Relativistic Dissipative Hydrodynamics and Heavy Ion Collisions: some recent developments

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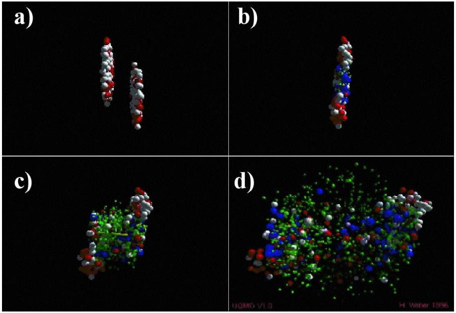
Outline

- Overview: Relativistic hydrodynamics and heavy-ion collisions [HICs].
- Conformal hydrodynamics: why is it important to HICs ?
- First- and second-order theories.
- Conformal hydrodynamics as a divergence-type theory.
- Hydrodynamics coupled to chiral fields.
- Some results.
- Extras.

Stages of a heavy-ion collision

a) Two ultrarelativistic heavy-ions collide.
b) A resulting hot volume is created after the collision (pre-thermalization), which eventually thermalizes ($t_{th} \lesssim 2.5$ fm/c).
c) This fireball expands and cools, going from QGP to hadrons.
d) The fireball reaches freeze-out, where hadrons become so diluted that local equilibrium cannot be achieved. Hadrons are then detected.



Relativistic hydrodynamics and HICs

- Space-time evolution of matter created at HICs: Very difficult nonequilibrium QFT problem.
- Relativistic Hydrodynamics succesfully describes the evolution of the matter created at HICs from the thermalization time until freeze-out. This stage cannot be described by kinetic theory.
- From freeze-out till detection: relativistic Boltzmann equation.

- Very good agreement between RH with very low η/s and RHIC experiments $\rightarrow$ QGP must be strongly-coupled.
- *Ideal* RH fails at more peripherial collisions and at high transverse momentum: viscous corrections are needed.
- 1st order RH (Navier-Stokes) is unstable and acausal.
- Thus, *one must use 2nd or higher order theories* (Israel-Stewart, Conformal Invariants, DTTs).

Why conformal hydrodynamics ?

- QGP is a strongly-coupled system: kinetic theory does not apply $\rightarrow$ AdS/CFT is the only theoretical tool available to extract transport coefficients.
- AdS/CFT correspondence represents a unique opportunity to study nonequilibrium dynamics of a strongly-coupled system (even beyond the hydrodynamic regime).
- At large T, QCD is approximately conformal $\rightarrow$ it is expected that some universal features of the QGP are reproduced by conformal theories (e.g., N=4 SYM).
- Number of transport coefficients in a conformal theory is relatively low (at 2nd order in curved space-time, we only need 5!).

1st and 2nd order theories (in velocity gradients)

- 1st order (Relativistic Navier-Stokes) theory is acausal and unstable, so one must use 2nd order theories.

- Only recently has the most general conformal **2nd-order** stress-energy tensor been constructed, which conforms with kinetic theory (of a massless gas) [e.g. Betz 1012.5772].

$$T^{\mu\nu} = \text{Perfect fluid} \\ + \eta \times \text{1st order dissipative corrections} \\ + (\tau_\pi, \lambda_{1,2,3}) \times \text{2nd order dissipative corrections} \tag{1}$$

- Transport coefficients (η, τ_π, and $\lambda_{1,2,3}$) must be calculated from microscopic theory (kinetic theory - when applicable - or AdS/CFT).

Israel-Stewart vs. Conformal invariants

$$T^{\mu\nu} = \text{Perfect fluid} + \overbrace{\Pi^{\mu\nu}}^{\text{Dissipative corrections}} \tag{2}$$

$$\Pi^{\mu\nu} = \overbrace{-\eta\sigma^{\mu\nu}}^{\text{1st Order}} + \overbrace{\tau_\pi \text{ (1st derivatives of } \sigma^{\mu\nu})}^{\text{Israel-Stewart '79}}$$
$$+ \overbrace{\lambda_1 \text{ (shear-shear)} + \lambda_2 \text{ (shear-vorticity)} + \lambda_3 \text{ (vort.-vort.)}}^{\text{Conformal Invariants Baier et al 2008}} \tag{3}$$

$$\sigma_{\alpha\gamma} = S_{(1)\alpha\gamma\mu\nu} u^{(\mu;\nu)} \quad \text{shear stress tensor} \tag{4}$$

Based on an expansion in velocity gradients.
See Baier et al, JHEP0804 (2008); Bhattacharyya et al, JHEP0245 (2008).

