Model for Color suppressed mode $\overline{B_d^0} \longrightarrow \pi^0\pi^0$

J.O.E. , Phys. Dept., Univ. Oslo

8th April 2011

Extended chiral quark model for light energetic quarks

Outline

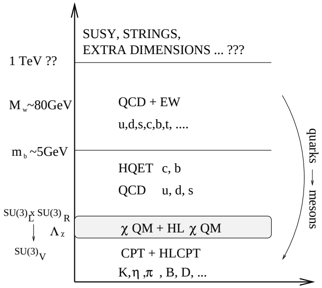
- Introduction
- Color suppression in non-leptonic decays
- Effective Theories at quark level (HQEFT and LEET $\longrightarrow SCET$)
- Mesonic picture - (HL)χPT
- Chiral Quark Models (χQM, HLχQM, LEχQM)
- Color suppression for $\overline{B_d^0} \longrightarrow \pi^0\pi^0$

Introduction

* Since 1970's : Non-leptonic decays always difficult-
Perturbative QCD worked well;
BUT: "Hadronic uncertainties" (use Lattice, Quark Models) (e.g. $K \to 2\pi$, ...)

From 1999, BBNS: *QCD factorization* For $B \to \pi\pi, \pi K, ...$
Corrections to factorization:
$\frac{\alpha_s}{\pi}$ (calculable), $\frac{\Lambda_{QCD}}{m_b}$ (not calculable).
For $\overline{B_d^0} \to \pi^0\pi^0$, tremendous effort (QCD fact, SCET, QCD sum rules) , but amplitude factor 2 off. Try new LEχQM !

Energy/Mass Scales of the SM



High mass particles may go in loops and affect low-energy decays!

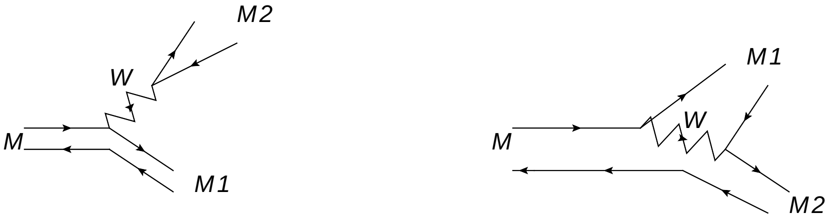
χQM and *HLχQM* bridges between the *quark* and *meson* picture

Bridge: (HL)χQM

Since 1990's used (HL)χQM (combined with χPT).
$K \to \gamma\gamma$, $K \to \pi\pi$,, $K - \overline{K}$ and $B - \overline{B}$ mixing, $D \to K\overline{K}$, $B \to D\overline{D}$, $D\eta'$, $D^*\gamma$, $K\eta'$, $D\pi$.
Collaborators: Ivica Picek, Marco Fabbrichesi, Stefano Bertolini, Svjetlana Fajfer, Aksel Hiorth, A.Polosa, J. Zupan, A. Prapotnik, J.A. Macdonald Sørensen, Krešimir Kumerički, L.E. Leganger

(HL)χQM also used by Bijnens et al, Pich and de Rafael, Ebert et al, Nardulli et al,.....

Quark Diagrams for Non-Leptonic Decays



Color suppression in Non-Leptonic Decays

Effective non-leptonic Lagrangian at quark level:

$$\mathcal{L}_W = \sum_i C_i(\mu)\, \hat{Q}_i(\mu) \ ,$$

$\mu =$ renormalization scale. C_i = Wilson coeff. Typically

$$\hat{Q}_i \ = j_L^\alpha(q_1 \rightarrow q_2)\, j_\alpha^L(q_3 \rightarrow q_4) \quad ; \ j_L^\alpha(q_i \rightarrow q_j) = \overline{(q_j)_L}\, \gamma^\alpha\, (q_i)_L$$

