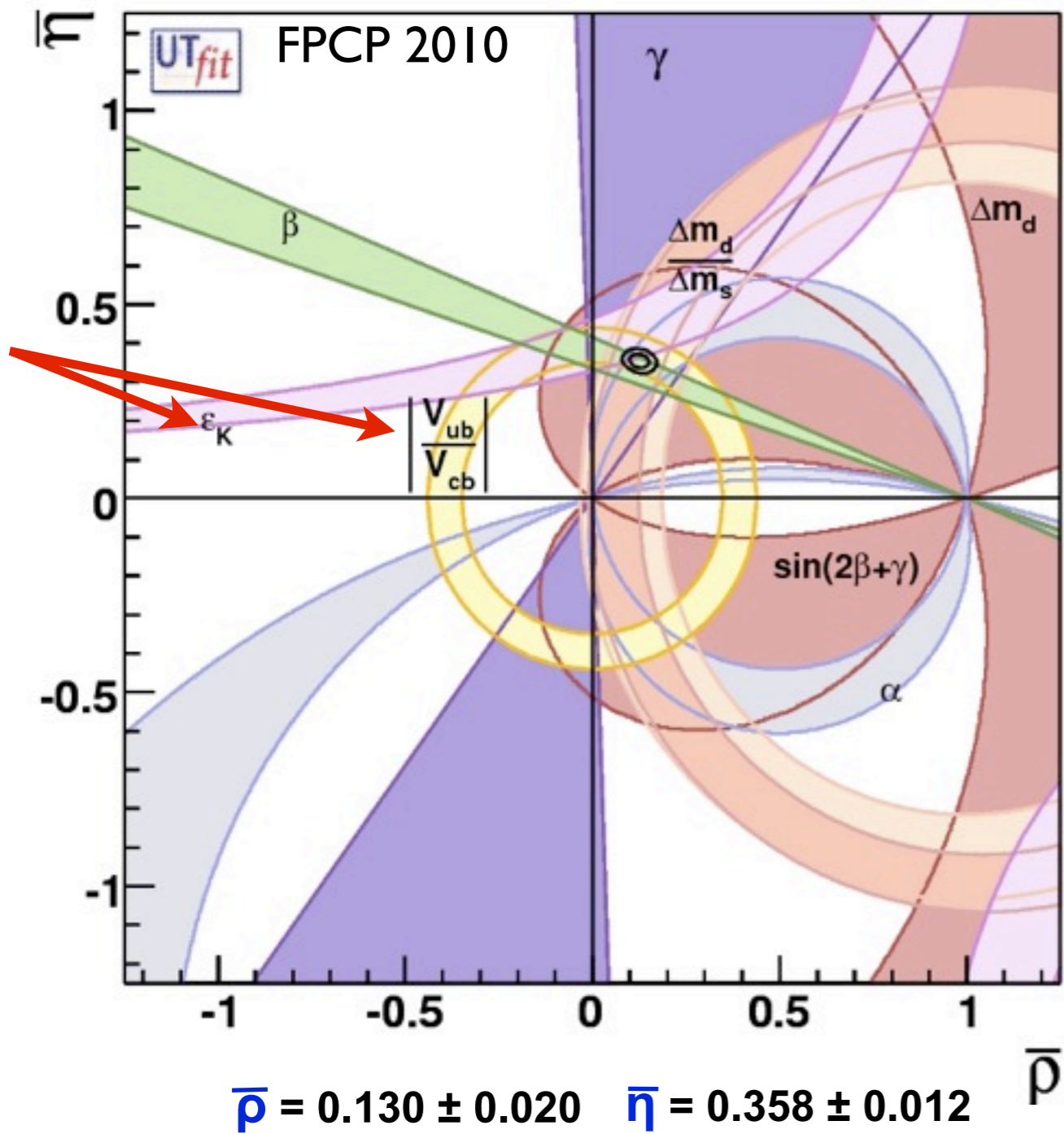


# Developments in semileptonic B decays

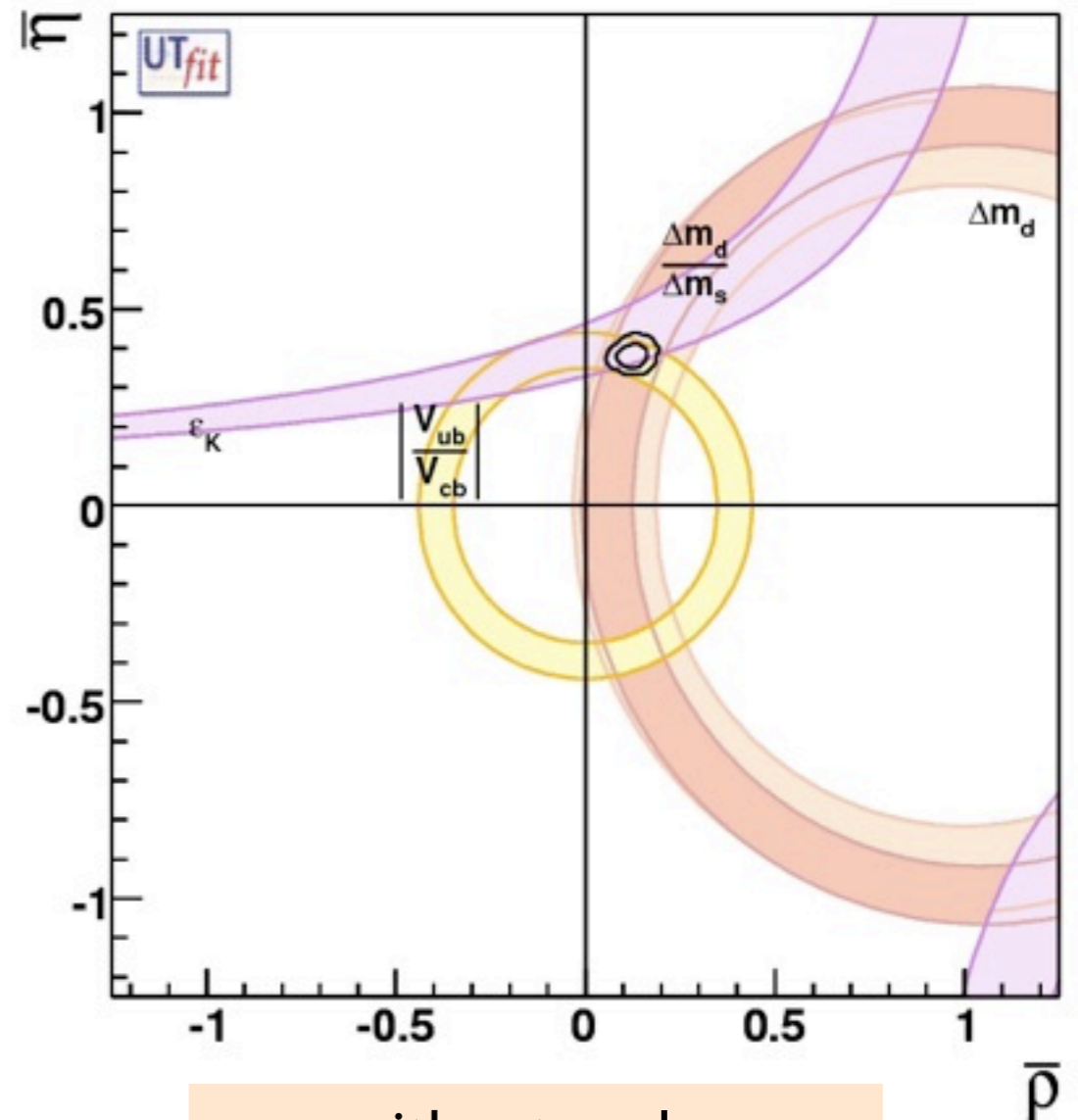
Paolo Gambino  
Università di Torino



# The UT and $V_{ub}, V_{cb}$



$\sin 2\beta_{\text{charmionium}} = 0.655 \pm 0.024$



without angles  
 UTfit inputs:  
 $\xi = 1.24(3)$     $B_K = 0.731(36)$   
 recent results  $B_K = 0.72(3)$   
 getting to 5% accuracy

# Tensions in the UTfit

## compatibility plots in the SM

measure the agreement of a single measurement with the indirect determination from the fit using all the other inputs

$$\sin 2\beta_{\text{exp}} = 0.655 \pm 0.024$$

$$\sin 2\beta_{\text{UTfit}} = 0.753 \pm 0.034$$

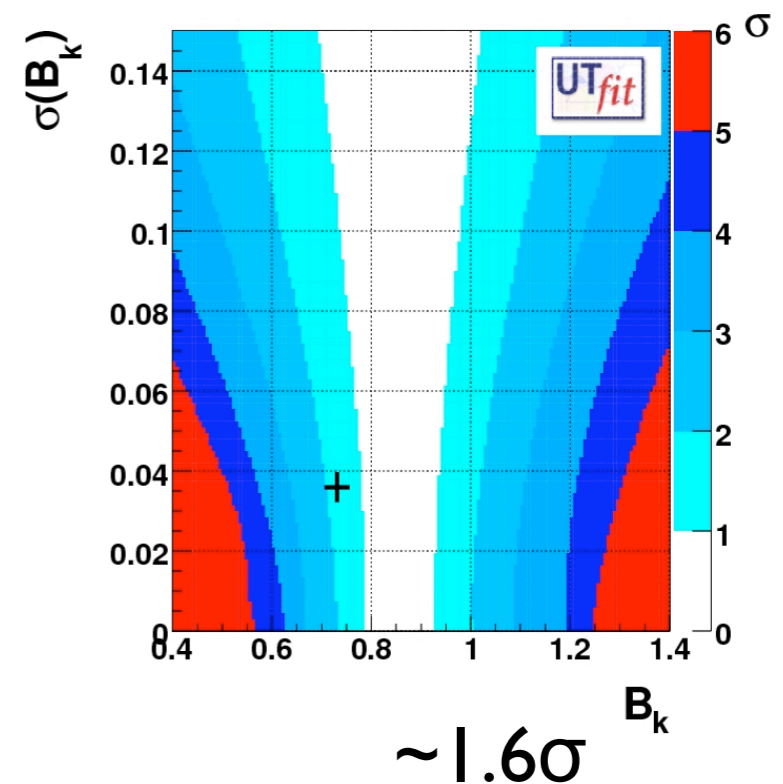
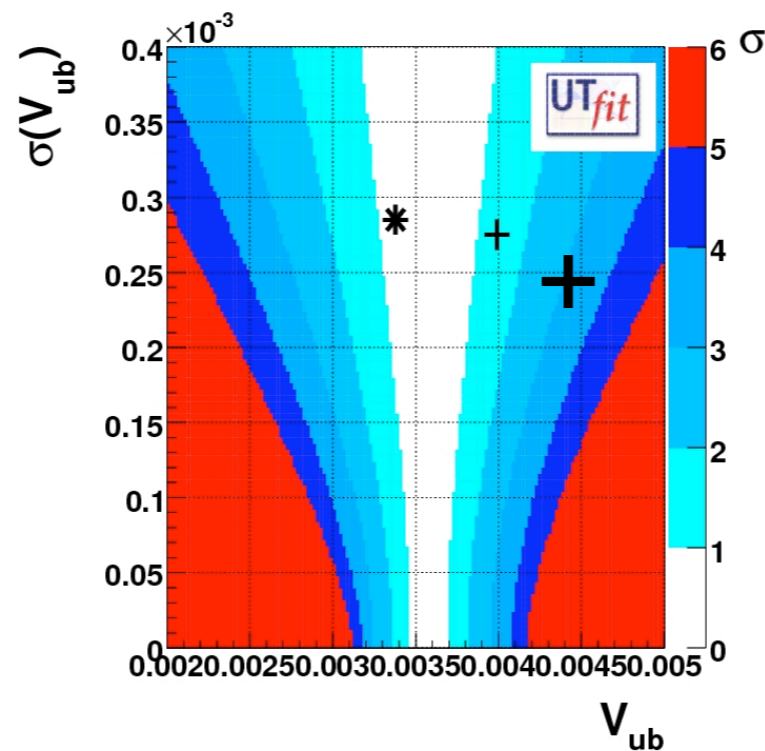
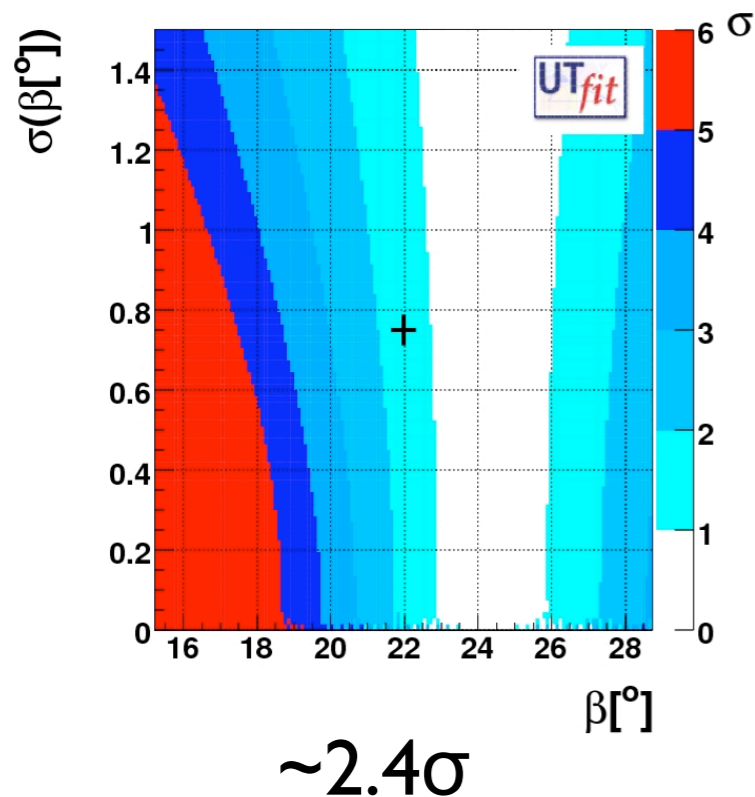
$$\left| \frac{V_{ub}}{V_{cb}} \right|_{\text{incl}} = 0.101 \pm 0.006$$

