# Determining the photon polarization of the $b \rightarrow s \gamma$ using the $B \rightarrow K_{1}(1270) \gamma \rightarrow(K \pi \pi) \gamma$ decay 

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## Outline

(1) Introduction: the $\mathbf{b} \rightarrow \mathbf{s} \gamma$ process and the photon polarization
(2) How to determine the polarization

- $B \rightarrow K_{1}(\rightarrow K \pi \pi) \gamma$ : new method
(3) Right-handed currents constraints

4 Conclusions and perspectives

## Why are we interested in measuring the photon polarization of $b \rightarrow s \gamma$ ?



$$
\mathcal{M}(b \rightarrow s \gamma)=\frac{4 G_{F}}{\sqrt{2}} V_{t s} V_{t b}^{*} F_{2} \frac{e}{(4 \pi)^{2}} \bar{s} \sigma_{\mu \nu} q^{\nu}(\underbrace{m_{b} \frac{1+\gamma_{5}}{2}}_{b_{R} \rightarrow s_{L} \gamma_{L}}+\underbrace{m_{s} \frac{1-\gamma_{5}}{2}}_{b_{L} \rightarrow s_{\boldsymbol{R}} \gamma_{R}}) \boldsymbol{b} \varepsilon^{\mu *}
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- In the SM since $m_{s} / m_{b} \ll 1$, photons are predominantly left (right)-handed in the $\bar{B}(B)$-decays.
- NP can induce new Dirac structures and lead to an excess of right(left)-handed photons, without contradicting with the measured $\mathcal{B}\left(B \rightarrow X_{s} \gamma\right)$.

The measurement of the photon polarization could provide a test of physics beyond the SM, namely right-handed currents.

## Right-handed currents in NP beyond the SM

How much right-handed currents is allowed?
At present, Br measurements of the inclusive and exclusive $b \rightarrow s \gamma$ processes do not put very strong constraints on the right-handed currents yet since it is not a direct measurement $\left(\operatorname{Br} \propto\left|\mathcal{M}_{L}^{S M}+\mathcal{M}_{L}^{N P}\right|^{2}+\left|\mathcal{M}_{R}^{S M}+\mathcal{M}_{R}^{N P}\right|^{2}\right)$.
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Some NP models predict significant contribution of the right-handed currents

- For example, in SUSY the squark mass can come from any combination of left and right couplings:

$$
\mathcal{L}_{\text {soft }}^{M S S M}=\tilde{Q}_{L}^{\dagger} m_{Q}^{2} \tilde{Q}_{L}+\tilde{u}_{R}^{\dagger} m_{\bar{u}}^{2} \tilde{\bar{u}}_{R}+\tilde{\bar{d}}_{R}^{\dagger} m_{\bar{d}}^{2} \tilde{\bar{d}}_{R}+v_{u} \tilde{\bar{u}}_{R} a_{u} \tilde{Q}_{L}+v_{d} \tilde{\bar{d}}_{R} a_{d} \tilde{Q}_{L}+\ldots
$$

- The soft SUSY breaking terms induce the chirality flip on the internal lines $\Rightarrow$ this leads to the enhancement factor $m_{\tilde{g}} / m_{b}$ compared to the SM.


$$
\begin{array}{r}
\mathcal{M}_{\tilde{\mathrm{g}}}^{\operatorname{SUSY}}\left(b_{L} \rightarrow s_{R} \gamma_{R}\right) \propto \\
m_{\tilde{g}} \times\left(\delta_{R L}^{d}\right)_{23} \times \text { loop }
\end{array}
$$

(1) Method 1: $C P$ asymmetry in $B_{q}(t) \rightarrow f^{C P} \gamma$

$$
S_{f \gamma}=-\xi_{f} \frac{2\left|\mathcal{M}_{L} \mathcal{M}_{R}\right|}{\left|\mathcal{M}_{L}\right|^{2}+\left|\mathcal{M}_{R}\right|^{2}} \sin \left(\phi_{M}-\phi_{L}-\phi_{R}\right)
$$

(2) Method 2: transverse asymmetry in $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \ell^{+} \ell^{-}$

$$
\mathcal{A}_{T}^{(2)}=-\frac{\mathcal{M}_{R} \mathcal{M}_{L}^{*}+\mathcal{M}_{R}^{*} \mathcal{M}_{L}}{\left|\mathcal{M}_{R}\right|^{2}+\left|\mathcal{M}_{L}\right|^{2}}
$$

(3) Method 3: $K_{1}$ three-body decay method in $B \rightarrow K_{1} \gamma$

$$
\lambda_{\gamma}=\frac{\left|\mathcal{M}_{R}\right|^{2}-\left|\mathcal{M}_{L}\right|^{2}}{\left|\mathcal{M}_{R}\right|^{2}+\left|\mathcal{M}_{L}\right|^{2}}
$$

## 1. CP-asymmery in $B^{0}$ <br> $\rightarrow K^{* 0}\left(\rightarrow K_{S \pi^{0}}\right)$



- Time-dependent CP-asymmetry in neutral $B$-mesons results from the interference of mixing and decay [Atwood et al.,Phys.Rev.Lett. 79 ('97)].

$$
\begin{aligned}
& \mathcal{A}_{C P}(t) \equiv \frac{\Gamma\left(\bar{B}_{q}(t) \rightarrow f \gamma\right)-\Gamma\left(B_{q}(t) \rightarrow f \gamma\right)}{\Gamma\left(\bar{B}_{q}(t) \rightarrow f \gamma\right)+\Gamma\left(B_{q}(t) \rightarrow f \gamma\right)} \simeq S_{f \gamma} \sin (\Delta m t) \\
& S_{f \gamma}=-\xi_{f} \frac{2\left|\mathcal{M}_{L} \mathcal{M}_{R}\right|}{\left|\mathcal{M}_{L}\right|^{2}+\left|\mathcal{M}_{R}\right|^{2}} \sin \left(\phi_{M}-\phi_{L}-\phi_{R}\right)
\end{aligned}
$$

where $\phi_{L, R}=\arg \left(\mathcal{M}_{L, R}\right)$ and $\phi_{M}$ is the $B_{q}-\bar{B}_{q}$ mixing phase.

