

# A relativistic model for three-body final state interaction in D decays

Karin S. F. F. Guimarães

*Instituto Tecnológico de Aeronáutica - Brazil*

## Collaborators:

*T. Frederico*

*A. C. dos Reis*

*ITA - Brazil (advisor)*

*CBPF - Brazil (advisor)*

*I. Bediaga*

*A. Delfino*

*L. Tomio*

*W. de Paula*

*M. Robillotta*

*P.C.Magalhães*

*CBPF - Brazil*

*UFF - Brazil*

*IFT/UFF - Brazil*

*ITA - Brazil*

*IFUSP - Brazil*

*IFUSP - Brazil*

# Outline

---

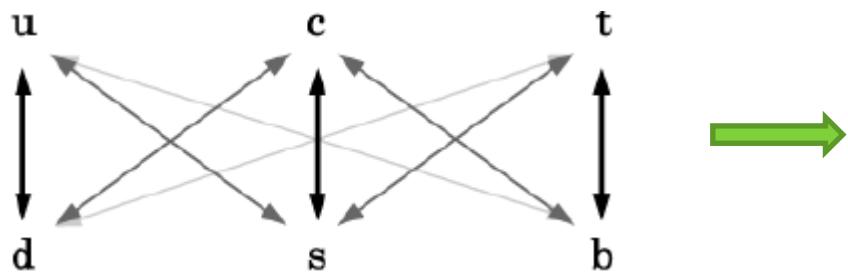
- ***General Motivation***
- ***Why  $D^+ \rightarrow K^-\pi^+\pi^+$  ?***
- ***On the analysis technique***
- $D^+ \rightarrow K^-\pi^+\pi^+$     ***versus***     $K^-\pi^+$
- ***Three-Body Final State Interaction in  $D^+ \rightarrow K^-\pi^+\pi^+$***
- ***Relativistic Three-Body Model***
- ***Separable model for the S-wave  $K\pi$  amplitude***
- ***Preliminary Results***
- ***Conclusion***
- ***Next Step***

# *Motivation*

## *General*

# General Motivation [1]

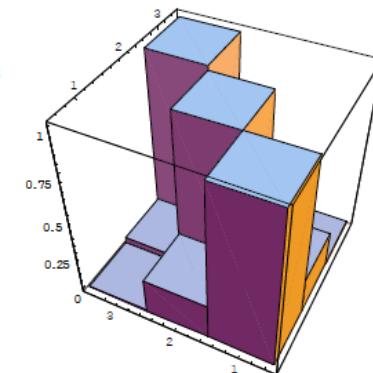
- B factory main task: measure CP Violation in the B physics
- Standard Model mechanism for CP Violation in weak interactions is given by a single complex phase in the CKM matrix.



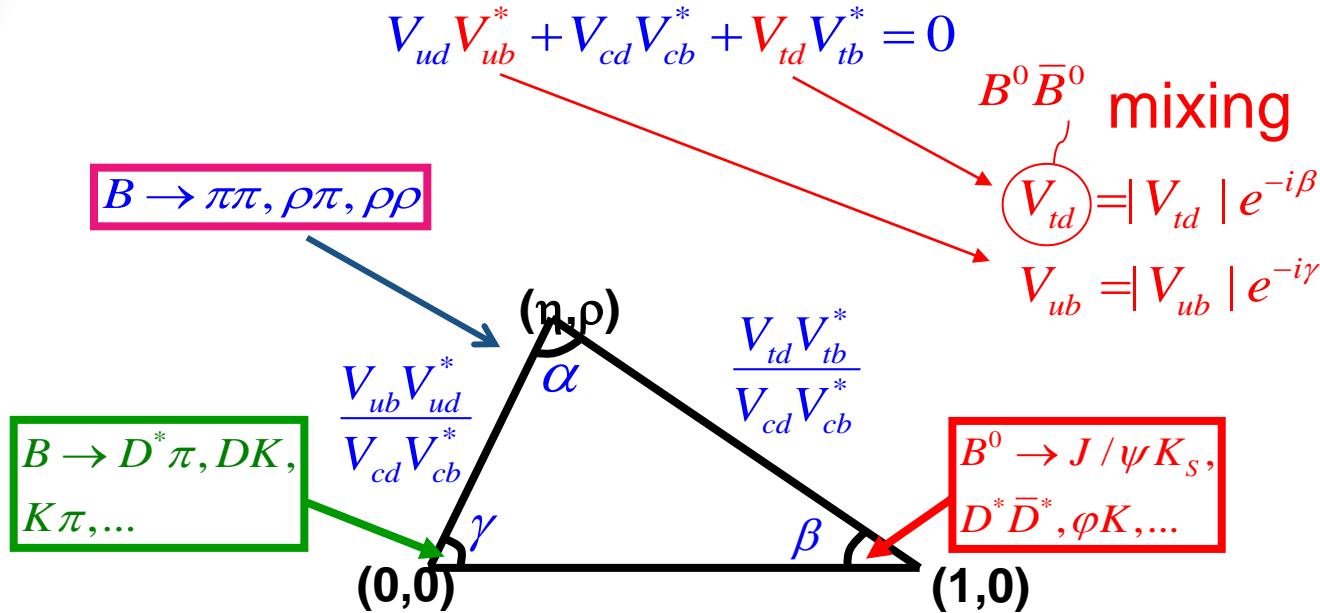
*Transitions between members of the same family more probable (=thicker lines) than others.*

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

CKM Matrix: almost a diagonal matrix, but not completely



# Why $D^+ \rightarrow K^- \pi^+ \pi^+$ ?



- $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$
- $B^\pm \rightarrow K^\pm \pi^+ \pi^-$
- $B^\pm \rightarrow \pi^\pm K^+ K^-$
- $B^\pm \rightarrow K^\pm K^+ K^-$
- $B^\pm \rightarrow \pi^\pm p \bar{p}$
- $B^\pm \rightarrow K^\pm p \bar{p}$

- $B^0 \rightarrow K^0 \pi^+ \pi^-$
- $B^0 \rightarrow K^0 K^+ K^-$
- $B^0 \rightarrow K^0 p \bar{p}$

- $D^\pm \rightarrow \pi^\pm \pi^+ \pi^-$
- $D^\pm \rightarrow K^\pm \pi^+ \pi^-$

BaBar Collab. Phys. Rev. D78:012004, 2008

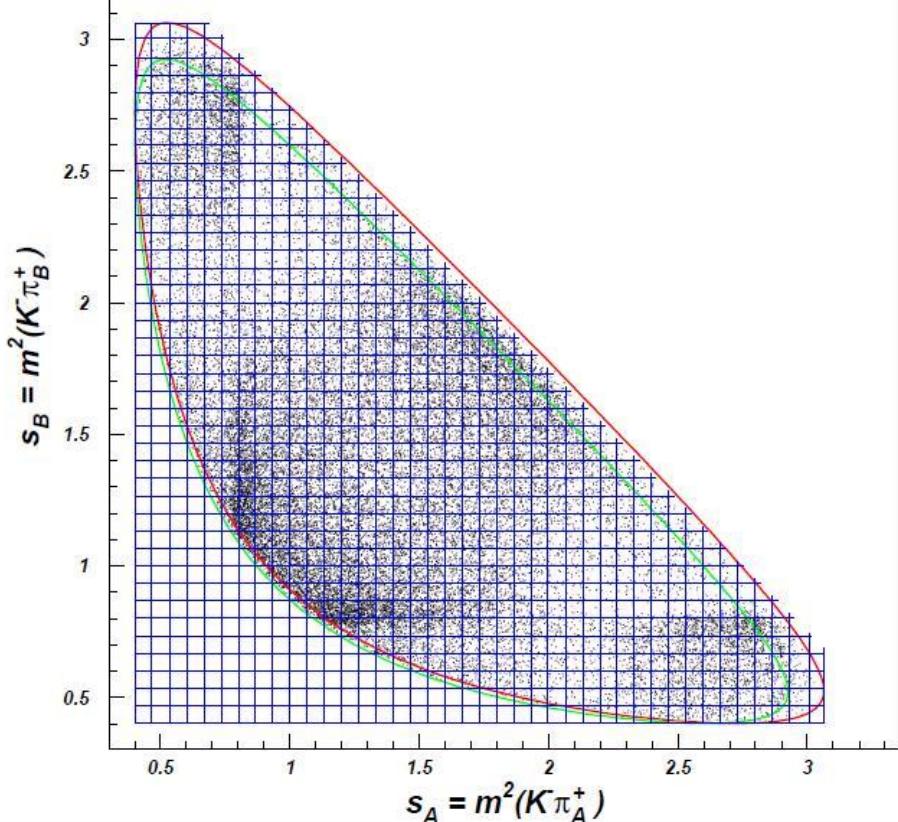
[6]

FSI

# Motivation Phenomenology

# Dalitz Plot<sub>[1]</sub>

E. Aitala Phys. Rev. D 73, 032004 (2006)



- Used in the study of resonance substructures on charmed meson decays,
- It represents the phase space of the decay and weighted by differential decay rate
- Decay amplitude dominated by S-wave channel.

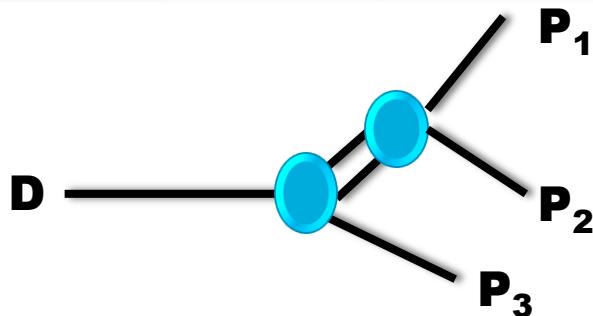
$$s_A = (p_K + p_{\pi_A})^2$$

$$s_B = (p_K + p_{\pi_B})^2$$

$$s_C = (p_{\pi_A} + p_{\pi_B})^2$$

$$m_{s_B}^2 = m_K^2 + m_{\pi_B}^2 - \frac{1}{2m_{s_A}^2} \left\{ \frac{\left( m_{s_A}^2 - M^2 + m_{\pi_B}^2 \right) \left( m_{s_A}^2 + m_K^2 - m_{\pi_A}^2 \right)}{\lambda^{1/2}(m_{s_A}^2, M, m_{\pi_B}^2) \lambda^{1/2}(m_{s_A}^2, m_{\pi_A}^2, m_{\pi_B}^2)} \mp \right\}$$

# Isobar Model [1]



- Decay matrix element is represented by a coherent sum of phenomenological amplitudes.
- These amplitudes correspond to the possible intermediate states in the decay chain  $D \rightarrow Rh$ ,  $R \rightarrow hh$  ( $h=K, \pi$ ), and grouped by orbital angular momentum  $L$ :

$$S_{pdf} = \left| \sum_L A_L \right|^2, A_L = \sum_k c_k^L A_k^L$$

The amplitudes are weighted by constant complex coefficients and the series can be truncated at  $L=2$ .

# Isobar Model [2]

In case of a resonance with spin, the standard procedure is to define the resonant amplitude  $A_k^L$  as product of:

- a relativistic Breit-Wigner function (BW)
- form factors (usually a Blatt-Weisskopf dumping factors) for D and the resonance vertices ( $F_D, F_R$ )
- And a function describing the angular distribution of the final state particles, accounting for the momentum and angular momentum conservation

$$A_k^L = \underbrace{F_{D,k} F_{R,k}}_{\text{Blatt -Weisskopf form factor}} \left( -2 \overrightarrow{\mathbf{p}_1} \cdot \overrightarrow{\mathbf{p}_2} \right)^{(L)} P_L(\cos \theta_{13}) BW_{k,12}$$

Breit-Wigner Function

[10 ]

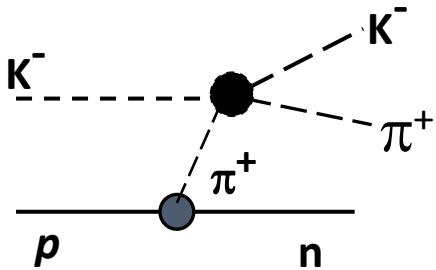
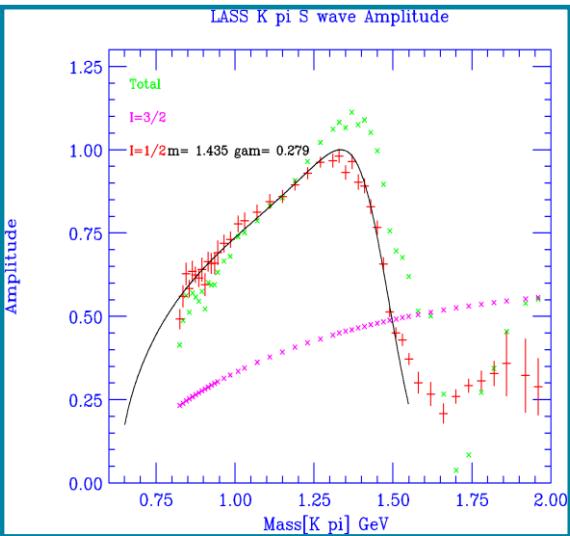
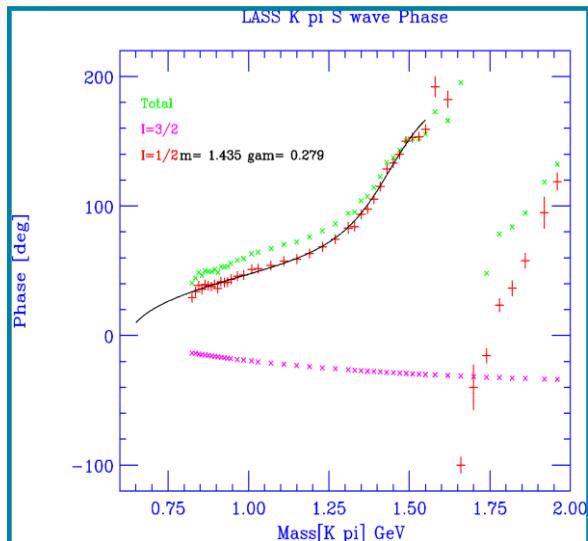
# MIPWA (Model Independent Partial Wave Analysis)

MIPWA → E791

E.M. Aitala et al. (E791 Collaboration), Phys. Rev. D73, 032004(2006)

- Decay matrix element is written as a sum of partial waves - truncated at D-wave
- No assumption is made on the nature of the S-wave,  $A_0(s) = a_0(s)e^{i\phi_0(s)}$  represented by :
- P- and D-waves are well described by a sum of Breit-Wigner amplitudes.
- MIPWA provides an inclusive measurement, since the  $K^-\pi^+$  system is embedded in a three-body strongly interacting final state.
- Extracting the pure  $K^-\pi^+$  amplitude is not a trivial task ...

