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Chirally enhanced corrections in the MSSM

Outline:

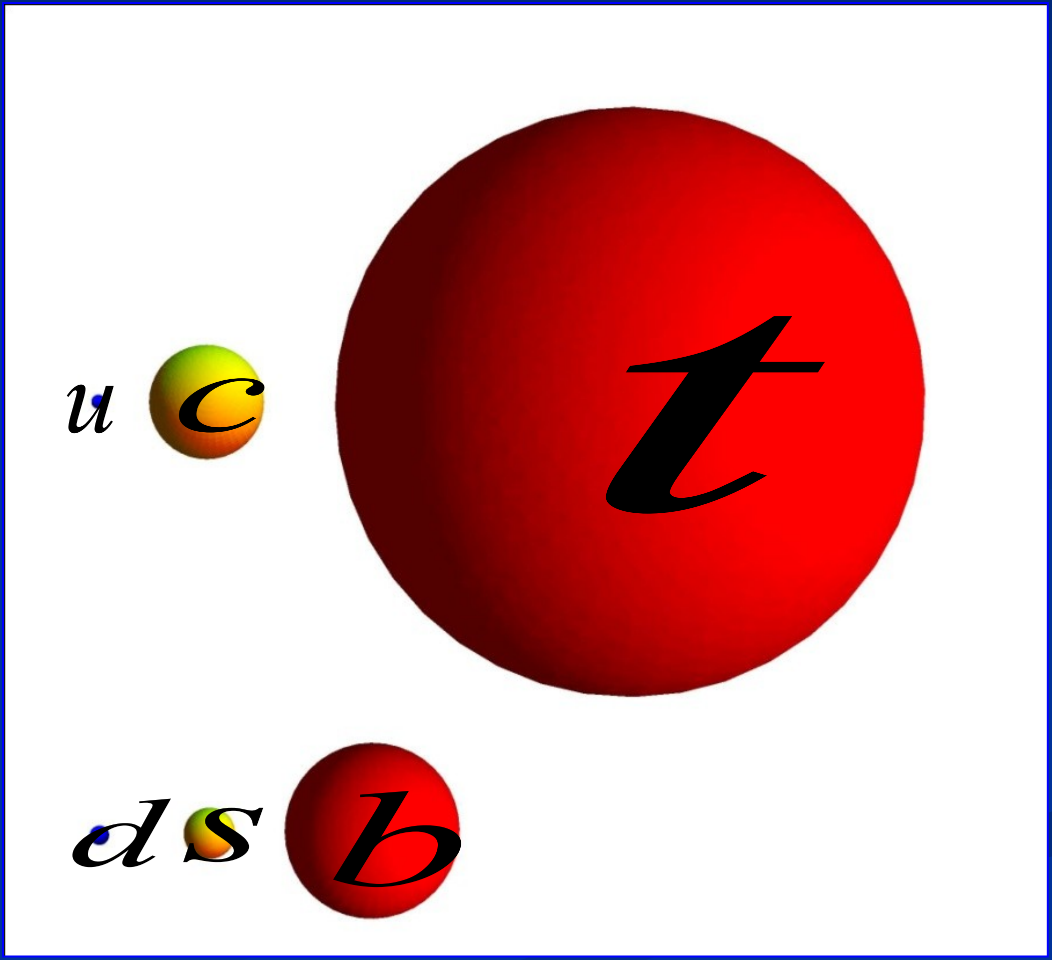
- Self-energies in the MSSM
- Resummation of chirally-enhanced corrections
- Effective higgsino and gaugino vertices
- Effective Higgs vertices
- Flavor-violation from SUSY

Introduction

Sources of flavor-violation in the MSSM

Quark masses

- Top quark is very heavy. $m_t \approx v$
- Bottom quark rather light, but Y^b can be big at large tan(β)
- All other quark masses are very small
  ➡ sensitive to radiative corrections



Squark mass matrix

$$M^2_{\tilde q} = \begin{pmatrix} M^{\tilde q}_{LL} & \Delta^{\tilde q LR} \\ \Delta^{\tilde q LR\dagger} & M^{\tilde q}_{RR} \end{pmatrix}$$

hermitian: ➡ $W^{\tilde q\dagger} M^2_{\tilde q} W^{\tilde q} = M^{2(D)}_{\tilde q}$

$M^{\tilde q}_{LL,RR}$ involves only bilinear terms (in the decoupling limit)

The chirality-changing elements are proportional to a vev

$$\Delta^{d\,LR}_{ij} = -v_d \left(\mu \tan(\beta) Y^{d(0)}_i \delta_{ij} + A^d_{ij}\right)$$

$$\Delta^{u\,LR}_{ij} = -v_u \left(\mu \cot(\beta) Y^{u(0)}_i \delta_{ij} + A^u_{ij}\right)$$

$$\tan(\beta) = \frac{v_u}{v_d}$$

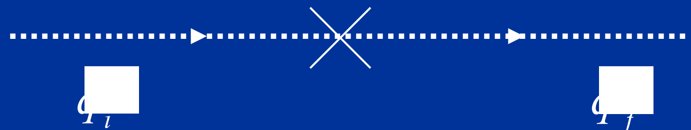
Mass insertion approximation

(L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B 267 (1986) 415.)

- Useful to visualize flavor-changes in the squark sector

  $\Delta_{ij}^{q\,AB}$ off-diagonal element of the squark mass matrix

- $q = u, d$
- i, j flavor indices 1,2,3
- A, B chiralitys L,R



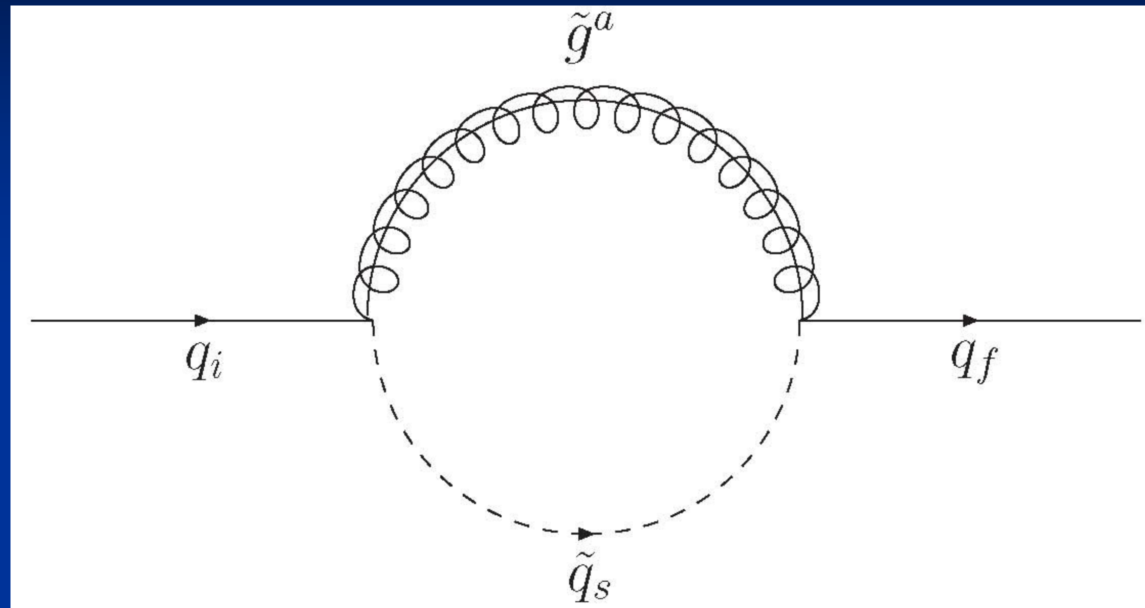
$$-i\Delta_{fi}^{q\,AB}$$

Self-energies

in the MSSM

SQCD self-energy:

$$-i\Sigma(0)_{fi}^{q\,LR} =$$



$$\Sigma_{fi}^{q\,LR} = \alpha_s \frac{2}{3\pi} m$$ [redacted]