$$\left| \frac{V_{ub}}{V_{cb}} \right|_{\text{excl}} = 0.084 \pm 0.008$$

$$B_{K_{\text{exp}}} = 0.731 \pm 0.036$$

$$B_{K_{\text{UTfit}}} = 0.855 \pm 0.069$$

$$B_{K_{\text{no lattice}}} = 0.869 \pm 0.079$$



$$V_{ub_{\text{exp}}} = (37.2 \pm 2.1) \cdot 10^{-4}$$

$$V_{ub_{\text{UTfit}}} = (35.8 \pm 1.1) \cdot 10^{-4}$$

!  
 $< 1\sigma$  (incl  $\sim 2.5\sigma$ )

M. Bona  
UTfit@FPCP 2010

# The total s.l. width in the OPE

$$\Gamma[\bar{B} \rightarrow X_c e \bar{\nu}] = \frac{G_F^2 m_b^5 |V_{cb}|^2 g(r)}{192\pi^3} \left[ 1 + \frac{\alpha_s}{\pi} p_c^{(1)}(r, \mu) + \frac{\alpha_s^2}{\pi^2} p_c^{(2)}(r, \mu) \right. \\ \left. - \frac{\mu_\pi^2}{2m_b^2} + \left( \frac{1}{2} - \frac{2(1-r)^4}{g(r)} \right) \frac{\mu_G^2 - \frac{\rho_{LS}^3 + \rho_D^3}{m_b}}{m_b^2} \right. \\ \left. + \left( 8 \ln r - \frac{10r^4}{3} + \frac{32r^3}{3} - 8r^2 - \frac{32r}{3} + \frac{34}{3} \right) \frac{\rho_D^3}{g(r) m_b^3} \right] \\ + O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

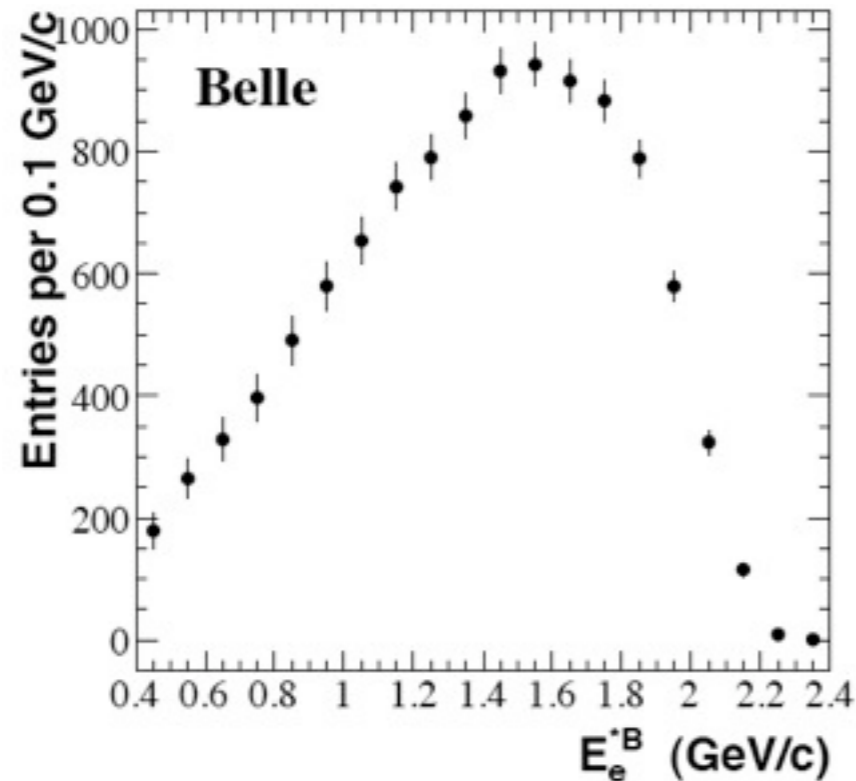
$$r = \frac{m_c^2}{m_b^2}$$

OPE valid for inclusive enough measurements, away from perturbative singularities  $\Rightarrow$  moments

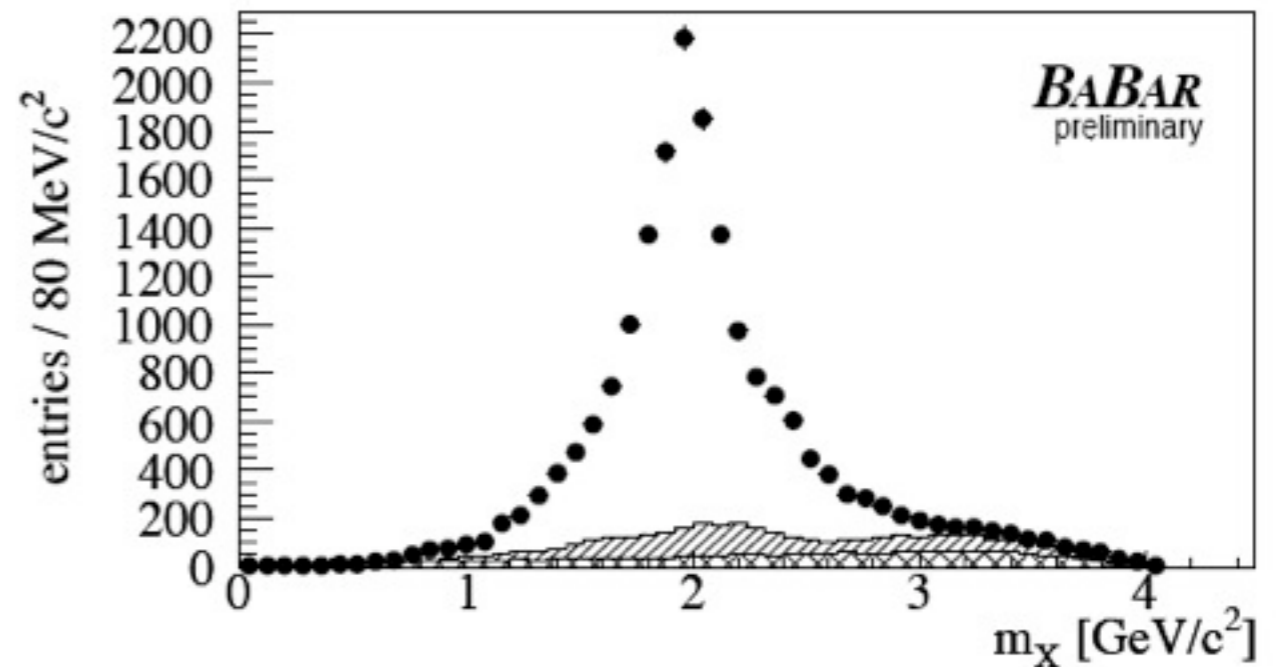
Present implementations include all terms through  $O(\alpha_s^2 \beta_0, 1/m_b^3)$ :  $m_{b,c}, \mu_{\pi,G}^2, \rho_{D,LS}^3$  6 parameters

# Fitting OPE parameters to the moments

$E_l$  spectrum



$m_x$  spectrum



Total **rate** gives  $|V_{cb}|$ , global **shape** parameters (moments of the distributions) tell us about  $B$  structure,  $m_b$  and  $m_c$

*OPE parameters describe universal properties of the  $B$  meson and of the quarks  $\rightarrow$  useful in many applications*

