Threshold Resummed Mass Corrected Spectra in Heavy Quark Decays

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Outline of the talk

1. Large logarithms in perturbation theory near threshold regions in phase space (Sudakov-like)

2. Resumming of large logarithms in QCD
   - ❖ semileptonic decays of heavy quarks
   - ❖ general formulas for
     - ○ massless final parton case
     - ○ massive case

3. Conclusions and future prospects

logarithms (of IR origin) in perturbative expansions LARGE at threshold regions

$$\frac{\alpha_s}{\pi}\int_{E_0}^{\Delta E}\frac{dE}{E}\int_{\theta_0}^{\Delta\theta}\frac{d\theta}{\theta}\sim\frac{\alpha_s}{\pi}\ln\frac{\Delta E}{E_0}\ln\frac{\Delta\theta}{\theta_0}$$

Sudakov-like double logs

“incomplete” cancellation of IR divergencies in virtual and real diagrams at threshold

“constrained“ at threshold

Common feature to many processes: at all orders

$$f\sim\sum_{n=0}^{\infty}\sum_{m=0}^{2n}R_{nm}\alpha^{n}L^{m}$$

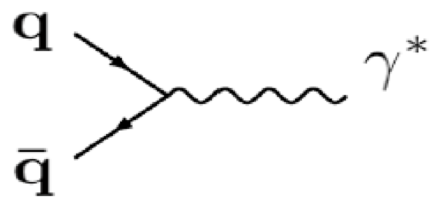
$$L=\ln(?)$$

Argument depends on the observable

Large logs spoil perturbative series

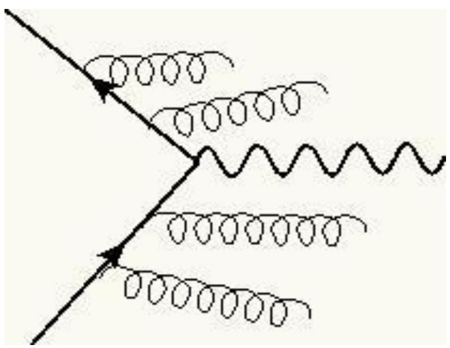
F.i. Drell Yan

$\hat{s}$



pair mass Q

$$z \equiv \frac{Q^2}{\hat{s}}$$



$$\hat{\sigma} \propto \alpha_s^k \left(\frac{\ln^{2k-1}(1-z)}{1-z} \right)_+$$

Similarly for other observables:

•inclusive cross sections $z = 4\, m^2$

•p_T distributions $z = 4\,(m^2 + p_T^2)$

….

Threshold resummation= factorization and (possibly) exponentiation of large threshold logs at all orders in QCD

Go als:

•Restore predictive power to the perturbative series and increase accuracy
•Hope to learn about non-perturbative physics from QCD resummation ambiguities (power corrections, better parameterization)

- Early works started in the eighties (Drell Yan, …)
  Sterman, Catani, Trentadue…

- Today a large collections of new derivations and applications (DIS, Event shapes, Higgs fragmentation, Inclusive B decays, top production and decays…)
  Catani, Mangano, Nason, Sterman, Kidonakis, Laenen, Matsuura, van Neerven, van der Marck, Vogt, Oderda, Grazzini , Corcella, Mitov, De Florian, Vogelsang, Vermaseren, Moch, Berger, Melnikov, Contopanagos, Cacciari, Frixione, Ridolfi, Bonciani, Banfi, Salam, ….

Inclusive heavy decays
$q_{heavy} \to q_{light(er)}$ + (non-QCD: $\gamma^{(*)}$, $W^{(*)}$, $Z^{(*)}$)

Summing virtual and unresolvable real gluon contribution

$$J(m_X) = 1 - \frac{C_F}{\pi}\alpha_S \int_0^1 \frac{d\omega}{\omega}\int_0^1 \frac{d\theta^2}{\theta^2} + \frac{C_F}{\pi}\alpha_S \int_0^1 \frac{d\omega}{\omega}\int_0^1 \frac{d\theta^2}{\theta^2}\,\Theta\left(\frac{m_X^2}{Q^2} - \omega\,\theta^2\right)$$

$$= 1 - \frac{C_F}{2\pi}\alpha_S \log^2\left(\frac{Q^2}{m_X^2}\right)$$

m_X jet invariant mass

Q hard scale

ω gluon energy

θ emission angle

Threshold region

$m_X^2 << Q^2 \quad (E_X)$

$$\sum_{n=1}^{\infty}\sum_{k=1}^{2n} c_{nk}\,\alpha_S^n(Q)\,\log^k\frac{Q^2}{m_X^2} = c_{12}\,\alpha_S(Q)\,\log^2\frac{Q^2}{m_X^2} + c_{11}\,\alpha_S(Q)\,\log\frac{Q^2}{m_X^2} + \cdots$$