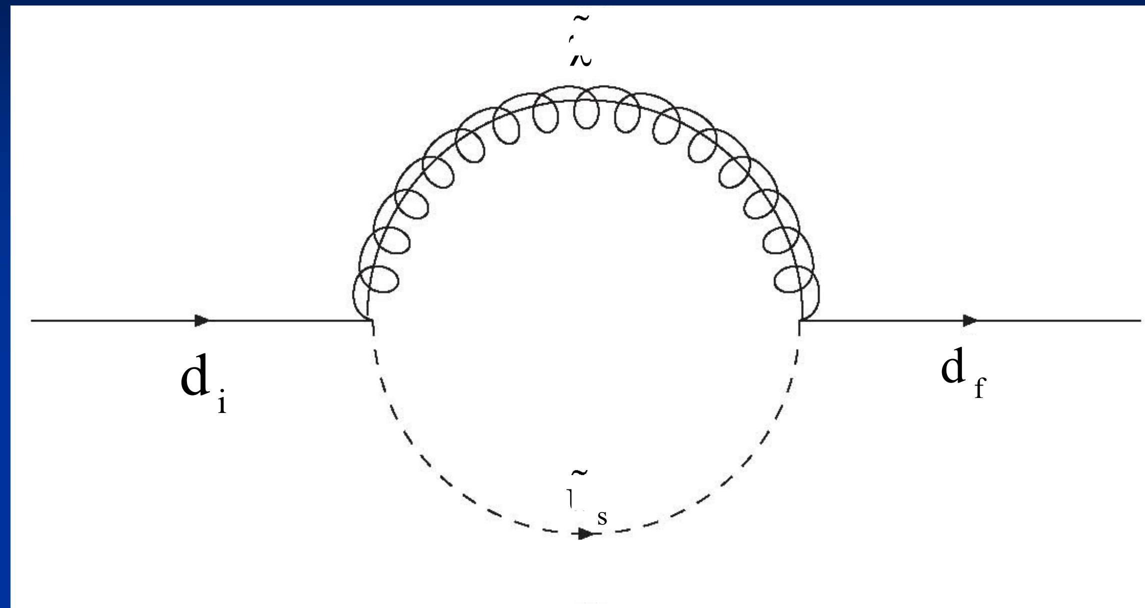
Finite and proportional to at least one power of $\Delta_{fi}^{q\,LR}$

$$\Sigma_{fi}^{q\,LR} = \alpha_s \frac{2}{3\pi} m$$ [redacted]

decoupling limit

Chargino self-energy:

$$-i\Sigma(0)_{d_f d_i} =$$

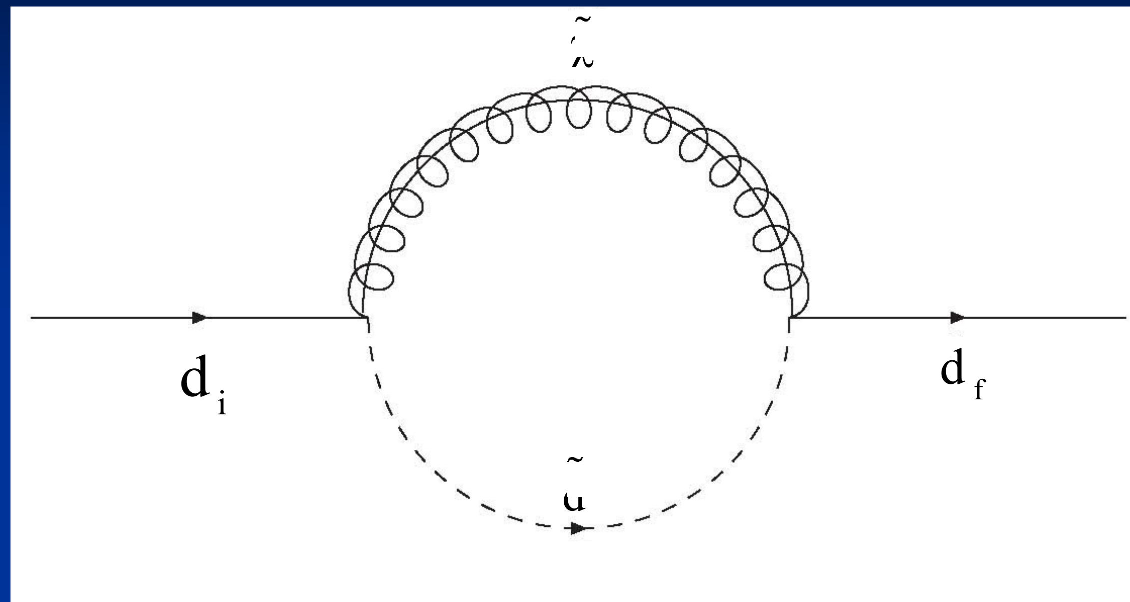


$$\Sigma_{d_f d_3} = \frac{-1}{16\pi^2}\mu Y^{d_3}\Bigg[V^{CKM(0)*}_{3f} Y^{u_3 *}\Delta^{u\,RL}_{33}\sum_{s,t=1}^{6} V^{u\,RR}_{s33} V^{d\,LL}_{t33} C_0\left(|\mu|^2, m^2_{s}, m^2_{t}\right)$$

$$-\sqrt{2}g_2\sin(\beta)M_W M_2\sum_{s=1}^{6} V^{d\,LL}_{sf3} C_0\left(m^2_{s}, |\mu|^2, |M_2|^2\right)\Bigg]$$

Neutralino self-energy:

$$-i\Sigma(0)^{d\,LR}_{fi\,\tilde{\chi}} =$$



$$\Sigma^{\tilde{\chi}}_{d_i d_j} = \frac{-1}{16\pi^2}\left\{\frac{2}{9} g_1^2 M_1 \Delta^{d\,LR}_{ij} C_0\left(\left|M_1^2\right|, m^2_{\tilde{d}_{iL}}, m^2_{\tilde{d}_{jR}}\right)\right.$$

$$+\sum_{s=1}^{3}\left[-\left(\frac{g_2^2}{2} v_u M_2 \mu C_0\left(\left|M_2^2\right|, \left|\mu^2\right|, m^2_{\tilde{d}_s}\right) \frac{1}{3}(1\to 2)\right) Y^{d_j} V^{d\,LL}_{s\,ij}\right.$$

$$\left.\left. + \frac{1}{3} g_1^2 v_u M_1 \mu C_0\left(\left|M_1^2\right|, \left|\mu^2\right|, m^2_{\tilde{d}_s}\right) Y^{d_i} V^{d\,RR}_{s\,ij}\right]\right\}$$

Decomposition of the self-energy
(flavour-conserving part)

$$\Sigma^{d\,LR}_{ii} = \Sigma^{d\,LR}_{ii\,Y} + \Sigma^{d\,LR}_{ii\,A}$$

into a holomorphic part proportional to an A-term

$$\Sigma^{d\,LR}_{fi\,A} = \Sigma^{d\text{[illegible]}}_{fi\,A} + \Sigma^{\text{[illegible]}}_{fi\,A}$$

non-holomorphic part proportional to a Yukawa

$$\Sigma^{d\,LR}_{fi} = \Sigma^{d\text{[illegible]}}_{fi\,Y} + \Sigma^{\text{[illegible]}}_{fi\,Y} + \Sigma^{\text{[illegible]}}_{fi}$$

