Neutrino Masses in a Two Higgs Doublet Model

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Introduction

Present status in the determination of neutrino parameters:

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	$2.28 - 2.64$ $-(2.17 - 2.54)$	$2.18 - 2.73$ $-(2.08 - 2.64)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	≤ 0.027 ≤ 0.031	≤ 0.035 ≤ 0.039

Schwetz, Tortola, Valle

WMAP, $0\nu 2\beta \rightarrow m_\nu \lesssim 0.5$ eV

No information about CP violation or about the neutrino mass spectrum

Even with this limited information about the neutrino sector, we can already notice some features:

- Neutrino masses are tiny, $m_\nu \lesssim \mathcal{O}$ (0.1 eV)
- Two large mixing angles ($\theta_{\text{atm}} \simeq \pi/4$, $\theta_{\text{atm}} \simeq \pi/6$)

  One small mixing angle ($\theta_{13} \simeq 0$)

$$U_{lep} \simeq \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

- The two heaviest neutrinos present a mild mass hierarchy

$$\Delta m^2_{atm} = m_3^2 - m_1^2 \longrightarrow m_3 = \sqrt{\Delta m^2_{atm} - m_1^2}$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 \longrightarrow m_2 = \sqrt{\Delta m^2_{sol} - m_1^2}$$

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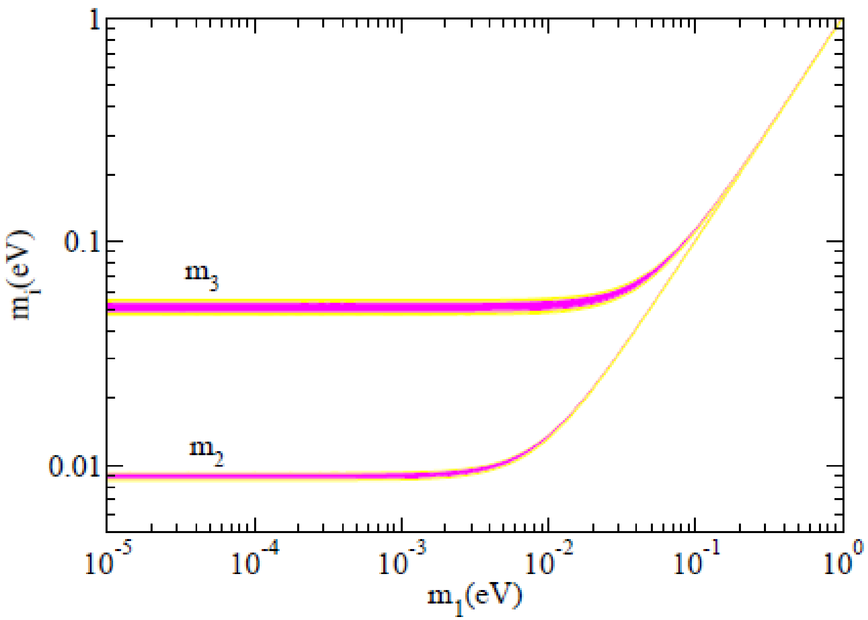

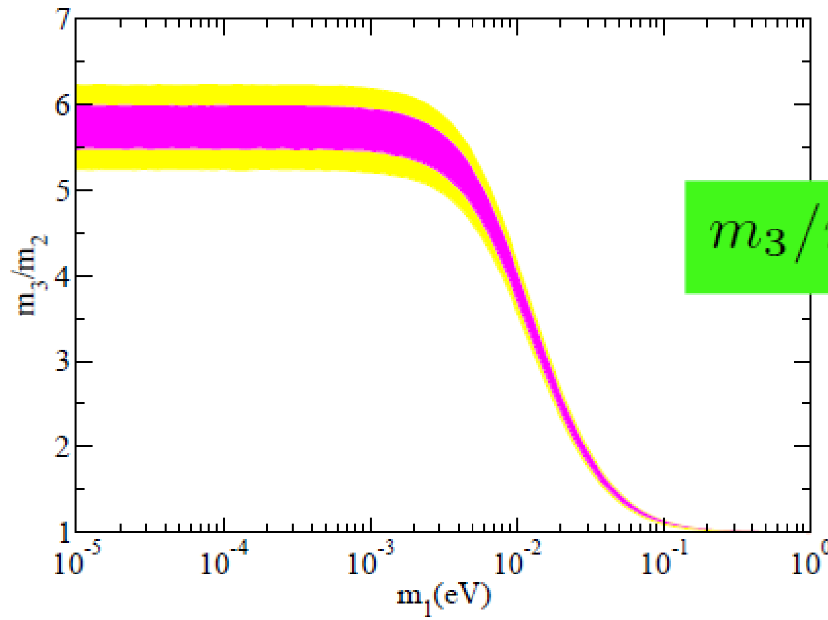

$m_3/m_2 \lesssim 6$

Compare to the quark sector

$$\left.\begin{aligned} m_u &= 1.7 \text{ to } 3.3 \text{ MeV} \\ m_c &= 1.27^{+0.07}_{-0.09} \text{ GeV} \\ m_t &= 172.0 \pm 0.9 \pm 1.3 \text{ GeV} \end{aligned}\right\} \begin{aligned} m_t/m_c &\simeq 140 \\ m_c/m_d &\simeq 500 \end{aligned}$$

$$\text{vs.} \quad m_3/m_2 \lesssim 6$$

$$\left.\begin{aligned} m_d &= 4.1 \text{ to } 5.8 \text{ MeV} \\ m_s &= 101^{+29}_{-21} \text{ MeV} \\ m_b &= 4.19^{+0.07}_{-0.09} \text{GeV} \end{aligned}\right\} \begin{aligned} m_b/m_s &\simeq 41 \\ m_s/m_d &\simeq 20 \end{aligned}$$

$$|U_{\rm CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.973 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix} \quad \text{vs.} \quad |U_{\rm lep}| \simeq \begin{pmatrix} 0.82 & 0.56 & 0 \\ 0.41 & 0.56 & 0.71 \\ 0.41 & 0.56 & 0.71 \end{pmatrix}$$

Compare also to the charged lepton sector

$$\left.\begin{aligned} m_e &= 0.51 \text{ MeV} \\ m_\mu &= 106 \text{ MeV} \\ m_\tau &= 1.78 \text{ GeV} \end{aligned}\right\} \begin{aligned} m_\tau/m_\mu &\simeq 17 \\ m_\mu/m_e &\simeq 208 \end{aligned}$$

The neutrino sector presents a completely different pattern

Any model of neutrino masses should address the following questions:

- **Why tiny masses?**
- **Why large mixing angles?**
- **Why mild mass hierarchy?**

And preferably, the model should be testable

A very popular neutrino mass model: the (type I) see-saw model

Add to the Standard Model at least two right-handed neutrinos

$$-\mathcal{L}^{\nu} = (Y_{\nu})_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\Phi} - \frac{1}{2}M_{\mathrm{M}ij}\bar{\nu}^{C}_{Ri}\nu_{Rj} + \mathrm{h.c.}$$

$$M_{\mathrm{Maj}} \gg M_Z$$

$$-\mathcal{L}^{\nu,\ \mathrm{eff}} = \frac{1}{2}\kappa_{ij}(\bar{l}_{Li}\tilde{\Phi})(\tilde{\Phi}^T l^{C}_{Lj}) + \mathrm{h.c.}$$

$$\kappa = (Y_{\nu}M_{\mathrm{M}}^{-1}Y_{\nu}{}^{T}) \quad \Longrightarrow \quad \mathcal{M}_{\nu} = \frac{v^2}{2}\kappa$$

Very compelling explanation to the small neutrino masses

Furthermore, this model predicts charged lepton flavour violation:

BR(μ→eγ)~10^{-57}, in *excellent* agreement with experiments.

Can the type I see-saw mechanism accommodate a mild mass hierarchy?

The high energy theory, spanned by $\{Y_\nu, M_{maj}\}$, depends on 18 parameters

The low energy theory, spanned by $\{\mathcal{M}_\nu\}$, depends on 9 parameters

There is a lot of freedom at high energies

It would not be surprising if the see-saw mechanism could accommodate $m_3/m_2<6$.

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It would not be surprising if the see-saw mechanism could accommodate $m_3/m_2<6$.

The answer is yes. In fact the see-saw mechanism can accommodate anything

$$Y_\nu = \frac{1}{\langle \Phi^0 \rangle} U^*_{\text{lep}} \sqrt{D_m} R^T \sqrt{D_M}$$

$$R = \begin{pmatrix} \hat{c}_2\hat{c}_3 & -\hat{c}_1\hat{s}_3 - \hat{s}_1\hat{s}_2\hat{c}_3 & \hat{s}_1\hat{s}_3 - \hat{c}_1\hat{s}_2\hat{c}_3 \\ \hat{c}_2\hat{s}_3 & \hat{c}_1\hat{c}_3 - \hat{s}_1\hat{s}_2\hat{s}_3 & -\hat{s}_1\hat{c}_3 - \hat{c}_1\hat{s}_2\hat{s}_3 \\ \hat{s}_2 & \hat{s}_1\hat{c}_2 & \hat{c}_1\hat{c}_2 \end{pmatrix} \qquad D_M = \text{diag}(M_1, M_2, M_3)$$

But there is a price...

The price is that the resulting Yukawa coupling could be "weird"

For example, taking $M_1=10^9$ GeV, $M_2=10^{11}$ GeV, $M_3=10^{13}$ GeV and $R(z_1=2i, z_2=0, z_3=0)$, one obtains the matrix

$$Y_\nu = \begin{pmatrix} 1.9\times 10^{-4} & 0.011 & 0.11i \\ -8.6\times 10^{-5} & 0.012-0.031i & 0.32+0.12i \\ 8.6\times 10^{-5} & -0.012-0.031i & 0.32-0.12i \end{pmatrix}$$

Which reproduces, *by construction*, the low energy neutrino data
($m_3=0.05$ eV, $m_2=0.0083$ eV, $\sin^2\theta_{12}=0.3$, $\sin^2\theta_{23}=1$, and $m_1=m_2/6$, $\theta_{13}=0$ and no CP violation)

However, the eigenvalues are

$$\left.\begin{aligned} y_3 &= 0.50 \\ y_2 &= 1.3\times 10^{-3} \\ y_1 &= 2.2\times 10^{-4} \end{aligned}\right\} \begin{aligned} y_3/y_2 &= 379 \\ y_2/y_1 &= 6 \end{aligned}$$

This Yukawa coupling does not seem to be generated by the same mechanism that generates Yu, Yd, Ye (whatever it is...)

A more interesting question is not whether the see-saw can accommodate the data, but whether the see-saw can accommodate the data *with our present (very limited) understanding of the origin of flavour.*

Can the see-saw mechanism accommodate the oscillation data when the neutrino Yukawa couplings are hierarchical?

