Multi-particle correlations for the new decade of QGP studies

27th Nordic Particle Physics Meeting

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INDEPENDENT RESEARCH FUND DENMARK

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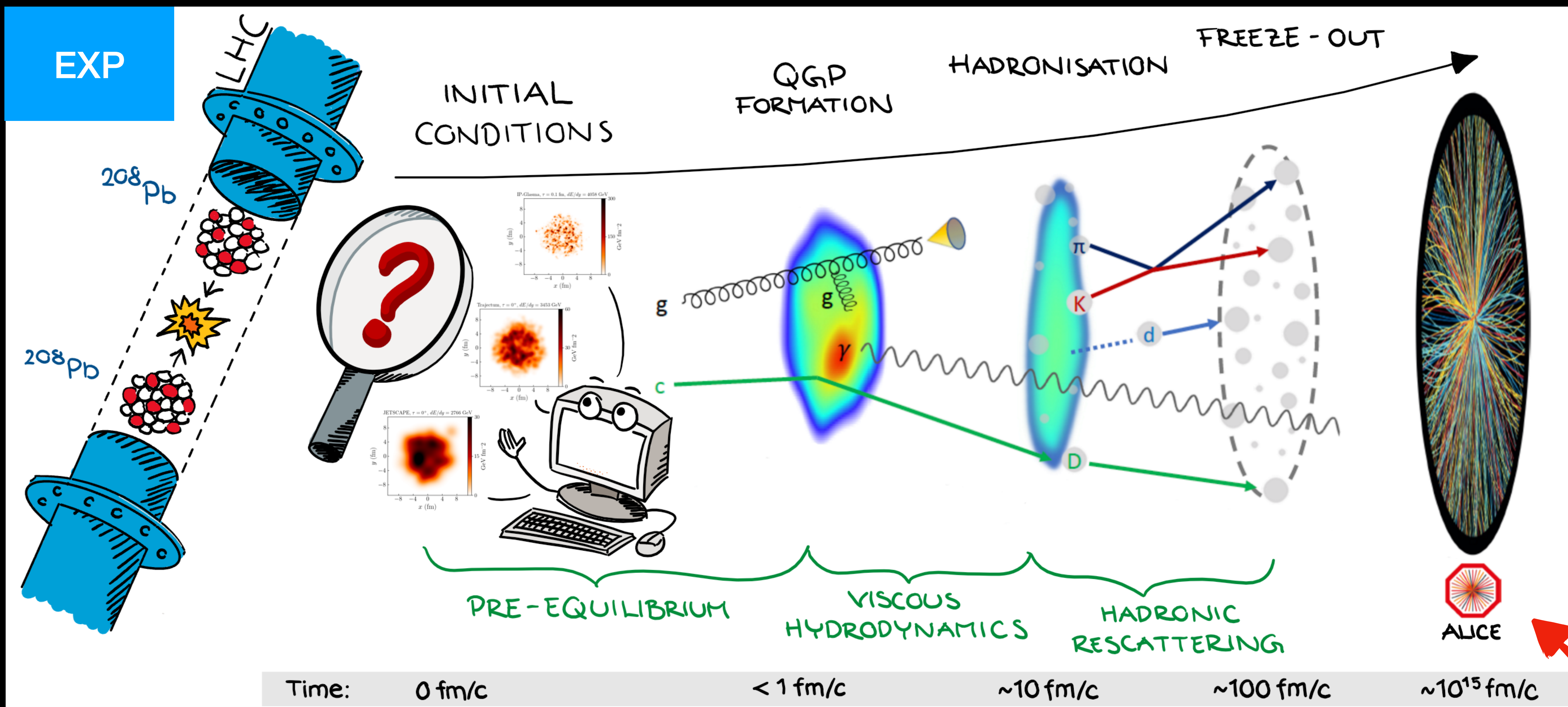
QGP study in heavy-ion collisions



TH — Initial State model → Hydrodynamic → Rescattering → Final

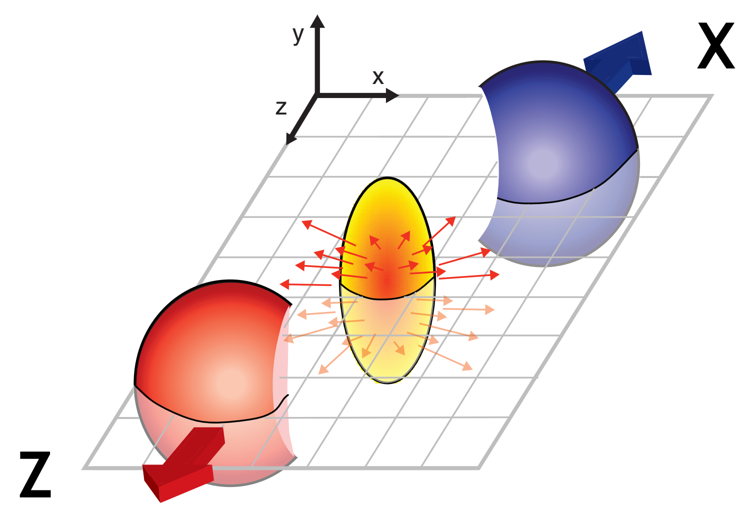
TH overview, see talk by E. A. Kurkela

EXP overview, see talk by A. Ohlson

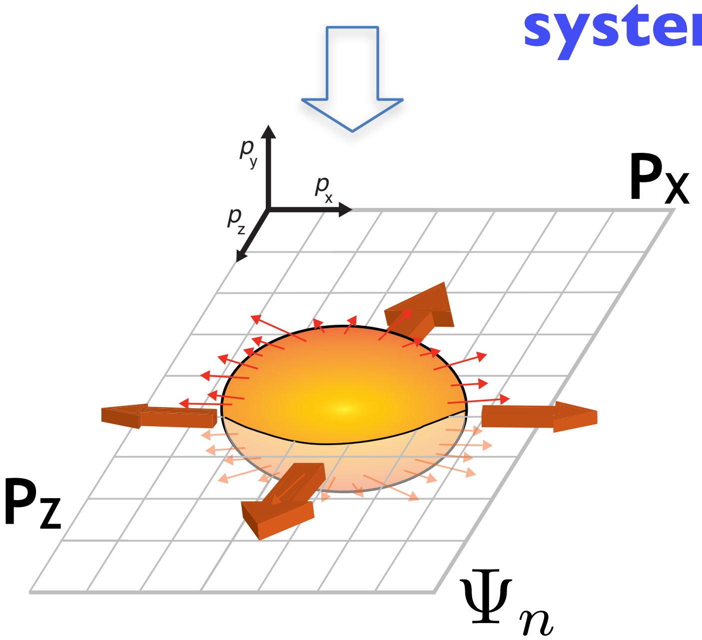
Anisotropic flow

❖ Spatial anisotropy in the initial state converted to momentum anisotropic particle distributions

