

RADIATIVE DECAY OF X(3872) AS A MIXED MOLECULE-CHARMONIUM STATE IN QCDSR

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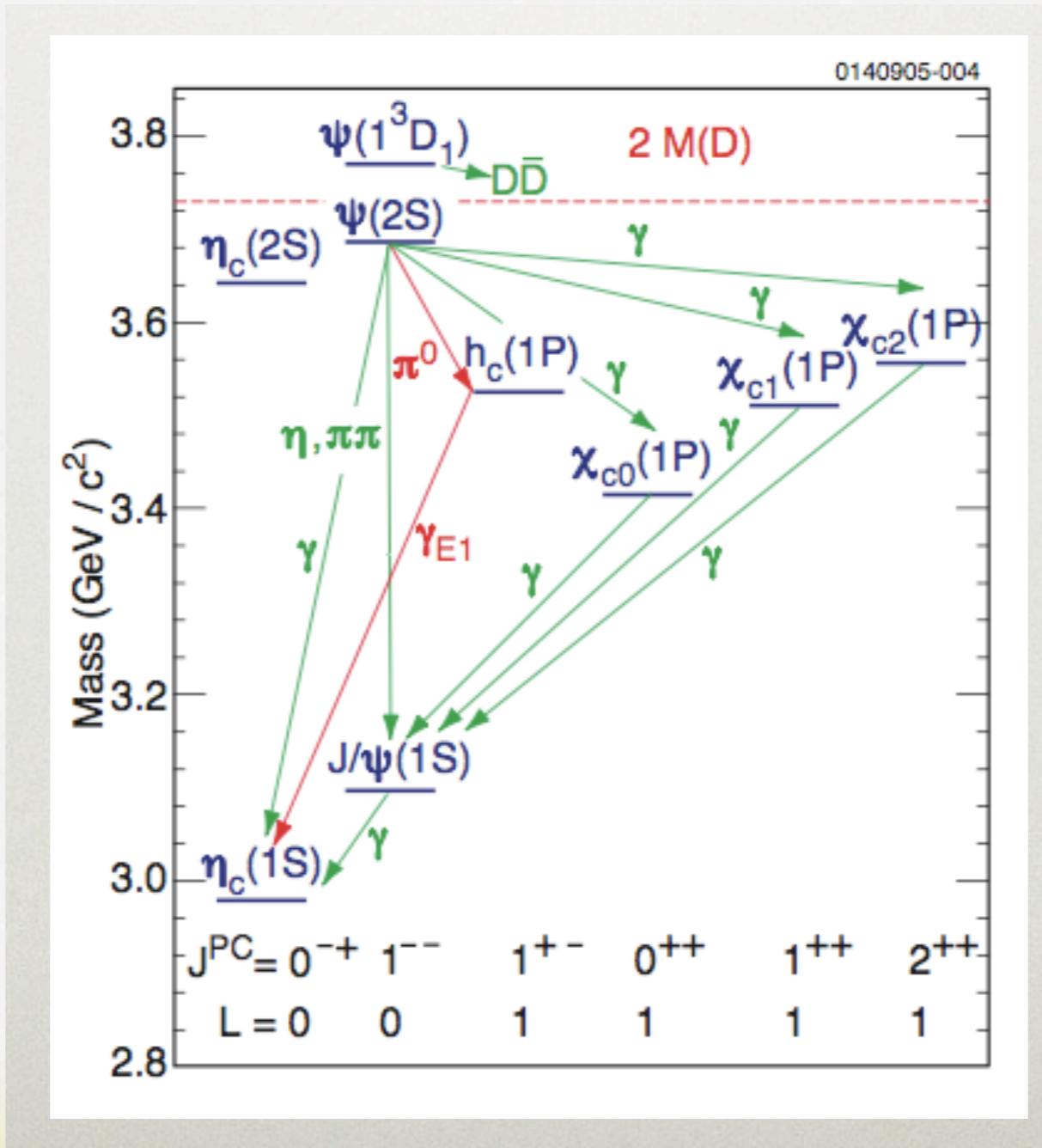
XXII RETINHA, 22/02/2011

OUTLINE

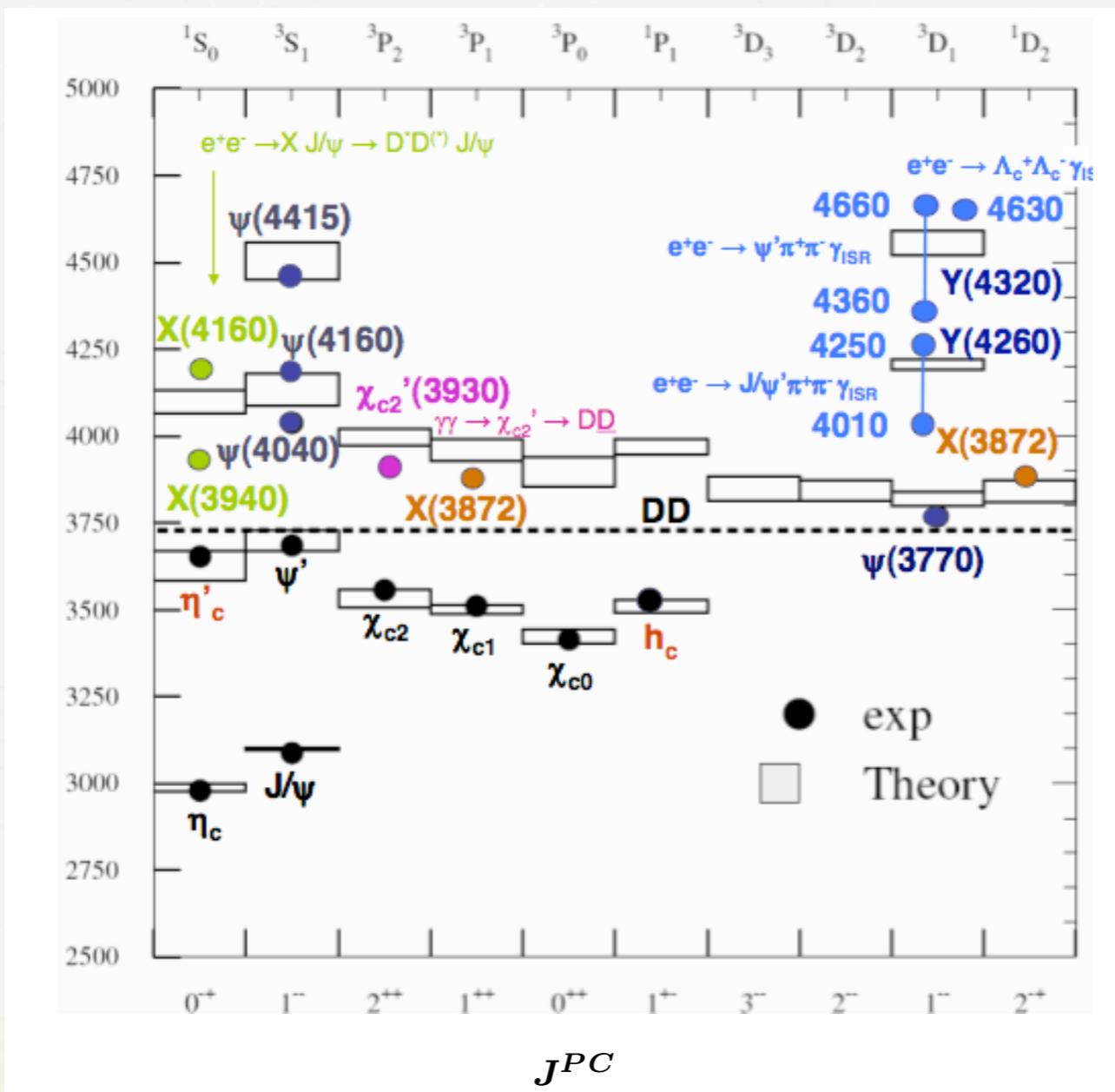
- ✿ Introduction
- ✿ QCD Sum Rules
- ✿ Mixed two and four quark current + Early Results
- ✿ Radiative decay
- ✿ Conclusion

