Heavy fermions and Kaon decays

The Role of Fermions in Fundamental Physics
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Phys. Rev. D 83 (2011) 034030 [arXiv:1009.0947]

Outline

- Introduction
- ε_K
  - Calculation
  - Results & error budget
- $K \to \pi\nu\nu$
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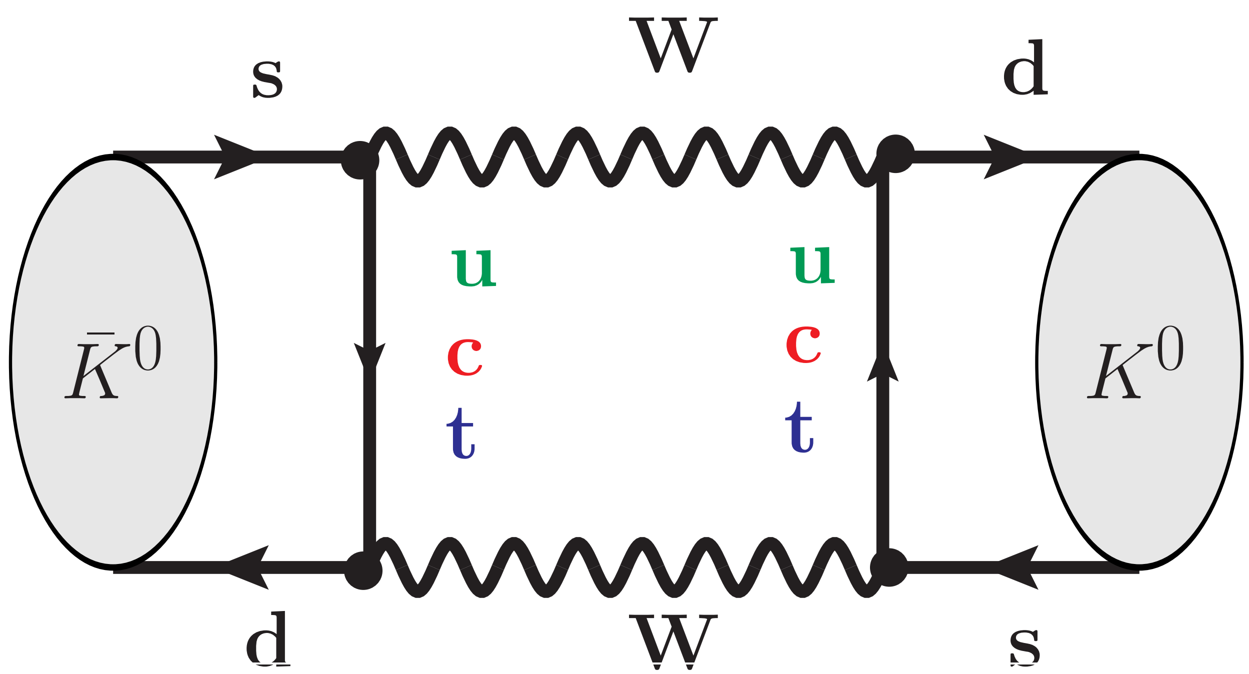
- Conclusion

Why ε_K and rare K decays?

The little things are infinitely the most important.
[Sherlock Holmes, A case of identity]

- Kaons and heavy fermions - a long story of success
  - Charm quark from $K_L \to \mu^+\mu^-$ (GIM 1970)
  - Third generation from CP violation (KM 1974)
- FCNC observables loop-induced in the Standard Model
  $\Rightarrow$ sensitive to virtual heavy particles

ε_K in the Standard Model



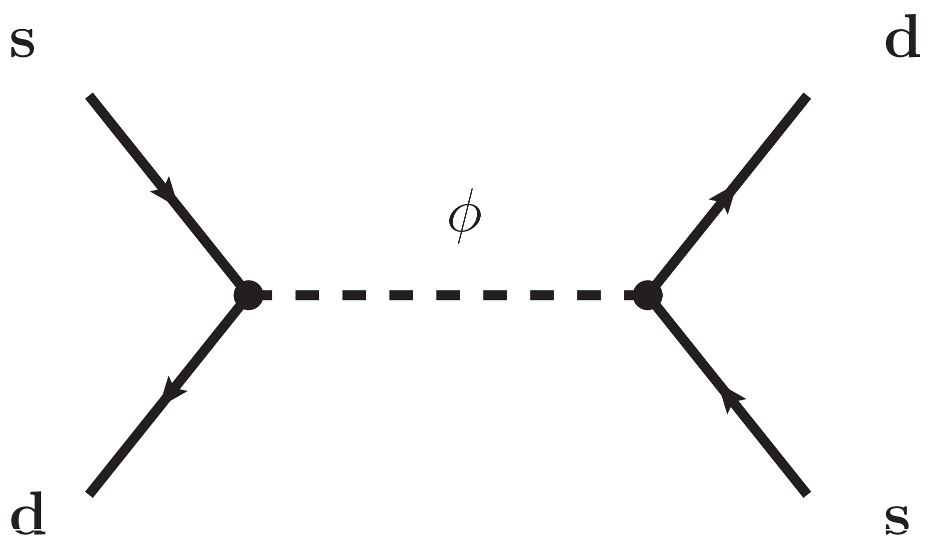
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

SM → CP violation by Complex Phase in the CKM matrix

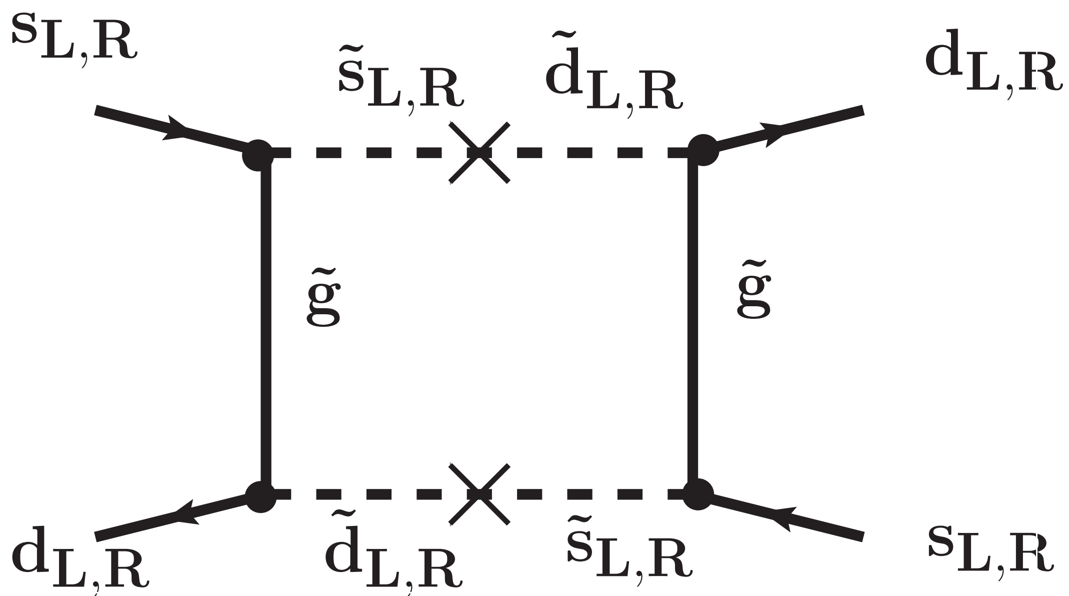
Top-quark contribution CKM suppressed
⇒ sensitivity to deviations from MFV

ε_K beyond the Standard Model

Many more sources of CP violation:



2HDM Type III



SUSY models

+Technicolour, extra dimensions, ...

Strong constraints from ε_K!

Chiral Enhancement of non-SM operators

Tension in the Data?

Experiment:

ε_K is measured precisely:

$|\varepsilon_K| = 2.228(11) \times 10^{-3}$.

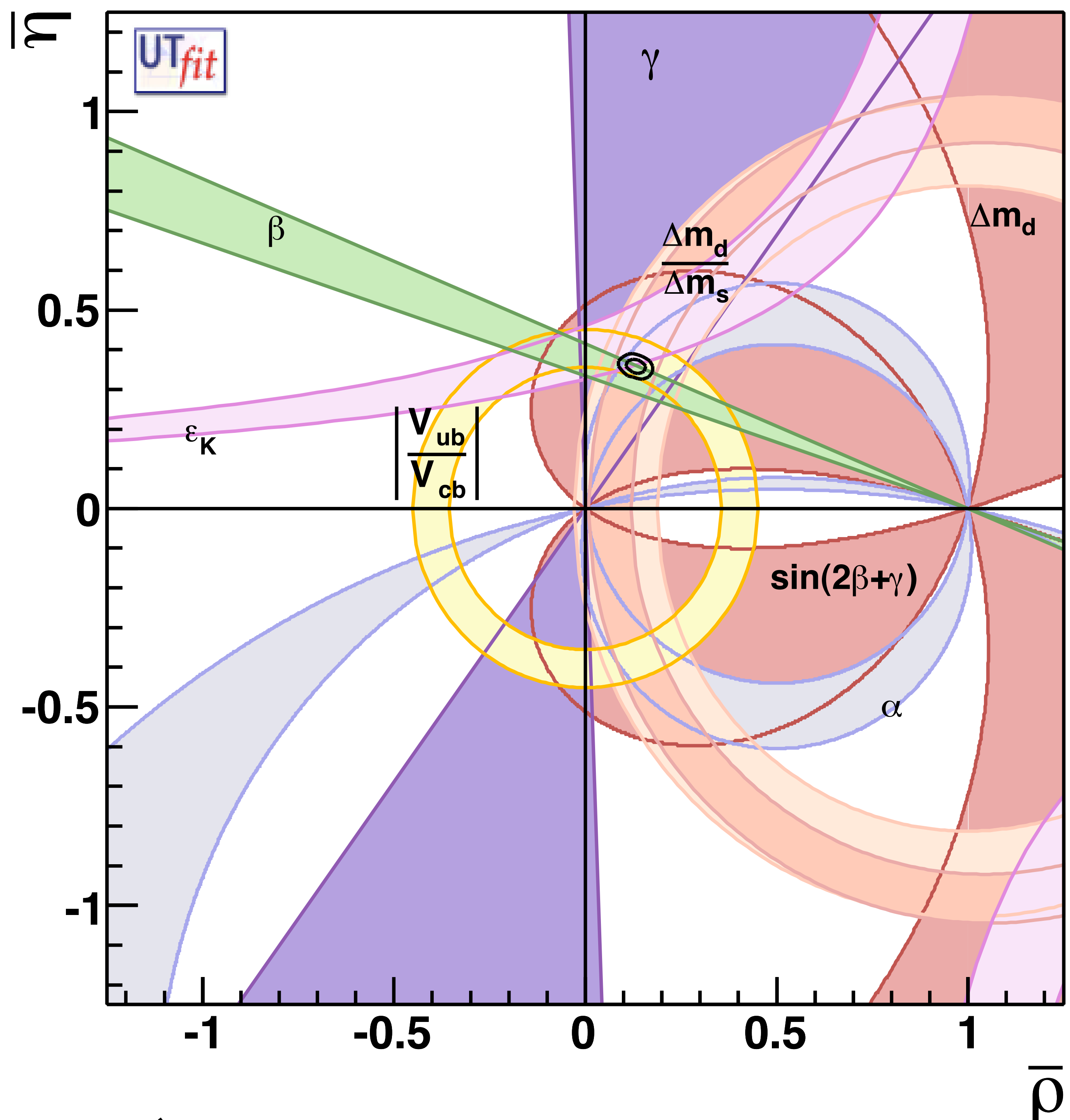
[PDG2010]

