Simulation and optimization of helical bundled meso-scale tube heat exchanger

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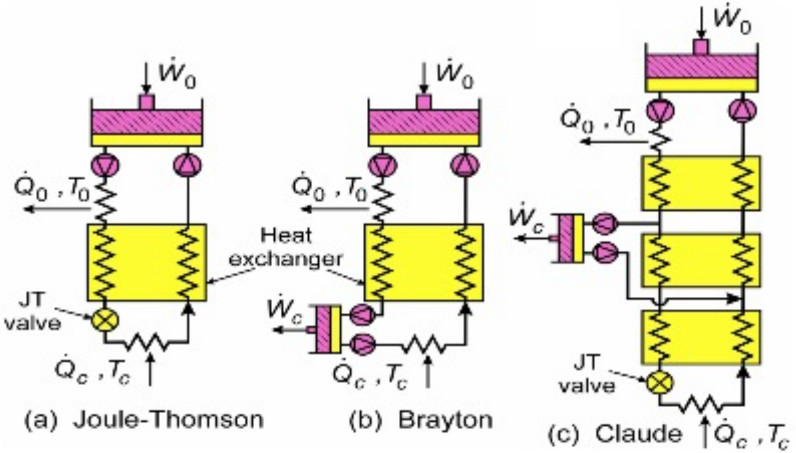
Content

- Recuperative cryocoolers are used for cooling infrared sensors, superconducting electronics, etc.



Schematics of the three most common recuperative cycles (Radebaugh)

- The recuperator strongly influences the overall system performance.
- Small size, high effectiveness, low pressure drop → Compact, uniform, continuous → Helical tubes

Recuperator of cryogenic systems

Working fluid:	Helium
Temperature span:	300K to 30K
Operating pressure:	320 psig (supply side), 100 psig (return side)
Pressure drop:	<0.5 bar for both sides (best), <1.0 bar (acceptable)
Flow rate:	13g/s
Materials:	Stainless steel or copper
Mass:	<60 lbs (best), <80 lbs (acceptable)
Approximate size:	<0.7m height, width/length<0.3m
Effectiveness:	>0.99 (best), >0.985 (acceptable)

Research objective

Objective: Design a helical bundled meso-scale tube heat exchanger and build a whole heat exchanger model to obtain its effectiveness and pressure drop.

Research method

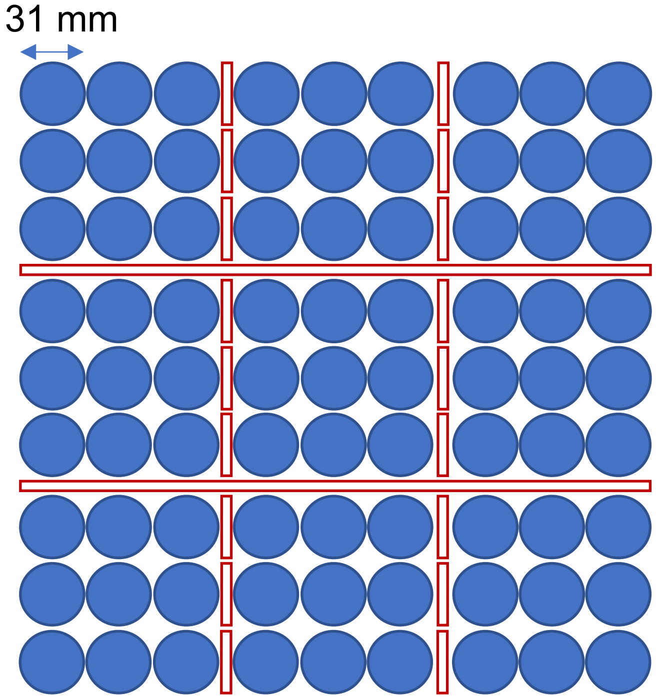
1. Use Fluent to simulate the flow of a short part of the heat exchanger, to get the geometry-based Nu and f correlations for both the shell and tube side.
2. Optimize the design by quantifying the effect of various geometry parameters.
3. Build the whole heat exchanger model using MATLAB.

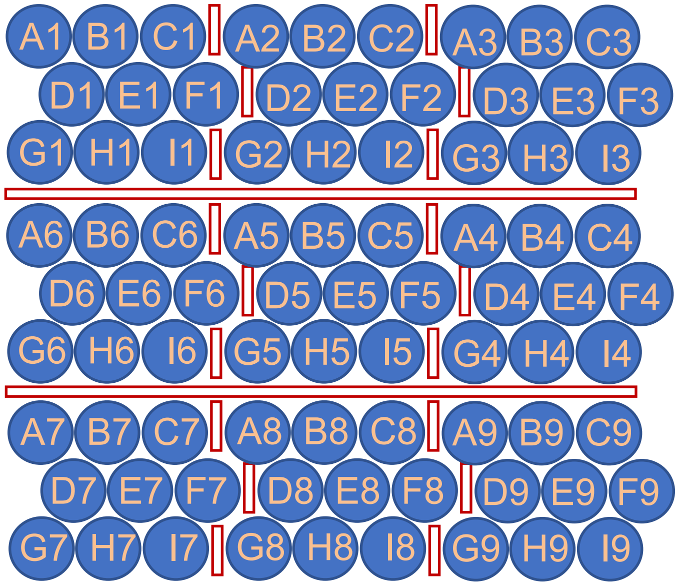
Overall heat exchanger design

9 groups in parallel, and each group will be put in its own shell.