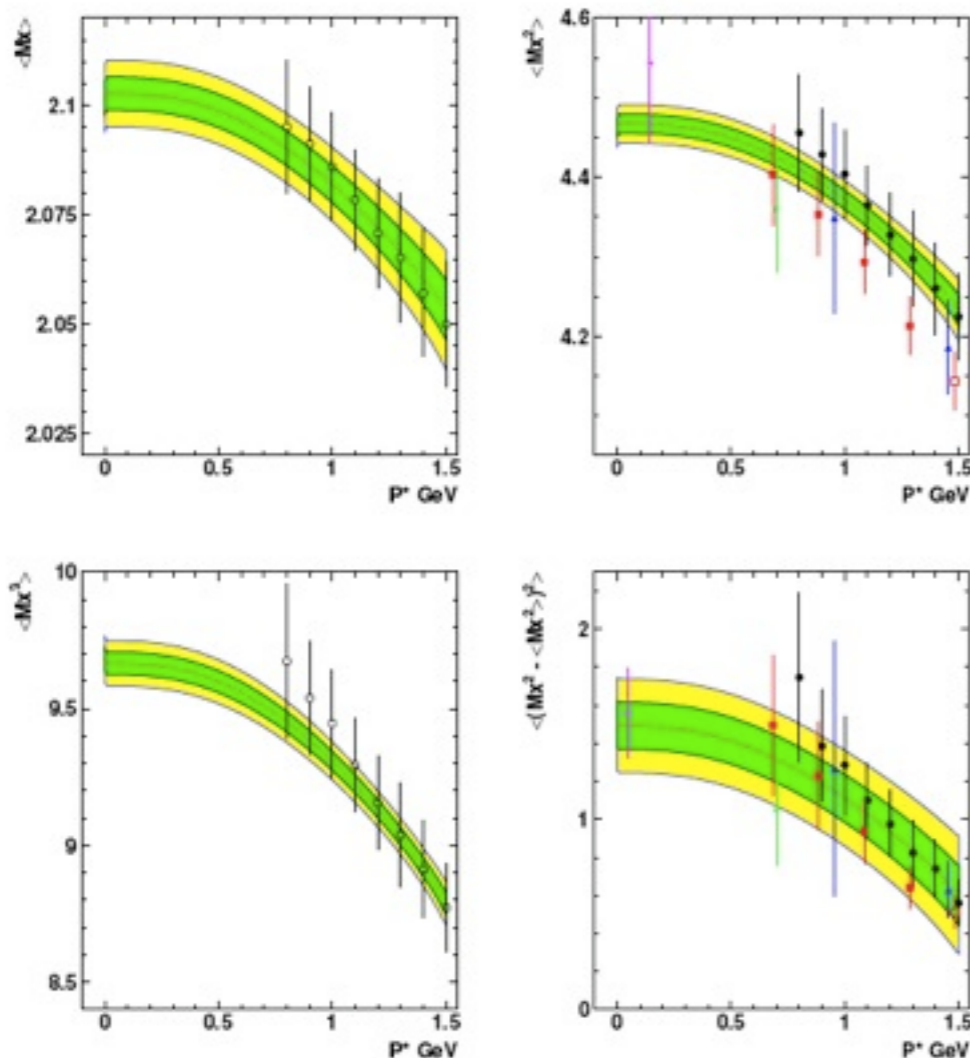
# Global HFAG fit (kinetic scheme)

Inputs	$ V_{cb}  \cdot 10^3$	$m_b^{\text{kin}}$	$\chi^2/\text{ndf}$
$b \rightarrow c$ & $b \rightarrow s\gamma$	41.85(44)(58)	4.590(31)	29.7/59
$b \rightarrow c$ only	41.68(48)(58)	4.646(47)	24.2/48

Based on PG, Uraltsev, Benson et al

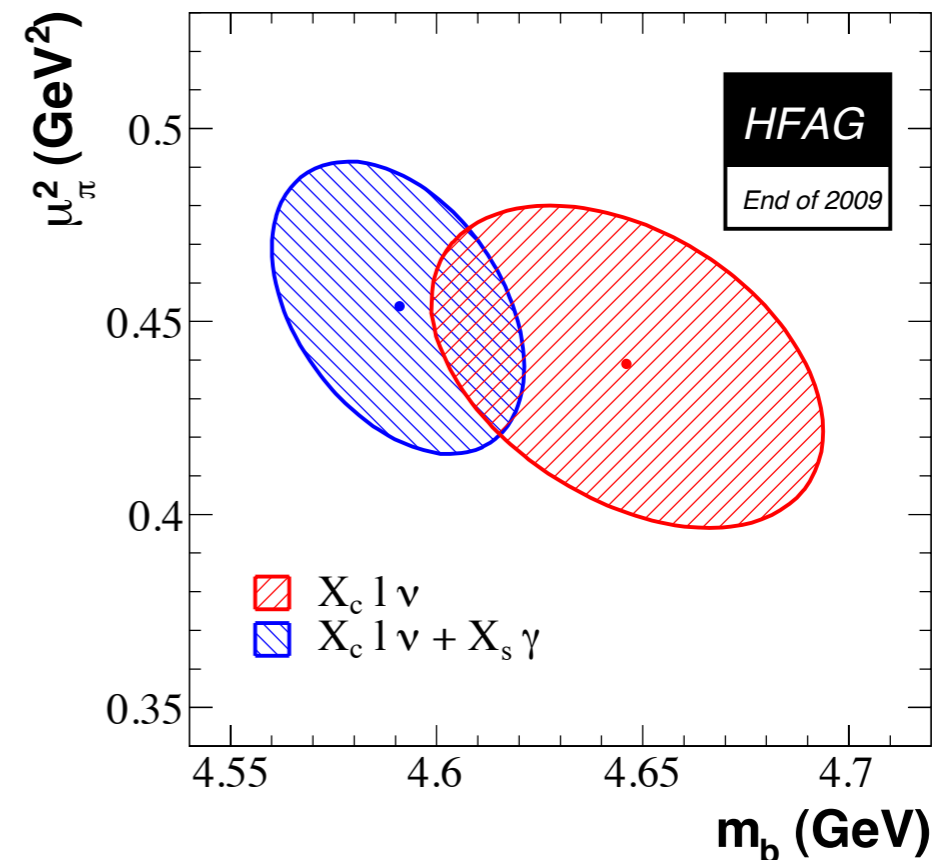
**In the kinetic scheme** the contributions of gluons with energy below  $\mu \approx 1 \text{ GeV}$  are absorbed in the OPE parameters

Here scheme means also a number of different assumptions, inclusion of different data, and a recipe for theory errors



Very close result for  $|V_{cb}|$  in  $1S$  scheme

Bauer Ligeti Luke Manohar Trott

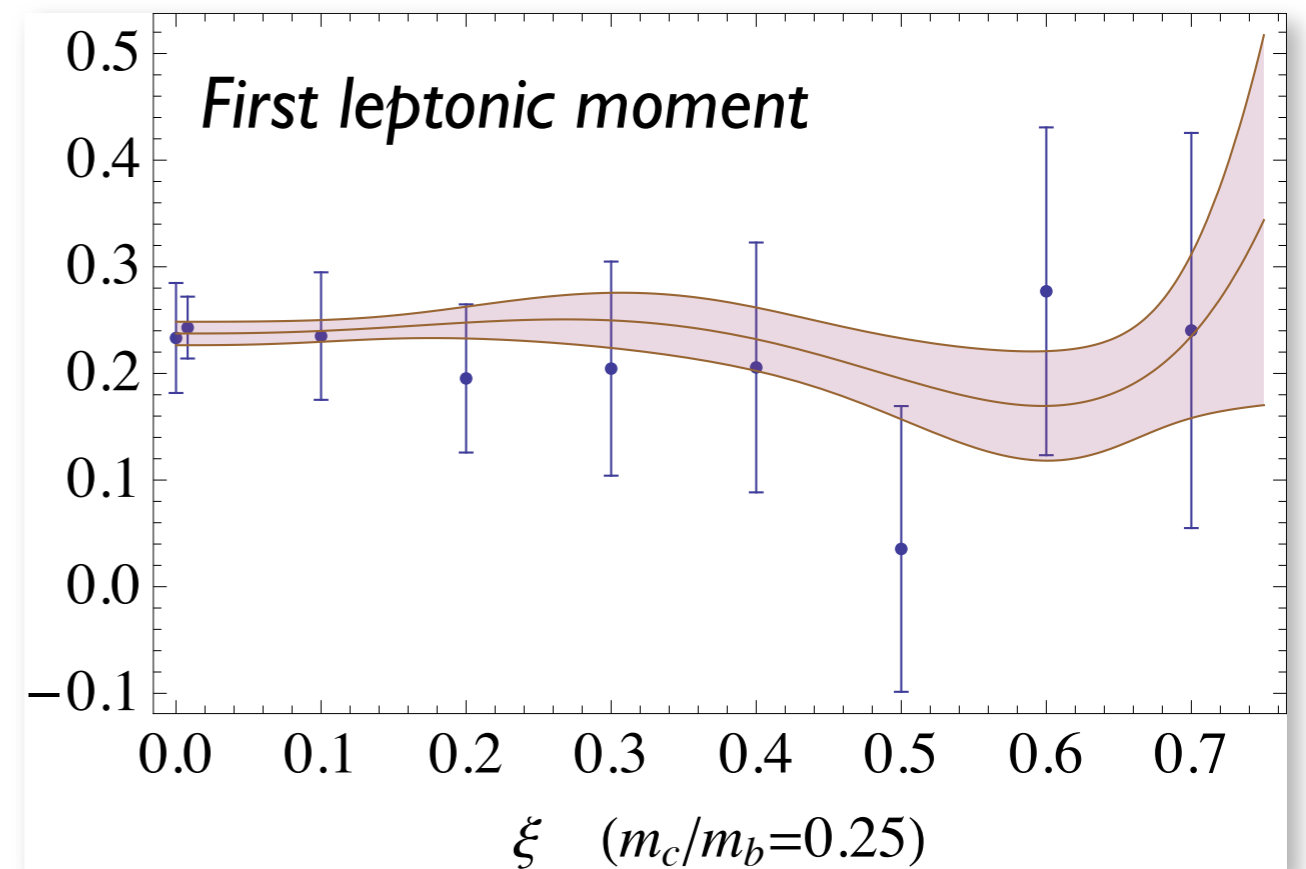


# OPE at NNLO

- \* Complete 2loop corrections to width and moments with cuts are now known, either in expansion  $m_c/m_b$  or numerically

Biswas-Melnikov, Pak, Czarnecki

- \* Minor corrections to BLM, residual th error on  $V_{cb}$   $O(0.5\%)$ .
- \* Competitive with pert corrections to power suppressed terms.  $O(1/m_b^3)$  corrections  $\sim 3\%$  in width, to have 1% accuracy we may need to compute through  $O(\alpha_s/m_b^3)$



*Available NNLO corrections have non-negligible uncertainty important in higher moments*

# Preliminary NNLO results

	$\ell_1$	$\ell_2$	$\ell_3$	$R^*$
	$\mu = 0$			
tree	1.5426	0.0848	-0.0010	0.8003
$O(\alpha_s)$	1.5398	0.0835	-0.0010	0.8009
$O(\beta_0\alpha_s^2)$	1.5343	0.0818	-0.0009	0.7992
$O(\alpha_s^2)$	1.5357(2)	0.0821(8)	-0.0014(25)	0.7992(1)
	$\mu = 1\text{GeV}$			
$O(\alpha_s)$	1.5455	0.0858	-0.0003	0.8029
$O(\beta_0\alpha_s^2)$	1.5468	0.0868	0.0005	0.8035
$O(\alpha_s^2)$	1.5470(2)	0.0866(8)	-0.0001(25)	0.8031(1)
$\delta\alpha_s = 0.04$	0.0011	0.0005	0.0003	0.0005
GU error	0.0113	0.0051	0.0022	

$E_{\text{cut}}=1\text{GeV}, m_c/m_b=0.25$

	$\mu = 1\text{GeV}, m_c^{\text{MS}}(2\text{GeV})$			
	$\ell_1$	$\ell_2$	$\ell_3$	$R^*$
tree	1.5536	0.0873	-0.0013	0.8058
$O(\alpha_s)$	1.5502	0.0869	-0.0003	0.8056
$O(\beta_0\alpha_s^2)$	1.5540	0.0884	0.0004	0.8073
$O(\alpha_s^2)$	1.5528(3)	0.0880(9)	-0.0006(26)	0.8061(1)
	$\mu = 1\text{GeV}, m_c^{\text{MS}}(3\text{GeV})$			
	$\ell_1$	$\ell_2$	$\ell_3$	$R^*$
tree	1.5748	0.0922	-0.0020	0.8159
$O(\alpha_s)$	1.5613	0.0894	-0.00043	0.8118
$O(\beta_0\alpha_s^2)$	1.5629	0.0904	0.00042	0.8125
$O(\alpha_s^2)$	1.5578(4)	0.0892(13)	-0.0014(40)	0.8093(2)