Theory:

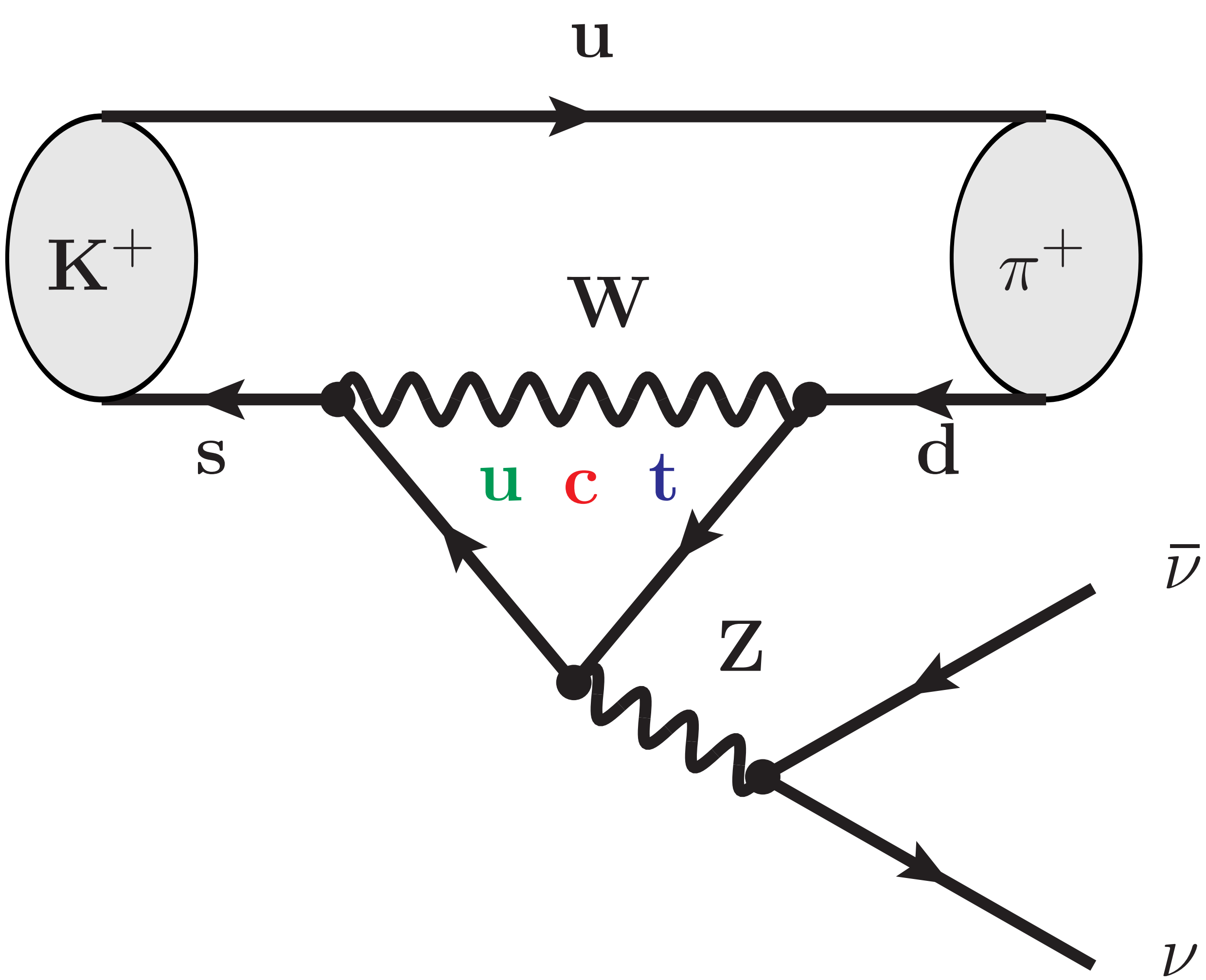
ε_K can be calculated reliably:

$|\varepsilon_K| = 1.83(27) \times 10^{-3}$.

[NLO SM prediction]



$K^+ \to \pi^+ \nu\nu$ und $K_L \to \pi^0 \nu\nu$ in the Standard Model



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Top-quark contribution CKM suppressed
⇒ sensitivity to deviations from MFV

Isospin Symmetry

[Marciano & Parsa '96]

$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

$K \to \pi \nu \bar{\nu}$ — unknown

$K \to \pi \ell \nu$ — well measured

... plus isospin-breaking corrections!

Experimental Prospect

NA62 (CERN), KOTO (JPARC), P996 (FNAL)

aim at measuring Br‘s with 10% (3%) precision!

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The Neutral Kaon System

Time evolution:

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix} = \left[\begin{pmatrix}M_{11} & M_{12} \\ M_{12}^* & M_{11}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11}\end{pmatrix}\right]\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix}$$

Diagonalise ⇒

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

CP is violated by the non-vanishing phase

$$\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) = \mathcal{O}\left(10^{-3}\right)$$ ← (in the Kaon system)

εK - Definition

ε_K describes indirect CP violation in the neutral Kaon system

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$$

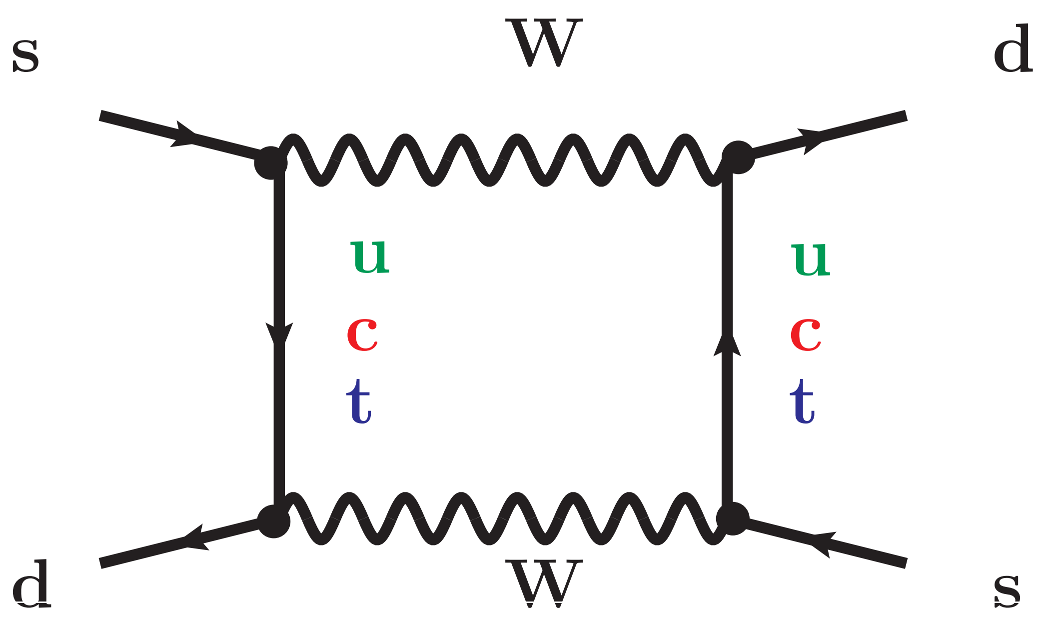
$$\frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle}$$

$$\epsilon_K = \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)$$

$$\phi_\epsilon = \arctan \frac{\Delta m_K}{\Delta \Gamma_K / 2}$$

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$



CKM parameters:

$$\lambda_i = V_{is}^* V_{id}$$

$$\lambda_u = -\lambda_c - \lambda_t$$

Re $\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$ ⇒ Δm_K Im $\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$ ⇒ ε_K

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

$$\approx \lambda_{\mathrm{Cabibbo}}^{10} \times \eta_{tt} \times \frac{m_t^2}{M_W^2}$$

$\lambda_{\mathrm{Cabibbo}} \approx 0.2$

Dominant contribution (≈+72%)

η_{tt} = 0.5765(65) at NLO QCD [Buras et al. ‘90]

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

$$\approx \lambda_{\mathrm{Cabibbo}}^6 \times \eta_{cc} \times \frac{m_c^2}{M_W^2}$$

Smallest contribution (≈-14%)

η_{cc} = 1.43(23) at NLO QCD [Herrlich, Nierste '94]