Overall heat exchanger design

Using hexagonal packing to reduce the heat exchanger size





279 mm without considering the thickness of the insulation layer.

254.1 mm without considering the thickness of the insulation layer.

Geometry

$$X_{1,t} = a \cdot \cos\left(\frac{2\pi t}{p}\right) + b \cdot \cos\left(\frac{2n\pi t}{p} + c \cdot \frac{\pi}{d}\right)$$

$$Y_{1,t} = a \cdot \sin\left(\frac{2\pi t}{p}\right) + b \cdot \sin\left(\frac{2n\pi t}{p} + c \cdot \frac{\pi}{d}\right)$$

$$Z_{1,t} = t$$

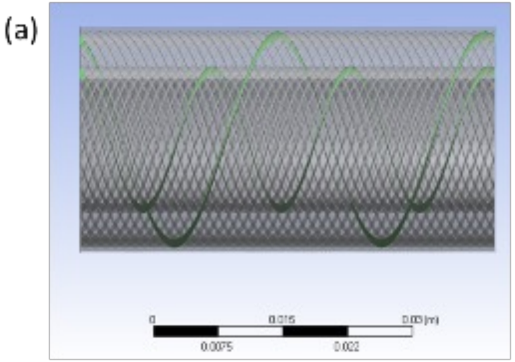
a, b: coil diameter

p: pitch length

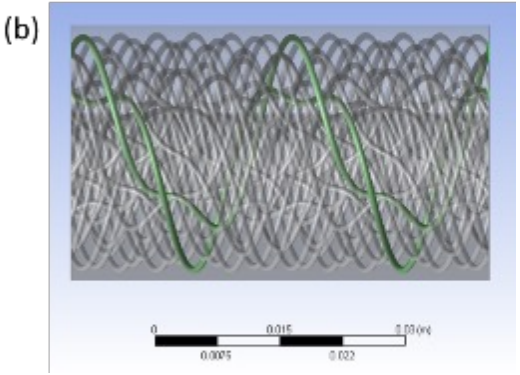
c: related to the initial phase

n: integer larger than 1

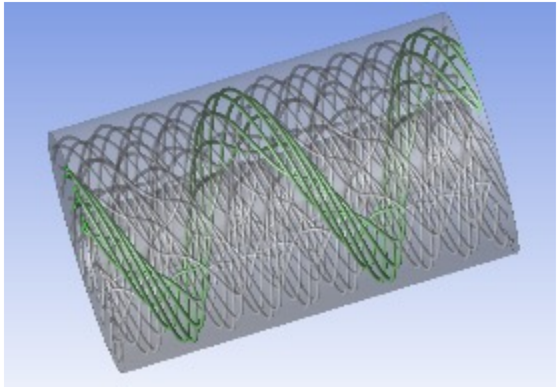
d: number of tubes in one bundle



Spiral geometry

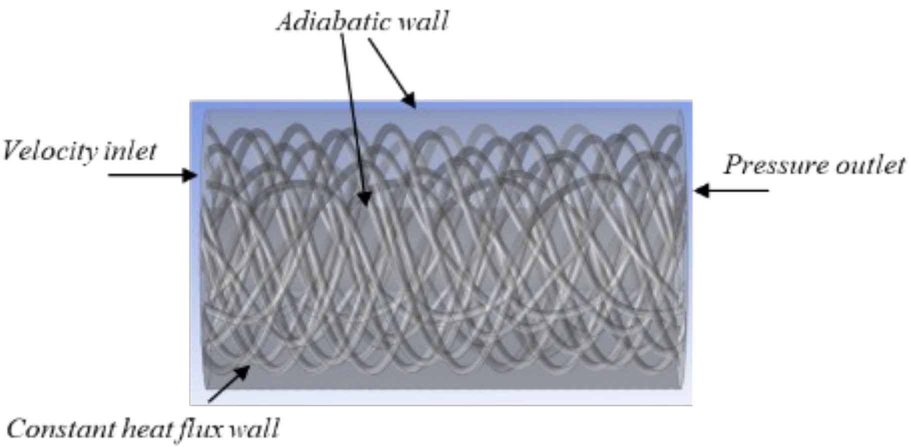


Twisted geometry



1. The curve is generated by different 3D sinusoidal equations, with the **same end-to-end length and coil diameter**.
2. The curve spiral inward and outward in the radial direction in one cycle.

Shell side boundary conditions



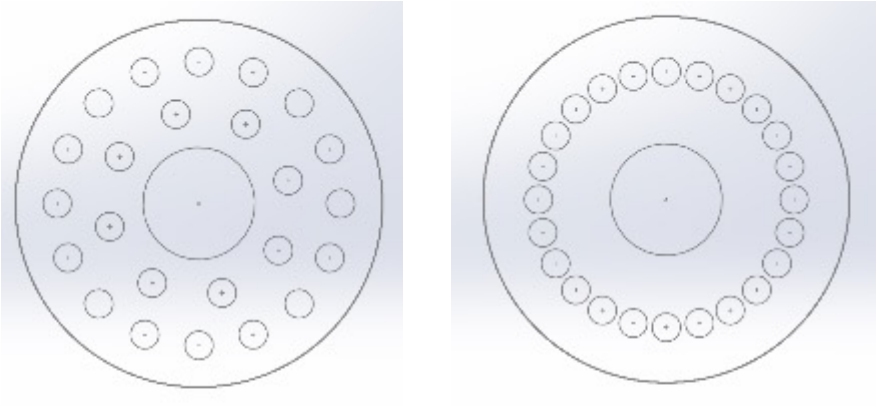
$$D_h = \frac{4V}{A_{exchange}} \qquad h = \frac{q}{\overline{T_{wall}} - \overline{T_{bulk}}}$$

1. SST K-ω accounts for the transport effects of the turbulent shear stress.
2. Predicting the onset and amount of flow separation accurately.