Small corrections, within theory errors.  
 NNLO code in any scheme almost finished, will allow for a more precise determination of masses etc

	$\langle M_X^2 \rangle$	
	on-shell	$\mu = 1\text{GeV}$
tree	4.452	4.452
$O(\alpha_s)$	4.563	4.426
$O(\beta_0\alpha_s^2)$	4.701	4.404
$O(\alpha_s^2)$	4.682(1)	4.424(1)



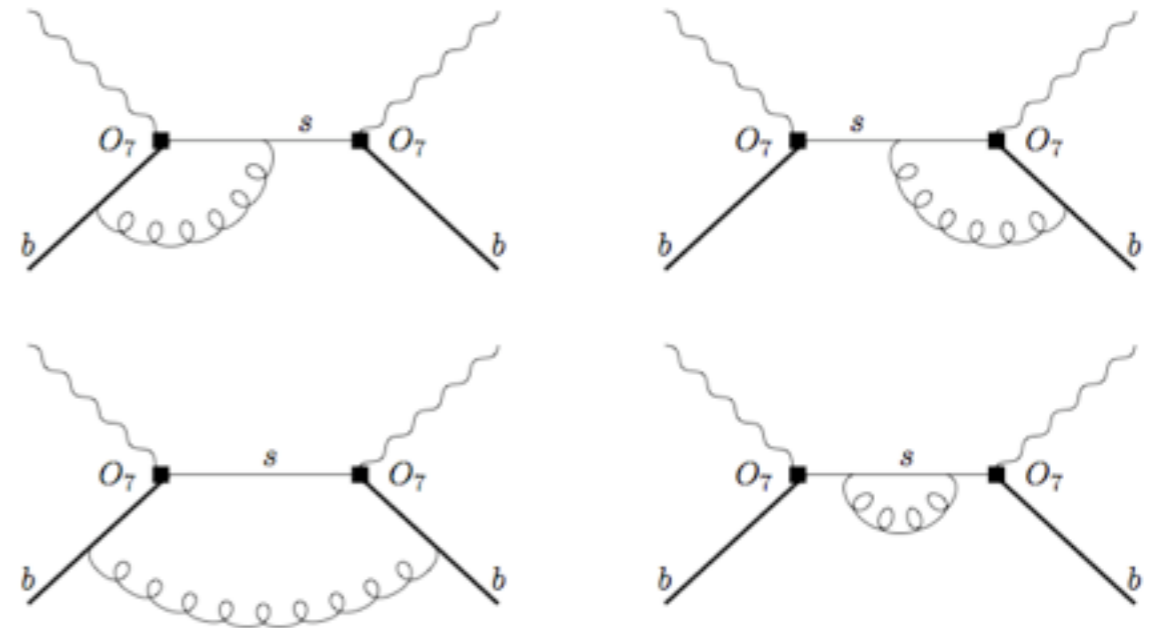
# $O(\alpha_s/m_b^2)$ effects in $B \rightarrow X_s \gamma$

Ewerth,Nandi,PG arXiv:0911.2175

$$T\{\bar{b}(x)\sigma_{\mu\nu}P_L s(x)\bar{s}(0)\sigma_{\alpha\beta}P_R b(0)\} = c_{\text{dim } 3} O_{\text{dim } 3} + \frac{1}{m_b} c_{\text{dim } 4} O_{\text{dim } 4} + \frac{1}{m_b^2} c_{\text{dim } 5} O_{\text{dim } 5} + \dots$$

$$O_b^\mu = \bar{b}\gamma^\mu b, \quad O_2^{\mu\nu} = \bar{b}_v \frac{1}{2} \{iD^\mu, iD^\nu\} b_v,$$

$$O_1^\mu = \bar{b}_v iD^\mu b_v, \quad O_3^{\mu\nu} = \bar{b}_v \frac{g_s}{2} G^{a\mu}{}_\alpha \sigma^{\alpha\nu} T^a b_v,$$



One-loop matching onto local operators with HQET fields in dim reg

$$\frac{d\Gamma_{77}}{dz} = \Gamma_{77}^{(0)} \left[ c_0^{(0)} + c_{\lambda_1}^{(0)} \frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(0)} \frac{\lambda_2(\mu)}{2m_b^2} + \frac{\alpha_s(\mu)}{4\pi} \left( c_0^{(1)} + c_{\lambda_1}^{(1)} \frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(1)} \frac{\lambda_2(\mu)}{2m_b^2} \right) \right]$$

$\lambda_{1,2}$  are HQET analogues of  $\mu_{\pi,G}^2$

The NLO effect 10-20% in coefficients of first few moments, leading to  $\delta m_b \sim 10 \text{ MeV}$ ,  $\delta \mu_{\pi}^2 \sim 0.04 \text{ GeV}^2$

Extension to semileptonic case almost complete:

$O(\alpha_s \mu_{\pi}^2 / m_b^2)$  to moments known numerically Becher,Boos,Lunghi

# Higher power corrections

Mannel, Turczyk, Uraltsev 1009.4622  
see also Bigi, Mannel, Turczyk, Uraltsev  
Bigi, Uraltsev, Zwicky

Proliferation of non-pert parameters: for ex at  $1/m_b^4$

$$\begin{aligned}2M_B m_1 &= \langle ((\vec{p})^2)^2 \rangle \\2M_B m_2 &= g^2 \langle \vec{E}^2 \rangle \\2M_B m_3 &= g^2 \langle \vec{B}^2 \rangle \\2M_B m_4 &= g \langle \vec{p} \cdot \text{rot } \vec{B} \rangle\end{aligned}$$

$$\begin{aligned}2M_B m_5 &= g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle \\2M_B m_6 &= g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle \\2M_B m_7 &= g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle \\2M_B m_8 &= g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle \\2M_B m_9 &= g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle\end{aligned}$$

can be estimated by Ground State Saturation

$$\frac{\delta\Gamma_{1/m^4} + \delta\Gamma_{1/m^5}}{\Gamma} \approx 0.013 \quad \frac{\delta V_{cb}}{V_{cb}} \approx +0.4\%$$

after inclusion of the corrections in the moments. While this might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.*

# Exclusive decays: $B \rightarrow D^* l \nu$

At zero recoil, where rate vanishes, the  $ff$  is

$$\mathcal{F}(1) = \eta_A(1 + \delta_{1/m^2})$$

Recent progress in the measurement of slopes and shape parameters *Despite extrapolation, exp error is only  $\sim 2\%$*

Main problem is the  $ff$   $F(1)$ : cannot be experimentally determined or constrained

New unquenched Lattice QCD (only group):

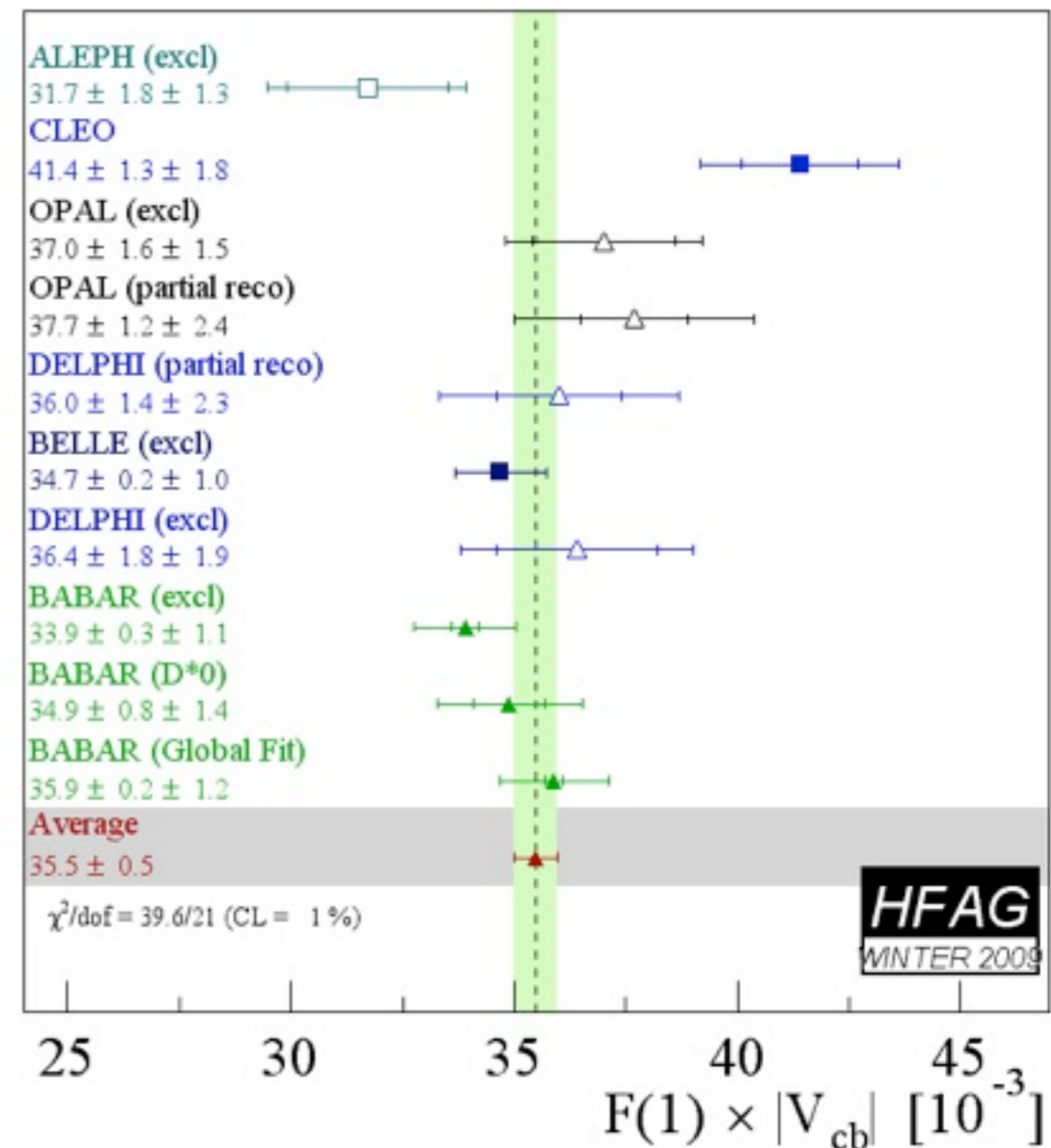
$$F(1) = 0.908(17) \quad \text{Laiho et al 2010}$$

$$|V_{cb}| = 39.1(1.1)(0.7) \times 10^{-3}$$

$\sim 1.8\sigma$  from inclusive determination

**2.5% error**

$B \rightarrow D l \nu$  gives consistent result with larger errors  $|V_{cb}| = 39.1(1.4)(1.3) \times 10^{-3}$



Promising alternative:  $w$  dependence, only quenched *de Divitiis et al*

# Fits are not only for $V_{cb}$ ...

- Results of fits to semileptonic & radiative moments are crucial input in inclusive  $|V_{ub}|$  determination (mostly  $m_b$  and  $\mu_\pi^2$ ) and in normalizing  $B \rightarrow X_s \gamma$  and  $B \rightarrow X_s l^+ l^-$
- $b$  quark mass determinations from  $e^+e^-$  have recently improved significantly: how do they compare with fits? do we understand/trust theory errors?
- Work in progress to implement new corrections, to use additional inputs (masses) in the fits, to control problems due to highly correlated theoretical inputs, to understand better various uncertainties. Role of radiative moments equivalent to loose PDG  $m_b$  constraint.