For “flavor mismatch”, use Fierz transf:

$$\hat{Q}_i \rightarrow \hat{Q}_i^{Fierz} = \frac{1}{N_c} j_L^\alpha(q_1 \rightarrow q_4)\, j_\alpha^L(q_3 \rightarrow q_2) + 2\, \hat{Q}_{color}$$

$$\hat{Q}_{color} \ = \ j_L^\alpha(q_1 \rightarrow q_4)^a\, j_\alpha^L(q_3 \rightarrow q_2)^a \quad ; \ j_L^\alpha(q_i \rightarrow q_j)^a = \overline{(q_j)_L}\, \gamma^\alpha\, t^a\, (q_i)_L$$

Colored operators might dominate!

Generic, for non-leptonic processes with two numerically relevant operators $\hat{Q}_{X,Y}$:

$$\begin{aligned}\langle M_1\, M_2 | \mathcal{L}_W | M \rangle &= \left(C_X + \frac{C_Y}{N_c} \right) \langle M_1 |\, j_L^{\alpha}(1) | 0 \rangle \, \langle M_2 | j_{\alpha}^L(2) | M \rangle \\ &+ \; 2 C_Y \, \langle M_1\, M_2 |\, \hat{Q}^{color} \, | M \rangle\end{aligned}$$

where $\hat{Q}^{color}$ is the product of the colored currents. Some cases: $\left(C_X + \frac{C_Y}{N_c} \right)$ close to zero. How to calculate $\langle M_1\, M_2 |\, \hat{Q}^{color} \, | M \rangle$?

Heavy Quark Effective Theory (*HQET*)

Project out movement of heavy quark ($p_Q = m_Q v + k$, $v^2 = 1$)

$$Q_v^{(\pm)}(x) = e^{\pm i m_Q v\cdot x} P_\pm(v) Q(x) \ , \ P_\pm(v) = \frac{1}{2}(1 \pm \gamma \cdot v)$$

$$\mathcal{L}_{HQET} = \pm \overline{Q_v^{(\pm)}} i v \cdot D Q_v^{(\pm)} + \mathcal{O}(m_Q^{-1})$$

Heavy quark propagator:

$$S(p_Q) \ \rightarrow \ \frac{P_+(v)}{k \cdot v}$$

Replacements in quark operators

$$b \rightarrow Q_{v_b}^{(+)} \quad , \ c \rightarrow Q_{v_c}^{(+)} \quad , \ \overline{c} \rightarrow Q_{\overline{v}}^{(-)}$$

Large Energy Eff. Th. (*LEET* $\rightarrow$ *SCET*)

Project out movement of light energetic quark: $p_q^\mu = E\, n^\mu + k^\mu$,

$$q_\pm(x) = e^{iEn\cdot x}\,\mathcal{P}_\pm\, q(x) \;\; ; \; n(\,\text{or}\; \tilde{n}) \; = \; (1, 0, 0, \pm\eta)$$

$$\mathcal{P}_+ = \frac{1}{4}\,\gamma\cdot n\,(\gamma\cdot\tilde{n}\,+\,\delta) \quad , \; \mathcal{P}_- = \frac{1}{4}\,(\gamma\cdot\tilde{n}\,-\,\delta)\,\gamma\cdot n$$

$$\eta = \sqrt{1-\delta^2}\,, \;\; n^2 = \tilde{n}^2 = \delta^2\,, \, v\cdot n = v\cdot\tilde{n} = 1.$$

$$\delta \sim \frac{\Lambda_{QCD}}{E} \quad ; \quad S(p_q) \rightarrow \frac{\gamma\cdot n}{2n\cdot k}$$

Project out q_- gives eff. Lagr. for $q_+ \; = \; q_n$ ($\sim$ as Orsay group):

$$\mathcal{L}_{LEET\delta} \; = \; \bar{q}_n \left(\frac{1}{2}(\gamma\cdot\tilde{n}+\delta) \right) (in\cdot D)\, q_n + \mathcal{O}(E^{-1}) \; ,$$