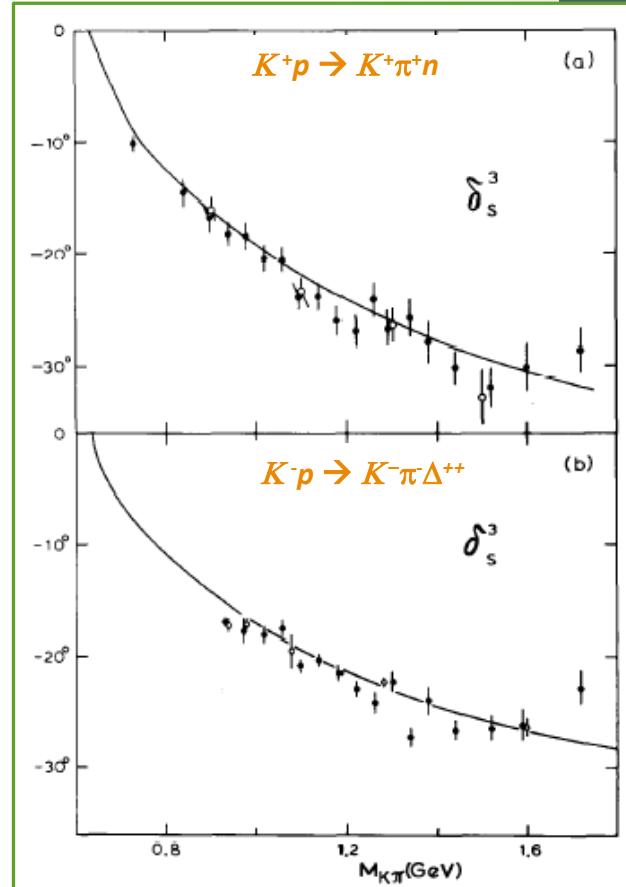
# $D^+ \rightarrow K^- \pi^+ \pi^+$ versus $K^- \pi^+$ [1]



D. Aston *et al.*, Nucl.  
Phys. B296(1988)493

SLAC/LASS experiment E135:  
 $K^- p \rightarrow K^- \pi^+ n$  (11 GeV/c)

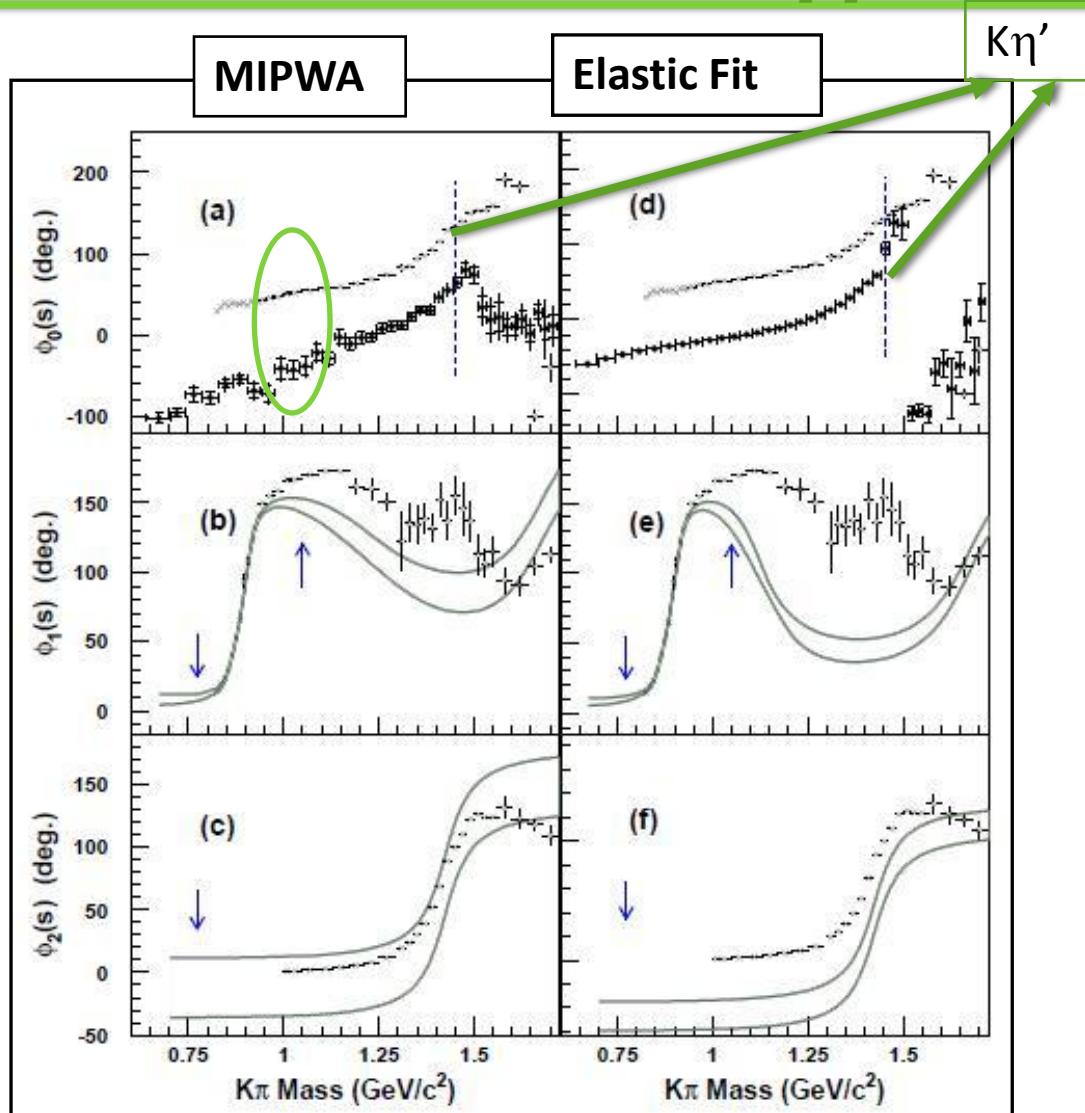
$I$ -spins are separated using  $I=3/2$  phases from  
 $K^+ p \rightarrow K^+ \pi^+ n$  and  $K^- p \rightarrow K^- \pi^- \Delta^{++}$   
(13 GeV/c)



Estabrooks, et al, NP B133,  
490(1978)

[12]

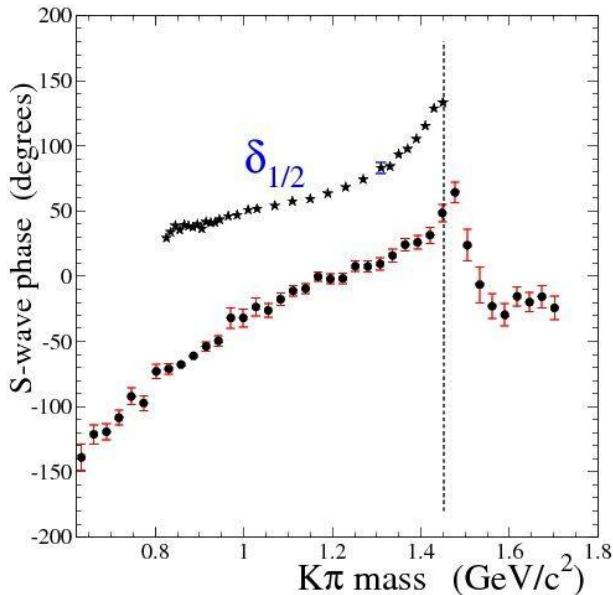
# $D^+ \rightarrow K^- \pi^+ \pi^+$ versus $K^- \pi^+_{[2]}$



E. Aitala *et al* Phys. Rev. D 73, 032004 (2006)

[13]

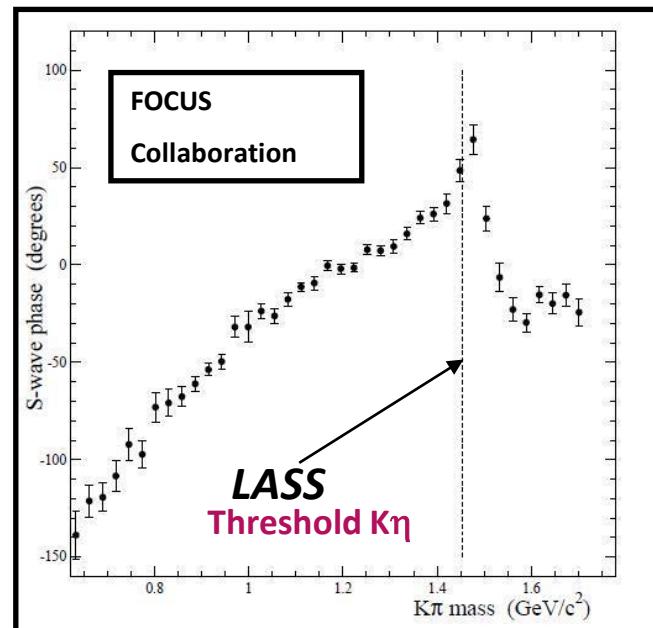
# $D^+ \rightarrow K^- \pi^+ \pi^+$ versus $K^- \pi^+$ [3]



The  $K^- \pi^+$  MIPWA S-wave (circles with error bars) from FOCUS  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay. The LASS  $I=1/2$  S-wave phase ( $\delta_{1/2}$ ) is shown as full stars. The  $K\pi$  amplitude is elastic up to 1.45  $\text{GeV}/c^2$ , indicated by the vertical line.

Why is so different?

One possible origin of this additional energy dependent phase is the three-body FSI



D. Aston *et al.*, Nucl. Phys.  
B296(1988)493

$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$

The Focus Collaboration arXiv:0905.4846v1  
[hep-ex]

(14)

# *Model*

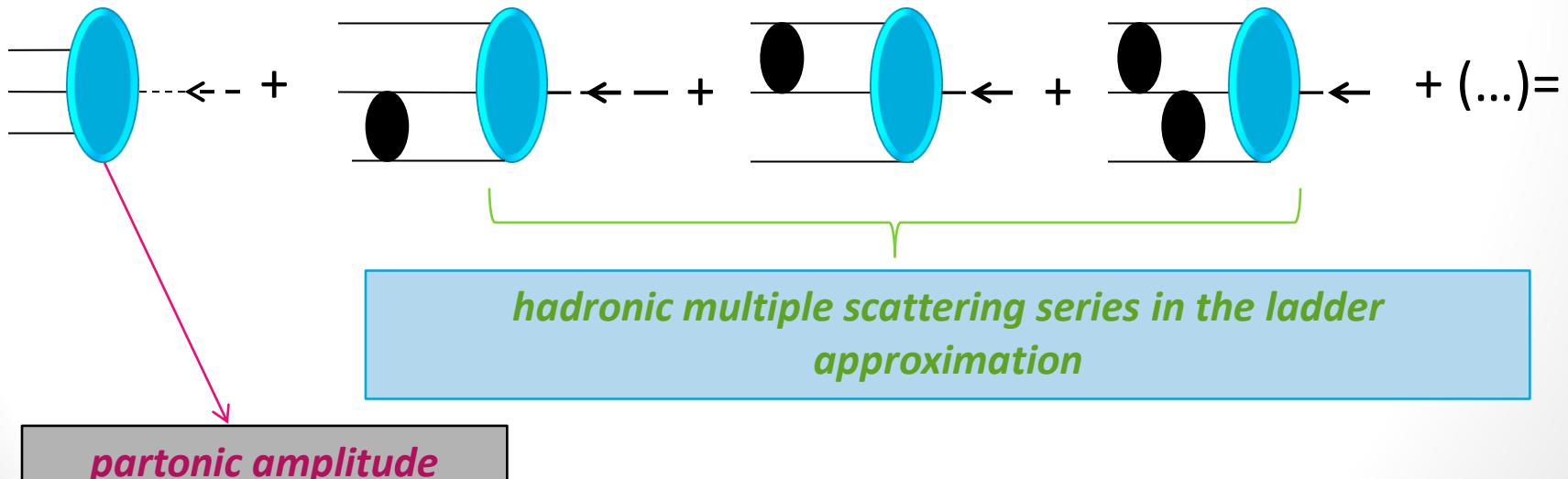
# *Relativistic 4d Three-body model*

# 3-body FSI in $D^+ \rightarrow K^- \pi^+ \pi^+$ [1]

The microscopic amplitude for the decay of the heavy meson into three light ones with off-shell momenta (the physical particles are on-mass-shell):

$$\mathcal{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) + \int \frac{d^4 q_\pi d^4 q_{\pi'}}{(2\pi)^8} T(k_\pi, k_{\pi'}; q_\pi, q_{\pi'}) S_\pi(q_\pi) S_{\pi'}(q_{\pi'}) S_K(K - q_{\pi'} - q_\pi) D(q_\pi, q_{\pi'})$$

