

Phenomenological aspects of the eikonal zero in the momentum transfer space

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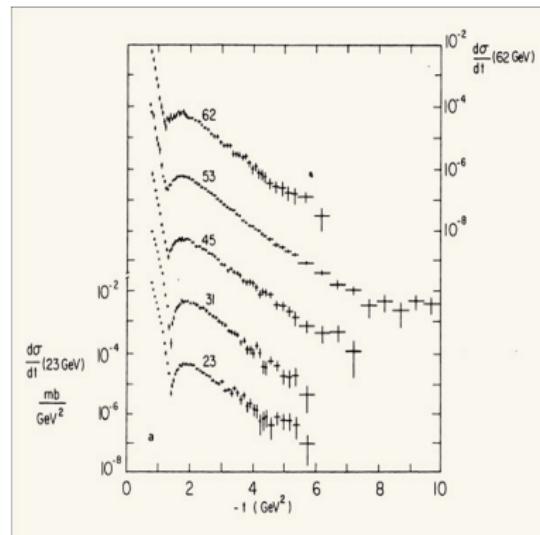
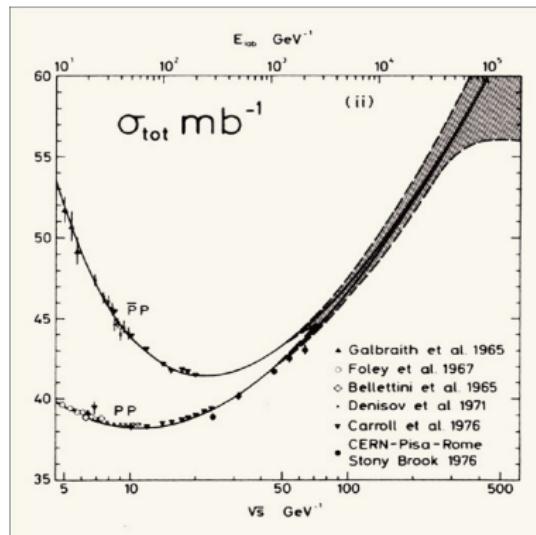
XXII Reunião de Trabalho sobre Interações Hadrônicas

Contents

1. Introduction
2. Eikonal and Impact Parameter Representation
3. Inverse Problem in pp Elastic Scattering
4. Opacity Function in Momentum Transfer Space
5. Consistency Tests - Differential Cross Section
6. Conclusions and Final Remarks

Introduction

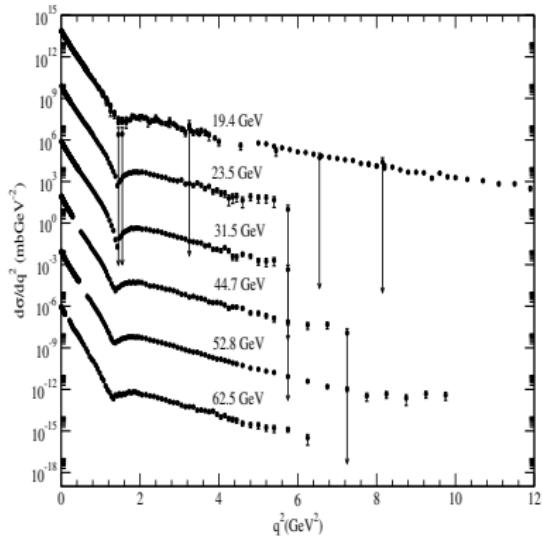
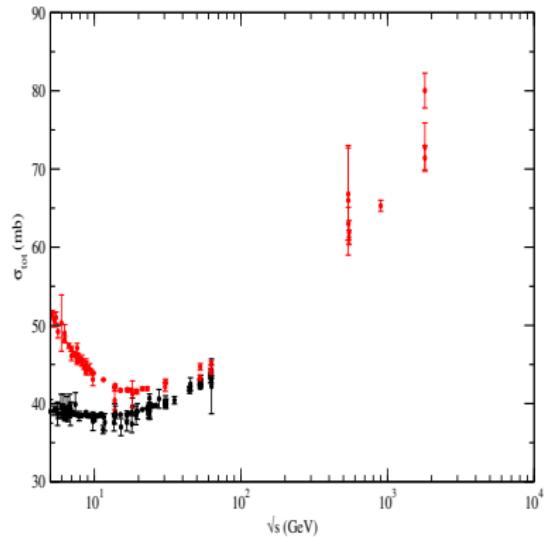
pp Collisions in ISR: how things looked like 40 years ago¹...