Define dimensionless quantity $\varepsilon^{d}_{i} = \dfrac{\Sigma^{d\,LR}_{ii\,Y^{d_i}}}{v_u Y^{d_i}}$

which is independent of a Yukawa coupling

Decomposition of the self-energy
(flavor-changing part)

$$\Sigma^{d\,LR}_{fi} = \Sigma^{d\,LR}_{fi\,\cancel{CKM}} + \Sigma^{d\,LR}_{fi\,CKM} = \Sigma^{d\,LR}_{fi\,\cancel{CKM}} + V^{CKM*}_{3f}\,\varepsilon^{d}_{FC}$$

into a part independent of the CKM matrix

$$\Sigma^{d\,LR}_{fi\,\cancel{CKM}} = \Sigma^{d\,\text{[illegible]}}_{fi} + \Sigma^{\text{[illegible]}}_{fi} + \Sigma^{\text{[illegible]}}_{fi\,\cancel{CKM}}$$

part proportional to CKM element

$$\Sigma^{d\,LR}_{fi\,CKM} = \Sigma^{d\tilde{\chi}\,\text{[illegible]}}_{fi\,CKM}$$

Define dimensionless quantity $\varepsilon^{d}_{FC} = \dfrac{\Sigma^{d\tilde{\chi}\,\text{[illegible]}}_{3f\,Y}}{m_b\,V^{CKM*}_{3f}}$

Finite Renormalization

and resummation of chirally enhanced corrections

AC, Ulrich Nierste, arXiv:0810.1613
AC, Ulrich Nierste, arXiv:0908.4404
AC, arXiv:1012.4840
AC, L. Hofer and J. Rosiek, arXiv:1103.4272

Renormalization I

- All corrections are finite; counter-term not necessary.
- Minimal renormalization scheme is simplest.

Mass renormalization

$$m_{d_i} = v_d Y^{d_i(0)} + \Sigma^{d\,LR}_{ii}$$

$$= v_d Y^{d_i(0)} + \Sigma^{q\,LR}_{ii\,A} + v_d \tan(\beta) Y^{d_i(0)} \varepsilon_{d_i}$$

$$\Rightarrow \quad Y^{d_i(0)} = \frac{m_{d_i} - \Sigma^{q\,LR}_{ii\,A}}{v_d \left(1 + \tan(\beta)\varepsilon^d_i\right)}$$

- $\tan(\beta)$ is automatically resummed to all orders

Renormalization II

- Flavour-changing corrections

important two-loop corrections
A.C. Jennifer Girrbach 2010

$$U^{qL} = \begin{pmatrix} 1 - \frac{\left|\Sigma^{q\,LR}_{12}\right|^2}{2 m_{q_2}^2} & \frac{1}{m_{q_2}} \Sigma^{q\,LR}_{12} & \frac{1}{m_{q_3}} \Sigma^{q\,LR}_{13} \\ \frac{-1}{m_{q_2}} \Sigma^{q\,RL}_{21} & 1 - \frac{\left|\Sigma^{q\,LR}_{12}\right|^2}{2 m_{q_2}^2} & \frac{1}{m_{q_3}} \Sigma^{q\,LR}_{23} \\ \frac{-1}{m_{q_3}} \Sigma^{q\,RL}_{31} + \frac{\Sigma^{q\,RL}_{32} \Sigma^{q\,RL}_{21}}{m_{q_2} m_{q_3}} & \frac{-1}{m_{q_3}} \Sigma^{q\,RL}_{32} & 1 \end{pmatrix}$$

Renormalization III

- Renormalization of the CKM matrix:

$$V^{(0)} = U^{u\,L} V \; U^{d\,L\dagger}$$

- Decomposition of the rotation matrices

$$U^{q\,L} = U^{q\,L}_{\cancel{CKM}} U^{q\,L}_{CKM}$$

- Corrections independent of the CKM matrix

$$V \quad = U_{\cancel{CKM}} V^{(0)} U^{d\,L}_{\cancel{CKM}}$$

- CKM dependent corrections

$$U^{u\,L\dagger}_{CKM} V \, U_{CKM}$$

$$\Rightarrow \quad V^{(0)}_{13,23} = \frac{\tilde{V}_{13,23}}{1+\varepsilon_{FC}}$$

Effective gaugino and higgsino vertices

- No enhanced genuine vertex corrections.

- Calculate $\varepsilon_{d_i}, \varepsilon^{d}_{FC}, \Sigma^{q\,LR}_{ii\,\cancel{Y_i}}, \Sigma^{q\,LR}_{ii\,\cancel{CKM}}$
- Determine the bare Yukawas and bare CKM matrix
- Insert the bare quantities for the vertices.
- Apply rotations $U^{q\,L,R}_{fi}$ to the external quark fields.
- Similar procedure for leptons (up-quarks)

Chiral enhancement

$$\Sigma^{d\,LR}_{fi} \approx \frac{1}{100}\frac{\Delta^{q\,LR}_{fi}}{M_{SUSY}} = \frac{-v_d}{100}\left(\tan(\beta)\, Y^{d(0)}_i \delta_{ij} + \frac{A^{d}_{ij}}{M_{SUSY}}\right)$$

- For the bottom quark only the term proportional to tan(β) is important.

  ➡ tan(β) enhancement

  Blazek, Raby, Pokorski, hep-ph/9504364

$$\Sigma^{d\,LR}_{33\,Y} = \frac{-1}{100} v_d \tan(\beta)\, Y^{b(0)} \;\ldots_{b}$$

$$O\left(\frac{\tan(\beta)}{100}\right)$$

- For the light quarks also the part proportional to the A-term is relevant.

$$\Sigma^{d\,LR}_{22\,A} = O(1) \;\ldots_{22} \;\ldots_{SUSY}$$

$$\Sigma^{d\,LR}_{11\,A} = O(1) \;\ldots_{11}\; \frac{1}{25} M_{SUSY}$$

Flavor-changing corrections

$$\frac{\Sigma^{q\,LR}_{fi}}{m_{q_{max(f,i)}}}$$

$$V^{CKM}_{cb}: \quad A^{q}_{23} \approx M_{SUSY}$$

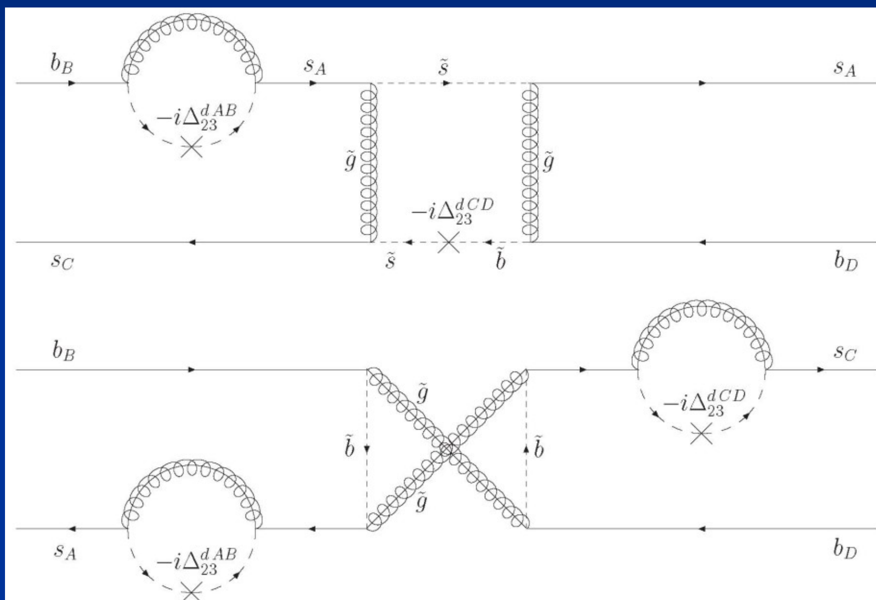
$$V^{CKM}_{ub}: \quad A^{q}_{13} \approx M_{SUSY} \times 10^{-1}$$