$|\Delta S|=2$ Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

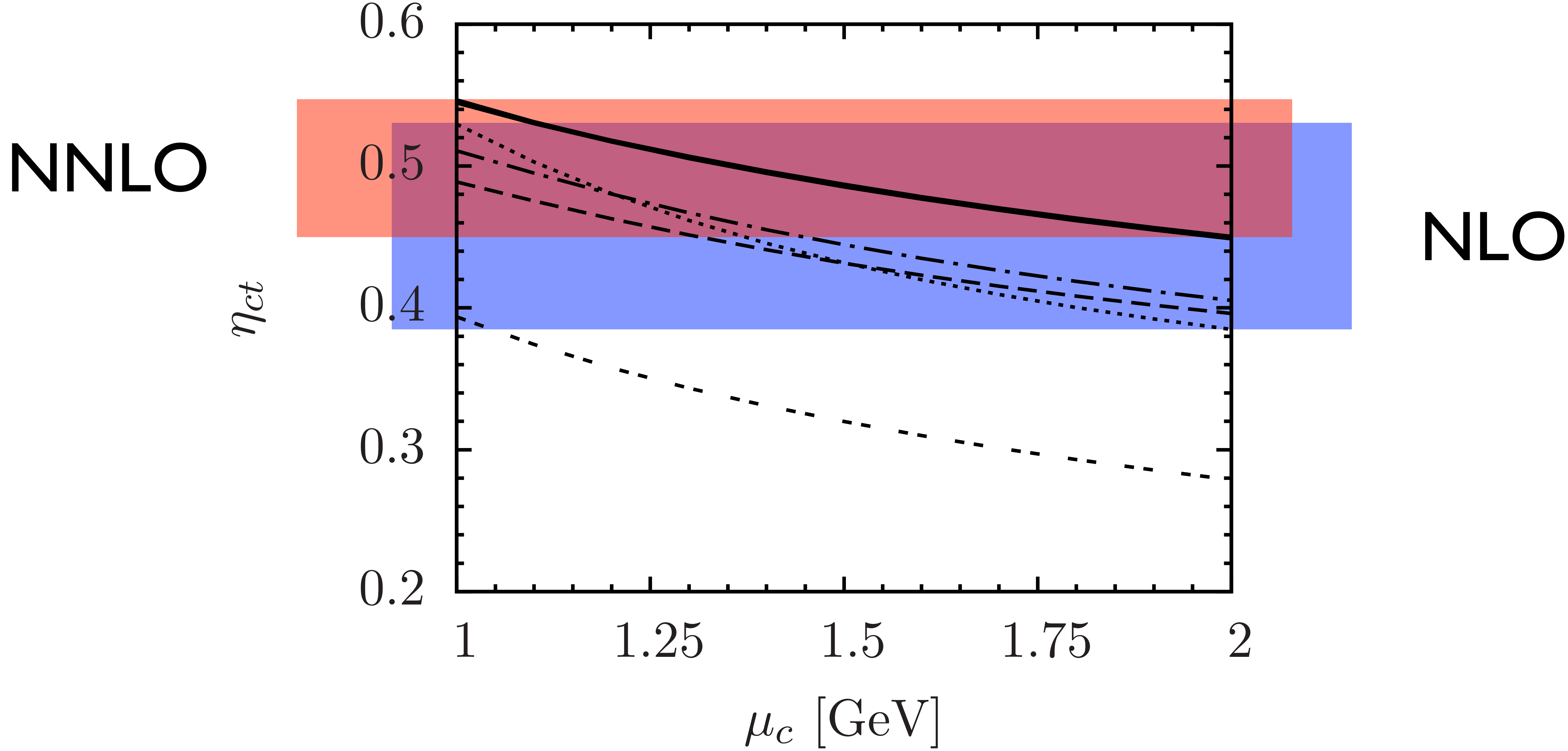
$$\approx \lambda_{\text{Cabibbo}}^6 \times \eta_{ct} \times \frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2}$$

Contributes $\approx$+42%

η_{ct} = 0.496(47) at NNLO QCD [Brod, Gorbahn '10]

New three-loop calculation: +7% shift w.r.t. NLO!

η_{ct} at NNLO: Scale Dependence

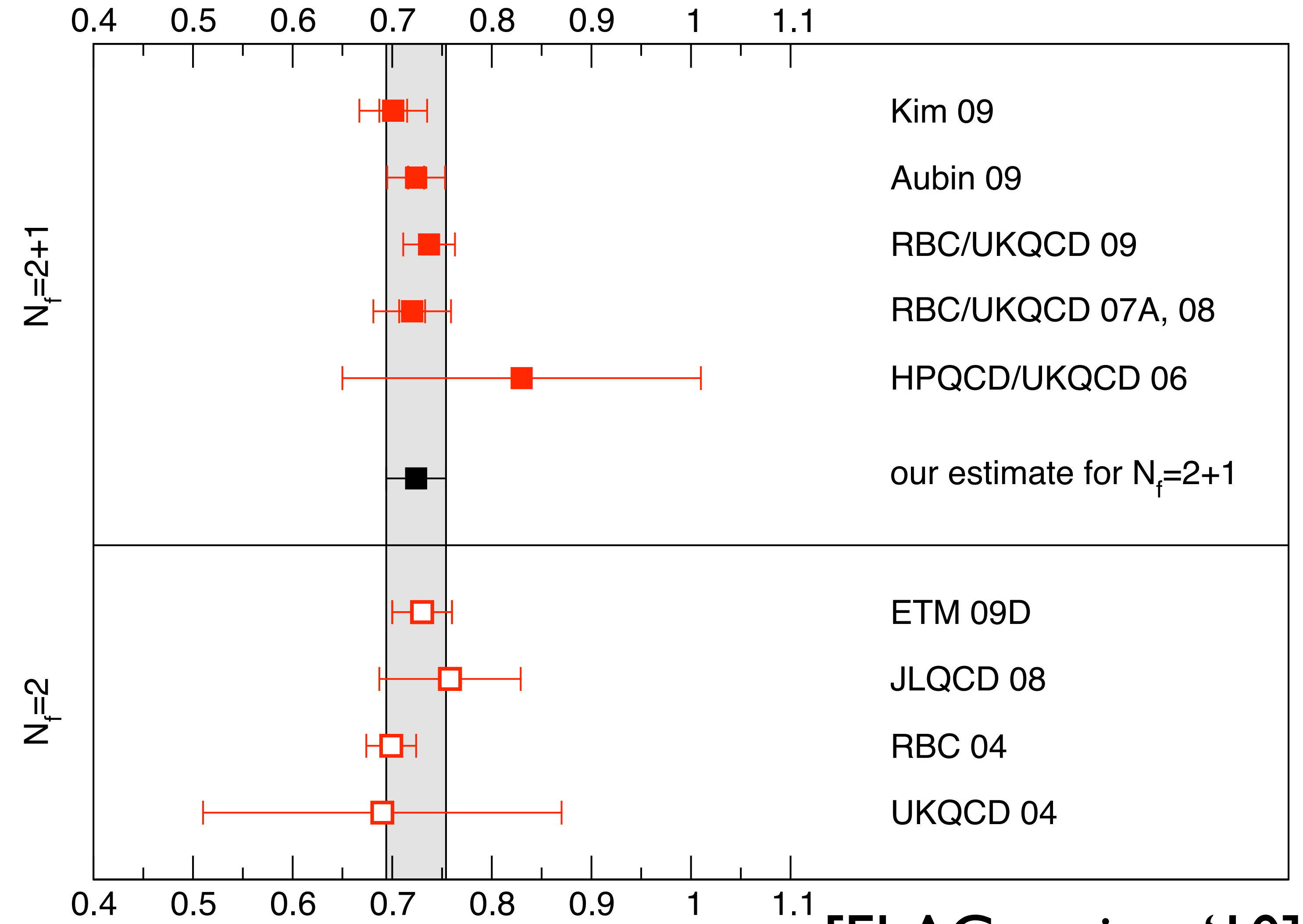


Determination of B_K

$$\langle K^0|\tilde{Q}^{|\Delta S|=2}|\bar{K}^0\rangle \propto \widehat{B}_K = 0.725(26)$$

Huge progress:

- Unquenched calculations
- Small pion masses
- Different lattice spacings
- Main error: matching to perturbative renormalisation



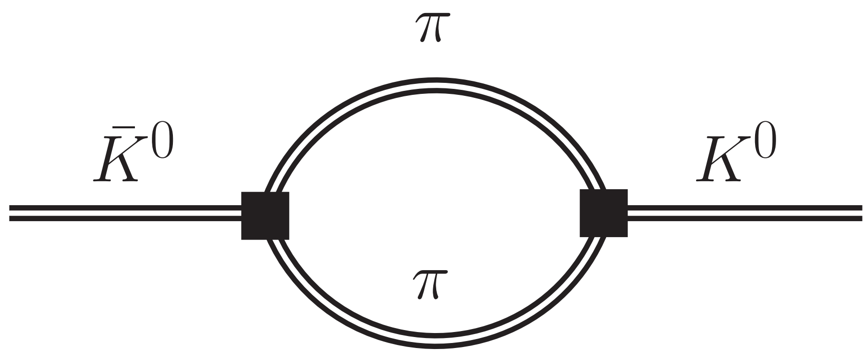
[FLAG review '10]

LD-Contributions from ΔS=1 Hamiltonian

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)$$

Dispersive (real) and

absorptive (imaginary) part of



$$\int d^4x \langle K^0 | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

- Estimate ξ from ε'/ε: -6% [Nierste '02, Buras et al. '09]
- compute absorptive part in ChPT: +2.4% [Buras et al. '10]

Combine with prefactor to κ_ε = 0.94(2)

LD-Contributions from ΔS=2 Hamiltonian

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)$$

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$$

Higher-dimensional operators contribute less than 1%

[Cata et al. '03]

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$|\varepsilon_K|$ - Numerics

using only experimental and lattice input:

$$|\varepsilon_K| \propto \kappa_\varepsilon B_K |V_{cb}|^2 \xi_s \sin\beta \times [|V_{cb}|^2 \xi_s \cos\beta \, \eta_{tt} S(x_t) + \eta_{ct} S(x_c, x_t) - \eta_{cc} S(x_c)]$$

$$\xi_s = \frac{F_{B_s}\sqrt{\hat{B}_s}}{F_{B_d}\sqrt{\hat{B}_d}} \qquad x_q = \frac{m_q^2}{M_W^2}$$

η_{ct}	0.496(47)
η_{cc}	1.47(23)
η_{tt}	0.5765(65)