INTRODUCTION

CHARMONIUM SPECTROSCOPY PRE - B FACTORIES



NEW CHARMONIUM SPECTROSCOPY



X(3872)

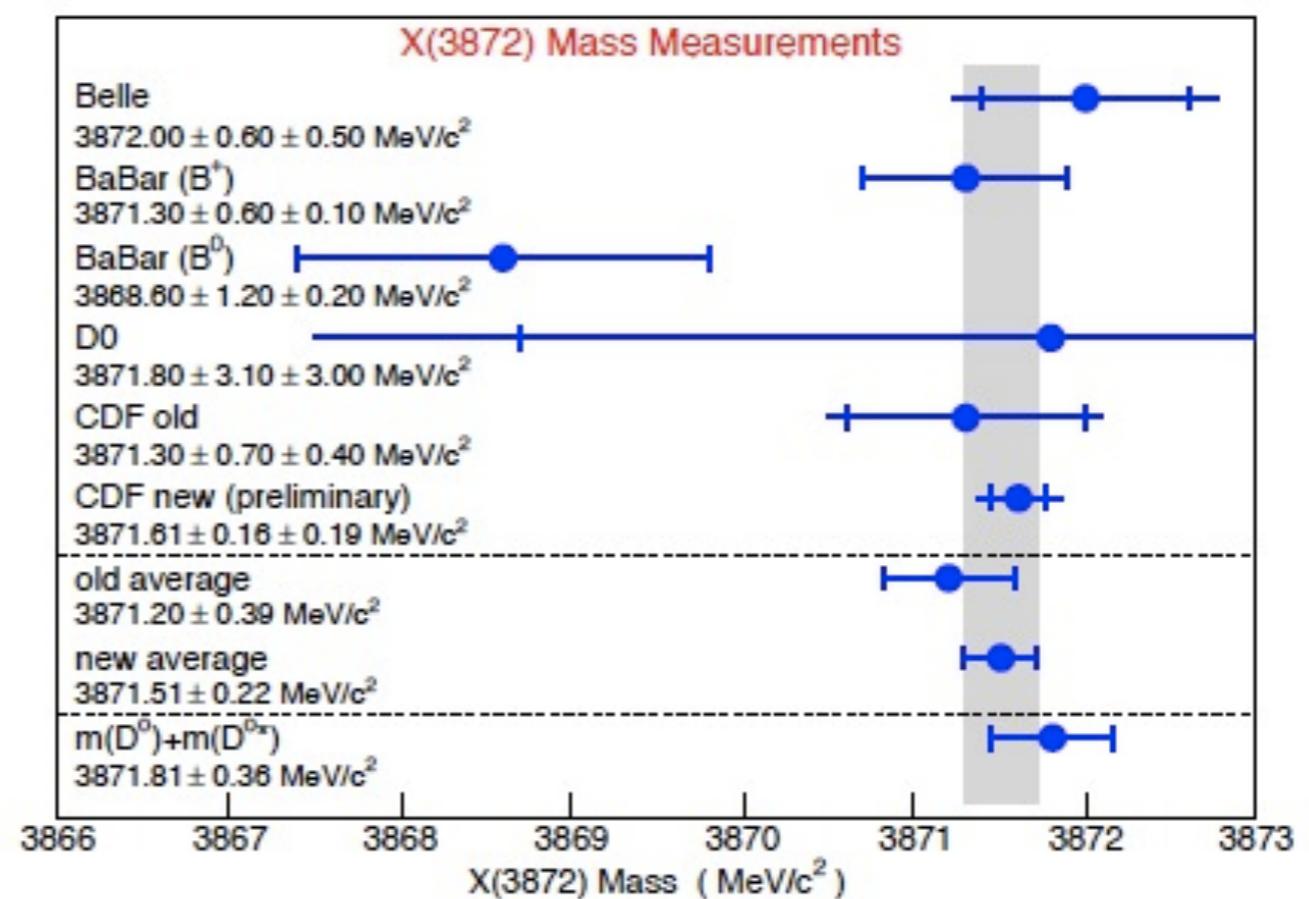
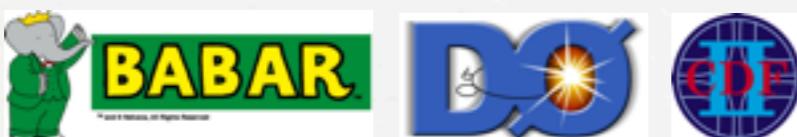
- * Discovery in 2003 @



$$M_X = (3871.20 \pm 0.39) \text{ MeV}$$
$$\Gamma < 2.3 \text{ MeV}$$

$$B^+ \rightarrow X(3872)K^+ \rightarrow J/\psi\pi^+\pi^-K^+$$

- * Later confirmed by



X(3872) IN THE QUARK MODEL

- ✿ Favored quantum numbers J^{PC} :
 1^{++} (*angular distribution + $\gamma J/\psi$*)
 2^{-+} (*not ruled out*)
- ✿ CQM predictions for charmonium states 1^{++}
 $2^3P_1(3990)$ $3^3P_1(4290)$
- ✿ Strong isospin violation:

Barnes & Godfrey PRD69 (2004)

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.0 \pm 0.4 \pm 0.3$$

Not a c-cbar state!

X(3872) AS A MULTIQUARK STATE

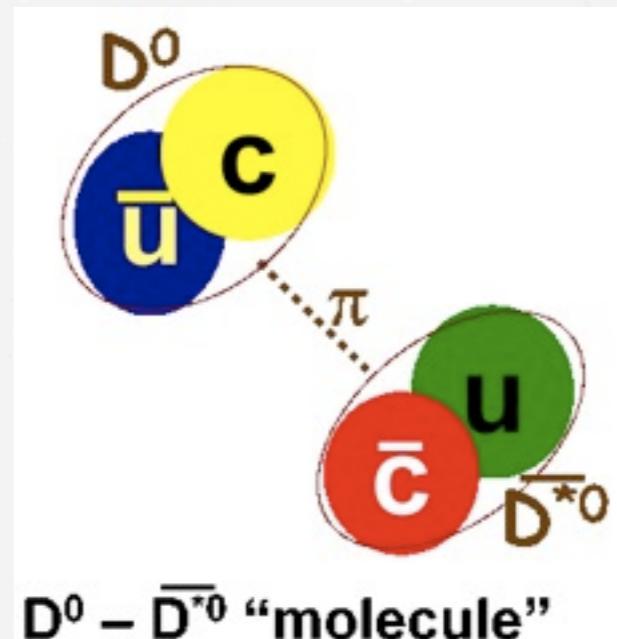
$$M(D^{*0}D^0) = (3871.81 \pm 0.36) \text{ MeV}$$