- In the SM

$$
\begin{array}{ll}
b \rightarrow s \gamma & \left|\mathcal{M}_{R} / \mathcal{M}_{L}\right| \simeq m_{s} / m_{b}, \\
B^{0}-\bar{B}^{0} & \phi_{L}=\phi_{R} \simeq 0 \\
B_{s}-\bar{B}_{s} & \phi_{M}=2 \beta \simeq 43^{\circ}
\end{array} \Rightarrow\left\{\begin{array}{l}
S_{K_{\boldsymbol{s}} \pi^{0} \gamma} \simeq-\left(2 m_{s} / m_{b}\right) \sin (2 \beta) \\
\left(S_{K_{\boldsymbol{s} \pi^{0} \gamma}}=-0.15 \pm 0.2\right) \\
S_{\phi \gamma} \simeq 0
\end{array}\right.
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\lambda_{\gamma}=\frac{\left|\mathcal{M}_{R}\right|^{2}-\left|\mathcal{M}_{L}\right|^{2}}{\left|\mathcal{M}_{R}\right|^{2}+\left|\mathcal{M}_{L}\right|^{2}}
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2. Transverse asymmetry in $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \ell^{+} \ell^{-}$

Analysis of the angular distributions in $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \ell^{+} \ell^{-}$in the low $\ell^{+} \ell^{-}$inv.mass region and measurement of the transverse asymmetry [Kruger\&Matias, Phys.Rev.D71('05)].

$$
\frac{d^{2} \Gamma}{d q^{2} d \phi}=\frac{1}{2 \pi} \frac{d \Gamma}{d q^{2}}\left[1+F_{T}\left(q^{2}\right)\left(A_{T}^{(2)}\left(q^{2}\right) \cos 2 \phi+A_{T}^{(\mathrm{im})}\left(q^{2}\right) \sin 2 \phi\right)\right]
$$

In the heavy quark and large $E_{K^{*}} \operatorname{limit}\left(\Leftrightarrow q^{2} \rightarrow 0\right)$

$$
\mathcal{A}_{T}^{(2)}=-\frac{\mathcal{M}_{R} \mathcal{M}_{L}^{*}+\mathcal{M}_{R}^{*} \mathcal{M}_{L}}{\left|\mathcal{M}_{R}\right|^{2}+\left|\mathcal{M}_{L}\right|^{2}}
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In [Phys.Rev.Lett.88,Phys.Rev.D66('02)] Gronau et al. proposed that the angular distribution of the three-body decay of $K_{\text {res }}$ in $B \rightarrow K_{\text {res }} \gamma$ decay carries the photon polarization information.

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- There are two known $K_{1}\left(1^{+}\right)$states, decaying into $K \pi \pi$ final state via $K^{*} \pi$ and $\rho K$ modes: $K_{1}(1270)$ and $K_{1}(1400)$.
- One of the decay channels $B \rightarrow K_{1} \gamma$, namely $B^{+} \rightarrow K_{1}^{+}(1270) \gamma$, is finally measured $\left(\mathcal{B}=(4.3 \pm 1.2) \times 10^{-5}\right)$, while $B^{+} \rightarrow K_{1}^{+}(1400) \gamma$ is suppressed ( $\mathcal{B}<1.5 \times 10^{-5}$ ) [Belle ('05)].
We investigate the feasibility of determining the photon polarization using the $B \rightarrow K_{1}(1270) \gamma$ channel.


## 3. $K_{1}$ three-body decay method

## Formalism

The decay distribution of $\bar{B} \rightarrow \bar{K}_{1} \gamma \rightarrow(\bar{K} \pi \pi) \gamma$ is given by the master formula:

$$
\frac{d^{3} \Gamma}{d s_{13} d s_{23} d \cos \theta} \propto \frac{1}{4}|\overrightarrow{\mathcal{J}}|^{2}\left(1+\cos ^{2} \theta\right)+\lambda_{\gamma} \frac{1}{2} \operatorname{lm}\left[\vec{n} \cdot\left(\overrightarrow{\mathcal{J}} \times \overrightarrow{\mathcal{J}}^{*}\right)\right] \cos \theta
$$

$\lambda_{\gamma}=\frac{\left|\mathcal{M}_{R}\right|^{2}-\left|\mathcal{M}_{L}\right|^{2}}{\left|\mathcal{M}_{R}\right|^{2}+\left|\mathcal{M}_{L}\right|^{2}}$
$\simeq-1(+1)+O\left(m_{s}^{2} / m_{b}^{2}\right) \quad$ in the SM for $\bar{B}(B)$ respectively.

## $K_{1}$-reference frame

$$
\overrightarrow{\mathcal{J}}=\mathcal{C}_{1}\left(s_{13}, s_{23}\right) \vec{p}_{1}-\mathcal{C}_{2}\left(s_{13}, s_{23}\right) \vec{p}_{2}
$$

$\Leftrightarrow K_{1}$-decay helicity amplitude.


