

Minimal Flavour Violation  
and Two-Body doublet Models

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The Role of Heavy Fermions in Fundamental  
Physics

Work in collaboration with

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arXiv : 0911.1753 ; arXiv : 1102.0520 ; work in progress

Neutral currents have played an important role in the construction and experimental tests of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

In the Standard Model Flavour Changing Neutral currents (FCNC) are forbidden at tree level

- in the gauge sector, ie no  $Z_{FCNC}$
- in the scalar sector, ie no  $H_{FCNC}$

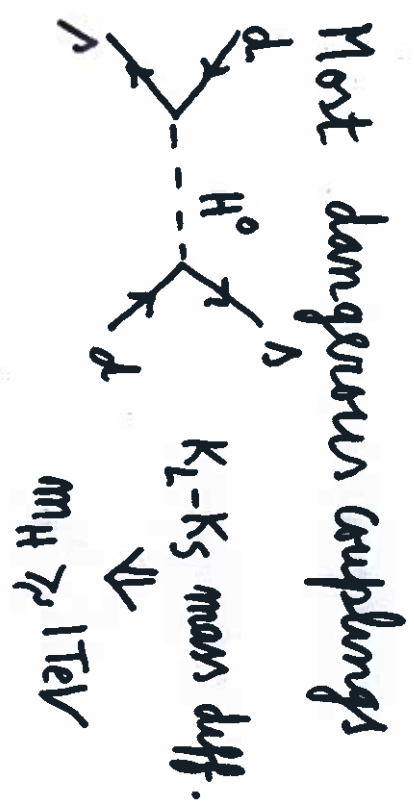
Models with two or more Higgs doublets potentially large  $H_{FCNC}$

strict limits on FCNC processes!

In the SM, FCNC are generated only at loop level  
 $\Rightarrow$  very suppressed

- $K^0 - \bar{K}^0$  mixing
- $D^0 - \bar{D}^0$  mixing
- $B_d^0 - \bar{B}_d^0$  mixing
- $B_s^0 - \bar{B}_s^0$  mixing
- rare Kaon decays
- rare B-meson decays
- CP violation

processes that play a crucial role in testing the SM and putting limits in Models for Physics Beyond the SM



CP violation  $\epsilon_K$   
 $m_H \gtrsim 30 \text{ TeV}$

Improved solutions, case of multi-Higgs models

NEC

Wenberg, Glashow (1977)

or

Paschos (1977)

existence of suppression factors in HFCNC

Antaramian, Hall, Rawn (1992)

Hall, Wenberg (1993)

Yoshida, Rindani (1991)

First models of this type with no ad-hoc assumptions suppression by small elements of

VCKM : BGL models

Braner, Gumm, Lawrence (1996)

More recently, we have generalized BGL models to "larger class of models of "Minimal Flavour Violation" type

# About Minimal Flavour Violation

Buras, Gambhir, Gorbahn, Jager, Salvendy (2001)  
D'Ambrosio, Giudice, Jurek, Strumia (2002)

Leptonic sector  
Cireigliano, Gounsbarn, Jurek, Wue (2005)

$G_F = U(3)^5$  largest asymmetry of the gauge sector  
Flavour violation completely determined by Yukawa couplings

## Our framework

- multi-Higgs models
- no Natural Flavour Conservation
- obey above condition (one of the defining ingredients of MFV framework)

"Higgs - mediated FCNC's: Natural Flavour Conservation vs.

Minimal Flavour Violation"

Buras, Carlucci, Gori, Jurek, arXiv:1005.5310

Question: Under what conditions the neutral Higgs couplings are only functions of  $V_{CKM}$ ?

