Flavor Symmetry and Grand Unification

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Major Flavor Puzzles

i) $m_t <<<< M_{GUT}$

ii) $m_u << m_c << m_t$

iii) quark and neutrino mixing angles and their difference.

literature: very large number of models on Flavor Symmetries,

but almost all models

need numerous ad hoc assumptions
have no connection between quarks and leptons
cannot be used for Grand Unification

The above puzzles remain unsolved. It is my aim to discuss attempts to combine GUT's with Flavor Symmmetry.

GUT + Flavor Symmetry require the direct product

$$GUT \times Flavor$$

with the consequence: the flavor representation of all fermions, quarks and leptons, must be identical.

For instance: by using the flavorgroup $SO(3)_F$ for 3 generations all fermions are 3 vectors with respect to this group.

This requirement leaves only very few models discussed in the literature:
These are rather complicated $SO(10)$ GUT's with the assignment of different flavor representations for different Higgs fields.

Question:

Can one construct a more simple and symmetric GUT$\times$ Flavor model in which all Higgs fields are flavor singlets and all flavon fields are GUT singlets ??

This appears difficult if not impossible for $SO(10)$ GUT's

but possible for the gauge groups

E_6 and "Trinification": $SU(3)_L \times SU(3)_R \times SU(3)_C \times Z_3$
the $(SU(3))^3$ subgroup of E_6.

some references to trinification: Y. Achiman, B.S. (1978-1979), A. de Rujula, H. Georgi, S.L. Glashow (1984), K.S. Babu, X.G. He, S.Pakvasa (1986), Ch. Canot, H.Paes, S. Wiesenfeldt (2010) hep/ph 1012.4083 .

There are several very good arguments for SUSY
but

i) the quadratic divergencies causing the hierarchy problem arise from tadpole diagrams which are momentum independent and can be subtracted.

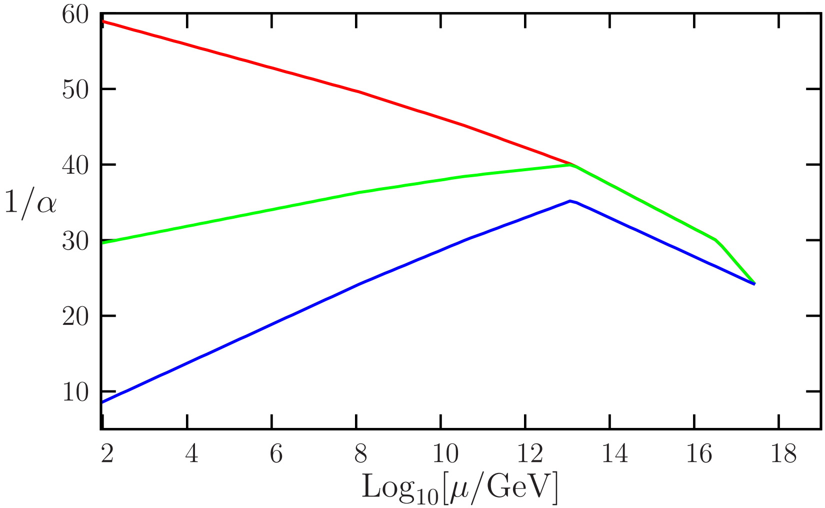
ii) non supersymmetric GUT's need fewer parameters to describe all fermion properties

iii) the neutrino masses most likely require a two step unification process which in non supersymmetric theories occur naturally by electroweak unification below the GUT scale at $M_I \approx 2 \cdot 10^{13}$ GeV .

$$SU(2)_L \times U(1) \times SU(3)_C \Rightarrow SU(3)_L \times SU(3)_R \times SU(3)_C \Rightarrow E_6$$

Phys. Rev. D77, 076009 (2008) Z. Tavartkiladze, B. S.