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$\simeq-1(+1)+O\left(m_{s}^{2} / m_{b}^{2}\right) \quad$ in the SM for $\bar{B}(B)$ respectively.

- In order to determine $\lambda_{\gamma}$, compared to the experimental decay distribution, we need a precise expression for $\mathcal{J}$.
- In principle, $\mathcal{J}$ can be extracted from data (e.g. Dalitz analysis of $B \rightarrow J / \psi K_{1}$ [Belle ('10)]) or computed within a quark model [Kou,Le Yaouanc\&A.T., in preparation].


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## 3. $K_{1}$ three-body decay method

New method

## Up-down asymmetry

In the original proposal by Gronau et al., only the $\theta$-dependence on the polarization was considered (up-down asymmetry)

$$
\mathcal{A}_{u p-d o w n}=\frac{\int_{0}^{1} d \cos \theta \frac{d \Gamma}{d \cos \theta}-\int_{-1}^{0} d \cos \theta \frac{d \Gamma}{d \cos \theta}}{\int_{-1}^{1} d \cos \theta \frac{d \Gamma}{d \cos \theta}}=\frac{3}{4} \lambda_{\gamma} \frac{\int d s_{13} d s_{23} \operatorname{lm}\left[\vec{n} \cdot\left(\overrightarrow{\mathcal{J}} \times \overrightarrow{\mathcal{J}}^{*}\right)\right]}{\int d s_{13} d s_{23}|\overrightarrow{\mathcal{J}}|^{2}}
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$$

## New method

In our work, we take into account the Dalitz variable $\left(s_{13}, s_{23}\right)$ dependence, which carries the further information of the polarization (it was pointed out in the ALEPH analysis of $\tau \rightarrow a_{1}(\rightarrow \pi \pi \pi) \nu$ [Davier et al., Phys.Lett.B306 ('93)J). In this method, we use the quantity, called $\omega$

$$
\omega\left(s_{13}, s_{23}, \cos \theta\right) \equiv \frac{2 / m\left[\vec{n} \cdot\left(\overrightarrow{\mathcal{J}} \times \overrightarrow{\mathcal{J}}^{*}\right)\right] \cos \theta}{|\overrightarrow{\mathcal{J}}|^{2}\left(1+\cos ^{2} \theta\right)}
$$

[Kou,Le Yaouanc\&A.T., hep-ph/1011.6593]
3. $K_{1}$ three-body decay method

Results: Monte Carlo simulation
$\lambda_{\gamma} \equiv \frac{\Gamma\left(B \rightarrow K_{1 R} \gamma_{R}\right)-\Gamma\left(B \rightarrow K_{1 L} \gamma_{L}\right)}{\Gamma\left(B \rightarrow K_{1} \gamma\right)} \simeq 1+O\left(m_{s}^{2} / m_{b}^{2}\right)$ in the SM
We estimate the sensitivity of future experiments to $\lambda_{\gamma}$ using "ideal" (i.e. detector effects and background are not taken into account) MC simulation.

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$$
\text { Expected } \lambda_{\gamma}^{(S M)} \text { from } B \rightarrow K_{1}(1270) \gamma \text { : }
$$

| $N_{\text {events }}$ | 1 k | 10 k |
| :---: | :---: | :---: |
| $B^{+} \rightarrow K^{+} \pi^{-} \pi^{+} \gamma$ | $1.00 \pm 0.18$ | $1.00 \pm 0.06$ |
| $B^{+} \rightarrow K^{0} \pi^{+} \pi^{0} \gamma$ | $1.00 \pm 0.12$ | $1.00 \pm 0.04$ |
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- For 10 k events the error on $\lambda_{\gamma}$ is $<10 \%$.
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Expected $\lambda_{\gamma}^{(S M)}$ from $B \rightarrow K_{1}(1270) \gamma$ :

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Exclusion plot for $\lambda_{\gamma}^{(S M)}$ for $B^{+} \rightarrow\left(K^{0} \pi^{+} \pi^{0}\right)_{K_{1}(1270)^{+}} \gamma$


## Future constraints on right-handed currents

- $\mathcal{B}\left(B \rightarrow X_{s} \gamma\right)^{\exp }=$ $(3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$

$$
\left(\left(\delta_{L R}^{d}\right)_{23}=\left(\delta_{L L}^{d}\right)_{23}=\left(\delta_{R R}^{d}\right)_{23}=0\right)
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$\sigma\left(\lambda_{\gamma}\right)_{\mathrm{th}} \sim 0.2$

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- $A_{T}^{(2)}$ potential measurement from angular analysis of $B^{0} \rightarrow K^{* 0}\left(\rightarrow K^{-} \pi^{+}\right) \ell^{+} \ell^{-}$. $\sigma\left(A_{T}^{(2)}\right)$ LHCb $\approx 0.2$ at $2 \mathrm{fb}^{-1}$.

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(2) We propose a new quantity $\omega\left(s_{13}, s_{23}, \cos \theta\right)$, which contains all the information on polarization in each event and allows to reduce the error on $\lambda_{\gamma}$ by a factor 2 , compared to the fit of pure $\cos \theta$-distribution.