In the formal limits $M_H \to \infty$ and $E \to \infty$, $\langle P \,|\, V^\mu \,|\, H \rangle$ of the form (Orsay group (Charles et al 1999)):

$$\langle P|V^\mu|H\rangle = 2E\left[\zeta^{(v)}(M_H,E)\,n^\mu + \zeta_1^{(v)}(M_H,E)\,v^\mu\right] \ ,$$

where

$$\zeta^{(v)} = C\frac{\sqrt{M_H}}{E^2} \quad , \quad C \sim (\Lambda_{QCD})^{3/2} \qquad , \qquad \frac{\zeta_1^{(v)}}{\zeta^{(v)}} \sim \delta \ \sim \ \frac{1}{E}$$

Behavior constistent with the energetic quark having x close to one, where x = quark momentum fraction of the outgoing pion.

Mesonic picture : (HL)χPT

$\Rightarrow$ Effective Theor. contains meson fields

Heavy meson field: $H^{(\pm)} = P_{\pm}(v)\,(P_{\mu}^{(\pm)}\gamma^{\mu} - iP_5^{(\pm)}\gamma_5)$

Light, soft meson fields (π, K, η_8). $\Lambda_{\chi} \sim 4\pi f \sim 1$ GeV

$$\mathcal{A}_{\mu} = \frac{1}{2i}(\xi^{\dagger}\partial_{\mu}\xi - \xi\partial_{\mu}\xi^{\dagger}) \quad ; \quad \xi \equiv \exp\left(\frac{i}{2f}\sum_a \lambda^a \pi^a(x)\right)$$

For HLχPT, need $(m_b - m_c) < \Lambda_{\chi}$, or $(m_b - 2m_c) < \Lambda_{\chi}$?!?!
Few ideal cases, but $B - \overline{B}$ mixing should work well...
Bosonization of currents (to lead. order for $H = B, D$).

$$\overline{q_L}\,\gamma^{\mu}\,Q_v \longrightarrow \frac{1}{2} f_H \sqrt{M_H}\, Tr\left[\xi^{\dagger}\gamma^{\mu} L\, H_v\right]$$

The Chiral Quark Model (χQM)

To be used for colored operators! Light $q = u, d, s$ sector:

$$\mathcal{L}_{\chi QM} = \mathcal{L}_{QCD} + \mathcal{L}_{\chi}$$

$$\mathcal{L}_{\chi} = -m \left(\overline{q}_R \, \Sigma \, q_L + \overline{q}_L \, \Sigma^{\dagger} q_R \right)$$

$\Rightarrow$ Meson-quark couplings. Modelling confinement!!

m = *constituent* light quark mass, due to chiral symmetry breaking.
“Rotated version” ; flavour rotated “constituent quark fields”

$$\chi_L = \xi q_L \quad , \quad \chi_R = \xi^{\dagger} q_R \quad ; \qquad \xi \cdot \xi = \Sigma$$

$$\mathcal{L}_{\chi QM} = \overline{\chi} \left[\gamma^{\mu} (i D_{\mu} + \mathcal{V}_{\mu} + \gamma_5 \mathcal{A}_{\mu}) - m \right] \chi + \ldots$$

Soft gluon emision

(HL)χQM: Loop momenta should be $< \Lambda_\chi$ Introducing gluon condensates (model dep.) by (Novikov et al.):

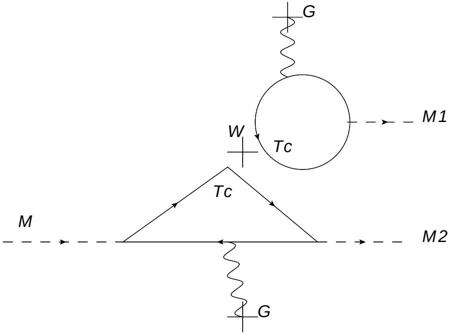
$$g_s^2 G^a_{\mu\nu} G^a_{\alpha\beta} \;\rightarrow 4\pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \, ,$$