The case of two Higgs doublets

Yukawa interactions

$$\mathcal{L}_Y = -\bar{Q}_L^0 \Gamma_1 \tilde{\Phi}_1 \mathcal{D}_R^0 - \bar{Q}_L^0 \Gamma_2 \tilde{\Phi}_2 \mathcal{D}_R^0 - \bar{Q}_L^0 \Delta_1 \tilde{\Phi}_1 \mathcal{U}_R^0 - \bar{Q}_L^0 \Delta_2 \tilde{\Phi}_2 \mathcal{U}_R^0 + h.c.$$

$$\tilde{\Phi}_i = -i\tau_2 \phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} (\nu_1 \Gamma_1 + \nu_2 e^{i\alpha} \Gamma_2) ; \quad M_u = \frac{1}{\sqrt{2}} (\nu_1 \Delta_1 + \nu_2 e^{-i\alpha} \Delta_2)$$

Diagonalized by

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag} (m_d, m_s, m_b)$$

$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag} (m_u, m_c, m_t)$$

Expansion around the vev's

$$\Phi_I = e^{i\alpha_j} \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}} (\eta_j + \theta + i\eta_j) \end{pmatrix} \quad j=1,2$$

We perform the following transformations

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = 0 \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} ; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = 0 \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} ; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = 0 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_2 \\ -\eta_1 \end{pmatrix} ; \quad \sqrt{2} = \sqrt{\eta_1^2 + \eta_2^2} = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV}$$

$O$  angles out

$H^0$  with couplings to quarks proportional to mass matrices

$G^0$  the neutral pseudo-goldstone boson

$G^+$  charged pseudo-goldstone boson

Physical neutral Higgs fields are combinations of  $H^0, R$  and  $I$

Yukawa couplings in terms of quark mass eigenstates

$$\begin{aligned} \delta \mathcal{L} = & \frac{\sqrt{2}}{v} H^+ \bar{u} (V_{Nd} \gamma_R - N_{uL}^\dagger V \gamma_L) d + h.c. - \frac{H^0}{v} (\bar{u} D_u u + \bar{d} D_d d) - \\ & - \frac{R}{v} [\bar{u} (N_{uR} \gamma_R + N_{uL}^\dagger \gamma_L) u + \bar{d} (N_d \gamma_R + N_{dL}^\dagger \gamma_L) d] + \\ & + i \frac{E}{v} [\bar{u} (N_{uR} \gamma_R - N_{uL}^\dagger \gamma_L) u - \bar{d} (N_d \gamma_R - N_{dL}^\dagger \gamma_L) d] \\ \gamma_L = & (1 - \gamma_5) / 2 \quad ; \quad \gamma_R = (1 + \gamma_5) / 2 \quad , \quad V \equiv V_{CKM} \end{aligned}$$

Flavour changing neutral currents controlled by  $N_d, N_u$

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger (\sqrt{2} \Gamma_1 - \sqrt{1} e^{i\alpha} \Gamma_2) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger (\sqrt{2} \Delta_1 - \sqrt{1} e^{-i\alpha} \Delta_2) U_{uR}$$

For generic two Higgs doublet models,  $N_u, N_d$  non-diagonal, arbitrary

For abelianers rewrite  $N_d$ :

$$N_d = \frac{\sqrt{2}}{\sqrt{1}} D_d - \frac{\sqrt{2}}{\sqrt{2}} \left( \frac{\sqrt{2}}{\sqrt{1}} + \frac{\sqrt{1}}{\sqrt{2}} \right) U_{dL}^\dagger e^{i\alpha} \Gamma_2 U_{dR}$$

sensory flavour

leads to FCNC



$$N_d = \frac{N_2}{N_1} D_d - \frac{N_2}{\sqrt{2}} \left( \frac{N_2}{N_1} + \frac{N_1}{N_2} \right) U_{dL}^T e^{i\alpha} \Gamma_2 U_{dR}$$

We want  $N_d$  entirely controlled by  $V_{CKM}$  elements  
 (together with ratios of  $N_1$  and  $N_2$  and quark masses)

$$V_{CKM} = U_{dL}^T U_{dL}$$

- Obstacles :
- (i) Dependence on  $U_{dL}$  rather than  $V_{CKM}$
  - (ii) Need to get rid of  $U_{dR}$

Solution to first difficulty :