## Conclusions and perspectives

(1) We study the $B \rightarrow K_{1} \gamma$ decay to determine the photon polarization in the $b \rightarrow \boldsymbol{s} \gamma$ process in order to search the effects of New Physics beyond the SM.
(2) We propose a new quantity $\omega\left(s_{13}, s_{23}, \cos \theta\right)$, which contains all the information on polarization in each event and allows to reduce the error on $\lambda_{\gamma}$ by a factor 2 , compared to the fit of pure $\cos \theta$-distribution.
(3) We obtain the statistical accuracy $<10 \%$ for the SM-prediction for $\lambda_{\gamma}$ for 10 k events of the $B \rightarrow K_{1}(1270) \gamma$ decay.

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(3) We obtain the statistical accuracy $<10 \%$ for the SM-prediction for $\lambda_{\gamma}$ for 10 k events of the $B \rightarrow K_{1}(1270) \gamma$ decay.
(4) Perspective: the right-handed currents will be very strictly constrained by the future experiments, LHCb and SuperB. I showed an example of SUSY with large RL mass insertion. It was demonstrated that combining the three methods, we will be able to constrain $\left(\delta_{R L}^{d}\right)_{23}$ at the level of $10^{-3}$.

## BACKUP SLIDES

The matrix element of the leading operator $\mathcal{O}_{7}$ can be parametrized in terms of hadronic form factors using the following conventions:

$$
\begin{aligned}
\left\langle K_{1}\right| \bar{s} \sigma_{\mu \nu} \gamma_{5} q^{\nu} b|\bar{B}\rangle & =i \epsilon_{\mu \nu \rho \sigma} \varepsilon_{K_{1}}^{\nu *} p_{B}^{\rho} p_{K_{1}}^{\sigma} 2 F_{1}^{K_{1}}\left(q^{2}\right) \\
\left\langle K_{1}\right| \bar{s} \sigma_{\mu \nu} q^{\nu} b|\bar{B}\rangle & =F_{2}^{K_{1}}\left(q^{2}\right)\left[\varepsilon_{K_{1} \mu}^{*}\left(m_{B}^{2}-m_{K_{1}}^{2}\right)-\left(\varepsilon_{K_{1}}^{*} \cdot p_{B}\right)\left(p_{B}+p_{K_{1}}\right)_{\mu}\right] \\
& +F_{3}^{K_{1}}\left(q^{2}\right)\left(\varepsilon_{K_{1}}^{*} \cdot p_{B}\right)\left[q_{\mu}-\frac{q^{2}}{m_{B}^{2}-m_{K_{1}}^{2}}\left(p_{B}+p_{K_{1}}\right)_{\mu}\right]
\end{aligned}
$$

- $F_{1}^{K_{1}}(0)=F_{2}^{K_{1}}(0)$ in order to avoid a kinematic singularity at $q^{2}=0$.
- Since the outgoing photon is on-shell, $q^{2}=0$ and $q_{\mu} \varepsilon^{* \mu}=0 \Rightarrow$ the last term, proportional to $F_{3}^{K_{1}}$, vanishes and hence the matrix element is parametrized with only one form factor $F_{1}^{K_{1}}(0)$ :

$$
\left\langle K_{1 L} \gamma_{L}\right| \mathcal{O}_{7 L}|\bar{B}\rangle=\left\langle K_{1 R} \gamma_{R}\right| \mathcal{O}_{7 R}|\bar{B}\rangle=i \frac{e}{8 \pi^{2}} m_{b}\left(m_{B}^{2}-m_{K_{1}}^{2}\right) F_{1}^{K_{1}}(0)
$$

- The form factors of the mass eigenstates are related to $F_{1}^{K_{1 A, B}}$, which can be calculated with LCSR, as following:

$$
\begin{aligned}
& F_{1}^{K_{1}(1270)}(0)=F_{1}^{K_{1 A}}(0) \sin \theta_{K_{1}}+F_{1}^{K_{1 B}}(0) \cos \theta_{K_{1}} \\
& F_{1}^{K_{1}(1400)}(0)=F_{1}^{K_{1 A}}(0) \cos \theta_{K_{1}}-F_{1}^{K_{1 B}}(0) \sin \theta_{K_{1}}
\end{aligned}
$$

- The decay amplitude of the axial-vector meson $A$ to some vector $\left(V_{i j}\right)$ and pseudoscalar $\left(P_{k}\right)$ mesons can be expressed in the following Lorentz invariant form:

$$
\mathcal{M}\left(A \rightarrow V_{i j} P_{k}\right)=\varepsilon_{\mu}^{(A)} T^{\mu \nu} \varepsilon_{\nu}^{\left(V_{k}\right) *}, \quad T^{\mu \nu}=f g^{\mu \nu}+h p_{V_{i j}}^{\mu} p_{A}^{\nu}
$$

The unknown effective couplings $f$ and $h$ can be related to the partial wave amplitudes as and $a_{D}$ as

$$
f=-\left(a_{S}+\frac{a_{D}}{\sqrt{2}}\right), \quad h=\left[\left(1-\frac{m_{V_{i j}}}{E_{V_{i j}}}\right) a_{S}+\left(1+2 \frac{m_{V_{i j}}}{E_{V_{i j}}}\right) \frac{a_{D}}{\sqrt{2}}\right] \frac{E_{V_{i j}}}{m_{A} \vec{p}_{k}^{2}}
$$

where $E V_{i j}$ and $\vec{p}_{k}\left(=-\vec{p} V_{i j}\right)$ are the energy of the vector meson and the momentum of pseudoscalar meson in the $A$-reference frame.