Molecule with a small binding energy

*Close & Page (2004)
Swanson (2006)*

Tetraquark state

Maiani et al (2005)



X(3872) AS A MULTIQUARK STATE

Good QCD Sum Rules Results for the mass

Molecule

$$M_X = (3.87 \pm 0.07) \text{ GeV}$$

R.D. Matheus, S. Narison,
M. Nielsen and J.-M. Richard,
PRD75 (2007)

Tetraquark

$$M_X = (3.92 \pm 0.13) \text{ GeV}$$

S.H. Lee, M. Nielsen and U. Wiedner, arXiv:
0803.1168

EVIDENCE OF A C \bar{C} COMPONENT



$$\frac{B(X \rightarrow \Psi(2S)\gamma)}{B(X \rightarrow \Psi\gamma)} = 3.4 \pm 1.4$$

- ✿ Radiative decays:

Swanson
(2004)

$$\frac{\Gamma(X \rightarrow \Psi(2S)\gamma)}{\Gamma(X \rightarrow \Psi\gamma)} \sim 4 \times 10^{-3}$$

- ✿ $\Gamma(X \rightarrow J/\psi (\pi \pi)) = 50 \text{ MeV}$

Nielsen & Navarra (2006)

- ✿ Production cross section of a bound DD* state with binding energy as small as 0.25 MeV is much smaller than the cross section obtained from the CDF data

C. Bignamini *et al* (2009)

QCD SUM RULES

QCD SUM RULES

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j(x)j^\dagger(0)] | 0 \rangle$$

Fundamental Assumption: Principle of Duality

Theoretical Side

Quarks and gluon fields

Wilson OPE

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j(x)j^\dagger(0)] | 0 \rangle = \sum_n C_n(Q^2) \hat{O}_n$$

Phenomenological Side

Hadronic fields

Pole + resonances (cont.)

$$\Pi(q^2) = - \int ds \frac{\rho(s)}{q^2 - s + i\epsilon} + \dots$$



$$\Pi^{OPE} = \Pi^{phen}$$

Aproximations required: truncation of the series at the OPE side;
One pole + continuum of excited states on the phenomenological side

$$\Pi^{phen}(q^2) = \frac{\lambda^2}{M_H^2 - q^2} + \int_0^\infty ds \frac{\rho^{cont}(s)}{s - q^2}; \quad \Pi^{OPE}(q^2) = \int_{s_{min}}^\infty ds \frac{\rho^{OPE}}{s - q^2}$$

$$\rho^{cont}(s) = \rho^{OPE}(s)\Theta(s - s_0)$$

s_0 -
continuum threshold

- Improving the match of $\Pi^{OPE}(Q^2) = \Pi^{phen}(Q^2)$

- Apply Borel Transform $\Pi(Q^2) \rightarrow \Pi(M^2)$

M - Borel Mass

*Suppresses higher order terms on the OPE side
and excited states on the phenomenological side*

- Pole > Continuum;
OPE Convergence;
Stability of the Borel Mass M

Constrains on the
parameters M and s_0

MIXED TWO AND FOUR QUARK STATES

TWO AND FOUR QUARKS CURRENT

$$(c\bar{c}) + (D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0) + (D^{*+}\bar{D}^- - \bar{D}^{*-}D^+)$$

$$J_\mu^q(x) = \sin(\theta) j_\mu^{(4q)}(x) + \cos(\theta) j_\mu^{(2q)}(x)$$

Sugiyama *et al*
PRD (2007)

$D(0)D^*(0)$ molecule ($q=u,d$):

$$j_\mu^{(4q)}(x) = \frac{1}{\sqrt{2}} \left[(\bar{q}_a(x)\gamma_5 c_a(x))(\bar{c}_b(x)\gamma_\mu q_b(x)) - (\bar{q}_a(x)\gamma_\mu c_a(x))\bar{c}_b(x)\gamma_5 u_b(x) \right]$$

Charmonium 1^{++} :

$$j^{(2q)}(x) = \frac{1}{6\sqrt{2}} \langle q\bar{q} \rangle (\bar{c}_a(x)\gamma_\mu\gamma_5 c_a(x))$$

$$(c\bar{c}) + (D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0) + (D^{*+}\bar{D}^- - \bar{D}^{*-}D^+)$$

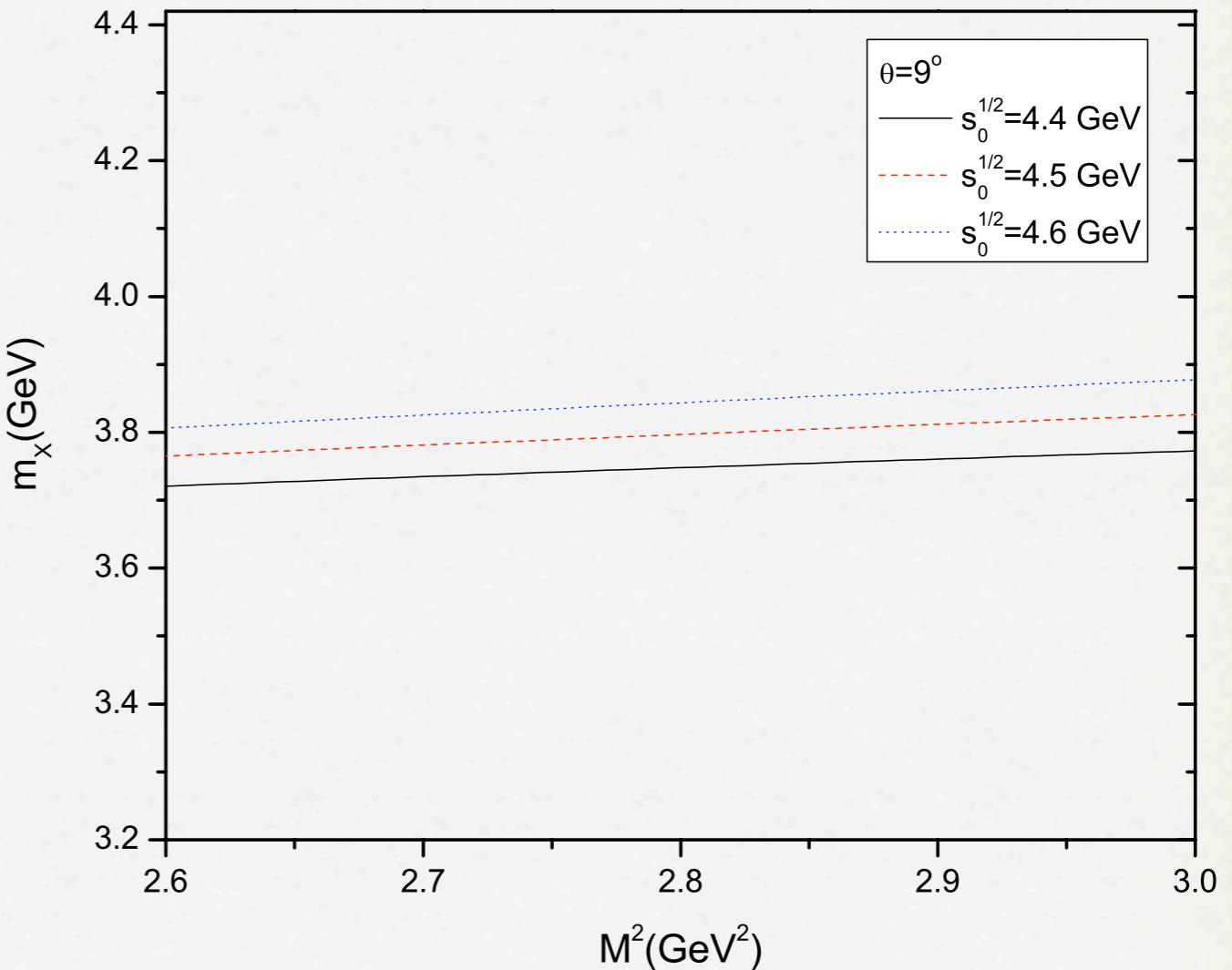
$$\dot{J}_\mu^X(x) = \cos\alpha J_\mu^u(x) + \sin\alpha J_\mu^d(x)$$