To factorize the kinematical constraint for multiple gluon emissions, a transformation to N-moment space or Mellin space is usually made

$$\left[\frac{\ln(1-x)}{1-x}\right]_+ = -\left[\int_0^1\int_0^1 \frac{d\omega}{\omega}\frac{dt}{t}\delta(1-x-\omega t)\right]_+$$

$$\int_0^1 dx\, x^{N-1}\left[\int_0^1\int_0^1 \frac{d\omega}{\omega}\frac{dt}{t}\delta(1-x-\omega t)\right]_+ \simeq -\frac{1}{2}\ln^2 N.$$

Mellin tranform

Treshold region $x \to 1 \leftrightarrow N \to \infty$

$$J_N \simeq \exp\left[-\frac{C_F}{2\pi}\alpha\log^2 N\right]$$

Exponentiation of large Sudakov logs at LO

At all orders in α and in L = log N

$$f \sim \sum_{n=0}^{\infty} \sum_{m=0}^{2n} R_{nm} \alpha^n L^m$$

$$f \sim C(\alpha) \; \exp\left[\sum_{n=1}^{\infty} \sum_{k=1}^{n+1} c_{n\,m}\, \alpha_S^n\, L^k\right] \; + D(\alpha)$$

Advantages : reliable predictions in larger region

$\alpha^n L^{2n}$ vs $\alpha^n L^{n+1}$

Resummation as reorganization in L

Sterman, Catani, Trentadue, Turnock, Webber, Marchesini,...

$$c_{12}\alpha_S\, L^2 + c_{11}\alpha_S\, L + c_{23}\alpha_S^2\, L^3 + c_{22}\alpha_S^2\, L^2 + c_{2\,1}\alpha_S^2\, L + c_{34}\alpha_S^3\, L^4 + \cdots$$

Same order in α L

$$= \; L\, g_1\left(\beta_0\alpha_S L\right) + g_2\left(\beta_0\alpha_S L\right) + \alpha_S\, g_3\left(\beta_0\alpha_S L\right) + \alpha_S^2\, g_4\left(\beta_0\alpha_S L\right) + \cdots$$

$$g_i\,(\lambda) = \sum_{n=1}^{\infty} g_{in}\lambda^n$$

$$\lambda = \beta_0 \alpha\, L$$

LO NLO NNLO

In the Mellin space

$$\sigma_N(\alpha) = e^{G_N(\alpha)}$$

$$G_N = L\, g_1(\lambda) + g_2(\lambda) + \alpha_s\, g_3(\lambda) + \cdots$$

Reals - Virtual terms

$$G_N(\alpha) = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{Q^2(1-z)^2}^{Q^2(1-z)} \frac{dk_t^2}{k_t^2} A\left[\alpha(k_t^2)\right] + B\left[\alpha(Q^2(1-z))\right] + D\left[\alpha(Q^2(1-z)^2)\right] \right\}$$

Sterman, Catani, Trentadue, Turnock, Webber, Marchesini,...

$$A(\alpha) = \sum_{n=1}^{\infty} A_n\, \alpha^n = A_1\alpha + A_2\alpha^2 + A_3\alpha^3 + A_4\alpha^4 + \cdots$$

Soft & Collinear gluon emission

$$B(\alpha) = \sum_{n=1}^{\infty} B_n\, \alpha^n = B_1\alpha + B_2\alpha^2 + B_3\alpha^3 + \cdots$$

Collinear emission at small angles

$$D(\alpha) = \sum_{n=1}^{\infty} D_n\, \alpha^n = D_1\alpha + D_2\alpha^2 + D_3\alpha^3 + \cdots$$

Soft Emission at large angles

Moch, Vogt,...

- Specific structure of the resummation formula varies with the specific process and the particular observable
  f.i. $t \to b\, W$
  collinear term B lacks in $x = 2\, E_b/m_t$ distributions

  Cacciari, Corcella, Mitov

- Resummation formula ill defined due to integrations over the Landau pole: get singular g_i

$$\lambda = \beta_0\, \alpha_s(Q^2)\, L,$$

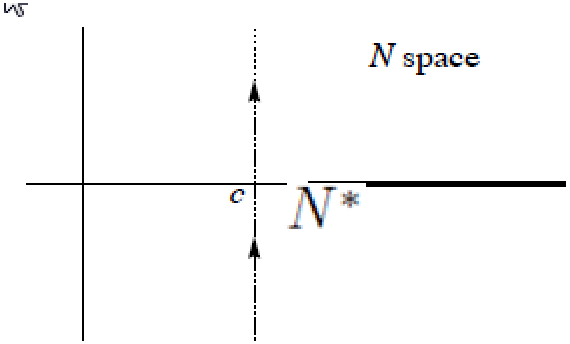
$$g_1(\lambda) = -\frac{A_1}{2\,\beta_0\,\lambda}\left[(1-2\,\lambda)\log(1-2\,\lambda) - 2\,(1-\lambda)\log(1-\lambda)\right]$$