Divergence-type theories [Geroch and Lindblom 1990]

- Do not rely on a gradient expansion.
- The conditions for hyperbolicity and causality of the full nonlinear evolution can be stated in very simple terms.
- Symmetries of underlying field theory are encoded in a single function χ (generating function).
- Introduce a new third-order tensor $A^{\mu\nu\rho}$ obeying a divergence-type equation, with source tensor $I^{\nu\rho}$, and a symmetric and traceless tensor which vanishes in equilibrium $\xi_{\mu\nu}$.
- Once we know χ and $I^{\mu\nu}$, we know the equations of motion. Once a solution for $\xi_{\mu\nu}$ is obtained, $T^{\mu\nu}$ can be calculated.

Conformal hydrodynamics as a DTT [w/ E. Calzetta]

- Motivation: to develop a non-linear hydrodynamic formalism including all orders in gradients to study e.g. conical flow in HICs.
- We set up a *quadratic* DTT (in $\xi^{\mu\nu}$):

$$\chi = A(u^\mu) + B(u^\mu)\xi^{\mu\nu} + C(u^\mu)[\xi^{\mu\nu}]^2 \ . \qquad (5)$$

- The tracelessness condition and the conformal weight of $T^{\mu\nu}$ allow us to find χ.
- We require that, at first order in velocity gradients, the DTT matches Navier-Stokes (this fixes $I_1^{\mu\nu}$).
- $I_2^{\mu\nu}$ is constructed by requiring that the Second Law holds exactly[1].

[1] It is not trivial to fulfill this.

- The DTT thus constructed is causal and satisfies the 2nd Law exactly.
- It contains all-order velocity gradients.
- It reduces, at 2nd order, to the 2nd order theory with $\lambda_{2,3} = 0$. Terms containing the vorticity cannot be reproduced (this is not a serious limitation to HICs).
- Numerical results (in 0+1 and 2+1) show that the DTT thermalizes faster and producing more entropy than 2nd order theory. Moreover, momentum anisotropy and elliptic flow calculated with the DTT saturate at lower p_T.
- The agreement between 2nd order and DTT results and RHIC data is very good. This means that the differences in hydrodynamic formalisms used to simulate RHICs introduce an additional uncertainty in the value of η/s that can be extracted from comparing models to data.

2+1 hydrodynamics coupled to chiral fields [w/ Gastão]

Motivation: To study the influence of chiral field dynamics on the hadronic observables of a HIC calculated in 2+1 dissipative hydrodynamics.

For simplicity we use a linear-sigma model since we aim at an estimation of the effect of the chiral fields on the viscous hydrodynamic evolution.

Split the EM tensor:

$$
\begin{aligned}
T_q^{\mu\nu} &= (\rho + p) u^\mu u^\nu - p g^{\mu\nu} + \Pi^{\mu\nu} \\
T_\phi^{\mu\nu} &= \sum_a \frac{\partial < \mathcal{L}_\phi >}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a - g^{\mu\nu} < \mathcal{L}_\phi >
\end{aligned}
\tag{6}
$$

where $\phi = \sqrt{\sigma^2 + \vec{\pi}^2}$, and $\rho = \rho(\phi, T) =< \mathcal{H}_q >$ and $p = p(\phi, T) = -V_e(\phi, T) + U(\phi)$ provide an equation of state for the quarks.