MASS OF THE X(3872) AS A MIXED STATE

$$5^\circ \leq \theta \leq 13^\circ$$

$$2.6 \text{ GeV}^2 \leq M^2 \leq 3.0 \text{ GeV}^2$$

$$m_X = (3.77 \pm 0.18) \text{ GeV}$$

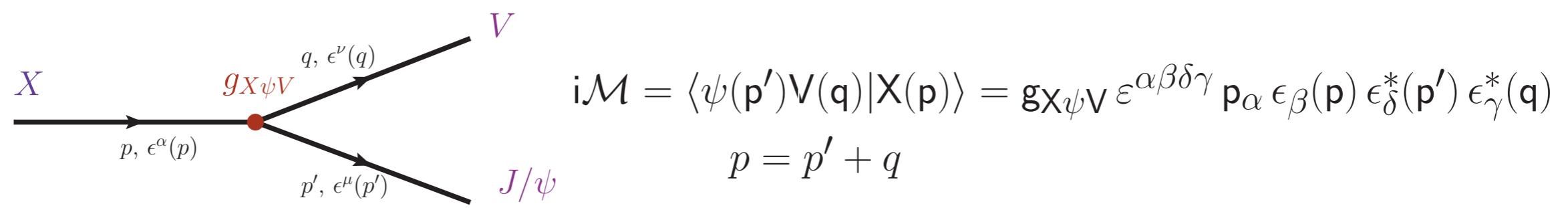


Matheus, Navarra, Nielsen & CMZ
PRD80 (2009)

DECAYS OF THE X(3872)

$$X \rightarrow J/\psi V \rightarrow J/\psi F, \quad F = \pi^+ \pi^- (\pi^+ \pi^- \pi^0) \rightarrow V = \rho, \omega$$

$$L_{eff} = g_{X\psi V} \epsilon^{\mu\nu\rho\sigma} p_\mu X_\nu \psi_\rho V_\sigma$$



$$\frac{d\Gamma}{ds}(X \rightarrow J/\psi f) = \frac{B_{V \rightarrow f}}{8\pi m_X^2} \frac{\Gamma_V m_V}{\pi} \frac{p(s)}{(s - m_V^2)^2 + (m_V \Gamma_V)^2} |M|^2$$

$g_{X\psi V}$ → QCDSR for the vertex $X\psi V$

NUMERICAL RESULTS

$$g_{X\psi\omega} = g_{X\psi\omega}(-m_\omega^2) = 5.4 \pm 2.4$$

$$\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0) = (9.3 \pm 6.9) \text{ MeV}$$

$$5^\circ \leq \theta \leq 13^\circ; \quad \alpha = 20^\circ$$

RADIATIVE DECAY OF THE X(3872)

$$\frac{\Gamma(X \rightarrow J/\psi \gamma)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)}_{Exp.} = 0.14 \pm 0.05$$

Nielsen & CMZ PRD82 (2010)

QCD SUM RULES FOR THE VERTEX $\chi J/\psi \gamma$

✿ Three-point function

$$\Pi_{\mu\nu\alpha}(p, p', q) = \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \langle 0 | T[j_\mu^\psi(x) j_\nu^\gamma(y) j_X^\dagger(0)] | 0 \rangle$$

✿ Interpolating currents

$$J_\mu^q(x) = \sin\theta j_\mu^{(4q)}(x) + \cos\theta j_\mu^{(2q)}(x) \quad j_\mu^X(x) = \cos\alpha J_\mu^u(x) + \sin\alpha J_\mu^d(x)$$

$$j_\mu^\psi = \bar{c}_a \gamma_\mu c_a$$

$$j_\nu^\gamma = \frac{2}{3} \bar{u} \gamma_\nu u - \frac{1}{3} \bar{d} \gamma_\nu d + \frac{2}{3} \bar{c} \gamma_\nu c$$