C.Schwanda, PG

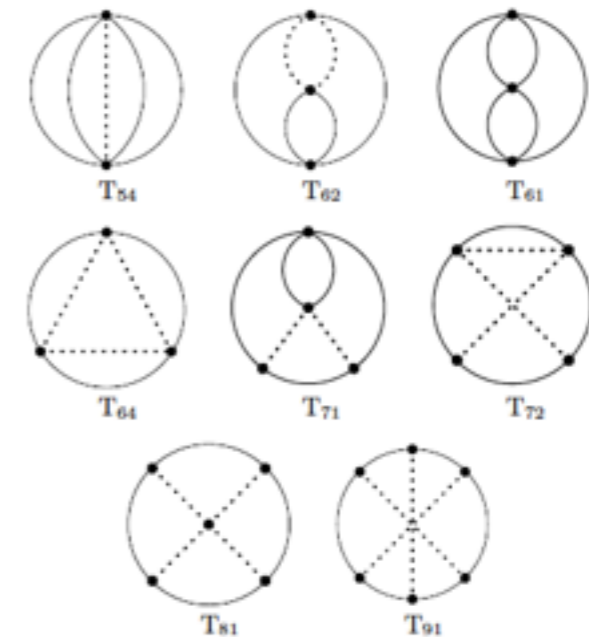
# c,b masses from SVZ sumrules

$$R(s) = 12\pi \text{Im} [\Pi(q^2 = s + i\epsilon)] \quad (-q^2 g_{\mu\nu} + q_\mu q_\nu) \Pi(q^2) \equiv i \int dx e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

Taylor expansion:  $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$

with  $z = q^2/(4m_Q^2)$  and  $m_Q = m_Q(\mu)$  the  $\overline{\text{MS}}$  mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_S}{\pi} \bar{C}_n^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 \bar{C}_n^{(3)} + \dots$$



Boughezal, Czakon, Schutzmeier (2006)  
Kuhn, Steinhauser, Sturm (2006)

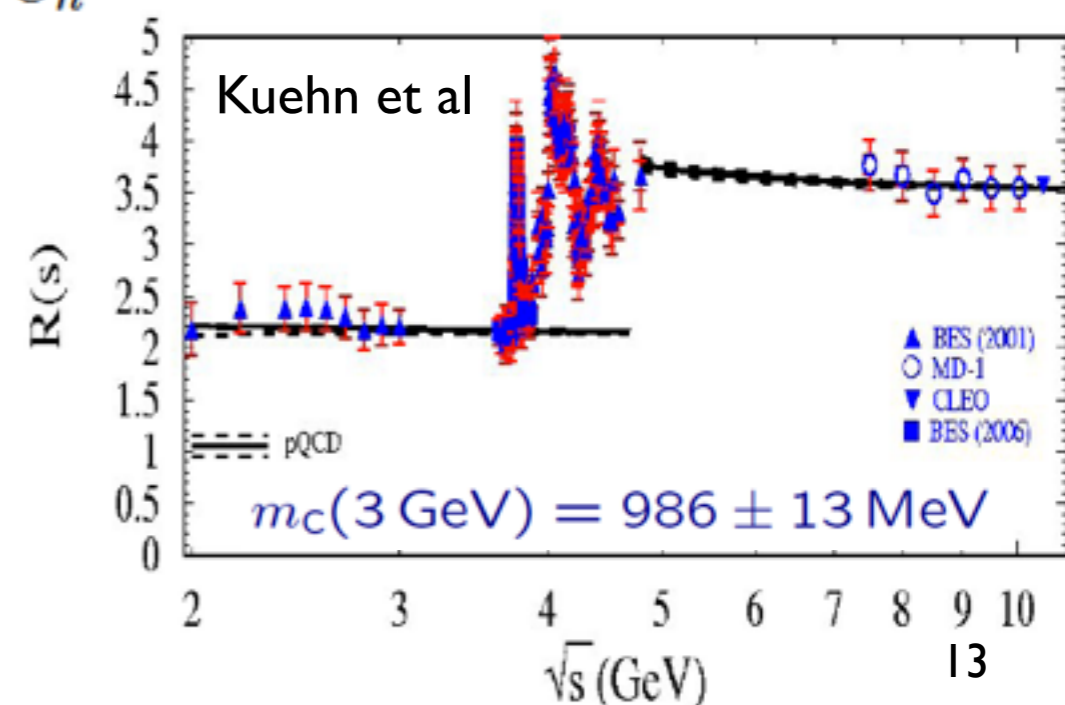
## Relation to measurements

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!} \left( \frac{d}{dq^2} \right)^n \Pi_c(q^2) \Big|_{q^2=0} = \frac{9}{4} Q_c^2 \left( \frac{1}{4m_c^2} \right)^n \bar{C}_n$$

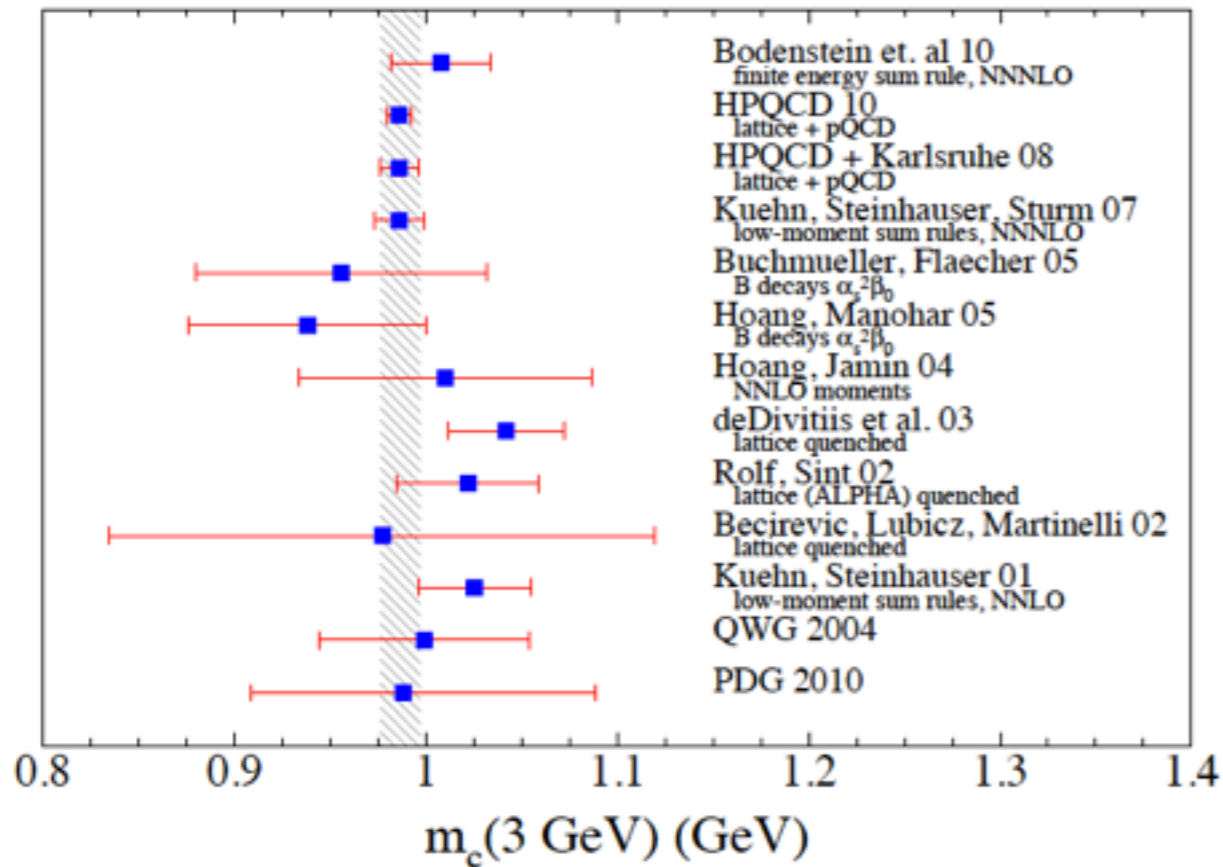
$$\Pi_c(q^2) = \frac{q^2}{12\pi^2} \int ds \frac{R_c(s)}{s(s - q^2)} + \text{subtraction}$$

$$\Leftrightarrow \mathcal{M}_n^{\text{exp}} = \int \frac{ds}{s^{n+1}} R_c(s) = \mathcal{M}_n^{\text{th}}$$

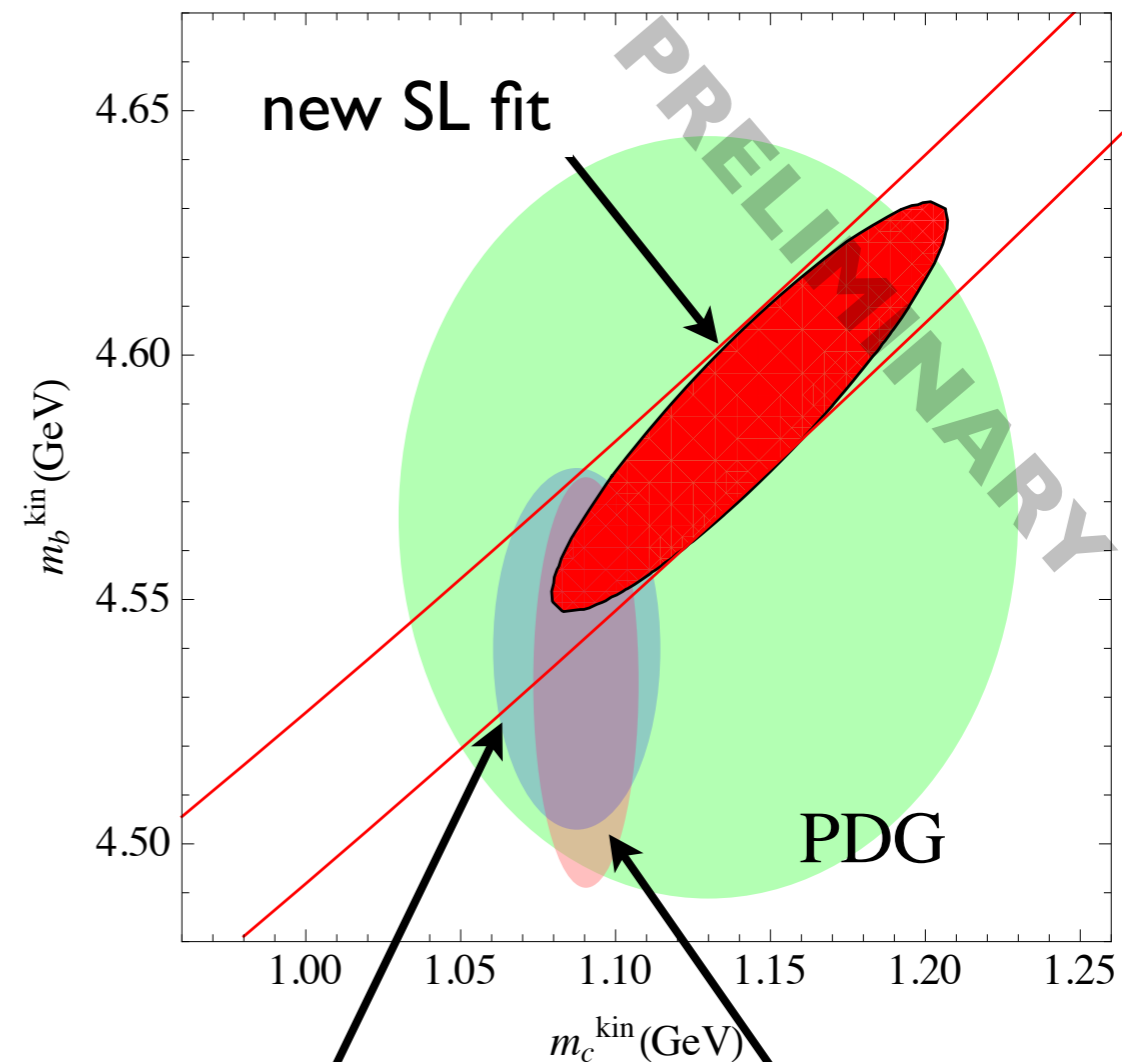
moments can also be *measured* on the lattice!