*Flavour asymmetry* constraining  $U_{dL} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$V_{CKM} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x & x & x \\ x & x & x \\ U_{d31} & U_{d32} & U_{d33} \end{pmatrix} = \begin{pmatrix} x & x & x \\ x & x & x \\ U_{d31} & U_{d32} & U_{d33} \end{pmatrix}$$

$$(V_{CKM})_{3j} = (U_{dL})_{3j}$$

together with

$$\Gamma_2 U_{dR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix} \Rightarrow$$

only third row of  $U_{dL}$  appears in  $N_d$

$$FCNC \propto U_L^\dagger e^{i\alpha} \Gamma_2 U_R$$

to get rid of  $U_R$ , choose  $\Gamma_2 \propto PM_L$ ,  $P$  hermitian

$$U_L^\dagger \Gamma_2 U_R \propto U_L^\dagger P M_L U_R \propto U_L^\dagger P U_L D_L$$

for  $P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ,  $\Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix}$

$$(U_L^\dagger \Gamma_2)_{ij} = (U_L^\dagger)_{i3} (\Gamma_2)_{3j} = (V_{CKM})_{i3} (\Gamma_2)_{3j}$$

$$(N_d)_{ij} = \frac{\sqrt{2}}{N_1} (D_d)_{ij} - \left( \frac{\sqrt{2}}{N_1} + \frac{\sqrt{1}}{\sqrt{2}} \right) (V_{CKM})_{i3} (V_{CKM})_{3j} (D_d)_{jj}$$

Symmetry BGL

$$Q_{L3}^0 \rightarrow e^{i\alpha} Q_{L3}^0; U_{R3}^0 \rightarrow e^{i\alpha} U_{R3}^0; \phi_2 \rightarrow e^{i\alpha} \phi_2 \quad \alpha \neq 0, \pi$$

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}; \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}; \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

Both Higgs doublets have non-zero Yukawa couplings in up and down sectors

$$N_u = -\frac{\sqrt{1}}{\sqrt{2}} \text{diag}(0, 0, m_E) + \frac{\sqrt{2}}{N_1} \text{diag}(m_u, m_c, 0)$$

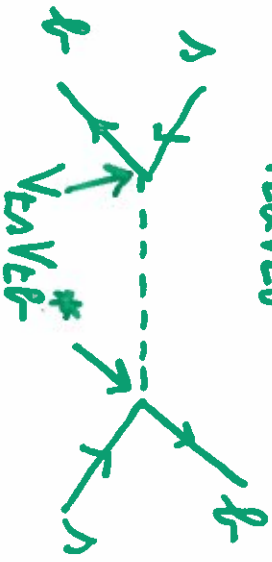
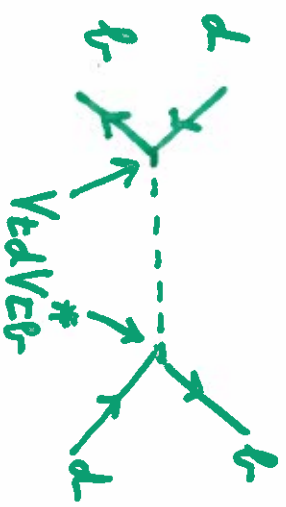
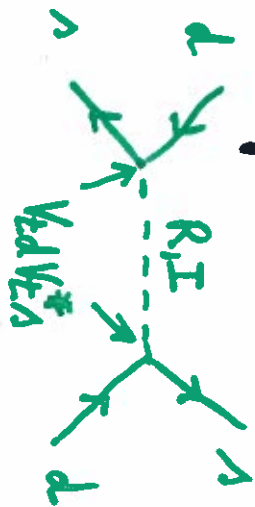
See different models

$$(Nd)_{ij} = \frac{\sqrt{2}}{N_1} (Dd)_{ij} - \left( \frac{\sqrt{2}}{N_1} + \frac{N_1}{\sqrt{2}} \right) \overbrace{(V_{ckn}^\dagger)_{i3} (V_{ckn})_{3j}}^{1+2+3} (Dd)_{ij}$$

$$Nu = -\frac{N_1}{\sqrt{2}} \text{diag}(0, 0, m_L) + \frac{\sqrt{2}}{N_1} \text{diag}(m_u, m_c, 0)$$