QCDSR: OPE SIDE

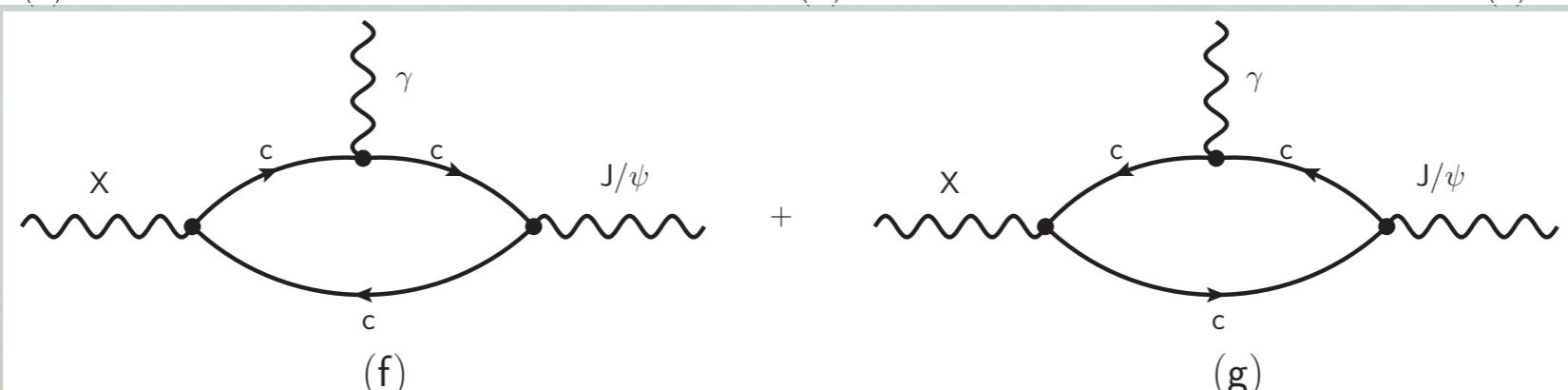
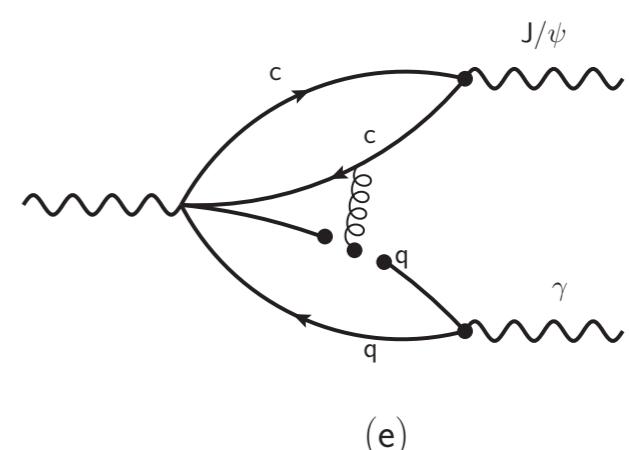
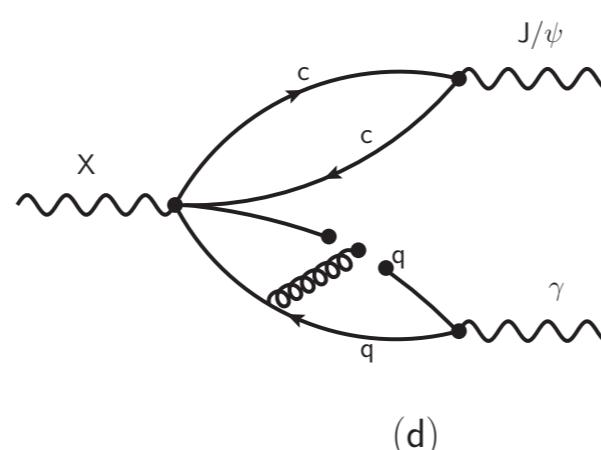
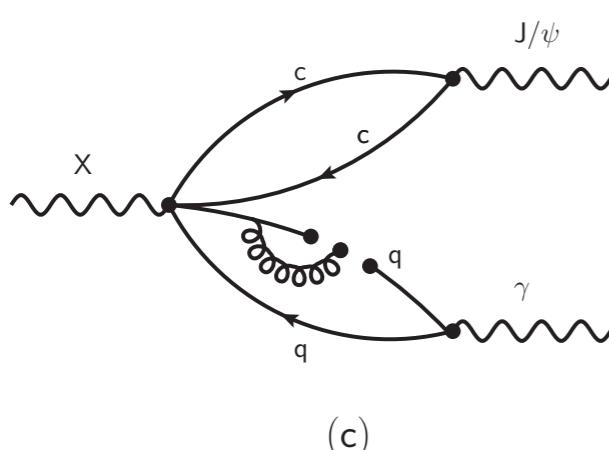
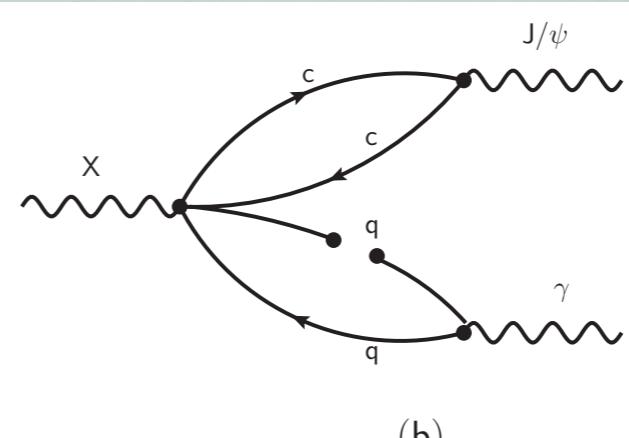
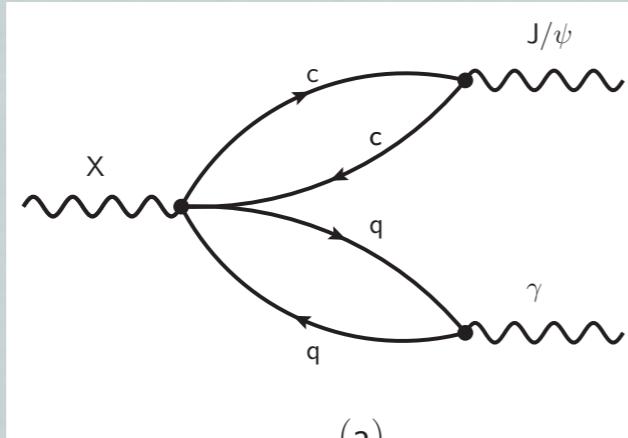
$$\begin{aligned}\Pi_{\mu\nu\alpha}(x,y) &= \frac{e \sin \theta}{3} (2 \cos \alpha - \sin \alpha) \Pi_{\mu\nu\alpha}^{mol}(x,y) \\ &+ \frac{e}{6\sqrt{2}} \cos \theta (\cos + \sin) \Pi_{\mu\nu\alpha}^{c\bar{c}}(x,y).\end{aligned}$$

$$\Pi_{\mu\nu\alpha}^{c\bar{c}}(x,y) = \langle 0 | T[j_\mu^\psi(x) j_\nu^\gamma(y) {j_\alpha^{'(2)}}^\dagger(0)] | 0 \rangle$$

$$\Pi_{\mu\nu\alpha}^{mol}(x,y) = \langle 0 | T[j_\mu^\psi(x) j_\nu^\gamma(y) {j_\alpha^{(4q)}}^\dagger(0)] | 0 \rangle$$

OPE SIDE CONTRIBUTION @ LO IN α_s AND DIM-5

Molecule
contribution
(a) - (e)



Charmonium
contribution
(f) - (g)

PHENOMENOLOGICAL SIDE

$$\Pi_{\mu\nu\alpha}^{\text{phen}}(p, p', q) = \langle 0 | j_\mu^\psi(p') | \psi(p') \rangle \frac{i}{p'^2 - m_\psi^2} \langle \psi(p') | j_\nu^\gamma(q) | X(p) \rangle \frac{i}{p^2 - m_X^2} \langle X(p) | j_X^\dagger(p) | 0 \rangle$$

$$\langle 0 | j_\mu^\psi | \psi(p') \rangle = m_\psi f_\psi \epsilon_\mu(p')$$

$$\langle X(p) | j_\alpha^X | 0 \rangle = (\cos\alpha + \sin\alpha) \lambda_q \epsilon_\alpha^*(p)$$

$$\langle \psi(p') | j_\nu^\gamma(q) | X(p) \rangle = i \epsilon_\nu^\gamma(q) M(X(p) \rightarrow \gamma(q) J/\psi(p'))$$

$$M(X(p) \rightarrow \gamma(q) J/\psi(p')) = e \epsilon^{\kappa\lambda\rho\sigma} \epsilon_X^\alpha(p) \epsilon_\psi^\mu(p') \epsilon_\gamma^\rho(q) \frac{q_\sigma}{m_X^2} (A g_{\mu\lambda} g_{\alpha\kappa} p \cdot q + B g_{\mu\lambda} p_\kappa q_\alpha + C g_{\alpha\kappa} p_\lambda q_\mu)$$