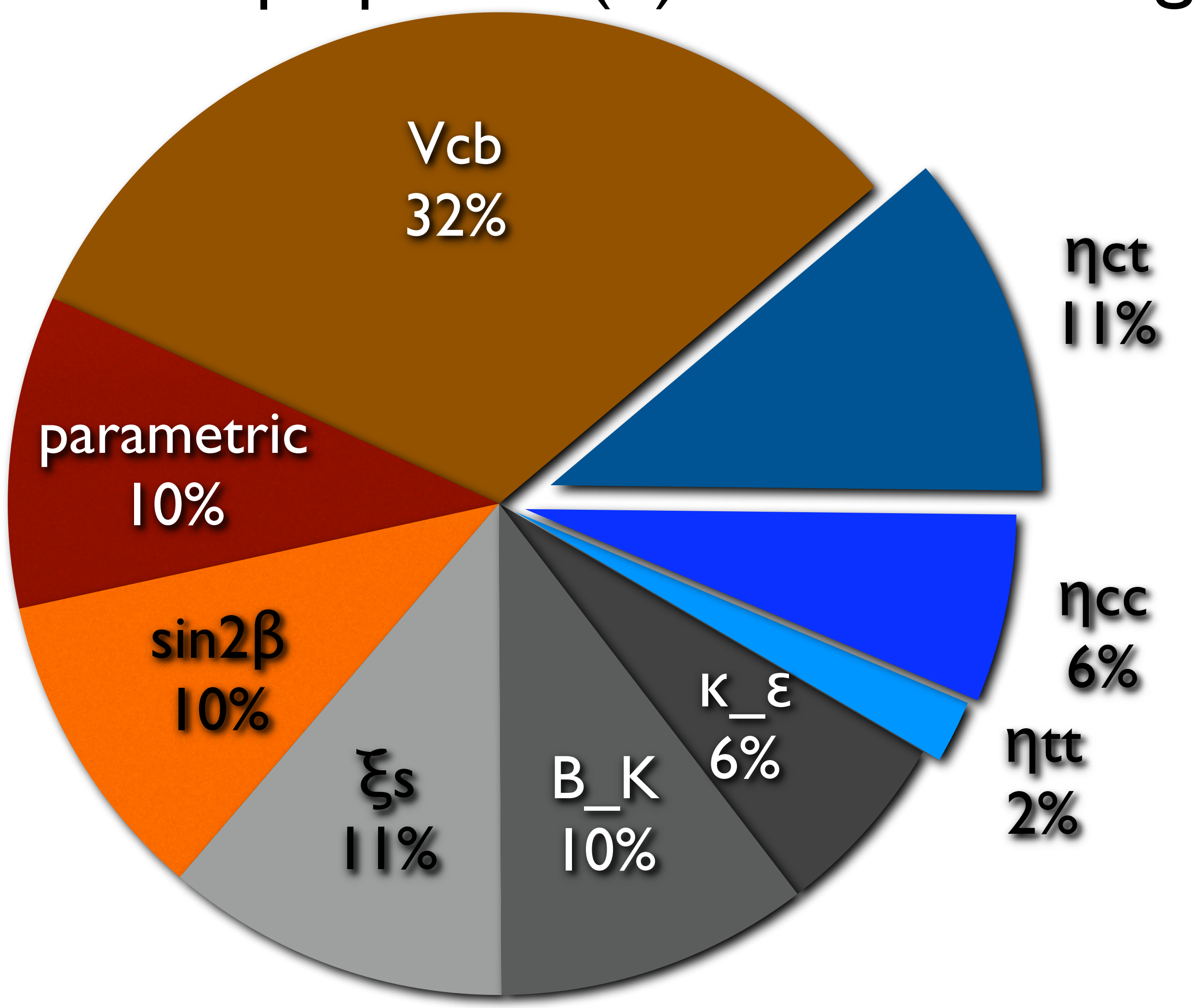
$\lvert V_{cb}\rvert$	0.0406(13)
$\sin(2\beta)$	0.671(23)
ξ_s	1.243(28)
B_K	0.725(26)
κ_ε	0.94(2)

$|\varepsilon_K|$ - Result & Error Budget

$|\varepsilon_K| = 1.90(26) \times 10^{-3}$ using $\eta_{ct} = 0.496(47)$

cf. $|\varepsilon_K| = 1.83(26) \times 10^{-3}$ using $\eta_{ct} = 0.457(73)$ (NLO!)

cf. $|\varepsilon_K| = 1.6(2) \times 10^{-3}$ using $|V_{cb}| = 0.0387(11)$ (exclusive)



→ better agreement with

Experiment [PDG '10]:

$$|\epsilon_K| \overset{\text{exp.}}{=} 2.228(11) \times 10^{-3}$$

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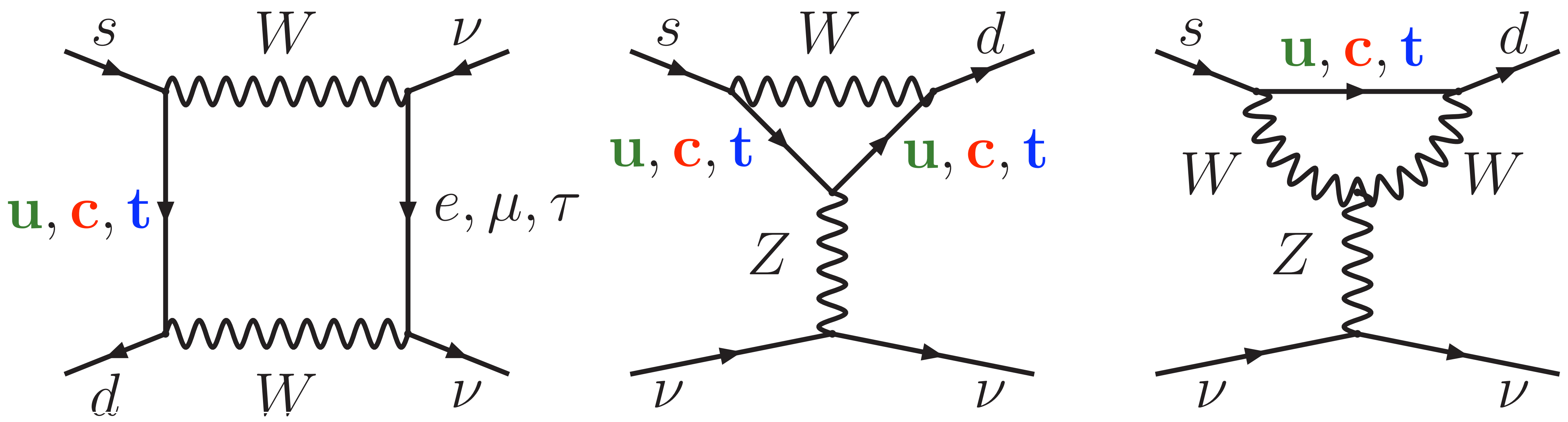
Effective Hamiltonian for $K \to \pi\nu\bar{\nu}$ decays



$$\mathcal{H}_{\text{eff}} \propto [\lambda_t(F(x_t) - F(x_u)) + \lambda_c(F(x_c) - F(x_u))](\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Quadratic GIM:

$$\lambda^5 \frac{m_t^2}{M_W^2} \qquad \lambda \frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2} \qquad \lambda \frac{\Lambda_{\text{QCD}}^2}{M_W^2}$$

Charm and Light Quarks

- P_c at NNLO QCD: ±2.5% theory uncertainty

  [Buras et al. '06]

- P_c NLO QED & Electroweak: +2% shift [Brod, Gorbahn '08]

Below the charm scale:
dimension-eight operators [Falk et al. '01]
and light-quark contributions [Isidori et al. '05]:
$\delta P_{c,u} = 0.04 \pm 0.02$

Top-quark Contribution

- X_t at NLO QCD: ±1% theory uncertainty

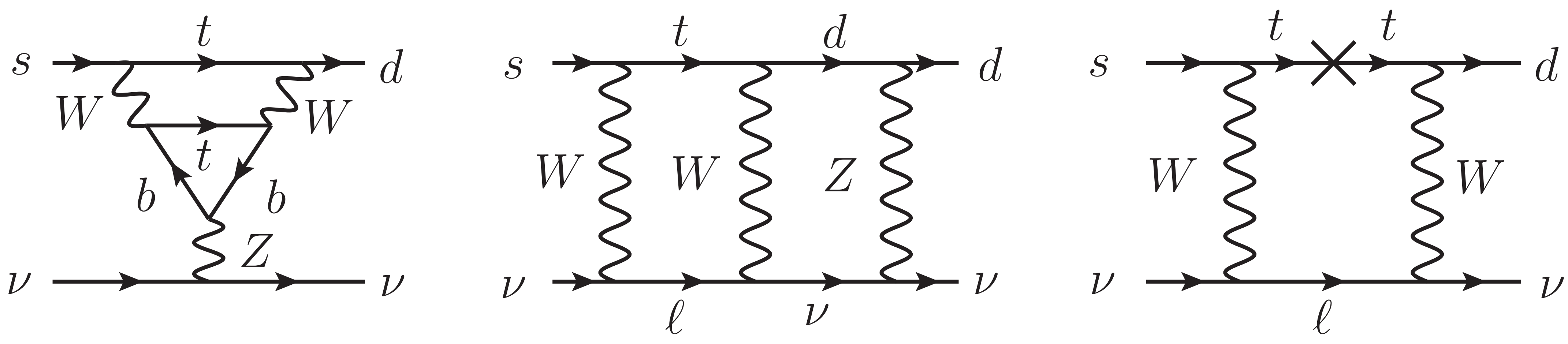
  [Misiak et al., Buchalla et al. '99]