Then

$$
\begin{aligned}
D\rho + (\rho + p) D_\mu u^\mu - \Pi^{\mu\nu}\sigma_{\mu\nu} &= g(\rho_s D\sigma + \vec{\rho}_{ps} \cdot D\vec{\pi}) \\
(\rho + p) D u^\gamma - \nabla^\gamma p + \Delta^\gamma_\nu D_\mu \Pi^{\mu\nu} &= g(\rho_s \nabla^\gamma \sigma + \vec{\rho}_{ps} \cdot \nabla^\gamma \vec{\pi})
\end{aligned} \tag{7}
$$

for the quarks and

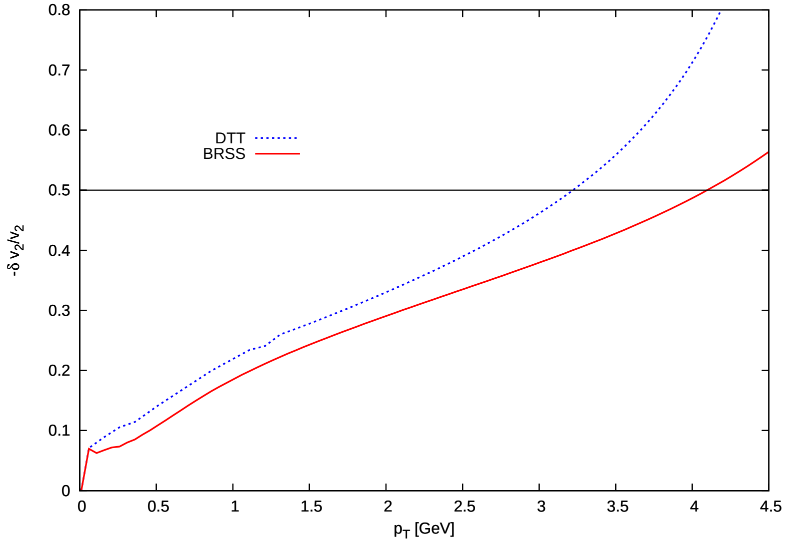
$$
\begin{aligned}
D_\mu D^\mu \sigma + \frac{\delta U}{\delta \sigma} &= -g\rho_s \\
D_\mu D^\mu \vec{\pi} + \frac{\delta U}{\delta \vec{\pi}} &= -g\vec{\rho}_{ps}
\end{aligned} \tag{8}
$$

for the chiral fields, where

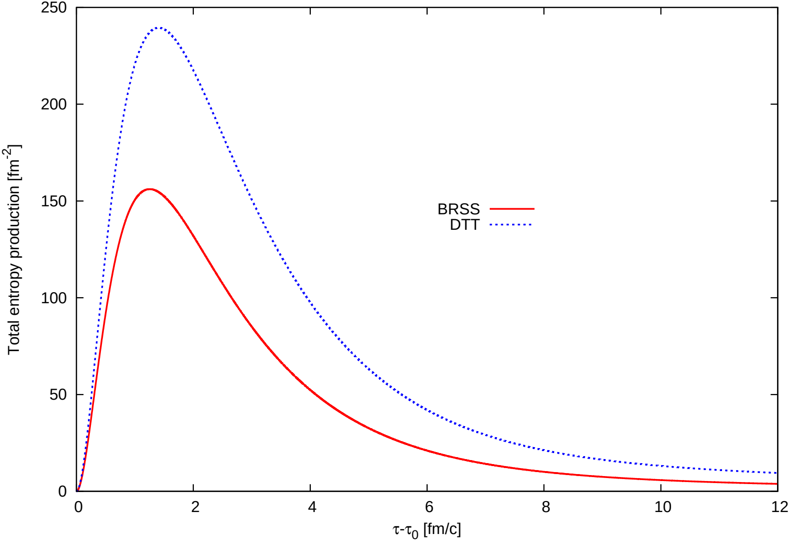
$$
\begin{aligned}
\rho_s &= g\sigma d_q \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{p^2 + g^2 \sum_a \phi_a^2}} f(p) \\
\vec{\rho}_{ps} &= g\vec{\pi} d_q \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{p^2 + g^2 \sum_a \phi_a^2}} f(p) \quad .
\end{aligned} \tag{9}
$$

SOME RESULTS.