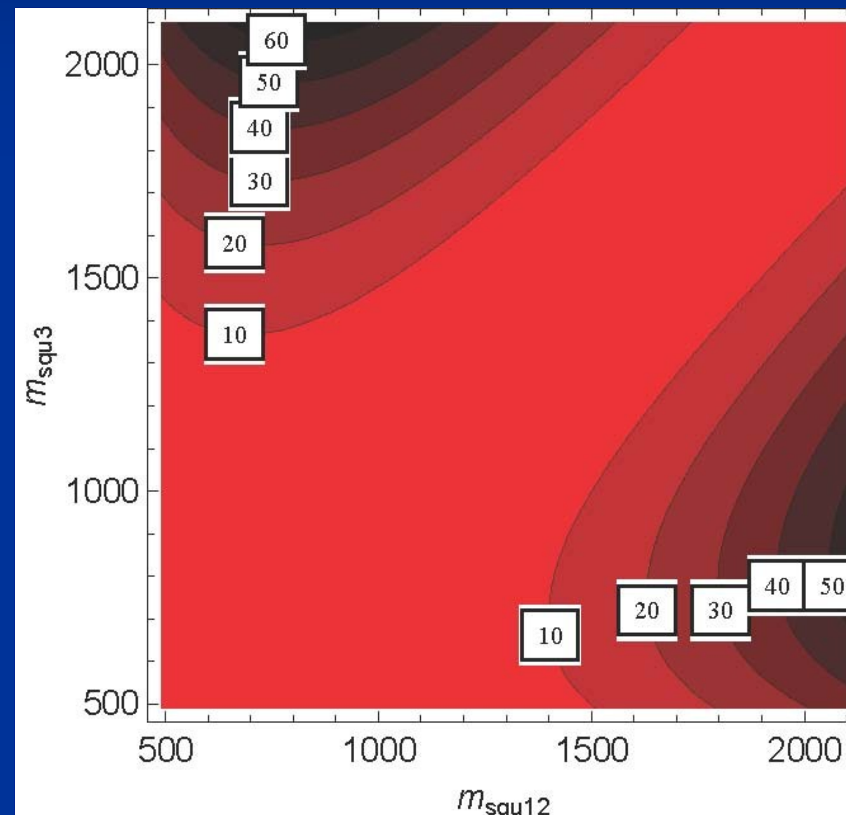
$$V^{CKM}_{us}: \quad A^{q}_{12} \approx M_{SUSY} \times 10^{-1}$$

- Flavor-changing A-term can easily lead to order one correction.

A.C., Ulrich Nierste, hep-ph/08101613

Effect of including the self-energies in ΔF=2 processes





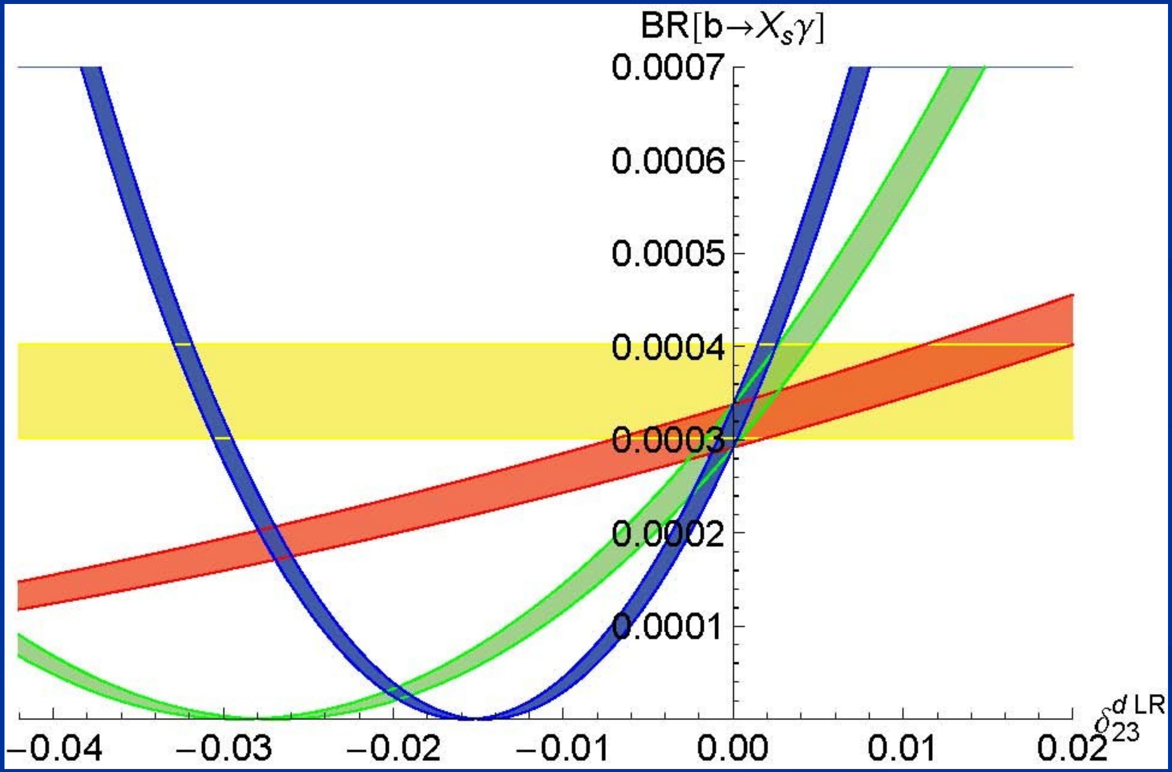
$$\frac{\Delta M_{B\,ren}}{\Delta M_B} \quad \text{for} \quad m_{\tilde{g}} = 1000\,GeV$$

AC, Ulrich Nierste, arXiv:0908.4404

b→sγ

Two-loop effects enter only if also $m_b\mu\cdot\tan(\beta)$ is large.