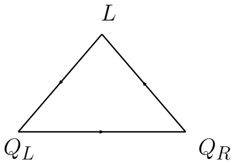
Concorde


60
50
40
30
20
10
$1/\alpha$
2
4
6
8
10
12
14
16
18
$\mathrm{Log}_{10}[\mu/\mathrm{GeV}]$

$E6 \supset SU(3)_L \times SU(3)_R \times SU(3)_C \times Z_3$

Single generation for fermions $\psi(27)$

$$\mathbf{27} = Q_L(x) + L(x) + Q_R(x)$$



$$(Q_L)^a_i = \begin{pmatrix} u^a \\ d^a \\ D^a \end{pmatrix}, \quad L^i_k = \begin{pmatrix} L^1_1 & E^- & e^- \\ E^+ & L^2_2 & \nu \\ e^+ & \hat{\nu} & L^3_3 \end{pmatrix}, \quad (Q_R)^k_a = \begin{pmatrix} \hat{u}_a, & \hat{d}_a, & \hat{D}_a \end{pmatrix},$$

$$Q_L(x) = \begin{pmatrix} 3, & 1, & \bar{3} \end{pmatrix}, \quad L(x) = \begin{pmatrix} \bar{3}, & 3, & 1 \end{pmatrix}, \quad Q_R(x) = \begin{pmatrix} 1, & \bar{3}, & 3 \end{pmatrix}$$

- mixing: $d \leftrightarrow D$ $\quad \mathcal{U}_L$-spin, $\quad \hat{d} \leftrightarrow \hat{D}$ $\quad \mathcal{U}_R$-spin

Higgs fields H_{27} , $\tilde{H}_{27}$

2 Higgs fields $(3^*, 3, 1)$ out of H_{27} and $\tilde{H}_{27}$ are needed to break the trinification group down to the standard model :

$$\langle H \rangle \simeq \begin{pmatrix} m_t & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & M_I \end{pmatrix} \qquad \langle \tilde{H} \rangle \simeq \begin{pmatrix} small & 0 & 0 \\ 0 & small & small \\ 0 & M & \kappa\, M \end{pmatrix}$$

Independent of generations !

$\langle H \rangle$ with $m_b = m_t = 0$ gives $SU(2)_L \times SU(2)_R \times U(1)$,
$\langle \tilde{H} \rangle$ with the small entries $= 0$ together with $\langle H \rangle$ leads to $SU(2)_L \times U(1)_Y \times SU(3)_C$
$\langle H \rangle$ is diagonal, $\langle \tilde{H} \rangle$ is not directly coupled to fermions.
12 high mass vector bosons
Problem: multi Higgs model

$E_6 \times$ Flavor $\supset$ Trinification $\times$ Flavor

Flavor $= SO(3)_F \times P_F$

$\Phi_{\alpha\beta}$: scalar flavor fields (GUT singlets)

$\frac{\langle \Phi_{\alpha\beta} \rangle}{M}$ = coupling matrix

$$\mathcal{L}_Y^{eff} = \frac{1}{M} \langle \Phi_{\alpha\beta} \rangle \left(\psi^{\alpha T} H \psi^{\beta} \right)$$

$\Rightarrow$ effective interaction to be understood on a deeper level

Flavor SO(3)

Flavor $SO(3)_g$

- All fermions transform as 3-vectors under this group
  The 3×3 coupling matrices in front of the Higgs fields are then obtained from the VEV's of $3 \times 3 = 9$ real scalar flavons which can be represented by the hermitian matrix field $\Phi_{\alpha\beta}(x)$:
- $\Phi_{\alpha\beta}(x) = \chi_{\alpha\beta}$ (symmetric) $+i\,\xi_{\alpha\beta}$ (antisymmetric)

$$\chi_{\alpha\beta} \sim \text{"1"} + \text{"5"} \qquad\qquad \xi_{\alpha\beta} \sim \text{"3"}$$

$$\mathcal{L}_Y^{eff} = \frac{\phi_{\alpha\beta}}{M}(\psi^{\alpha T} H \psi^{\beta}) + \ldots$$

