(Meta)stability of the supersymmetry breaking vacuum and gaugino masses

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in collaboration with E. Dudas and J. Parmentier [arXiv:1011.4001]

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Introduction

Renewed interest in gauge-mediated supersymmetry breaking (GMSB) in the past few years:

- flavour physics experiments: no or very small deviations from MFV (i.e. close to flavour-universal soft terms)
- theoretical progress on supersymmetry breaking [Intriligator, Seiberg, Shih (2006): dynamical supersymmetry breaking in a metastable vacuum]

→ intense activity on the construction of complete models of supersymmetry breaking and its mediation to the observable (MSSM) sector, as well as on the phenomenology of general gauge mediation [Meade, Seiberg, Shih '08]

Generic problem: explicit models often lead to suppressed gaugino masses

Metastability seems to be an important ingredient of viable models

→ can radiative corrections promote the supersymmetry breaking vacuum to the ground state of the theory?

Short review of gauge mediation

Supersymmetry breaking is parametrized by a spurion field X with

$$\langle X \rangle = M + F\theta^2$$

X couples to messenger fields in vector-like representations of the SM gauge group [often complete GUT representations, e.g. $(\mathbf{5}, \bar{\mathbf{5}})$ of SU(5)]:

$$W_{\text{mess}} = \lambda_X X \Phi \tilde{\Phi}$$

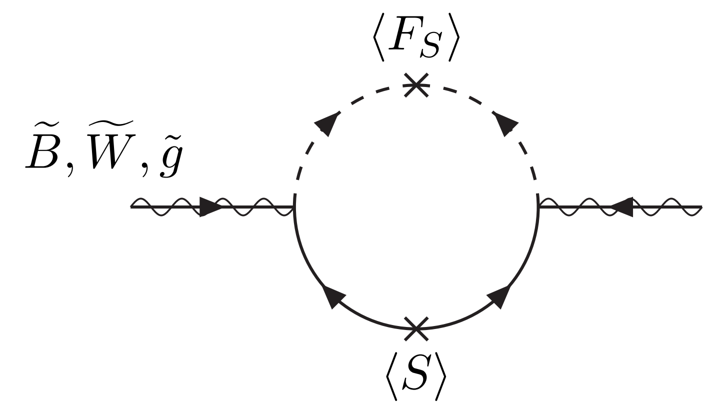
$\Rightarrow$ supersymmetric messenger mass M + supersymmetry breaking mass term $F \phi \tilde{\phi} + \text{h.c.}$ for the scalar messengers:

$$\begin{pmatrix} \phi^* & \tilde{\phi} \end{pmatrix} \begin{pmatrix} M^2 & -F^* \\ -F & M^2 \end{pmatrix} \begin{pmatrix} \phi \\ \tilde{\phi}^* \end{pmatrix} \quad \Rightarrow \text{ scalar masses } M^2 \pm |F|$$

$|F| < M^2$ required (no tachyon among scalar messengers)

$\Rightarrow$ soft terms in the observable sector via gauge loops

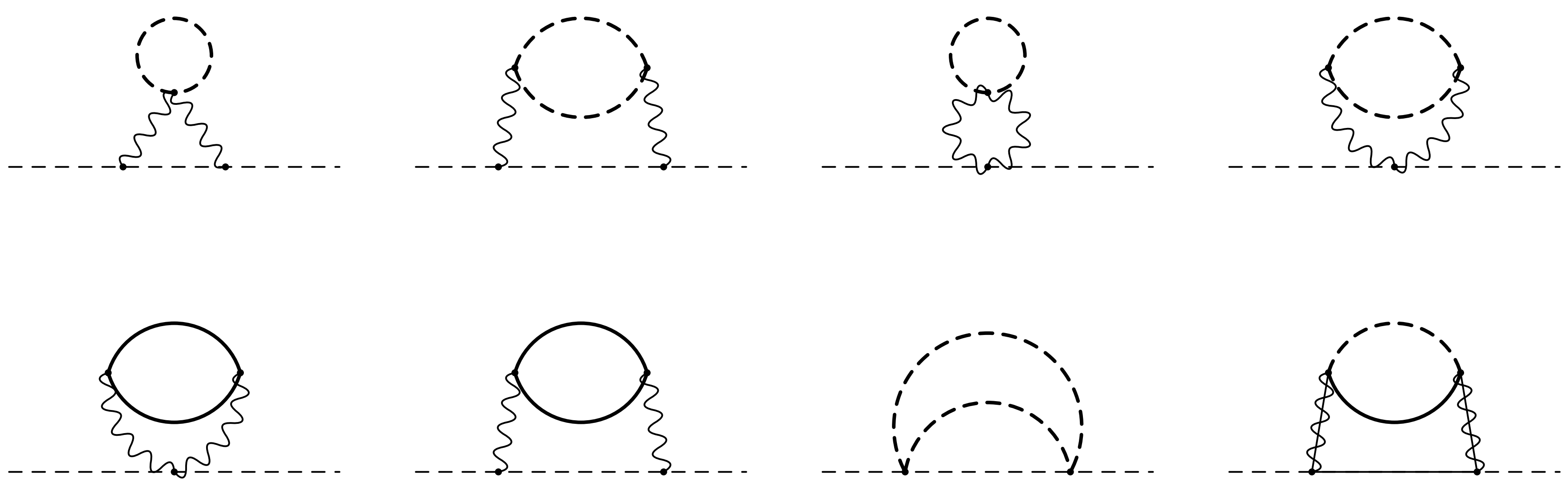
Gaugino masses arise at one loop:



$$M_a \;=\; \frac{\alpha_a}{4\pi}\, 2\, T_a(R)\, \frac{F}{M}$$

R = messenger representation, $T_a(R)$ = Dynkin index

Scalar masses arise at two loops:



$$m_\chi^2 \;=\; \sum_a C_\chi^a \left(\frac{\alpha_a}{4\pi}\right)^2 2\, T_a(R) \left|\frac{F}{M}\right|^2$$

C_χ^a = second Casimir coefficient for the superfield χ

Main advantage of GMSB: since gauge interactions are flavour blind, the induced soft terms do not violate flavour

$\Rightarrow$ solves the SUSY flavour problem

Black box: dynamics that generates M and F

$\rightarrow$ hidden (supersymmetry breaking) sector

can be perturbative (O'raifeartaigh) or non-perturbative (e.g. SQCD)

Simplest example: O'Raifeartaigh $W = X\left(f + Y^2\right) + m\,ZY$

scalar potential: $V = \left|f + Y^2\right|^2 + |2XY + mZ|^2 + |mY|^2$

$f + Y^2 = 0$ and $mY = 0$ incompatible $\Rightarrow$ supersymmetry broken

minimum for $Y = Z = 0$ (assuming $m^2 > 2f$)

$\Rightarrow V = |F_X|^2 = f^2$ for any value of X $\qquad F \equiv F_X = -f$

X not fixed at tree level $\rightarrow$ flat direction

Next step: couple the supersymmetry breaking field X to messengers

$$W = X\left(f + Y^2\right) + m\,ZY + \Phi\,(\lambda X + M)\,\tilde{\Phi}$$

$$V = \left|f + Y^2 + \lambda\Phi\tilde{\Phi}\right|^2 + |2XY + mZ|^2 + |mY|^2 + |\lambda X + M|^2\left(|\Phi|^2 + |\tilde{\Phi}|^2\right)$$

As long as $|\lambda X + M|^2 > |\lambda f|$, the O'R vacuum (with $\Phi = \tilde{\Phi} = 0$) remains a local minimum