- known as ***anisotropic flow***
- reflect initial **anisotropy** and **transport properties** of QGP



system expansion



Initial state

$$\varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}$$

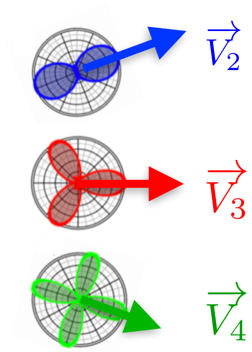
Initial spatial Anisotropy

Final state

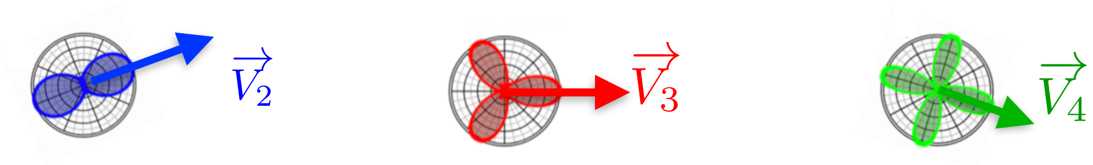
$$v_n = \langle \cos n(\varphi - \Psi_n) \rangle$$

momentum space Anisotropic Flow

$$\vec{V_n} = v_n \, e^{in\Psi_n}$$



Two-particle azimuthal correlations



- What we *want* to measure:

$$v_n = \langle \cos n(\varphi - \Psi_n) \rangle$$

Ψ_n is unknown in experiment

- What we *hope* to get from 2-particle azimuthal correlation

$$\langle\langle \cos n(\varphi_1 - \varphi_2) \rangle\rangle = \langle\langle \cos n\,[(\varphi_1 - \Psi_n) - (\varphi_2 - \Psi_n)] \rangle\rangle$$

$$= \langle\langle \cos n(\varphi_1 - \Psi_n) \cdot \cos n(\varphi_2 - \Psi_n) \rangle\rangle ~~+ \langle\langle \sin n(\varphi_1 - \Psi_n) \cdot \sin n(\varphi_2 - \Psi_n) \rangle\rangle~~$$

= 0 due to symmetry

$$= \langle v_n^2 \rangle$$

  - Get the RMS value of v_n distribution without knowing Ψ_n

- What we *actually get* from 2-particle azimuthal correlation in experiment

$$\langle\langle \cos n(\varphi_1 - \varphi_2) \rangle\rangle = \langle v_n^2 + \delta_2 \rangle$$

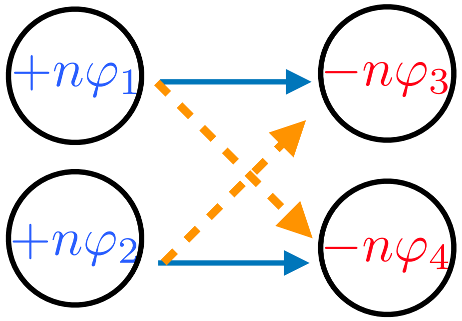
→ *Nonflow (resonance decay, jets etc)*

$$\delta_2 \sim 1/M$$

Multi-particle correlation/cumulant

Flow analysis from multiparticle azimuthal correlations

Nicolas Borghini, Phuong Mai Dinh, and Jean-Yves Ollitrault
Phys. Rev. C **64**, 054901 – Published 25 September 2001



- Example: 4-particle cumulant

$$c_n\{4\} = \langle\langle \cos n(\varphi_1+\varphi_2-\varphi_3-\varphi_4)\rangle\rangle - \langle\langle \cos n(\varphi_1-\varphi_3)\rangle\rangle\,\langle\langle \cos n(\varphi_2-\varphi_4)\rangle\rangle - \langle\langle \cos n(\varphi_1-\varphi_4)\rangle\rangle\,\langle\langle \cos n(\varphi_2-\varphi_3)\rangle\rangle$$

$$= \langle v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 + \delta_4\rangle - 2\langle (v_n^2+\delta_2)^2\rangle = \langle -v_n^4 + \delta_4 \rangle = -v_n\{4\}^4$$

Nonflow (of 4-particles) $\delta_4 \sim 1/M^3$

- Using multi-particle cumulant, one can largely suppress nonflow contaminations

$$v_n\{2\} = \langle v_n^2\rangle^{1/2},$$

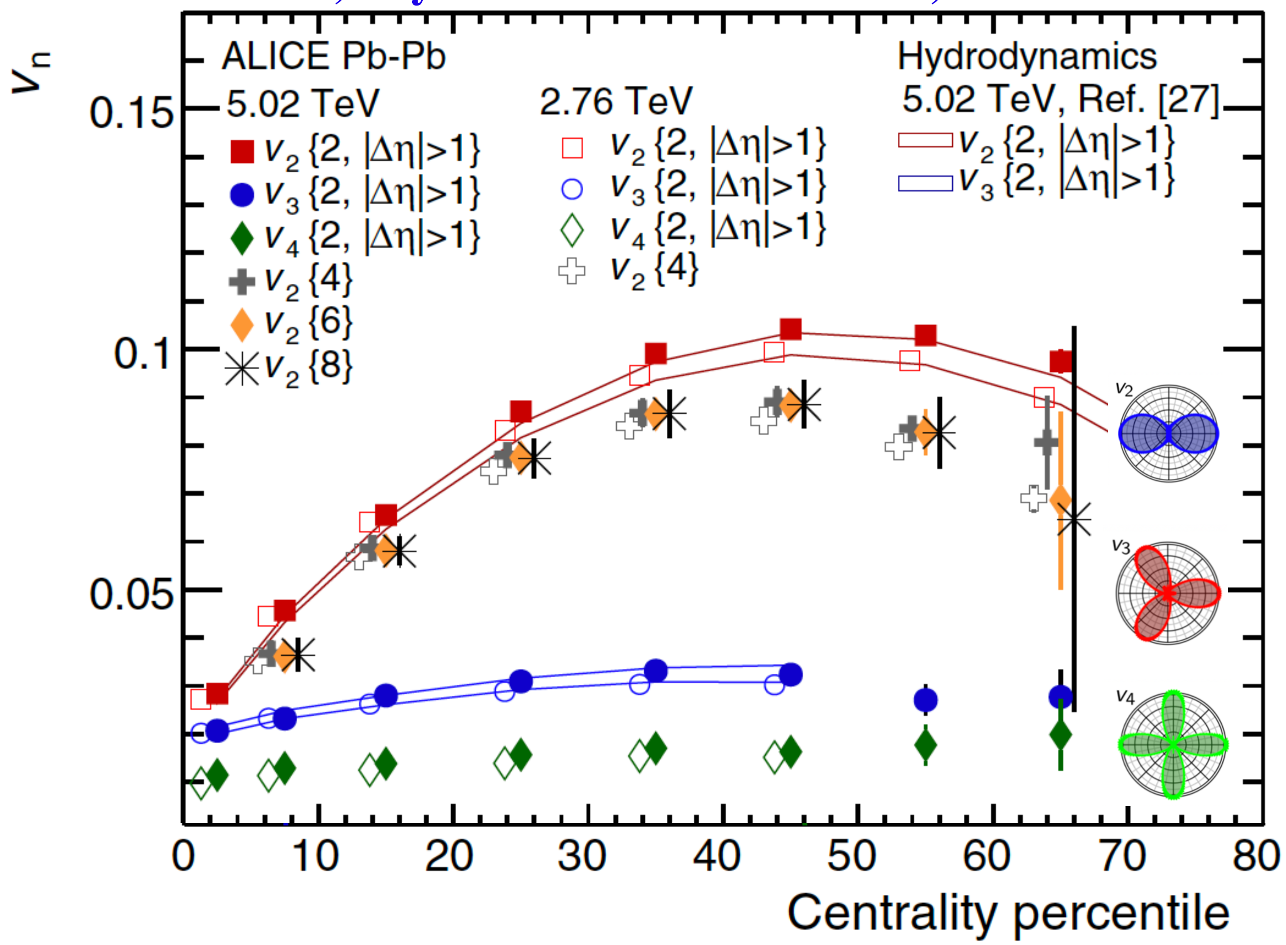
$$v_n\{4\} = \left[2\langle v_n^2\rangle^2 - \langle v_n^4\rangle\right]^{1/4},$$

$$v_n\{6\} = \left[(1/4)\cdot\left(\langle v_n^6\rangle - 9\langle v_n^4\rangle\langle v_n^2\rangle + 12\langle v_n^2\rangle^3\right)\right]^{1/6},$$