χ can be taken diagonal

$$\chi = \begin{pmatrix} \chi_1 & 0 & 0 \\ 0 & \chi_2 & 0 \\ 0 & 0 & \chi_3 \end{pmatrix} , \quad \xi = \begin{pmatrix} 0 & \xi_3 & -\xi_2 \\ -\xi_3 & 0 & \xi_1 \\ \xi_2 & -\xi_1 & 0 \end{pmatrix}$$

subgroup of SO(3) leaving χ diagonal is the discrete group $S4$
$S4 \Rightarrow$ simple permutations of $\psi's$, $\chi's$ and $\xi's$.
however: no breaking to S4 occurs !

one gets spontaneous symmetry breaking of $SO(3)$ in one step by appropriate SO(3) invariant potentials. Each S4 transformation of $\langle\chi\rangle$ and $\langle\xi\rangle$ values will also be a minimum of the potential.

Phenomenology

The coupling matrix $G = \frac{\langle\chi\rangle}{M_I}$ determines the mass hierarchy

$$G = \frac{\langle\chi\rangle}{M_I} = \begin{pmatrix} m_u & 0 & \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \frac{1}{m_t} = \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{at } \mu=M_I}$$

$\sigma = 0.050 \Rightarrow$ correct up quark masses

The coupling matrix $A = i\frac{\langle\xi\rangle}{M'}$ describes particle mixings.
It is antisymmetric and hermitian, 1 real parameter:

$$A = i\frac{\langle\xi\rangle}{M'} = i\begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & 1/2 \\ \sigma & -1/2 & 0 \end{pmatrix}$$

the generation matrices $G_{\alpha\beta}$ and $A_{\alpha\beta}$ appear in the effective Yukawa interaction

$$\mathcal{L}_Y^{\text{eff}} = G_{\alpha\beta}(\psi^{\alpha T} H \psi^{\beta}) + A_{\alpha\beta}(\psi^{\alpha T} H_A \psi^{\beta}) + \frac{(G^2)_{\alpha\beta}}{M_N}\Big((\psi^{\alpha T} H^{\dagger})_1 (\tilde{H}^{\dagger}\psi^{\beta})_1\Big) + h.c.$$

- hierarchy of masses and the mixings of all fermions are now fully determined
- The A term is CP odd $\rightarrow$ unique $\not{CP}$

The Masses and Mixings of all 3×27 fermions are obtained from the mass matrix

$$M_{ij}^{\alpha\beta} = G_{\alpha\beta}\langle H\rangle_{ij} + A_{\alpha\beta}\langle H_A\rangle_{ij} + (G^2)_{\alpha\beta}\langle H^\dagger\rangle_i \frac{1}{M_N}\langle \tilde{H}^\dagger\rangle_j$$

$$\langle H\rangle = \begin{pmatrix} m_t & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & M_I \end{pmatrix} \qquad \langle \tilde{H}\rangle = \begin{pmatrix} small & 0 & 0 \\ 0 & small & small \\ 0 & M & \kappa\, M \end{pmatrix}$$

$$\langle H_A\rangle_{\text{q}uarks} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & f_2^2 & f_3^2 \\ 0 & f_2^3 & f_3^3 \end{pmatrix} \qquad \langle H_A\rangle_{\text{l}eptons} = \text{s}imilar\ parameters$$

$$m_D = m_L = M_I$$

- **Results** :

$$m_u = G\, m_t$$

$$m_d \simeq G\, m_b + A\, f_2^2 - f_0\, A \cdot G^{-1} \cdot A \quad \text{small mixing and } \not{CP}$$

$$m_\nu \simeq \frac{m_t^2}{M_I}\left(\kappa\, \mathbb{1} + \frac{f_2^3}{M_I}(G^{-1} \cdot A - A \cdot G^{-1})\right) \quad \text{large mixing and } \not{CP}$$

- heavy neutrino masses $M_L \approx G^2 M_I \quad 1 : \sigma^4 : \sigma^8$
  very large hierarchy $2 \cdot 10^{13}$, $1 \cdot 10^8$, $8 \cdot 10^2$ GeV!
- 9 parameters:
  $m_t, m_b, m_\tau, f_2^2, f_0, f_2^3, \kappa, \sigma, 1/2$
- observables:
  quarks → 10, charged leptons → 3, light neutrinos → 9,
  heavy fermions → 15 22 + 15 = 37