Diagrammatic representation of the heavy meson decay process into  $K\pi\pi$

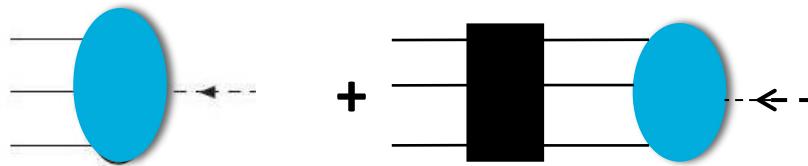


partonic amplitude

hadronic multiple scattering series in the ladder approximation

# 3-body FSI in $D^+ \rightarrow K^- \pi^+ \pi^+$ [2]

$$\mathcal{D}(k_\pi, k_{\pi'}) =$$

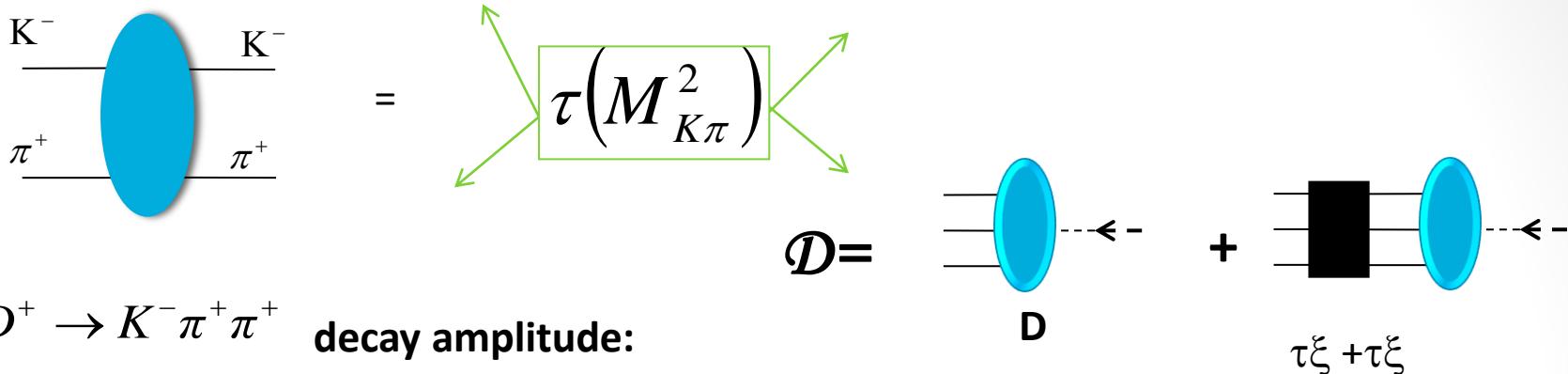


Resumming the 3-body rescattering with T-matrix

$$T = \text{[black rectangle]} = \text{[black oval]} + \text{[empty horizontal line]} + \text{[black oval, black oval]} + (\dots)$$

$$\mathcal{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) + \int \frac{d^4 q_\pi d^4 q_{\pi'}}{(2\pi)^8} T(k_\pi, k_{\pi'}; q_\pi, q_{\pi'}) S_\pi(q_\pi) S_\pi(q_{\pi'}) S_K(K - q_{\pi'} - q_\pi) D(q_\pi, q_{\pi'})$$

# Relativistic Three-Body Model [1]



S-wave contact interaction is chosen for the  $K\pi$  system to allow the physically intuitive decomposition of the decay amplitude as

$$\mathcal{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) + \tau(M_{K\pi}^2) \xi(k_{\pi'}) + \tau(M_{K\pi'}^2) \xi(k_\pi)$$

Smooth background given by the partonic decay amplitude

Carry the full effect of the FSI through 2 meson resonant amplitude  $\tau$  times a spectator amplitude  $\xi$

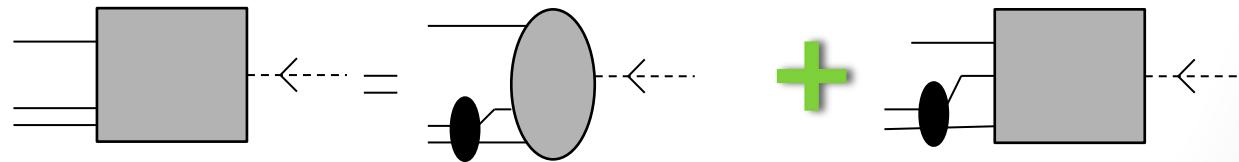
S-wave  $K\text{-}\pi$  scattering amplitude  $I=1/2$

[19]

# *Relativistic 4d Three-body model<sub>[2]</sub>*

- Spectator amplitude satisfies Bethe-Salpeter like equation
- The re-summation of the scattering series by the function  $\xi$  can be done by a integral equation showed bellow and taking account the isospin recoupling coefficients:

$$\tau(M_{K\pi'}^2)\xi(k_\pi) =$$



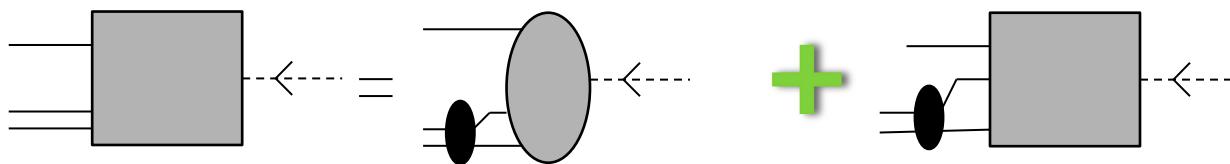
# *Relativistic 4d Three-body model<sub>[4]</sub>*

- ❖ Assuming the dominance of the isospin doublet S-wave in the state  $K\pi$  scattering
- ❖ Isospin preserving property of the operator  $\tau$  the 3 body scattering can happens only in isospin  $3/2$  and  $1/2$  states

$$D(q_\pi, q_{\pi'}) = D_{3/2}(q_\pi, q_{\pi'}) \left[ \left| \frac{3}{2}, I_{K\pi} = \frac{1}{2}, I_{\pi'} = 1 \right\rangle + \left| \frac{3}{2}, I_{K\pi'} = \frac{1}{2}, I_{\pi} = 1 \right\rangle \right]$$

$$D_{3/2}(q_\pi, q_{\pi'}) \approx const.$$

# Relativistic 4d Three-body model<sub>[5]</sub>



$$\xi_{3/2}(k) = \frac{5}{3} \xi_0(k) + \frac{2}{3} \int \frac{d^4 q}{(2\pi)^4} \tau_{1/2}((K-q)^2) \times S_K(K-k-q) S_\pi(q) \xi_{3/2}(q)$$

$$\xi_0(k) = \int \frac{d^4 q}{(2\pi)^4} S_\pi(q) S_K(K-k-q) D_{3/2}(k, q)$$

The spectator amplitude:

- Is built by mixing resonances of the two possible  $K\pi$  pairs
- It is a function of the momentum of the spectator pion.

# Separable model for the S-wave $K\pi$ amplitude [1]

## S-wave $K\pi$ amplitude

$K_0^*(1430)$

$K_0^*(1630)$

$K_0^*(1950)$

BaBar parametrization

This class of models comes after Maldacena's Conjecture that introduced new perspectives by treating strong-interaction physics with gauge/string dualities

Proposal to interpret the scalar mesons  $f_0$  family as radial excitations of sigma within a Dynamical AdS/QCD model

[W. dePaula, T. Frederico Phys. Lett. B693 (2010) 287]

Radial excitations of  $K^*(800)$  with slope  $\sim 0.6 \text{ GeV}^2$

[\\*\\*BaBar Collab. Phys. Rev. D78:012004,2008](#)

[23]

W. De Paula, T. Frederico, H. Forkel and M. Beyer, Phys. Rev. D 79 (2009) 075019

# Separable model for the S-wave $K\pi$ amplitude [2]

## Light Scalar Mesons

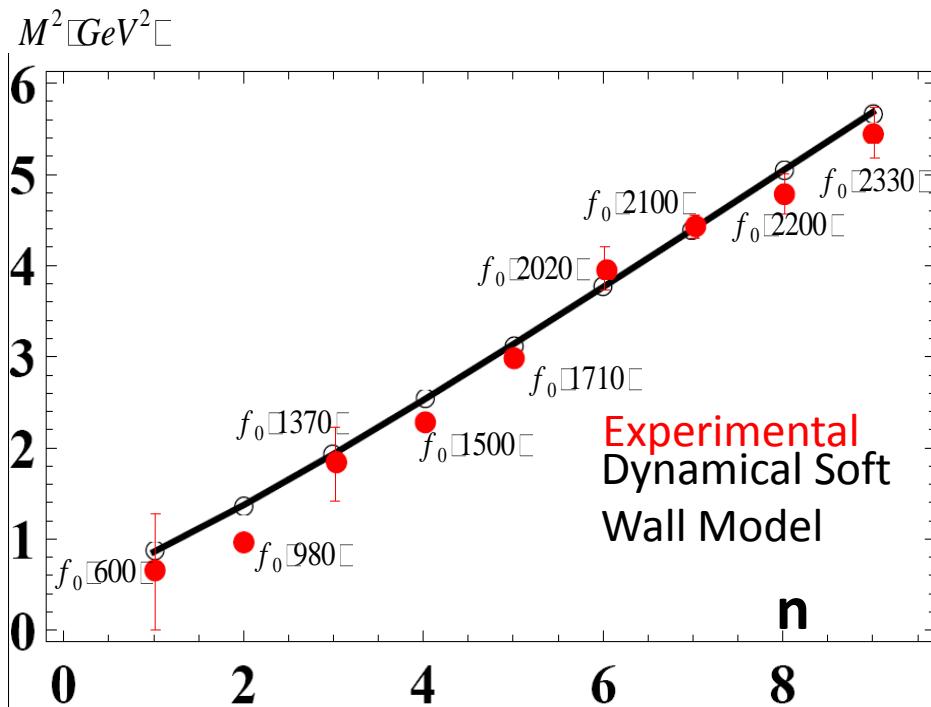


TABLE I: Two-pion decay width and masses for the  $f_0$  family. Experimental values from PDG[1]. (<sup>†</sup>Mixing angle of  $20^\circ$ .)

Meson	$M_{exp}(\text{GeV})$	$M_{th}(\text{GeV})$	$\Gamma_{\pi\pi}^{exp}(\text{MeV})$	$\Gamma_{\pi\pi}^{th}(\text{MeV})$
$f_0(600)$	0.4 - 1.2	0.86	600 - 1000	602
$f_0(980)$	$0.98 \pm 0.01$	1.10	$\sim 15 - 80$	$47^\dagger$
$f_0(1370)$	1.2 - 1.5	1.32	$\sim 41 - 141$	159
$f_0(1500)$	$1.505 \pm 0.006$	1.52	$38 \pm 3$	42
$f_0(1710)$	$1.720 \pm 0.006$	1.70	$\sim 0 - 6$	6
$f_0(2020)$	$1.992 \pm 0.016$	1.88	—	0.0
$f_0(2100)$	$2.103 \pm 0.008$	2.04	—	1.4
$f_0(2200)$	$2.189 \pm 0.013$	2.19	—	2.8
$f_0(2330)$	2.29 - 2.35	2.33	—	3.2

W. de Paula, T. F. PLB693 (2010) 287

# Separable model for the S-wave $K\pi$ amplitude [3]

We introduce  $K^*(1630)$  and  $K^*(1950)$  extending the parametrization given in BaBar:

$$\tau(M_{K\pi}^2) = 4\pi \frac{M_{K\pi}}{k} (S_{K\pi} - 1)$$

With the S-matrix given by:

$$S_{K\pi} = \frac{k \cot \delta + i k}{k \cot \delta - i k} \prod_{r=1}^3 \frac{M_r^2 - M_{K\pi}^2 + i z_r \bar{\Gamma}_r}{M_r^2 - M_{K\pi}^2 - i z_r \bar{\Gamma}_r}$$

$$k \cot \delta = -a^{-1} + \frac{1}{2} r_0 k^2$$

$$z_r = k M_r^2 / (k_r M_{K\pi})$$

Rest-frame momentum:  $k = \left[ \left( \frac{M_{K\pi}^2 + m_\pi^2 - m_K^2}{2 M_{K\pi}} \right)^2 - m_\pi^2 \right]^{\frac{1}{2}}$

$$e^{i\delta_{1/2}} \sin \delta_{1/2} = -\frac{i}{8\pi} \frac{k}{M_{K\pi}} \tau_{1/2}(M_{K\pi}^2)$$

# *Separable model for the S-wave $K\pi$ amplitude [4]*

$$S_{K\pi} = \frac{k \cot \delta + i k}{k \cot \delta - i k} \prod_{r=1}^3 \frac{M_r^2 - M_{K\pi}^2 + i z_r \bar{\Gamma}_r}{M_r^2 - M_{K\pi}^2 - i z_r \Gamma_r}$$

$$k \cot \delta = -a^{-1} + \frac{1}{2} r_0 k^2$$

$$\begin{aligned} a &= -2.07 \text{ GeV}^{-1} \\ r_0 &= 3.32 \text{ GeV}^{-1} \end{aligned}$$