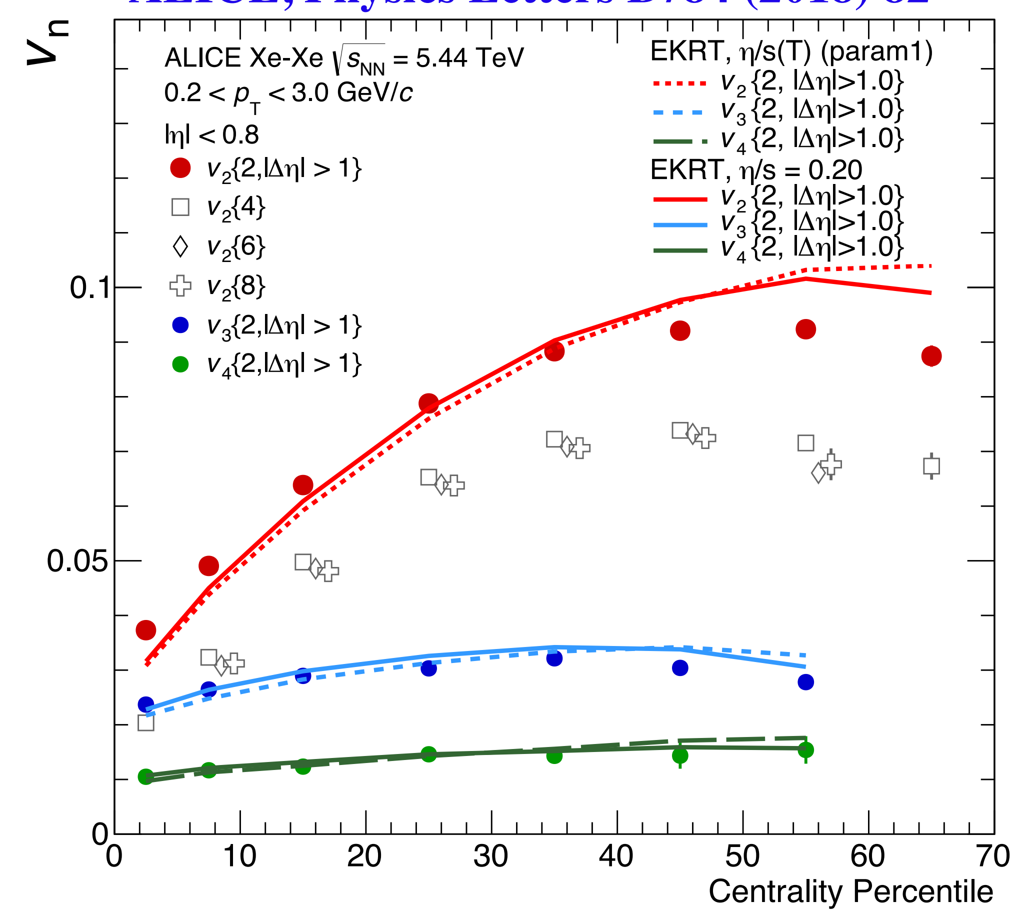
$$v_n\{8\} = \left[-(1/33)\cdot\left(\langle v_n^8\rangle - 16\langle v_n^6\rangle\langle v_n^2\rangle - 18\langle v_n^4\rangle^2 + 144\langle v_n^4\rangle\langle v_n^2\rangle^2 - 144\langle v_n^2\rangle^4\right)\right]^{1/8},$$

QGP with multi-particle correlations

ALICE, Physical Review Letters 116, 132302



ALICE, Physics Letters B784 (2018) 82



❖ Experimental data:

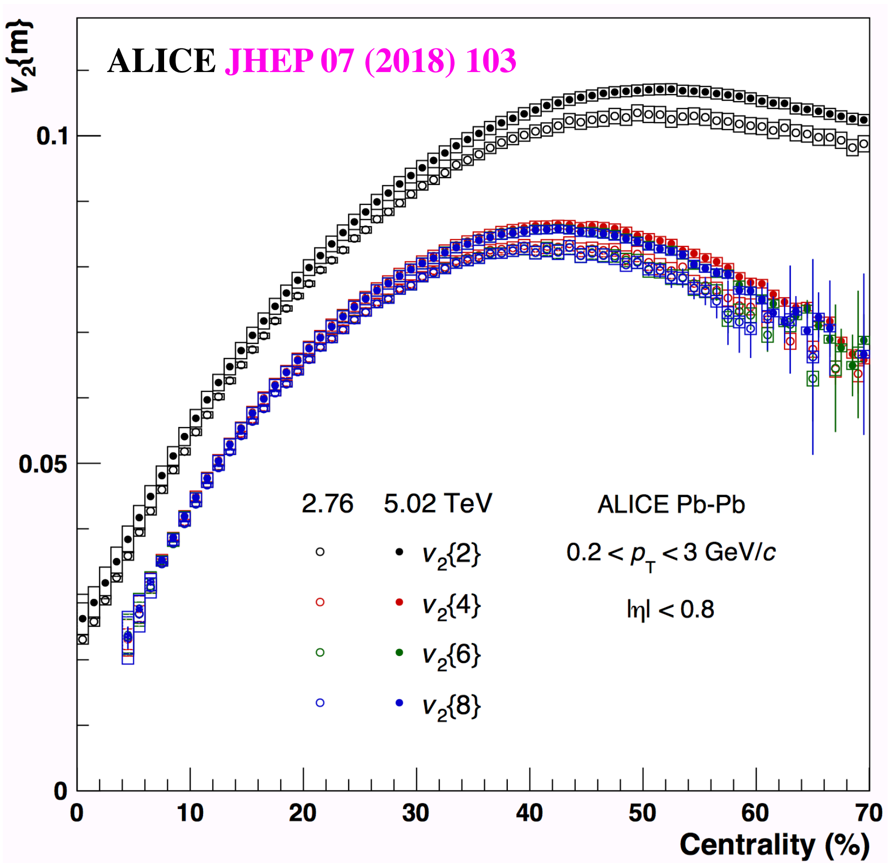
- $v_2 > v_3 > v_4$
- $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$

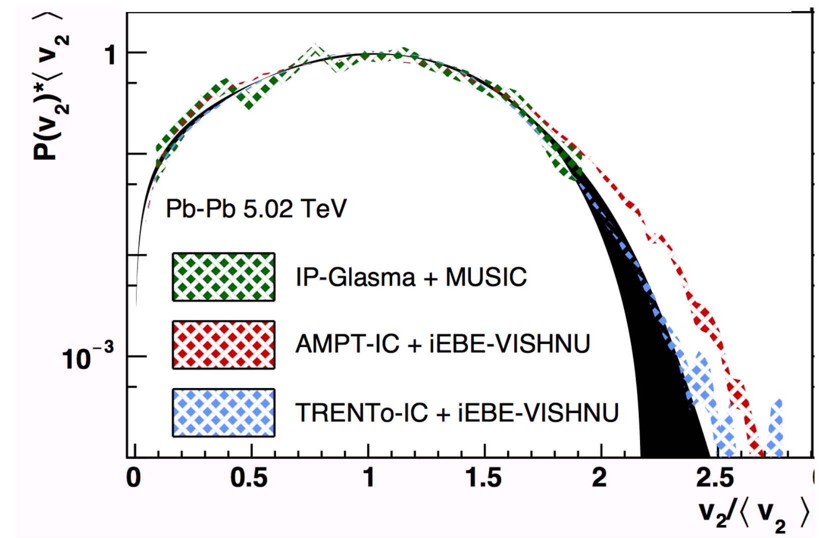
❖ Flow measurements at the top LHC energies agree with hydrodynamic predictions

