Heavy Fermions and Some Aspects of Physics Beyond the SM

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We shall concentrate our analysis on extensions of the SM with vector-like isosinglet quarks of $Q = -1/3$ or $Q = 2/3$.

Question: Why study them? what can vector-like quarks do for us?

- Provide a simple self-consistent framework with naturally small violations of 3×3 unitarity of V^{CKM}

$$\Downarrow$$

- Lead to naturally small Flavour Changing Neutral Currents (FCNC) in the down and/or up sector (mediated by Z)

- Provide the simplest framework to have spontaneous CP violation which generates a non-trivial CKM phase. This is an important requirement since there is experimental evidence of a complex V_{CKM} even if one allows for the presence of New Physics
- Provide New Physics contributions to $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ mixings.
- Provide a simple solution to the Strong CP-problem, which does not require Axions

- Provide a framwork where there is a common origin of all CP violations:

  (i) CP violation in the quark sector

  (ii) CP violation in the lepton sector, detectable through neutrino oscillations

  (iii) CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis

All these "manifestations" of CP Violation originate in a single phase of the vev of a complex singlet neutral scalar S:

$$\langle S \rangle = V e^{i\alpha}$$

A Minimal Model

Consider an extension of the SM where the following fields are added to the SM:

- A vectorial quark D^0:
  Both D_L^0 and D_R^0 are $SU(2)_L$ singlets, with charge $Q = -1/3$
- Three right-handed neutrinos $\nu_{R_j}^0$
- A neutral complex singlet S

- Impose CP invariance at the Lagrangian level
- Introduce a Z_4 symmetry on the Lagrangian, under which:

$$\psi_\ell^0 \to i\,\psi_\ell^0 \qquad e_{R_j}^0 \to i\,e_{R_j}^0 \qquad \nu_{R_j}^0 \to i\,\nu_{R_j}^0$$

$$D^0 \to -D^0 \quad ; \quad S \to -S$$

All other fields are invariant under Z_4

Since we impose CP invariance at the Lagrangian level, all couplings are real. CP is spontaneously broken by the vacuum:

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence:

$$V_{phase} = \left(\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^\dagger \phi\right)\left(S^2 + S^{*2}\right) + $$
$$+ \lambda_3 \left(S^4 + S^{*4}\right)$$

There is a range of the parameters of the Higgs potential, where the minimum is:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general $SU(2)\times U(1)\times Z_4$ invariant

Yukawa couplings in the quark sector:

$$\mathcal{L}_Y = -(\bar{u}^\circ \bar{d}^\circ)_{Li}\left[g_{ij}\, d^\circ_{Rj} + h_{ij}\,\tilde{\phi}\, u^\circ_{Rj}\right] - \bar{M}(\bar{D}^\circ_L D^\circ_R)$$
$$-(f_i S + f_i' S^*)\,\bar{D}^\circ_L\, d^\circ_{Ri} + h.c.$$

Quark mass-matrix for down-type quarks:

3x3, real

$$\left[\bar{d}^\circ_{1L}\ \bar{d}^\circ_{2L}\ \bar{d}^\circ_{3L}\ \bar{D}^\circ_L\right]\left[\begin{array}{ccc|c} & m_d & & 0 \\ \hline M_1 & M_2 & M_3 & \bar{M}\end{array}\right]\left[\begin{array}{c} d^\circ_{1R}\\ d^\circ_{2R}\\ d^\circ_{3R}\\ D^\circ_R\end{array}\right]$$

$\mathcal{M} \to 4\times 4$

$$M_j = f_j V e^{i\theta} + f_j' V e^{-i\theta}$$

$$\mathcal{U}_L^\dagger\,(\mathcal{M}\mathcal{M}^\dagger)\,\mathcal{U}_L = \begin{bmatrix} d^2 & \\ & D^2\end{bmatrix}$$

$$\mathcal{U} = \begin{bmatrix} K & R \\ S & T\end{bmatrix}$$

$$\boxed{K^{-1}\left[m_d m_d^\dagger - \frac{m_d M^\dagger M\, m_d^\dagger}{MM^\dagger + \bar{M}^2}\right]K = d^2}$$

A remarkable feature of the Model:

The phase θ arising from $\langle S \rangle$ generates a non-trivial CKM phase, provided $|M_j|$ and $\overline{M}$ are of the same order of magnitude.

$$K^{-1}\left(m_{eff}\, m_{eff}^{\dagger}\right)K = \mathrm{diag.}\left(m_d^2, m_s^2, m_b^2\right)$$

$$m_{eff}\, m_{eff}^{\dagger} = m_d\, m_d^{\dagger} - \frac{m_d\, M^{\dagger} M\, m_d^{\dagger}}{M M^{\dagger} + \overline{M}^2}$$

$$M_j = \left(f_i\, V e^{i\theta} + f_j'\, V e^{-i\theta}\right)$$

$|M_j|$, $|\overline{M}|$ can naturally be of the same order of magnitude

Violating 3×3 unitarity:

Suppose that one drops the requirement of 3×3 unitarity.