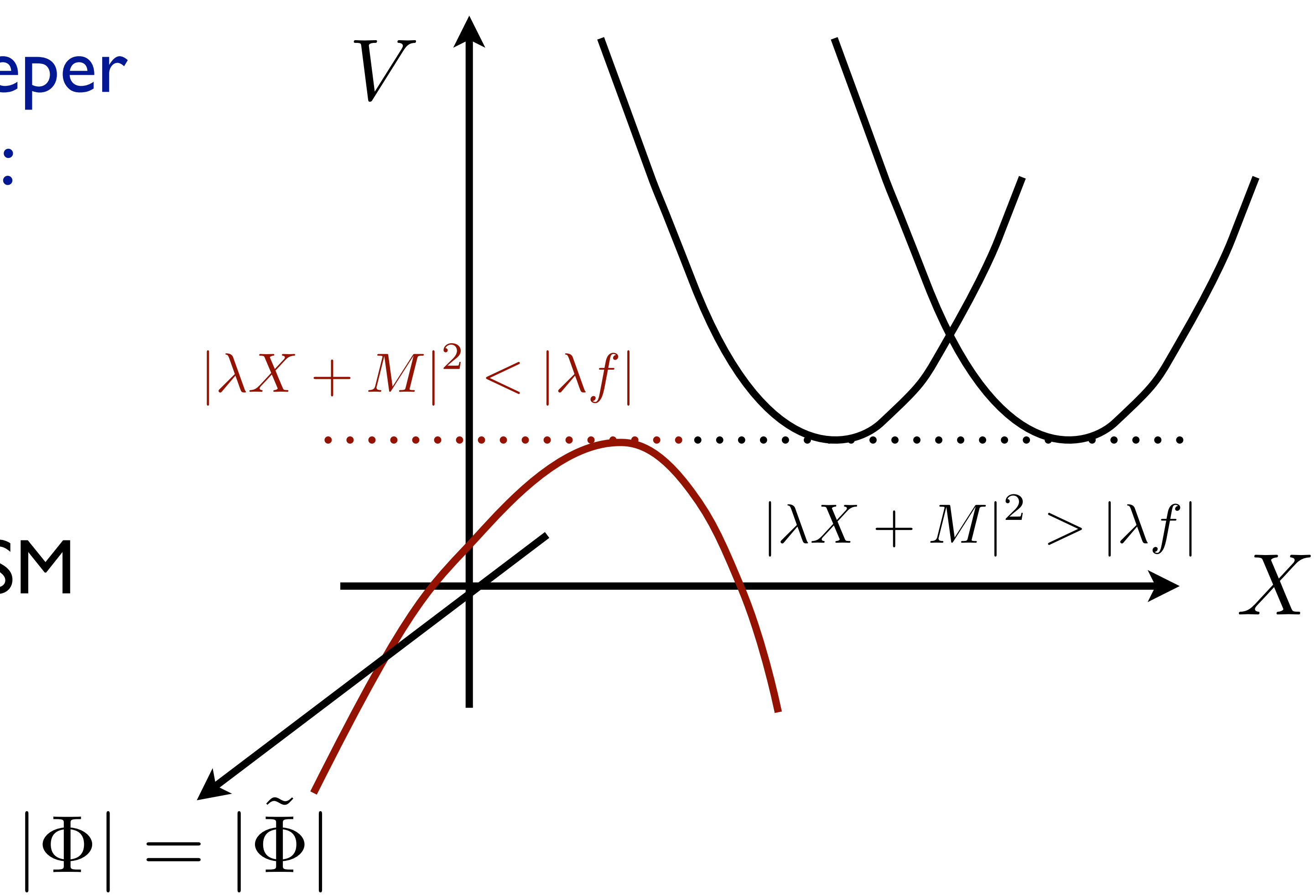
For $|\lambda X + M|^2 < |\lambda f|$, an instability appears in the messenger direction

$\Rightarrow$ signals the presence of a deeper minimum with messenger VEVs:

$$\lambda X + M = 0\,,\;\; Y = Z = 0\,,$$

$$\Phi\tilde{\Phi} = -f/\lambda$$

unacceptable since breaks the SM gauge symmetry



Ways out: (i) 1-loop corrections may stabilize X at a value X_0 such that $|\lambda X_0 + M|^2 > |\lambda f|$ $\Rightarrow$ supersymmetry broken in a metastable vacuum (lower vacuum still exists)

(ii) models in which the messenger fields are part of the supersymmetry breaking sector (direct GM models) can avoid instabilities

However, gaugino masses vanish at leading order in such models [Polchinski, Susskind '82 - Komargodski, Shih '09]

Proof [KS]: consider renormalizable models of the form (with canonical K)

$$W \;=\; fX + \tfrac{1}{2}\,(\lambda_{ab}X + m_{ab})\phi_a\phi_b + \tfrac{1}{6}\,\lambda_{abc}\phi_a\phi_b\phi_c$$

To ensure stability in all ϕ_a directions, $\det\,(\lambda X + m)$ must not vanish for any X $\Rightarrow$ constant polynomial in X

$$\Rightarrow \quad m_\lambda \;\propto\; \frac{\partial \ln\det(\lambda X + m)}{\partial X}\,F \;=\; 0$$

To avoid a huge gaugino/sfermion mass hierarchy, must admit tree-level instabilities $\rightarrow$ back to (i)

(iii) can radiative corrections promote the supersymmetry breaking vacuum to the ground state of the theory?