- **The Quark-Gluon Plasma behaves like a fluid**

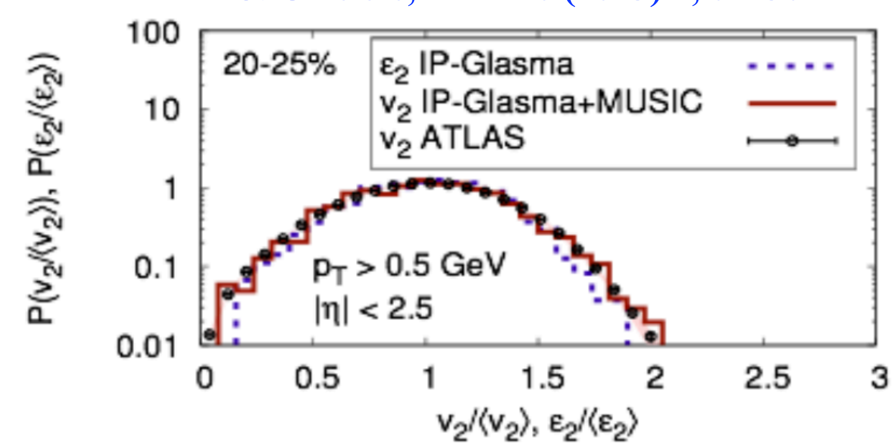
P(v_n) and P(ε_n)

$v_n\{m\}$ → $p(v_n)$ → $p(\varepsilon_n)$





C. Gale etc, PRL110 (2013) 1, 012302



$$v_n\{2\} = \sqrt[2]{\langle v_n^2\rangle},$$
$$v_n\{4\} = \sqrt[4]{2\langle v_n^2\rangle^2 - \langle v_n^4\rangle},$$
$$v_n\{6\} = \sqrt[6]{\langle v_n^6\rangle - 9\langle v_n^2\rangle\langle v_n^4\rangle + 12\langle v_n^2\rangle^3},$$
$$v_n\{8\} = \sqrt[8]{\langle v_n^8\rangle - 16\langle v_n^2\rangle\langle v_n^6\rangle - 18\langle v_n^4\rangle^2 + 144\langle v_n^2\rangle^2\langle v_n^4\rangle - 144\langle v_n^2\rangle^4}.$$

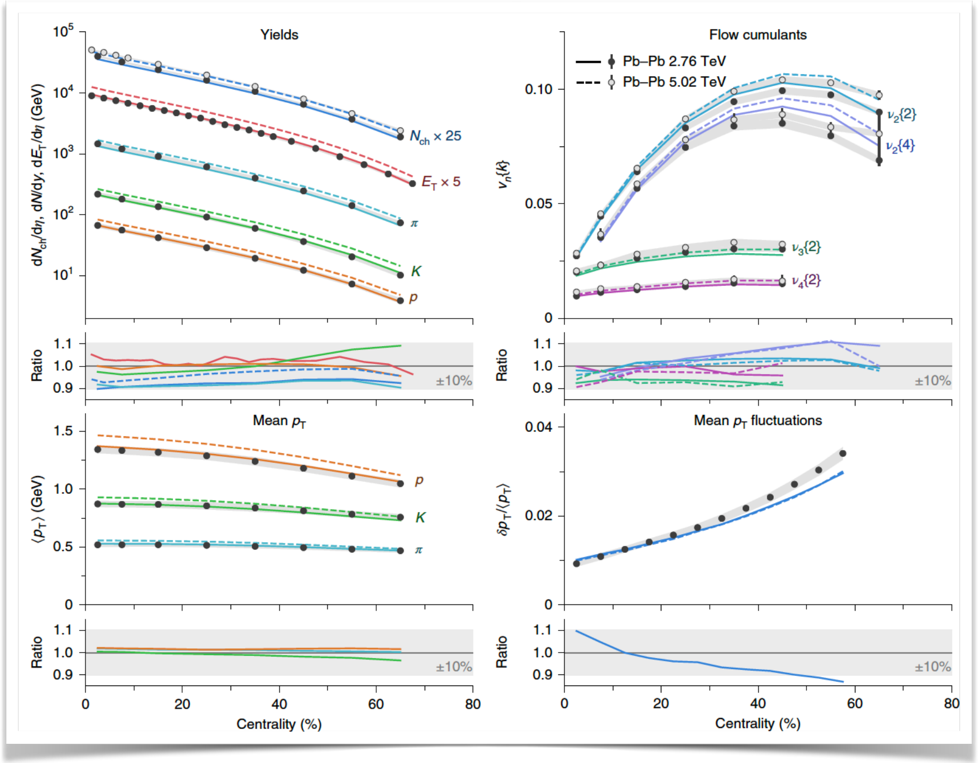
$$v_n \propto \varepsilon_n$$
$$P(v_n/\langle v_n\rangle) \approx P(\varepsilon_n/\langle\varepsilon_n\rangle)$$

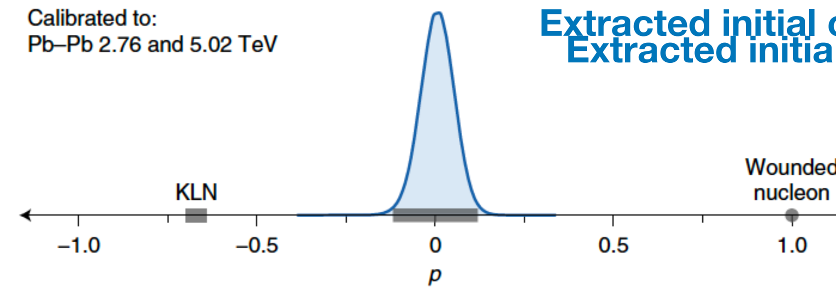
❖ Investigating p(v_2) with multi-particle cumulants

- Ultra-higher order cumulants e.g. $v_2\{10\}\{12\}\{14\}\{16\}$ is implemented for HL-LHC,
- Possibility to construct a more precise p.d.f. with higher moments

Bayesian analyses with simple v_n

J.E. Bernhard etc, Nature Physics,15, 1113 (2019)