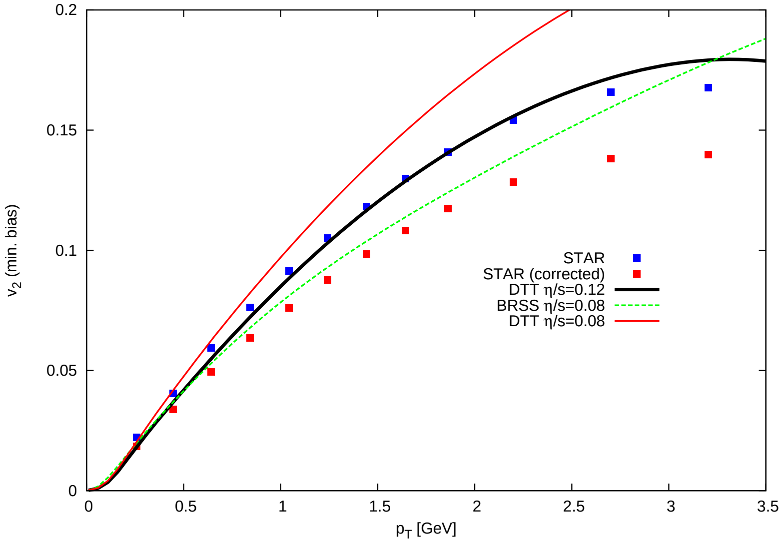
DTT vs 2nd order



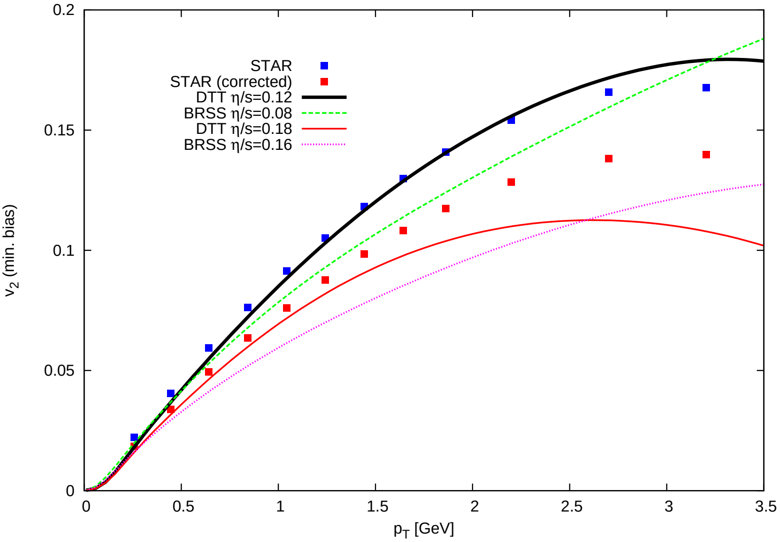
DTT vs 2nd order



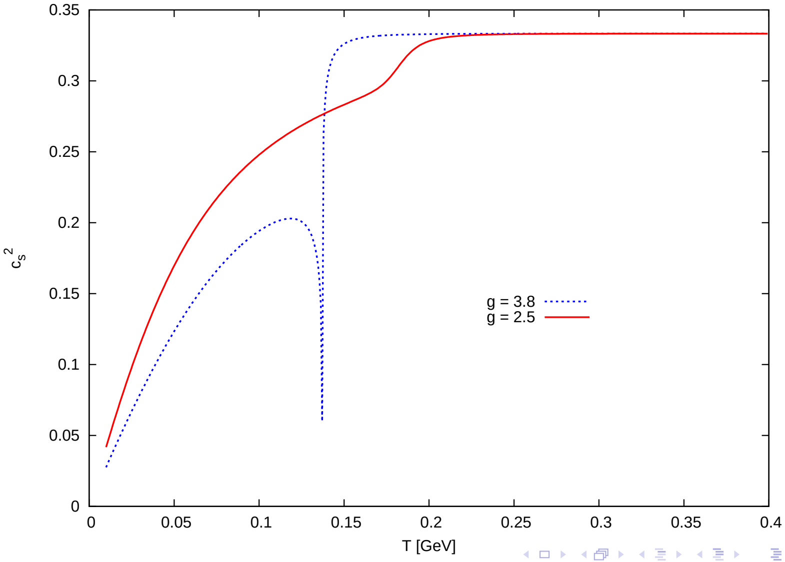
DTT vs 2nd order

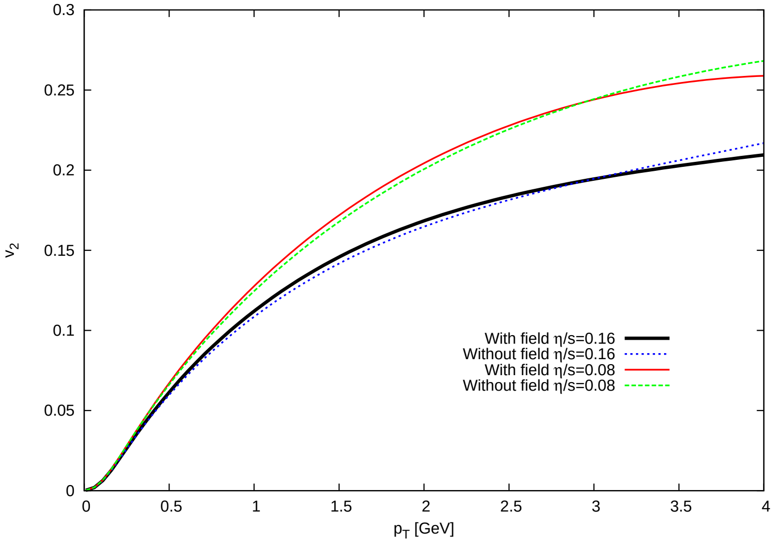


DTT vs 2nd order

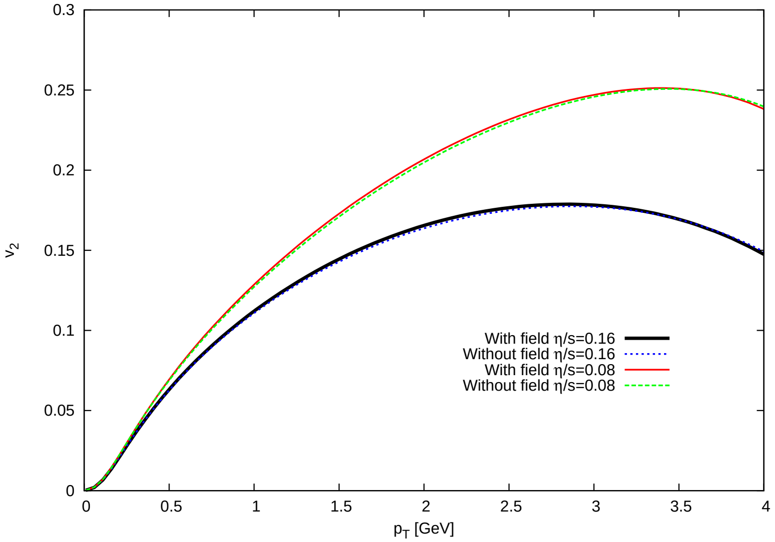


Coupled chiral-hydrodynamics [preliminar]




0.3
0.25
0.2
0.15
0.1
0.05
0
v_2
0
0.5
1
1.5
2
2.5
3
3.5
4
p_T [GeV]
With field η/s=0.16
Without field η/s=0.16
With field η/s=0.08
Without field η/s=0.08

Coupled chiral-hydrodynamics [preliminar]



Interesting issues in HIC physics

- $\eta[\phi]$ and $\zeta[\phi]$ calculated dynamically: interplay with hydro evolution (we are currently working on it).
- Some recently observed phenomena (e.g. conical flow) seem to require hydrodynamic theories going beyond 2nd order velocity gradients. DTTs may come in handy [see also work by Lublinsky and Shuryak PRD **80**, 065026 (2009) where they develop an all-order yet *linear* hydrodynamic formalism].
- Coupling of kinetic theory of non-abelian fields to hydrodynamics to fully describe e.g. jet quenching.
- Use of AdS/CFT to describe hydrodynamic evolution (and beyond) of QGP.
- Pre-equilibrium physics: not very much is known.

