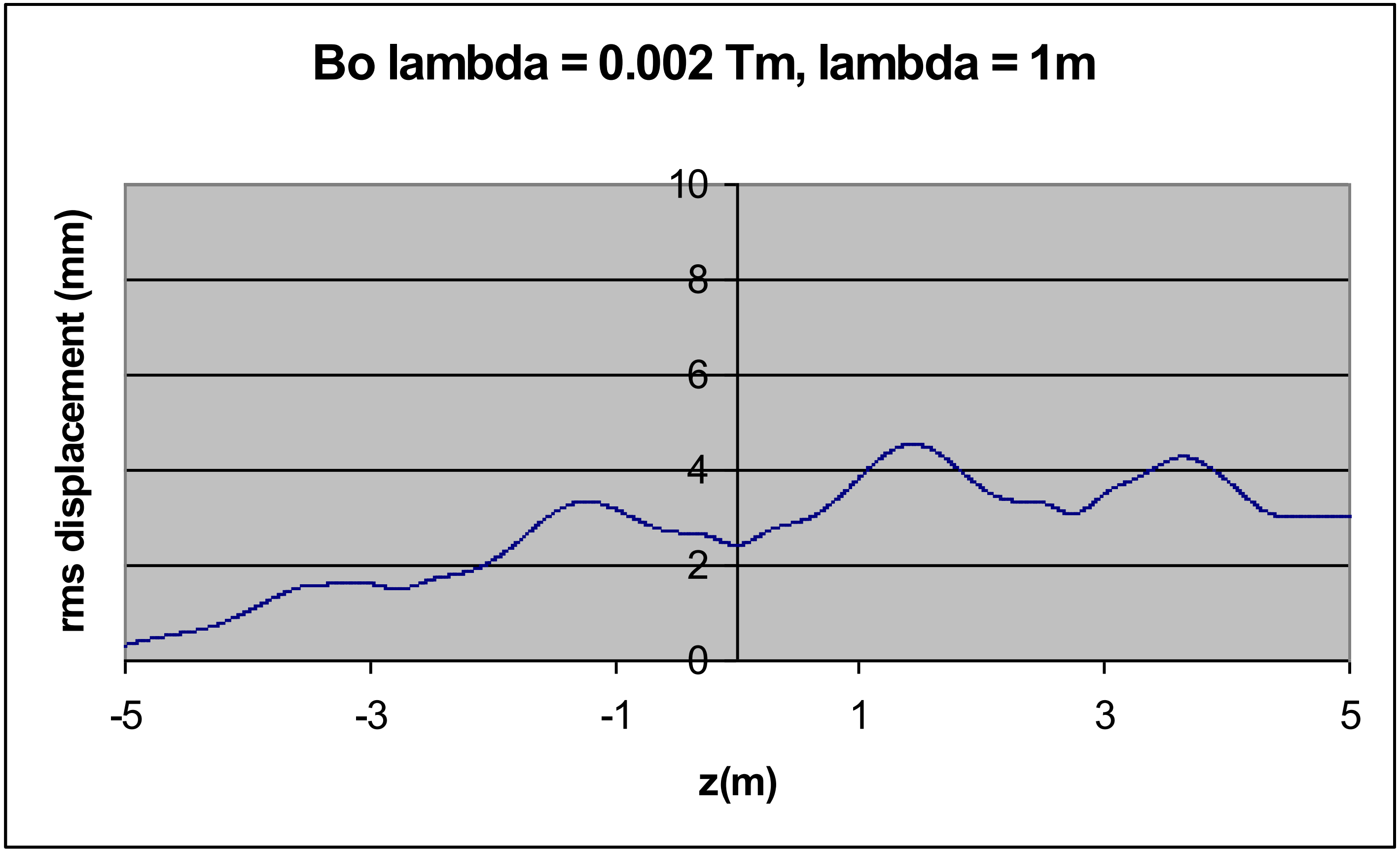
- Each has same average field errors, different in detail
  - Beams will all be deflected differently

- Add the results for one muon through each experiment
  - The ensemble of muons has emittance growth as above
  - Expected mean square deflection of beam at Tracker-2 in one experiment is then