# Using mass determinations



Recent sum rules determinations converted to kin scheme



Comparisons and combinations for  $m_{b,c}$  penalized by changes of scheme.

Direct fit to  $m_c(3\text{GeV})$  with Karlsruhe constraint on  $m_c$  leads to

$$m_b^{\text{kin}} = 4.535(21)\text{GeV}$$

$$\Rightarrow m_b(m_b) = 4.165(45)\text{GeV} \text{ Consistent!}$$

Hoang et al 2010  
Hoang ( $m_b$ )

Kuhn et al 2009

# The total $B \rightarrow X_u l \bar{\nu}$ width in the OPE

$$\Gamma[\bar{B} \rightarrow X_u e \bar{\nu}] = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3} \left[ 1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r, \mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2} \right. \\ \left. + \left( \frac{77}{6} + 8 \ln \frac{\mu_{\text{WA}}^2}{m_b^2} \right) \frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\text{WA}}(\mu_{\text{WA}}) \right] \\ + O\left(\alpha_s \frac{\mu_{\pi, G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

Life could be relatively easy  
with the total width...

Weak Annihilation

# Weak Annihilation

Spectator dependent non-pert contribution localized at max  $E_l$  (or max  $q^2$ )

Bigi, Uraltsev 1993

*In principle affects  $B^+$  only but WA mixes with Darwin operator at  $O(1)$ .*

*Isosinglet component can be as large as isotriplet*

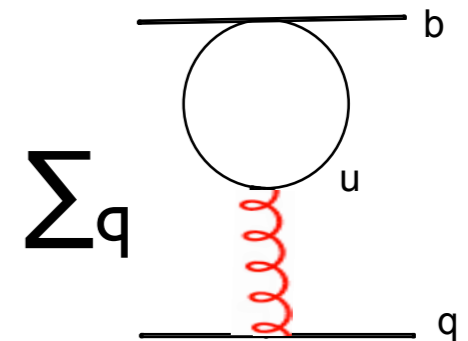
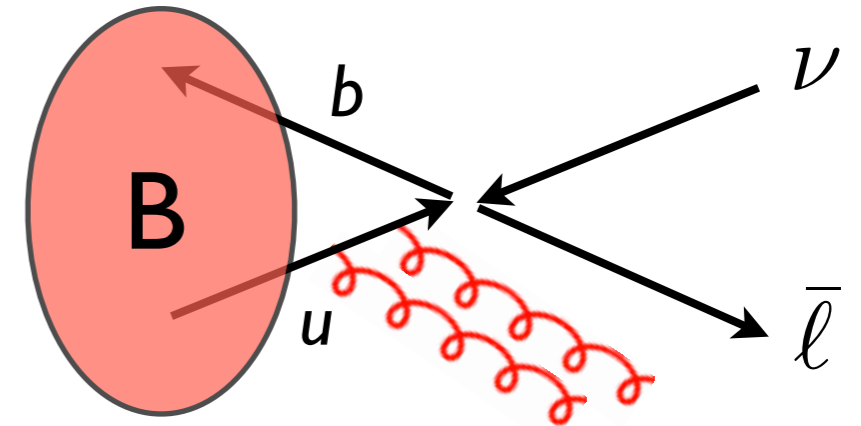
*Difficult to study on the lattice, can be constrained experimentally Rosner et al [Cleo coll]*

*WA may pollute all present inclusive determinations of  $V_{ub}$  and more severely the less inclusive ones ( $E_l$  endpoint, high  $q^2$ )*

$D_s$  and  $D_0$  rates differ significantly, (Cleo-c [arXiv:0912.4232](https://arxiv.org/abs/0912.4232))

$$\Gamma(D^+ \rightarrow Xe^+\nu)/\Gamma(D^0 \rightarrow Xe^+\nu) = 0.985(28)$$

$$\Gamma(D_s^+ \rightarrow Xe^+\nu)/\Gamma(D^0 \rightarrow Xe^+\nu) = 0.828(57)$$



Valence WA Cabibbo suppressed in  $D^+$ , absent in  $D^0$ , is it a sign of WA?

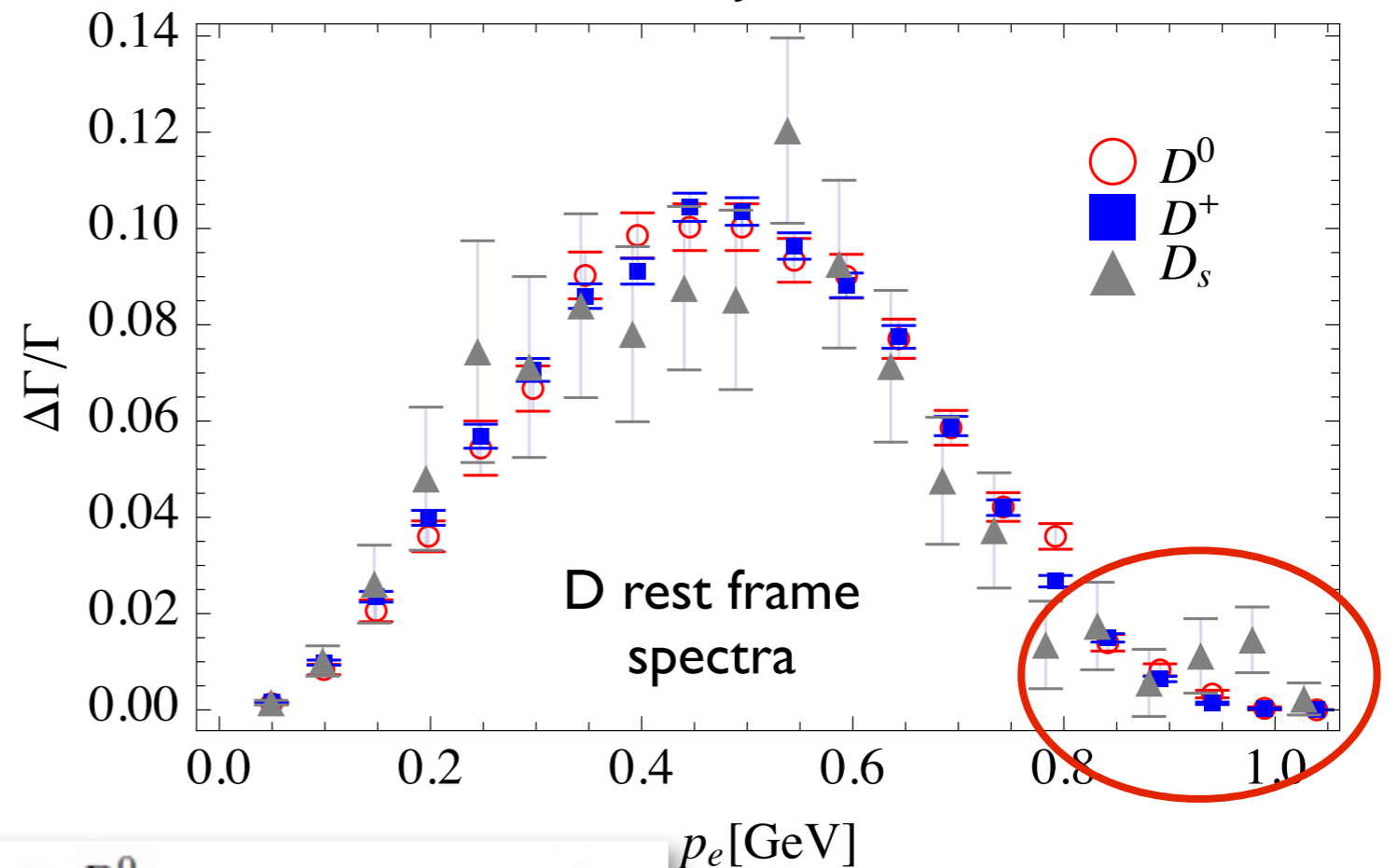
Bigi, Mannel, Turczyk, Uraltsev 0911.3322 Ligeti, Luke, Manohar 1003.1351



# Cleo-c electron spectra

JF Kamenik, PG, arXiv:1004.0114

- Cleo also measured the electron spectra for  $p > 0.2 \text{ GeV}$ . We extrapolated them to  $p=0$ , computed their first moments, and boosted to the  $D$  rest frame
- Moments should follow OPE, but are less sensitive to power and pert corrections than widths



$$\langle E_\ell \rangle_{exp}^{D^0} = 0.459(3) \text{ GeV},$$

$$\langle E_\ell^2 \rangle_{exp}^{D^0} = 0.240(2) \text{ GeV}^2,$$

$$\langle E_\ell \rangle_{exp}^{D^+} = 0.455(1) \text{ GeV},$$

$$\langle E_\ell^2 \rangle_{exp}^{D^+} = 0.236(1) \text{ GeV}^2,$$

$$\langle E_\ell \rangle_{exp}^{D_s} = 0.456(11) \text{ GeV},$$

$$\langle E_\ell^2 \rangle_{exp}^{D_s} = 0.239(12) \text{ GeV}^2,$$

**Moments**

No evidence for spectator effects! Is there really evidence for WA?