How many independent parameters are there in V_{CKM}?

$$
\begin{bmatrix}
V_{ud} & V_{us} & V_{ub} & \cdots \\
V_{cd} & V_{cs} & V_{cb} & \cdots \\
V_{td} & V_{ts} & V_{tb} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{bmatrix}
$$

9 moduli + 4 rephasing invariant phases

$= 13$ parameters

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^{*} V_{td}\right)$$

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^{*} V_{cd}^{*}\right)$$

$$\chi \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^{*} V_{tb}^{*}\right)$$

$$\chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^{*} V_{cs}^{*}\right)$$

The SM with 3 generations predicts a series of exact relations among these 13 measurable (in principle) quantities. Violation of any of these exact relations signals the presence of New Physics which may involve deviations of 3×3 unitarity or not.

The presence of New Physics contributions to $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings affects the extraction of $|V_{td}|$, $|V_{ts}|$ from the data, even in the framework of New Physics which respects 3×3 unitarity

Example:

SUSY extensions of the SM

In many of the extensions of the SM the dominant effect of New Physics arises from New contributions to $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ mixings, which is convenient to parametrize as:

$$M_{12}^q = \left(M_{12}^q\right)^{SM} r_q^2 \, e^{2i\theta_q} \qquad q = d, s$$

$$\Downarrow$$

$$\Delta M_{B_d} = r_d^2 \left(\Delta M_{B_d}\right)^{SM} \rightarrow$$ affects the extraction of $|V_{td}|$ from ΔM_{B_d}

$$\Delta M_{B_s} = r_s^2 \left(\Delta M_{B_s}\right)^{SM} \rightarrow$$ affects the extraction of $|V_{ts}|$ from ΔM_{B_s}

$$S_{J/\psi K_S} = \sin(2\beta + 2\theta_d) = \sin(2\bar{\beta})$$

$$S_{\rho^+\rho^-} = \sin(2\alpha - 2\theta_d) = \sin(2\bar{\alpha})$$

How to detect the presence of New Physics?

Answer: Use the exact relations predicted by the SM.

(db) $$|V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|}\,\frac{\sin\beta}{\sin(\gamma+\beta)} \rightarrow \text{extraction of } \theta_d$$

(sb) $$\sin\chi = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|}\sin(\gamma-\chi+\chi') \rightarrow \text{extraction of } \theta_s$$

$$\sin\chi = \frac{|V_{us}||V_{cd}||V_{cb}|}{|V_{ts}||V_{tb}||V_{ud}|}\,\frac{\sin\beta\,\sin(\gamma+\chi')}{\sin(\gamma+\beta)}$$

well approximated by: $$\boxed{\sin\chi \simeq \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{\sin\beta\,\sin\gamma}{\sin(\gamma+\beta)}}$$

Silva, Wolfenstein

$$\boxed{\sin\chi = \frac{|V_{td}|}{|V_{ts}|}\,\frac{|V_{cd}|}{|V_{cs}|}\sin\beta}$$

$\Rightarrow$ If either (γ, χ) or $\left(\left(\frac{\Delta M_{Bd}}{\Delta M_{Bs}}\right), \chi\right)$ are measured with some precision, one has novel stringent tests of the SM where contribution of New Physics can be large

Extraction of θ_d :

$$\boxed{\tan\theta_d = \frac{R_u \sin(\gamma+\bar{\beta}) - \sin\bar{\beta}}{\cos\bar{\beta} - R_u \cos(\gamma+\bar{\beta})}}$$

where

$$\boxed{R_u = \frac{|V_{ud}|\,|V_{ub}|}{|V_{cd}|\,|V_{cb}|}}$$

Extraction of θ_s :

$$\boxed{\tan\theta_s = \frac{\sin\bar{\chi} - C\sin(\gamma-\bar{\chi})}{C\cos(\gamma-\bar{\chi}) + \cos\bar{\chi}}}$$

$$\boxed{C = \frac{|V_{us}|\,|V_{ub}|}{|V_{cs}|\,|V_{cb}|}}$$

Naturally small deviations of 3 x 3 unitarity $\longleftrightarrow$ Naturally small Flavour-Changing Neutral Currents

For definiteness consider the case of one isosinglet $Q = -1/3$ quark.