Note: singular at $\lambda = 1/2$ or $N = N^* = \exp(1/2\beta_0\alpha_s)$

The inverse Mellin transform

$$\sigma(u;\,\alpha) \;=\; \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}(1-u)^{-N}\,\sigma_N(\alpha)$$

Branch cut due the Landau singularity for N › $N^* = \exp(1/2\beta_0\alpha_s)$
The Mellin Inverse does not exists

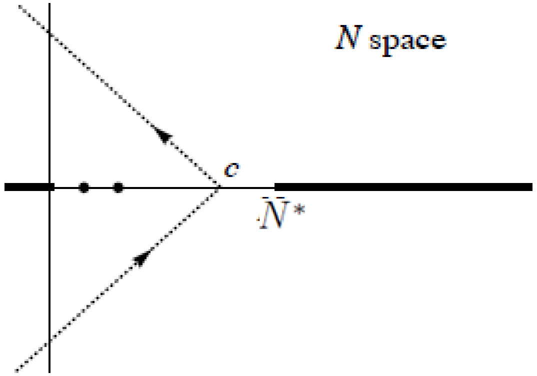


Several different prescriptions
Catani, Mangano, Nason, Trentadue; Contopanagos, Sterman; Vogt…

Minimal Prescription

Catani, Mangano, Nason, Trentadue

Mellin chosen contour to the *left of N**



•PT series defined in this way has no factorial growth

•asymptotic to the original divergent series

•Spurious effects originated by neglecting sub-leading terms (unwanted extra factorial growth) in the inverse transform can be avoided by taking an appropriate truncated expansion

•Difference between the truncated & the entire expansion is exponentially suppressed

Large Sudakov logarithms can be summed also by soft collinear effective theory, that works by separating the degrees of freedom of QCD into soft modes and collinear modes

Same logarithmic expansion at all orders expected

f.i. $B \to X_s \gamma$

$$\Gamma(N) = \int_0^1 x^{N-1} \frac{d\Gamma}{dx} \quad \equiv \Gamma_0 \, C(N;\tilde{\mu}) \, \langle O(N;\tilde{\mu}) \rangle$$

$$\Gamma(N) = \Gamma_0 f\,(N; m_b n_0/N) \left(\frac{\alpha_s(m_b \sqrt{\frac{n_0}{N}})}{\alpha_s} \right)^{\frac{C_F}{\beta_0}\left(5 - \frac{8\pi}{\beta_0 \alpha_s}\right)} \left(\frac{\alpha_s(m_b \frac{n_0}{N})}{\alpha_s(m_b \sqrt{\frac{n_0}{N}})} \right)^{\frac{2C_F}{\beta_0}\left(1 + \frac{4\pi}{\beta_0 \alpha_s} - 2\log\frac{N}{n_0}\right)}$$

resums leading logarithms and the same class of next-to-leading logarithms as in the QCD approach