Mass Matrices

Up quark matrix $M_u = G\, m_t$

Down quark matrix $\langle H_A \rangle \sim (\bar{3}, 3, 1)$

$$M_{d,D} = \begin{array}{c} \\ d \\ D \end{array} \begin{array}{c} \begin{array}{cc} \hat{d} & \hat{D} \end{array} \\ \begin{pmatrix} m_b^0 G + f_2^2 A \,, & f_3^2 A \\ f_2^3 A \,, & M_I G \end{pmatrix} \end{array},$$

$$\begin{aligned} f_2^2 &= \langle H_A \rangle_2^2 \\ f_3^2 &= \langle H_A \rangle_3^2 = f_0 / x_f \\ f_2^3 &= \langle H_A \rangle_2^3 = x_f M_I \end{aligned}$$

$5 \rightarrow 3$ parameters   7 observables

6×6 3×3

$$M_d = m_b^0 G + f_2^2 A - f_0\, A G^{-1} A$$

This gives a very good fit for *CKM* unitarity triangle

Results: Quarks

- Fit: $m_b^0 = 2.95\ GeV$, $f_2^2 = -0.23$ GeV, $f_0 = 1.30$ GeV

$$m_b = 2.89\ \text{GeV}, \qquad m_s = 50\ \text{MeV}, \qquad m_d = 2.6\ \text{MeV}$$

- exp: 2.89 ± 0.09 $\quad 55 \pm 15$ $\quad 2.9 \pm 1.2$

$$|V_{cd}| = 0.228, \qquad |V_{cb}| = 0.042, \qquad |V_{ub}| = 0.0039$$

$$\alpha_q = 97^o, \qquad \beta_q = 23^o, \qquad \gamma_q = 60^o.$$

- all results are within experimental errors
- lightest D quark for $M_I = 2 \cdot 10^{13}$ GeV
  $m_{D_1} \simeq 1 \cdot 10^8$ GeV

Neutral Lepton Mass Matrix

- Neutral Lepton Mass Matrix 15×15

$$M_L = \begin{array}{c} \\ L_3^2 \\ L_1^1 \\ L_2^2 \\ L_2^3 \\ L_3^3 \end{array} \begin{array}{c} \begin{array}{ccccc} L_3^2 & L_1^1 & L_2^2 & L_2^3 & L_3^3 \end{array} \\ \left(\begin{array}{ccccc} 0 & -f_2^3 A & 0 & -m_t G & 0 \\ -f_2^3 A & 0 & M_I G & 0 & 0 \\ 0 & M_I G & 0 & 0 & m_t G \\ -m_t G & 0 & 0 & 0 & M_I G^2 + F_A A \\ 0 & 0 & m_t G & M_I G^2 + F_A A & \kappa\, M_I G^2 \end{array} \right) \end{array} .$$

- New element: $F_A = \langle H_A \rangle_{33,\,1} = \sigma^5 M_I\, x_A$
  $15 \times 15 \quad \Rightarrow \quad 3 \times 3$ *multiple* see-saw

Results: Neutrinos

- one has with $m = \frac{m_t^2}{M_I} x_g \quad \rho = \frac{\kappa}{x_g}, \quad (x_g \simeq 0.039$ from $\langle H_A \rangle)$

$$m_\nu \Rightarrow \quad m \begin{pmatrix} \rho & \mathrm{i} & -\mathrm{i} \\ \mathrm{i} & \rho & \mathrm{i}\frac{\sigma}{2} \\ -\mathrm{i} & \mathrm{i}\frac{\sigma}{2} & \rho \end{pmatrix}$$