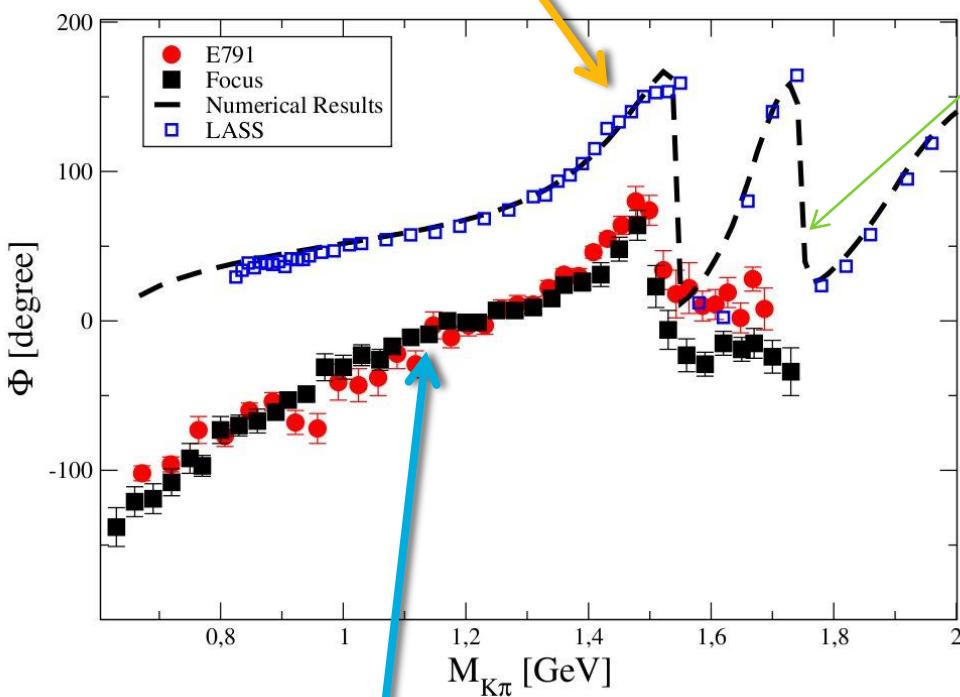
- The resonance parameters  $M_r, \Gamma_r, \bar{\Gamma}_r$  in GeV for

$$K_0^*(1430)$$
$$(1.48, 0.25, 0.25)$$
$$K_0^*(1630)$$
$$(1.67, 0.1, 0.1)$$
$$K_0^*(1950)$$
$$(1.9, 0.2, 0.14)$$

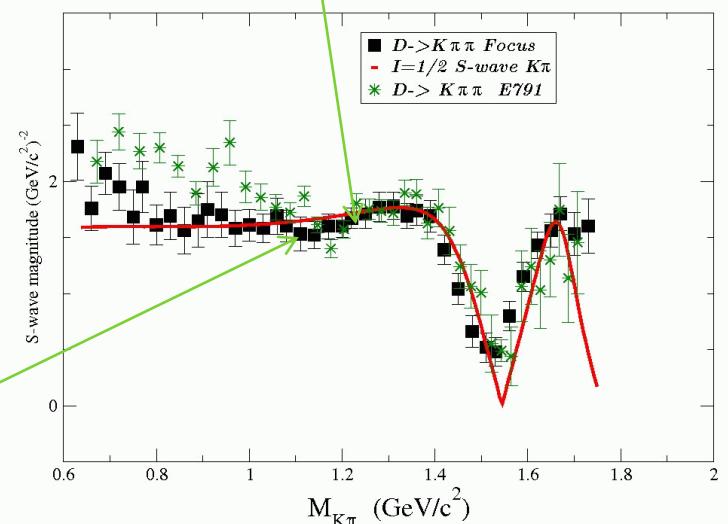
# Model for the S-wave $K\pi$ and $K\pi\pi$

LASS data for  $K\pi$  elastic S-wave scattering phase-shift

Preliminary Results (4d)

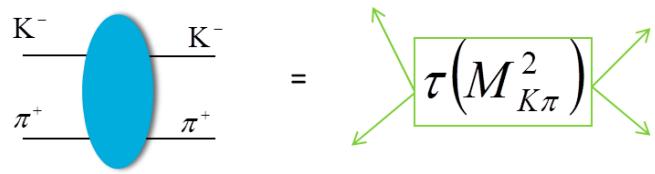


$$S_{K\pi} = \frac{k \cot \delta + i k}{k \cot \delta - i k} \prod_{r=1}^3 \frac{M_r^2 - M_{K\pi}^2 + i z_r \bar{\Gamma}_r}{M_r^2 - M_{K\pi}^2 - i z_r \Gamma_r}$$



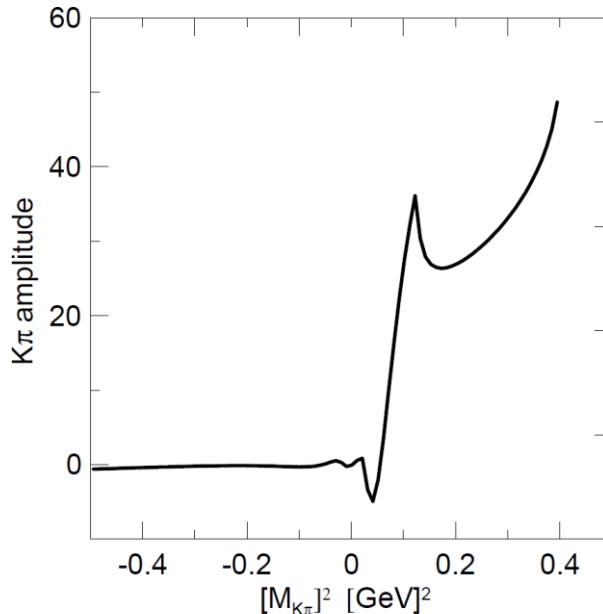
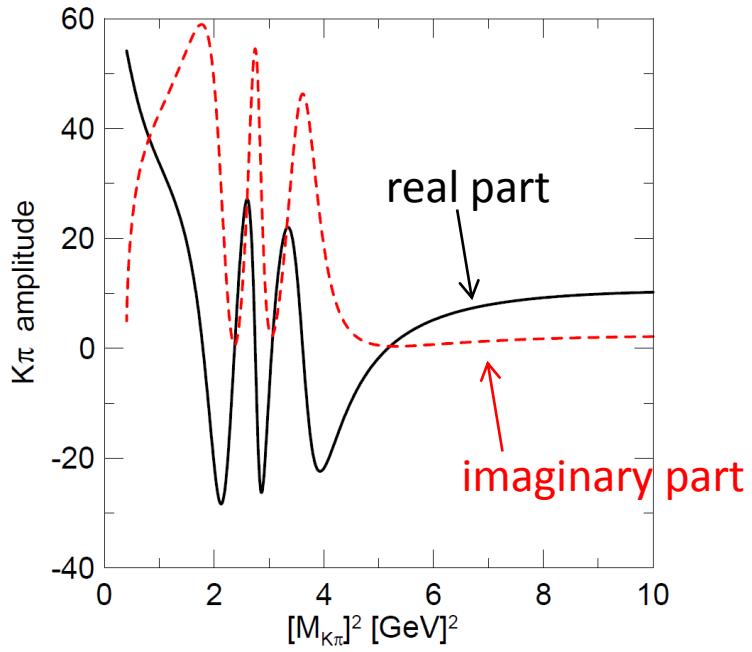
$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$

# Separable model for the S-wave $K\pi$ amplitude [6]



Isospin  $\frac{1}{2}$

$$f_{1/2}(M_{K\pi}^2) = -i \tau_{1/2}(M_{K\pi}^2)$$



# *Relativistic Three-body Model in Light-Front Coordinates*

# A Snapshot on Light-Front dynamics: $x^+ = t+z=0$

Projection of the Bethe-Salpeter equation on the LF (Weinberg 1966)

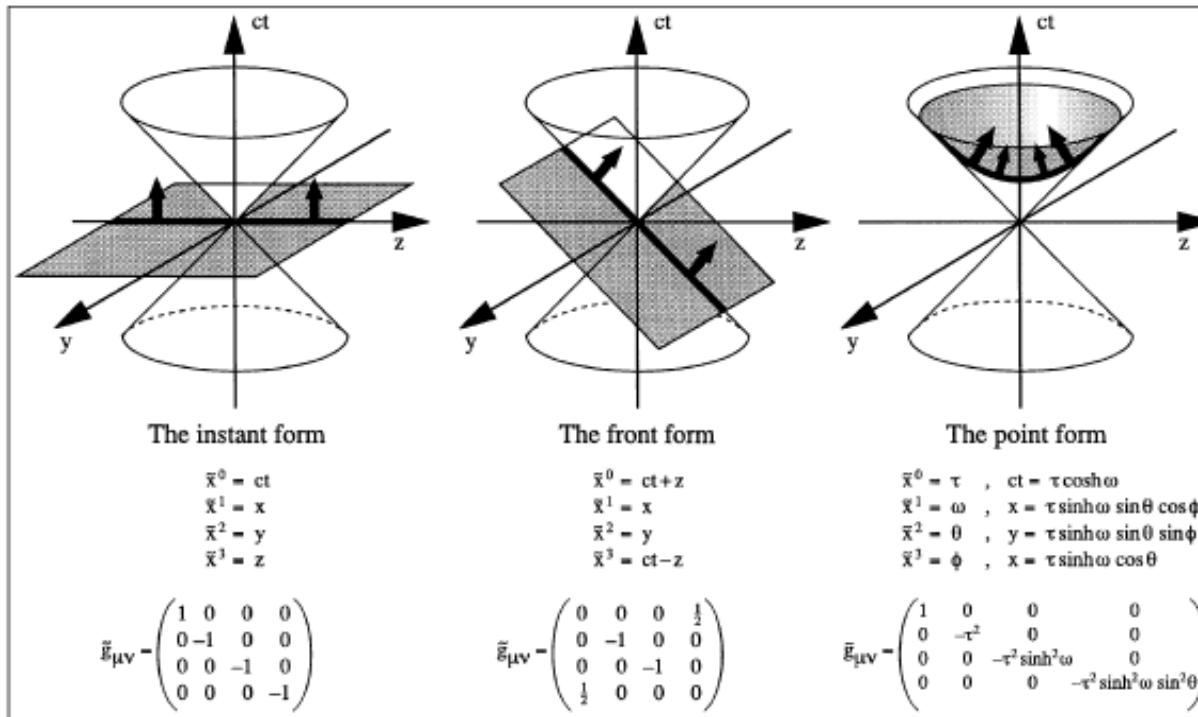
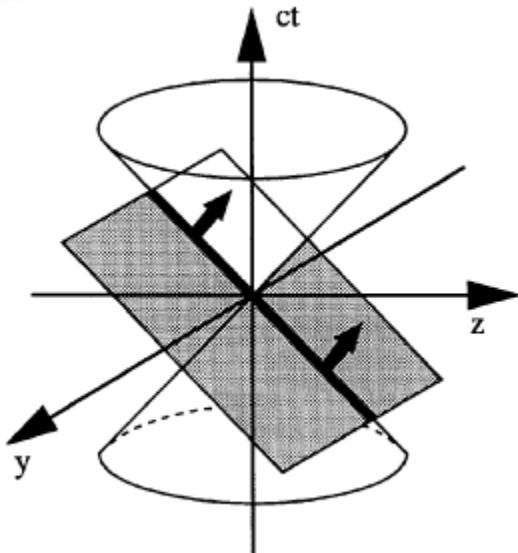


Fig. 1. Dirac's three forms of Hamiltonian dynamics.

S.J. Brodsky et al. / Physics Reports 301 (1998) 299–486

[30]

# Properties of LF quantization



1. Trivial vacuum - perturbative (except for zero modes);
2. Maximal number of 7 kinematical transf. (3 boosts + 1 rot. + 3 transl.)
3. Truncation in the Fock-space not stable under rotations around transverse directions (non-kinematical boosts).

Light-Front time  $x^+$ :  $\tau = t + z/c$

Generator of LF time translations

$$\hat{p} = (\hat{p}^+, \hat{p}^-, \hat{\mathbf{p}}_\perp) \quad \hat{p}^\pm = \hat{p}^0 \pm \hat{p}^3$$

Kinematical generators of translations

Mass square Fock-space operator

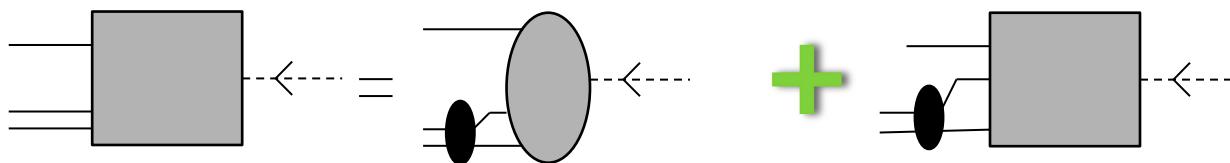
$$H_{LF} = \hat{p}^- \hat{p}^+ - \hat{\mathbf{p}}_\perp^2$$

**Particles:** eigenstates

$$H_{LF}|p\rangle = \mathcal{M}_H^2|p\rangle$$

[31 ]

# Light-Front Three-Body Model [1]



$$\xi_{3/2}(y, \vec{k}_\perp) = \frac{5}{3} \xi_0(y, \vec{k}_\perp) + \frac{i}{3} \int_0^{1-y} \frac{dx}{x(1-y-x)} \int \frac{d^2 q_\perp}{(2\pi)^3} \frac{\tau_{1/2}(M_{K\pi}^2) \xi_{3/2}(x, \vec{q}_\perp)}{M_D^2 - M_{0,K\pi\pi}^2 + i\varepsilon}$$

$$M_{K\pi}^2 = (1-x)(M_D^2 - \frac{q_\perp^2 + m_\pi^2}{x}) - q_\perp^2$$

$$M_{0,K\pi\pi}^2 = \frac{k_\perp^2 + m_\pi^2}{y} + \frac{q_\perp^2 + m_\pi^2}{x} + \frac{(\vec{k}_\perp + \vec{q}_\perp)^2 + m_K^2}{1-y-x}$$