FCNC only in the down sector  
 suppression by the 3rd row of  $V_{ckn}$

Strong and Natural suppression of the most constrained processes



$\Delta S = 2$  processes

$$|V_{cd}V_{cs}^*| \sim \lambda^5 \quad (\lambda^{10} \text{ suppression})$$

$$\sim 10^{-4}$$

may contribute significantly to  $B_d - \bar{B}_d$  mixing

contribution to  $B_s - \bar{B}_s$  mixing

How to find a general expansion of  $N_d^0, N_u^0$  which conforms to the MFV requirements?

$$N_d^0 = U_{dL} N_d U_{dR}^\dagger = \frac{1}{\sqrt{2}} (\nu_2 \Gamma_1 - \nu_1 e^{i\alpha} \Gamma_2)$$

$$N_u^0 = U_{uL} N_u U_{uR}^\dagger = \frac{1}{\sqrt{2}} (\nu_2 \Delta_1 - \nu_1 e^{i\alpha} \Delta_2)$$

Necessary condition  $N_d^0, N_u^0$  to be of MFV type:

Should be functions of  $M_d, M_u$  and other flavour dependence  
 Furthermore,  $N_d^0, N_u^0$  should transform under WB appropriate from

$$Q_L^0 \rightarrow W_L Q_L^0, \quad d_R^0 \rightarrow W_R^d d_R^0, \quad u_R^0 \rightarrow W_R^u u_R^0$$

$$M_d \rightarrow W_L^\dagger M_d W_R^d, \quad M_u \rightarrow W_L^\dagger M_u W_R^u$$

$$U_{dL} \rightarrow W_L^\dagger U_{dL}; \quad U_{uL} \rightarrow W_L^\dagger U_{uL}; \quad U_{dR} \rightarrow W_R^{d\dagger} U_{dR}; \quad U_{uR} \rightarrow W_R^{u\dagger} U_{uR}$$

$$H_{d,u} \equiv (M_{d,u})(M_{d,u}^\dagger), \quad H_{d,u} \rightarrow W_L^\dagger H_{d,u} W_L$$

$N_d^0, N_u^0$  Transform as  $M_d, M_u$

It is convenient to write  $H_d, H_u$  in terms of projection operators  
 Botella, Nebot, Vives 2004

$$H_d = \sum_i m_{d_i}^2 P_i^{dL} ; \quad P_i^{dL} = U_{dL} P_i U_{dL}^\dagger ; \quad (P_i)_{jk} = \delta_{ij} \delta_{ik} \quad u \leftrightarrow d$$

MFV expansion for  $N_d^0$  and  $N_u^0$

$$N_d^0 = \lambda_1 M_d + \lambda_{2i} U_{dL} P_i U_{dL}^\dagger M_d + \lambda_{3i} U_{uL} P_i U_{uL}^\dagger M_d + \dots$$

$$N_u^0 = \tau_1 M_u + \tau_{2i} U_{uL} P_i U_{uL}^\dagger M_u + \tau_{3i} U_{dL} P_i U_{dL}^\dagger M_u + \dots$$

In green terms that do not lead to FCNC

In red terms that lead to FCNC

In the quark eigenstate basis

$$N_d = \lambda_1 D_d + \lambda_{2i} P_i D_d + \lambda_{3i} (V_{CKM})^\dagger P_i V_{CKM} D_d + \dots$$

$$N_u = \tau_1 D_u + \tau_{2i} P_i D_u + \tau_{3i} V_{CKM} P_i (V_{CKM})^\dagger D_u + \dots$$

At this stage  $\lambda$  and  $\tau$  coefficients appear as free parameters, MFV  
 Need for additional symmetries in order to constrain these coeff.

BGL example again

corresponds to the following truncation of our NEV expansion

$$N_d^0 = \frac{\sqrt{2}}{\sqrt{1}} M_d - \left( \frac{\sqrt{2}}{\sqrt{1}} + \frac{\sqrt{1}}{\sqrt{2}} \right) U_{uL} P_3 U_{uL}^\dagger M_d$$

$$N_u^0 = \frac{\sqrt{2}}{\sqrt{1}} M_u - \left( \frac{\sqrt{2}}{\sqrt{1}} + \frac{\sqrt{1}}{\sqrt{2}} \right) U_{uL} P_3 U_{uL}^\dagger M_u$$