Identification of logarithmic divergent loop integral I_2 and quadratic div. int. I_1 (Esprui and Taron, Bijnens et al, Pich and de Rafael,...):

$$f^2 = -i4m^2 N_c I_2 + \frac{1}{24m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

$$\langle \overline{q} q \rangle = -4imN_c I_1 - \frac{1}{12m} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

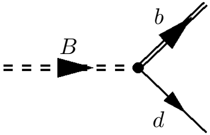
Color suppressed $M \rightarrow M_1 M_2$ in χ QM



The $HL\chi QM$

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQET} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int}$$

$HL\chi$QM ansatz (also: Ebert et al, Bardeen and Hill, Nardulli et al):



$$\mathcal{L}_{Int} = -G_H \left[\overline{\chi}_f \, \overline{H_v^f} \, Q_v + \overline{Q_v} \, H_v^f \, \chi_f \right]$$

Integrating out quarks (by loop diagrams) should give the known HLχPT terms! $\Rightarrow$ Physical and model dep. param. $f_\pi, \langle \overline{q}q \rangle, f_H, g_A$ linked to (divergent) loop integrals in HLχQM!

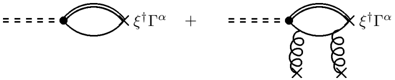


Figure: Bosonization of left handed current

Fit in strong sector: $m \sim 220$ MeV, $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim 315$ MeV, ${G_H}^2 = \frac{2m}{f^2}\,\rho$ where $\rho \sim 1$ and $\rho = \rho(f_\pi, \langle \frac{\alpha_s}{\pi} G^2 \rangle, m, g_A)$

$$B - \overline{B}\text{-mixing}: \quad \hat{B}_{B_q} = \frac{3}{4}\,\widetilde{b}\left[1 + \frac{1}{N_c}\left(1 - \delta_G^B\right) + \frac{\tau_b}{m_b} + \frac{\tau_\chi}{32\pi^2 f^2}\right]$$

The $LE\chi QM$

Ansatz(with L.E. Leganger, PRD 82 (2010)):

$$\mathcal{L}_{intq} = G_A \bar{q}\, \gamma_\mu \gamma_5 (\partial^\mu M)\, q_n \;+\; h.c \qquad , \; M = \text{meson fields}$$

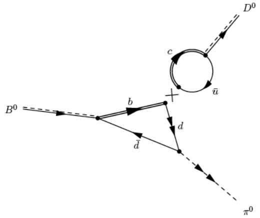


Figure: Factorizable contribution to the $B^0 \to D^0\pi^0$ decay, including the current matrix element for $B \to \pi$. Here: Small Wilson coeff. combination for factorized term. Colored operator dominates.

Coupling G_A determined by loop diagram for $\zeta^{(v)}$.
$\zeta_1^{(v)}$ suppr. $\sim \delta \sim 1/E$.

$B \longrightarrow \pi_n$ current (M_n = 3 $\times$ 3 matrix of energetic mesons):

$$J^{\mu}_{tot}(H_v \rightarrow M_n) = -i\frac{G_H\, G_A}{2}\, m^2\, F\, \mathrm{Tr}\left\{\gamma^{\mu} L H_v\, [\gamma \cdot n]\, \xi^{\dagger} M_n\right\} \;,$$

Use $C = \hat{c}m^{3/2}$ (where $m \sim \Lambda_{QCD}$) and $\delta = m/E$:

$$G_A \;=\; \frac{4\zeta^{(v)}}{m^2\, G_H\, F}\sqrt{\frac{E}{M_H}} \;=\; \left(\frac{4\hat{c}f_{\pi}}{m\, F\, \sqrt{2\rho}}\right)\, \frac{1}{E^{\frac{3}{2}}} \;,$$