Frederico 1992, Sales et al 2000, Karmanov and Carbonell 2003,  
Marinho and Frederico 2008, Karmanov and Maris 2008 & 2009

[32 ]

# Light-Front Three-Body Model [2]

Driving term:

$$\begin{aligned}\xi_0(y, \vec{k}_\perp) &= - \int \frac{d^4 q}{(2\pi)^3} \frac{1}{q^2 - m_\pi^2 + i\varepsilon} \frac{1}{(K - k - q)^2 - m_K^2 + i\varepsilon} \\ &= \frac{i}{2} \int_0^1 dx \int \frac{d^2 q_\perp}{(2\pi)^3} \left[ \frac{1}{M_{K\pi}^2 - M_{0,K\pi}^2 + i\varepsilon} - \frac{1}{\mu^2 - M_{0,K\pi}^2 + i\varepsilon} \right] + i\lambda(\mu^2)\end{aligned}$$

Renormalization parameter

$M_{K\pi}^2 = (M_D^2 - \frac{k_\perp^2 + m_\pi^2}{y}) - k_\perp^2 \quad , \quad M_{0,K\pi}^2 = \frac{q_\perp^2 + m_\pi^2}{x} + \frac{q_\perp^2 + m_K^2}{1-x}$

$$\mathcal{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) + \tau(M_{K\pi}^2) \xi(y', \vec{k}_{\perp\pi'}) + \tau(M_{K\pi'}^2) \xi(y, \vec{k}_{\perp\pi})$$

Kinematics : z direction transverse to the decay plane

# Light-Front Three-Body Model [3]

Comparison with data parameterization

$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$

$$A_0(M_{K\pi}^2) = a_0(M_{K\pi}^2) e^{i\phi_0(M_{K\pi}^2)} = N \left( 1 + \tau_{\gamma_2}(M_{K\pi}^2) \xi_{\beta_2}(y, \vec{k}_{\perp\pi}) \right)$$

$$\left| \vec{k}_{\perp\pi} \right| = \left[ \left( \frac{M_D^2 + m_\pi^2 - M_{K\pi}^2}{2M_D} \right)^2 - m_\pi^2 \right]^{\frac{1}{2}}, \quad y = \frac{\sqrt{\vec{k}_{\perp\pi}^2 + m_\pi^2}}{M_D}$$

Assumptions:

- ❖ cut-off ( $\Lambda$ ) in  $k_{\perp\pi}$
- ❖  $\epsilon = 0.1$  GeV.
- ❖ partonic amplitude  $D_{\beta_2}(k_\pi, k_{\pi'}) = 1$  - momentum independent

# *Light-Front Three-Body Model [3]*

Two types of calculations with  $\lambda=0$  (renormalization parameter):

$$\xi_{3/2}(y, \vec{k}_\perp) = \frac{5}{3} \xi_0(y, \vec{k}_\perp) + \frac{i}{3} \int_0^{1-y} \frac{dx}{x(1-y-x)} \int \frac{d^2 q_\perp}{(2\pi)^3} \frac{\tau_{1/2}(M_{K\pi}^2) \xi_{3/2}(x, \vec{q}_\perp)}{M_D^2 - M_{0,K\pi\pi}^2 + i\varepsilon}$$

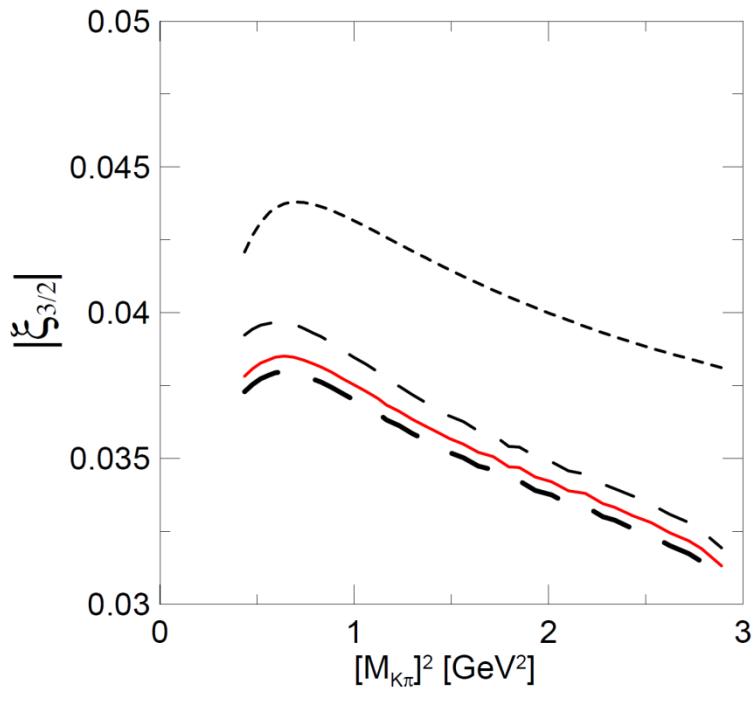
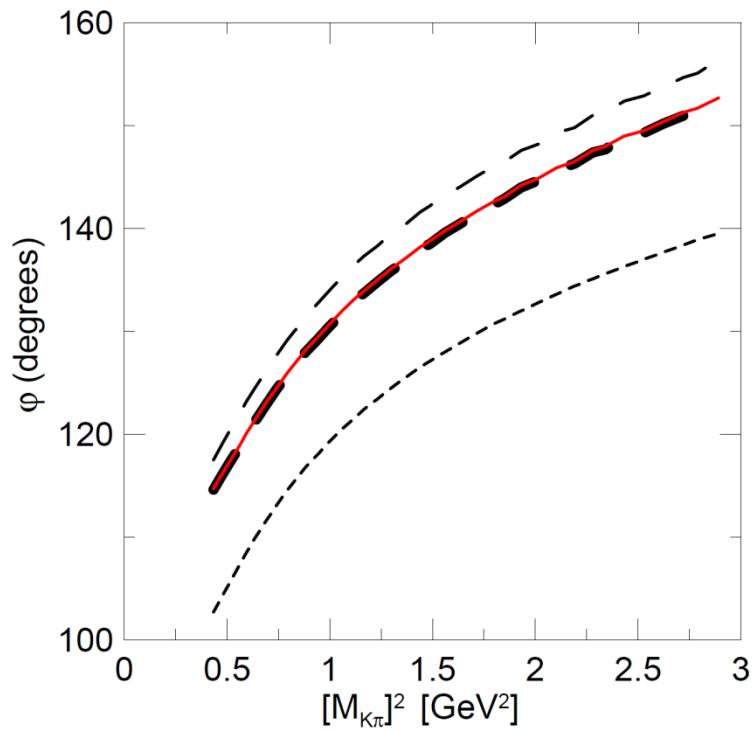
(I) Subtraction in the kernel:

$$\frac{1}{M_D^2 - M_{0,K\pi\pi}^2 + i\varepsilon} \rightarrow \frac{1}{M_D^2 - M_{0,K\pi\pi}^2 + i\varepsilon} + \frac{1}{\mu^2 + M_{0,K\pi\pi}^2}$$

(II) Mass of the virtual  $K\pi$  system real:

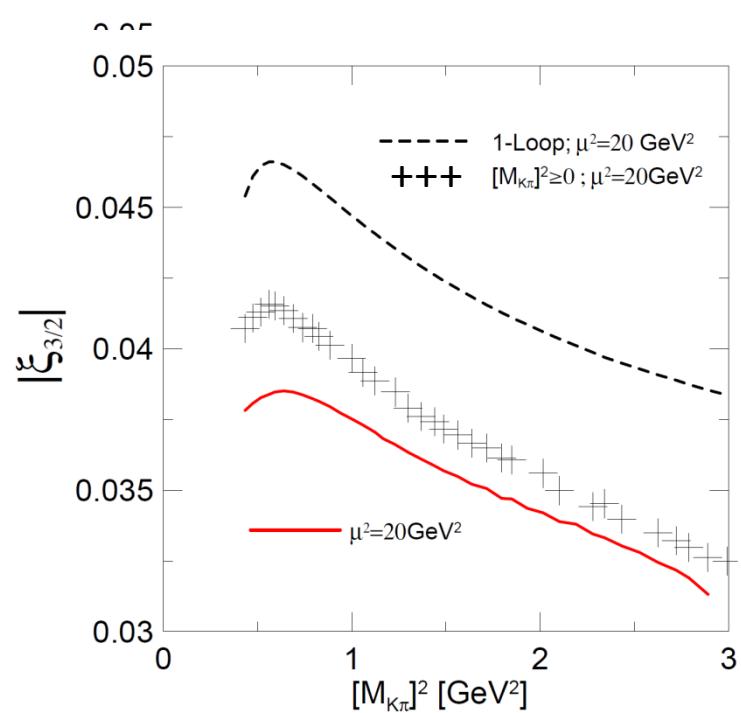
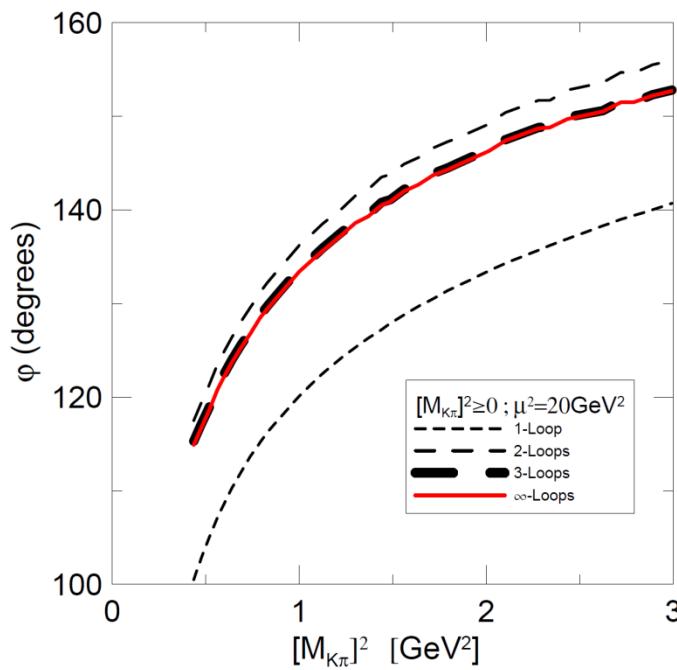
$$M_{K\pi}^2 = (P_D - k_\pi)^2 = (1-x)(M_D^2 - \frac{k_\perp^2 + m_\pi^2}{x}) - k_\perp^2 \geq 0$$

# Preliminary Results $K\pi$ [1]c



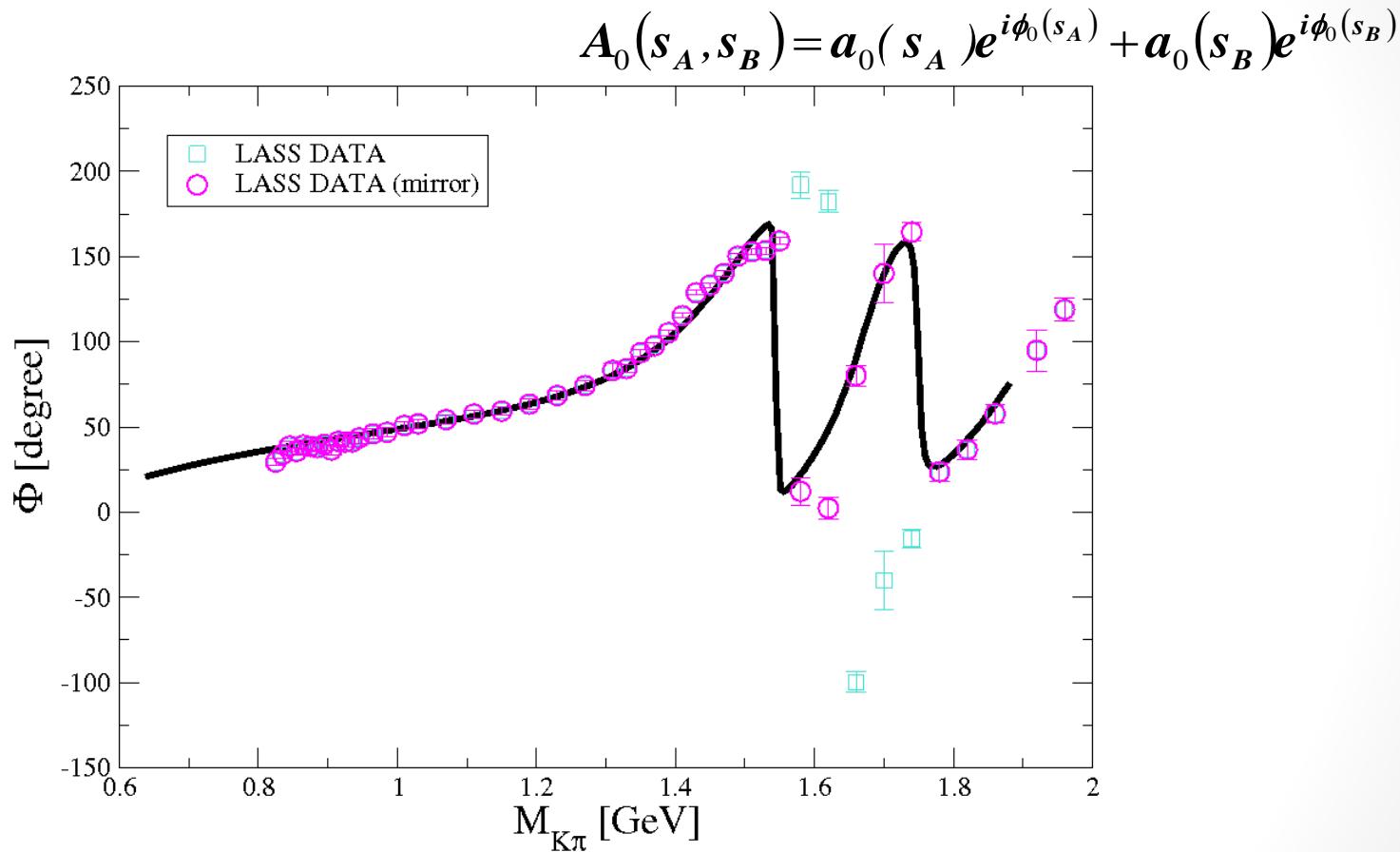
— Solution of the LF integral equation     $\mu^2 = 20 \text{ GeV}^2$