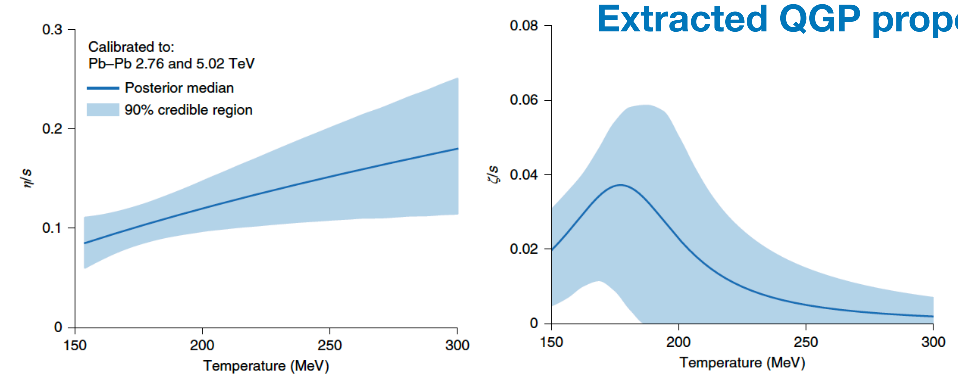




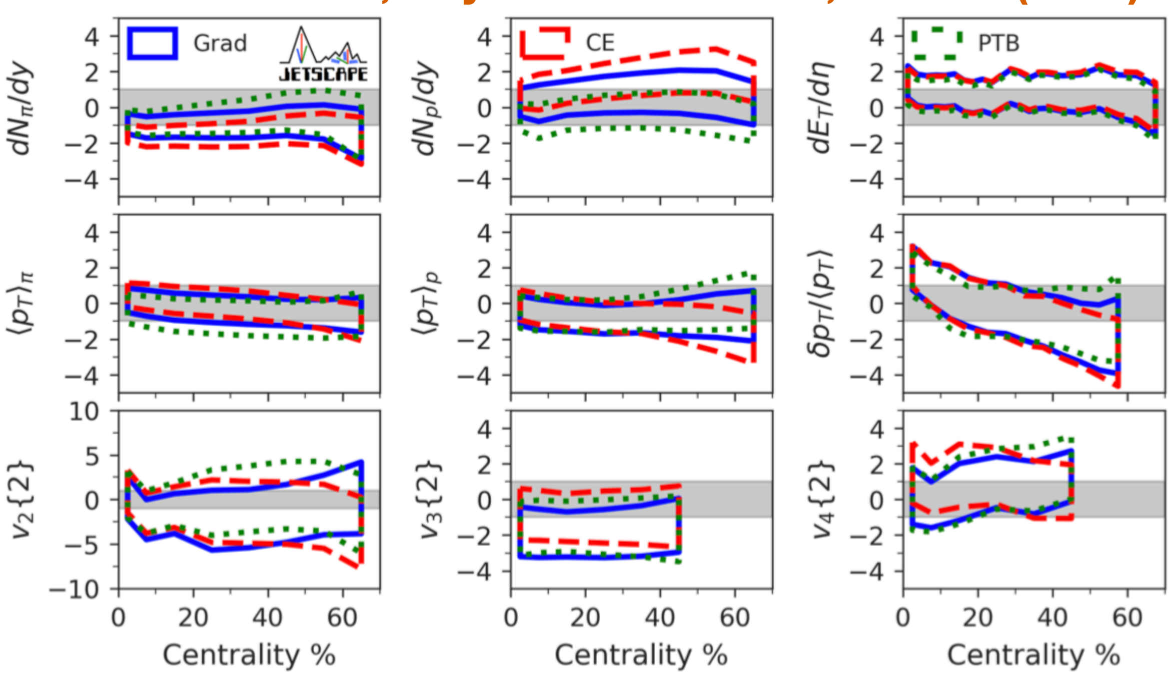
Extracted initial conditions

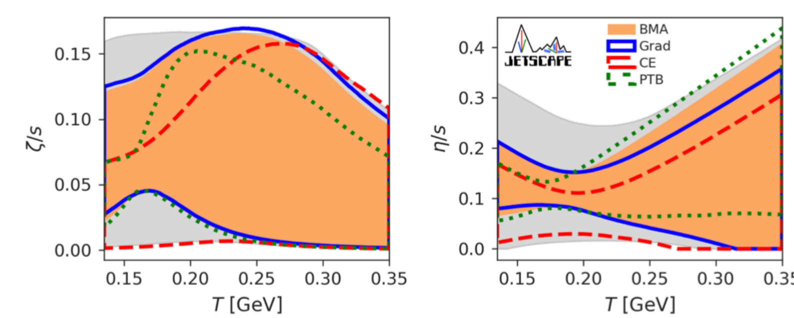
Extracted initial conditions

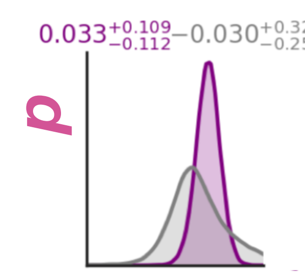
Extracted QGP properties



JETSCAPE, Phys. Rev. Lett. 126, 242301 (2021)





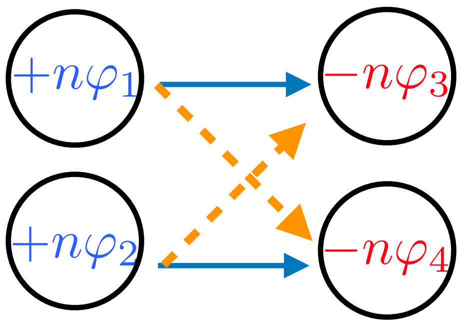


Extracted QGP properties

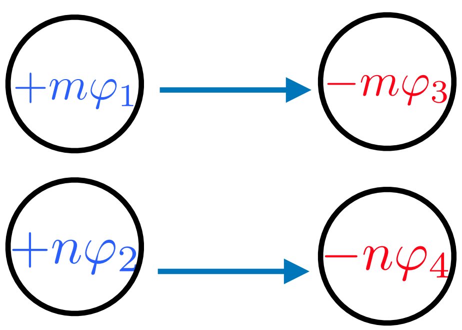
Extracted initial conditions

Note: there is no constraint on flow fluctuations in JETSCAPE

From single harmonic to mixed harmonics



$$c_n\{4\} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\rangle\rangle - \langle\langle \cos n(\varphi_1 - \varphi_3)\rangle\rangle \, \langle\langle \cos n(\varphi_2 - \varphi_4)\rangle\rangle - \langle\langle \cos n(\varphi_1 - \varphi_4)\rangle\rangle \, \langle\langle \cos n(\varphi_2 - \varphi_3)\rangle\rangle$$



$$\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle - \langle\langle \cos(m\varphi_1 - m\varphi_3)\rangle\rangle - \langle\langle \cos(n\varphi_2 - n\varphi_4)\rangle\rangle$$
$$= \langle v_m^2 \, v_n^2\rangle - \langle v_m^2\rangle \, \langle v_n^2\rangle$$

Symmetric cumulants:

$$SC(m,n) = \langle v_m^2 \, v_n^2\rangle - \langle v_m^2\rangle \, \langle v_n^2\rangle$$

PHYSICAL REVIEW C **89**, 064904 (2014)

Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations

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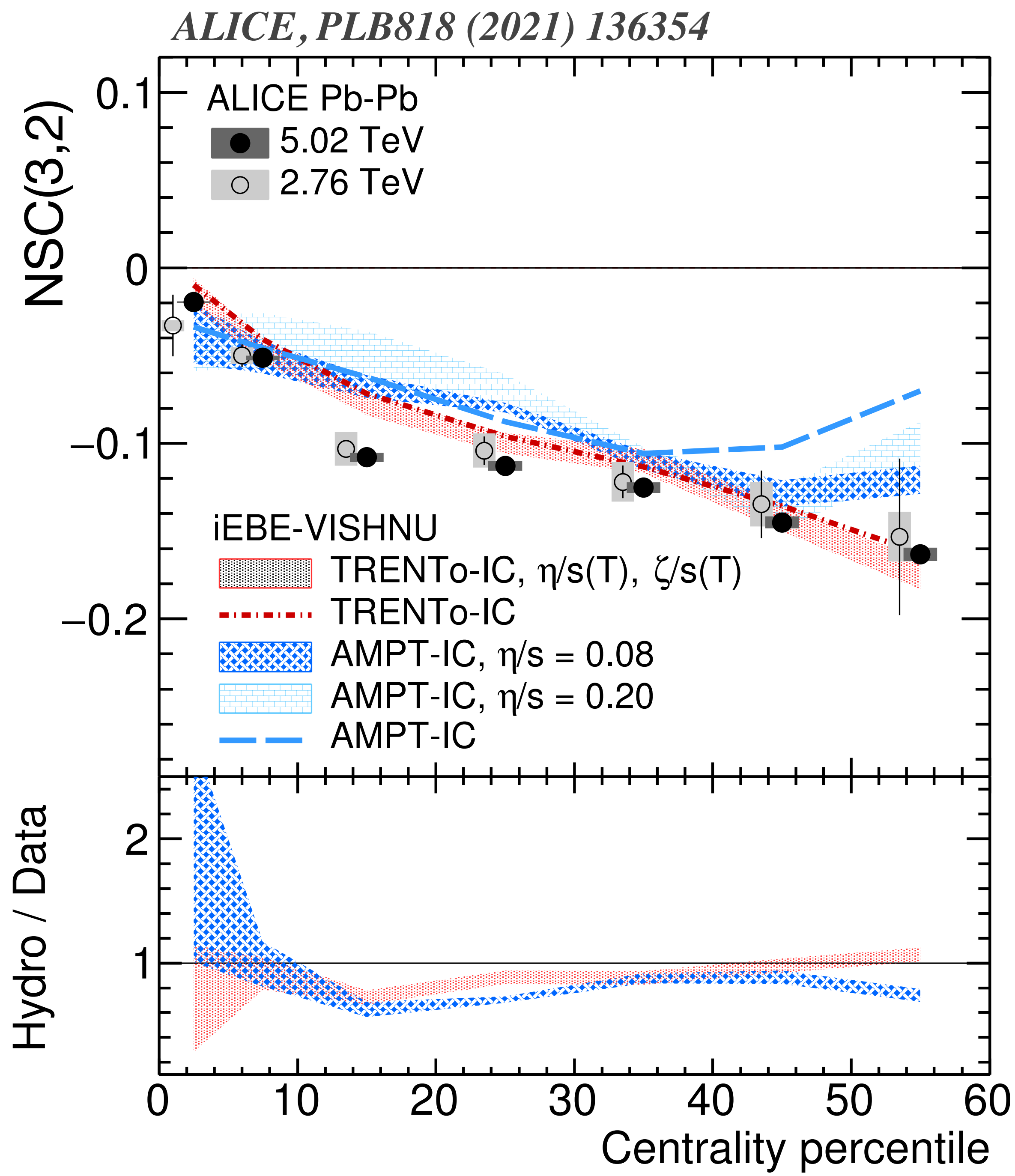
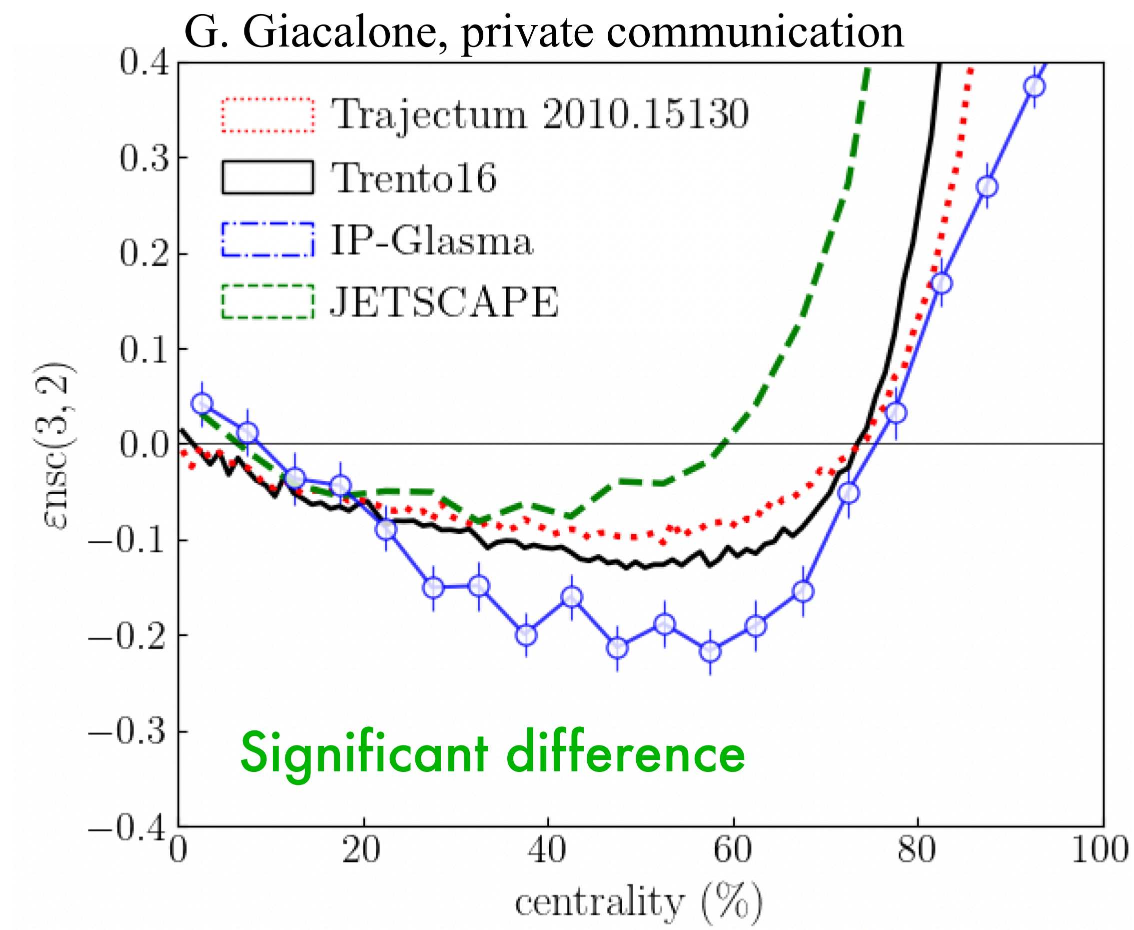
[2] *Nikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands*
[3] *Utrecht University, P.O. Box 80000, 3508 TA Utrecht, The Netherlands*

Probe IC with NSC(3,2)



$$NSC^{v}(3,2) = NSC^{\varepsilon}(3,2)$$



- Precise NSC(3,2) data at Run2 provides tight constraints on the initial state models
  - I predict JETSCAPE & IP-Glasma will fail to describe the NSC(3,2) data
- what is the general correlation between any order of v_n^k and v_m^p and the correlations among multiple flow coefficients

The full p.d.f. P(v_m, v_n, v_k, . . ., Ψ_m, Ψ_n, Ψ_k, . . .)