¹font: <http://cerncourier.com/cws/article/cern/44857>

Introduction

pp Collisions: how things look like nowadays...mostly the same.



However, a lot has been learned up to now and much is about to!

Introduction

Importance to look at elastic scattering in the LHC

1. Experimental: TOTEM Collaboration - pp elastic scattering over wide $|t|$ range (**up to 10 GeV^2**) + highest energies so far → good test for models.
2. Theoretical: soft processes → large distance interactions + absence of full QCD description + wide variety of models with *essentially different* physical scenarios → opportunity for improvements in the theory.

Eikonal and Impact Parameter Representation

Scattering Amplitude - Complex Function

$$A(s, q) = A_R(s, q) + iA_I(s, q)$$

$s, q^2 = -t$: Mandelstam Variables

Physical Quantities

1. Differential Cross Section

$$\frac{d\sigma}{dq^2}(s, q) = \pi |A(s, q)|^2$$

2. Total Cross Section (Optical Theorem)

$$\sigma_{tot}(s) = 4\pi A_I(s, q=0)$$

Eikonal and Impact Parameter Representation

3. ρ parameter (related to the phase of the forward amplitude)

$$\rho(s) = \frac{A_R(s, q=0)}{A_I(s, q=0)}$$

Eikonal Representation

$$A(s, q) = i \int_0^\infty b db J_0(qb) \Gamma(s, b),$$

where $\Gamma(s, b) = 1 - e^{i\chi(s, b)}$ is the *profile function*.

$$\chi(s, b) = \chi_R(s, b) + i\chi_I(s, b)$$

Eikonal and Impact Parameter Representation

Unitarity in the b-space demands:

$$2\Gamma_R(s, b) = |\Gamma(s, b)|^2 + G_{in}(s, b)$$



$$G_{in}(s, b) = 1 - e^{-2\chi_I(s, b)}$$



$\Omega(s, b) \equiv \chi_I(s, b)$ - opacity function



$\Omega(s, b) \geq 0 \Rightarrow G_{in}(s, b) \leq 1$ - probability for inelastic event

Inverse Problem in pp Elastic Scattering

Main ideas

Parametrization for $A(s, q)$ as sum of exponentials in q^2



Fits to experimental data ($\frac{d\sigma}{dq^2}, \sigma_{tot}, \rho$) with best $\chi^2/\text{DOF} (\approx 1.0)$



Extraction: $\Gamma(s, b), \Omega(s, b) \rightarrow \tilde{\Omega}(s, q)$



Not that simple, though...

Inverse Problem in pp Elastic Scattering

Main problems

1. $d\sigma/dq^2$ data covers finite range in q^2 while integration requires: $q^2 \rightarrow \infty$ and $b \rightarrow \infty$
2. contributions from $A_R(s, q)$ and $A_I(s, q)$ known **only** in forward direction ($q^2 = 0$) $\rightarrow \rho(s)$
3. only formal result concerning $A(s, q) \rightarrow$ real part of even amplitude has a change of sign below $\approx 1 \text{ GeV}^2$