Shear stress transport k-omega model

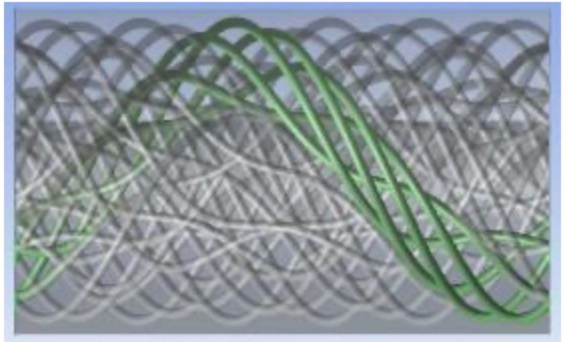
Tube diameter (mm)	0.6, 0.8, 1
Pitch length (mm)	25, 40, 50
Tube number and distribution	36 (6×6, 4×9), 30 (6×5, 6×5) 27 (3×9), 25 (5×5), 20 (2×10)

Limitation of dimensionless pitch length and hydraulic diameter

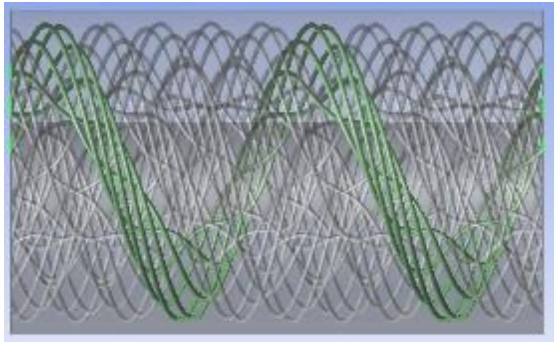


Two geometries with the same D_h

Conduction shape factor



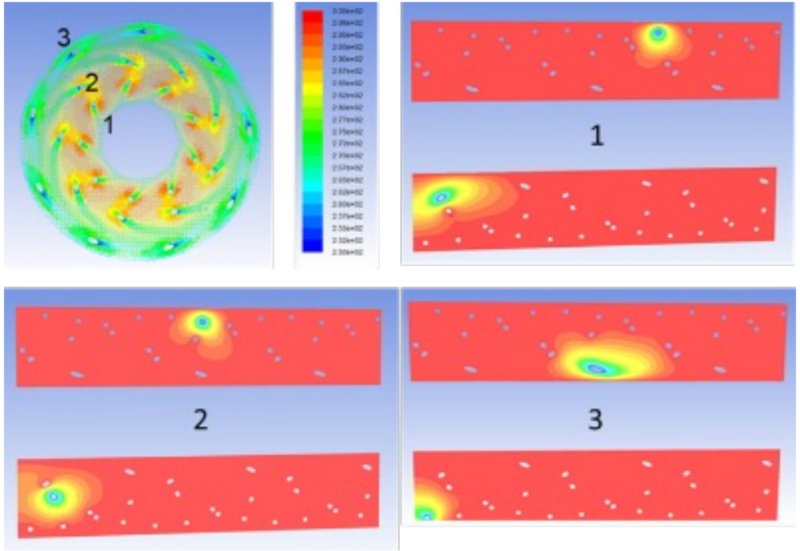
p=50 mm
D=1 mm



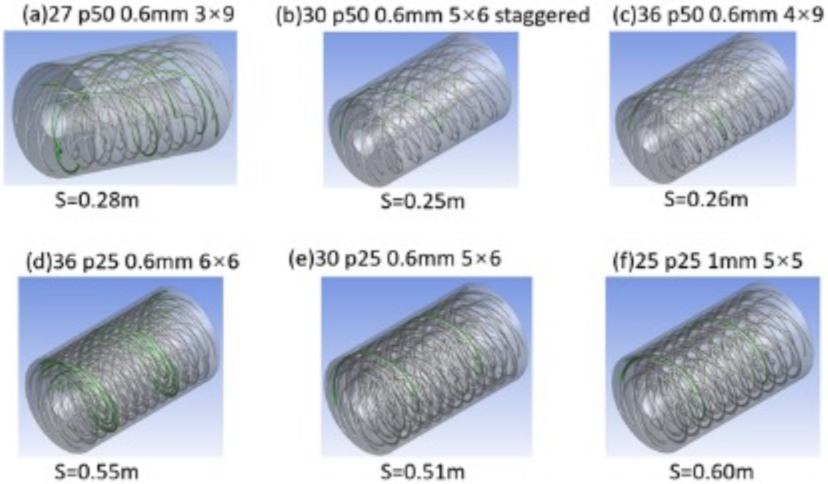
p=25 mm
D=0.5 mm

Two geometries with the same p/D

Conduction shape factor



Temperature distribution of a steady state conduction model


(a)27 p50 0.6mm 3×9
S=0.28m

(b)30 p50 0.6mm 5×6 staggered
S=0.25m

(c)36 p50 0.6mm 4×9
S=0.26m

(d)36 p25 0.6mm 6×6
S=0.55m

(e)30 p25 0.6mm 5×6
S=0.51m

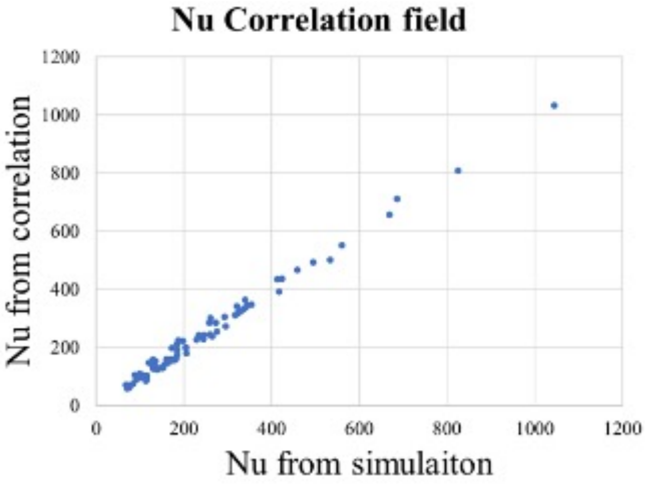
(f)25 p25 1mm 5×5
S=0.60m

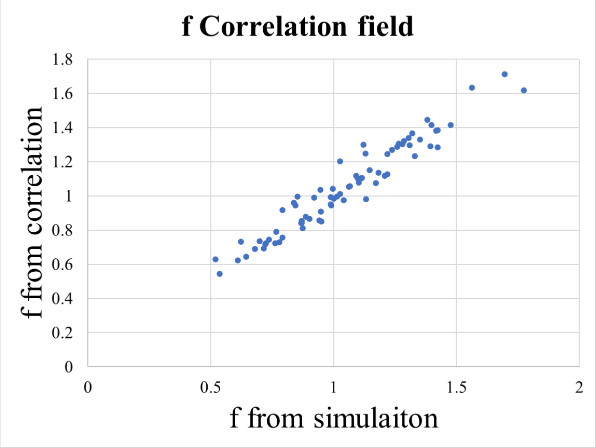
Shape factor of different geometries

1. Pitch length and hydraulic diameter are not enough to specify the geometry.