Not so easy... The see-saw mechanism tends to produce very large neutrino mass hierarchies

Casas, AI, Jimenez-Alburquerque

"Naïve see-saw" (no mixing)

$$m_1 \sim \frac{y_1^2}{M_1}\langle\Phi^0\rangle^2, \quad m_2 \sim \frac{y_2^2}{M_2}\langle\Phi^0\rangle^2, \quad m_3 \sim \frac{y_3^2}{M_3}\langle\Phi^0\rangle^2 \qquad \frac{m_3}{m_2} \sim \frac{y_3^2}{y_2^2}\frac{M_2}{M_3}$$

- Assume hierarchical Yukawa couplings
  $y_1 : y_2 : y_3 \sim 1 : 20 : 20^2$ (down-type quark Yukawas)
  $y_1 : y_2 : y_3 \sim 1 : 300 : 300^2$ (up-type quark Yukawas)

- For the right-handed neutrino masses, we don't know

Hierarchy in ν_R as in Y_ν — Degenerate ν_R

$\frac{m_3}{m_2} \sim 20 - 300$ — $\frac{m_3}{m_2} \sim 400 - 90000$ — far from $\frac{m_3}{m_2} \lesssim 6$

A more rigorous analysis shows that generically

$$\frac{m_3}{m_2} \gtrsim \frac{y_3^2}{y_2^2}\frac{M_3}{M_2}$$

Hierarchical ν_R $\quad \frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{3-7})$

Degenerate ν_R $\quad \frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{2-5})$

far from $\frac{m_3}{m_2} \lesssim 6$

Consider a model with just two right-handed neutrinos

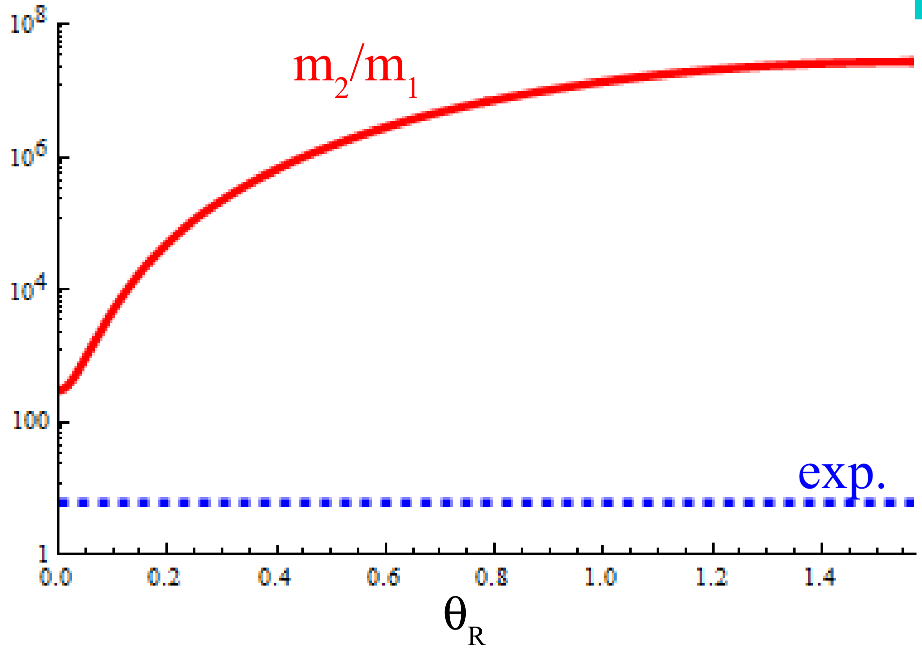
Assume

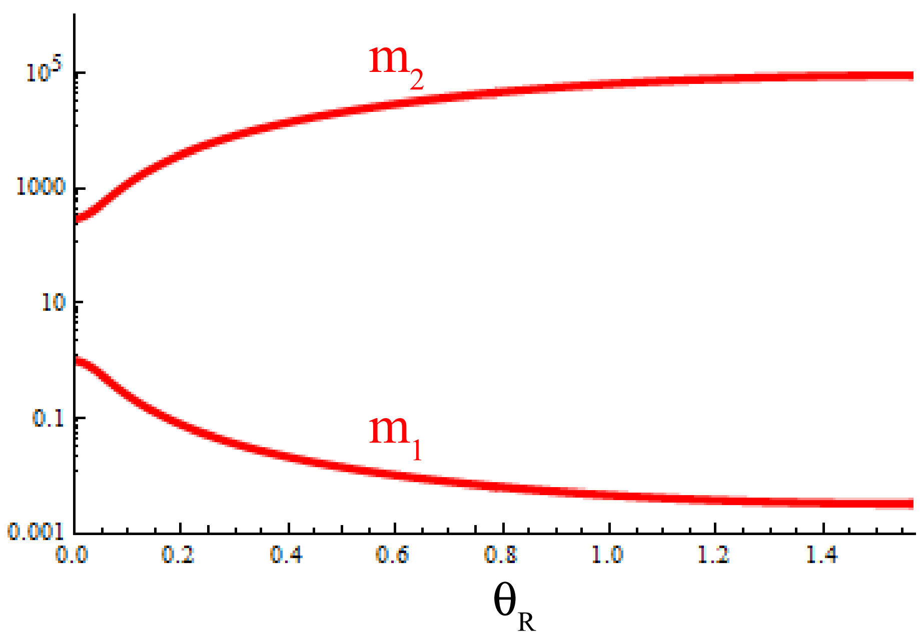
$$\boxed{\begin{aligned} y_1 : y_2 &= 1 : 300 \\ M_1 : M_2 &= 1 : 300 \end{aligned}}$$

(inspired by the hierarchy in the up-quark sector)

The neutrino mass hierarchy depends just on the mixing angle in the right-handed sector, θ_R, and on the Majorana phase α.

$\alpha=0$





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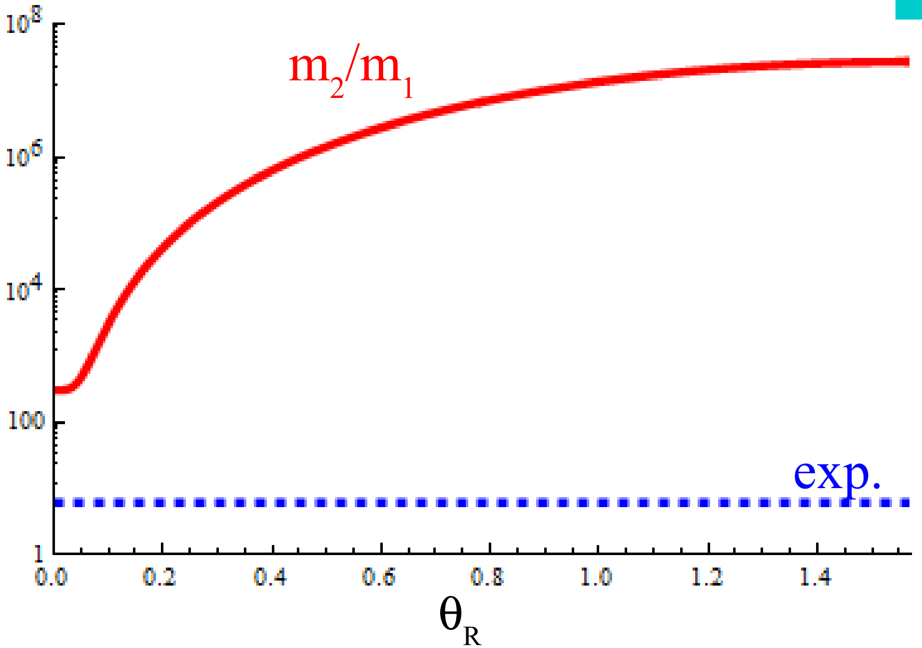

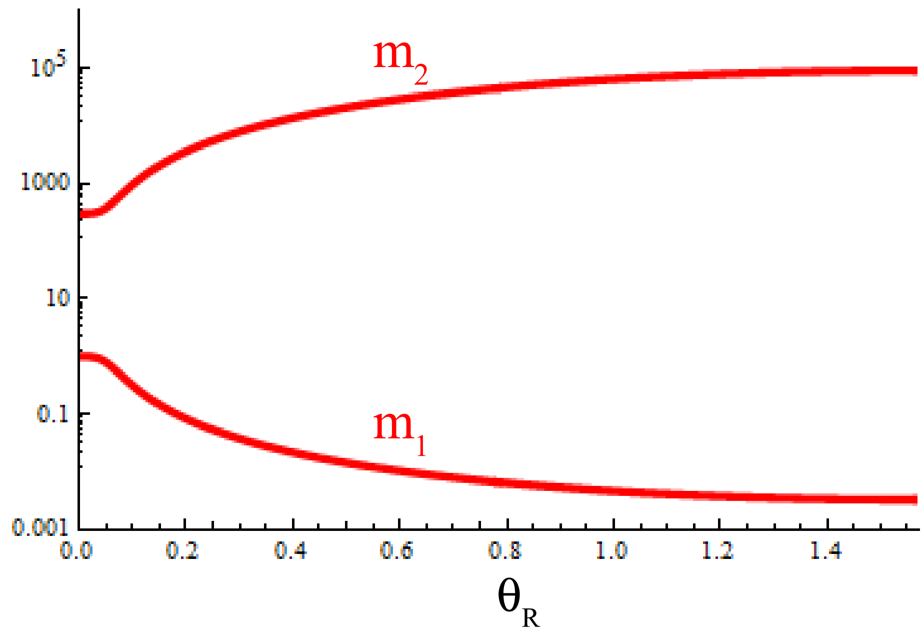

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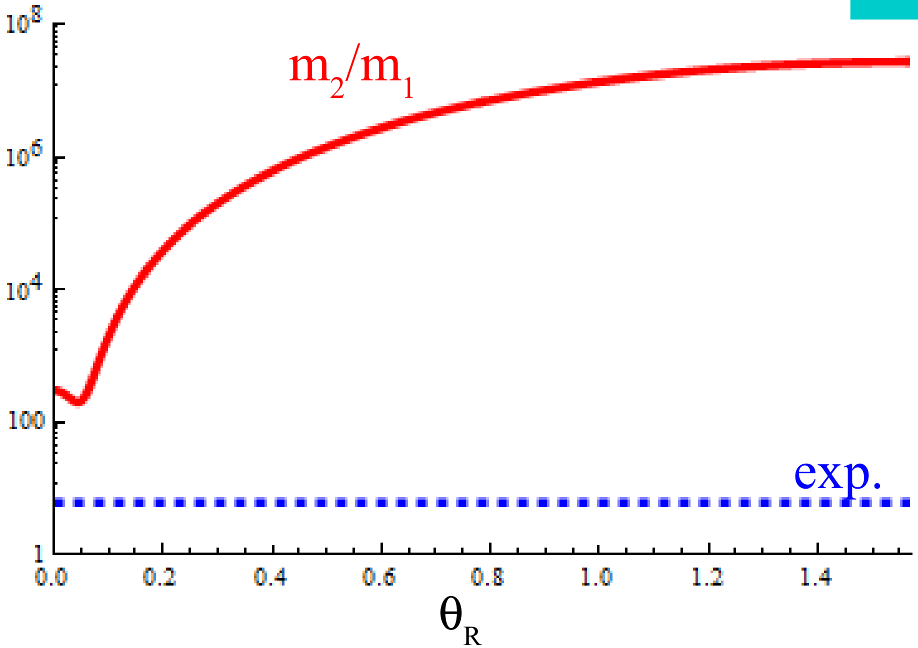