Inverse Problem in pp Elastic Scattering

Main Strategies

1. only $d\sigma/dq^2$ at the largest intervals in $q^2 \Rightarrow$ pp elastic scattering (CERN-SPS + Fermilab + ISR data)
2. independence with s of data at ISR region ($23.5 - 62.5 \text{ GeV}^2$) at $q^2 > 3.5 \text{ GeV}^2 \Rightarrow$ addition of 27.4 GeV^2 data
3. unknown contributions from $A_R(s, q)$ and $A_I(s, q) \Rightarrow$ two parametrizations analyzed

Inverse Problem in pp Elastic Scattering

Parametrizations

- i. Variant I - constrained (A_R to A_I) parametrization²

$$\begin{aligned} A(s, q) &= \left\{ \left[\frac{\rho \sigma_{tot}}{4\pi \sum_{i=1}^m a_i} \right] \sum_{i=1}^m a_i e^{-b_i q^2} \right\} \\ &+ i \left\{ \sum_{j=1}^n a_j e^{-b_j q^2} \right\}, \quad \sum_{j=1}^n a_j = \frac{\sigma_{tot}}{4\pi}. \end{aligned}$$

²R. F. Ávila, M. J. Menon, EPJC **54**, 555 (2008)

Inverse Problem in pp Elastic Scattering

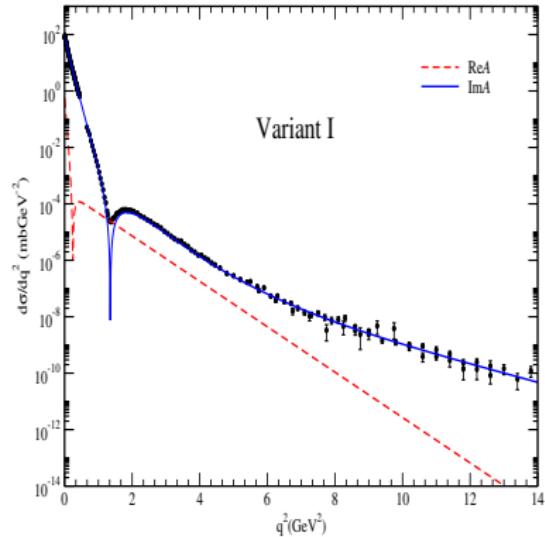
ii. Variant II - representation for the Martin's formula³

$$A(s, q) = \left\{ \left[\frac{\rho \sigma_{tot}}{4\pi \sum_{i=1}^n a_i} \right] \frac{d}{dq^2} \left[q^2 \sum_{i=1}^n a_i e^{-b_i q^2} \right] \right\} \\ + i \left\{ \left[\frac{\sigma_{tot}}{4\pi \sum_{i=1}^n a_i} \right] \sum_{i=1}^n a_i e^{-b_i q^2} \right\}.$$

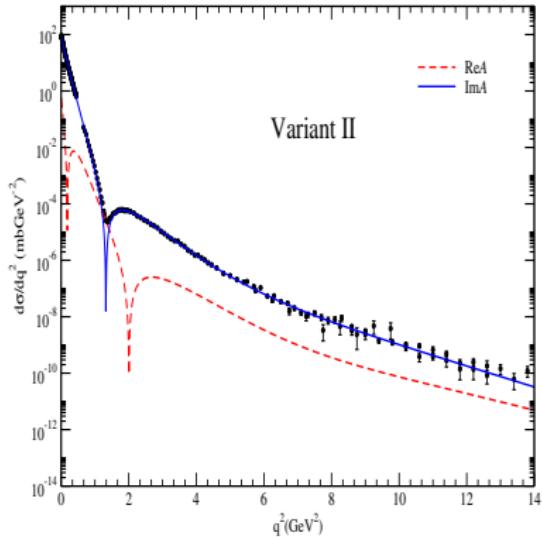
³D. A. Fagundes, M. J. Menon, AIP Conf. Proc. **1296**, p. 282-285 (2010) ↗ ↘ ↙ ↛

Inverse Problem in pp Elastic Scattering

Fit Results - typical pp 52.8 GeV² ($\chi^2/\text{DOF} = 1.7$)