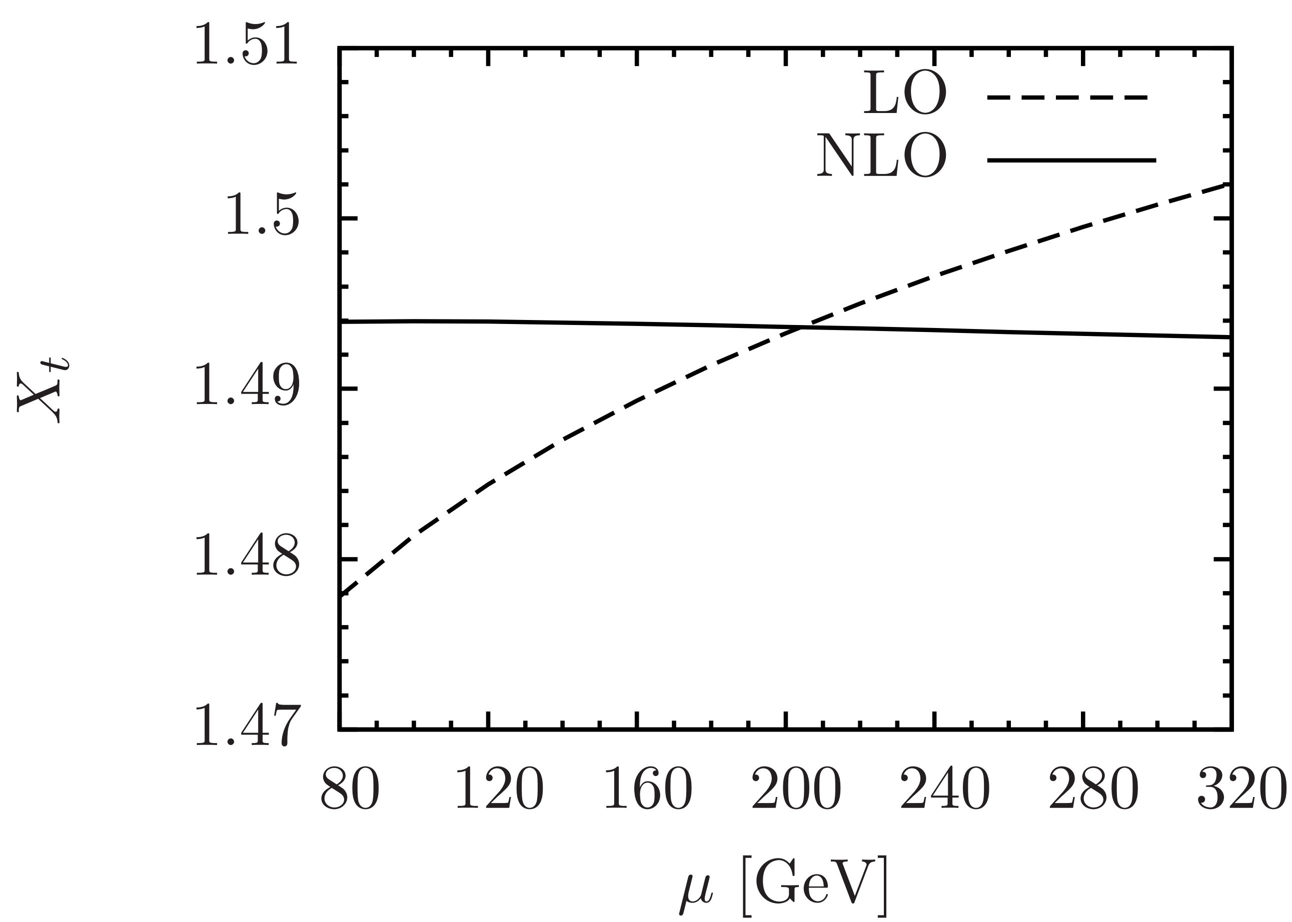
- X_t NLO Electroweak: ±0.1% uncertainty

  [Brod, Gorbahn, Stamou '10]

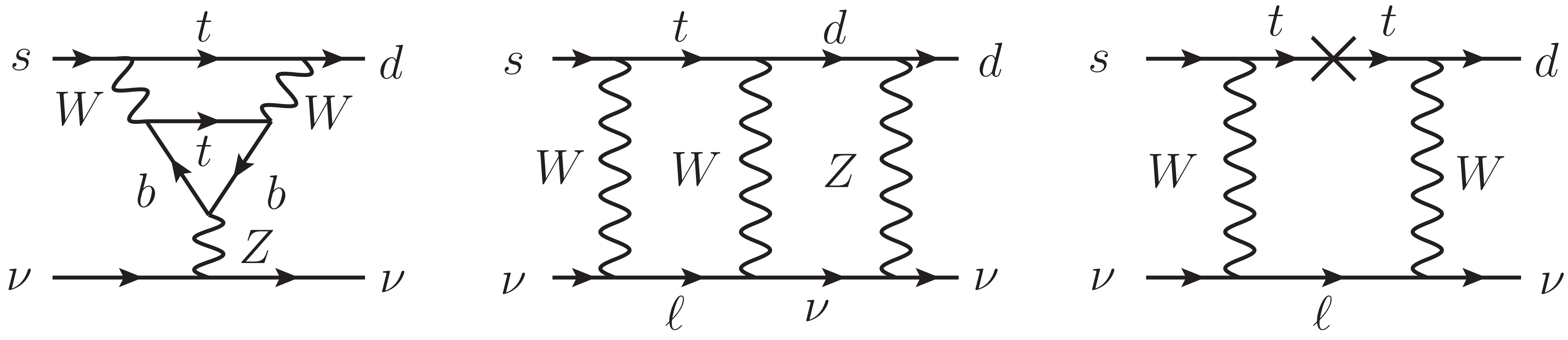
B.t.w.: X_t contributes also to $B \to X_{d,s} \nu\nu$ decays!