Behavior of the branching ratio for $\delta_{23}^{d\,LR}$

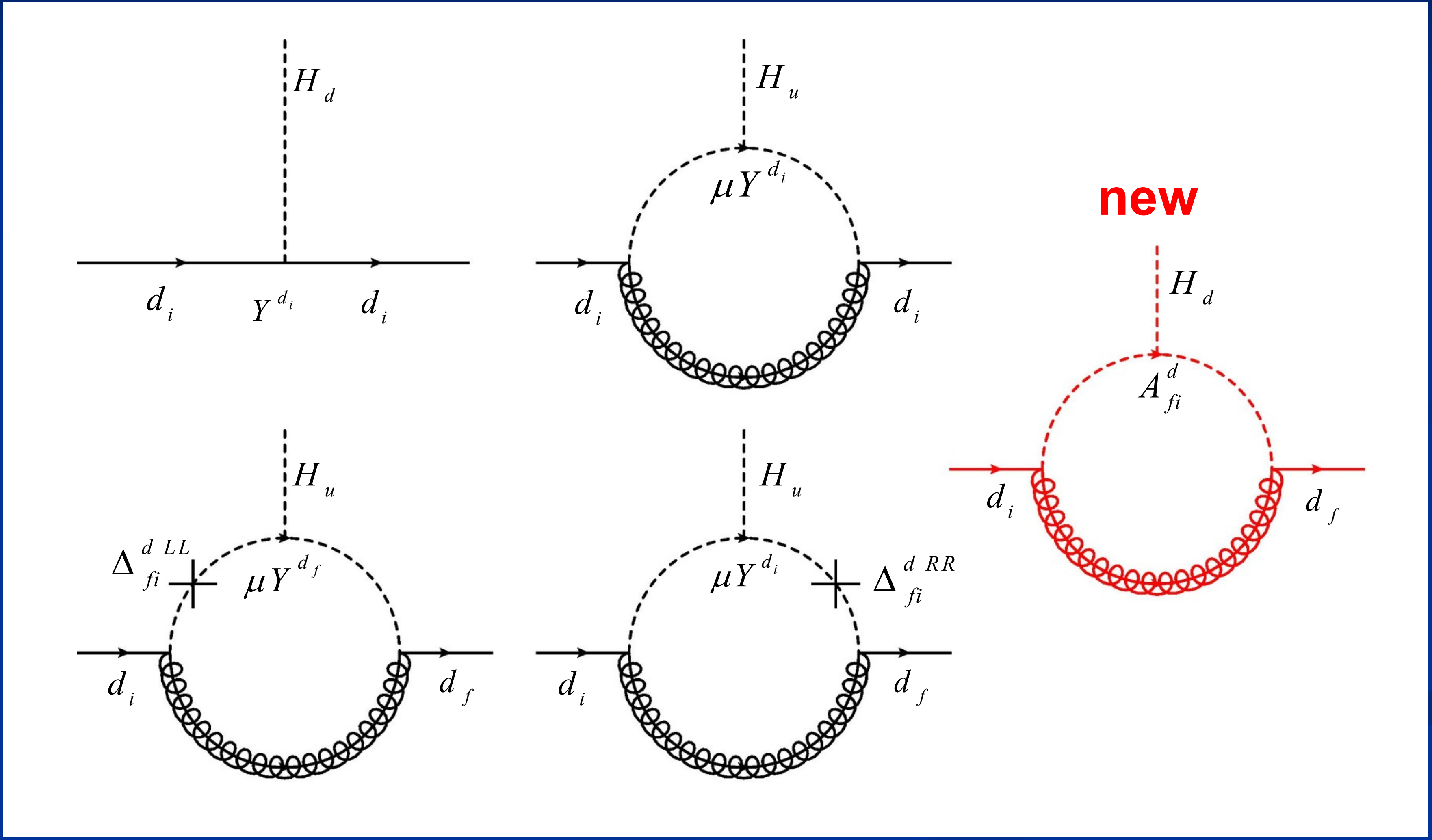


- experimentally allowed range
- $m_b\mu\ \tan(\beta) = 0\,\mathrm{TeV}$
- $m_b\mu\ \tan(\beta) = 30\,\mathrm{TeV}$
- $m_b\mu\ \tan(\beta) = -30\,\mathrm{TeV}$

Effective Higgs vertices

AC, arXiv:1012.4840
AC, L. Hofer and J. Rosiek, arXiv:1103.4272

Higgs vertices in the EFT I



Higgs vertices in the EFT II

$$L_Y^{eff} = \bar{Q}^a_{f\,L}\left(\left(Y_i^d \delta_{fi} + E^d_{fi}\right)\varepsilon_{ba} H^b_d + E'^d_{fi} H^{a*}_u\right) d_{i\,R}$$

- Non-holomorphic corrections $E'^d_{fi} = \dfrac{\Sigma^{d\,LR}_{fi\,Y}}{v_u}$
- Holomorphic corrections $E^d_{fi} = \dfrac{\Sigma^{d\,LR}_{fi\,A}}{v_d}$
- The quark mass matrix $m^d_{fi} = v_d\left(Y^{d_i}\delta_{fi} + E^d_{fi}\right) + v_u E'^d_{fi}$ is no longer diagonal in the same basis as the Yukawa coupling

⇒ Flavor-changing neutral Higgs couplings

Effective Yukawa couplings

- Final result:

$$\Sigma^{d\,LR}_{fi\,Y} = \Sigma^{d\,\text{[illegible]}}_{fi\,Y} + \Sigma^{\text{[illegible]}}_{fi} + \Sigma^{\text{[illegible]}}_{fi\,Y}$$

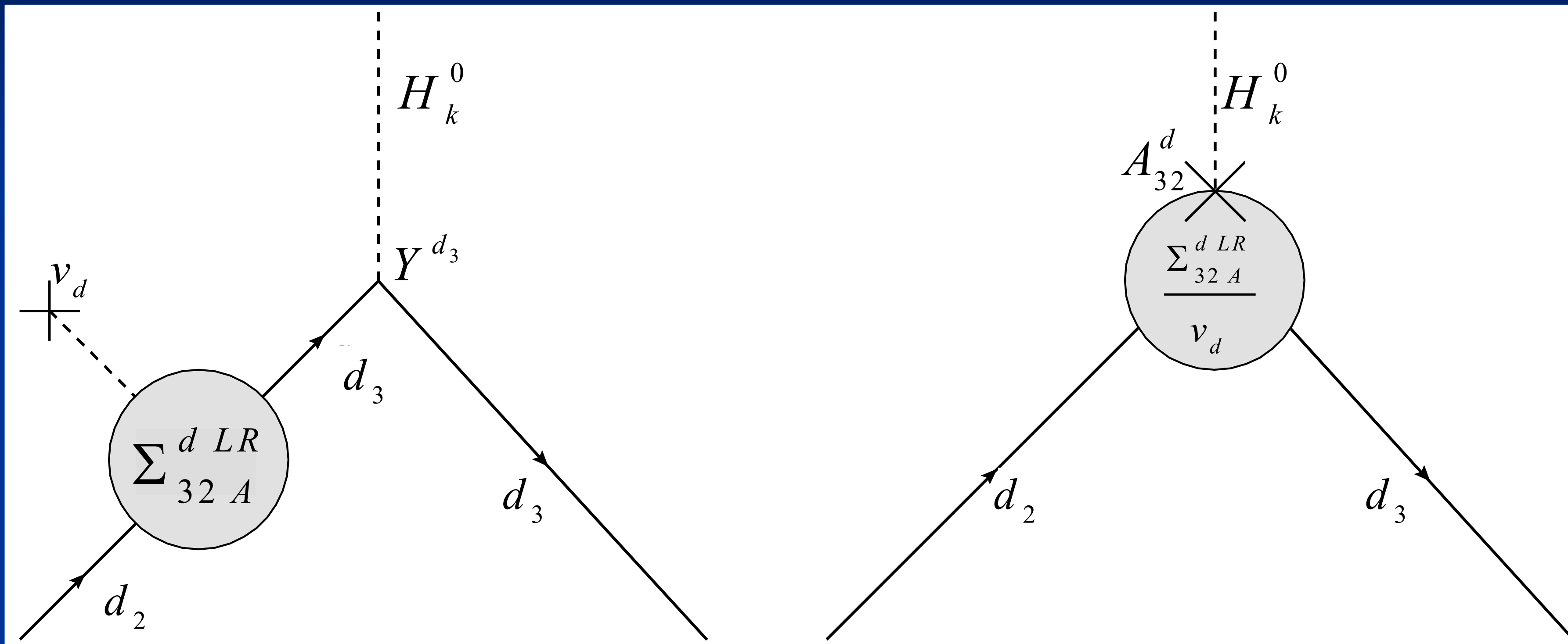
$$Y^{d\,\mathrm{eff}}_{ij} = \frac{1}{v_d}\left(m_{d_i}\,\delta_{ij} - \tilde{\Sigma}^{\text{[illegible]}}_{ij\,Y} \right)$$

with

$$\text{[illegible]}_{jk\,Y} \text{[illegible]}^{L*}_{jf}\,\Sigma^{d\,LR}_{jk\,Y}\,U^{d\,R}_{ki} \approx \Sigma^{d\,LR}_{fi\,Y} - \begin{pmatrix} 0 & \dfrac{\Sigma^{d\,LR}_{22\,Y}}{m_{d_2}}\Sigma^{d\,LR}_{12} & \dfrac{\Sigma^{d\,LR}_{33\,Y}}{m_{d_3}}\Sigma^{d\,LR}_{13} \\ \dfrac{\Sigma^{d\,LR}_{22\,Y}}{m_{d_2}}\Sigma^{d\,LR}_{21} & 0 & \dfrac{\Sigma^{d\,LR}_{33\,Y}}{m_{q_3}}\Sigma^{d\,LR}_{23} \\ \dfrac{\Sigma^{d\,LR}_{33\,Y}}{m_{d_3}}\Sigma^{d\,LR}_{31} & \dfrac{\Sigma^{d\,LR}_{33\,Y}}{m_{q_3}}\Sigma^{d\,LR}_{32} & 0 \end{pmatrix}$$

Diagrammatic explanation in the full theory:

Higgs vertices in the full theory



- Cancellation incomplete since $v_d Y^{d_3} \neq m_{d_3}$
  Part proportional to $\Sigma_{33\,Y}^{d\,LR}$ is left over.