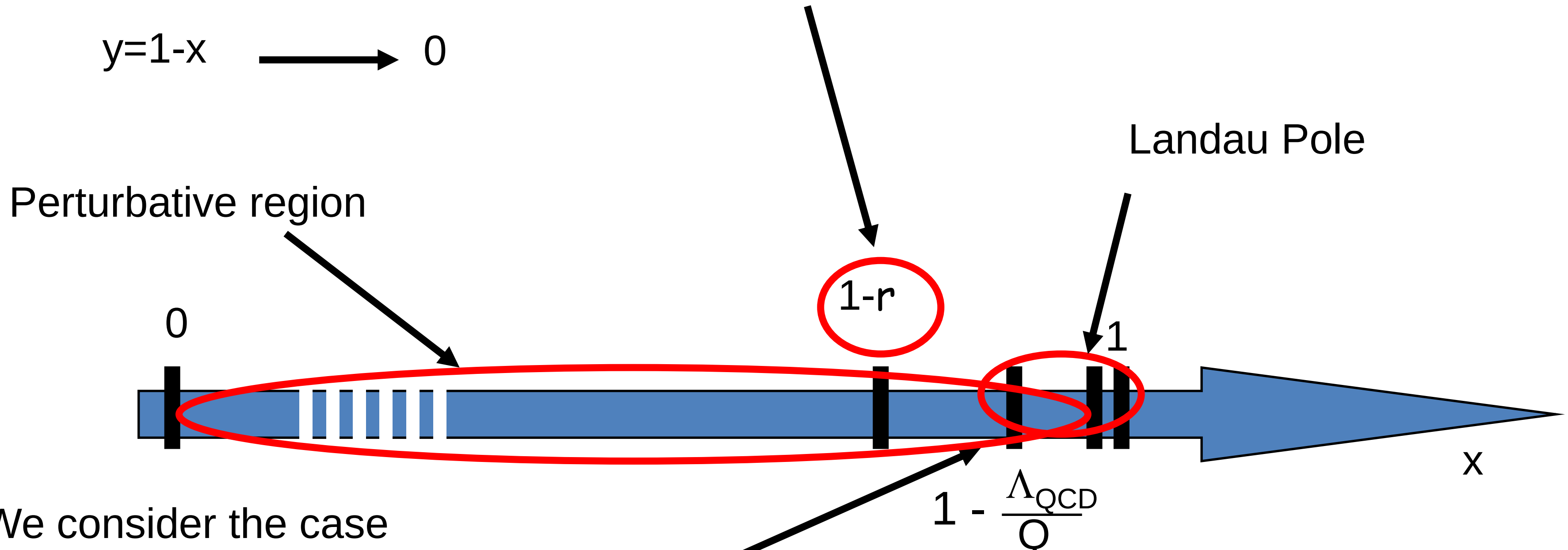
Bauer, Fleming, Luke 00

What changes if we consider also the mass of the final parton?

Aglietti, Di Giustino, Ferrera,
Renzaglia, GR, Trentadue,
PhysLettB653, 38 (07);
Di Giustino, GR, Trentadue,
arXiv:1102.0331 [hep-ph]

The massive case: a multi-scale problem

- The threshold region is affected by large logs and also by non perturbative effects
- A new scale is added in the P.R.: the quark mass scale $r = m^2/Q^2$

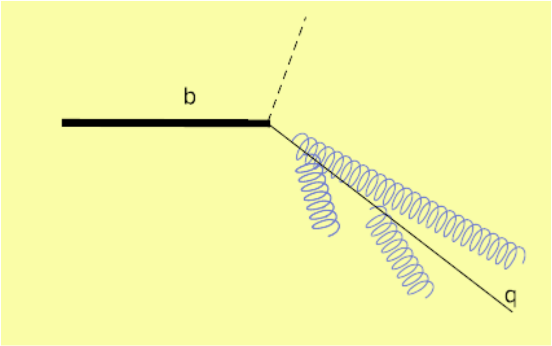


We consider the case

$$\frac{\Lambda_{QCD}}{Q} < r << 1$$

The non perturbative contributions can be factorized by the shape function or described in a model by an effective coupling

Logarithmic structure in the massive case



- For massless partons amplitudes contain terms proportional to:

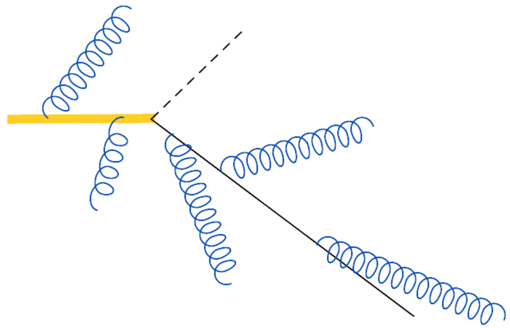
$$\alpha_s \frac{dE}{E} \frac{d\theta}{\theta}$$

- With (final) massive parton of mass m:

$$\alpha_s \frac{dE}{E} \frac{d\theta}{\theta^2 + \frac{m^2}{Q^2}} \sim \alpha_s \frac{dE}{E} \Theta \left(\theta^2 - \frac{m^2}{Q^2} \right) \frac{d\theta}{\theta}$$

dead cone effect + soft quanta isotropically radiated in its rest frame

$$B \to X_c + l + \nu_l$$



soft — collinear

$$\alpha_S^n \log^k\left(\frac{m_{X_c}^2 - m_c^2}{m_b^2 - m_c^2}\right) \log^l\left(\frac{m_{X_c}^2}{m_b^2}\right)$$

The soft terms in the amplitude are factorized by the eikonal current and the collinear by using the dipole factorization formulae for the massive case

Catani, Dittmaier, Seymour ,Trocsanyi, …

$$\sigma_N(\rho, Q^2) = \exp \int_0^1 dy \left[(1-y)^{N-1} - 1\right] \left\{ \frac{1}{y} \int_{\frac{Q^2 y^2}{1+\rho}}^{\frac{Q^2 y^2}{y+\rho}} A\left[\rho; \alpha_S(k^2)\right] \frac{dk^2}{k^2} + \frac{1}{y} D\left[\alpha_S\left(\frac{Q^2 y^2}{1+\rho}\right)\right] + \right.$$
$$\left. + \left(\frac{1}{y} - \frac{1}{y+\rho}\right) \Delta\left[\alpha_S\left(\frac{Q^2 y^2}{y+\rho}\right)\right] + \frac{1}{y+\rho} B\left[\alpha_S\left(\frac{Q^2 y^2}{y+\rho}\right)\right] \right\}$$