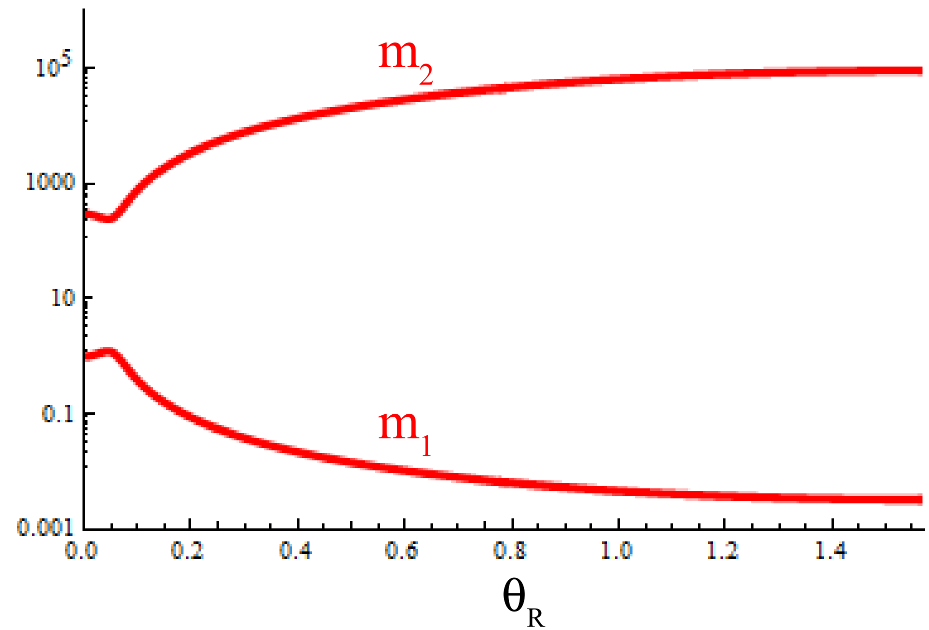

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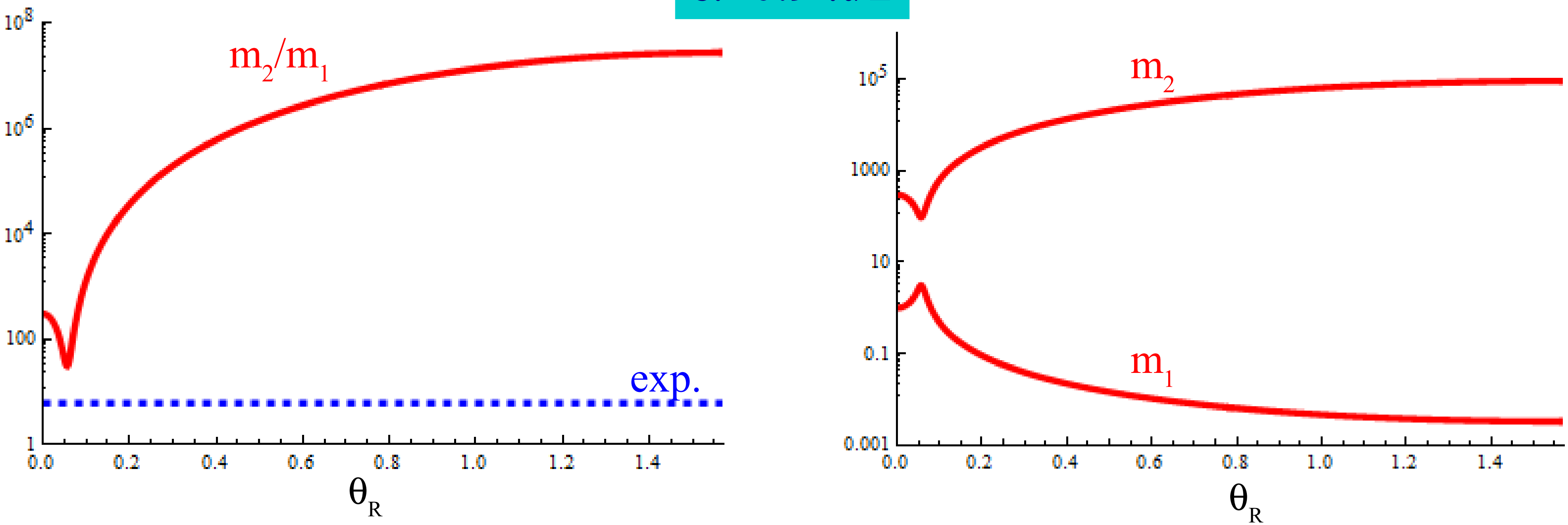

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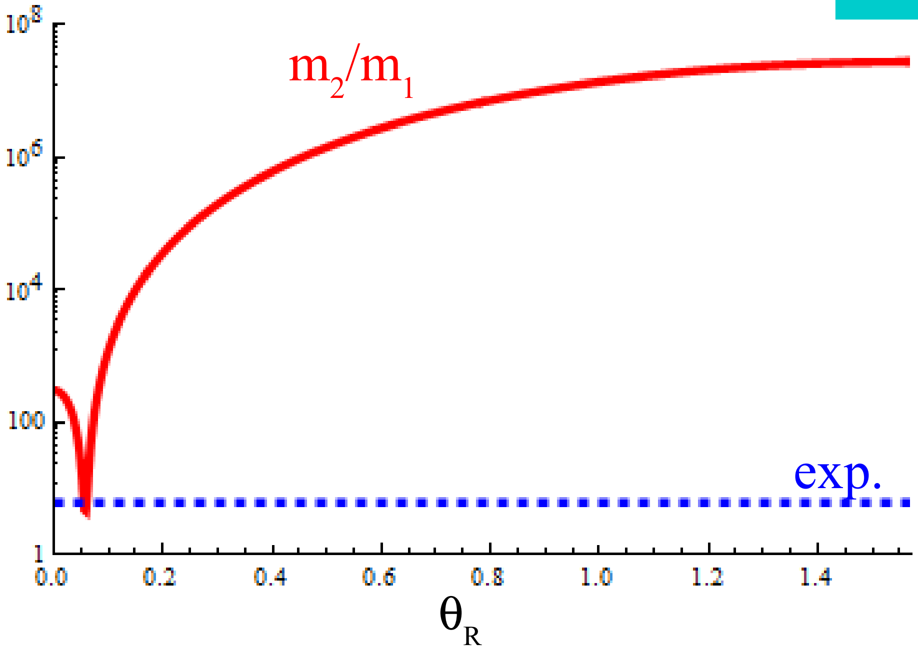

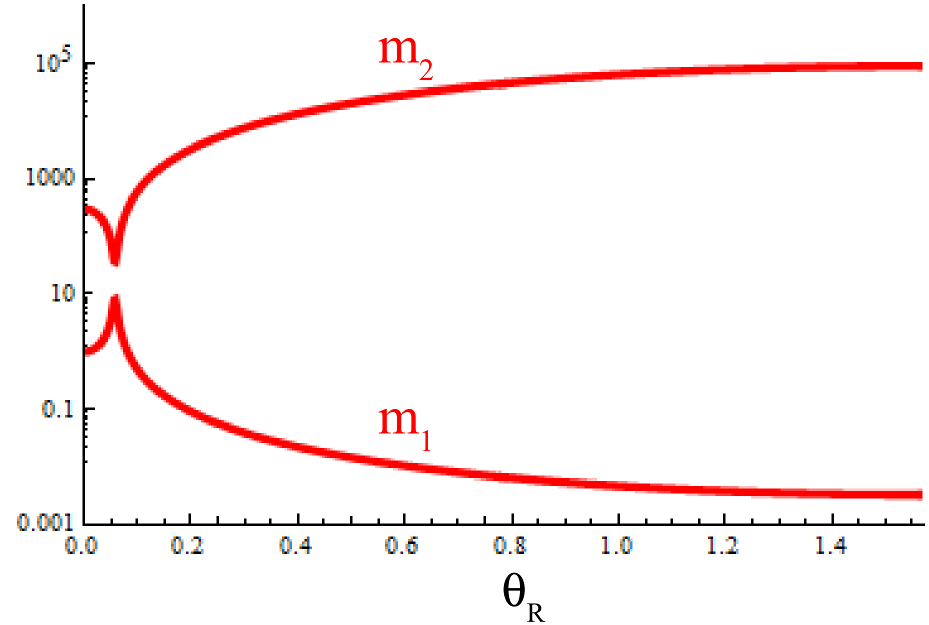

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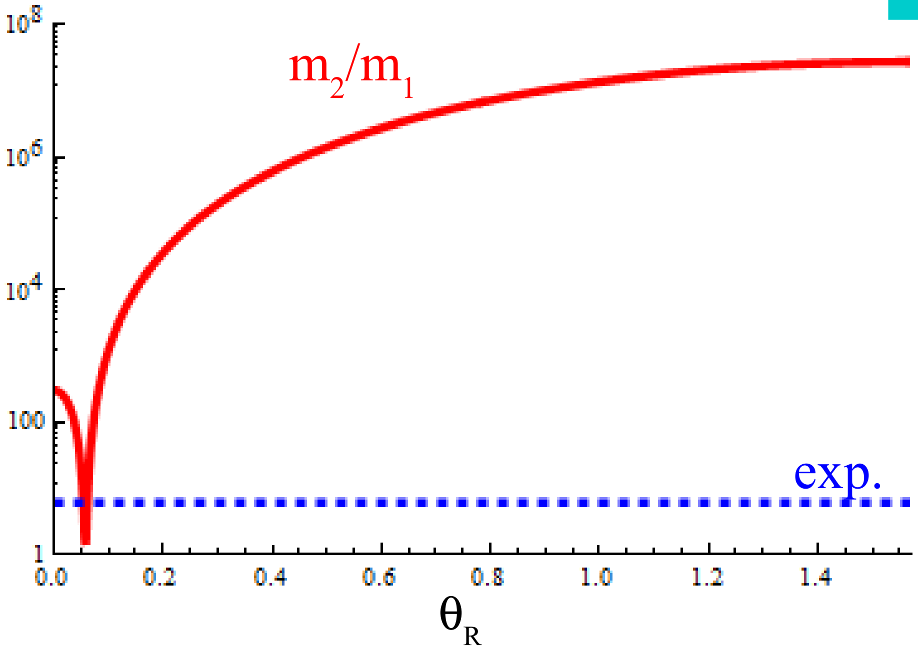

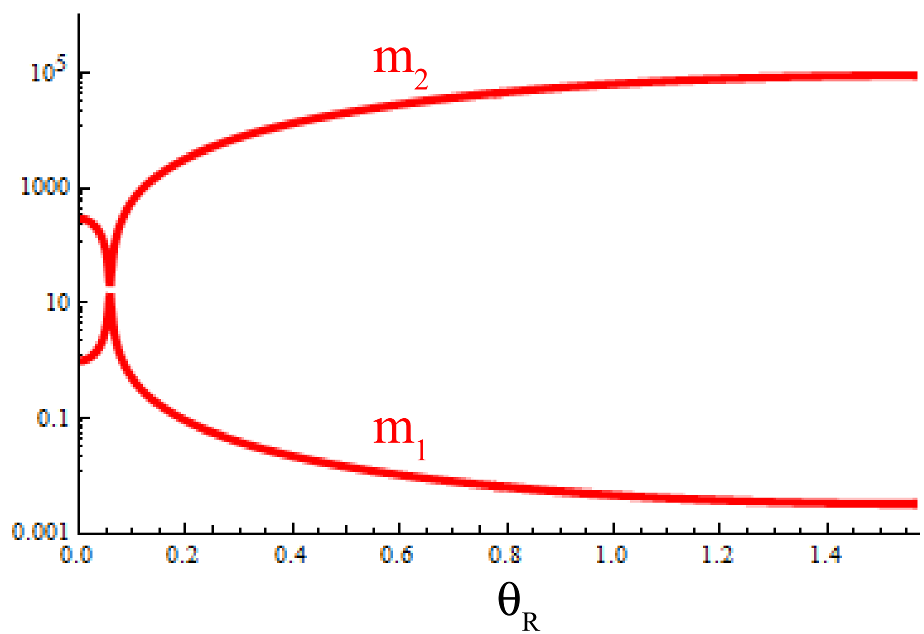

The see-saw mechanism (with two right-handed neutrinos) with hierarchical Yukawa eigenvalues and RH masses can accommodate the observed neutrino mass hierarchy, but *only for very special choices of parameters.*

$$\boxed{\cos^2\theta_R \simeq \frac{M_2 y_2^2 - M_1 y_1^2}{(M_1+M_2)(y_2^2-y_1^2)} \quad \text{and} \quad \alpha \simeq \pi/2}$$

α=π/2

There are two cases where the see-saw mechanism can naturally generate a mild neutrino mass hierarchy (without tunings):

"naive see-saw" $$\frac{m_2}{m_1} \sim \frac{y_2^2}{y_1^2}\frac{M_1}{M_2} \lesssim 6$$