➡ A-terms generate flavor-changing Higgs couplings

Radiative generation of light quark masses and mixing angels

AC, Ulrich Nierste, arXiv:0908.4404
AC, Jennifer Girrbach, Ulrich Nierste, arXiv:1010.4485
AC, Ulrich Nierste, Lars Hofer, Dominik Scherer arXiv:1104.XXXX

Radiative flavor-violation

SU(2)³ flavor-symmetry in the MSSM superpotential:

- CKM matrix is the unit matrix.
- Only the third generation Yukawa coupling is different from zero.

$$V^{(0)}_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Y^{q} = \frac{1}{v_q} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_{q_3} \end{pmatrix}$$

All other elements are generated radiatively using the trilinear A-terms!

Features of the model

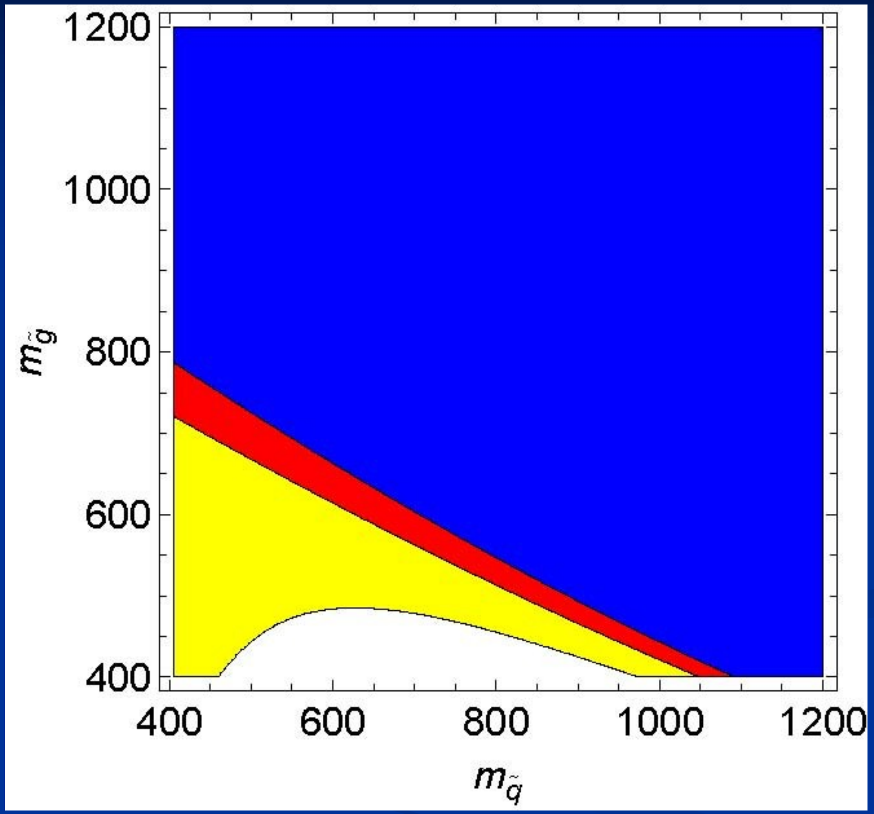
- Additional flavor symmetries in the superpotential.
- Explains small masses and mixing angles via a loop-suppression.
- Deviations from MFV if the third generation is involved.
- Solves the SUSY CP problem via a mandatory phase alignment. (Phase of μ enters only at two loops)
  Borzumati, Farrar, Polonsky, Thomas 1999.
- The SUSY flavor problem reduces to the elements $\delta_{32}^{q\,LR}, \delta_{31}^{q\,LR}$
- Can explain the B_s mixing phase

CKM generation in the down-sector

$$\Sigma^{d\,LR}_{13} \overset{!}{=} m_b V_{ub}$$

$$\Sigma^{d\,LR}_{23} \overset{!}{=} m_b V_{cb}$$

- Constraints from $b \to s\gamma$. Chirally enhanced corrections important
- $\delta^{d\,LR}_{31}, \delta^{d\,LR}_{32}$ less constrained since they contribute to C7‘,C8‘.
- $\delta^{d\,LR}_{32}$ can explain the CP phase in B_s mixing. (not possible in MFV)