Variant I



Variant II

Different contributions from $A_R(s, q)$ and $A_I(s, q)!!$

Extracted Opacity Function in q^2 -Space

Opacity in b-Space

$$\Omega(s, b) = \ln \left\{ \frac{1}{\sqrt{[1 - \operatorname{Re}\Gamma(s, b)]^2 + [\operatorname{Im}\Gamma(s, b)]^2}} \right\}$$

But, from variants I and II...

$$r(s, b) \equiv \left[\frac{\operatorname{Im}\Gamma(s, b)}{1 - \operatorname{Re}\Gamma(s, b)} \right]^2 \ll 1$$

Extracted Opacity Function in q^2 -Space

Therefore,

$$\Omega(s, b) \approx \ln \left\{ \frac{1}{1 - \text{Re}\Gamma} \right\}$$



$$\Omega(s, b) = \text{Re}\Gamma(s, b) + R(s, b)$$

$R(s, b)$ is the “residual” function:

$$R(s, b) = \left[\ln \left\{ \frac{1}{1 - \text{Re}\Gamma} \right\} - \text{Re}\Gamma(s, b) \right] \rightarrow R_i(s, b) \pm \Delta R_i(s, b)$$

Extracted Opacity Function in q^2 -Space

Opacity in q-Space

Points $R_i(s, b) \pm \Delta R_i(s, b)$ (~ 1000) parametrized

$$R(s, b) = \sum_{j=1}^m A_j e^{-B_j b^2}$$



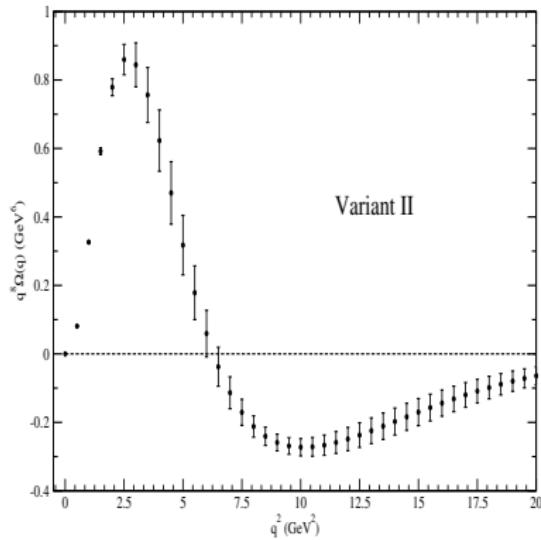
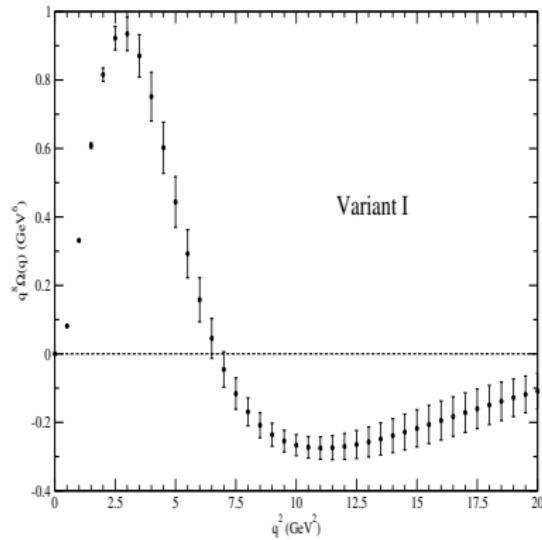
$$\tilde{\Omega}(s, q) = \text{Im}A(s, q) + \tilde{R}(s, q)$$



$$\tilde{\Omega}_i(s, q) \pm \Delta \tilde{\Omega}_i(s, q)$$

Extracted Opacity Function in q^2 -Space

Empirical points - typical pp 52.8 GeV 2 - $q^8 \tilde{\Omega}(s, q) \times q^2$



Change of sign around 6.3 GeV 2 !!