PHENOMENOLOGICAL SIDE SIX DIFFERENT DIRAC STRUCTURES

$$\begin{aligned}
 \Pi_{\mu\nu\alpha}^{\text{phen}}(p, p', q) &= \frac{ie(\cos\alpha + \sin\alpha)\lambda_q m_\psi f_\psi}{m_X^2(p^2 - m_X^2)(p'^2 - m_\psi^2)} \\
 &\times \left(\epsilon^{\alpha\mu\nu\sigma} q_\sigma A + \epsilon^{\mu\nu\lambda\sigma} p'_\lambda q_\sigma q_\alpha B - \epsilon^{\alpha\nu\lambda\sigma} q_\mu q_\sigma p'_\lambda C \right. \\
 &+ \epsilon^{\alpha\nu\lambda\sigma} p'_\lambda p'_\mu q_\sigma (C - A) \frac{p \cdot q}{m_\psi^2} \\
 &\left. - \epsilon^{\mu\nu\lambda\sigma} p'_\lambda q_\sigma (q_\alpha + p'_\alpha) (A + B) \frac{p \cdot q}{m_X^2} \right)
 \end{aligned}$$

$P^2 = P'^2$

After Borel Transform: 1 Sum Rule for each structure

$$G_i(Q^2) \left(e^{-m_\psi^2/M^2} - e^{-m_X^2/M^2} \right) + H_i(Q^2) e^{-s_0/M^2} = \bar{\Pi}_i^{(OPE)}(M^2, Q^2)$$

FOR STRUCTURE 1: $\epsilon^{\alpha\mu\nu\sigma} q_\sigma$.

$$\bar{\Pi}_1^{\text{OPE}}(M^2, Q^2) = -\langle q\bar{q} \rangle \left[\frac{\sin\theta(2\cos\alpha - \sin\alpha)}{3Q^4} \bar{\Pi}_1^{4q}(M^2, Q^2) + \frac{\cos\theta}{2Q^2} (\cos\alpha + \sin\alpha) \bar{\Pi}_1^{\bar{c}c}(M^2, Q^2) \right]$$

$$\bar{\Pi}_1^{\text{mol}}(M^2, Q^2) = \left(1 - \frac{m_0^2}{3Q^2}\right) \int_{4m_c^2}^{u_0} du e^{-u/M^2} u \sqrt{1 - \frac{4m_c^2}{u}} \left(\frac{1}{2} + \frac{m_c^2}{u}\right) + \frac{m_c^2 m_0^2}{16} \int_0^1 d\alpha \frac{1+3\alpha}{\alpha^2(1-\alpha)} e^{-\frac{m_c^2}{\alpha(1-\alpha)M^2}}$$

$$\bar{\Pi}_1^{\bar{c}c}(M^2, Q^2) = - \int_{4m_c^2}^{s_0} ds \int_{u_-}^{u_+} du e^{-\frac{u+s}{M^2}} \frac{2}{\sqrt{\lambda(s,t,u)}} \left(m_c^2 + \frac{tu(t-u)}{\lambda(s,t,u)} \right)$$

RHS

LHS

Form Factor (Photon is off-shell in the vertex):

$$G_1(Q^2) = \frac{3\sqrt{2}\pi^2(\cos\alpha + \sin\alpha)\lambda_q m_\Psi f_\Psi}{m_X^2(m_X^2 - m_\Psi^2)} A(Q^2)$$

FOR STRUCTURE 2: $\epsilon^{\mu\nu\sigma\lambda} p'_\sigma p'_\alpha q_\lambda$.

(only molecule contribution)

RHS

$$\bar{\Pi}_2^{\text{OPE}}(M^2, Q^2) = \frac{m_0^2 \langle q\bar{q} \rangle}{Q^4} \int_0^1 d\alpha \frac{1-\alpha}{\alpha} e^{-\frac{m_c^2}{\alpha(1-\alpha)M^2}}$$

LHS

$$G_2(Q^2) = \frac{3^2 2^4 \sqrt{2} \pi^2 (\cos \alpha + \sin \alpha) \lambda_q m_\Psi f_\Psi}{\sin \theta (2 \cos \alpha - \sin \alpha) m_X^4 (m_X^2 - m_\Psi^2)} (A(Q^2) + B(Q^2))$$

FOR STRUCTURE 3: $\epsilon^{\alpha\nu\lambda\sigma} p'_\lambda q_\sigma q_\mu$.

(only charmonium contribution)

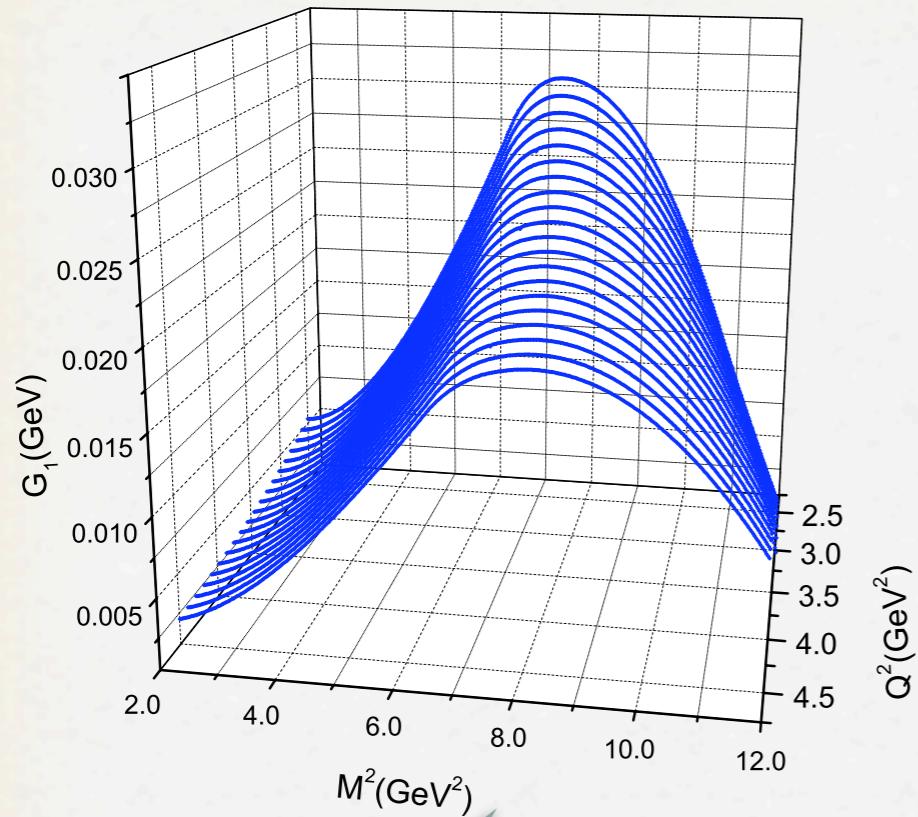
$$\bar{\Pi}_3^{\text{OPE}}(M^2, Q^2) = \langle q\bar{q} \rangle \int_{4m_c^2}^{s_0} ds \int_{u_-}^{u_+} du e^{-\frac{u+s}{M^2}} \frac{2}{\lambda(s,t,u)^{3/2}} \left[tu + m_c^2(-s+t+u) + \frac{3 s t u (-s+t+u)}{\lambda(s,t,u)} \right]$$