# SU(3) violation in charm

- Decay constants on the lattice:

$$f_D=260(10)\text{MeV} \quad \text{vs} \quad f_{D_s}=217(10)\text{MeV} \quad \text{Bazavov et al}$$

- Hyperfine splittings  $\Delta_{D_q}^{hf} = 3(m_{D_q^*}^2 - m_{D_q}^2)/4 \approx \mu_G^2$

$$\Delta_{D^+}^{hf} = 0.409(1)\text{GeV}^2, \quad \Delta_{D^0}^{hf} = 0.413(1)\text{GeV}^2, \quad \Delta_{D_s}^{hf} = 0.440(2)\text{GeV}^2$$

- SU(3) violation can be as large as 20%. Widths get much larger power corrections than moments and this might partially explain the observed width difference without WA

# Results and implications J Kamenik, PG 1004.0114

*Allowing for 20% SU(3) violation in the OPE parameters*

$$\Delta B_{\text{WA}}^{(0),s} \equiv B_{\text{WA}}^{(0),s}(D_s) - B_{\text{WA}}^{(0),s}(D^0) \quad \text{Valence component}$$
$$\Delta B_{\text{WA}}^{(0),s} = 0.0000(12)(3)\text{GeV}^3 \quad \text{always compatible with zero}$$

$$B_{\text{WA}}^s = -0.0003(25)\text{GeV}^3 \quad \text{Singlet component}$$

*In worst dilution scenario from the moments alone (linearly adding errors)  
equivalent to 30% error on rate*

*A factor ~2.5 in going to B:*  $|B_{\text{WA}}^b(\mu_{\text{WA}} = 0.8\text{GeV})| \lesssim 0.006\text{GeV}^3$  **Singlet**

$$-0.004\text{GeV}^3 \lesssim \Delta B_{\text{WA}}^b(\mu_{\text{WA}} = 0.8\text{GeV}) \lesssim 0.002\text{GeV}^3 \quad \text{Valence}$$

**Max 1% effect on  $V_{ub}$  for most inclusive analyses**  
 *$B^0$  and  $B^+$  inclusive widths should not differ more than about 1%*

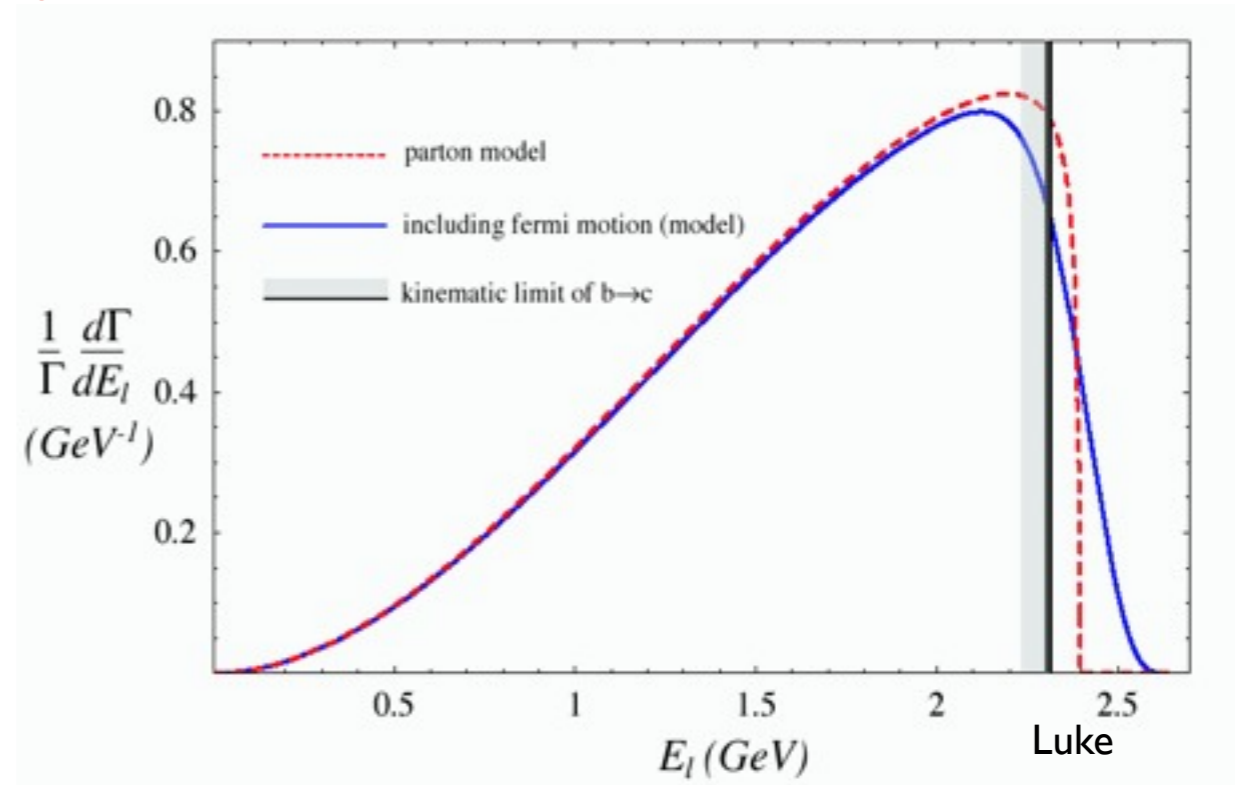
# The problems with cuts

Experiments generally need kinematic cuts to avoid the  $\sim 100x$  larger  $b \rightarrow cl\nu$  background:

$$m_X < M_D \quad E_l > (M_B^2 - M_D^2)/2M_B \quad q^2 > (M_B - M_D)^2 \dots$$

*The cuts destroy convergence of the OPE that works so well in  $b \rightarrow c$ . OPE expected to work only away from pert singularities*

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION**  $f(k_+)$



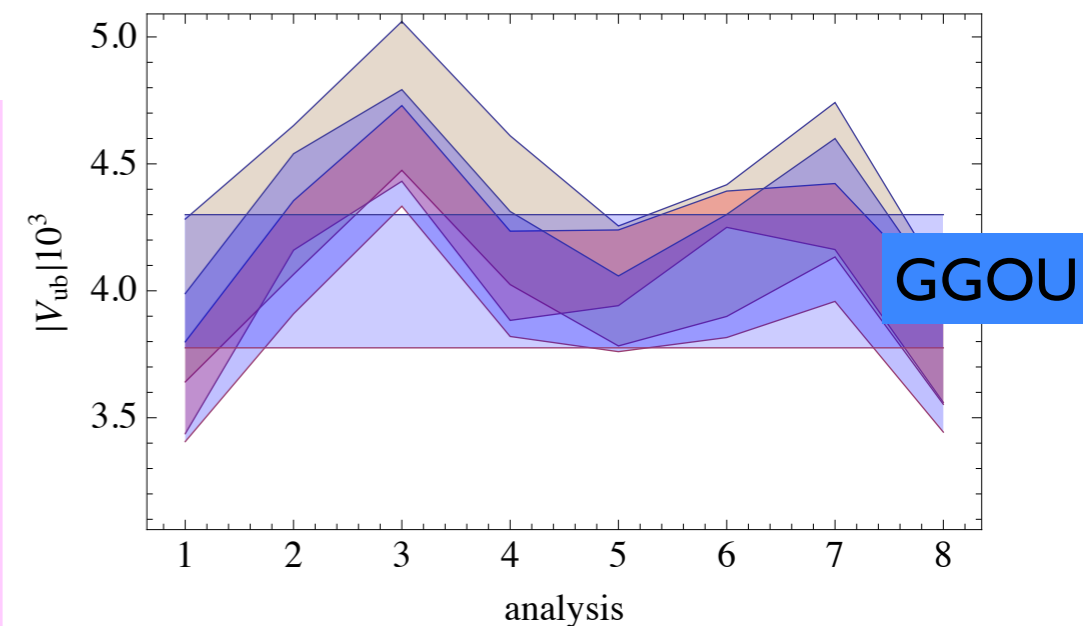
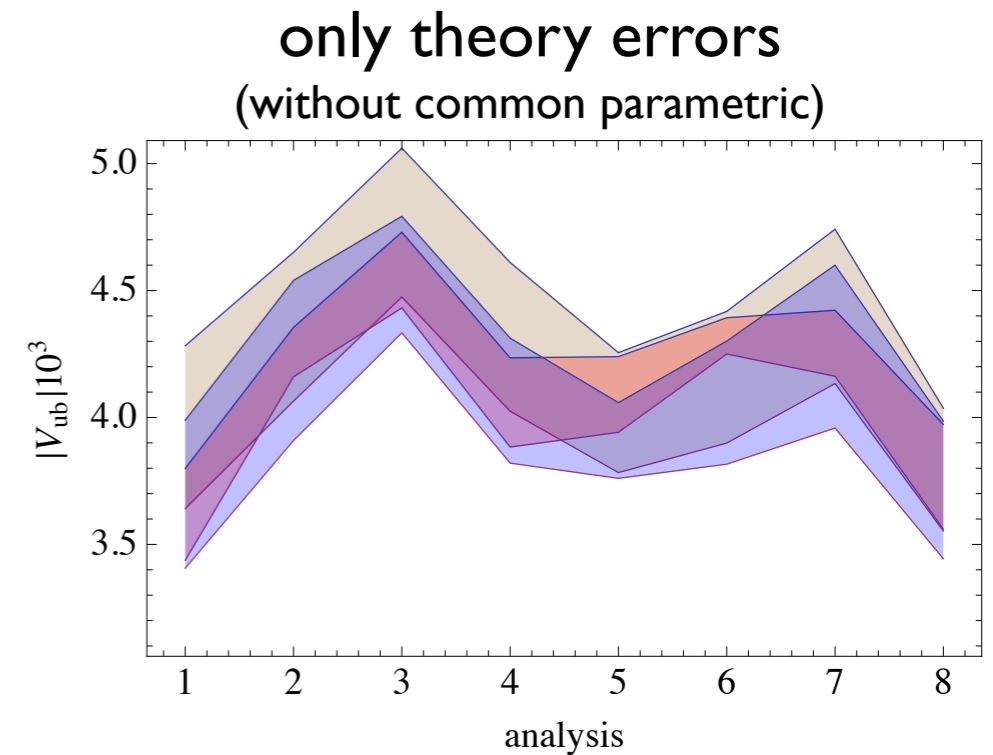
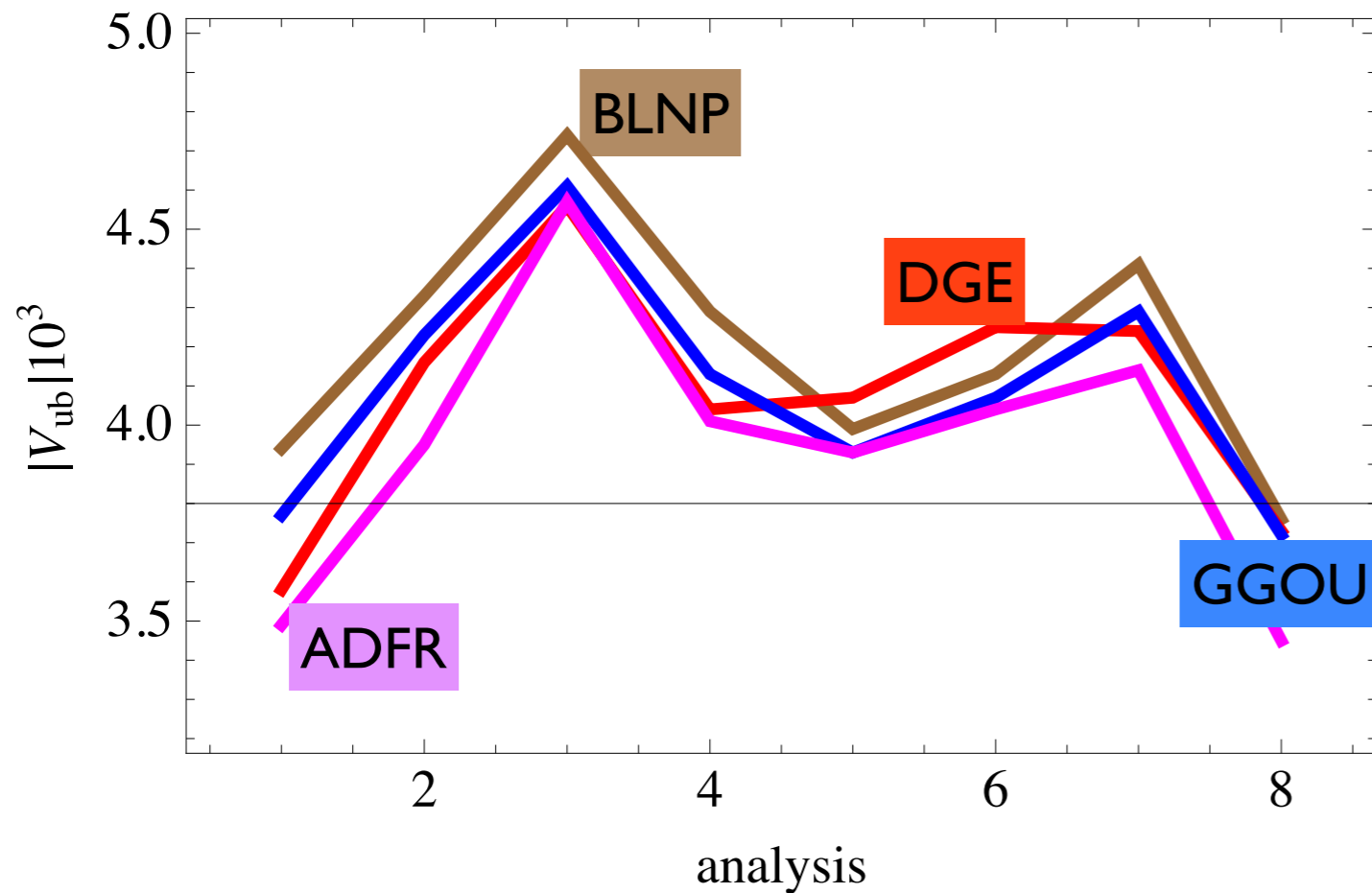
# How to access the SF?