# Preliminary Results $K\pi$ [1]



# Preliminary Results $K\pi$ [1]

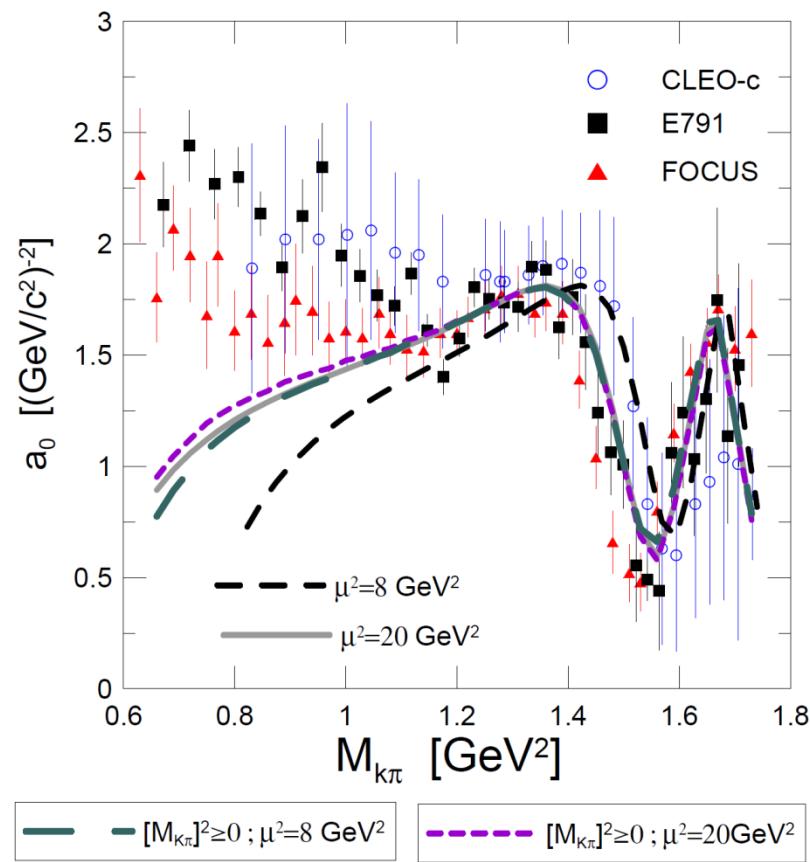
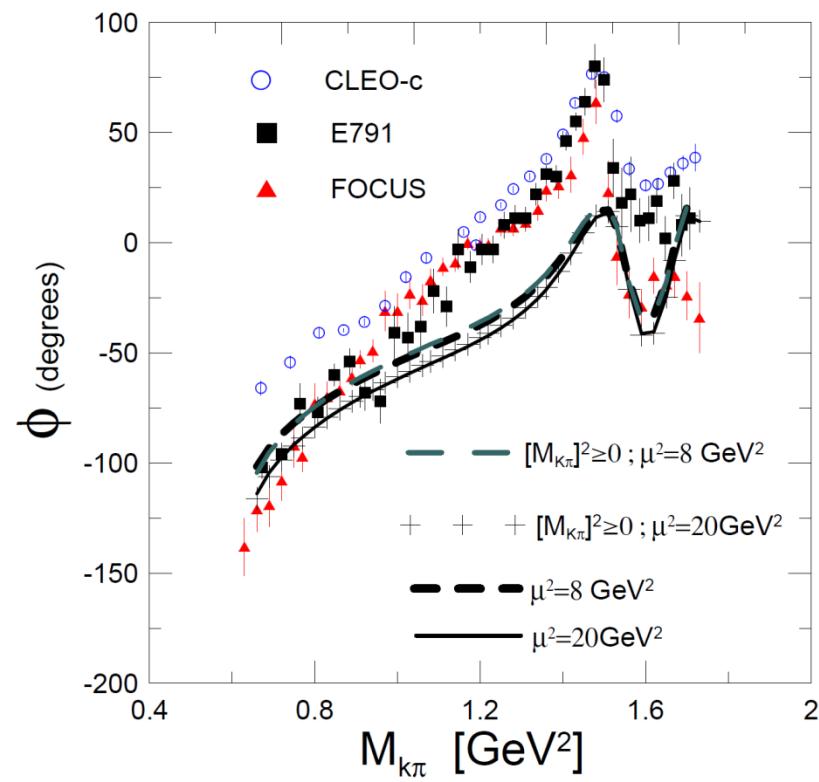
LASS data for  $K\pi$  elastic S-wave scattering phase-shift



$$S_{K\pi} = \frac{k \cot \delta + i k}{k \cot \delta - i k} \prod_{r=1}^3 \frac{M_r^2 - M_{K\pi}^2 + i z_r \bar{\Gamma}_r}{M_r^2 - M_{K\pi}^2 - i z_r \bar{\Gamma}_r}$$

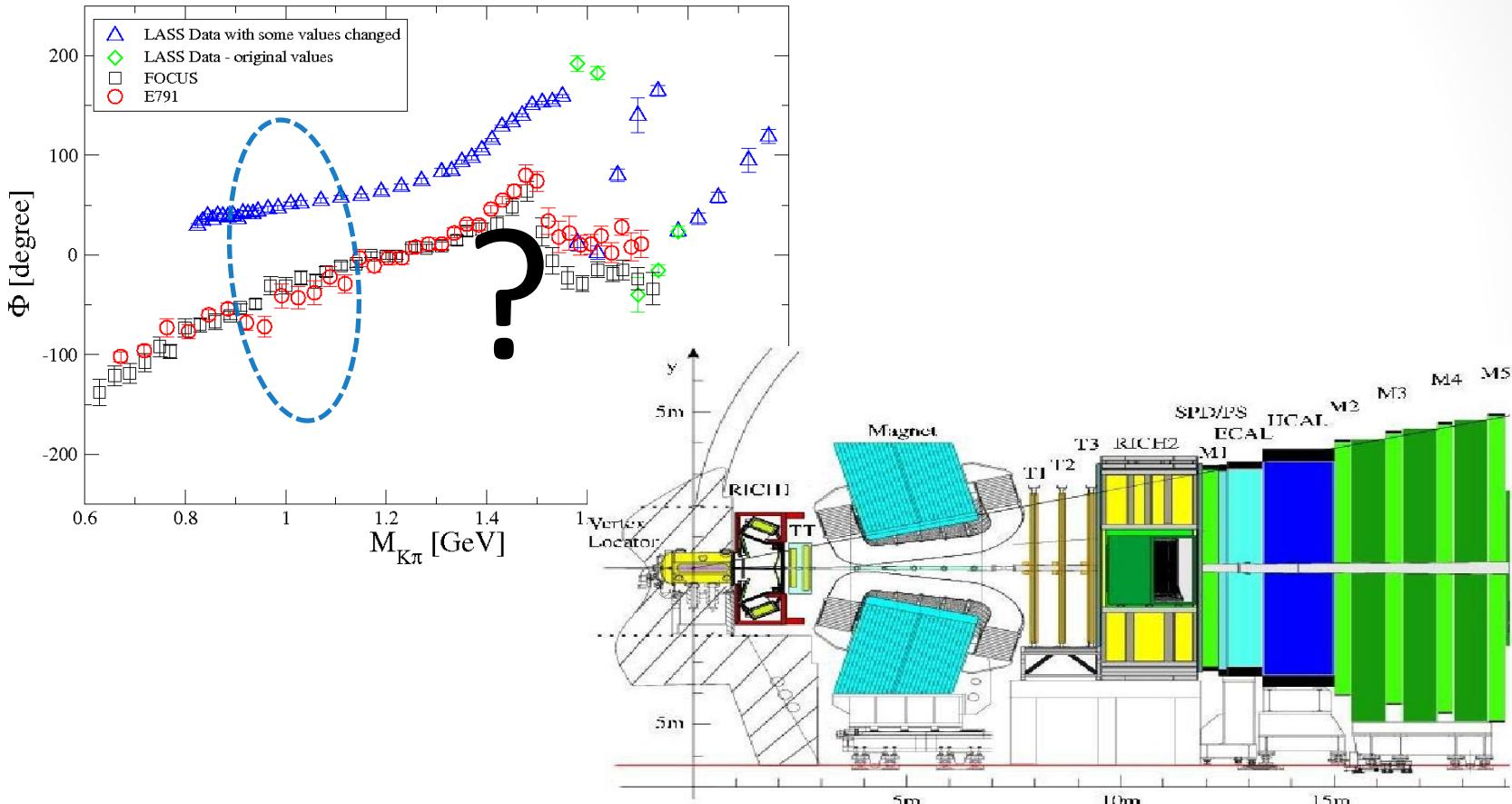
# Preliminary Results $K \pi \pi$

$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$

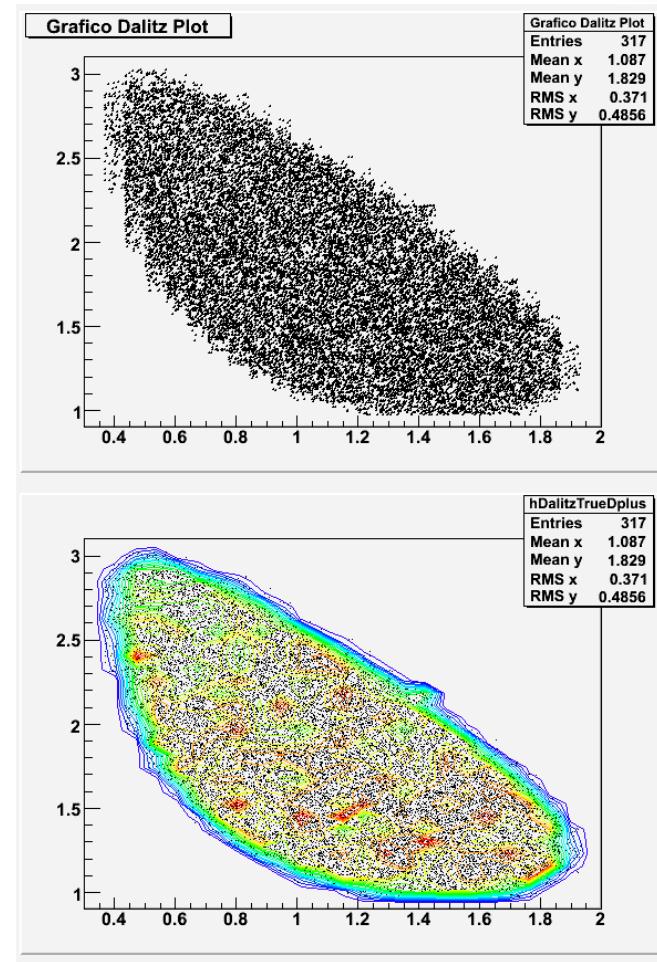
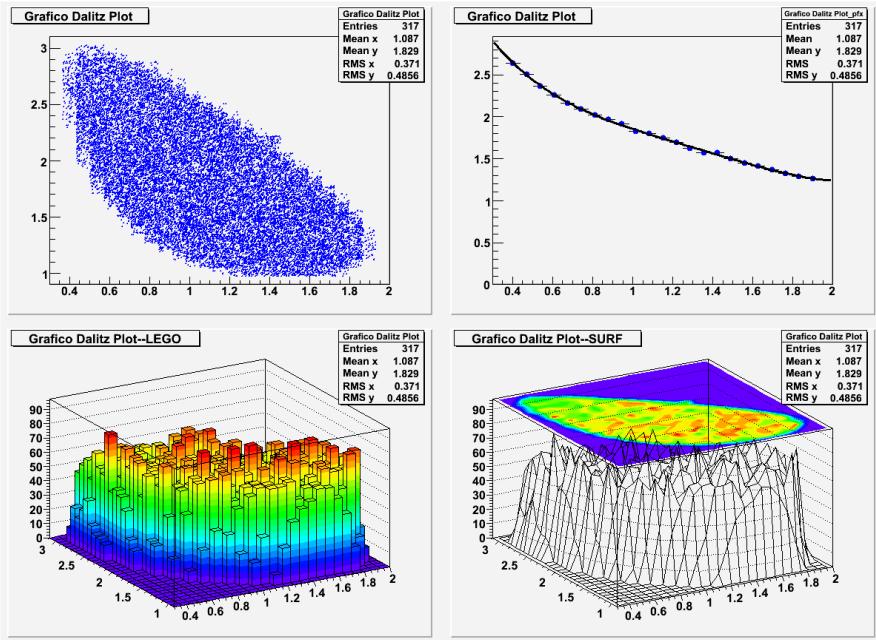