Soft emission from the final quark

$$y \equiv \frac{m_{X_c}^2 - m_c^2}{Q^2 - m_c^2} \qquad \rho \equiv \frac{m_c^2}{Q^2 - m_c^2}$$

in the threshold region $y \ll 1$

1. *very slow charm quark:* $u_c \gtrsim 0$ or, equivalently, $\rho \gg 1$.

$$u_c = p_c/E_c$$

$$\rho \simeq \frac{1-u_c}{2u_c}$$

$$\sigma_N(\rho; Q^2) \to 1 \quad \text{for} \quad \rho \to +\infty,$$

2. *non-relativistic charm quark.* $u_c \approx \frac{1}{3}$ or $\rho \approx 1$.

$$\sigma_{S,N}(\rho, Q^2) = \exp \int_0^1 \frac{dy}{y}\left[(1-y)^{N-1}-1\right]\left\{\int_{\frac{Q^2y^2}{1+\rho}}^{\frac{Q^2y^2}{\rho}} A\left[\rho; \alpha_S(k^2)\right]\frac{dk^2}{k^2} + D\left[\alpha_S\left(\frac{Q^2y^2}{1+\rho}\right)\right] + \Delta\left[\alpha_S\left(\frac{Q^2y^2}{\rho}\right)\right]\right\}$$

3. *fast charm quark:* $u_c \lesssim 1$ or $\rho \ll 1$.

$$\sigma_N(0, Q^2) = \exp \int_0^1 \frac{dy}{y}\left[(1-y)^{N-1}-1\right]\left\{\int_{Q^2y^2}^{Q^2y} A\left[\alpha_S(k^2)\right]\frac{dk^2}{k^2} + D\left[\alpha_S\left(Q^2y^2\right)\right] + B\left[\alpha_S\left(Q^2y\right.\right.$$

$$y \to u = \frac{E_X - p_X}{E_X + p_X} \simeq \frac{m_X^2}{4E_X^2}; \qquad Q \to E_X + p_X \simeq 2E_X,$$

Factorize into
massless x universal jet factor

$$\sigma_N(\rho, Q^2) \simeq \sigma_N(0, Q^2)\,\delta_N(\rho, Q^2) \qquad \text{for} \quad \rho \ll 1,$$

$$\delta_N(\rho, Q^2) = \exp \int_0^1 dy \frac{(1-y)^{\rho(N-1)}-1}{y}\left\{-\int_{\rho Q^2y^2}^{\rho Q^2 y}\frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] - B\left[\alpha\left(\rho Q^2 y\right)\right] + D\left[\alpha\left(\rho Q^2y^2\right)\right]\right\}.$$

Mass effects in Heavy Quark Decays, (*t, b*)

- mass terms can be relegated into a factor $\delta_N(Q^2;\, m^2)$:
  - at NLO in the Mellin space

$$J_N(Q^2;\, m^2) = J_N(Q^2)\ \delta_N(Q^2;\, m^2)$$

$J_N(Q^2)$ is the massless jet distribution

$$J_N(Q^2) = \exp \int_0^1 \frac{dy}{y}\left[(1-y)^{N-1} - 1\right]\left\{\int_{Q^2y^2}^{Q^2y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] + B\left[\alpha\left(Q^2 y\right)\right]\right\}$$

$\delta_N(Q^2;\, m^2)$ is the mass-correction factor

$$\delta_N(Q^2; m^2) = \exp \int_0^1 dy \frac{(1-y)^{r\,(N-1)} - 1}{y}\left\{-\int_{m^2y^2}^{m^2y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] - B\left[\alpha\left(m^2 y\right)\right] + D\left[\alpha\left(m^2 y^2\right)\right]\right\}$$

conjecture its general validity beyond NLO

The mass correction factor has a similar structure than the massless case: use of similar approach (formulas)

$$\delta_N(Q^2;\, m^2) = e^{F_N(Q^2;m^2)}$$

where

$$F_N\left(Q^2; m^2\right) = \theta\,(N - 1/r) \sum_{n=1}^{\infty} \sum_{k=1}^{n+1} F_{nk}\, \alpha^n \log^k(Nr)$$

The exponent can be expanded in towers of logarithms as:

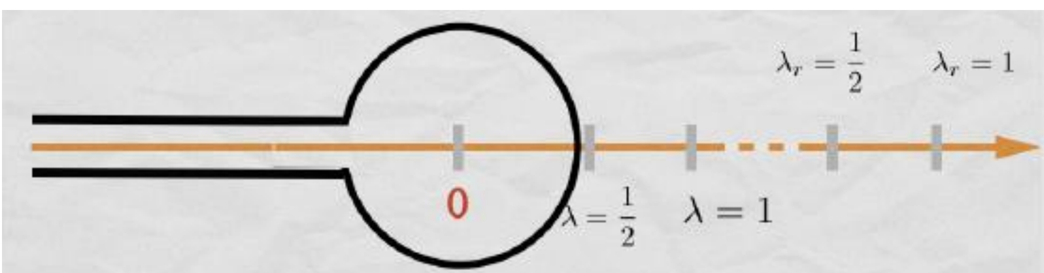
$$\begin{aligned} F_N\left(Q^2; m^2\right) &= L\, d_1(\rho) + \sum_{n=0}^{\infty} \alpha^n\, d_{n+2}(\rho) \\ &= L\, d_1(\rho) + d_2(\rho) + \alpha\, d_3(\rho) + \alpha^2\, d_4(\rho) + \cdots \end{aligned}$$

where

$$\rho \equiv \beta_0 \alpha(\mu^2)\, L \qquad L = \theta\,(N - 1/r)\, \log\,(N\, r)$$

- There are two regions dominated respectively by a massless / massive behaviors: N>1/r | N<1/r;
- The Minimal Prescription is still valid
- 2 ways to proceed: Numerical /Analytical

$$J\left(x;\, Q^2, r\right) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\, x^{-N}\, J_N(Q^2;\, r) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\, x^{-N} J_N(Q^2)\ \delta_N(Q^2$$



Landau Poles of the Massive Jet distribution.

The MP guarantees the numerical Mellin inverse to be free of factorial growth

Landau pole $x = 1 - e^{\frac{1}{2\alpha_S\ \beta_0}}$

The Jet - functions in physical space

$$J(x;Q^2) = -x\,\frac{d}{dx}\left\{\theta\left(1-x-\epsilon\right)\,\Sigma\left(x;\,Q^2\right)\,\right\}$$

$$\Sigma\left(x;\,Q^2\right)\;=\;\frac{e^{l\,g_1(\tau)\,+\,g_2(\tau)}}{\Gamma\left[1-h_1(\tau)\right]}\;\delta\Sigma$$

$$l \equiv -\ln(-\ln x)$$

$$l \rightarrow -\ln(1-x)\ \text{when}\ x\rightarrow 1$$

$$\tau \equiv \beta_0\alpha l\,, \qquad h_1(\tau) \equiv \frac{d}{d\tau}(\tau\, g_1(\tau))$$

$$\delta\left(x;\,Q^2,m^2\right)\;=\;-x\,\frac{d}{dx}\left\{\theta(1-x-\epsilon)\,\Delta\left(x;\,Q^2,m^2\right)\,\right\}$$

$$\Delta\left(x;\,Q^2,m^2\right)\;=\;\frac{e^{l'\,d_1(\tau')\,+\,d_2(\tau')}}{\Gamma\left[1-h_1(\tau')\right]}\delta\Delta$$

$$l' \equiv -\log(-\log x^{1/r})$$

$$\cong \theta(\boldsymbol{x}-1+\boldsymbol{r})\log\frac{\boldsymbol{r}}{1-\boldsymbol{x}}$$

$$\tau' \equiv \beta_0\alpha l'.$$

The factor $\theta(1-x-\epsilon)$ ensures the unitary normalization of the distributions in the (0,1) interval in the limit $\epsilon \longrightarrow 0$, this term can be omitted for a regular distribution, but it is mandatory for the mass correction factor.

The Frozen Coupling Limit

$$\begin{aligned} \log J_N(Q^2, m^2) &= f_N(Q^2) + F_N(Q^2, m^2) \qquad \beta_0 \longrightarrow 0 \\ &\simeq L g_1 + L_r d_1 = \\ &= -\frac{A_1}{2\,\beta_0}\,(\lambda\,L - \rho\,L_r), \end{aligned}$$

The leading log behavior is given by

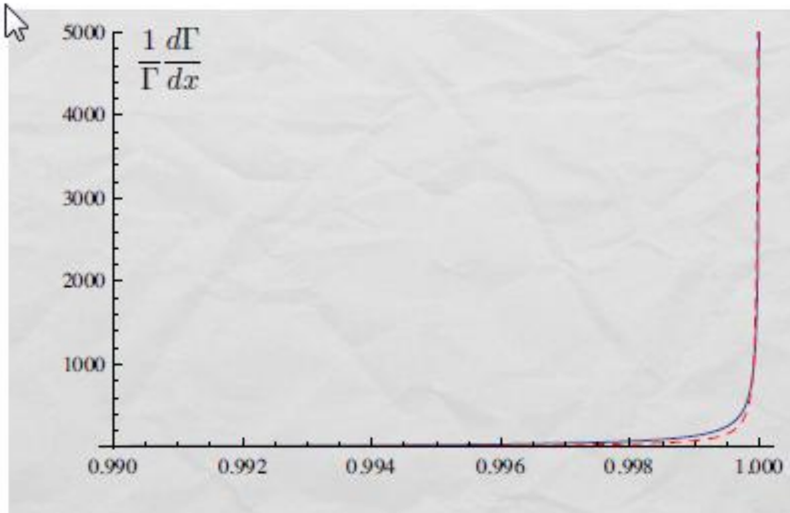
$$\log J_N(Q^2, m^2) \simeq A_1 \alpha_S \cdot \log N \cdot \log r$$