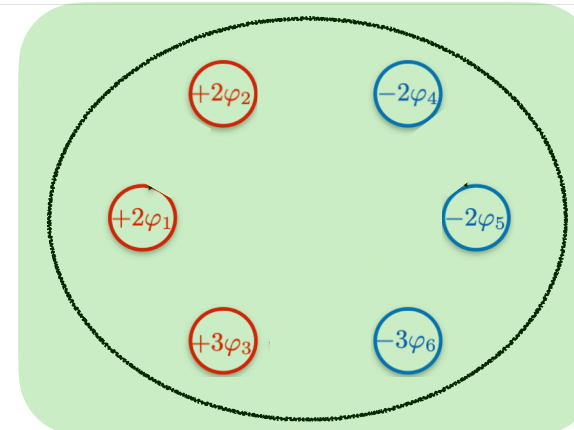
A reminder

J. Jia, JPG41 (2014) 124003

	pdfs	cumulants
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k=1,2,\ldots$
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle,\ n\neq m$ ...
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2\rangle + 2\langle v_n^2\rangle\langle v_m^2\rangle\langle v_l^2\rangle - \langle v_n^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_m^2 v_l^2\rangle\langle v_n^2\rangle - \langle v_l^2 v_n^2\rangle\langle v_m^2\rangle$ $n\neq m\neq l$ ...
	...	Obtained recursively as above
EP-correlation	$p(\Phi_n, \Phi_m, \ldots)$	$\langle v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} \ldots \cos(c_n n\Phi_n + c_m m\Phi_m + \ldots)\rangle$ $\sum_k kc_k = 0$
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, \ldots)$	$\langle v_l^2 v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} \ldots \cos(c_n n\Phi_n + c_m m\Phi_m + \ldots)\rangle - \langle v_l^2\rangle\langle v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} \ldots \cos(c_n n\Phi_n + c_m m\Phi_m + \ldots)\rangle$ $\sum_k kc_k = 0,\ n\neq m\neq l\ldots$

- Example: 6-particle cumulants with mixed harmonics

$$\langle\langle\cos(2\varphi_1+2\varphi_2+3\varphi_3-2\varphi_4-2\varphi_5-3\varphi_6)\rangle\rangle_c$$
$$= \langle\langle\cos(2\varphi_1+2\varphi_2+3\varphi_3-2\varphi_4-2\varphi_5-3\varphi_6)\rangle\rangle$$
$$-4\,\langle\langle\cos(2\varphi_1+3\varphi_2-2\varphi_4-3\varphi_6)\rangle\rangle\,\langle\langle\cos(2\varphi_1-2\varphi_2)\rangle\rangle$$
$$-\,\langle\langle\cos(2\varphi_1+2\varphi_2-2\varphi_4-2\varphi_5)\rangle\rangle\,\langle\langle\cos(3\varphi_1-3\varphi_2)\rangle\rangle$$
$$+4\,\langle\langle\cos(2\varphi_1-2\varphi_2)\rangle\rangle^2\,\langle\langle\cos(3\varphi_1-3\varphi_2)\rangle\rangle$$



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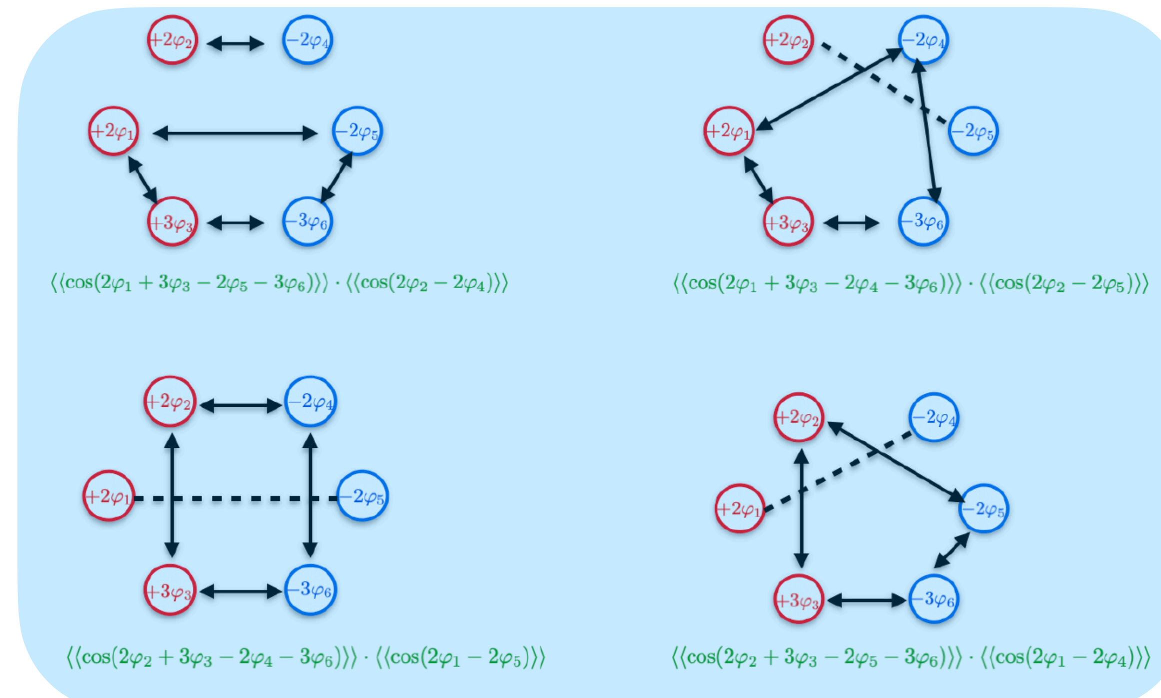

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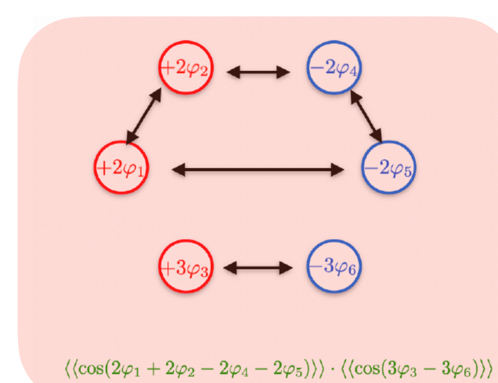


Generic algorithm for azimuthal correlations

PHYSICAL REVIEW C **103**, 024913 (2021)

Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

Zuzana Moravcova, Kristjan Gulbrandsen,* and You Zhou†
Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark

m-particle correlation

```
complex Correlator(int* harmonic, int n, int mult = 1, int skip = 0)
 {
  int har_sum = 0;
  for (int i = 0; i<mult; ++i) har_sum += harmonic[n-1+i];
  complex c(Q(har_sum, mult));
  if (n == 1) return c;
  c *= Correlator(harmonic, n-1);
  if (n == 1+skip) return c;

  complex c2 = 0;
  int h_hold = harmonic[n-2];
  for (int counter = 0; counter <= n-2-skip; ++counter)
  {
   harmonic[n-2] = harmonic[counter];
   harmonic[counter] = h_hold;
   c2 += Correlator(harmonic, n-1, mult+1, n-2-counter);
   harmonic[counter] = harmonic[n-2];
  }
  harmonic[n-2] = h_hold;
  return c-mult*c2;
 }
```