- The amplitude of the subsequent decay $V_{i j} \rightarrow P_{i} P_{j}$ can be parametrized in terms of the effective coupling $g V_{i j} P_{i} P_{j}$ (which can be determined from the measured partial decay width of $V_{i j}$ ):

$$
\mathcal{M}\left(V_{i j} \rightarrow P_{i} P_{j}\right)=g V_{i j} P_{i} P_{j} \cdot \varepsilon_{\mu}^{\left(V_{i j}\right)}\left(p_{i}-p_{j}\right)^{\mu}
$$

Parametrizing the propagation of $V_{i j}$ with the relativistic Breit-Wigner form $B W v_{i j}\left(s_{i j}\right)=1 /\left(s_{i j}-m_{V_{i j}}^{2}-i m_{V_{i j}} \Gamma v_{i j}\right)$, one can write the total amplitude of the $A$-decay chain as
$\mathcal{M}\left(A \rightarrow\left(P_{i} P_{j}\right) v_{i j} P_{k}\right)=\varepsilon_{\mu}^{(A)}\left(f g^{\mu \nu}+h p_{V_{i j}}^{\mu} p_{A}^{\nu}\right) \varepsilon_{\nu}^{\left(V_{i j}\right) *} B W_{v_{i j}}\left(s_{i j}\right) g V_{i j} P_{i} P_{j} \varepsilon_{\sigma}^{\left(V_{i j}\right)}\left(p_{i}-p_{j}\right)^{\sigma}$
Summing over the $V_{i j}$-polarizations, one obtains the total Lorentz invariant amplitude:

$$
\begin{gathered}
\mathcal{M}\left(A \rightarrow\left(P_{i} P_{j}\right) v_{i j} P_{k}\right)=\varepsilon_{\mu}^{(A)} J_{i j k}^{\mu}, \quad J_{i j k}^{\mu}=c_{k}\left(s_{i j}\right) p_{k}^{\mu}-c_{i}\left(s_{i j}\right) p_{i}^{\mu} \\
c_{k}\left(s_{i j}\right)=g v_{i j} P_{i} P_{j}\left[-\left(f+h\left(m_{A}^{2}-p_{A} \cdot p_{k}\right)\right)\left(1+\frac{m_{i}^{2}-m_{j}^{2}}{m_{V_{i j}}^{2}}\right)+2 h\left(p_{A} \cdot p_{i}\right)\right] B W_{v_{i j}}\left(s_{i j}\right) \\
c_{i}\left(s_{i j}\right)=2 g v_{i j} P_{i} P_{j} f B W_{v_{i j}}\left(s_{i j}\right)
\end{gathered}
$$

If there are several possible channels of the $A$-decay to the same charged final state $P_{1} P_{2} P_{3}$, one has to sum over the all possible diagrams with different intermediate vector resonance states:

$$
\begin{array}{r}
\mathcal{M}\left(A \rightarrow P_{1} P_{2} P_{3}\right)=\sum_{V_{i j}}\left(I_{i}, I_{i}^{z} ; I_{j}, I_{j}^{z} \mid I_{V_{i j}}, I_{V_{i j}}^{z}\right)\left(I_{V_{i j}}, I_{V_{i j}}^{z} ; I_{k}, I_{k}^{z} \mid I_{A}, I_{A}^{z}\right) \\
\times \mathcal{M}\left(A \rightarrow\left(P_{i} P_{j}\right) v_{i j} P_{k}\right)=\varepsilon_{\mu}^{(A)} \mathcal{J}^{\mu}=\varepsilon_{\mu}^{(A)}\left(\mathcal{C}_{1}\left(s_{13}, s_{23}\right) p_{1}^{\mu}-\mathcal{C}_{2}\left(s_{13}, s_{23}\right) p_{2}^{\mu}\right)
\end{array}
$$

## $\mathcal{J}$-function represents the $K_{1} \rightarrow K \pi \pi$ decay amplitude.

Assuming that this process comes from the vector-pseudoscalar meson intermediate state, $K_{1} \rightarrow V P_{1} \rightarrow P_{1} P_{2} P_{3}, \mathcal{J}$ contains

- two form factors for $K_{1} \rightarrow V P_{1}$ (one can express them in terms of $S$ and $D$ partial wave amplitudes)
- one coupling for $V \rightarrow P_{2} P_{3}$
which, in principle, can be determined from the experiment. But due the non sufficient amount of data we have to use some model to predict $\mathcal{J}$.
- Therefore in the following, we estimate the $K_{1} \rightarrow V P_{1}$ (namely $K_{1} \rightarrow K^{*} \pi, K_{1} \rightarrow \rho K$ ) form factors in the framework of the ${ }^{3} P_{0}$ quark-pair-creation model [Kou,Le Yaouanc\&A.T., in preparation].
[Daum et al., Nucl.Phys.B187 ('81)]:

$$
\begin{aligned}
\mathcal{B}\left(K_{1}(1400) \rightarrow \rho K\right) / \mathcal{B}\left(K_{1}(1400) \rightarrow K^{*} \pi\right) & =0.01 \pm 0.01 \\
\mathcal{B}\left(K_{1}(1400) \rightarrow\left(K^{*} \pi\right)_{D}\right) / \mathcal{B}\left(K_{1}(1400) \rightarrow\left(K^{*} \pi\right)_{S}\right) & =0.04 \pm 0.01 \\
\mathcal{B}\left(K_{1}(1270) \rightarrow \rho K\right) / \mathcal{B}\left(K_{1}(1270) \rightarrow K^{*} \pi\right) & =4.16 \pm 1.56 \\
\mathcal{B}\left(K_{1}(1270) \rightarrow\left(K^{*} \pi\right)_{D}\right) / \mathcal{B}\left(K_{1}(1270) \rightarrow\left(K^{*} \pi\right)_{S}\right) & =0.54 \pm 0.15
\end{aligned}
$$

In order to compute $\mathcal{J}$, we use QPCM [Le Yaouanc et al., Phys.Rev.D8 ('73), Phys.Rev.D9 ('74)] to describe the intermediate $K_{1} \rightarrow K^{*} \pi, \rho K$ decays.
(1) QPCM is one of the simplest and most successful quark models which has a good predictive power.
(2) The model has just one(!) universal phenomenological parameter- the quark pair-creation constant $\gamma$.
(3) It is very good especially to compute the $P$-wave particles (and in this sense, better than the flux-tube-breaking model, for some case).