Coupling $G_A \sim N_c^{-1/2}$ fixed. $F = N_c/(16\pi) + \ldots \sim 10^{-1}$
Bosonization of colored current for outgoing D-meson:

$$(\overline{q_L}\, t^a\, \gamma^{\alpha}\, Q_v)_{1G} \;\longrightarrow -\frac{G_H\, g_s}{64\pi}\, G^a_{\mu\nu} Tr\left[\xi\gamma^{\alpha} L\, \sigma_{\mu\nu}\, \overline{H_v}\right] \;+ \ldots$$

Color suppression in $\overline{B^0} \to D^0\pi^0$

Factorized amplitude dominates for $\overline{B^0} \to D^+\pi^-$. BUT: $\overline{B^0} \to D^0\pi^0$ has very small factorized amplitude (-small Wilson coeff.)

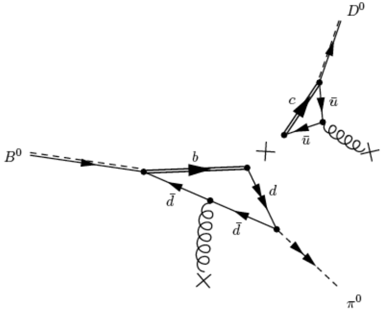


Figure: Non-factorizable contributions to $\overline{B^0} \to D^0\pi^0$ from the colored operators within LEχQM. Ampl. account for 2/3 of the experimental amplitude. Additional meson loop ampl.

$\overline{B^0} \to \pi^0\pi^0$ in LEχQM

with T. Palmer, PRD 83 (2011)

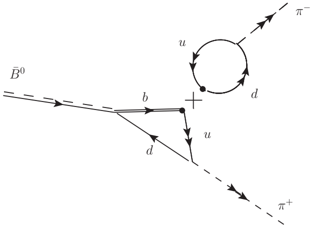


Figure: Factorized contribution to the $\overline{B_d^0} \to \pi^+\pi^-$ decay, as described in combined χQM, HLχQM and LEχQM. Factorized contrib to $\overline{B_d^0} \to \pi^0\pi^0$ small due to small Wilson coeff.

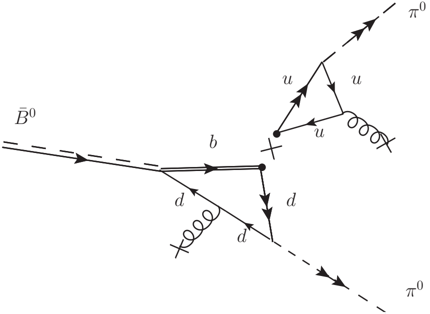


Figure: Non-factorizable contribution to $B \longrightarrow \pi^0\pi^0$ containing large energy light fermions and mesons. Also corresponding diagram where the outgoing anti-quark $\overline{u}$ is hard.

The colored $B \longrightarrow \pi_n$ current

$$J^{\mu}_{1G}(H_b \longrightarrow M)^a \; = \; g_s \, G^a_{\alpha\beta} \, \frac{G_H \, G_A}{128\pi} \epsilon^{\sigma\alpha\beta\lambda} \, n_\sigma \mathrm{Tr} \left(\gamma^\mu L H_v \gamma_\lambda \, \xi^\dagger M_n \right) \; ,$$

The colored current for outgoing hard $\pi_{\tilde{n}}$:

$$J^{\mu}_{1G}(M_{\tilde{n}})^a \; = \; g_s G^a_{\alpha\beta} \; 2 \, \left(-\frac{G_A E}{4} \right) \; Y \; \tilde{n}_\sigma \epsilon^{\sigma\alpha\beta\mu} \; \mathrm{Tr} \left[\lambda^X \, M_{\tilde{n}} \right] \quad ,$$

λ^X = appropriate SU(3) flavor matrix. Loop factor:

$$Y \; = \; \frac{f_\pi^2}{4m^2 N_c} \left(1 \; - \; \frac{1}{24 m^2 f_\pi^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) \; .$$

$$r \;\equiv\; \frac{\mathcal{M}(\overline{B_d^0} \to \pi^0\pi^0)_{\text{Non-Fact}}}{\mathcal{M}(\overline{B_d^0} \to \pi^+\pi^-)_{\text{Fact}}} \;=\; \frac{c_A}{c_f}\,\frac{\kappa}{N_c}\,\frac{E\,\zeta^{(v)}}{\sqrt{m\,M_B}}\;,$$