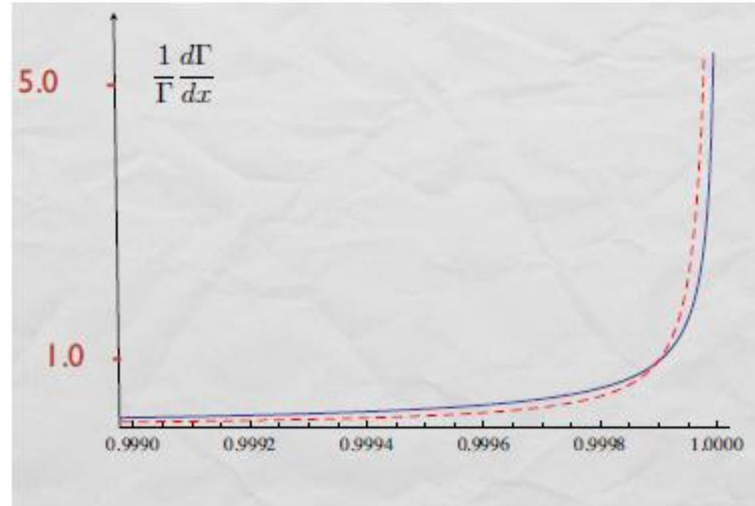
- *Log r* is negative and restores a finite limit when $x \to 1$ in the physical space.

- We no longer have spurious effects generated by neglecting sub-leading terms: the approximation *Log* N → -*Log x* works well for the Numerical convolution

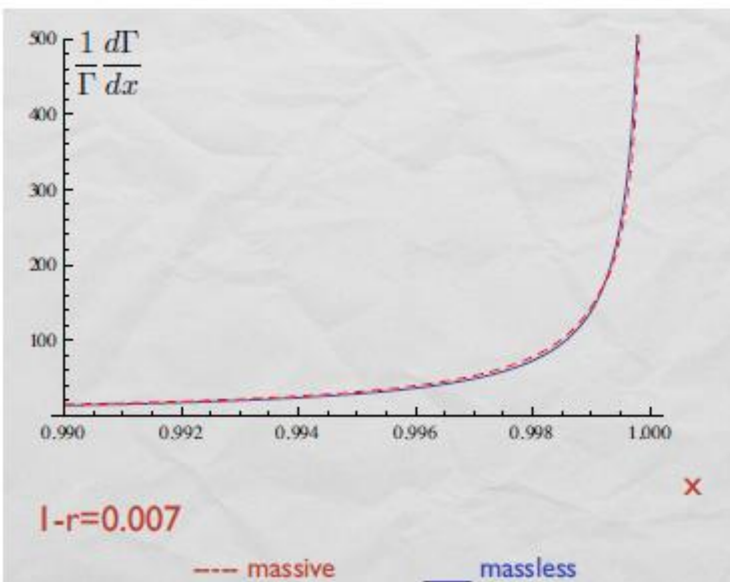
In the Frozen coupling approximation the Numerical and Physical Convolutions give the same results

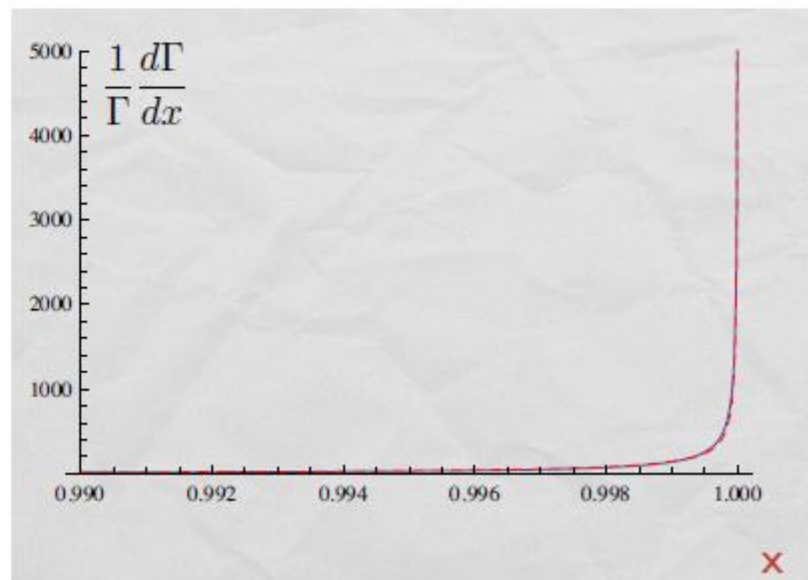
Results in frozen coupling ($\beta \to 0$) ($x = 1 - y$) in order to estimate the mass effects $b \to c$