    $\sigma_{xx} = \beta\varepsilon = 330\text{mm} \times 0.0075\text{mm} = 2.5\text{mm}^2$

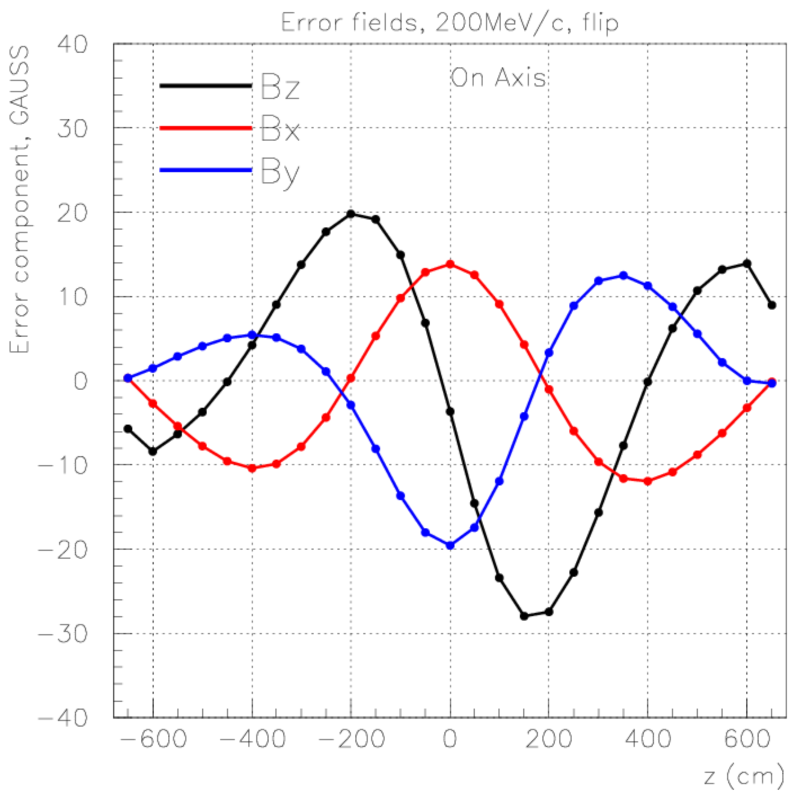
  - rms displacement of beam due to unknown field errors of 160 Gauss^2-m and λ = 1m is ~ 1.6mm
  - Does this make any sense so far?
    - Example ➔

RMS displacement of Reference Muon



$(B_o\ \lambda)$ = (20 Gauss metres), λ = 1m
as previous example for emittance growth