Very mild hierarchies

$$y_1 \simeq y_2$$
$$M_1 \simeq M_2$$

Very strong hierarchies

$$M_1 : M_2 \simeq y_1^2 : y_2^2$$

$$y_1 : y_2 = 1 : 2$$
$$M_1 : M_2 = 1 : 6$$

$$y_1 : y_2 = 1 : 300$$
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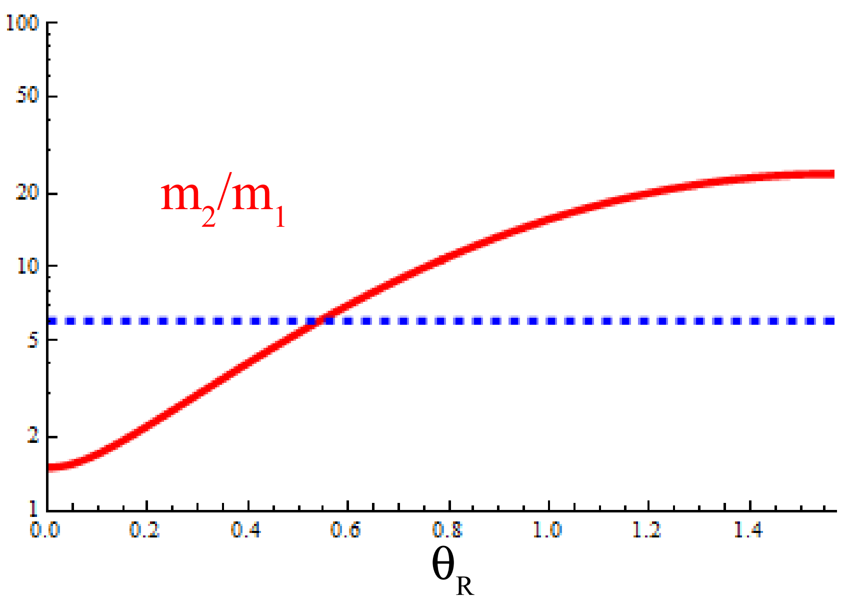
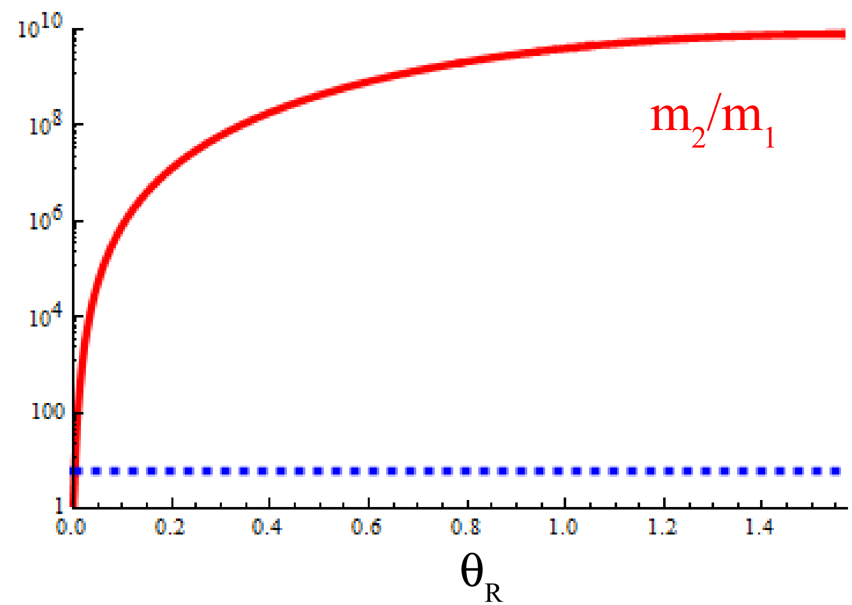


$\alpha=0$



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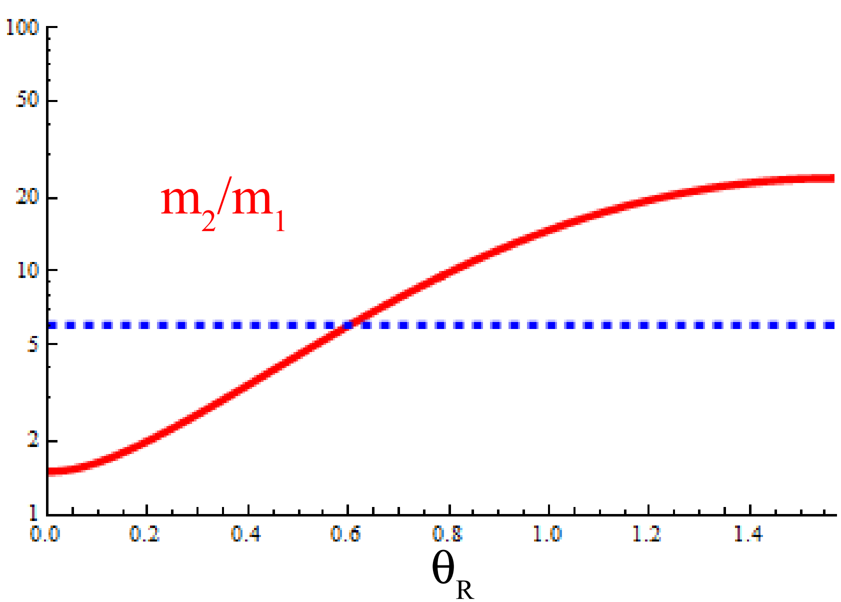

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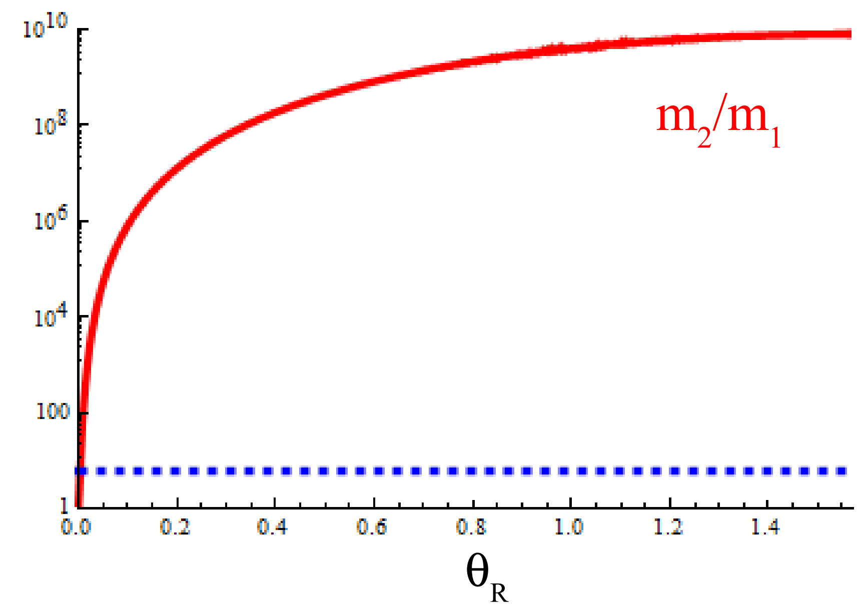

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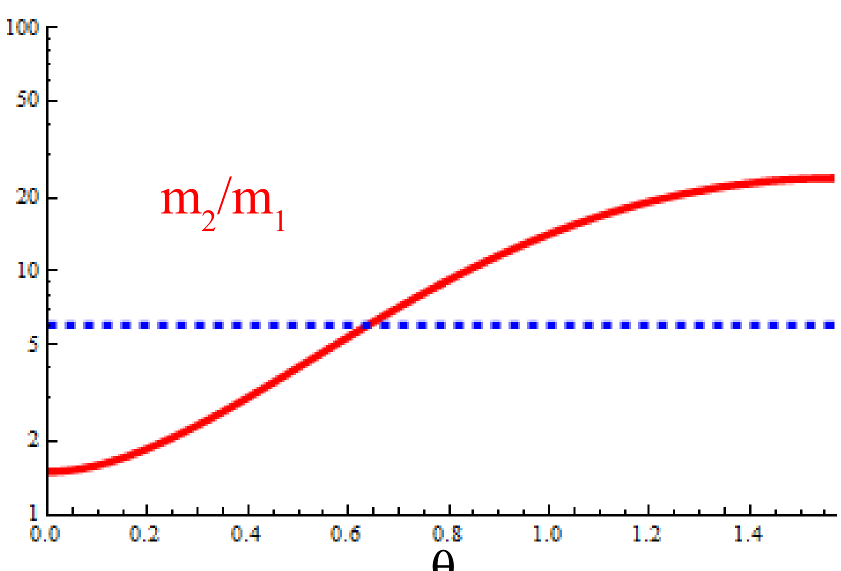
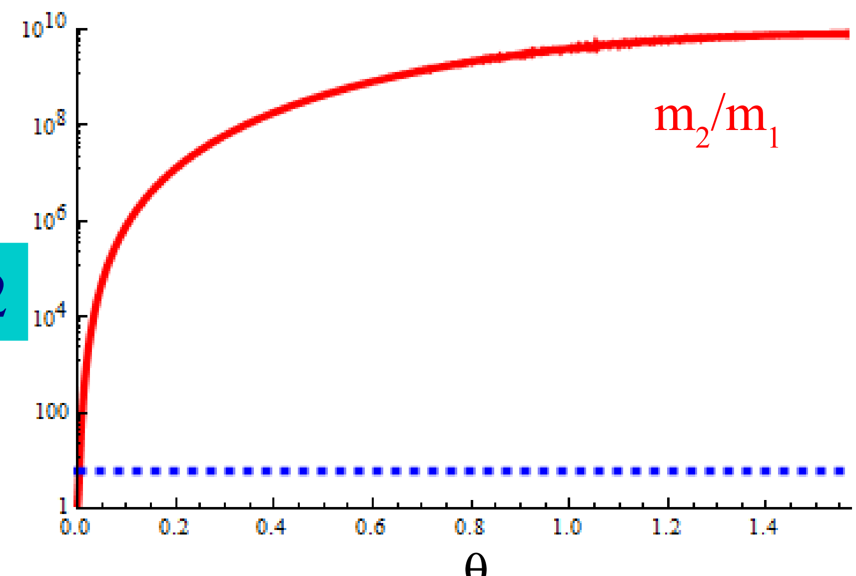


α=0.7 π/2



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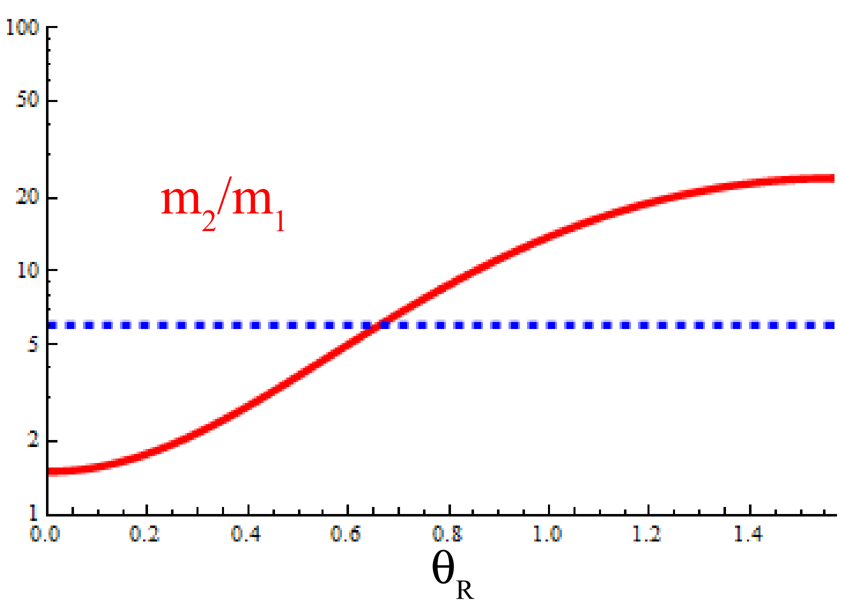