2. Conduction shape factor is used to reveal and quantify the distribution of tubes.
3. In each geometry, the shape factor of every tube is the same.
4. The difference between geometries exist and is obvious.

$$S = \frac{1}{kR} = \frac{q}{k(T_{others} - T_{one})}$$

Shell side Nu and friction factor correlations





$$Nu = 0.5833Re^{0.60825}\left(\frac{p}{D}\right)^{0.66005}N^{-0.618}$$

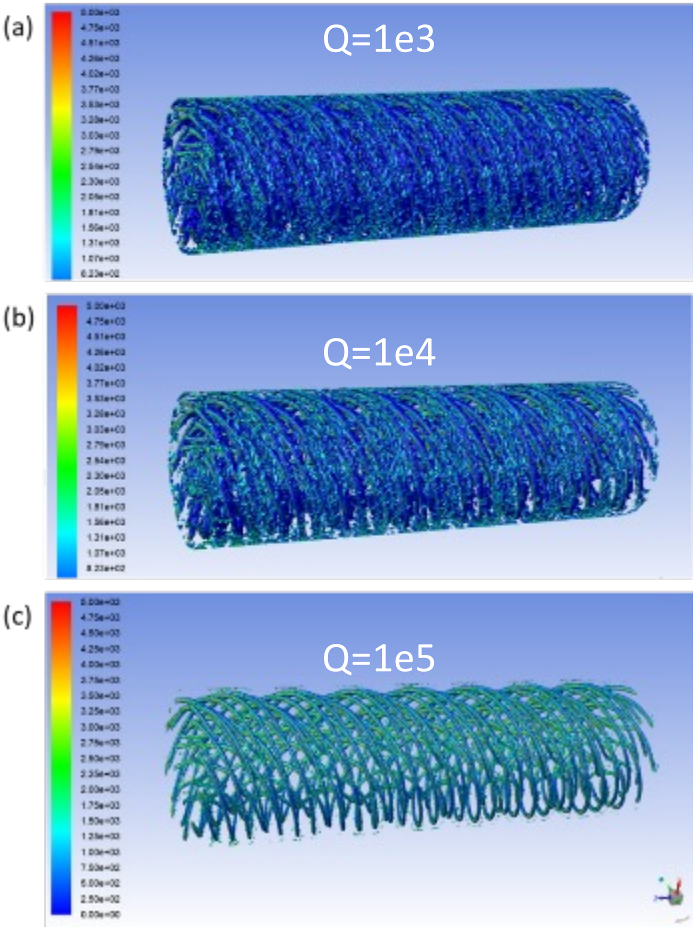
R-Square=0.99

$$f = 6.7026Re^{-0.39059}\left(\frac{p}{D}\right)^{0.38582}\left(\frac{DD}{D}\right)^{0.27427}\left(\frac{S}{p}\right)^{0.26130}N^{-0.39882}$$

R-Square=0.9287

1. The correlations are developed based on results of 17 geometries under 6 working conditions.
2. p : pitch length, DD: distance between two adjacent tubes in one bundle
   S: conduction shape factor

Vortex visualization



← The vorticity magnitude contour on the Q-criterion surface

1. Higher values of Q result in limited vortical structures, while lower ones result in excessive number of structures.
2. The average vorticity magnitude on the iso-surface is explored.