Consider the following class of models [$i = 1 \cdots Q\,,\ a = 1 \cdots P$]:

$$W \;=\; X_i \left(f_i + \tfrac{1}{2} h_{ai} \varphi_a^2 \right) \,+\, m_a \,\varphi_a Y_a \,+\, \Phi \left(\lambda_i X_i + M \right) \tilde{\Phi}$$

φ_a, Y_a stabilized at $\varphi_a = Y_a = 0$ provided that $|h_{ai} f_i| < m_a^2$

then $\quad V \;=\; \sum_i \left| f_i + \lambda_i \Phi \tilde{\Phi} \right|^2 + |\lambda_i X_i + M|^2 \left(|\tilde{\Phi}|^2 + |\Phi|^2 \right)$

→ 2 vacua (flat directions)

- the O'R vacuum, in which $\Phi = \tilde{\Phi} = 0$

$F_{X_i} = - f_i \qquad\qquad V_1 = \sum_i |f_i|^2 \equiv f^2 \qquad\qquad$ Xi = flat directions

- the "messenger" vacuum, with broken SM gauge symmetry

$\lambda \cdot X + M = 0 \qquad\qquad \Phi \tilde{\Phi} = - \frac{1}{|\lambda|^2}\, \bar{\lambda} \cdot f \qquad\qquad \bar{\lambda} \cdot f \equiv \sum_i \lambda_i^* f_i$

$-F_{X_i} = f_i - \frac{\lambda_i}{|\lambda|^2}\, \bar{\lambda} \cdot f \qquad\qquad V_2 = f^2 - \frac{|\bar{\lambda} \cdot f|^2}{|\lambda|^2}$

$$\Delta V \equiv V_1 - V_2 = \frac{|\bar{\lambda} \cdot f|^2}{|\lambda|^2}$$ $\rightarrow$ messenger vacuum lower

for 1-loop corrections to dominate over ΔV, need $|\bar{\lambda} \cdot f|^2 \ll |\lambda|^2 f^2$

Stabilizing the ~~SUSY~~ vacuum with radiative corrections

Compute the 1-loop effective potential $V^{(1)}(X_i)$ in the vicinity of the two vacua, then minimize it and compare the two energies

Small supersymmetry breaking limit $\Rightarrow$ use effective Kaehler potential

$$V_{eff} = V^{(0)} + V^{(1)} = (K^{-1})_{ij}\, F_i \bar{F}_j$$

with $K_{ij}^{(0)} \equiv \frac{\partial^2 K^{(0)}}{\partial\phi_i \partial\bar{\phi}_j} = \delta_{ij}$ and $K^{(1)} = -\dfrac{1}{32\pi^2}\,\mathrm{Tr}\left(\mathcal{M}\mathcal{M}^\dagger \ln \dfrac{\mathcal{M}\mathcal{M}^\dagger}{\Lambda^2}\right)$

where $\mathcal{M}_{ij} \equiv \partial^2 W/\partial\phi_i\partial\phi_j$ is the superpotential mass matrix

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix}$$

$\mathcal{M}_1 :\quad (\varphi_a, Y_a)$ mass matrix

$\mathcal{M}_2 :\quad (\Phi, \tilde{\Phi}, X_i)$ mass matrix

$$\mathcal{M}_1 = \begin{pmatrix} h_{ai}X_i & m_a \\ m_a & 0 \end{pmatrix} \qquad \mathcal{M}_2 = \begin{pmatrix} 0 & \lambda\cdot X + M & \lambda_j\tilde{\phi} \\ \lambda\cdot X + M & 0 & \lambda_j\phi \\ \lambda_i\tilde{\phi} & \lambda_i\phi & 0 \end{pmatrix}$$

Case without messengers: no $\mathcal{M}_2$ matrix

$$K_{ij} = \delta_{ij} + Z_a(\chi_a)\, h_{ai}\bar{h}_{aj} \qquad\qquad \chi_a \equiv \sum_i h_{ai} X_i$$

$$\Rightarrow \quad V^{(1)} = -Z_a(\chi_a)\, |\bar{h}_a \cdot f|^2$$

where Za is a decreasing function of $|\chi_a|$:

$$Z_a\left(|\chi_a| \ll m_a\right) \simeq -\frac{1}{32\pi^2}\left(2 + \ln\frac{m_a^2}{\Lambda^2} + \frac{2|\chi_a|^2}{3m_a^2}\right)$$

$$Z_a\left(|\chi_a| \gg m_a\right) \simeq -\frac{1}{32\pi^2} \ln\frac{|\chi_a|^2}{\Lambda^2}$$

The Xi are stabilized at the origin (no remaining flat direction if $P \geq Q$) and the vacuum energy is increased by an amount

$$V^{(1)} \simeq \frac{1}{32\pi^2}\left(2 + \ln\frac{m_a^2}{\Lambda^2}\right) |\bar{h}_a \cdot f|^2$$