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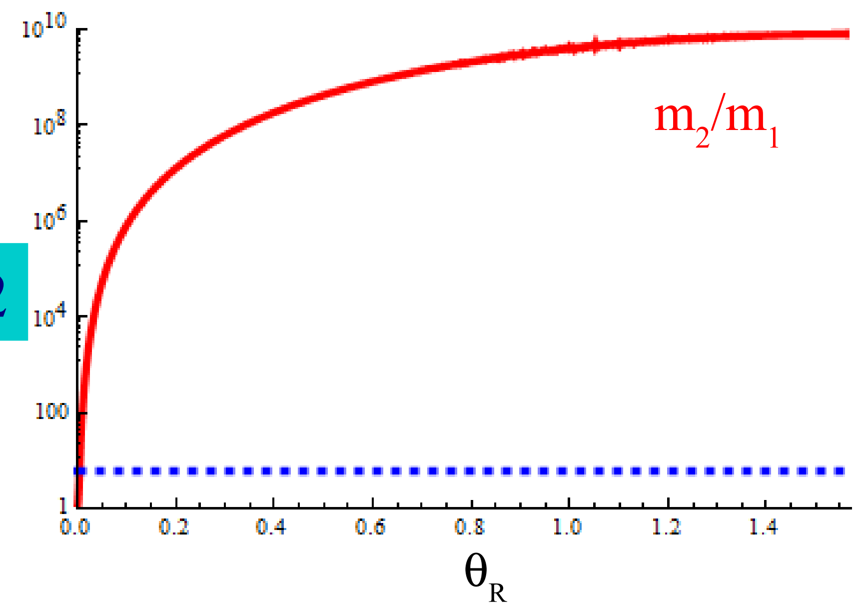

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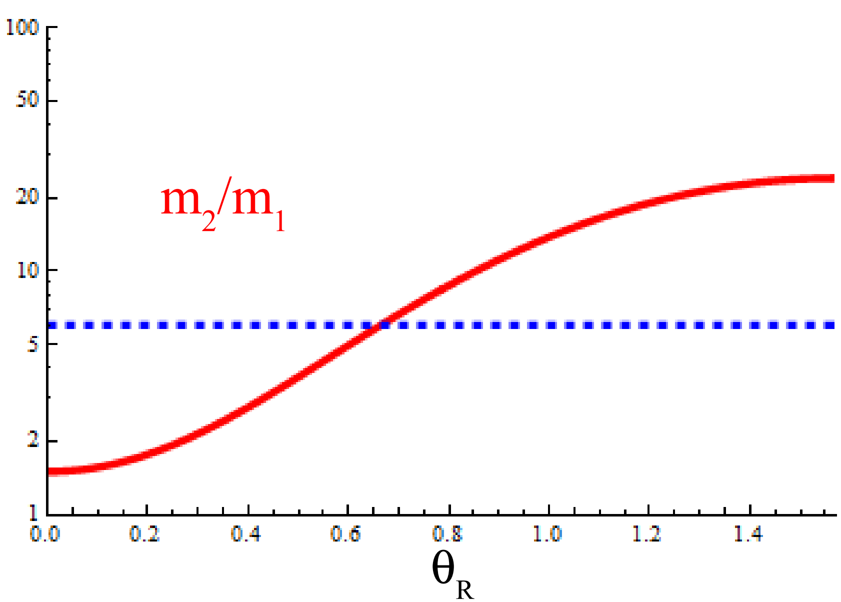

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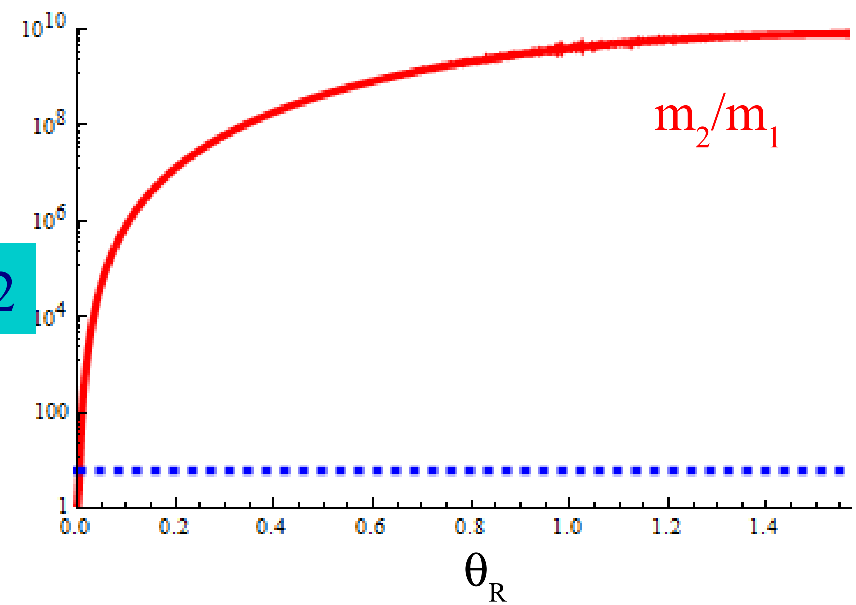

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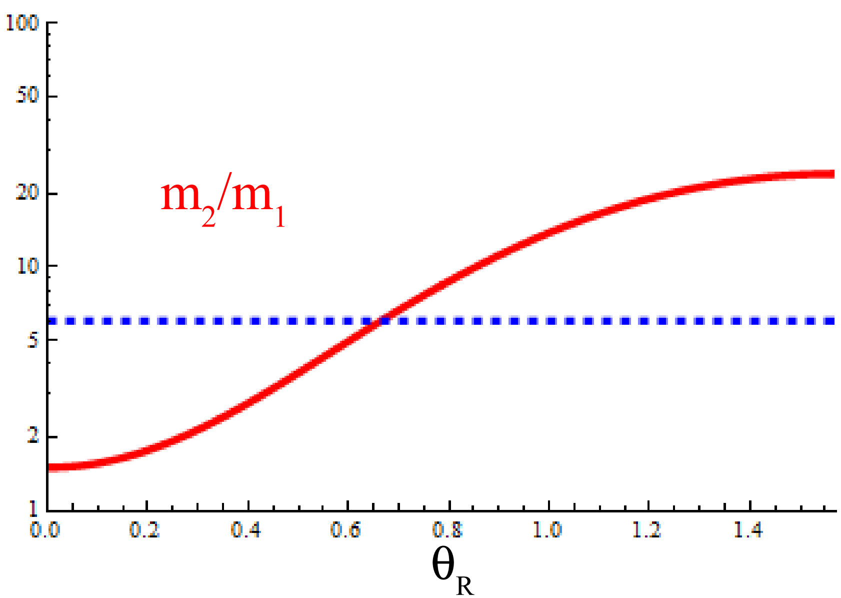

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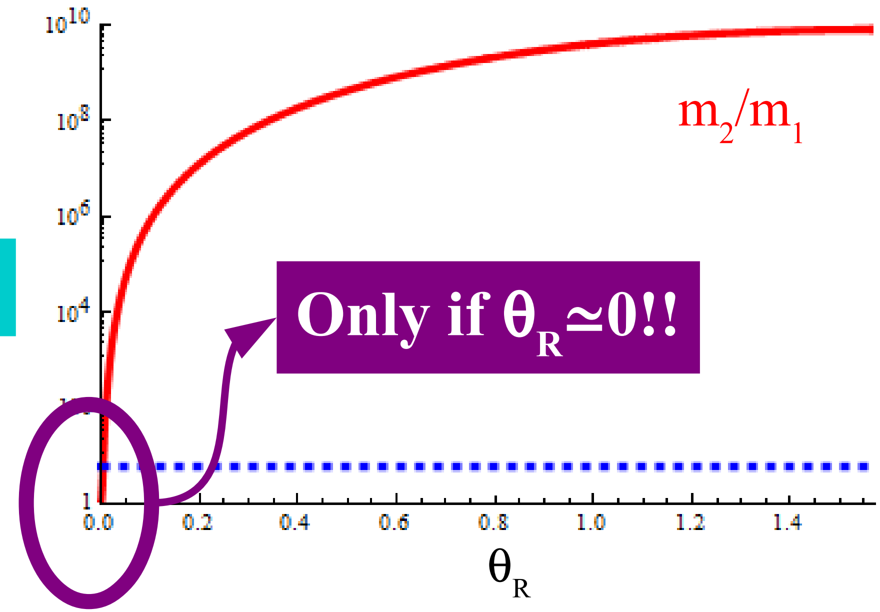
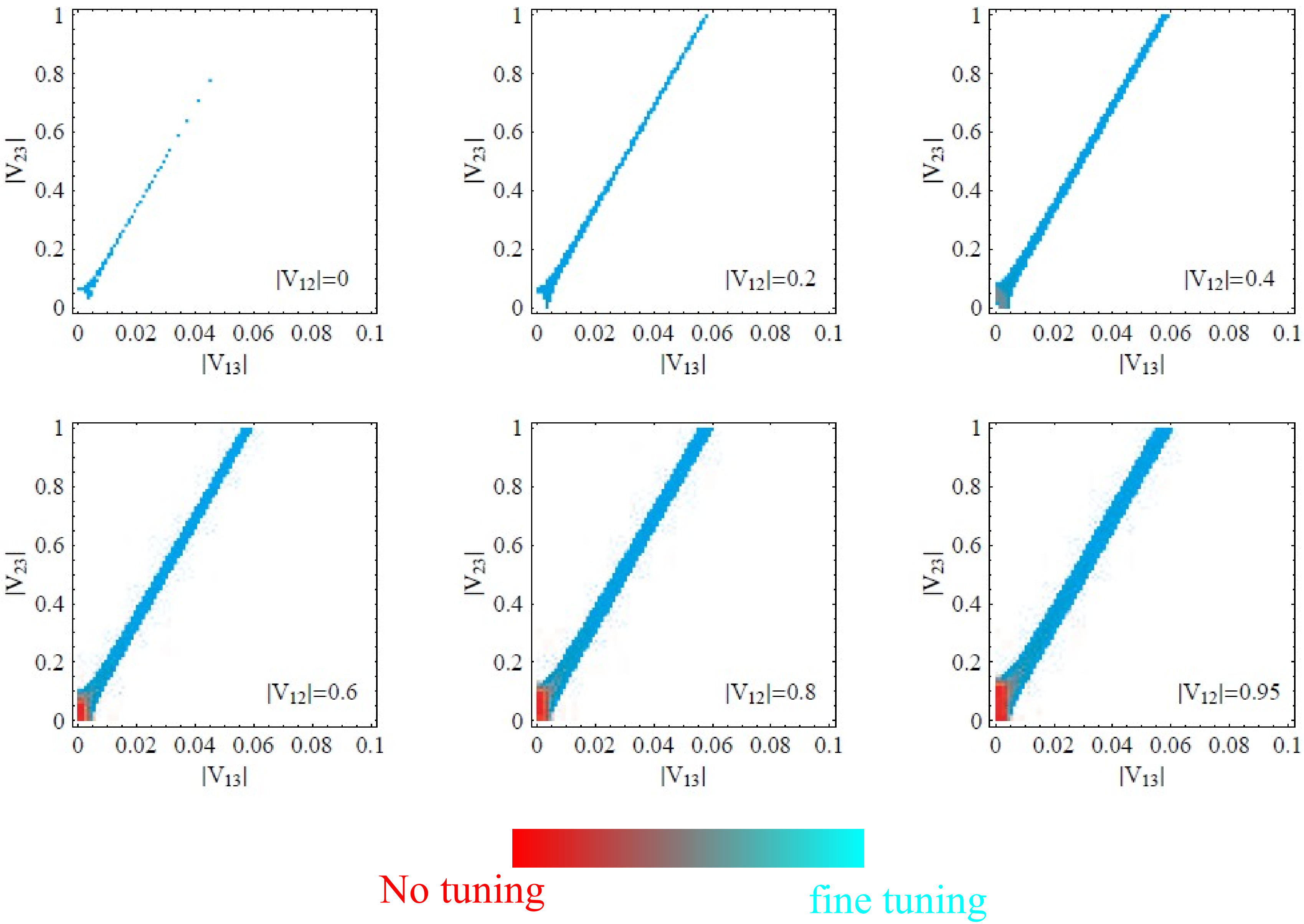


The case with three right-handed neutrinos

$$y_1 : y_2 : y_3 = 1 : 300 : 300^2$$

$$M_1 : M_2 : M_3 = 1 : 300 : 300^2$$



The see-saw mechanism generates a neutrino mass hierarchy much larger than the observed experimentally, except:

- When the Yukawa eigenvalues and right-handed masses present a mild mass hierarchy.