## Basic idea

Instead of being created from quark lines, $q \bar{q}$ is created from anywhere within the hadronic matter and has the quantum numbers of the vacuum $\Rightarrow$ $q \bar{q}$-pair must be in a ${ }^{3} P_{0}$ state, $S U(3)$ singlet and of null momentum.


- To understand the $K_{1} \rightarrow K^{*} \pi, \rho K$ decays, first one has to explain the observed hierarchy [Daum et al., Nucl.Phys.B187 ('81)]:

$$
\begin{aligned}
\mathcal{B}\left(K_{1}(1400) \rightarrow \rho K\right) / \mathcal{B}\left(K_{1}(1400) \rightarrow K^{*} \pi\right) & =0.01 \pm 0.01 \\
\mathcal{B}\left(K_{1}(1400) \rightarrow\left(K^{*} \pi\right)_{D}\right) / \mathcal{B}\left(K_{1}(1400) \rightarrow\left(K^{*} \pi\right)_{s}\right) & =0.04 \pm 0.01 \\
\mathcal{B}\left(K_{1}(1270) \rightarrow \rho K\right) / \mathcal{B}\left(K_{1}(1270) \rightarrow K^{*} \pi\right) & =4.16 \pm 1.56 \\
\mathcal{B}\left(K_{1}(1270) \rightarrow\left(K^{*} \pi\right)_{D}\right) / \mathcal{B}\left(K_{1}(1270) \rightarrow\left(K^{*} \pi\right)_{s}\right) & =0.54 \pm 0.15
\end{aligned}
$$

- It can be explained with the help of the $K_{1}$ mixing angle: mass eigenstates $K_{1}(1270)$ and $K_{1}(1400)$ are considered as mixtures of $1^{3} P_{1}\left(K_{1 A}\right)$ and $1^{1} P_{1}\left(K_{1 B}\right)$ states [Suzuki, Phys.Rev.D47 ('93)]:

$$
\begin{aligned}
\left|K_{1}(1270)\right\rangle & =\left|K_{1 A}\right\rangle \sin \theta_{K_{1}}+\left|K_{1 B}\right\rangle \cos \theta_{K_{1}} \\
\left|K_{1}(1400)\right\rangle & =\left|K_{1 A}\right\rangle \cos \theta_{K_{1}}-\left|K_{1 B}\right\rangle \sin \theta_{K_{1}}
\end{aligned}
$$

- Most interestingly, this mixing angle can give a good explanation of the observed suppression of the $B \rightarrow K_{1}(1400) \gamma$ channel [Yang et al., Phys.Rev.Lett. 94 ('05); Hatanaka\& Yang, Phys.Rev.D77 ('08)]:

$$
\mathcal{B}\left(B \rightarrow K_{1}(1400) \gamma\right) / \mathcal{B}\left(B \rightarrow K_{1}(1270) \gamma\right)<0.35
$$

## Partial wave amplitudes

$$
\begin{gathered}
a_{S}\left(K_{1}(1270) \rightarrow K^{*} \pi / \rho K\right)=\mathcal{S}_{K^{*} / \rho}\left(\sqrt{2} \sin \theta_{K_{1}} \mp \cos \theta_{K_{1}}\right) \\
a_{D}\left(K_{1}(1270) \rightarrow K^{*} \pi / \rho K\right)=\mathcal{D}_{K^{*} / \rho}\left(-\sin \theta_{K_{1}} \mp \sqrt{2} \cos \theta_{K_{1}}\right) \\
a_{S}\left(K_{1}(1400) \rightarrow K^{*} \pi / \rho K\right)=\mathcal{S}_{K^{*} / \rho}\left(\sqrt{2} \cos \theta_{K_{1}} \pm \sin \theta_{K_{1}}\right) \\
a_{D}\left(K_{1}(1400) \rightarrow K^{*} \pi / \rho K\right)=\mathcal{D}_{K^{*} / \rho}\left(-\cos \theta_{K_{1}} \pm \sqrt{2} \sin \theta_{K_{1}}\right) \\
\mathcal{S}_{V}=\gamma \sqrt{\frac{3}{2}} \frac{2 \mathcal{I}_{1}^{V}-\mathcal{I}_{0}^{V}}{18}, \quad \mathcal{D}_{V}=\gamma \sqrt{\frac{3}{2}} \frac{\mathcal{I}_{1}^{V}+\mathcal{I}_{0}^{V}}{18} \\
\mathcal{I}_{m=0, \pm 1}^{V}=\frac{1}{8} \int d^{3} \vec{k} \mathcal{Y}_{1}^{m}\left(\vec{k} \vec{k}_{P}-\vec{k}\right) \psi_{0}^{(P)}(\vec{k}) \psi_{0}^{(V)}(-\vec{k}) \psi_{1}^{-m\left(K_{1}\right)}\left(\vec{k}_{P}+\vec{k}\right)
\end{gathered}
$$

- Fitting the combination of the ratios of measured branching fractions, we found $\theta_{K_{1}} \simeq 50^{\circ}$.
- Using this model and the fitted value of $\theta_{K_{1}}$, we obtained the $K_{1} \rightarrow K^{*} \pi, \rho K$ form factors and thus the $\mathcal{J}$-function.