Case with messengers:

- around the O'R vacuum, the Xi are stabilized close to the origin (assuming $m_a \ll M$), and the 1-loop vacuum energy is given by:

$$V_1 \simeq f^2 + \frac{1}{32\pi^2}\left[\sum_a |\bar{h}_a \cdot f|^2 \left(\ln\frac{m_a^2}{\mu^2} + 2\right) + 2\,|\bar{\lambda}\cdot f|^2 \left(\ln\frac{M^2}{\mu^2} + 2\right)\right]$$

- around the messenger vacuum, one has $V^{(1)}_{\min} \geq V^{(1)}(\chi_a = 0)$, hence

$$V_2 \geq \sum_i \left| f_i - \frac{\lambda_i}{|\lambda|^2}\,\bar{\lambda}\cdot f\right|^2 + \frac{1}{32\pi^2}\sum_a \left(\ln\frac{m_a^2}{\mu^2} + 2\right)\left|\bar{h}_a \cdot f - (\bar{h}_a \cdot \lambda)\,\frac{\bar{\lambda}\cdot f}{|\lambda|^2}\right|^2$$

Can now derive a sufficient condition for $\Delta V = V_1 - V_2 < 0$, which for $m_a \equiv m$ reads (with all couplings evaluated at $\mu = m$):

$$|\bar{\lambda}\cdot f|^2 < -\frac{1}{8\pi^2}\,\mathrm{Re}\left[(\bar{\lambda}\cdot f)\sum_a (h_a \cdot \bar{f})(\bar{h}_a \cdot \lambda)\right]$$

This is possible only if $|\bar{\lambda}\cdot f| < \frac{1}{8\pi^2}\left|\sum_a (h_a \cdot \bar{f})(\bar{h}_a \cdot \lambda)\right|$

The vacuum stability condition $|\bar{\lambda}\cdot f| < \frac{1}{8\pi^2}\left|\sum_a (h_a\cdot\bar{f})(\bar{h}_a\cdot\lambda)\right|$

has a simple interpretation in terms of Goldstino couplings

Goldstino superfield: $X \equiv \frac{1}{f}\sum_i f_i X_i$

Redefine $\{X_i\}_{i=1\cdots Q} \rightarrow \{X, Z_I\}_{I=1\cdots Q-1}$, then $F_X = -f, \quad F_{Z_I} = 0$

The vacuum stability condition can be rewritten:

$$|\lambda_X| < \frac{1}{8\pi^2}\left|\sum_a (\lambda\cdot\bar{h}_a)h_{aX}\right|$$

where $\lambda_X \equiv (\lambda\cdot\bar{f})/f$ is the Goldstino-messenger coupling

→ the Goldstino superfield should have suppressed couplings to the messengers with respect to the other O'R fields Z_I

Implications:

1) the vacuum stability condition + perturbativity imply $|\lambda\cdot\bar{f}| \ll |\lambda| f$

since $m_{\rm soft} \sim \frac{\alpha}{4\pi}\,\frac{|\bar{\lambda}\cdot f|}{M}$, this results in an (irrelevant) overall suppression of soft masses relative to $\frac{|\lambda| f}{M}$

→ no suppression of gaugino masses relative to scalar masses

2) constraints on the various mass scales:

$$m_a \ll M\,, \quad |\bar{h}_a\cdot f| < m_a^2\,, \quad |\bar{\lambda}\cdot f| < M^2\,, \quad m_{\rm soft} \sim 1\,\mathrm{TeV}$$

+ vacuuum stability condition

→ minimal allowed values:

$$m_a \sim 10^5\,\mathrm{TeV}\,, \quad M \sim 10^6\,\mathrm{TeV}\,, \quad f \sim \lambda_X^{-1}\,(10^4\,\mathrm{TeV})^2$$

[corresponding to $m_{3/2} \sim 2\,\mathrm{MeV}\,(10^{-2}/\lambda_X)$]

→ no light messenger / O'R fields allowed

Conclusions

Gauge mediation is an attractive mechanism for transmitting supersymmetry breaking to the observable (MSSM) sector, but the simplest ~~SUSY~~ sectors lead to vanishing gaugino masses at leading order

This problem can be cured by allowing the supersymmetry breaking vacuum to be metastable

In some models, radiative corrections can cure the tree-level instabilities of the ~~SUSY~~ vacuum and promote it to the ground state of the theory, allowing for non-vanishing gaugino masses without metastability. This requires a suppressed coupling of the Goldstino superfield to the messengers