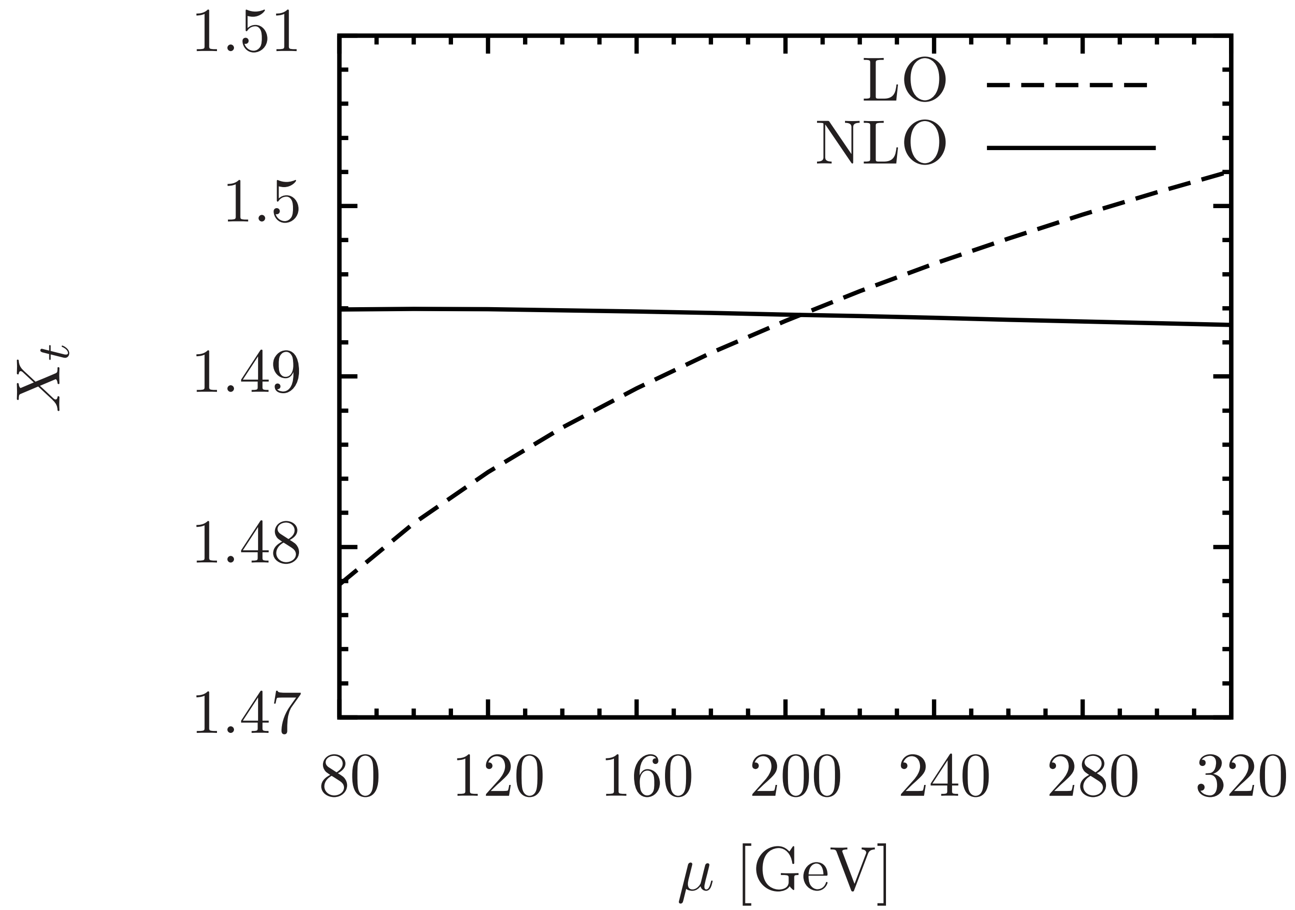
NLO Electroweak Corrections to X_t

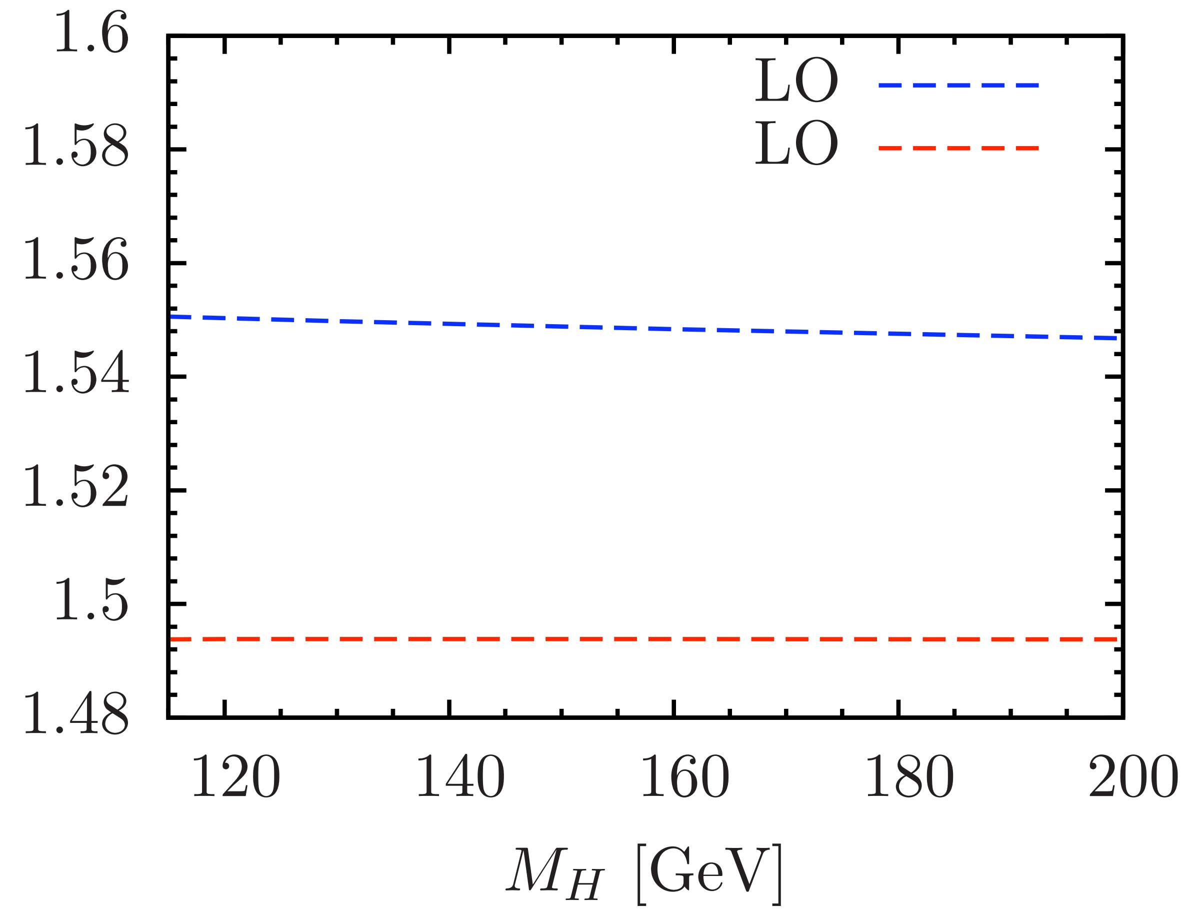




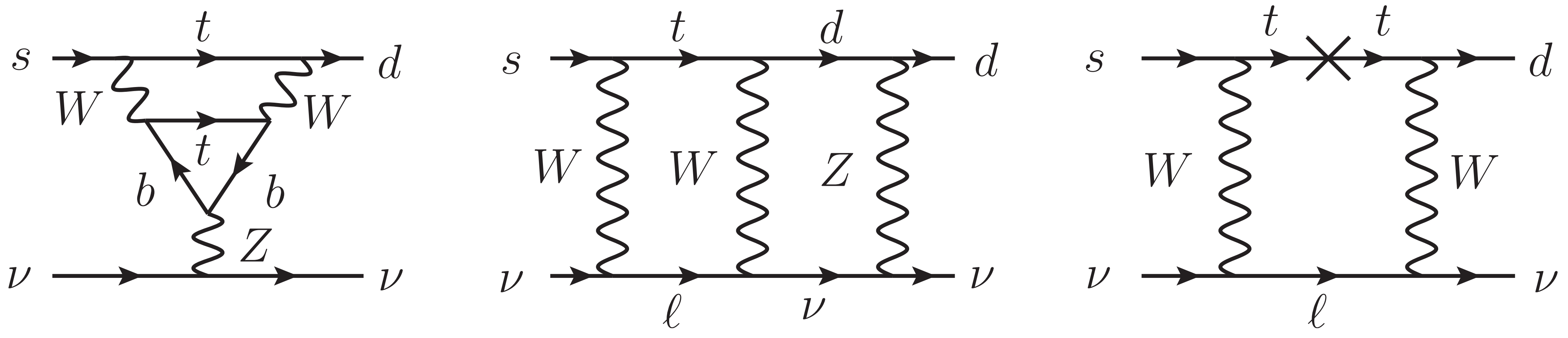
NLO Electroweak Corrections to X_t

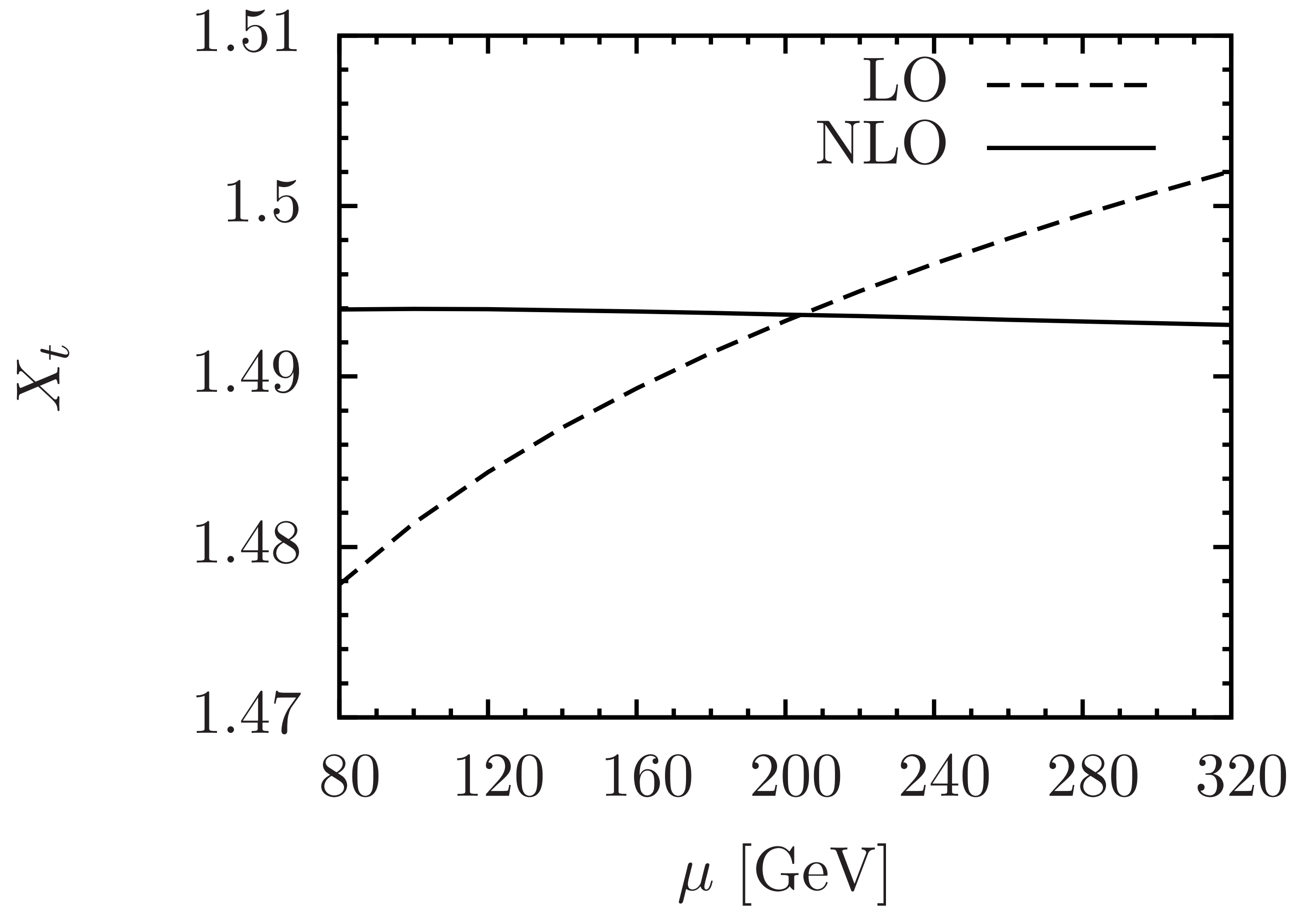


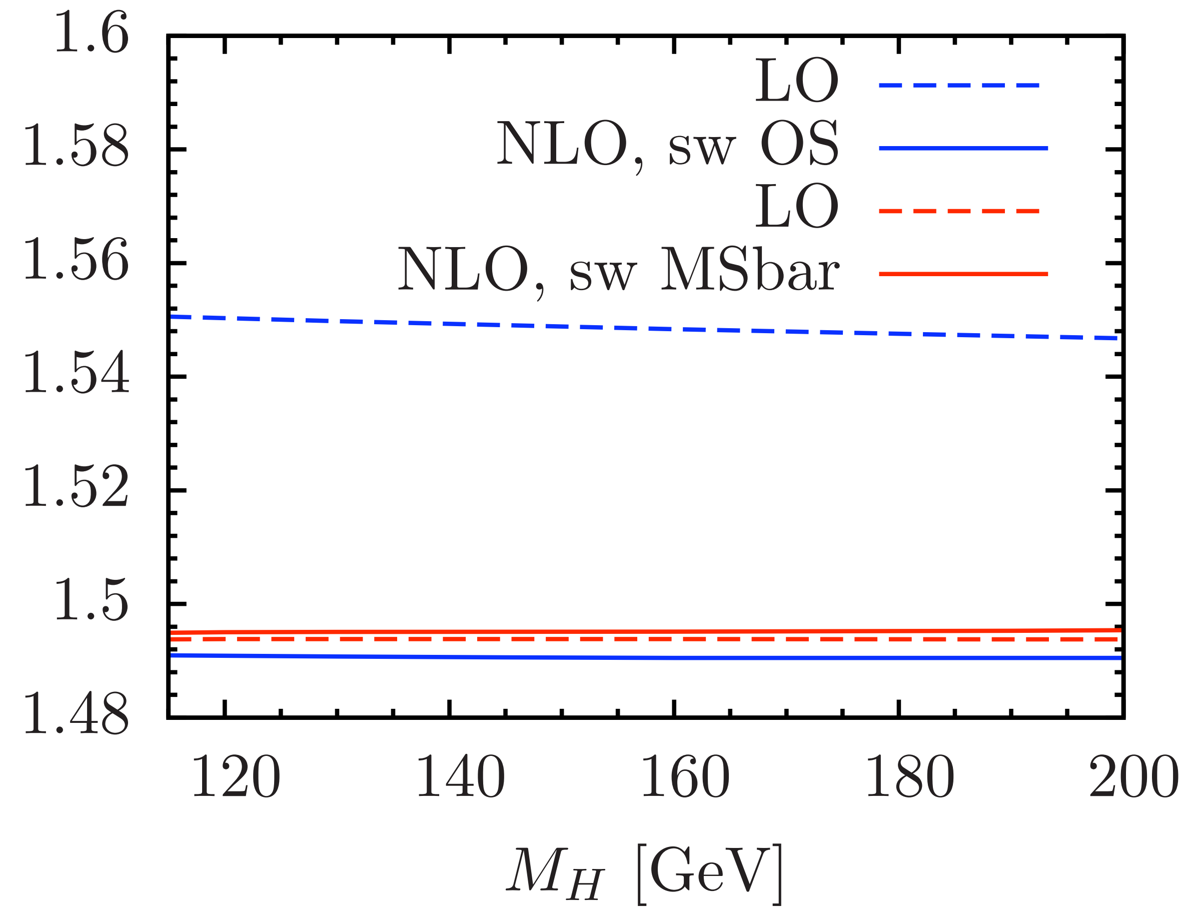




NLO Electroweak Corrections to X_t







$K \to \pi\nu\nu$ Branching Ratios

Matrix element extracted from K_{l3} decays
at (N)NLO ChPT,
QED radiative corrections [Mescia, Smith '07]