Prediction <i>based</i> on resummed pQCD  DGE, ADFR	OPE constraints + parameterization without/with resummation  GGOU, BLNP
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SIMBA fits radiative data for leading SF &  $m_b$ , parameterizes subleading only. No  $V_{ub}$  yet Bernlocher et al

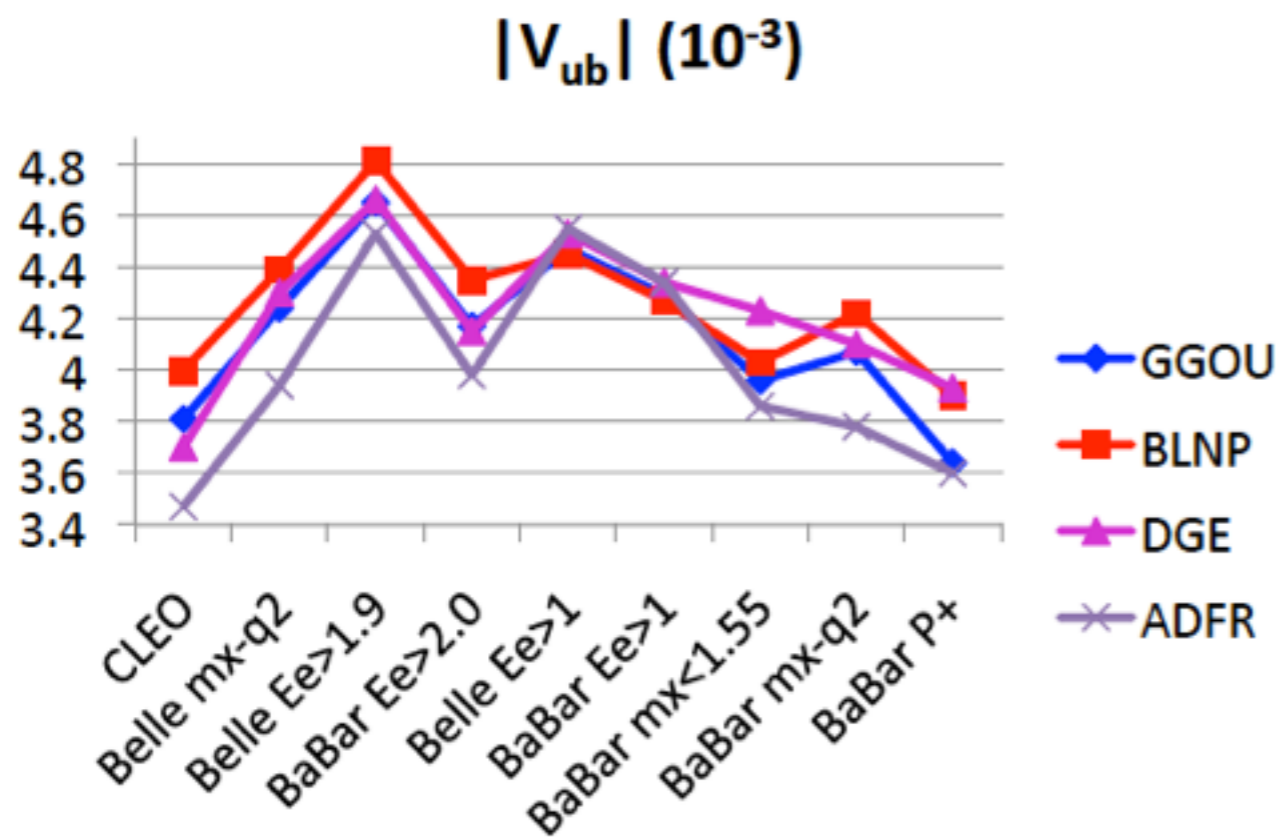
# A global comparison

0907.5386, Phys Rept



- \* Overall good agreement with common inputs  
SPREAD WITHIN TH ERRORS
- \* Recent BLNP at NNLO (in SF region) Asatrian, Greub, Neubert, Pecjak, Bonciani, Ferroglia Beneke, Huber, Li, Bell *Strong impact in BLNP (+10%), not yet included, unlikely in other approaches.  $O(\alpha_s^2)$  calculation in the full phase space necessary*
- \* Not all observables are equally clean.

# Inclusive $|V_{ub}|$ averages



HFAG 2010	Average $ V_{ub}  \times 10^3$
DGE	$4.44(16)_{\text{ex}}^{+18}_{-17}$
BLNP	$4.31(16)_{\text{ex}}^{+22}_{-23}$
GGOU	$4.33(16)_{\text{ex}}^{+15}_{-22}$

2.1 $\sigma$  from  $B \rightarrow \pi l \nu$  (MILC-FNAL)  
 2.5 $\sigma$  from UTFit (because of  $\sin 2\beta$ )

*Belle+Babar multivariate analysis,  $E_l > 1 \text{ GeV}$*

$$|V_{ub}| \approx (4.35 \pm 0.18_{-0.17}^{+0.13}) \times 10^{-3}$$

*Includes about 90% of the rate: really inclusive measurement, no need for SF.  
 Crucial input  $m_b$ , needs to be confirmed!*

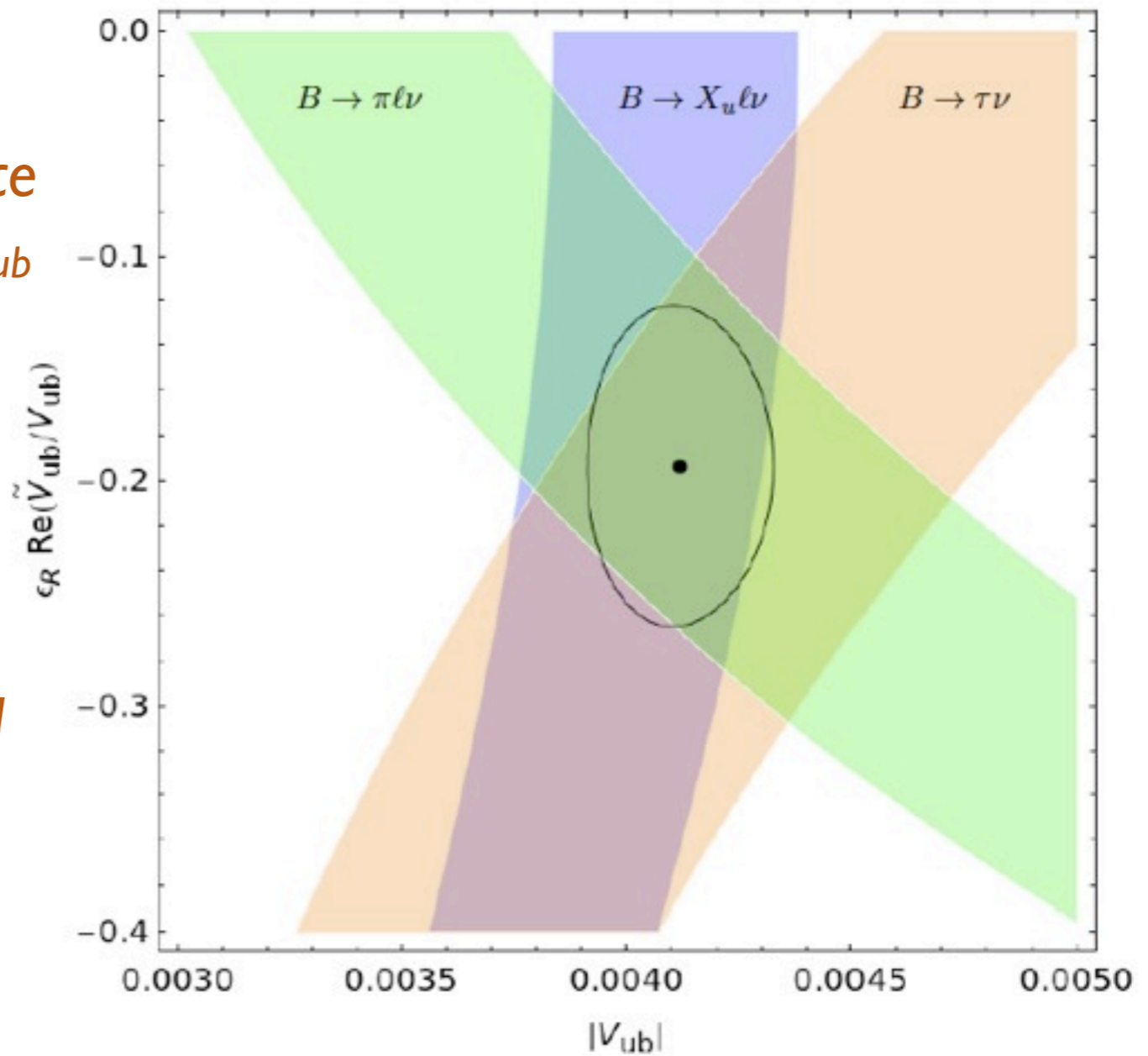
- **7-8% total error**
- More inclusive measurements, less dependence on  $m_b$

# New physics?

*LR models can explain a difference between inclusive and exclusive  $V_{ub}$  determinations* Chen,Nam

*Also in MSSM Crivellin*

*BUT the RH currents affect predominantly the exclusive  $V_{ub}$ , making the conflict between  $V_{ub}$  and  $\sin 2\beta$  ( $\psi K_S$ ) stronger...*



Buras, Gemmler, Isidori 1007.1993



# Conclusions

- *Semileptonic B decays provide us with a lot of information:  $V_{cb}, V_{ub}$ , constraints on  $m_{b,c}$  (consistent with sum rules)*
- *Slow but steady progress in inclusive  $|V_{cb}|$ , NNLO and higher power corrections, good prospects for the error reduction*
- *Some tension persists between exclusive and inclusive  $|V_{cb}|$*
- *Inclusive  $V_{ub}$  moves slightly up,  $\sim 2\sigma$  clash with exclusive one and UT fit, but latest results should be described by local OPE.  
experimental problem or new physics?*