27 tubes, pitch=40 mm, D=0.6 mm

Average vorticity magnitude (1/s) on iso-surface Q= 1E4

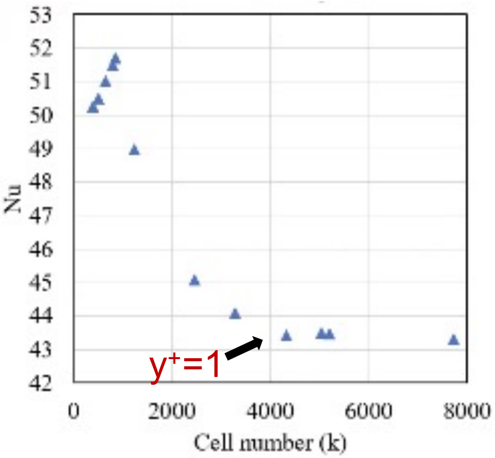
LES: 795.5

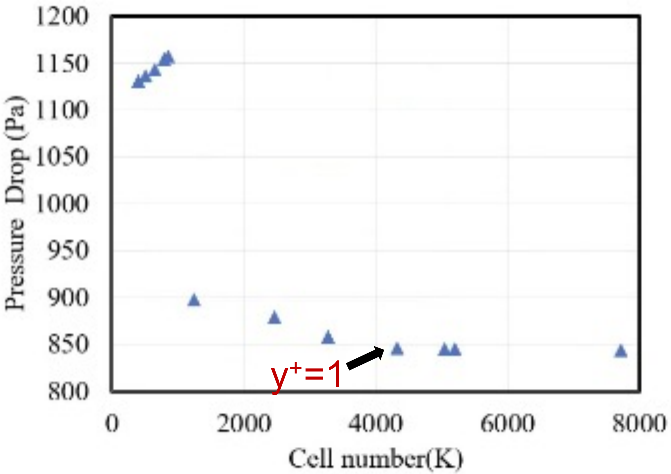
Transient k-omega SST: 790.1

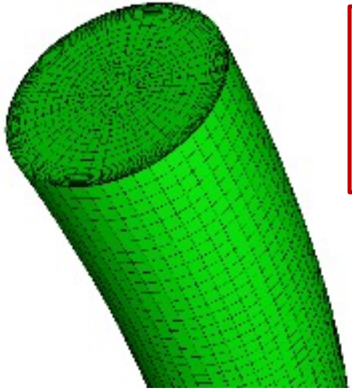
Steady k-omega SST: 805.1

Tube side mesh

2D projection of the twisted geometry

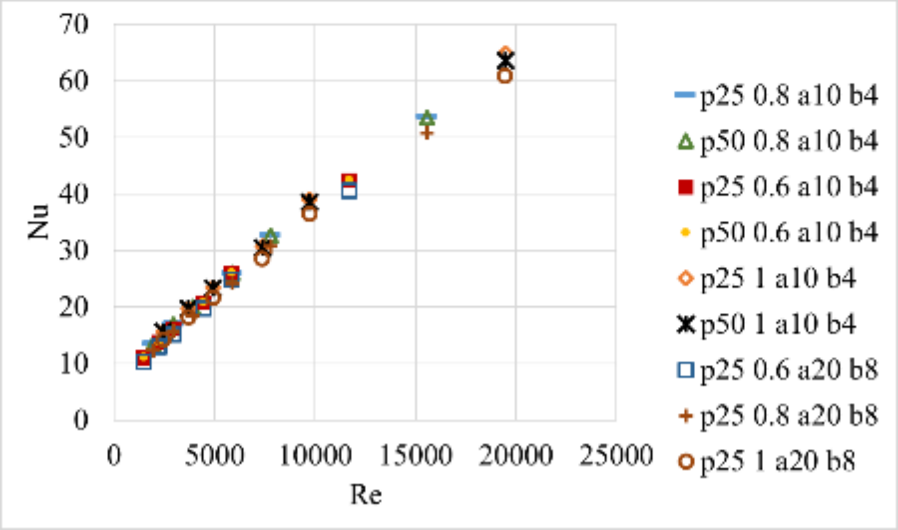


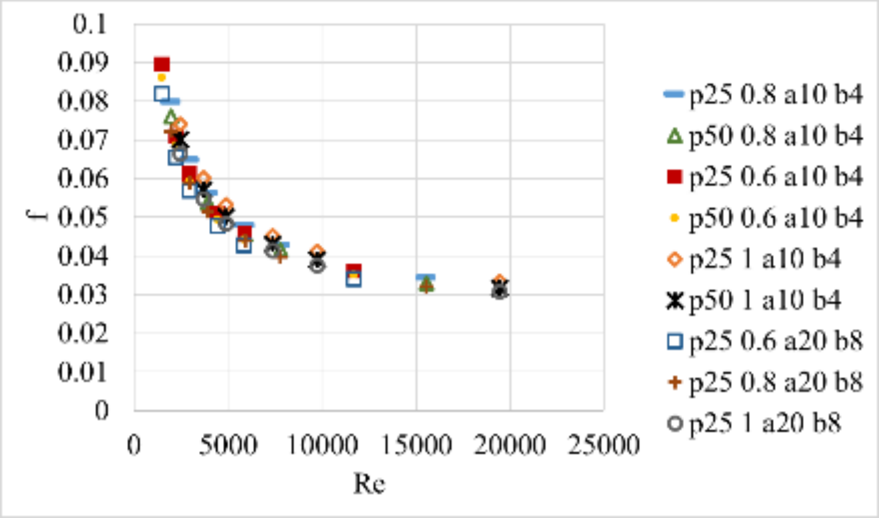




1. The SST k-ω model can resolve the viscous sublayer, when the first cell is in the viscous layer.
2. Enough inflation layers to make sure y^+ be about 1.
3. The Nu and pressure drop values reach steady when y^+ is 1.