Thank you for your attention !

Jeronimo Peralta Ramos.

EXTRAS.

Conformal hydrodynamics: A few facts

- Stress-energy tensor is traceless and must trasform as

$$T^{\mu\nu} \to e^{(d+2)\omega} T^{\mu\nu} \tag{10}$$

under Weyl transformations $g_{\mu\nu} \to e^{-2\omega(x^\gamma)} g_{\mu\nu}$.

- Tracelessness and conformal weight of $T^{\mu\nu}$ $(= (d+2))$ imply:

$$\rho = (d-1)p \quad \text{and} \quad \zeta = 0 \ , \tag{11}$$

where ρ is the energy density in the local frame, p is the thermodynamic pressure, and ζ is the bulk viscosity, and

$$\rho \to e^{d\omega} \rho \ , \qquad u^\mu \to e^\omega u^\mu \ , \qquad T \to e^\omega T \tag{12}$$

Up to *2nd order in velocity gradients* (D_α is the covariant derivative, $\nabla^\alpha = \Delta^{\alpha\gamma} D_\gamma$ is the spatial gradient, and $D = u_\alpha D^\alpha$ is the convective time derivative):

$$\begin{aligned} \tau^{\mu\nu}_{c.i.} &= -\eta\sigma^{\mu\nu} + \eta\tau_\pi \left(S^{\mu\nu\rho\sigma}_{(1)} D\sigma_{\rho\sigma} + \frac{1}{3}\sigma^{\mu\nu}(D_\delta u^\delta) \right) \\ &\quad + \frac{\lambda_1}{\eta^2} S^{\mu\nu\rho\sigma}_{(1)} \sigma^\lambda_\rho \sigma_{\sigma\lambda} + \frac{\lambda_2}{\eta} S^{\mu\nu\rho\sigma}_{(1)} \sigma^\lambda_\rho \Omega_{\rho\lambda} + \lambda_3 S^{\mu\nu\rho\sigma}_{(1)} \Omega^\lambda_\rho \Omega_{\rho\lambda} \end{aligned} \tag{13}$$

$$S^{\mu\nu\rho\sigma}_{(1)} = \Delta^{\mu(\rho}\Delta^{\sigma)\nu} - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\rho\sigma} \ , \quad \Delta^{\mu\nu} = g^{\mu\nu} u^\mu u^\nu \tag{14}$$

$$\begin{aligned} \sigma^{\mu\nu} &= S^{\mu\nu\rho\sigma}_{(1)} D_{(\rho} u_{\sigma)} \ \ \text{shear tensor} \\ \Omega^{\mu\nu} &= -\nabla^{[\mu} u^{\nu]} \ \ \text{vorticity tensor} \end{aligned} \tag{15}$$

where () and [] denote symmetrization and anti-symmetrization.

$$(\rho + p)Du^i = \frac{1}{3}(g^{ij}\partial_j \rho - u^i u^\alpha \partial_\alpha \rho) - \Delta^i_\alpha D_\beta \Pi^{\alpha\beta}$$
$$D\rho = -(\rho + p)\nabla_\mu u^\mu + \Pi^{\mu\nu}\sigma_{\mu\nu} \tag{16}$$

where

$$D_\beta \Pi^{\alpha\beta} = \Pi^{i\alpha}\partial_\tau \frac{u_i}{u_\tau} + \frac{u_i}{u_\tau}\partial_\tau \Pi^{i\alpha} + \partial_i \Pi^{i\alpha}$$
$$+ \Gamma^\alpha_{\beta\gamma}\Pi^{\beta\gamma} + \Gamma^\beta_{\beta\gamma}\Pi^{\alpha\gamma} \ . \tag{17}$$