Error fields due to shield walls, 200 MeV/*c*, Flip mode, Step VI



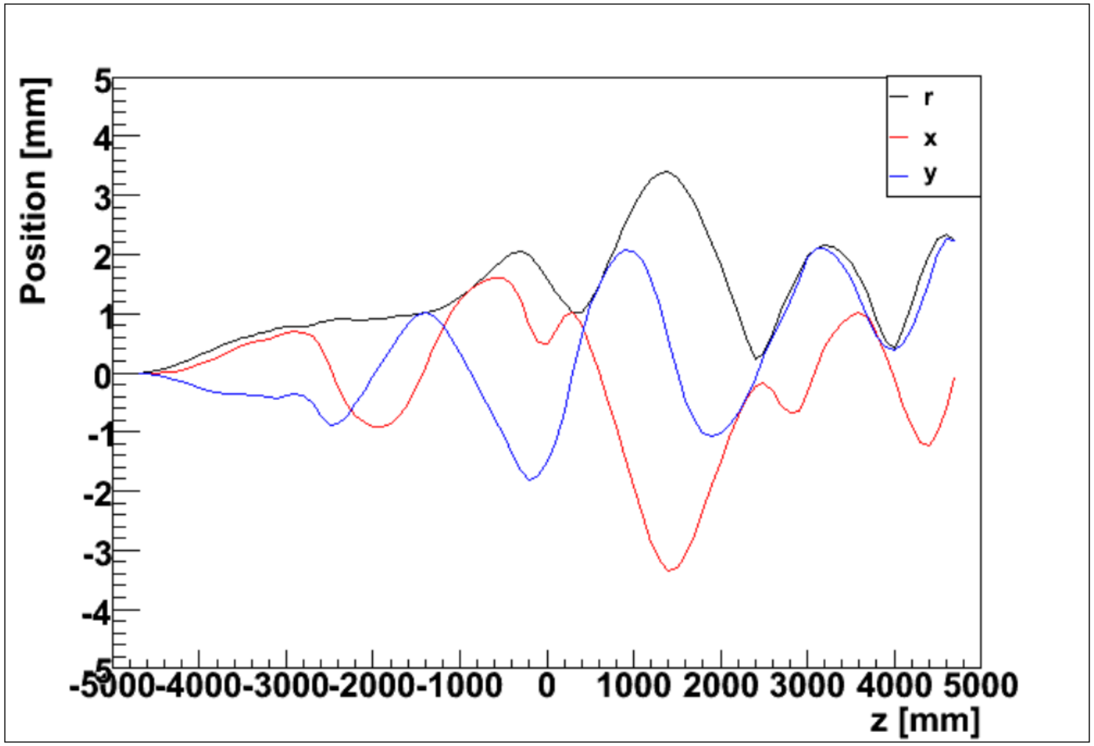
B_x ~ 12 sin (2 π x / 7 metres) Gauss

< B_x^2> = 72 $Gauss^2$

λ ~ 1 - 2 metres

➔ < B^2> λ ~ 100 $Gauss^2$-metre

Rather extreme long-range correlations



Displacement of reference muon due to shield error fields

~2.5 mm for < B^2> λ ~ 100 Gauss^2-metre and λ ~ 1 - 2 metres

Not inconsistent with simple model
– though *extreme case of only two kicks*
– Gives some confidence, perhaps

SUMMARY SO FAR

- **Hand-waving model suggests**
  - **Field errors should be described by $B_o^2\ \lambda$**
    - **or something similar**
  - **Need $B_o^2\ \lambda$ < 160 Gauss^2 – metres and λ > 1m to limit deflection of reference particle to < 1 – 2 mm at end**

- **However, this description is not very satisfactory**
  - **Parameter λ is unknown**
    - **Could have a wide range**

- **Try to include correlations correctly**

INCLUDING CORRELATIONS

Without much mathematical rigour and using B for error field ($\times qc$)

- Consider set B_i of N equally spaced measurements over length L
  $\Delta = L/N$
- p_t^2 acquired by muon in traversing L is

$$p_t^2 = \Delta^2(\sum B_i)^2 = \Delta^2(\sum B_i \sum B_j)$$
$$= \Delta^2(\sum_{i=1,N} B_i^2 + \sum_{i \neq j} B_i B_j)$$

so

$$\overline{\frac{dp_t^2}{dz}} = \frac{1}{N\Delta}\left(\Delta^2(\sum_{i=1,N} B_i^2 + \sum_{i \neq j} B_i B_j)\right)$$
$$= \Delta\sigma_B^2 + \frac{\Delta}{N}\sum_{i \neq j} B_i B_j$$
$$= \Delta\sigma_B^2 + (N-1)\Delta\overline{\sigma_{B_i B_j}}$$

First term (like stochastic kicks) is rms field error or measurment error

Second term includes correlations

TODAY’S CONCLUSION

If arguments are correct a piece of field (or magnet) can be ‘Qualified’ from a set of *N* measurement spaced by Δ if

$$\Delta\sigma_B^2 + \frac{\Delta}{N}\sum_{i\neq j} B_i B_j \quad < 160 \text{ G}^2\text{-m}$$

i.e. by cross-correlating the measurements

This is model-independent, which is desirable

Places some constraint on Δ for given measurement error

From F. Bergsma’s talk at CM28 rms(?) error = 2mT = 20 Gauss

➔ Δ < (or <<) 0.4 m – fine, 5cm or better is planned

Not clear what scale or errors would be – probably many cm

THE END

(for the moment)