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u}\ \bar{c}\ \bar{t})_L \gamma^\mu \left[\underset{\uparrow}{K} \quad R \right] \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} W^+_\mu$$

$(V^{CKM})_{3\times3}$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W} \Big\{ (\bar{u}\ \bar{c}\ \bar{t})_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix}_L -$$

$$- \left[\bar{d}\ \bar{s}\ \bar{b}\ \bar{D} \right]_L \gamma^\mu \begin{bmatrix} K^\dagger K & K^\dagger R \\ R^\dagger K & R^\dagger R \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} - \sin^2\theta_W J^\mu_{em} \Big\} Z_\mu$$

From unitarity of $\mathcal{U}_L \equiv \begin{bmatrix} K & R \\ S & T \end{bmatrix}$ one obtains:

$$K^\dagger K + S^\dagger S = \mathbb{1}$$

but $S \approx -\frac{M' m_d^\dagger K}{M^2} \rightarrow O(m/M)$; $K^\dagger K = \mathbb{1} - O(m^2/M^2)$

Similar analysis can be done for

$Q = 2/3$ vector like quarks

Without loss of generality, one may choose to work in a WB where the down quarks mass matrix is diagonal. Let $\mathcal{U}$ be the 4×4 unitary basis which diagonalizes the up-quark mass matrix. The quark mixing matrix consists of the first 3 columns of $\mathcal{U}$

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{bmatrix};$$

From orthogonality of the 2^{nd} and 3^{rd} column, one obtains:

$$\sin\chi = \frac{V_{ub}V_{us}}{V_{cb}V_{cs}} \sin(\gamma - \chi + \chi') + \frac{|V_{Tb}||V_{Ts}|}{|V_{cb}||V_{cs}|} \sin(\sigma - \chi)$$

$$\sigma = \arg\left(V_{Ts} V_{cb} V_{Tb}^* V_{cs}^*\right)$$

If $V_{Tb} \approx O(\lambda)$
$V_{Ts} \approx O(\lambda^2)$
$\sigma \approx O(1)$; $\boxed{\chi = \lambda}$

Leptonic Sector

Recall that the relevant fields transform under Z_4 as:

$$\psi_\ell^0 \to i\psi_\ell^0 \quad ; \quad e_R^0 \to i e_R^0 \quad ; \quad \nu_R^0 \to i\nu_R^0$$

Leptonic Yukawa terms:

$$\mathcal{L}_\ell = \overline{\psi_\ell^0}\, G_\ell\, \phi\, e_R^0 + \overline{\psi_\ell^0}\, G_\nu\, \tilde{\phi}\, e_R^0 +$$

$$+ \frac{1}{2}\, \nu_R^{0T} C \left(f_\nu S + f_\nu' S^* \right) \nu_R^0 + h.c.$$

Leptonic mass matrices:

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & m \\ m^T & M \end{bmatrix} \quad ; \quad m_\ell = \frac{v}{\sqrt{2}} G_\ell$$

$$m = \frac{v}{\sqrt{2}} G_\nu$$

$$M = \frac{V}{\sqrt{2}} f_\nu^+ \cos\alpha + i f_\nu^- \sin\alpha$$

$$f_\nu^\pm \equiv f_\nu \pm f_\nu'$$

Leptonic Mixing

In the weak-basis where m_ℓ is diagonal, real, the light neutrino masses and the low energy leptonic mixing are obtained from the diagonalization of the effective neutrino mass matrix:

$$m_{eff} = - m \frac{1}{M} m^T$$

$$-K^\dagger m \frac{1}{M} K^* = d_\nu$$

m is real, but since M is a generic complex matrix, m_{eff} is also a generic complex matrix. Therefore K will have three complex phases, one Dirac-type, two Majorana-type. It can be shown that one has viable leptogenesis.

Conclusions

Vector-like quarks provide a very interesting scenario for New Physics.

1) They provide a consistent framework where there are naturally small violations of 3×3 unitarity in V^{CKM}, leading to naturally small FCNC.

2) They provide a framework for having a common origin for all CP violations.

3) They play a crucial rôle in the simplest model of spontaneous CP violation where a complex V^{CKM} is generated

4) They provide a simple solution to the strong CP problem.