In the second-order theory:

$$\partial_\tau \Pi^{i\alpha} = -\frac{4}{3u^\tau}\Pi^{i\alpha}\nabla_\mu u^\mu - \frac{1}{\tau_\pi u^\tau}\Pi^{i\alpha} + \frac{\eta}{\tau_\pi u^\tau}\sigma^{i\alpha} - \frac{\lambda_1}{2\tau_\pi \eta^2 u^\tau}\Pi_\mu^{<i}\Pi^{\alpha>\mu} - \frac{u^i \Pi_\mu^\alpha + u^\alpha \Pi_\mu^i}{u^\tau} D u^\mu - \frac{u^j}{u^\tau}\partial_j \Pi^{i\alpha} \tag{18}$$

$$\sigma^{\mu\nu} = \nabla^{<\mu} u^{\nu>} \tag{19}$$

$$\nabla_\mu u^\mu = \partial_\tau u^\tau + \partial_i u^i + \frac{u^\tau}{\tau}$$

$$\nabla_{<x} u_{x>} = \Delta^{\tau x}\partial_\tau u^x + \Delta^{ix}\partial_i u^x - \frac{1}{3}\Delta^{xx}\nabla_\mu u^\mu$$

$$\nabla_{<x} u_{y>} = \frac{1}{2}\Delta^{\tau x}\partial_\tau u^y + \frac{1}{2}\Delta^{\tau y}\partial_i u^x + \frac{1}{2}\Delta^{ix}\partial_i u^y + \frac{1}{2}\Delta^{iy}\partial_i u^x - \frac{1}{3}\Delta^{xy}\nabla_\mu u^\mu \tag{20}$$

$$\nabla_{<\psi} u_{\psi>} = \tau^4 \Delta^{\psi\psi}\Gamma^\psi_{\tau\psi} u^\tau - \frac{1}{3}\tau^4\Delta^{\psi\psi}\nabla_\mu u^\mu \ .$$

Conformal hydrodynamics as a divergence-type theory

- The entropy current is extended to

$$S^\mu = \Phi^\mu - \beta_\nu T^{\mu\nu} - \alpha N^\mu - A^{\mu\nu\rho}\xi_{\nu\rho} \qquad (21)$$

where $\Phi^\mu = \frac{\partial\chi}{\partial\beta_\mu}$ is the thermodynamic potential, and $\alpha = \mu/T$ is the affinity. χ is the *generating function*.

- Introducing the symbol ζ^A to denote the set $(\alpha, \beta_\mu, \xi_{\mu\nu})$, A^μ_B the set $(N^\mu, T^{\mu\nu}, A^{\mu\nu\rho})$ and I_B the set $(0, 0, I_{\mu\nu})$, the theory is summed up in the equations

$$A^\mu_B = \frac{\partial\Phi^\mu}{\partial\zeta^B} \quad , \quad S^\mu_{;\mu} = -I_B\zeta^B \quad , \quad A^\mu_{B;\mu} = I_B \qquad (22)$$

- Once we know χ and $I^{\mu\nu}$, we know N^μ, $T^{\mu\nu}$, $A^{\mu\nu\rho}$ and the entropy production.

In the DTT 2+1, $D_\mu A^{\mu\rho\sigma} = I^{\rho\sigma}$ yields

$$
\begin{aligned}
\partial_\tau \xi^{i\alpha} = & -\frac{2}{3u^\tau}\xi^{i\alpha}\nabla_\mu u^\mu - \frac{1}{\tau_\pi u^\tau}\xi^{i\alpha} + \frac{1}{\tau_\pi u^\tau}\sigma^{i\alpha} \\
& - \frac{\lambda_1}{3\tau_\pi \eta u^\tau}\xi_\mu^{< i}\xi^{\alpha > \mu} - \frac{u^i\xi_\mu^\alpha + u^\alpha \xi_\mu^i}{u^\tau} D u^\mu \\
& - \frac{u^j}{u^\tau}\partial_j \xi^{i\alpha} \ .
\end{aligned}
\tag{23}
$$

The shear tensor is calculated from the nonequilibrium tensor $\xi^{\alpha\gamma}$ as follows:

$$
\Pi^{\mu\nu} = \eta \xi^{\mu\nu} - \frac{\lambda_1 \tau_\pi T^4}{3\eta}\left(\xi^{\mu\alpha}\xi_\alpha^\nu - \frac{1}{3}\Delta^{\mu\nu}\xi^{\alpha\gamma}\xi_{\alpha\gamma}\right) \ . \tag{24}
$$