together with

$$N_d^0 = \frac{\sqrt{2}}{\sqrt{1}} M_d - \frac{\sqrt{2}}{\sqrt{2}} \left( \frac{\sqrt{2}}{\sqrt{1}} + \frac{\sqrt{1}}{\sqrt{2}} \right) e^{i\alpha} \Gamma_2$$

$$N_u^0 = \frac{\sqrt{2}}{\sqrt{1}} M_u - \frac{\sqrt{2}}{\sqrt{2}} \left( \frac{\sqrt{2}}{\sqrt{1}} + \frac{\sqrt{1}}{\sqrt{2}} \right) e^{-i\alpha} \Delta_2$$

implies BGL model fully defined in convenient way under WB transform.

$$\frac{\sqrt{2}}{\sqrt{2}} e^{i\alpha} \Gamma_2 = U_{uL} P_3 U_{uL}^\dagger M_d ; \frac{\sqrt{2}}{\sqrt{2}} e^{-i\alpha} \Delta_2 = U_{uL} P_3 U_{uL}^\dagger M_u$$

we have

$$U_{uL} P_3 U_{uL}^\dagger \Gamma_2 = \Gamma_2 ; U_{uL} P_3 U_{uL}^\dagger \Gamma_1 = 0 ; U_{uL} P_3 U_{uL}^\dagger \Delta_2 = \Delta_2$$

$$U_{uL} P_3 U_{uL}^\dagger \Delta_1 = 0$$

# The leptonic sector

Required for completeness

- study of experimental implications
- study of stability under RGE

Case of Dirac neutrinos, straightforward

Seesaw framework

$$\begin{aligned} \mathcal{L}_{y+mass} = & -\bar{L}_L^0 \pi_1 \tilde{\Phi}_L \nu_R^0 - \bar{L}_L^0 \pi_2 \tilde{\Phi}_L \nu_R^0 - \\ & - \bar{L}_L^0 \Sigma_1 \tilde{\Phi}_1 \nu_R^0 - \bar{L}_L^0 \Sigma_2 \tilde{\Phi}_2 \nu_R^0 + \\ & + \frac{1}{2} \nu_R^{0T} C^{-1} M_R \nu_R^0 + h.c. \end{aligned}$$

$$L_{\text{max}} = -\bar{L}_L^0 m_E \bar{L}_R^0 + \frac{1}{2} (V_L^{0T}, (V_R^0)^{cT}) C^{-1} \mathcal{H}^* \begin{pmatrix} V_L^0 \\ (V_R)^c \end{pmatrix} + h_c$$

$$\mathcal{H} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad (\Psi_L)^c \equiv C \delta_0^T (\Psi_L)^*$$

BGL type example,  $Z_4$  symmetry

$$L_{L3}^0 \rightarrow \exp(i\alpha) L_{L3}^0, \quad V_{R3}^0 \rightarrow \exp(i2\alpha) V_{R3}^0, \quad \phi_2 \rightarrow \exp(i\alpha) \phi_2$$

$$\alpha = \frac{2\pi}{N}$$

$$\Pi_1 = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix}, \quad \Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix}$$

$$\Sigma_1 = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{pmatrix}, \quad M_R = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix}$$

New feature  $m_{Vi}$  from  $m_{eff} \equiv -m_D \frac{1}{M_R} m_D^T$   $M_{33} \neq 0$



Three right neutrinos  $\nu_i$ , plus heavy neutrinos  $N_j$

Right-Right, Right-heavy, Heavy-heavy couplings

$H^0, R, I$  couplings

$U^T m_{eff} U^* = d$ ,  $m_D \frac{1}{D} m_D^T = -U d U^T$  (WB  $m_D$  diag)

$m_D = i U \sqrt{2} O^c \sqrt{D}$  Casas and Ibarra, 2001

$$(N_e)_{ij} = \frac{\sqrt{2}}{N_i} (D_e)_{ij} - \left( \frac{\sqrt{2}}{N_i} + \frac{\sqrt{I}}{\sqrt{2}} \right) (U_U^T)_{i3} (U_U)_{3j} (D_e)_{jj}$$