Extracted Opacity Function in q^2 -Space

Analytical Parametrizations

- General structure (multiple diffraction formalism)

$$\tilde{\Omega}(q) = CG^2(q)f(q)$$

with

$$G(q) = \frac{1}{(1 + q^2/\alpha^2)} \frac{1}{(1 + q^2/\beta^2)}$$

$$f(q) = \frac{1 - q^2/q_0^2}{1 + (q^2/q_0^2)^n}$$

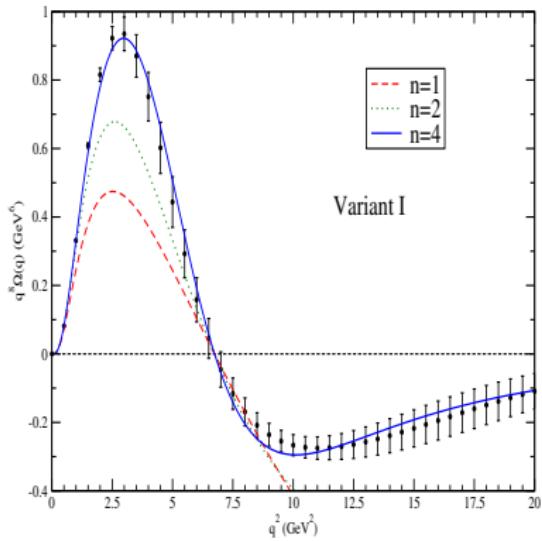
C, α, β - fit parameters n - fixed

Extracted Opacity Function in q^2 -Space

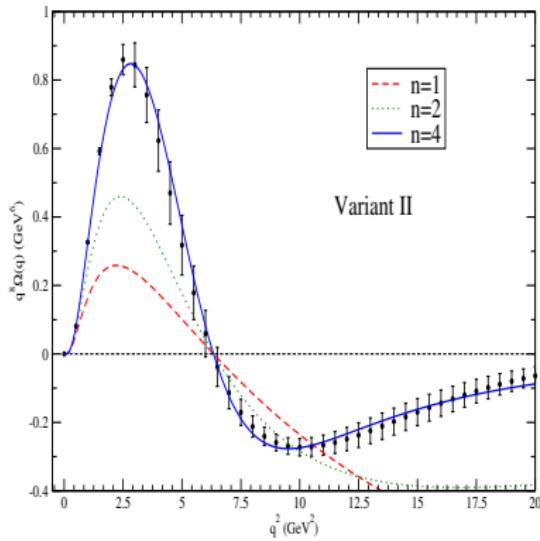
- Models - corresponding opacities
 - i. QCD inspired models $\Rightarrow f = 1$ and $\alpha^2 = \beta^2$ fit parameter (dipole)
 - ii. Bourrely-Soffer-Wu model $\Rightarrow f(q)$ with $n = 1$, and α^2, β^2 fit parameters (unconstrained product of two simple poles)
 - iii. Multiple diffraction and hybrid models $\Rightarrow f(q)$ with $n = 2$ and α^2, β^2 fit parameters (unconstrained product of two simple poles)

Extracted Opacity Function in q^2 -Space

Fits results - typical pp 52.8 GeV 2 - $q^8 \tilde{\Omega}(s, q) \times q^2$



Variant I



Variant II

Consistency Tests - First Order

A consistency test: check of $d\sigma/dq^2$

In *first order*:

$$A_I(s, q) = \tilde{\Omega}(s, q)$$



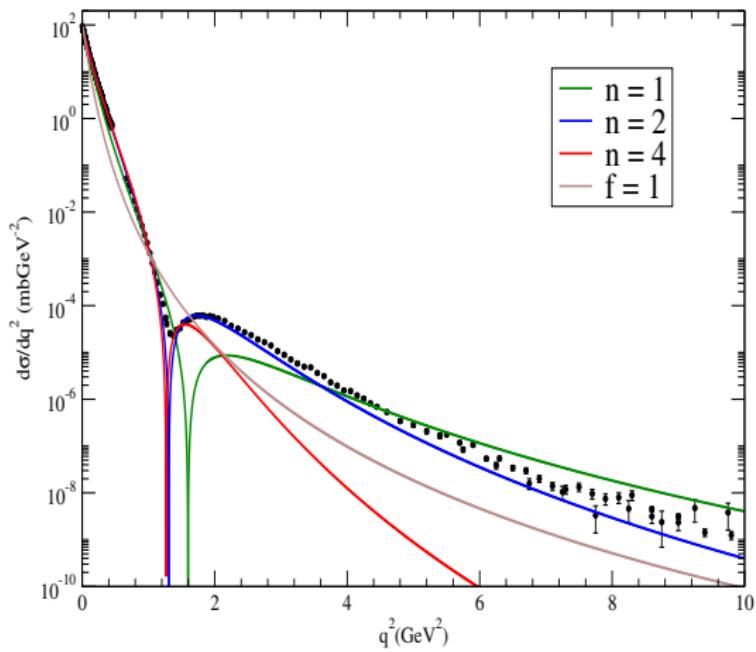
$$\frac{d\sigma}{dq^2}(s) \approx \pi |\tilde{\Omega}(s, q)|^2$$



for $n = 1, 2, 4$ and $f=1$ (QCD inspired)

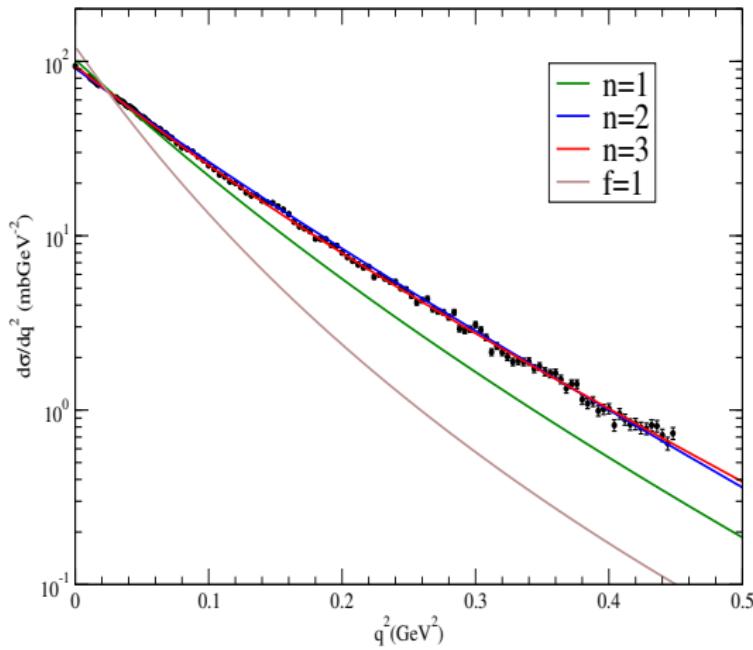
Consistency Tests - First Order

Fit Results - typical pp 52.8 GeV²



Consistency Tests - First Order

Fit Results - typical pp 52.8 GeV² - diffraction peak



Conclusions and Final Remarks

From this analyses

1. model-independent analyses and fits to experimental data
2. extraction of empirical opacity in momentum transfer space with zero (change of sign)
3. comparison with model assumptions
4. good reproduction of the differential cross section for opacities with zero, even in first order
5. QCD-inspired models inconsistent with empirical opacity and with first order approach

Conclusions and Final Remarks

Work in progress

- study of the influence of higher orders of the opacity to the amplitude
- application of empirical results to a QCD-inspired model using a non-perturbative approach (dynamical gluon mass) - *in collaboration with A. A. Natale and E. G. S. Luna*

Acknowledgments

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