κ = model-dependent hadronic factor, dimension-less and $\sim (N_c)^0$:

$$\kappa \;=\; \left(\frac{\pi\, N_c\, \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2\,F^2\, m^4 \sqrt{2\rho}} \right)\; Y.$$

From scaling behaviour of $\zeta^{(v)}$ with $C = \hat{c}\, m^{\frac{3}{2}}$

$$r \;\simeq\; \left(\frac{c_A}{c_f}\, \kappa\, \hat{c} \right) \frac{1}{N_c}\, \frac{m}{E}\;.$$

i.e. Our calculations show that the ratio $r \sim 1/N_c$ and
$r \sim m/2E \simeq m_b/\Lambda_{QCD}$ as it should acc. to BBNS

Experimental value of $\overline{B_d^0} \to \pi^0\pi^0$ ampl. can be accomodated for $m \sim$ 220 Mev and $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim 315$ MeV. - as in previous work. BUT: Result very sensitive to variations of m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, as seen by loop factor Y. In addition,- meson loops:

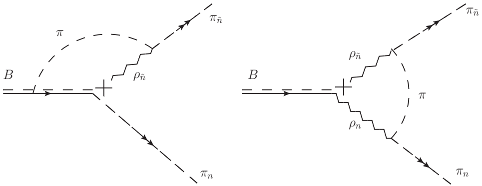


Figure: Meson loops for $\overline{B_d^0} \to \pi\pi$.

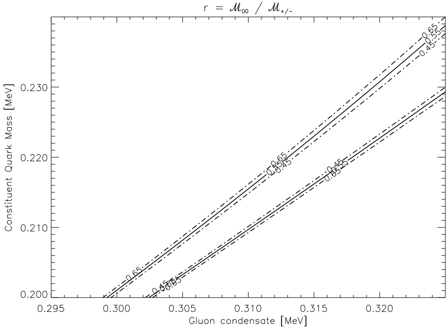


Figure: Plot for the ratio r in terms of m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4}$. For reasonable values of these parameters the ratio r can take a wide range of values such that fine-tuning is required to reproduce the experimental value.

Triangle anomaly?

Anomaly rather tricky when going from $\pi^0 \to 2\gamma$ to higher energies where some cancellations occurr (Pham and Pham, 1990), to $\gamma^* \to \pi^0\gamma$ and $Z \to \pi^0\gamma$;- “anomaly tail”

$$J^{\mu}_{1G}(M_{\tilde{n}})^a(An) \; = \; g_s \, G^a_{\alpha\beta} \frac{I_{An}}{4\pi^2 f_\pi \sqrt{2}} \, p^\pi_\sigma \, \epsilon^{\sigma\alpha\beta\mu} \quad ,$$

$$I_{An} \; = \; \int_0^1 \frac{x \, dx}{\eta \, x(1-x) - 1} \quad \sim \; \ln \eta / \eta$$

where $\eta \equiv p_\pi^2 / m_q^2$.

Numerically favorable ,- BUT:

Hybrid description without $m/2E \sim \Lambda_{QCD}/m_b$ suppression

Conclusions

- Have constructed LEχQM in accordance with $\langle \pi_n | j_V^\mu | B \rangle$
- 2/3 of $B \longrightarrow \pi^0 \overline{D^0}$ described. Rest meson int.?
- Good news: Color suppressed ($\sim 1/N_c$) ampl. $\overline{B_d^0} \longrightarrow \pi^0\pi^0$ can be accomodated numerically! And: amp. $\sim m/2E \simeq \Lambda_{QCD}/m_b$
- Bad news: Obtained ampl. very sensitive to m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$