Right-Right neutral couplings: diag,  $d$

Right-heavy neutral couplings: sensitive to  $O^c, d, D$

Heavy-heavy neutral couplings: diag, sensitive to  $O^c, d, D$

$H^+$  couplings

$$\frac{\sqrt{2} H^+}{\nu} (\bar{\nu}_L^0 N_e^0 R - \bar{\nu}_R^0 N_\nu^{0\dagger} L_L^0) + h.c.$$

# Scalar Potential

$$Z_4 \text{ forbids } \phi_1^\dagger \phi_2, \phi_1^\dagger \phi_2, \phi_2^\dagger \phi_1, \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2$$

ungauged accidental continuous symmetry  
not a symmetry of full Lagrangian  
after spontaneous gauge symmetry breaking  $\rightarrow$   
 $\rightarrow$  pseudo Goldstone boson  
solution: soft symmetry breaking  $m_{12} \phi_1^\dagger \phi_2 + \text{h.c.}$

CP violation

Explicit - Complex Yukawa couplings

Spontaneous - not possible without extension  
of scalar sector

## Conclusions

Multi-Higgs models are very interesting candidates for NP

There are new mechanisms beyond NFC to obtain strong suppression of FCNC as required by experiment

LHC results may bring surprises for the Higgs sector

Models with three Higgs doublets

Yukawa interactions

$$\begin{aligned}
 \mathcal{L}_Y = & -\bar{Q}_L^0 \Gamma_1 \Phi_1 d_R^0 - \bar{Q}_L^0 \Gamma_2 \Phi_2 d_R^0 - \bar{Q}_L^0 \Gamma_3 \Phi_3 d_R^0 - \\
 & -\bar{Q}_L^0 \Delta_1 \tilde{\Phi}_1 u_R^0 - \bar{Q}_L^0 \Delta_2 \tilde{\Phi}_2 u_R^0 - \bar{Q}_L^0 \Delta_3 \tilde{\Phi}_3 u_R^0 + \text{h.c.}
 \end{aligned}$$

$$\tilde{\Phi}_i = -i \tau_2 \Phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} (\nu_1 e^{i\alpha_1} \Gamma_1 + \nu_2 e^{i\alpha_2} \Gamma_2 + \nu_3 e^{i\alpha_3} \Gamma_3)$$

$$M_u = \frac{1}{\sqrt{2}} (\nu_1 e^{-i\alpha_1} \Delta_1 + \nu_2 e^{-i\alpha_2} \Delta_2 + \nu_3 e^{-i\alpha_3} \Delta_3)$$

after spontaneous symmetry breakdown

$$\begin{aligned}
 & -4 - \Phi_i = e^{i\alpha_i} \\
 & \left( \frac{1}{\sqrt{2}} (\nu_i + \rho_i + i\eta_i) \right)
 \end{aligned}$$

We perform the following transformation

$$\begin{pmatrix} H^0 \\ R^i \end{pmatrix} = 0 \quad , \quad \begin{pmatrix} P^1 \\ P^2 \\ P^3 \end{pmatrix} \quad , \quad \begin{pmatrix} G^0 \\ F^i \end{pmatrix} = 0 \quad \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

$$0 = \begin{pmatrix} \frac{\nu_1}{\sqrt{5}} & \frac{\nu_2}{\sqrt{5}} & \frac{\nu_3}{\sqrt{5}} \\ \frac{\nu_1}{\sqrt{5}} & -\frac{\nu_1}{\sqrt{5}} & 0 \\ \frac{\nu_1}{\sqrt{5}} & \frac{\nu_2}{\sqrt{5}} & \frac{-(\nu_1^2 + \nu_2^2)/\sqrt{5}}{\nu_3} \end{pmatrix} \quad , \quad \begin{matrix} \nu = \sqrt{\nu_1^2 + \nu_2^2 + \nu_3^2} \\ \nu^i = \sqrt{\nu_1^2 + \nu_2^2} \\ \nu^{ii} = \sqrt{\nu_1^2 + \nu_2^2 + (\nu_1^2 + \nu_2^2)^2 / \nu_3^2} \end{matrix}$$