Tube side Nu and friction factor correlations





$$Nu = 0.06105 Re^{0.71054} \left(\frac{p}{D}\right)^{-0.02156}$$

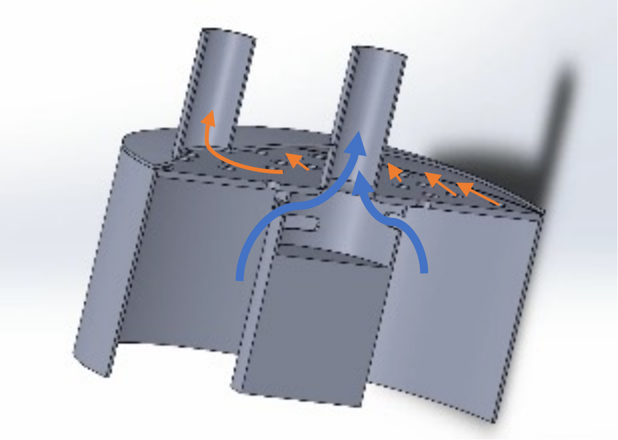
R-Square=0.99686

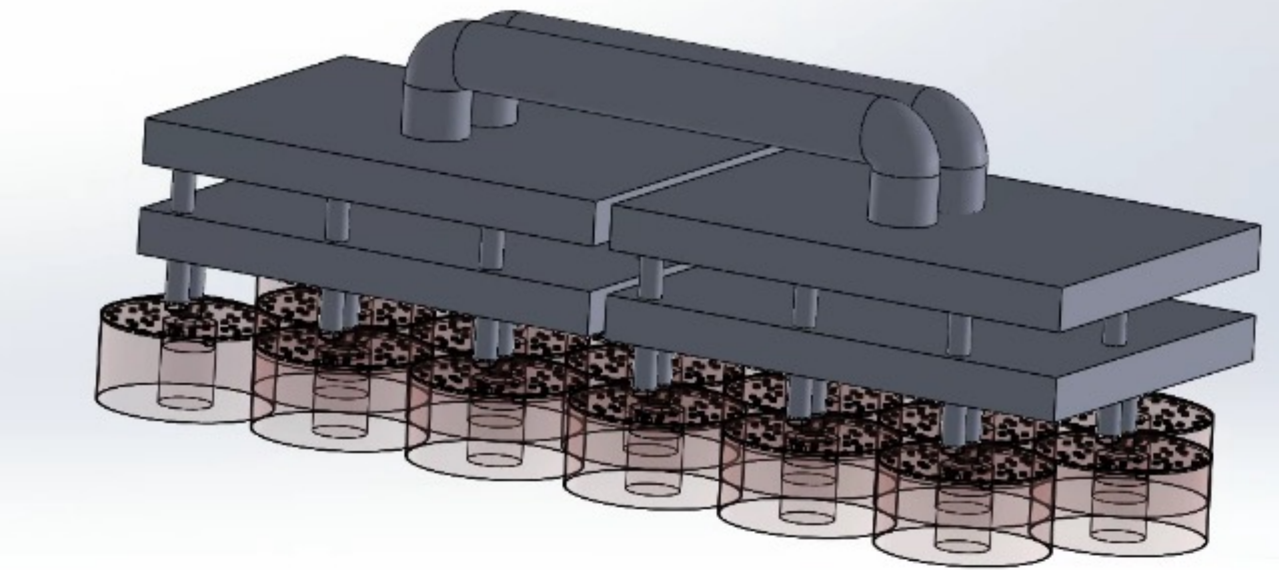
$$f = 2.68097 Re^{-0.43137} \left(\frac{p}{D}\right)^{-0.07549}$$