G. Bonvicini et al. (CLEO-c Coll.) ,Phys. Rev. D78 (2008) 052001

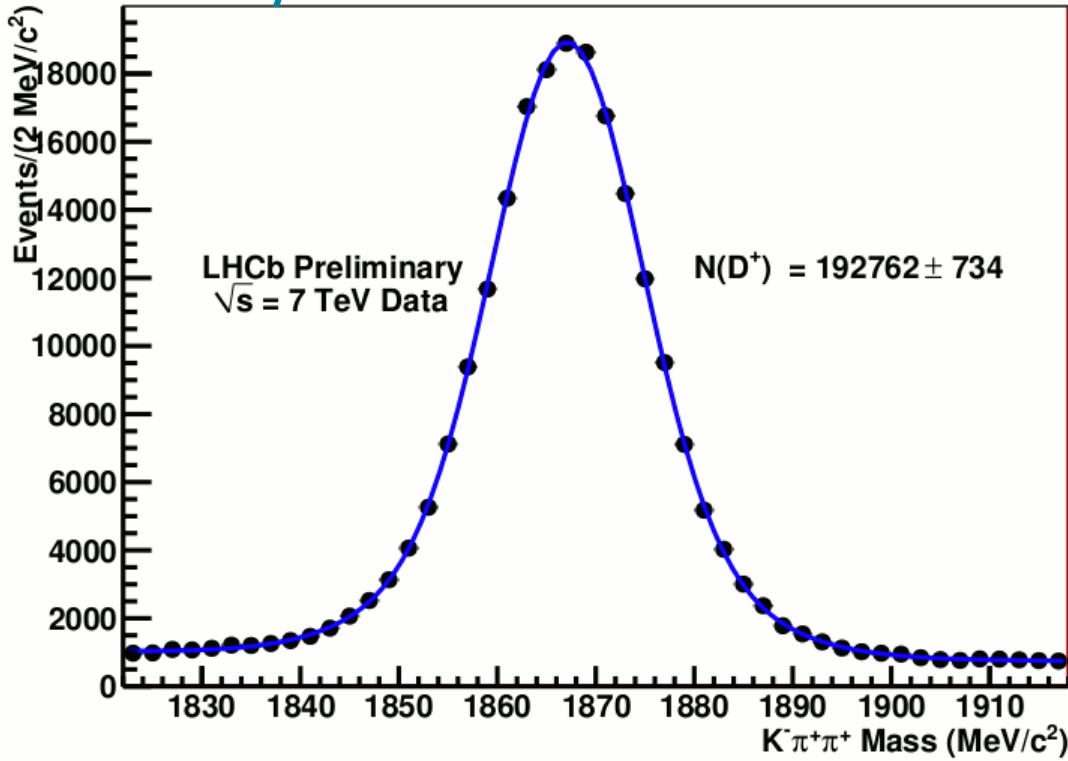
# *Taking data* **LHCb**



**LHCb → Large Hadron Collider Beauty experiment**



“tiny fraction” of LHCb data



Although we had a large number of events



Limited by systematic errors

- One way to decrease the systematic errors is to fit the P-wave with the S-wave fixed
- Uncertainties on P-wave parameters → systematic errors

## OUR ACTIVITIES:

- 1) Reproduce the analysis performed before in order to make a sort of a calibration;
- 2) Introduce the calculation showed before in order to extract the pure amplitude  $K^-\pi^+$ ;

Remembering that :  $K^-\pi^+$  system is embedded in a three-body strongly interacting final state.

# *Next Steps*

- Introduce an absorptive effective three-body interaction to account for the missing channels
- Analysing the data, using the same methodology as used by other collaborations;
- In a next step , include our calculation showed before to extract the Kpi amplitude.

# Conclusions

- Three-body final state interaction in  $\xi$  is dominated by the first scattering (1-Loop); two-binary process (2-Loops)  $\sim 30\%$ ; three-binary process (3-Loops)  $\sim 10\% \dots$
  - $\xi$  is a complex function that depends on momentum !
  - Phase  $\phi(s)$  can be moved towards negative values depending on  $\mu$ .
- 
- Other decay channels compete with  $K^- \pi^+ \pi^+$  (BR=9%):

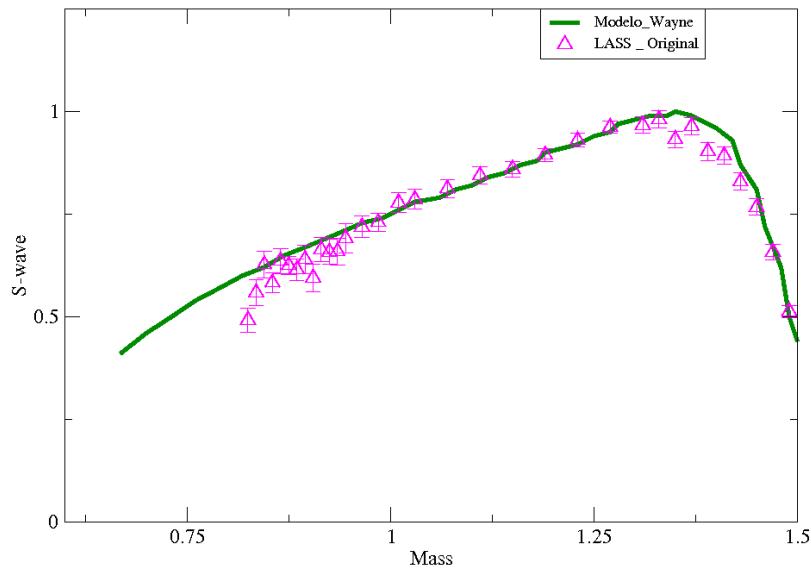
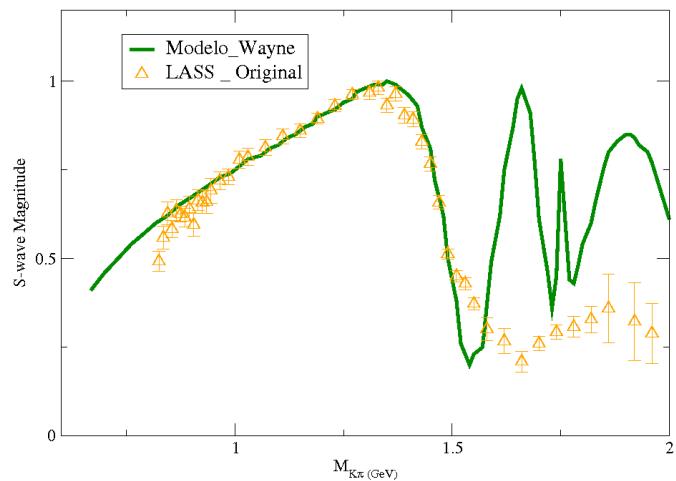
$$K_s \pi^+ \pi^0 \rightarrow (6.8\%) \quad K_s \pi^+ \pi^- \pi^+ \rightarrow (3.0\%) \quad K_s \pi \rightarrow (1.45\%) \quad K^+ \pi^+ \pi^- \rightarrow (6.0 \cdot 10^{-4}\%)$$

- Kpi amplitude physically constrained : chiral symmetry;
- Interaction in all isospin states;
- Introduce an absorptive effective three-body interaction to account for the missing channels...

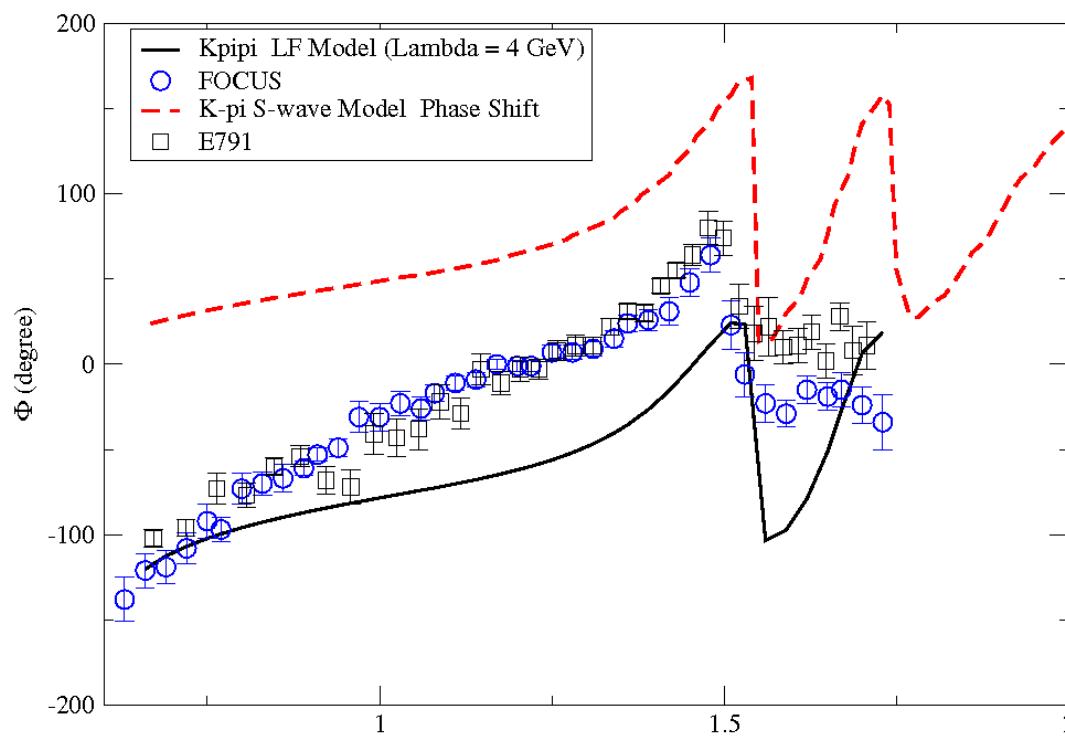
**THANK YOU!**

# Preliminary Results $K\pi$ [2]

LASS data for  $K\pi$  elastic S-wave amplitude



# Preliminary Results $K\pi\pi$ [3]



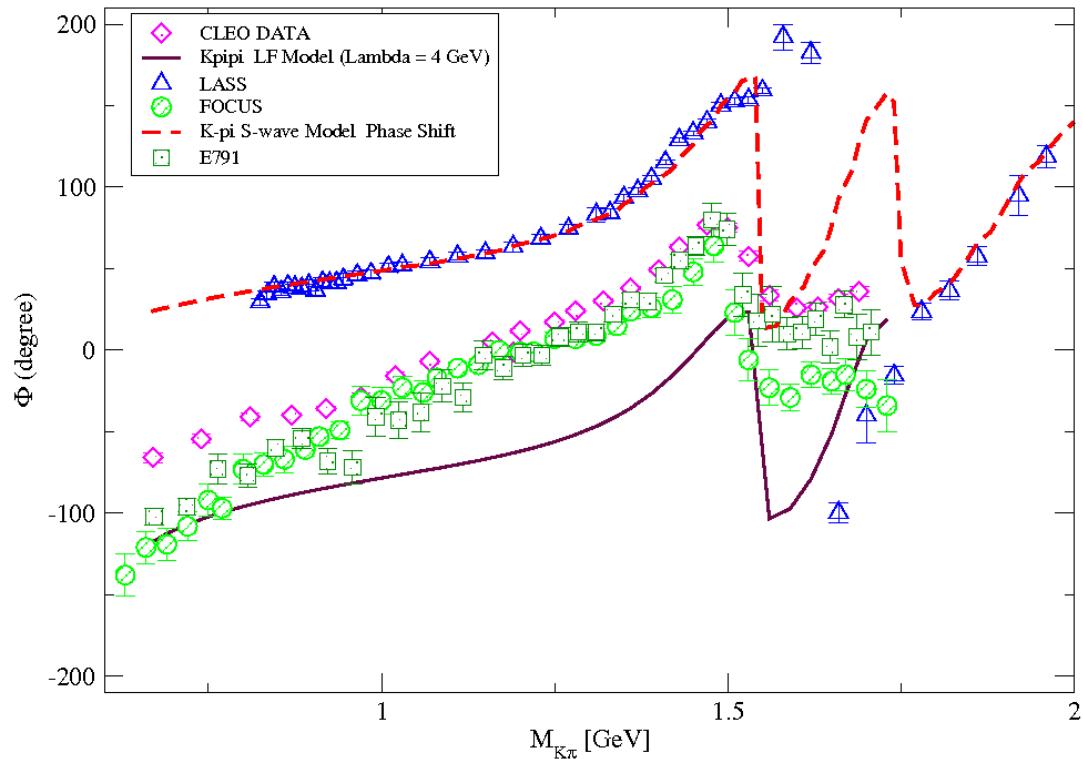
$$\Lambda = 4 \text{ GeV}$$

$$\lambda(0) = 0.12 + i0.06$$

$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$

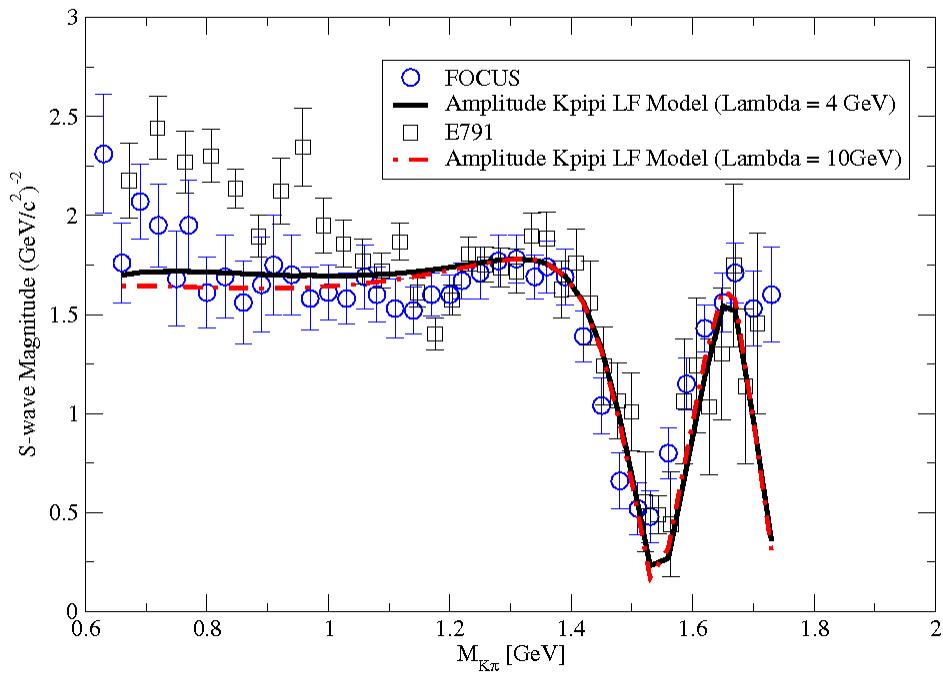
[49 ]