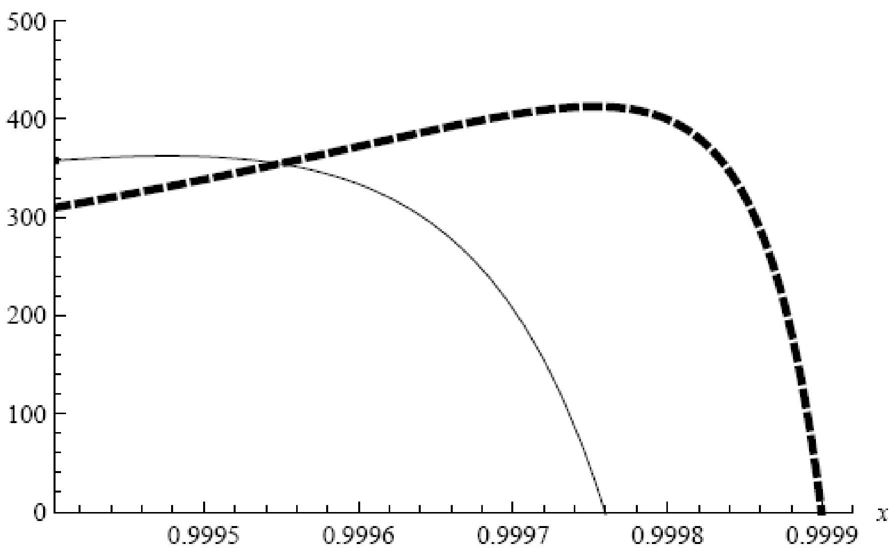
Results in frozen coupling ($\beta \to 0$) in order to estimate the mass effects $t \to b$



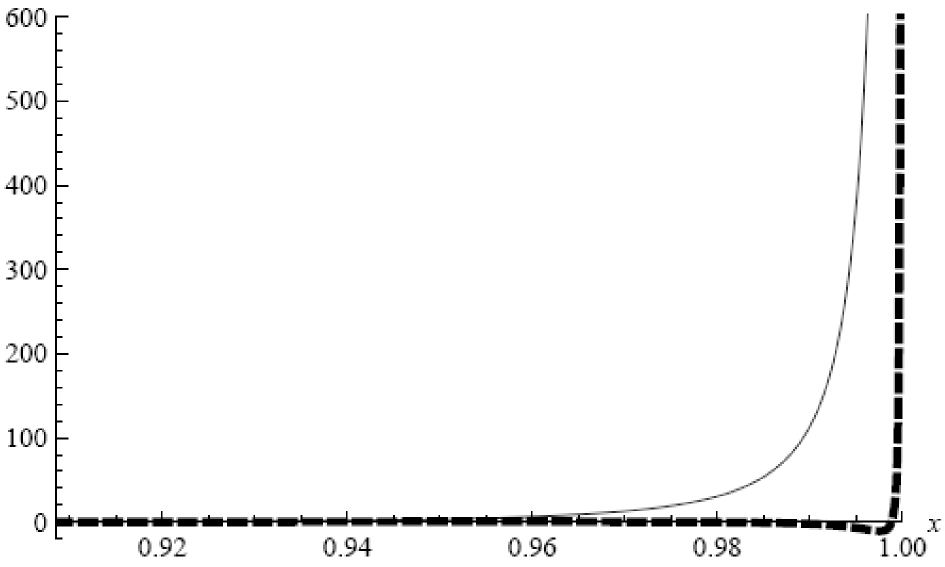


Running Coupling: Jet rates in the massive case (dotted line) compared to the massless case (continuous line) at NNLL;

$\alpha s= 0.11;$ Top decay



$\alpha s= 0.219$; Beauty decay



Conclusions

- The regularization of a generalized resummed approach to mass corrections in HQ decays in the threshold region up to NNLL terms

- In the frozen coupling constant case the massive Minimal Prescription Formula regularization, for the leading terms, shows a milder divergent behaviour (shadow of the singularity in the massless case) with respect to the massless one

approach the massive case is potentially extensible *to any other massive perturbative evaluation at threshold region*

Work in progress: exploring different prescriptions and comparison with data