- In the case of hierachical Yukawa eigenvalues, for very special choices of the parameters.

The see-saw mechanism provides a very compelling explanation to the smallness of neutrino masses, while keeping all the successes of the Standard Model. However, it fails to provide a compelling explanation to why the neutrino mass hierarchy is so mild.

A possible solution: Introduce a second higgs doublet

With a second higgs doublet, quantum corrections can soften the neutrino mass hierarchy.

Even if at tree level m_3/m_2 is very large, as generically expected in the (standard) see-saw mechanism, the quantum corrections can generate $m_3/m_2 \sim 6$.

Grimus, Neufeld
AI, Simonetto

Neutrino masses in the see-saw model extended with one extra Higgs

Consider the Standard Model extended by right-handed neutrinos and at least one extra Higgs doublet

$$-\mathcal{L}^{\nu} = (Y_\nu^a)_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\Phi}_a - \frac{1}{2}M_{\mathrm{M}ij}\bar{\nu}_{Ri}^C\nu_{Rj} + \mathrm{h.c.}$$

$$\Big\Downarrow \quad M_{\mathrm{Maj}} \gg m_H, M_Z$$

$$-\mathcal{L}^{\nu,\ \mathrm{eff}} = \frac{1}{2}\kappa_{ij}^{ab}(\bar{l}_{Li}\tilde{\Phi}_a)(\tilde{\Phi}_b^T l_{Lj}^C) + \mathrm{h.c.}$$

$$\kappa^{ab}(M_1) = (Y_\nu^a M_{\mathrm{M}}^{-1} Y_\nu^{b\,T})(M_1)$$

Work in the basis where only Φ_1 acquires a vev

$$\mathcal{M}_\nu(M_1) = \frac{v^2}{2}\kappa^{11}(M_1)$$

The neutrino mass matrix is affected by quantum corrections below M_1

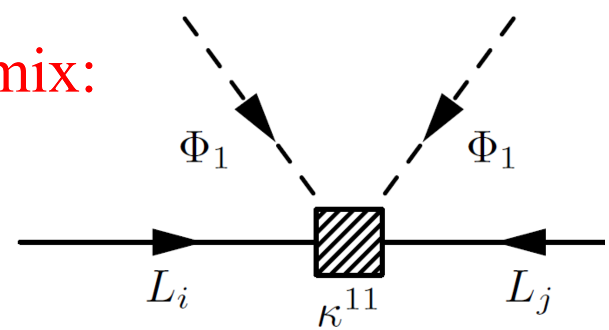
RGE effects

Quantum effects generate a correction to the coefficient of the dimension 5 operator which generates neutrino masses:

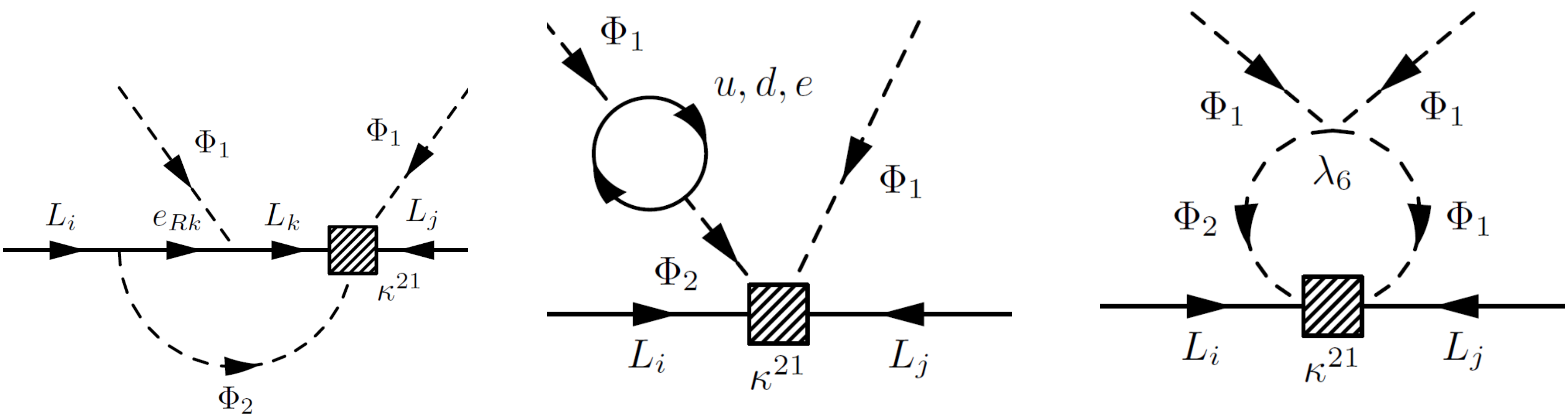
$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$

Grimus, Lavoura

Different operators mix:



Is corrected by $B_{12}\,\kappa^{21}$



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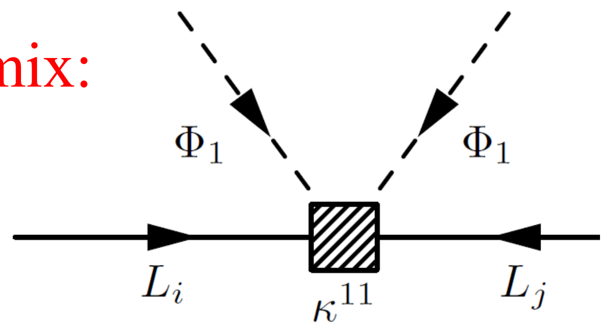

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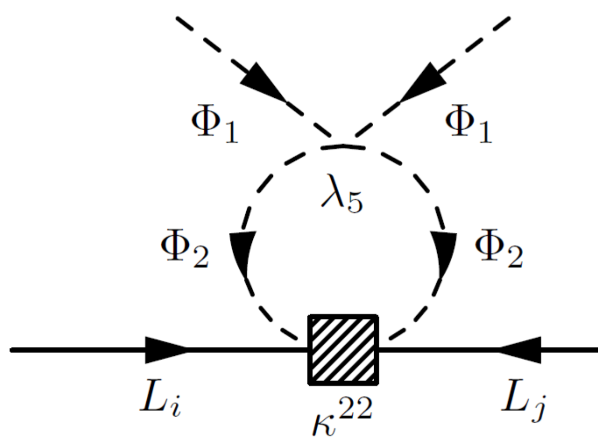

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Grimus, Lavoura

Different operators mix.

Compare to the correction in the "one Higgs doublet model":

$$\delta\kappa \simeq B\kappa + \kappa B^T$$

Babu, Leung, Pantaleone

New qualitative features?

To highlight the new features, consider a model with one right-handed neutrino and two higgs doublets (no discrete symmetries imposed):

$$-\mathcal{L}^{\nu} = (Y_\nu^1)_i \bar{l}_{Li} \nu_R \tilde{\Phi}_1 + (Y_\nu^2)_i \bar{l}_{Li} \nu_R \tilde{\Phi}_2 - \frac{1}{2} M_{\rm Maj} \bar{\nu}_R^C \nu_R + \text{h.c.}$$

$$\Big\Downarrow \quad M_{\rm Maj} \gg m_H, M_Z$$

$$-\mathcal{L}^{\nu,\ \rm eff} = \frac{1}{2} \kappa_{ij}^{ab} (\bar{l}_{Li} \tilde{\Phi}_a)(\tilde{\Phi}_b^T l_{Lj}^C) + \text{h.c.}$$

Work in the basis where only Φ_1 acquires a vev

$$\mathcal{M}_\nu(M_{\rm Maj}) = \frac{v^2}{2} \kappa^{11}(M_{\rm Maj})$$

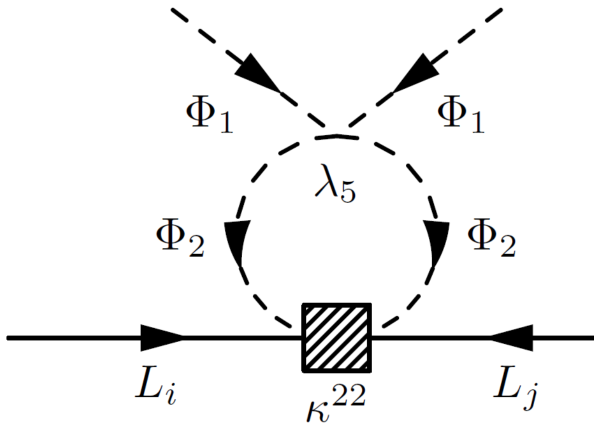
Rank 1. At tree level

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2 M_{\rm Maj}}$$

$$m_2,\ m_1 = 0$$

RGE effects

$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$



$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}} + \text{small corrections}$$

$$m_2 = -\frac{1}{16\pi^2}\frac{|\lambda_5|v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\left(\frac{M_{\rm maj}}{m_H}\right)$$

$$m_1 = 0$$

A second neutrino mass is generated from the same right-handed neutrino mass scale $M_{\rm maj}$ → a mild mass hierarchy might be naturally accommodated.

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\text{maj}}}$$

$$m_2 = -\frac{1}{16\pi^2}\frac{|\lambda_5|v^2}{M_{\text{maj}}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\left(\frac{M_{\text{maj}}}{m_H}\right)$$

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H \ll M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)
- $|Y_\nu^1| \sim |Y_\nu^2|$
- $\lambda_5 \sim O(1)$

$$\boxed{\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\text{maj}}}{m_H}\right) \simeq 0.2}$$

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$$m_2 = -\frac{1}{1\ldots}\frac{|\lambda_5| v^2}{\ldots}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger} Y_\nu^1|^2}{\ldots}\right]\log\left(\frac{M_{\rm maj}}{\ldots}\right)$$

LEPTON FLAVOUR VIOLATION?
ELECTRIC DIPOLE MOMENTS?

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- $\lambda_5 \sim \mathcal{O}(1)$

Logarithmic dependence with m_H, while the rate for $\mu \to e\gamma$ decreases as m_H^4