$m_b \mu \tan(\beta) = 0.12 TeV^2$

$m_b \mu \tan(\beta) = 0 TeV^2$

$m_b \mu \tan(\beta) = 0.12 TeV^2$

Higgs effects: $B_s \rightarrow \mu\mu$

- Constructive contribution due to

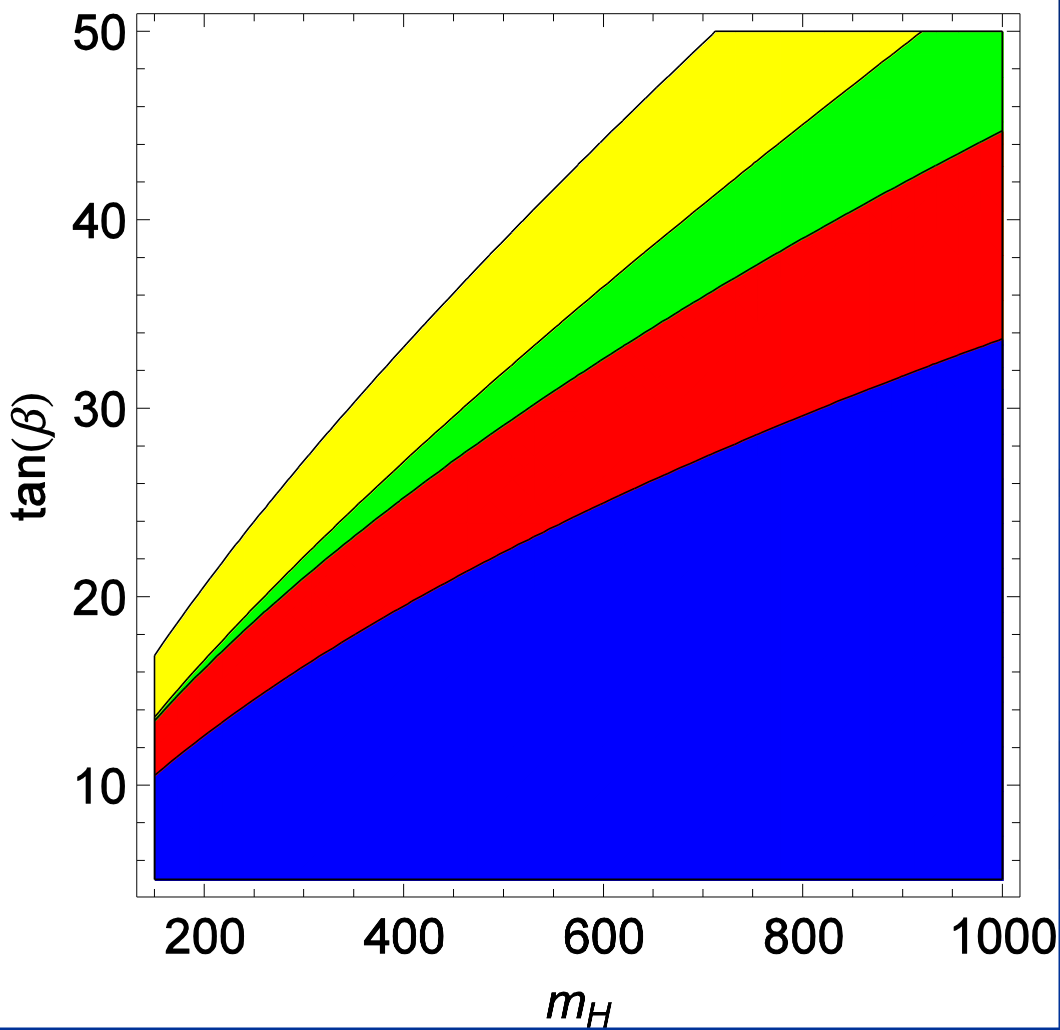
$$\Sigma_{23}^{d\ LR} = m_b V_{cb}$$

$\varepsilon_b = 0.005$

$\varepsilon_b = 0.01$

$\varepsilon_b = -0.005$

$\varepsilon_b = -0.01$

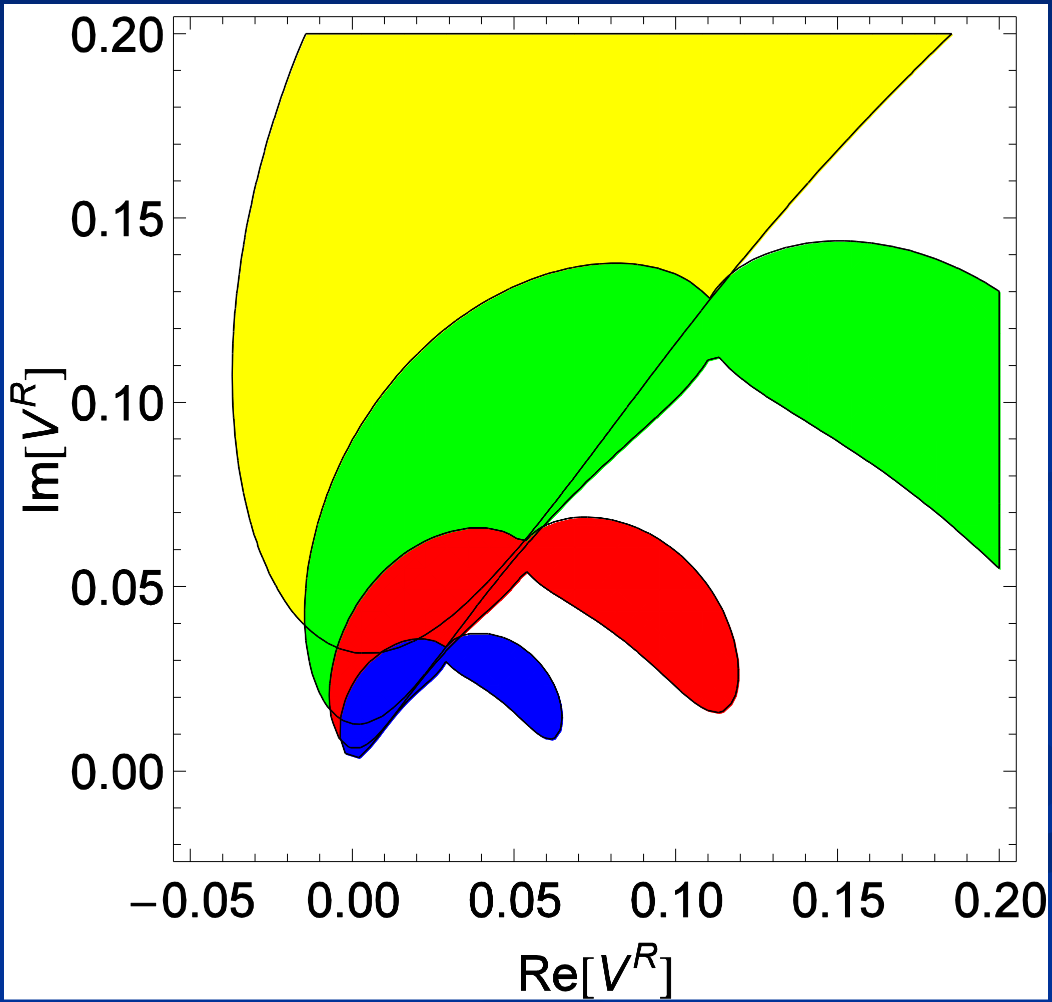


Higgs effects: B_s mixing

- Contribution only if

$$V_{23}^{R} = \frac{\Sigma_{23}^{d\ RL}}{m_b} \neq 0$$

due to Peccei-Quinn symmetry



$\tan(\beta) = 11$

$\tan(\beta) = 14$

$\tan(\beta) = 17$

$\tan(\beta) = 20$

Correlations between B_s mixing and $B_s \to \mu\mu$

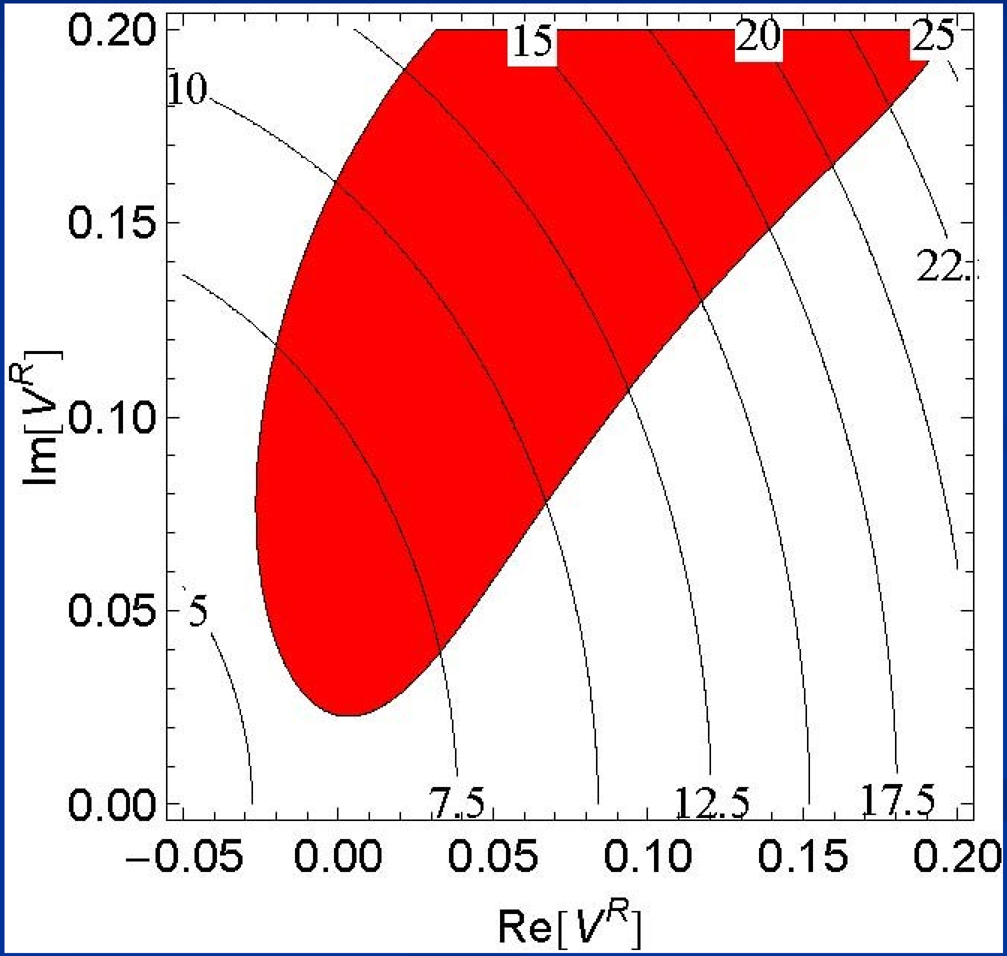
$$Br\left[B_s \to \mu^+\mu^-\right]\times 10^9$$