R-Square=0.98518

1. The bundle radius is (a+b) mm.
2. The bundle radius has little influence on the tube side heat transfer performance.

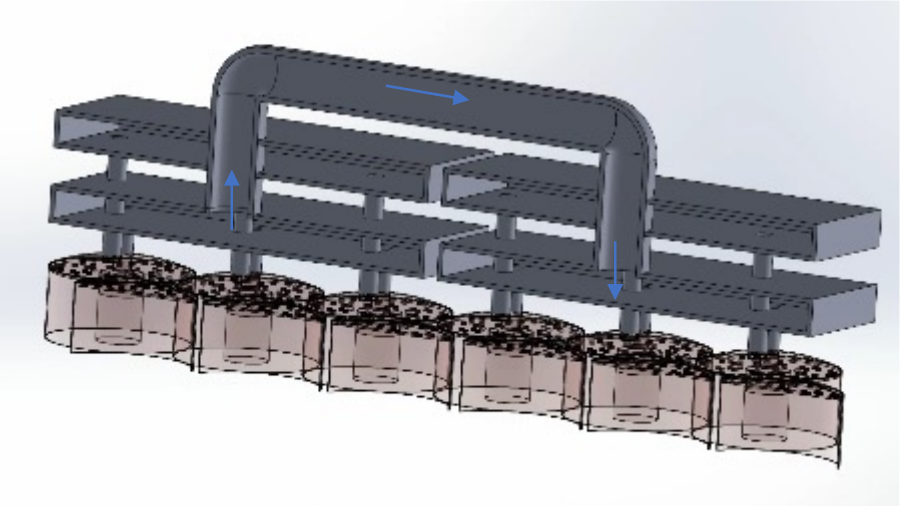
Overall heat exchanger design

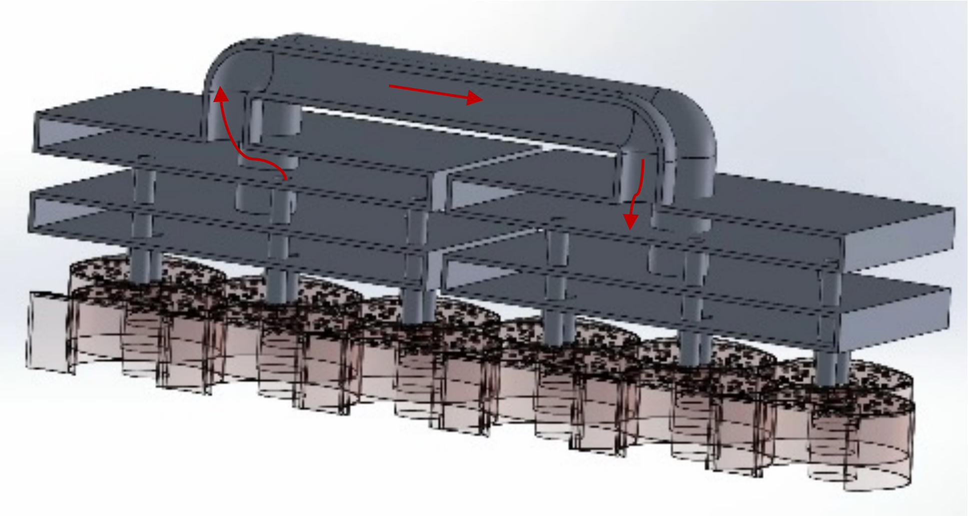




Double header design: redistribute both sides flow at every turn.

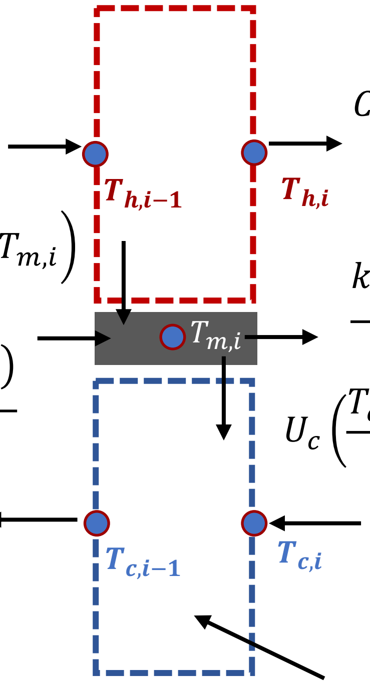
Whole heat exchanger design





1. Shell side flow is redistributed in the lower header and goes straight through the upper header.
2. Tube side flow goes straight through the lower header and is redistributed in the upper header.

Overall heat exchanger finite difference model



$$C_h\left(\frac{T_{h,i}+T_{h,i-1}}{2}\right)\cdot T_{h,i-1}$$

$$C_h\left(\frac{T_{h,i}+T_{h,i-1}}{2}\right)\cdot T_{h,i}$$

$$U_h\left(\frac{T_{h,i}+T_{h,i-1}}{2}\right)\cdot \Delta A\cdot\left(\frac{T_{h,i}+T_{h,i-1}}{2}-T_{m,i}\right)$$

$$\frac{k_m\left(\frac{T_{m,i}+T_{m,i+1}}{2}\right)\cdot A_{c,m}\cdot\left(T_{m,i}-T_{m,i+1}\right)}{\Delta x}$$

$$\frac{k_m\left(\frac{T_{m,i}+T_{m,i-1}}{2}\right)\cdot A_{c,m}\cdot\left(T_{m,i-1}+T_{m,i}\right)}{\Delta x}$$

$$U_c\left(\frac{T_{c,i}+T_{c,i-1}}{2}\right)\cdot\Delta A\cdot\left(T_{m,i}-\frac{T_{c,i}+T_{c,i-1}}{2}\right)$$

$$C_c\left(\frac{T_{c,i}+T_{c,i-1}}{2}\right)\cdot T_{c,i-1}$$

$$C_c\left(\frac{T_{c,i}+T_{c,i-1}}{2}\right)\cdot T_{c,i}$$

$$q'_h\left(\frac{T_{c,i}+T_{c,i-1}}{2}\right)\cdot\Delta x$$

Axial conduction and parasitic heat load is considered.

Overall heat exchanger finite difference model

Δx=L/n
B.C. : $T_{h,in}$, $T_{c,in}$, metal hot end adiabatic, metal cold end adiabatic
3n+4 equations and unknows.