$$\boxed{\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2}$$

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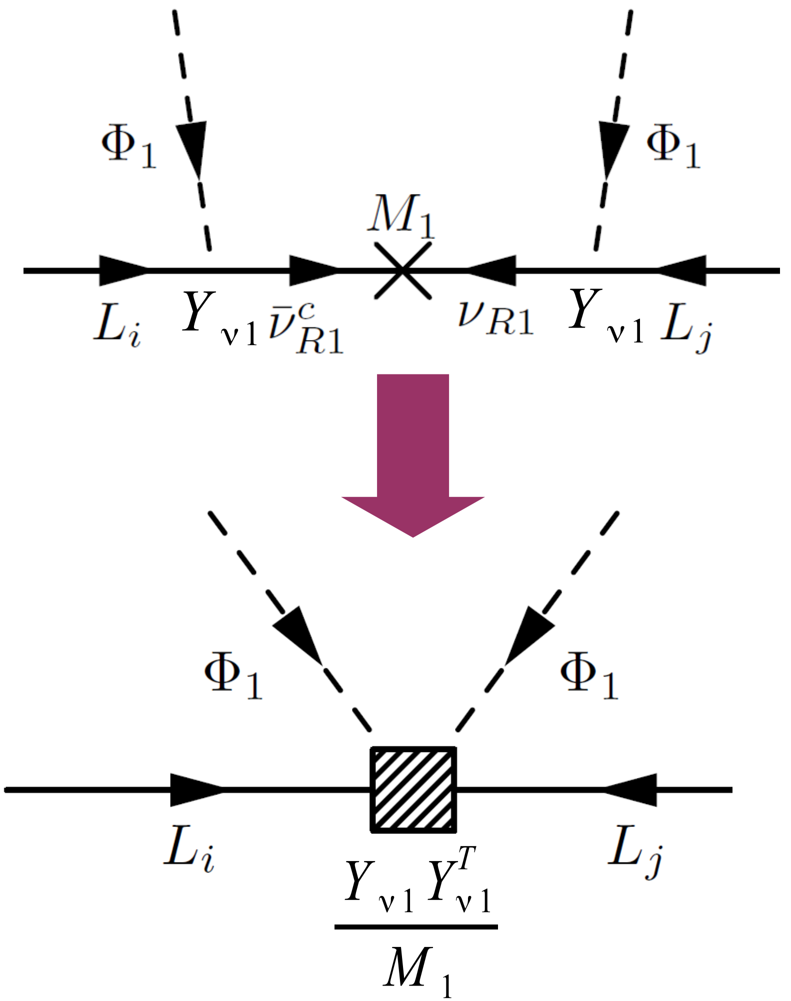
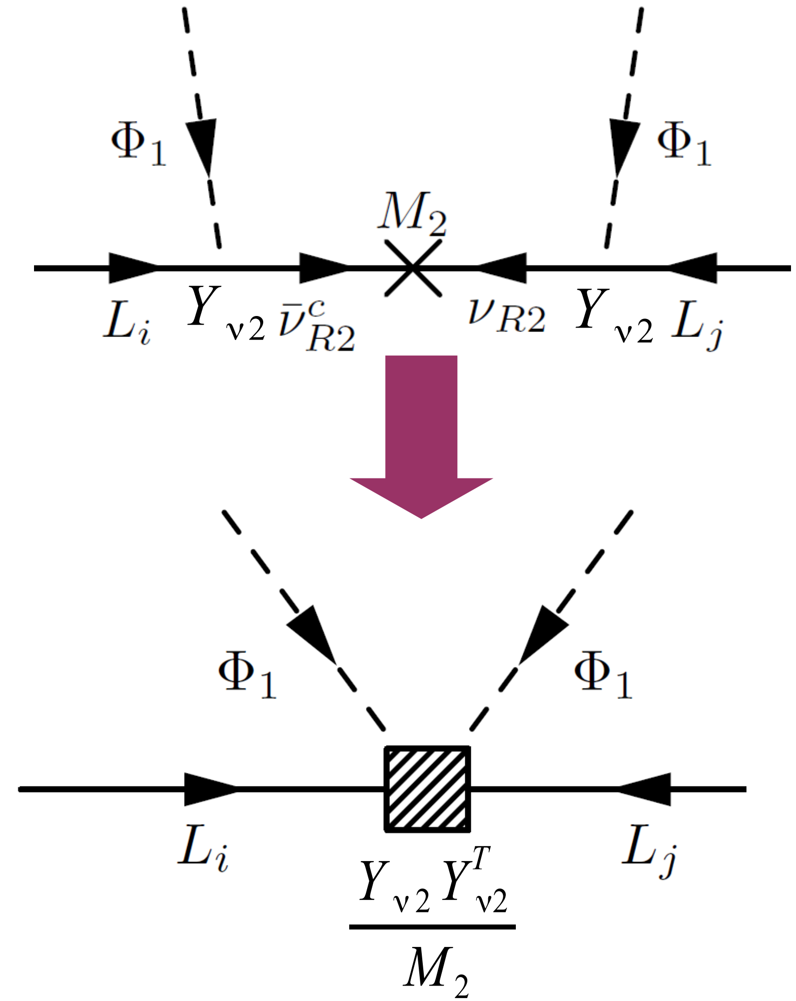
Comparison to the two right-handed neutrino model

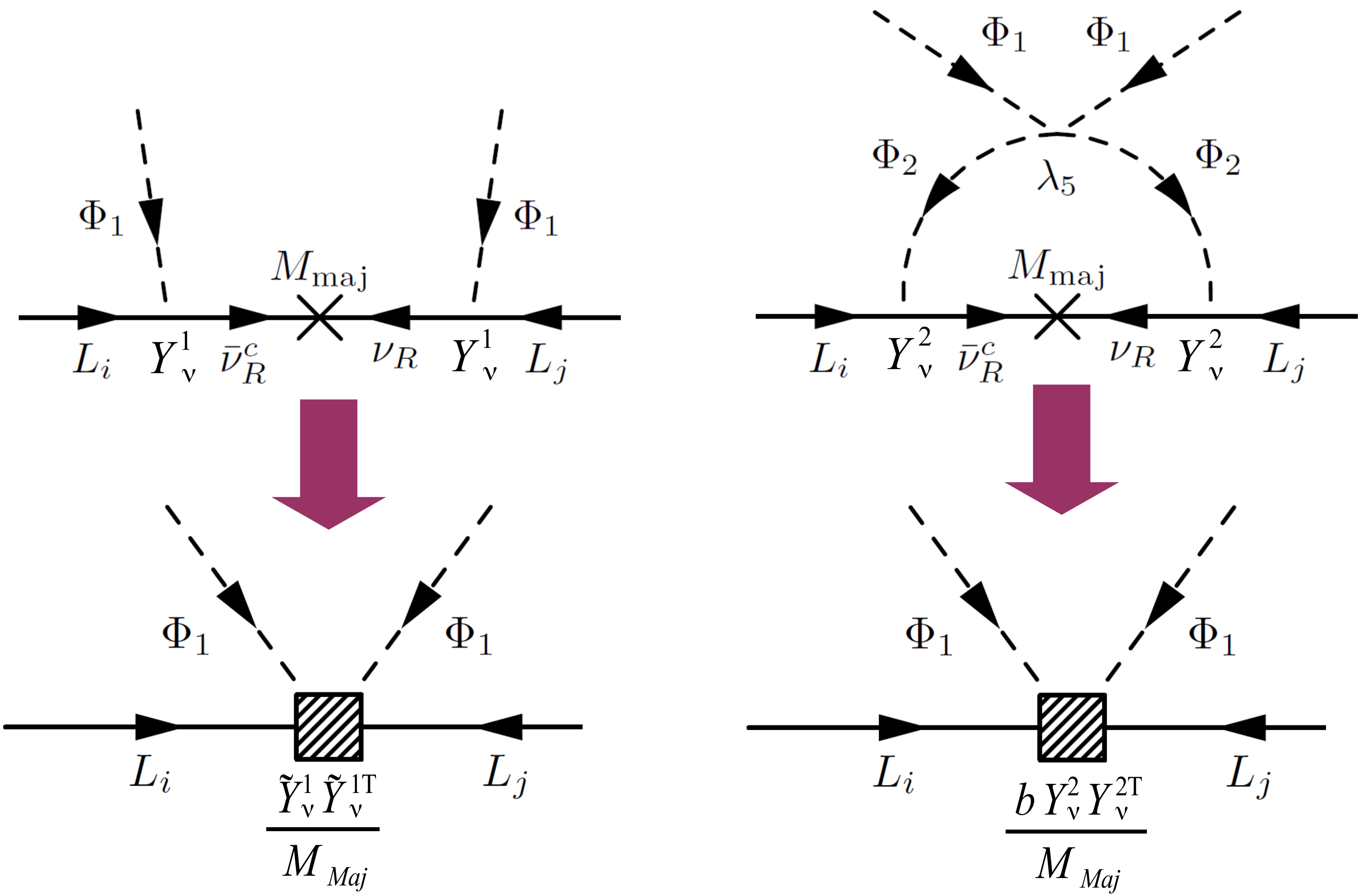
Effective theory of the 2RHN-1HDM





Comparison to the two right-handed neutrino model

Effective theory of the 1RHN-2HDM



$$\tilde Y^1_\nu = Y^1_\nu + B_{1a} Y^a_\nu$$

The effective theories are identical

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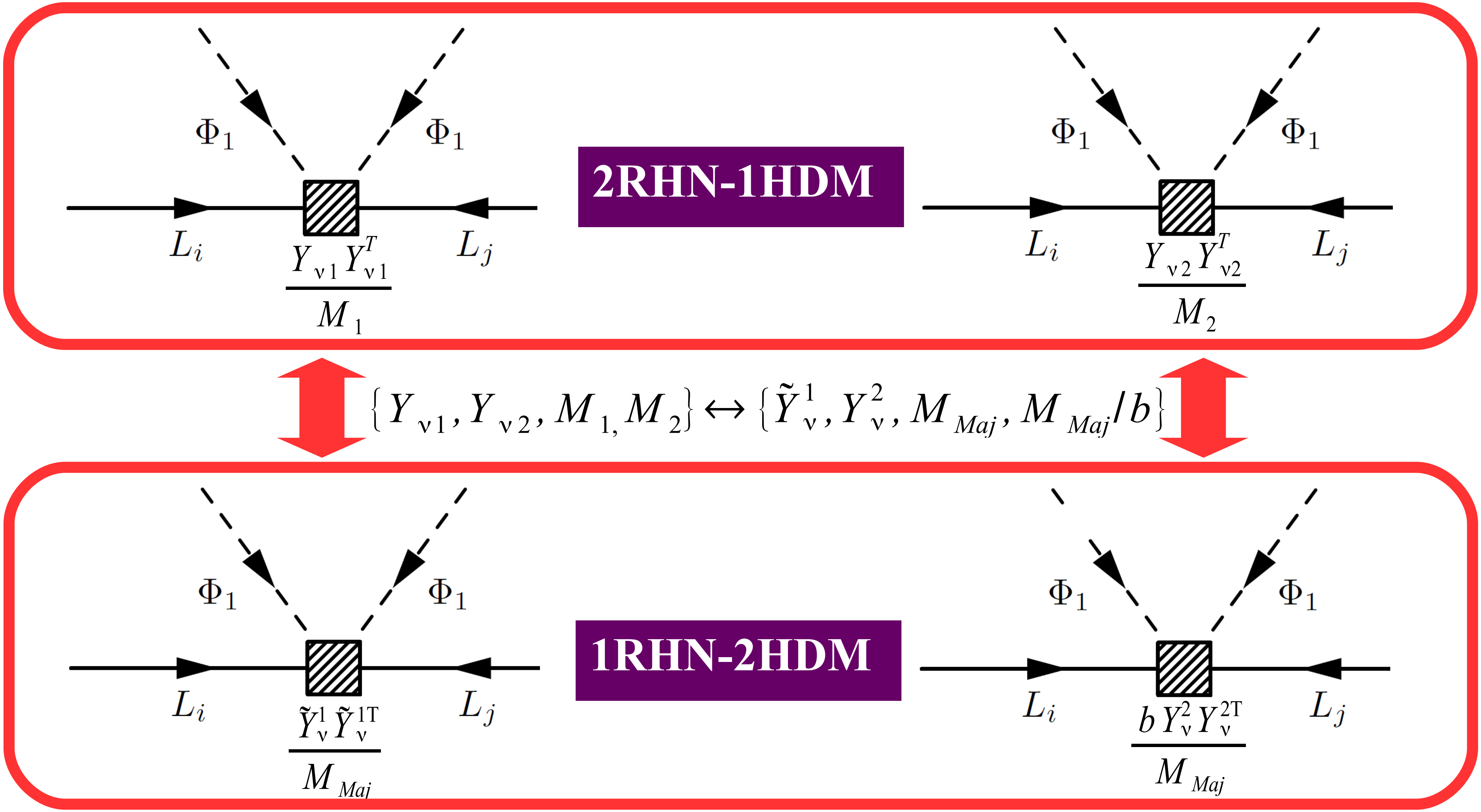

However, there are important differences in the way the can generate the mild neutrino mass hierarchy.

First part of the talk: the 2RHN-1HDM can generate a neutrino mass hierarchy in agreement with experiments when:

- When the Yukawa eigenvalues *and* right-handed masses present a mild hierarchy.

- When there are hierachical Yukawa eigenvalues, only for very special choices of the parameters.