- Eigenvalues of $m_\nu m_\nu^\dagger$:

$$(m_2)^2 \simeq (\rho^2 + 2 + \frac{\sigma}{\sqrt{2}})\, m^2 \ ,$$
$$(m_1)^2 \simeq (\rho^2 + 2 - \frac{\sigma}{\sqrt{2}})\, m^2 \ ,$$
$$(m_3)^2 \simeq \rho^2 m^2$$

i) Inverted hierarchy

ii) degeneracy in the no mixing limit $x_g \to 0$

iii) $R = (m_2^2 - m_1^2)/(m_2^2 - m_3^2) \simeq \sigma/\sqrt{2} \simeq 0.035$

iv) m and x_g fixed from $\Delta m^2_{\mathrm{atm}}$

$$m \simeq \frac{1}{\sqrt{2}}\sqrt{\Delta m^2_{\mathrm{atm}}} \simeq 0.035\ eV\ , \quad m = \frac{m_t^2}{M_I}x_g \ \Rightarrow\ x_g \simeq 0.04$$

v) M_I together with σ
fixes masses of all heavy fermions

vi) In the limit of no RG effects one gets bimaximal neutrino mixing.

Neutrino properties

examples : setting $\kappa = 0.23$ and alternatively $\kappa = 0.08$
fixing $m_{3,3}$ to get $\Theta_{12} \simeq 34^\circ$ one finds

- masses:
  $m_2 = 0.2028, \quad m_1 = 0.2026, \quad m_3 = 0.1973$ eV
  $m_2 = 0.0830, \quad m_1 = 0.0825, \quad m_3 = 0.0687$ eV
- mixing angles:
  $\Theta_{12} \simeq 34^\circ, \quad \Theta_{23} \simeq 45^\circ, \quad \Theta_{13} \simeq 4^\circ$,
  $\Theta_{12} \simeq 34^\circ, \quad \Theta_{23} \simeq 48^\circ, \quad \Theta_{13} \simeq 3.6^\circ$
- $\not{C}\not{P}$: $\delta_\nu = 265^\circ, \quad \delta_\nu = 255^\circ$
- Neutrinoless double β decay parameter:
  $|m_{\beta\beta}| = 0.20$ eV , $|m_{\beta\beta}| = 0.073$ eV

  very satisfactory ! $|m_{\beta\beta}|$ scales with m_2

Potentials

for spontaneous symmetry breaking $\Rightarrow$ large hierarchies

Example :

$2\ H_{27}$ Higgs fields $\quad SU(3)_L \times SU(3)_R = H^i_k$

$$\langle H \rangle \Rightarrow \begin{pmatrix} m_t & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & M_I \end{pmatrix} \qquad \langle \tilde{H} \rangle \Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M & \kappa\, M \end{pmatrix}$$

?? Potential giving this hierarchy with $M_I, M \ggg$ m$_t$??

Potentials

Coleman-Weinberg type

Invariants

$$Y_1 = (Tr[H \cdot H^\dagger])^2, \;\; Y_2 = M_I(\det H + \det H^\dagger), \;\; Y_3 = Tr[H \cdot H^\dagger \cdot H \cdot H^\dagger],$$

$$Y_4 = (Tr[\tilde{H} \cdot \tilde{H}^\dagger])^2, \;\; Y_5 = Tr[\tilde{H} \cdot \tilde{H}^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger], \;\; + \ldots$$

speculation :

$$V_i = \frac{c_i}{16} \, Y_i \, (\log \frac{Y_i}{\mu_i^4} - 1),$$

advantage: V_i has minimum at $\mu_i^4 = \langle Y_i \rangle$

natural normalization: c_i can be fixed by the requirement

mass at minimum: $m_i^4 = (Tr[\frac{\partial^2 V_i}{\partial h_a \partial h_b}])^2 = \mu_i^4 = \langle Y_i \rangle$.

example: $c_1 = c_3 = 1$.