For the axial meson decay $\left(A=K_{1 A, B}\right)$ into the ground states of vector ( $V=K^{*} / \rho$ ) and pseudoscalar ( $P=\pi / K$ ) mesons, the spacial integrals are given by

$$
\begin{aligned}
\mathcal{I}_{\boldsymbol{m}=\mathbf{0}, \pm \mathbf{1}}^{\boldsymbol{V}}=\int d^{\mathbf{3}} \vec{k}_{\mathbf{1}} d^{\mathbf{3}} \vec{k}_{\mathbf{2}} d^{\mathbf{3}} \vec{k}_{\mathbf{3}} d^{\mathbf{3}} \vec{k}_{\mathbf{4}} \delta & \left(\vec{k}_{\mathbf{1}}+\vec{k}_{\mathbf{2}}-\vec{k}_{\mathbf{A}}\right) \delta\left(\vec{k}_{\mathbf{2}}+\vec{k}_{\mathbf{3}}-\vec{k}_{\boldsymbol{V}}\right) \delta\left(\vec{k}_{\mathbf{4}}+\vec{k}_{\mathbf{1}}-\vec{k}_{\boldsymbol{P}}\right) \delta\left(\vec{k}_{\mathbf{3}}+\vec{k}_{\mathbf{4}}\right) \\
& \times \mathcal{Y}_{1}^{\boldsymbol{m}}\left(\vec{k}_{\mathbf{3}}-\vec{k}_{\mathbf{4}}\right) \psi^{(\boldsymbol{A})}\left(\vec{k}_{\mathbf{1}}-\vec{k}_{\mathbf{2}}\right) \psi^{(\boldsymbol{V})}\left(\vec{k}_{\mathbf{2}}-\vec{k}_{\mathbf{3}}\right) \psi^{(\boldsymbol{P})}\left(\vec{k}_{\mathbf{4}}-\vec{k}_{\mathbf{1}}\right) \\
& =\frac{1}{8} \int d^{\mathbf{3}} \vec{k} \mathcal{Y}_{\mathbf{1}}^{\boldsymbol{m}}\left(\vec{k}_{\boldsymbol{P}}-\vec{k}\right) \psi_{0}^{(\boldsymbol{P})}(\vec{k}) \psi_{0}^{(\boldsymbol{V})}(-\vec{k}) \psi_{\mathbf{1}}^{-\boldsymbol{m}(\boldsymbol{A})}\left(\vec{k}_{\boldsymbol{P}}+\vec{k}\right)
\end{aligned}
$$

where $\psi_{L}^{L_{z}}$ are the normalized Fourier transforms of harmonic oscillator meson wave functions:

$$
\psi_{0}^{(i)}(\vec{k})=\frac{R_{i}^{3 / 2}}{\pi^{3 / 4}} \exp \left(-\frac{\vec{k}^{2} R_{i}^{2}}{8}\right), \quad \psi_{1}^{m(i)}(\vec{k})=\sqrt{\frac{2}{3}} \frac{R_{i}^{5 / 2}}{\pi^{1 / 4}} \mathcal{Y}_{1}^{m}(\vec{k}) \exp \left(-\frac{\vec{k}^{2} R_{i}^{2}}{8}\right)
$$

Here $\mathcal{Y}_{1}^{m}(\vec{k})=|\vec{k}| Y_{1}^{m}(\hat{\vec{k}})=\left(\vec{\varepsilon}_{m} \vec{k}\right) \sqrt{3 / 4 \pi}, R_{i}$ is the meson wave function radius and $\vec{\varepsilon}_{m}$ are the polarization vectors, defined as $\vec{\varepsilon}_{0}=(0,0,1), \vec{\varepsilon}_{ \pm 1}=\mp \frac{1}{\sqrt{2}}(1, \pm i, 0)$.
$\mathcal{I}_{0}^{V}=-\frac{4 \sqrt{3}}{\pi^{5 / 4}} \frac{R_{A}^{5 / 2}\left(R_{V} R_{P}\right)^{3 / 2}}{\left(R_{A}^{2}+R_{V}^{2}+R_{P}^{2}\right)^{5 / 2}}\left(1-\vec{k}_{P}^{2} \frac{\left(2 R_{A}^{2}+R_{V}^{2}+R_{P}^{2}\right)\left(R_{V}^{2}+R_{P}^{2}\right)}{4\left(R_{A}^{2}+R_{V}^{2}+R_{P}^{2}\right)}\right) \exp \left[-\vec{k}_{P}^{2} \frac{R_{A}^{2}\left(R_{V}^{2}+R_{P}^{2}\right)}{8\left(R_{A}^{2}+R_{V}^{2}+R_{P}^{2}\right)}\right]$
$\mathcal{I}_{1}^{V}=\frac{4 \sqrt{3}}{\pi^{5 / 4}} \frac{R_{A}^{5 / 2}\left(R_{V} R_{P}\right)^{3 / 2}}{\left(R_{A}^{2}+R_{V}^{2}+R_{P}^{2}\right)^{5 / 2}} \exp \left[-\vec{k}_{P}^{2} \frac{R_{A}^{2}\left(R_{V}^{2}+R_{P}^{2}\right)}{8\left(R_{A}^{2}+R_{V}^{2}+R_{P}^{2}\right)}\right]$