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1RHN-2HDM

$$\{\underbrace{Y_{\nu 1}, Y_{\nu 2}}, \underbrace{M_{1}, M_{2}}\}$$

Yukawa couplings to different generation of RH neutrinos.
Possibly hierarchical eigenvalues

Two different right-handed neutrino masses.
Hierarchy completely unknown

First part of the talk: the 2RHN-1HDM can generate a neutrino mass hierarchy in agreement with experiments when:

- When the Yukawa eigenvalues *and* right-handed masses present a mild hierarchy. *implausible*
- When there are hierachical Yukawa eigenvalues, only for very special choices of the parameters. *tuned?*

1RHN-2HDM

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1RHN-2HDM 1RHN-2HDM

$$\{Y_{\nu 1}, Y_{\nu 2}, M_{1,} M_{2}\} \leftrightarrow \{\underbrace{\tilde{Y}^{1}_{\nu}, Y^{2}_{\nu}}, \underbrace{M_{Maj}, M_{Maj}/b}\}$$

$$b = -\frac{\lambda_5}{8\pi^2}\log\left(\frac{M_{\mathrm{Maj}}}{m_H}\right)$$

Yukawa couplings to *the same* generation of RH neutrinos.
Eigenvalues expected to be similar

The "equivalent" right-handed neutrino masses naturally have a mild hierarchy.

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Generic situation in 1RHN-2HDM

- When there are hierachical Yukawa eigenvalues, only for very special choices of the parameters.

1RHN-2HDM 1RHN-2HDM

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Yukawa couplings to *the same* generation of RH neutrinos. Eigenvalues expected to be similar

The "equivalent" right-handed neutrino masses naturally have a mild hierarchy.

A remarkable difference with respect to the two right-handed neutrino model:

Possibly, new phenomena at low energies, apart from neutrino masses

LFV processes could be observable, if not too suppressed by m_H.

$$\mathrm{BR}(\mu \to e\,\gamma) = \frac{8\alpha^3}{3\pi^3} \frac{|Y^2_{e12}|^2 + |Y^2_{e21}|^2}{|Y^1_{e22}|^2} \left| f\left(\frac{m_t^2}{m_h^2}\right) \cos\alpha - \frac{Y^2_{u33}}{Y^1_{u33}} \frac{m_t^2}{m_H^2} \log^2 \frac{m_t^2}{m_H^2} \right|^2$$

Paradisi
Hisano, Sugiyama, Yamanaka

Could be present at tree level.
If not, generated radiatively by the neutrino Yukawa couplings

Mixing angles

Operator mixing has also effects on the mixing angles

Assume that at tree level, $\theta_{13}=0$, $\theta_{23}=\pi/4$.

New flavour structures in κ^{22} and Y_e^2 can induce radiatively a non-vanishing θ_{13} and a deviation from maximal atmospheric mixing, through quantum corrections to the neutrino mass operator κ^{11} and the charged lepton Yukawa coupling Y_e^1.

$$\begin{aligned}
\delta U_{13} = \frac{1}{16\pi^2}\frac{Y^{2*}_{\nu 1}}{|Y^1_\nu|}\Big\{ & -3\mathrm{Tr}(Y_u^{1\dagger}Y_u^2 + Y_d^1 Y_d^{2\dagger})\log\frac{\Lambda}{m_H} \\
& -\left(2\lambda_6^* + 2\lambda_5^*\frac{Y_\nu^{2\dagger}Y_\nu^1}{|Y_\nu^1|^2}\right)\log\frac{M_{\rm maj}}{m_H} \\
& \left(-\mathrm{Tr}(Y_\nu^2 Y_\nu^{1\dagger}) - 2Y_\nu^{1\dagger}(Y_e^1)^{-1}Y_e^{2\dagger}Y_\nu^1\right)\log\frac{\Lambda}{M_{\rm maj}}\Big\} \\
+ \frac{1}{16\pi^2}\frac{(Y_\nu^{1\dagger}(Y_e^1)^{-1}Y_e^{2\dagger})_1}{|Y_\nu^1|} & Y_\nu^{2\dagger}Y_\nu^1\log\frac{\Lambda}{M_{\rm maj}}
\end{aligned}$$

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\end{aligned}
$$

Similar to m_2/m_3 $\rightarrow$ δU_{13} can easily be ~ 0.2

Some speculations about the mixing angles

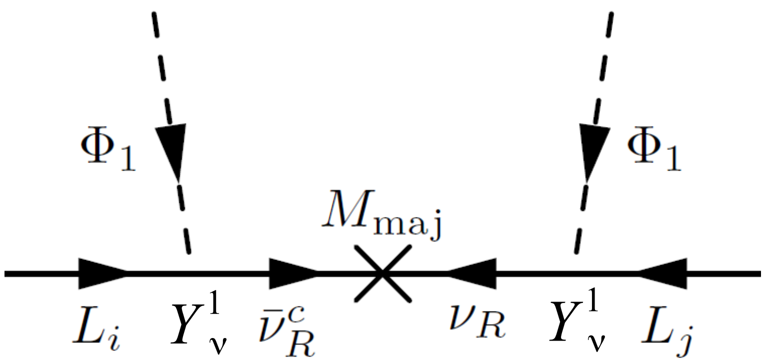
The third column of the leptonic mixing matrix seems to follow a pattern, at least at lowest order: $\theta_{13}=0$, $\theta_{23}=\pi/4$.

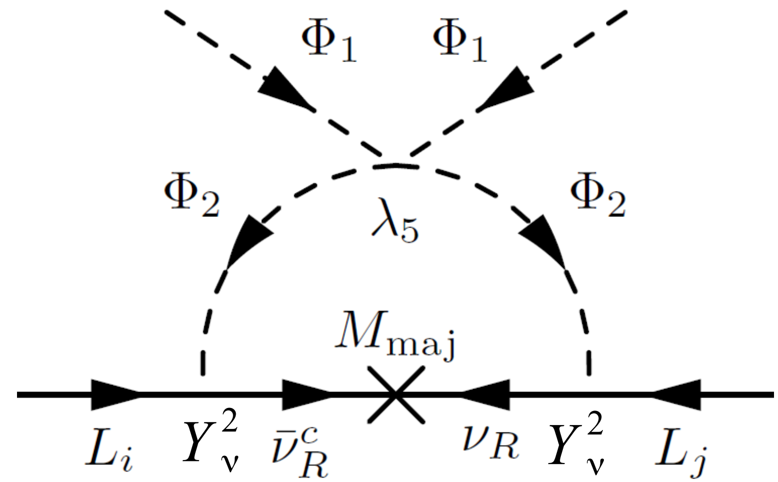
$$U_{i3} \approx \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

The second column does not seem to follow any pattern: the solar mixing angle is neither minimal nor maximal.

$$U_{i2} \approx \begin{pmatrix} O(1) \\ O(1) \\ O(1) \end{pmatrix}$$

In the 1RHN-2HDM





Third column of U_{lep}
Possibly with a pattern
(if Y_ν^1 has a pattern)

second column of U_{lep}
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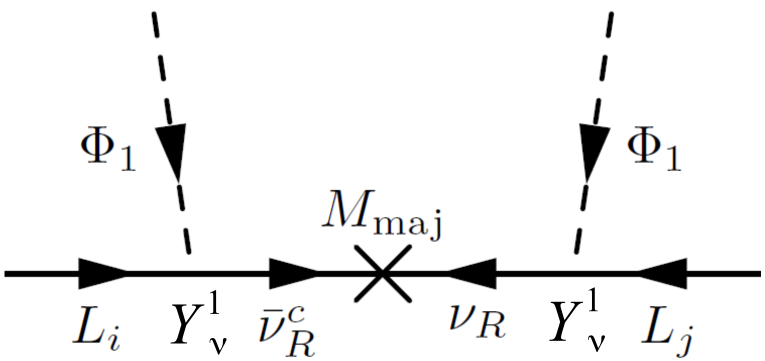
In the 1RHN-3HDM,
(more higgs doublets!)



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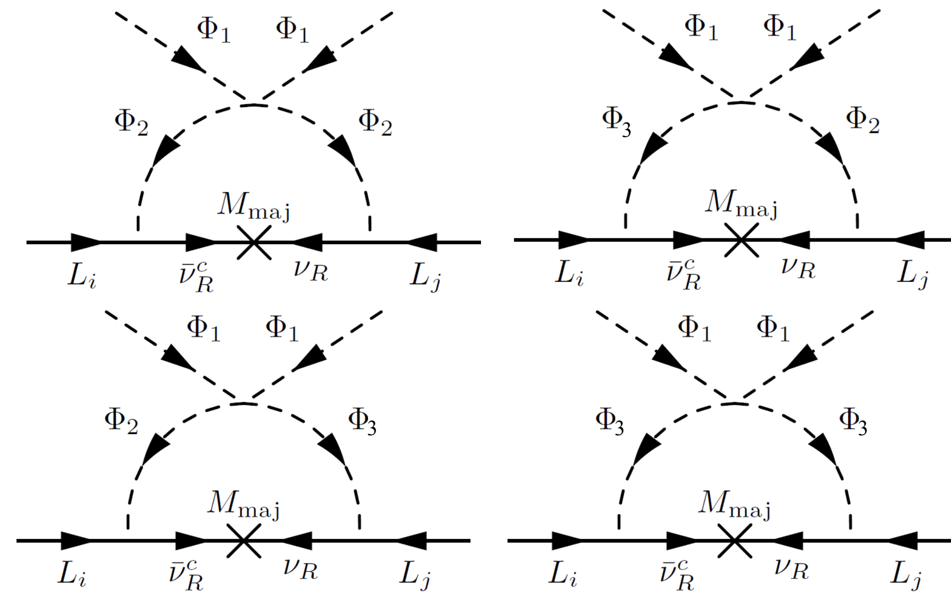

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Even if each Yukawa coupling had an structure, the combination of them gives a "structureless" U_{i2}.

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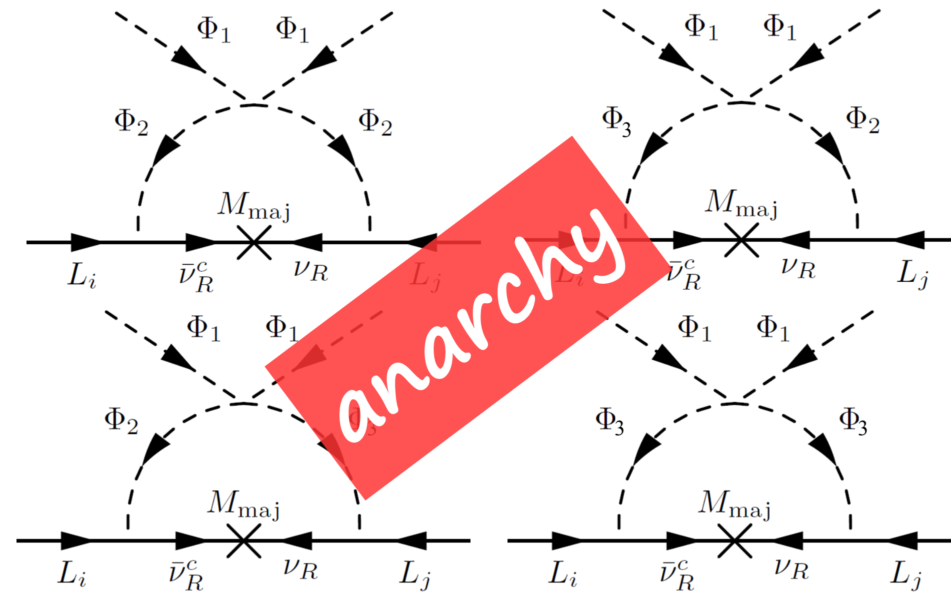

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Conclusions

- The Standard Model extended with right-handed neutrinos can explain the smallness of neutrino masses while preserving the successes of the Standard Model
- However, it is challenging to explain in this framework the neutrino mass hierarchy inferred from experiments.
- We have considered the Standard Model extended with at least one right-handed neutrino and at least one extra Higgs doublet, with the following pattern of masses:

$$M_Z \ll M_H \ll M_{maj}$$

This model generates:

- Small neutrino masses.
- Mild neutrino mass hierarchy.
- New insight into mixing angles ("anarchic" solar mixing).
- No tension with FCNC, LFV, EDMs.