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ \left| \lambda_t X_t + \mathrm{Re}\lambda_c (P_c + \delta P_{c,u}) \right|^2$$

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left| \mathrm{Im}\lambda_t X_t \right|^2$$

DCPV:ICPV:CPC = 1:10^{-2}:$\leq 10^{-4}$

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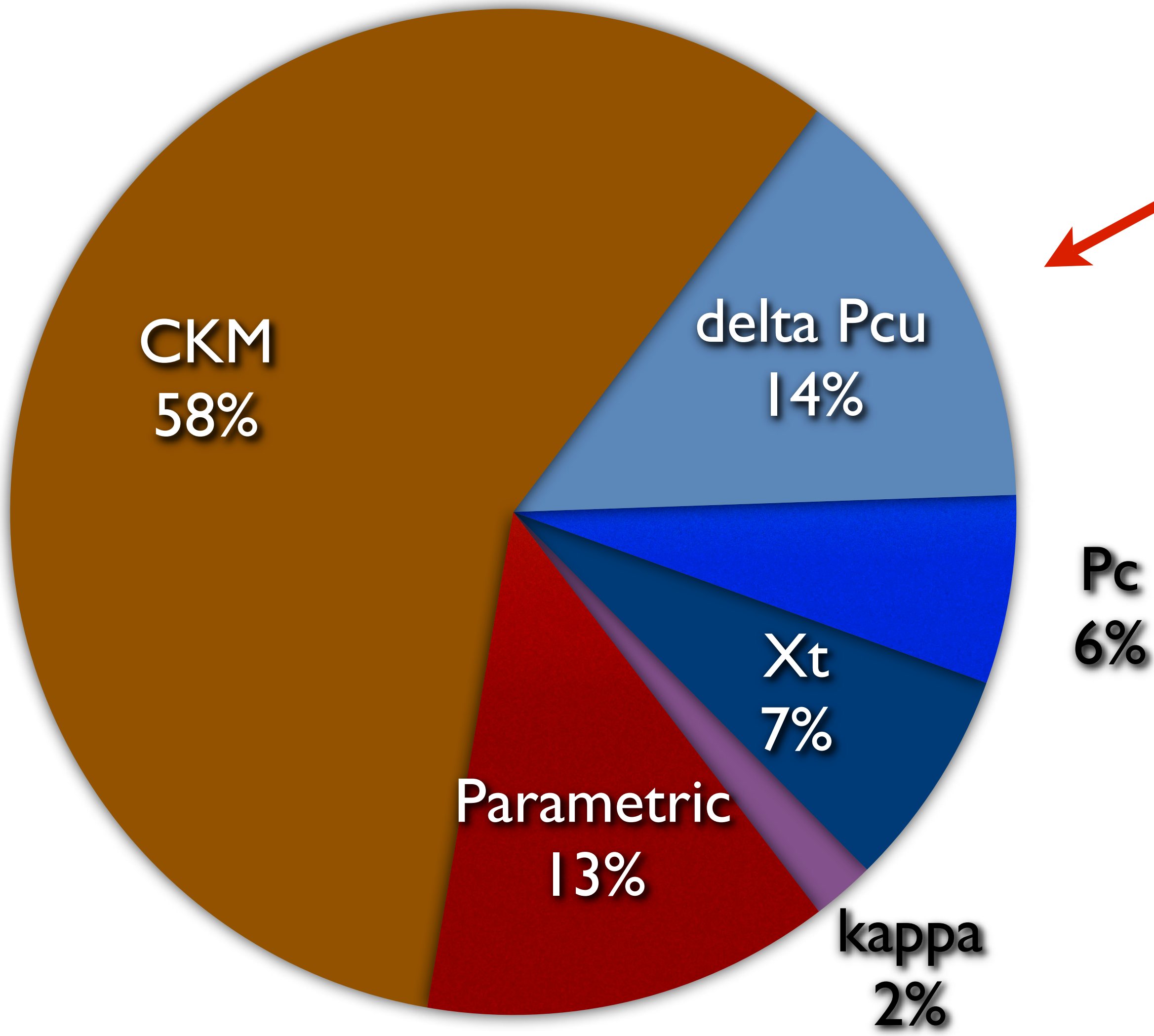
$K^+ \to \pi^+ \nu\nu$ - Result and Error Budget

$Br(K^+ \to \pi^+ \nu\nu) = 7.81(75)(29) \times 10^{-11}$



Improve by lattice QCD [Isidori et al. '05]

Experimental Result

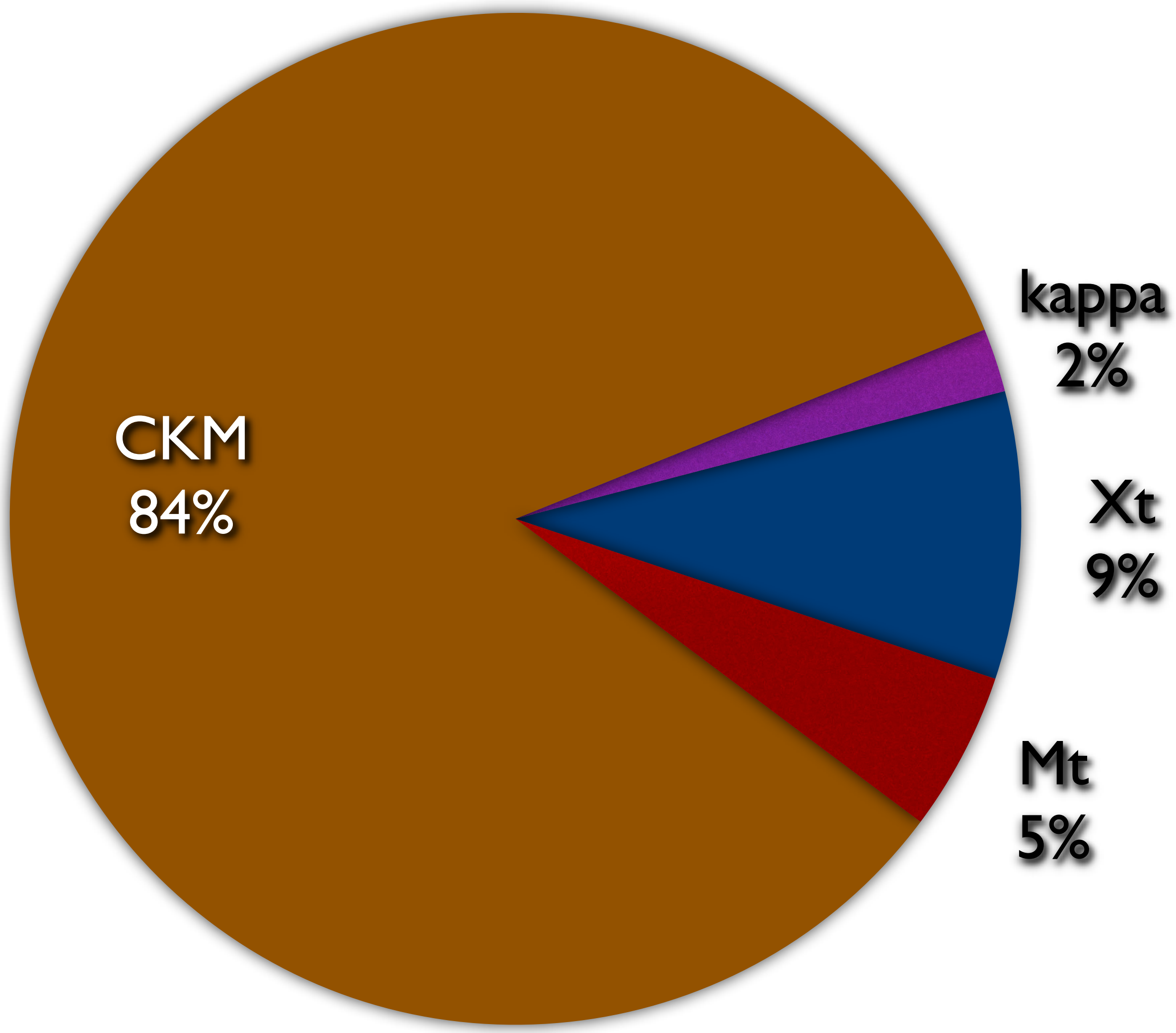
[E787, E949 '08]

$$\mathrm{Br} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

NA62 (CERN) aims at 10% measurement!

$K_L \to \pi^0 \nu\nu$ - Result and Error Budget

$$Br(K_L \to \pi^0 \nu\nu) = 2.43(39)(6) \times 10^{-11}$$



Experimental upper bound [E391a '08]

$Br < 6.7 \times 10^{-8}$

10% measurement at KOTO (JPARC) experiment?

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Conclusion

- FCNC Kaon decays provide information on high-energy scales (e.g. heavy fermions).
- ε_K receives +3% shift by NNLO QCD corrections to charm-top contribution.
- Rare K decays are exceptionally clean:
  Theory uncertainty
  < 4% (charged mode),
  < 2% (neutral mode).

Backup

Effective Field Theory

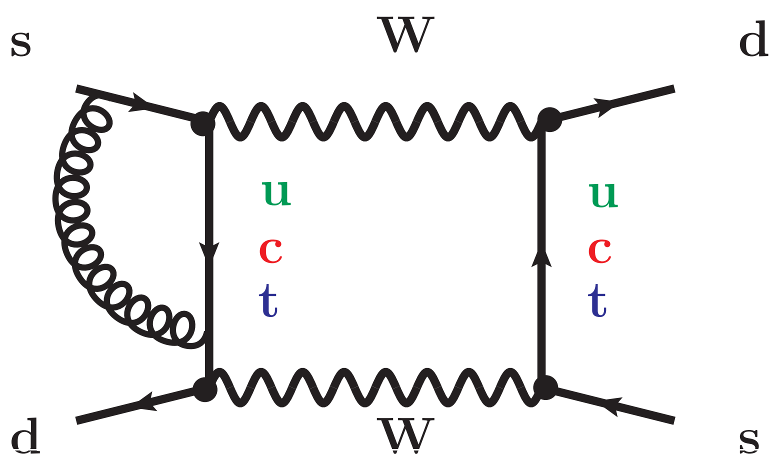
- Wilson coefficients
- Short-distance
- Perturbative

$$\langle H^{|\Delta S|=2} \rangle = C \times \langle \tilde{Q}^{|\Delta S|=2} + Q^{|\Delta S|=1} Q^{|\Delta S|=1} \rangle$$

- Hadronic matrix elements
- Long-distance
- Non-perturbative QCD - lattice, ChPT

η_{ct} at NNLO: Calculation

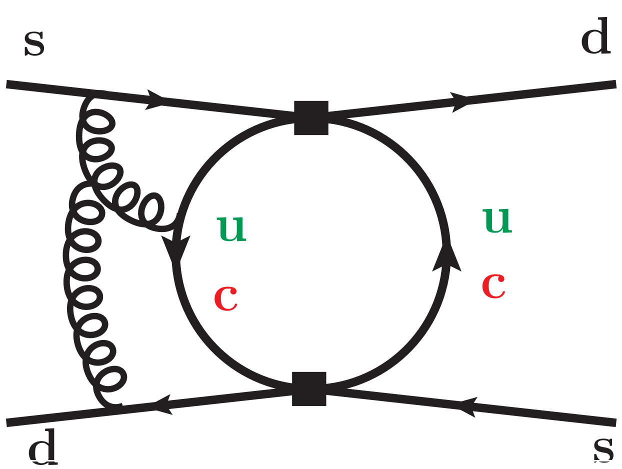
μ_{NP}



Matching at M_W: initial condition

- Here: SM
- NP can change initial conditions!