# Preliminary Results $K\pi\pi$ [4]



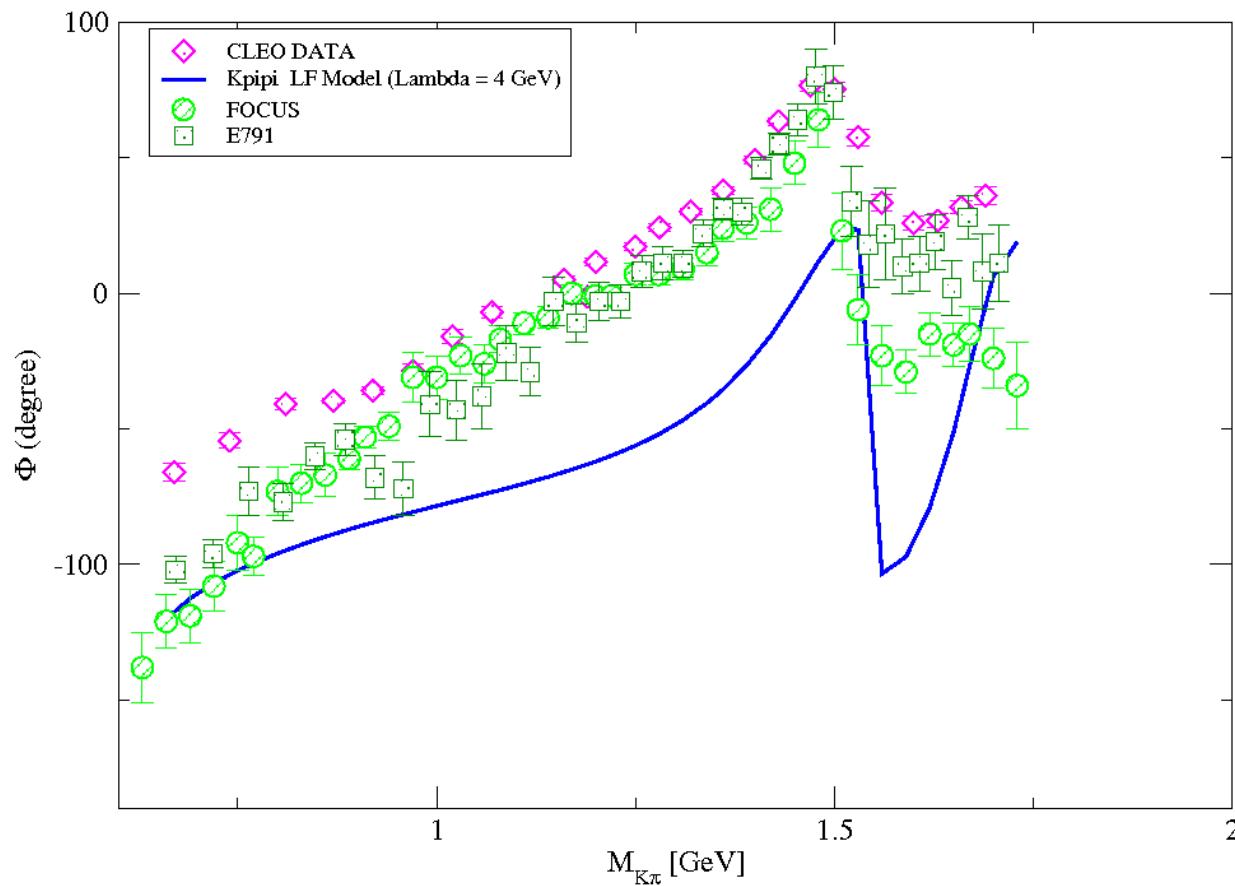
$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$

# Preliminary Results $K\pi\pi$ [5]

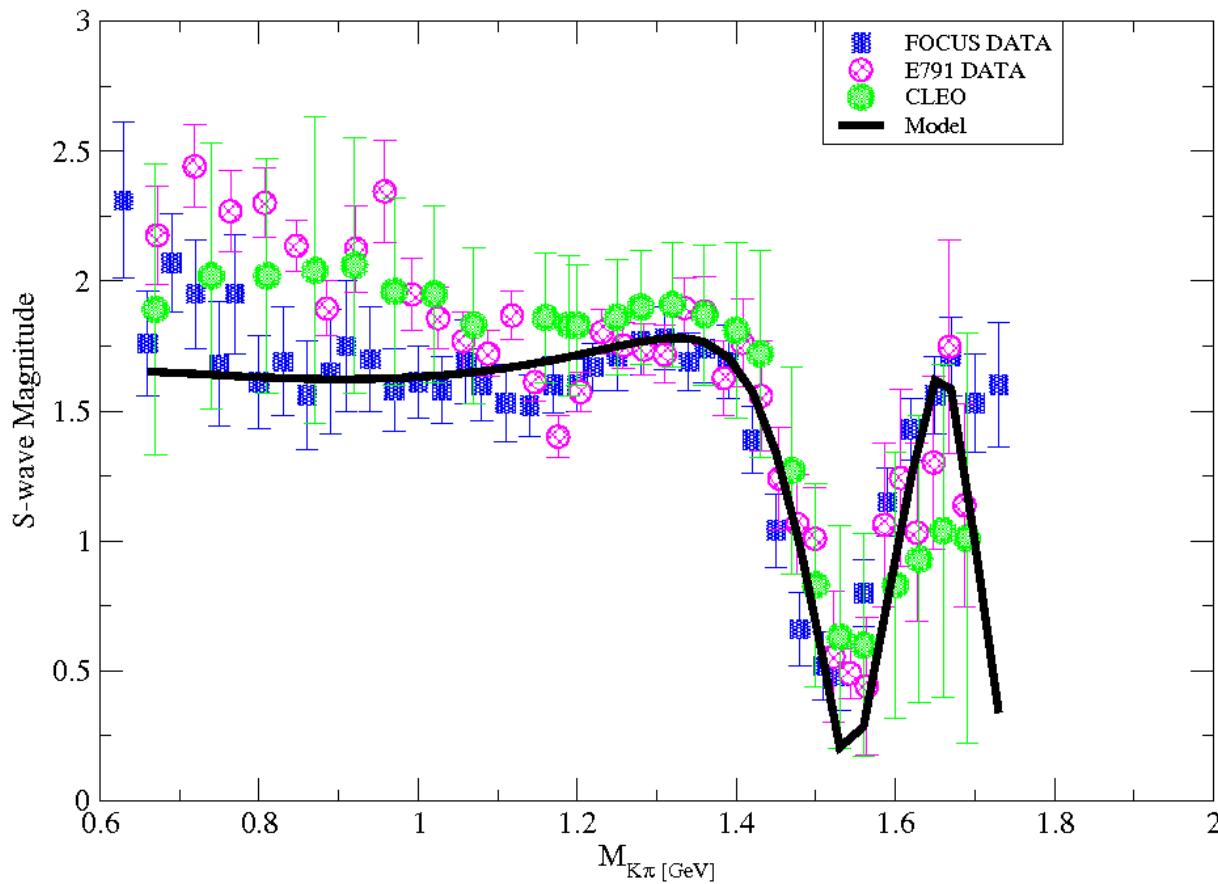


[51 ]

# Preliminary Results $K\pi\pi$ [6]



# Preliminary Results $K\pi\pi$ [4]





# Conclusions

- Three-body final state interaction correction depends on the cut-off  
→ short-range effect but  $\xi$  depends weakly on momentum;
- Phase  $\phi(s)$  can be moved towards negative values depending on  $\lambda$ ;
- Other decay channels compete with  $K^- \pi^+ \pi^+$  (BR=9%):

$$K_s \pi \rightarrow (1.45\%)$$

$$K_s \pi^+ \pi^0 \rightarrow (6.8\%)$$

$$K^+ \pi^+ \pi^- \rightarrow (6.0 \cdot 10^{-4}\%)$$

$$K_s \pi^+ \pi^- \pi^+ \rightarrow (3.0\%)$$

# Three- Body Relativistic Model

$$\mathcal{D}(k_\pi, k_{\pi'}) = D(k_\pi, k_{\pi'}) + \tau(M_{K\pi}^2) \xi(k_{\pi'}) + \tau(M_{K\pi'}^2) \xi(k_\pi)$$

Bose-symmetrized complex function with respect to the identical pions:

$$\mathcal{A}_0 = A_0(M_{K\pi}^2, M_{K\pi'}^2) + A_0(M_{K\pi'}^2, M_{K\pi}^2)$$

$A_0$  are complex functions of the two invariant masses squared

$$M_{K\pi}^2 = (K - k_{\pi'})^2$$

→ Decay kinematics

$$M_{K\pi'}^2 = (K - k_\pi)^2$$

# Three- Body Relativistic Model

The dependence on the  $K\pi$  subsystem mass of  $A_0(M_{K\pi'}^2, M_{K\pi}^2)$  can be reduced in our model to a complex function of only one variable, so we can write:

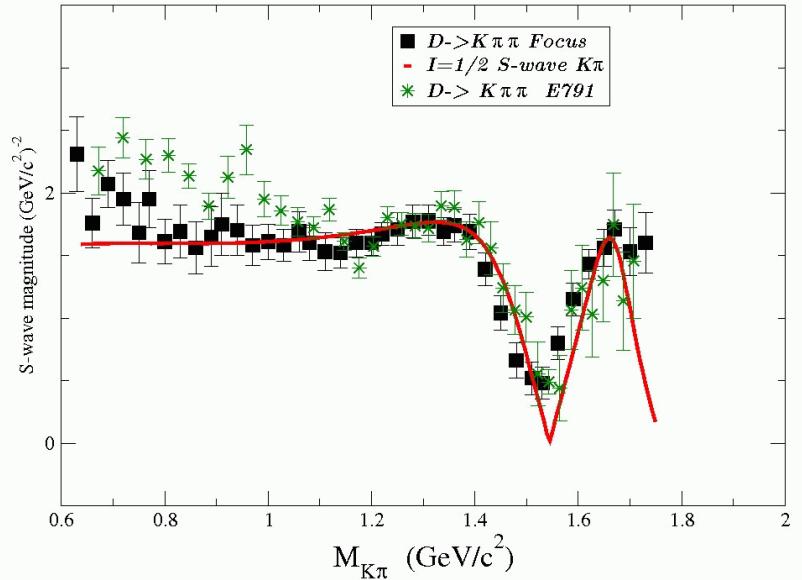
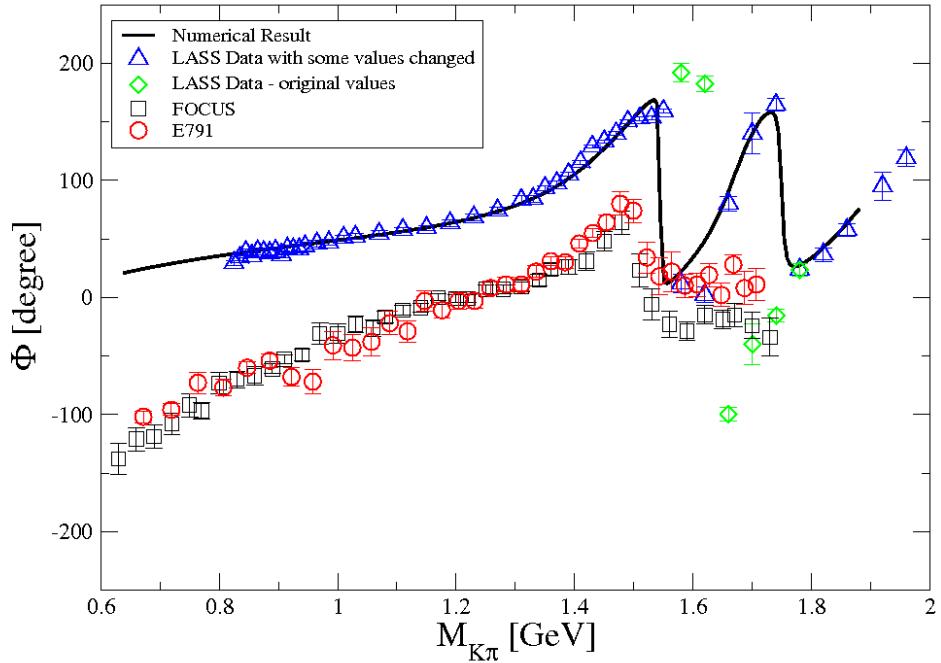
$$A_0(M_{K\pi'}^2) = a_0(M_{K\pi'}^2) e^{i\phi_0(M_{K\pi'}^2)} = N \left( -\frac{i}{2} + \tau_{\frac{1}{2}}(M_{K\pi'}^2) \xi_{\frac{1}{2}}(k_\pi) \right)$$

With the pion on mass shell momentum given by:

$$|\vec{k}_\pi| = \left[ \left( \frac{M_D^2 + m_\pi^2 - M_{K\pi'}^2}{2M_D} \right)^2 - m_\pi^2 \right]^{\frac{1}{2}}$$

The normalization constant  $N$  is thought to be real and positive.

# Numerical Results



$$A_0(s_A, s_B) = a_0(s_A) e^{i\phi_0(s_A)} + a_0(s_B) e^{i\phi_0(s_B)}$$