Allowed region from B_s mixing

$$\tan(\beta) = 12$$

$$M_H = 400\,GeV$$

$$V_{23}^R = \frac{\Sigma_{23}^{d\,RL}}{m_b}$$



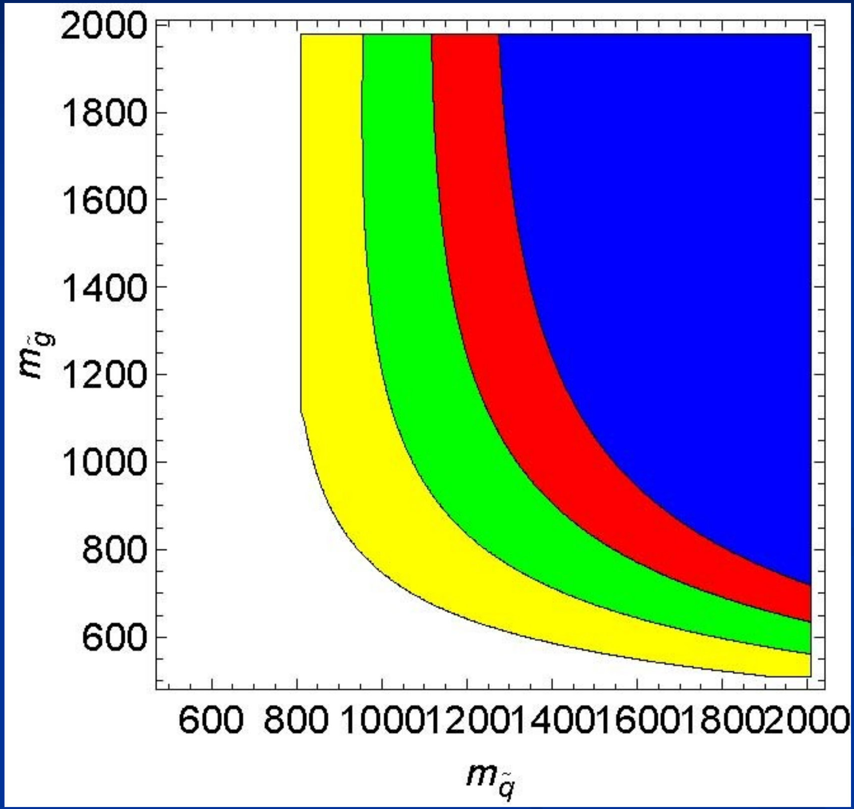
CKM generation in the up-sector:

$$\Sigma_{13}^{u\,LR} \overset{!}{=} m_t V_{td}^{*}$$

$$\Sigma_{23}^{u\,LR} \overset{!}{=} m_t V_{cb}^{*}$$

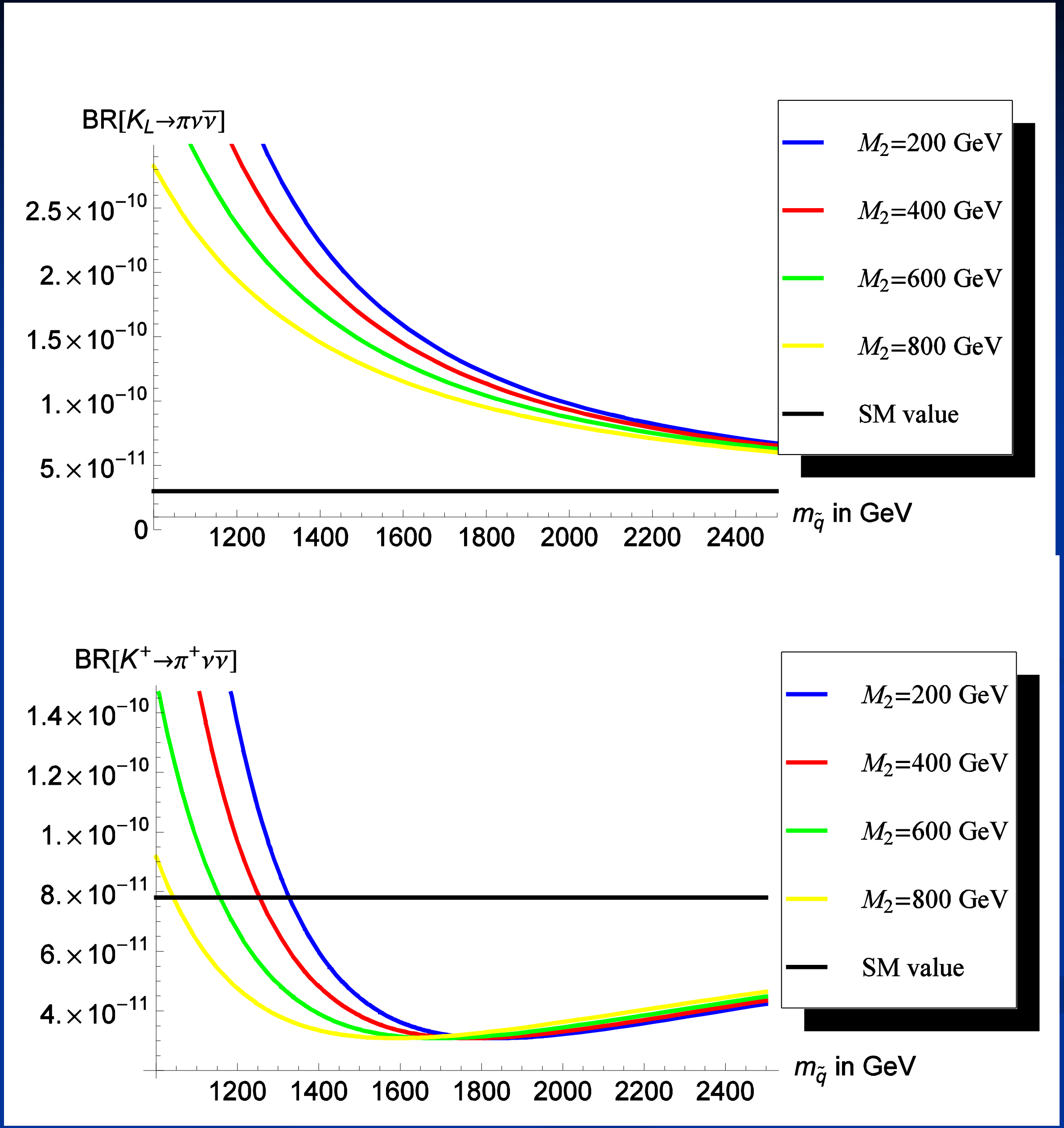
- Constraints from Kaon mixing.
- $\delta_{31}^{u\,LR}, \delta_{32}^{u\,LR}$ unconstrained from FCNC processes.
- $\delta_{31}^{u\,LR}$ can induce a sizable right-handed W coupling.

A.C. 2009



$M_2 = 200\text{GeV}$ (blue), $M_2 = 400\text{GeV}$ (red), $M_2 = 400\text{GeV}$ (green), $M_2 = 800\text{GeV}$ (yellow)

- Effects in K→πνν
- Verifiable predictions for NA62



Conclusions

- Self-energies in the MSSM can be of order one.
- Chirally enhanced corrections must be taken into account in FCNC processes.
- A-terms generate flavor-changing neutral Higgs couplings.
- Radiative generations of light fermion masses and mixing angles solves the SUSY flavor and the SUSY CP problem. It can explain B_s mixing.