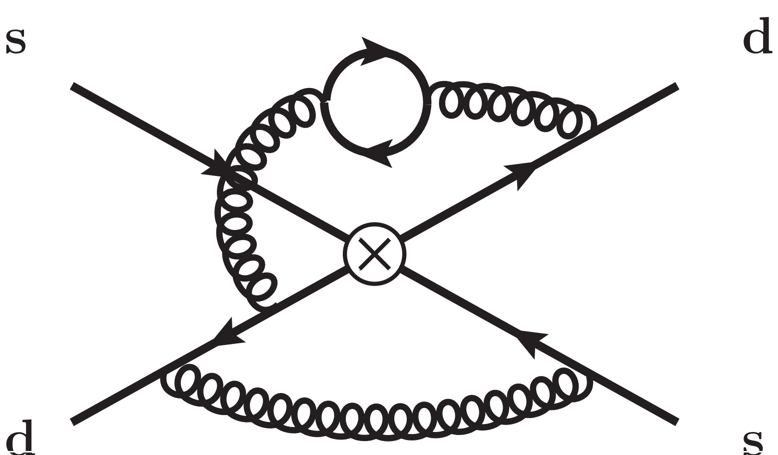
μ_{Wt}



Running to m_c

- O(100 000) diagrams
- RGE for double insertions
- Include threshold corrections at m_b

μ_b



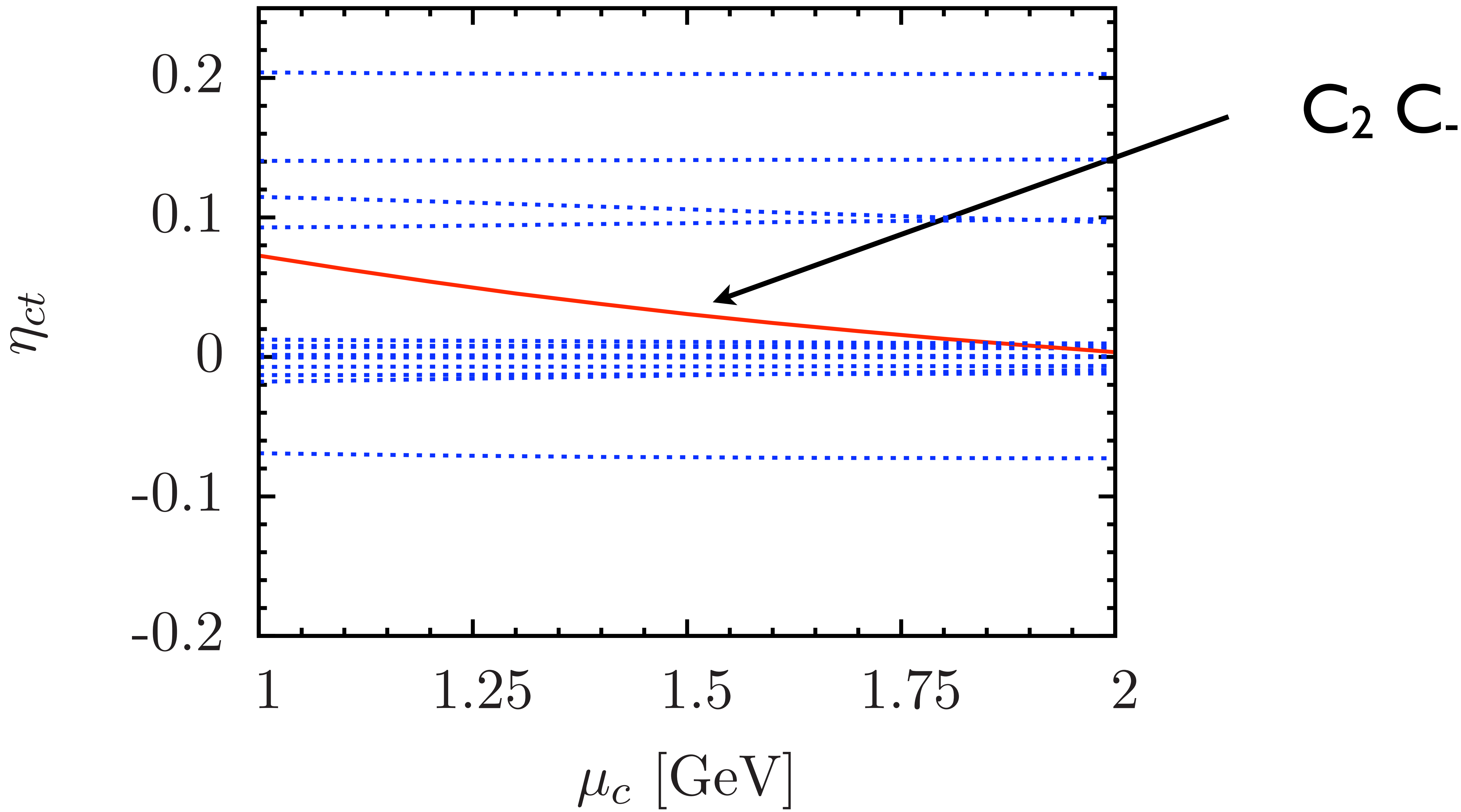
Matching at m_c

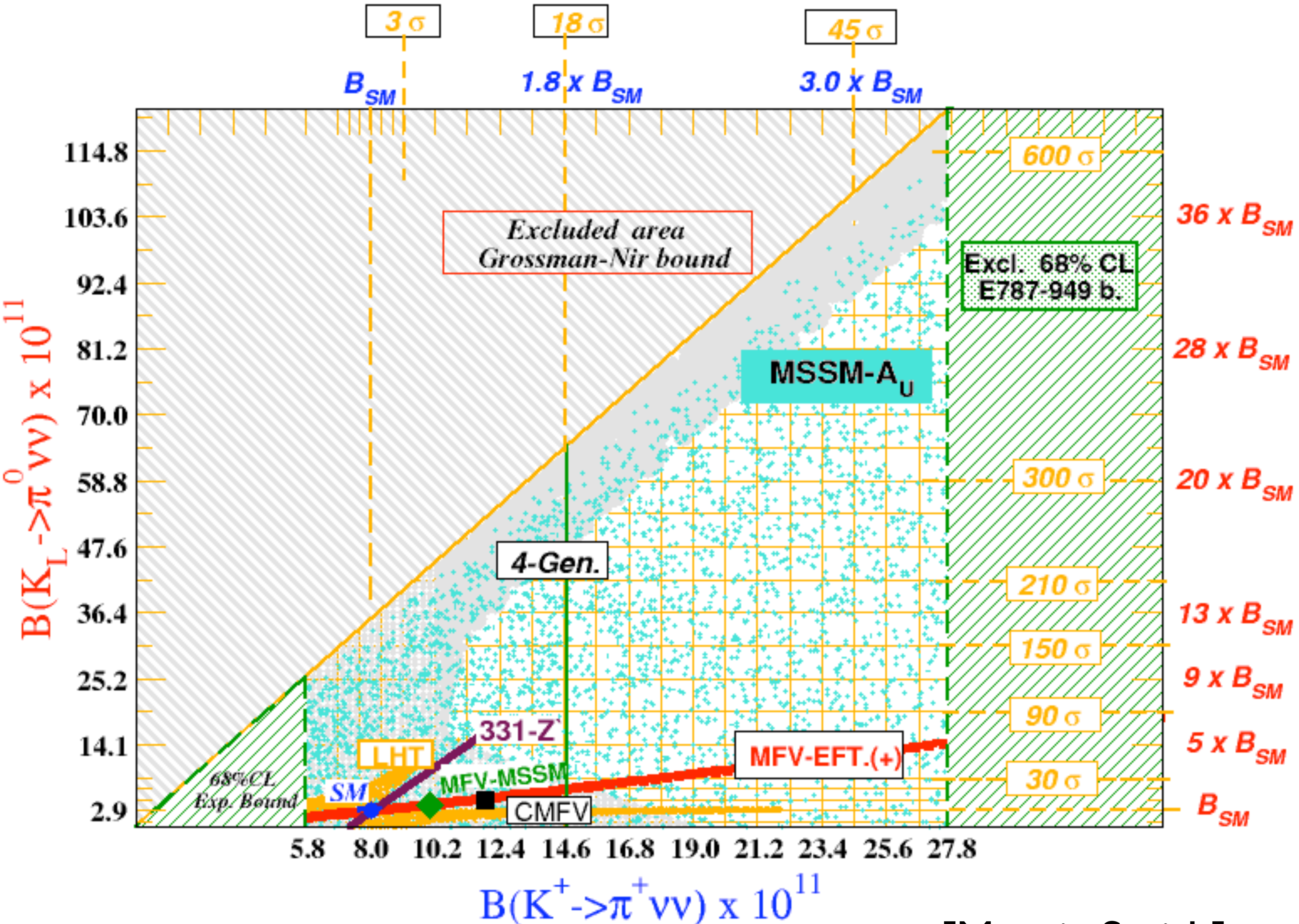
- Match to effective 3-flavour theory

μ_c

Λ_{QCD}

η_{ct} at NNLO: Scale Dependence





[Mescia,Smith]