RHS

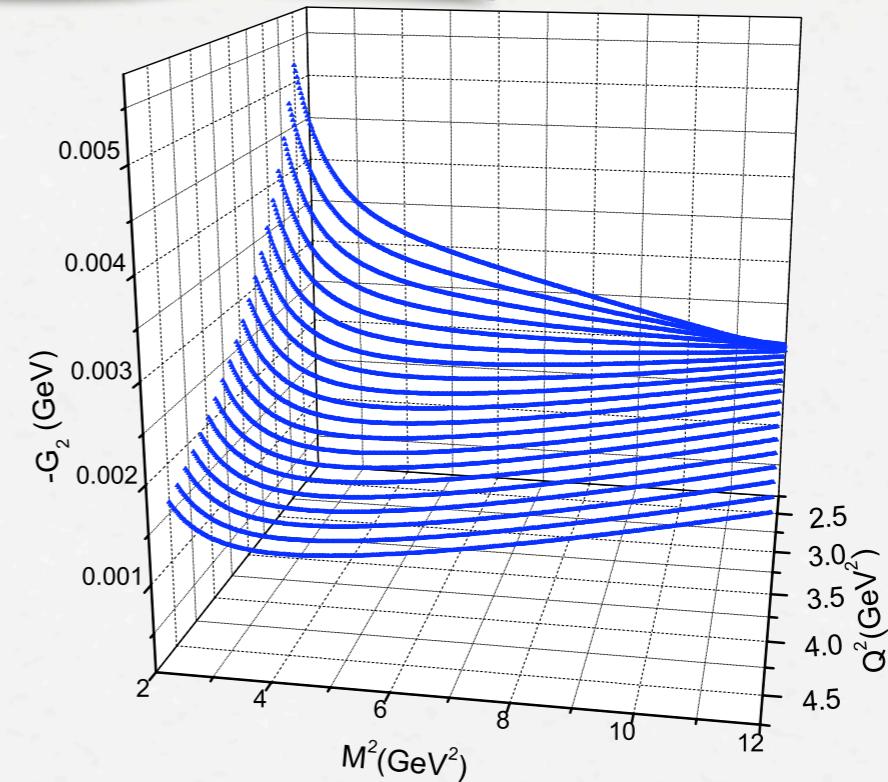
LHS

$$G_3(Q^2) = \frac{6\sqrt{2}\pi^2 \lambda_q m_\psi f_\psi}{\cos\theta m_X^2 (m_X^2 - m_\psi^2)} C(Q^2)$$

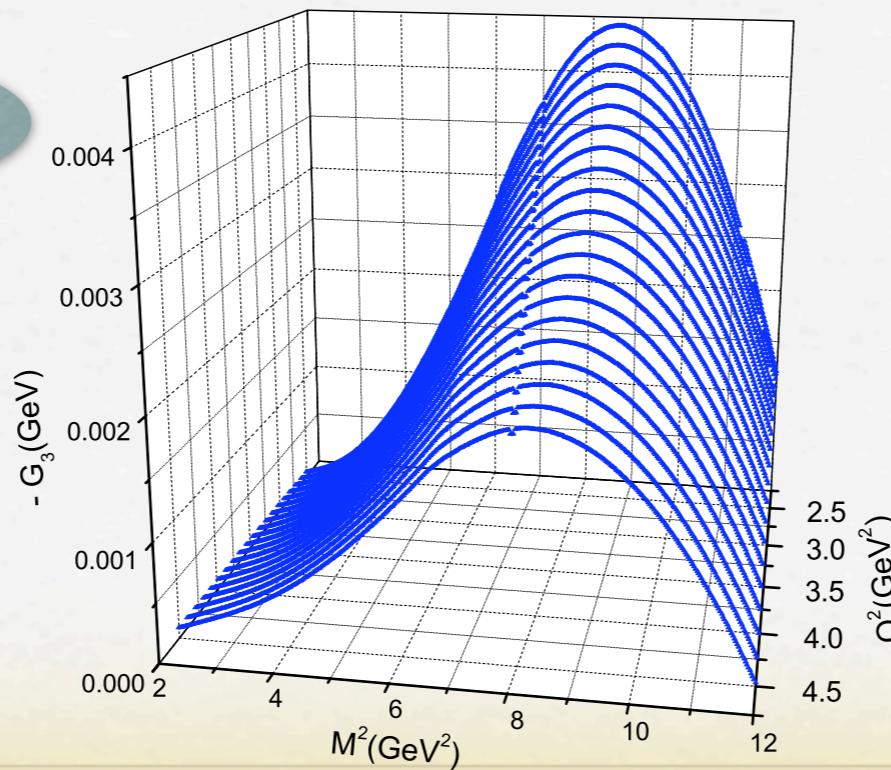
$7.0 \text{ GeV}^2 \leq M^2 \leq 8.5 \text{ GeV}^2$