- Our probability density function (i.e. the normalized decay width distribution) can be written as

$$
W\left(s_{13}, s_{23}, \cos \theta\right)=f\left(s_{13}, s_{23}, \cos \theta\right)+\lambda_{\gamma} g\left(s_{13}, s_{23}, \cos \theta\right)=f\left(1+\lambda_{\gamma} \omega\right)
$$

- Then, the log-likelihood function for a sample of $N$ measurements is:

$$
\ln \mathcal{L}=\ln \prod_{i=1}^{N} W\left(s_{13}^{i}, s_{23}^{i}, \cos \theta_{i}\right)=\sum_{i=1}^{N} \ln \left(1+\lambda_{\gamma} \omega_{i}\right)
$$

+other terms independent of $\lambda_{\gamma}$

- Using the maximum likelihood method, we obtain $\lambda_{\gamma}$ as a solution of the following equation:

$$
\frac{\partial \ln \mathcal{L}}{\partial \lambda_{\gamma}}=\sum_{i=1}^{N} \frac{\omega_{i}}{1+\lambda_{\gamma} \omega_{i}}=N\left\langle\frac{\omega}{1+\lambda_{\gamma} \omega}\right\rangle=0
$$

Notice: resulting solution does not depend on $f$ and $g$ separately but only on their ratio $\omega$.

Since $W$ depends on $\lambda_{\gamma}$ linearly, one can reduce a multi-dimensional fit to a one-dimensional, using variable $\omega \equiv g / f$ ! [Davier et al., Phys.Lett.B306('93)]

## Approximate solution for $\lambda_{\gamma}$

Example of $\omega$-distribution for 10 k of $B^{+} \rightarrow$ $\left(K^{+} \pi^{-} \pi^{+}\right)_{K_{1}(1270)} \gamma$ events with purely right-handed (red) and left-handed (blue) photons.


## NO fit is needed to extract $\lambda_{\gamma}$

When $\lambda_{\gamma} \omega \ll 1$ :

$$
\begin{array}{r}
\frac{\partial \ln \mathcal{L}}{\partial \lambda_{\gamma}} \simeq N\left(\langle\omega\rangle-\lambda_{\gamma}\left\langle\omega^{2}\right\rangle\right)=0 \\
\Rightarrow \lambda_{\gamma} \simeq \frac{\langle\omega\rangle}{\left\langle\omega^{2}\right\rangle} \\
\frac{1}{\sigma_{\lambda}^{2}}=-\frac{\partial^{2} \ln \mathcal{L}}{\partial \lambda_{\gamma}{ }^{2}}=N\left\langle\left(\frac{\omega}{1+\lambda_{\gamma} \omega}\right)^{2}\right\rangle \\
\Rightarrow \sigma_{\lambda}^{2} \simeq \frac{1}{N\left(\left\langle\omega^{2}\right\rangle-2 \frac{\langle\omega\rangle\left\langle\omega^{3}\right\rangle}{\left\langle\omega^{2}\right\rangle}\right)}
\end{array}
$$

We only has to sum $\omega$ and $\omega^{2}$ over all the events (no fit is needed).

## Comparison with the other methods: $\mathcal{A}_{C P}\left(B \rightarrow f^{C P} \gamma\right)$

Here we compare the precision of $x \equiv\left|\mathcal{M}_{L} / \mathcal{M}_{R}\right|$ measurement, using the methods of the $\mathcal{A}_{C P}(t)$ measurement in $B^{0} \rightarrow\left(K_{S} \pi^{0}\right)_{K^{*}} \gamma, B_{s} \rightarrow \phi \gamma$ and $\lambda_{\gamma}=\frac{1-x^{2}}{1+x^{2}}$ determination in $B \rightarrow(K \pi \pi)_{K_{1}} \gamma$.

The error of $x$ determination will be dependent on the measured value of $\lambda_{\gamma}(\Leftrightarrow x)$ :

$$
\sigma_{x}=\frac{\left(1+x^{2}\right)^{2}}{4 x} \sigma_{\lambda_{\gamma}}
$$

For some values of $x$, considerably different from the SM (i.e. 0), one can obtain a better sensitivity, compared to the $\mathcal{A}_{C P}$-method.


For instance, one can see from the Fig. that if we measure $\lambda_{\gamma}<0.9(\Leftrightarrow x>0.3)$ with the error $\sigma_{\lambda_{\gamma}} \approx 0.1$, we can have a smaller error on $x$, compared to the estimated $\sigma_{x} \simeq 0.1$ from potential measurement of $\mathcal{A}_{C P}$ at LHCb [LHCB-ROADMAP4-001].
$\operatorname{Im}\left[\vec{n} \cdot\left(\overrightarrow{\mathcal{J}} \times \overrightarrow{\mathcal{J}}^{*}\right)\right]$ is sensitive to the relative phase between $K^{*} \pi$ and $\rho K$.

- The relative sign of two amplitudes, predicted by QPCM, can be verified using the recent exp. data on the $B \rightarrow K \pi \pi \psi$ decay [Belle ('10)].

- The interference between the $\rho K$ and $K^{*} \pi$ amplitudes is responsible for the abrupt fading of the $K^{*}(892)$ signal at $M_{K \pi}>M_{K^{*}(892)}$.
- We confirm the sign, predicted by QPCM.