Higgs masses

speculation!! parameter : $M/M_I \leq 1/5, \;\; \kappa \leq 1/5$

$$V(H,\tilde{H}) = \sum_{i=1}^{11} \;\; V_i(Y_i) \; + \; j^2 \; (Y_3 - Y_4)$$

Higgs mass matrix : $m_{ab}^2 = \frac{\partial^2 V}{\partial h_a \partial h_b}$

$m_1 \simeq 2.8 \cdot 10^{13}\; GeV, \;\; m_2 \simeq 1.3 \cdot 10^{13}\; GeV$
$m_3 \ldots m_{10} \simeq 8.2\, j \cdot 10^{12}\; GeV, \;\; m_{11} \simeq 3.0 \cdot 10^{12}\; GeV,$
$m_{12} \simeq 2.3 \cdot 10^{7}\; GeV, \;\; m_{13} = 123\; GeV,$
$m_{14} \simeq 0.13\; GeV$

m_{13} is independent of M_I and of m_b for $m_b << v_0 \simeq m_t$.

$$m_{13} \; standard\ model\ Higgs = \frac{v_0}{\sqrt{2}} = 123\; GeV.$$

Summary

$$\mathrm{E}_6 \times \mathit{Flavor} \supset \; \mathit{Trinification} \times SO(3)_F \times P_F$$

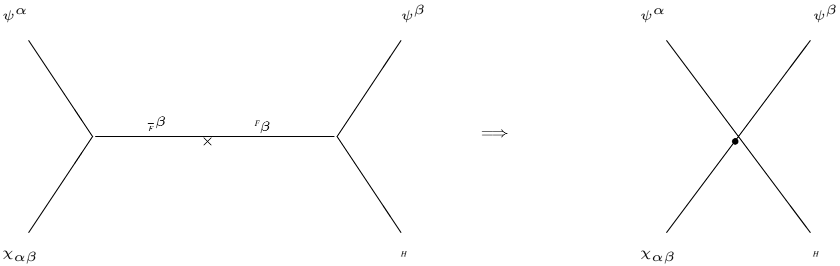
- The effective Yukawa interaction at the weak scale has a very simple form with only 3 flavor singlet Higgs fields.
- $M_I \simeq 2 \cdot 10^{13}$ GeV, the meeting point of g_1 and g_2, fixes the mass scales of light and heavy neutrinos.
- few symmetry breaking parameters allow for a quantitative fit of all fermion masses and mixings !
- Vev's of Higgs and flavons cause the observed large mass hierarchies.
- Higgs masses need further attention, some are not coupled to the standard model fermions!!

- mixing of generations and $\not{CP}$ is combined with the mixing of standard model states with heavy particles.
- The known quark properties determine to a large extent the neutrino properties,
  Neutrino hierarchy, $\not{CP}$ and Majorana phases as well as the mass parameter for $O\nu\beta\beta$ decays.
- Speculative: Spontaneous breaking of generation symmetry and $SU(3)_L \times SU(3)_R$ from logarithmic potentials. Needs justification ore falsification.

it's great fun to work on non fashionable models!

- **Renormalizable** interaction: introduce massive spinor fields

Vertices : $$\chi_{\alpha\beta}\psi^{\alpha}\bar{F}^{\beta} + \psi^{\alpha}HF_{\alpha} + MF_{\alpha}\bar{F}^{\alpha}$$



$$\frac{\langle\chi_{\alpha\beta}\rangle}{M}\left(\psi^{\alpha T}H\psi^{\beta}\right) = G_{\alpha\beta}\left(\psi^{\alpha T}H\psi^{\beta}\right)$$

- Masses for the 'right handed' neutrinos $\hat{\nu}$ are not obtained from $\langle H \rangle$.
- Coupling with two H needed: $H(-), \tilde{H}(+)$
  via a massive spinor singlet field
  $N^\alpha(1,3,+), \bar{N}^\alpha(1,3,-)$
  Vertices: $F^\alpha H^\dagger N^\alpha + F^\alpha \tilde{H}^\dagger \bar{N}^\alpha + M_N N^\alpha \bar{N}^\alpha$

$$\Rightarrow \frac{\langle \chi_{\alpha\beta} \rangle}{M} \frac{1}{M_N} \left(\psi^\alpha H^\dagger \tilde{H} \psi^\gamma \right) \frac{\langle \chi_{\gamma\beta} \rangle}{M}$$