Hot fluid energy balance

$$\frac{\mu_h(\Theta_{h,i-1})+\mu_h(\Theta_{h,i})}{2}\cdot\Theta_{h,i-1}+\frac{\chi_h(\Theta_{h,i})+\chi_h(\Theta_{h,i-1})}{2}\Delta X_i$$
$$=\frac{\mu_h(\Theta_{h,i-1})+\mu_h(\Theta_{h,i})}{2}\cdot\Theta_{h,i}$$
$$+\frac{\beta_h(\Theta_{h,i})+\beta_h(\Theta_{h,i-1})}{2}\Delta X_i$$
$$\cdot\left(\frac{\Theta_{h,i}+\Theta_{h,i-1}}{2}-\Theta_{m,i}\right),\quad i=1,\ldots,n$$

Cold fluid energy balance

$$\frac{\nu(\Theta_{c,i})+\nu(\Theta_{c,i-1})}{2}\cdot\Theta_{c,i}+\frac{\chi_c(\Theta_{c,i})+\chi_c(\Theta_{c,i-1})}{2}\Delta X_i$$
$$+\frac{\beta_c(\Theta_{c,i})+\beta_c(\Theta_{c,i-1})}{2}\Delta X_i\cdot\left(\Theta_{m,i}-\frac{\Theta_{c,i}+\Theta_{c,i-1}}{2}\right)$$
$$=\frac{\nu(\Theta_{c,i})+\nu(\Theta_{c,i-1})}{2}\cdot\Theta_{i-1},\quad i=1,\ldots,n$$

Metal energy balance

$$\chi_m(\Theta_{m,i})\Delta X_i+\frac{\beta_h(\Theta_{h,i-1})+\beta_h(\Theta_{h,i})}{2}\Delta X_i$$
$$\cdot\left(\frac{\Theta_{h,i-1}+\Theta_{h,i}}{2}-\Theta_{m,i}\right)+\frac{[\lambda(\Theta_{m,i})+\lambda(\Theta_{m,i-1})]}{[\Delta X_i+\Delta X_{i-1}]}$$
$$\cdot(\Theta_{m,i-1}-\Theta_{m,i})=\frac{\beta_c(\Theta_{c,i-1})+\beta_c(\Theta_{c,i})}{2}\Delta X_i$$
$$\cdot\left(\Theta_{m,i}-\frac{\Theta_{c,i-1}+\Theta_{c,i}}{2}\right)+\frac{[\lambda(\Theta_{m,i})+\lambda(\Theta_{m,i+1})]}{[\Delta X_i+\Delta X_{i+1}]}$$
$$\cdot(\Theta_{m,i}-\Theta_{m,i+1}),\quad i=1,\ldots,n$$

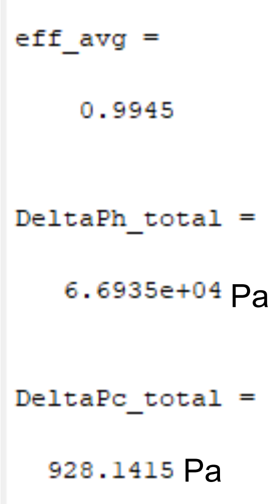
Θ: *dimensionless temperature*
X: *dimensionless parasitic parameter*
μ, ν: *capacity ratios*
λ: *axial conduction parameter*
β: *dimensionless NTU*

Effectiveness and pressure drop

P50 1.2 mm 1 mm 48 tubes m_dot=0.013 kg/s

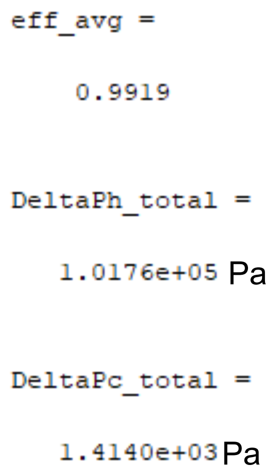
9 groups in parallel, total length 5.4 m

Without considering radiation parasitic heat load



```
eff_avg =

    0.9945


DeltaPh_total =

   6.6935e+04 Pa


DeltaPc_total =

  928.1415 Pa
```

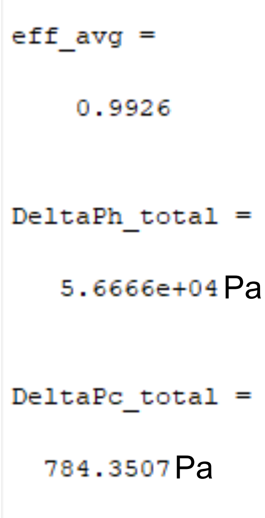
Without insulation layer



```
eff_avg =

    0.9919


DeltaPh_total =

   1.0176e+05 Pa


DeltaPc_total =

   1.4140e+03 Pa
```

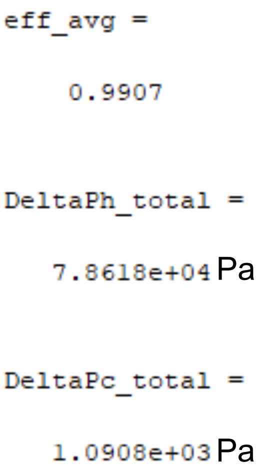
9 groups in parallel, total length 4.5 m

Without considering radiation parasitic heat load



```
eff_avg =

    0.9926


DeltaPh_total =

   5.6666e+04 Pa


DeltaPc_total =

  784.3507 Pa
```

Without insulation layer



```
eff_avg =

    0.9907


DeltaPh_total =

   7.8618e+04 Pa


DeltaPc_total =

   1.0908e+03 Pa
```

Conclusion:

1. A design of helical bundled meso-scale tube heat exchanger is proposed.
2. Nu and friction factor correlations are developed as inputs for the overall HX model.
3. Including axial conduction and parasitic heat load, the effectiveness and pressure drop requirements can be satisfied.

Thanks,
Questions?