0 angles out

$H^0$  with couplings to quarks proportional to mass  
 $G$  the neutral pseudo - Goldstone boson

$$\begin{aligned}
 \mathcal{L}_y (\text{neutral}) = & -\frac{H^0}{N} (\bar{d}_L D d_R + \bar{u}_L D u_R) - \\
 & -\bar{d}_L \frac{1}{N_1} M_d (R+iI) d_R - \bar{u}_L \frac{1}{N_1} M_u (R-iI) u_R - \\
 & -\bar{d}_L \frac{1}{N_1'} M_d' (R'+iI') d_R - \bar{u}_L \frac{1}{N_1'} M_u' (R'-iI') u_R + h.c.
 \end{aligned}$$

with

$$M_d = \frac{1}{\sqrt{2}} U_L^T (v_2 e^{i\alpha_1} \Gamma_1 - v_1 e^{i\alpha_2} \Gamma_2) U_{dR}$$

$$M_u = \frac{1}{\sqrt{2}} U_L^T (v_2 e^{-i\alpha_1} \Delta_1 - v_1 e^{-i\alpha_2} \Delta_2) U_{uR}$$

$$M_d' = \frac{1}{\sqrt{2}} U_L^T (v_1 e^{i\alpha_1} \Gamma_1 + v_2 e^{i\alpha_2} \Gamma_2 + \kappa e^{i\alpha_3} \Gamma_3) U_{dR}$$

$$M_u' = \frac{1}{\sqrt{2}} U_L^T (v_1 e^{-i\alpha_1} \Delta_1 + v_2 e^{-i\alpha_2} \Delta_2 + \kappa e^{-i\alpha_3} \Delta_3) U_{uR}$$

$$\kappa = -(\sqrt{1^2 + \sqrt{2}^2}) / \sqrt{3}$$

Improving the following discrete symmetry on the Lagrangian

$$\begin{aligned}
 Q_{L1}^0 &\rightarrow w Q_{L1}^0, & Q_{L2}^0 &\rightarrow w^2 Q_{L2}^0, & Q_{L3}^0 &\rightarrow w^4 Q_{L3}^0 \\
 \Phi_1 &\rightarrow w \Phi_1, & \Phi_2 &\rightarrow w^2 \Phi_2, & \Phi_3 &\rightarrow w^4 \Phi_3 \\
 U_{R1}^0 &\rightarrow w^2 U_{R1}^0, & U_{R2}^0 &\rightarrow w^4 U_{R2}^0, & U_{R3}^0 &\rightarrow w^8 U_{R3}^0 \\
 d_{Rf}^0 &\rightarrow d_{Rf}^0 & & & & \text{with } w = \exp i\pi/4
 \end{aligned}$$

restricts the Yukawa coupling matrices. Following structure

$$\begin{aligned}
 \Gamma_1 &= \begin{bmatrix} x & x & x \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; & \Gamma_2 &= \begin{bmatrix} 0 & 0 & 0 \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}; & \Gamma_3 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix} \\
 \Delta_1 &= \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; & \Delta_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 0 \end{bmatrix}; & \Delta_3 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}
 \end{aligned}$$

all three Higgs doublets have non-zero Yukawa couplings  
both in the up and down sectors

In this case there are Higgs mediated FCNC only down sector

$$(M_d)_{ij} = \frac{\sqrt{2}}{v_1} (D_d)_{ij} - \left( \frac{\sqrt{2}}{v_1} + \frac{v_1}{\sqrt{2}} \right) (V_{CKM})_{i2}^{\dagger} (V_{CKM})_{2j} (D)_{ij} - \frac{\sqrt{2}}{v_1} (V_{CKM})_{i3}^{\dagger} (V_{CKM})_{3j} (D)_{ij} \quad \alpha = -(\sqrt{v_1^2 + v_2^2})/\sqrt{3}$$

$$(M_d)_{ij} = (D_d)_{ij} - \frac{\sqrt{3-x}}{\sqrt{3}} (V_{CKM})_{i3}^{\dagger} (V_{CKM})_{3j} (D_d)_{ij}$$

$M_d$  includes FCNC terms where the suppression factor in  $\Delta S = 2$  transitions is only  $(V_{cd}^{\dagger} V_{cs})^2$ , which then requires quite heavy neutral Higgs