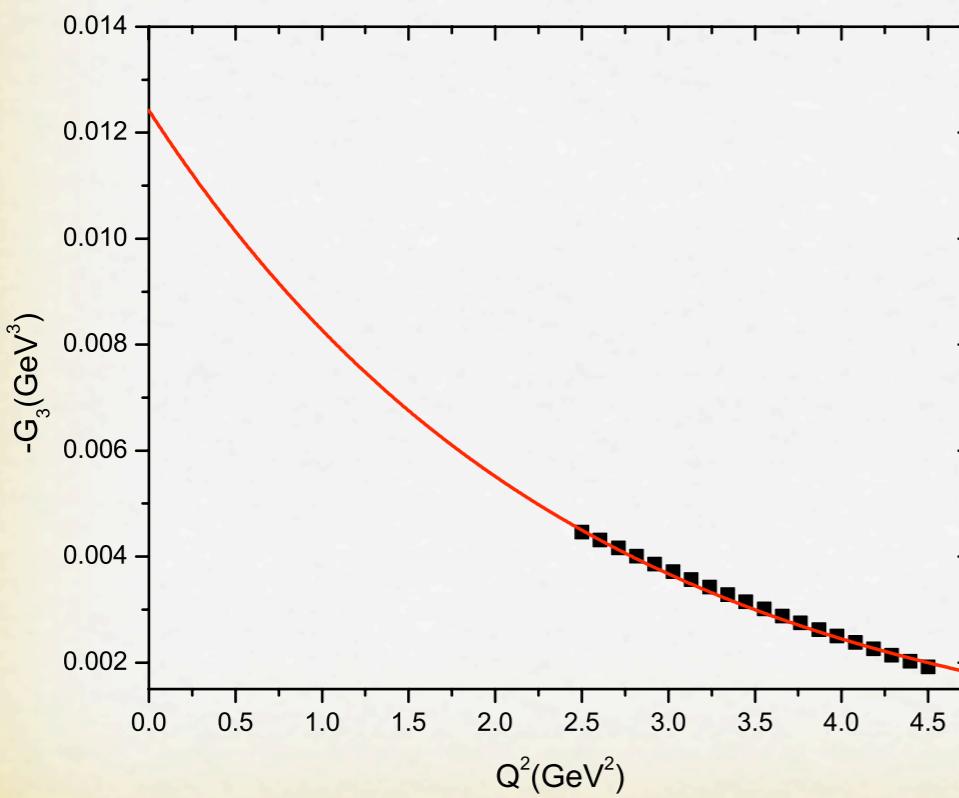
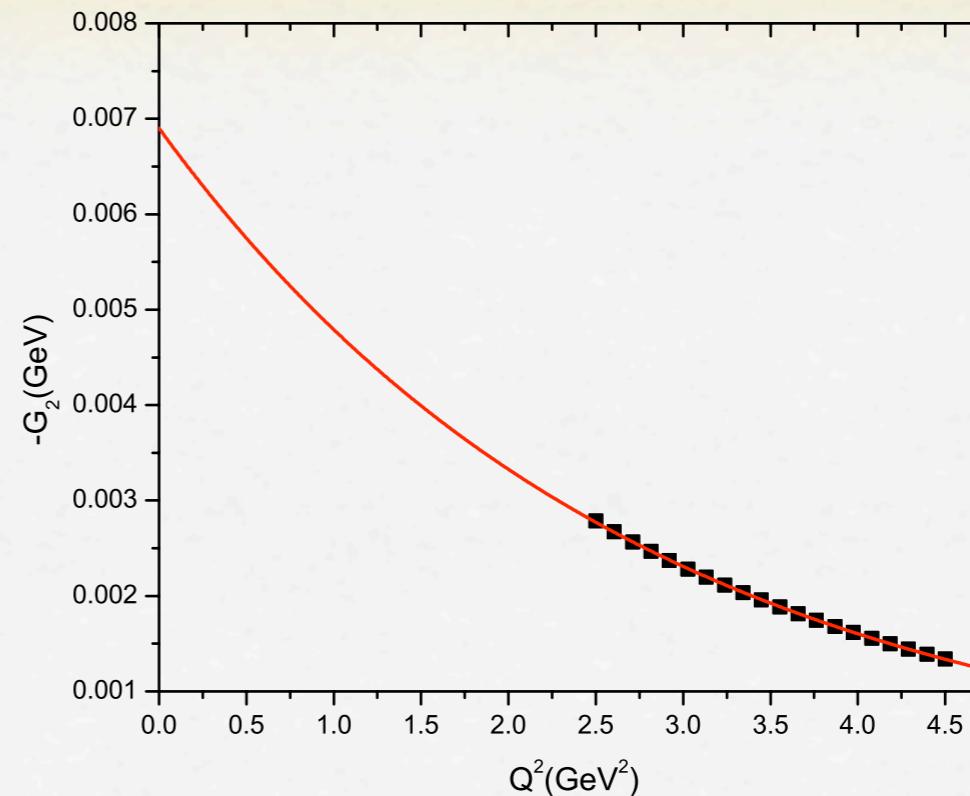
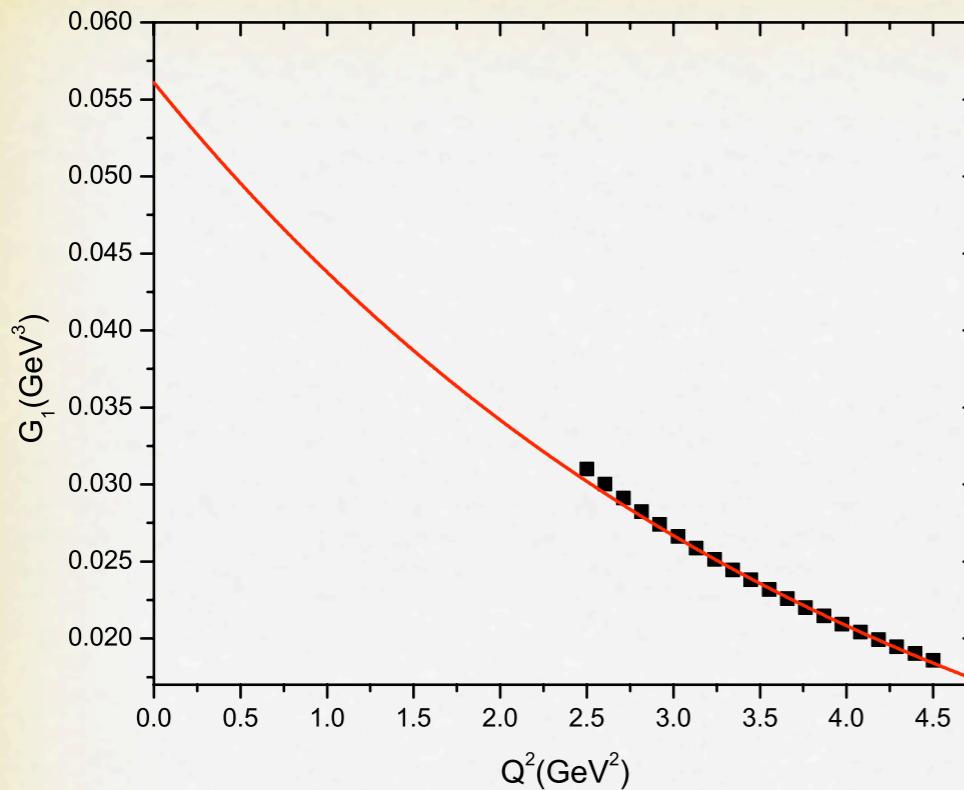
$7.0 \text{ GeV}^2 \leq M^2 \leq 8.5 \text{ GeV}^2$



$7.0 \text{ GeV}^2 \leq M^2 \leq 8.5 \text{ GeV}^2$



$2.5 \text{ GeV}^2 \leq Q^2 \leq 4.5 \text{ GeV}^2$



$$G_i(Q^2) = g_1 e^{-g_2 Q^2}$$

	G₁	G₂	G₃
g₁	0.056 GeV ³	-0.0069 GeV	-0.013 GeV ³
g₂	0.25 GeV ⁻²	0.365 GeV ⁻²	0.41 GeV ⁻²

$G(Q^2) \rightarrow$ FORM FACTORS $A(Q^2)$, $B(Q^2)$, $C(Q^2)$

Coupling Constant \rightarrow Form Factors at the
Photon Pole $Q^2=0$:

$$A = A(Q^2 = 0) = 18.65 \pm 0.94;$$

$$A+B = (A+B)(Q^2 = 0) = -0.24 \pm 0.11;$$

$$C = C(Q^2 = 0) = -0.843 \pm 0.008.$$

$$5^\circ \leq \theta \leq 13^\circ; \quad \alpha = 20^\circ$$

THE DECAY WIDTH

$$\begin{aligned}\Gamma(X \rightarrow J/\psi \gamma) &= \frac{\alpha}{3} \frac{p^{*5}}{m_X^4} \left((A+B)^2 + \frac{m_X^2}{m_\psi^2} (A+C)^2 \right), \\ p^* &= (m_X^2 - m_\psi^2)/(2m_X)\end{aligned}$$

$$\frac{\Gamma(X \rightarrow J/\psi \gamma)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 0.19 \pm 0.13$$

$$\frac{\Gamma(X \rightarrow J/\psi \gamma)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)}_{Exp.} = 0.14 \pm 0.05$$

CONCLUSION

- ✿ QCD Sum Rules calculations for the radiative decay of the X(3872) as a mixed state
$$(c\bar{c}) + (D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0) + (D^{*+}\bar{D}^- - \bar{D}^{*-}D^+)$$
- ✿ Mass and decays tested are compatible with mixing angles
$$5^\circ \leq \theta \leq 13^\circ; \quad \alpha = 20^\circ$$
- ✿ Next... $B^+ \rightarrow X(3872)K^+$