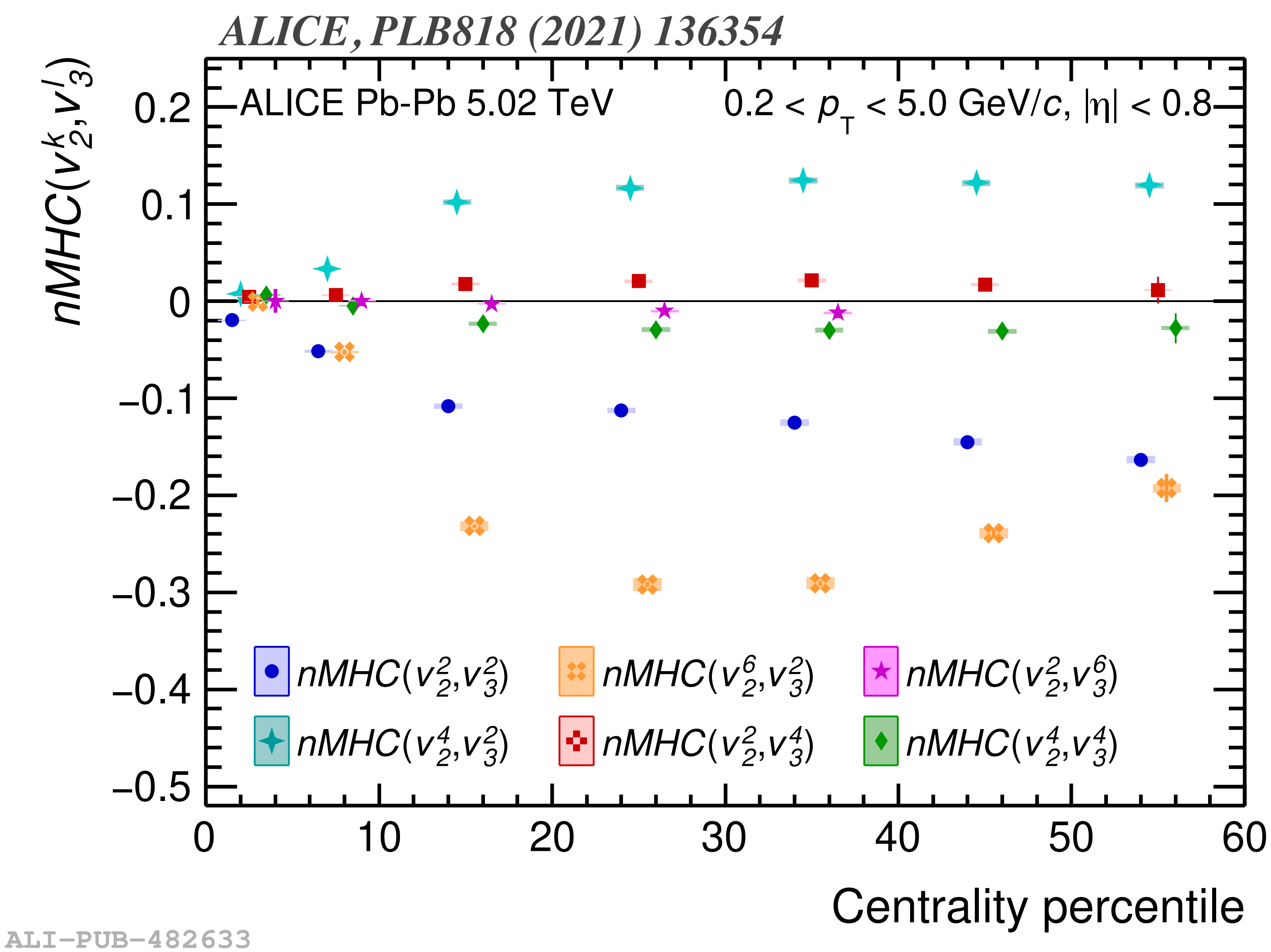
m-particle cumulant

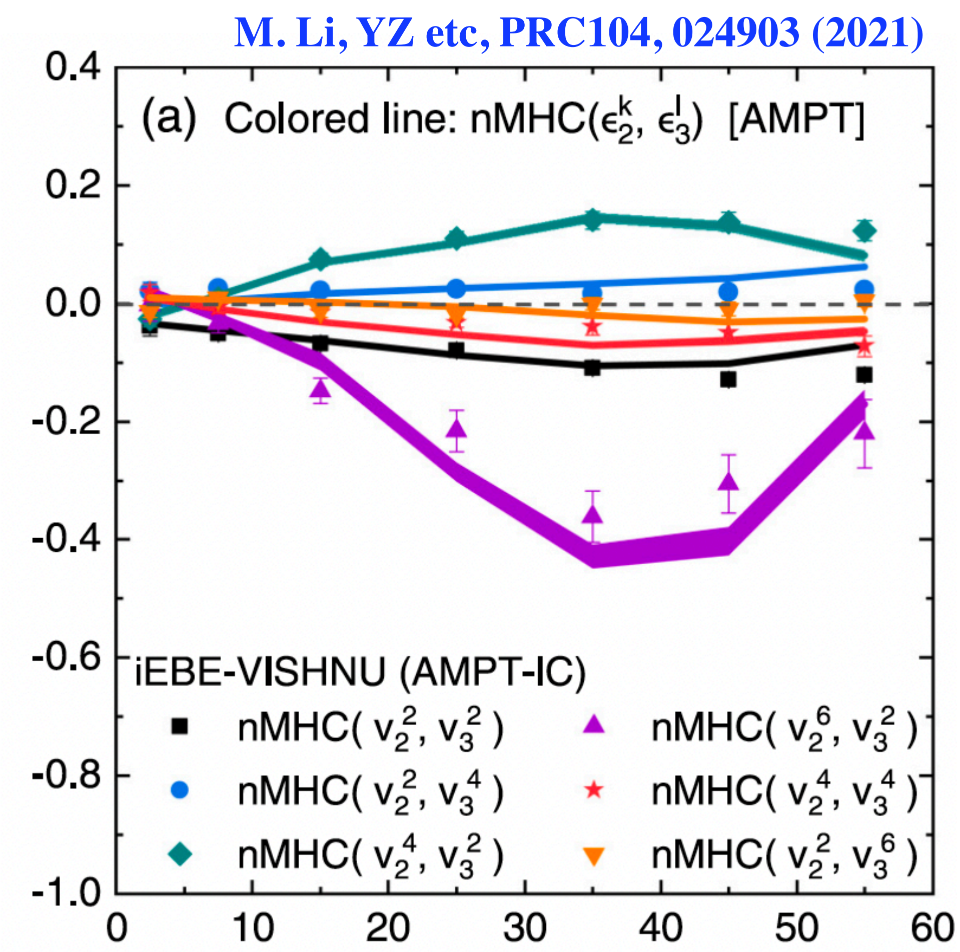
```
complex Cumulant(int* harmonic, int n, bool remove_zeros=true, int negsplit=-1,
     int mult = 1, int skip = 0)
    {
     bool remove_term = false;
     if (remove_zeros)
     {
      int har_sum = 0;
      for (int i = 0; i<mult; ++i) har_sum += harmonic[n-1+i];
      if (har_sum != 0) remove_term = true;
     }
     complex c = 0;
     if (!remove_term)
     {
      c = Corr(harmonic+(n-1), mult);
      if (n == 1) return c;
      c *= negsplit*Cumulant(harmonic, n-1, remove_zeros, negsplit-1);
     }

     int h_hold = harmonic[n-2];
     for (int counter = 0; counter <= n-2-skip; ++counter)
     {
      harmonic[n-2] = harmonic[counter];
      harmonic[counter] = h_hold;
      c += Cumulant(harmonic, n-1, remove_zeros, negsplit, mult+1, n-2-counter);
      harmonic[counter] = harmonic[n-2];
     }
     harmonic[n-2] = h_hold;
     return c;
    }
```

- One algorithm for any particle cumulant
  - Multi-particle mixed harmonic cumulants
  - correlation between v_m^k, v_n^l and v_p^q
  - correlation between v_m^k and v_n^l
  - No need of any package !

Correlations between v_2^k and v_3^L

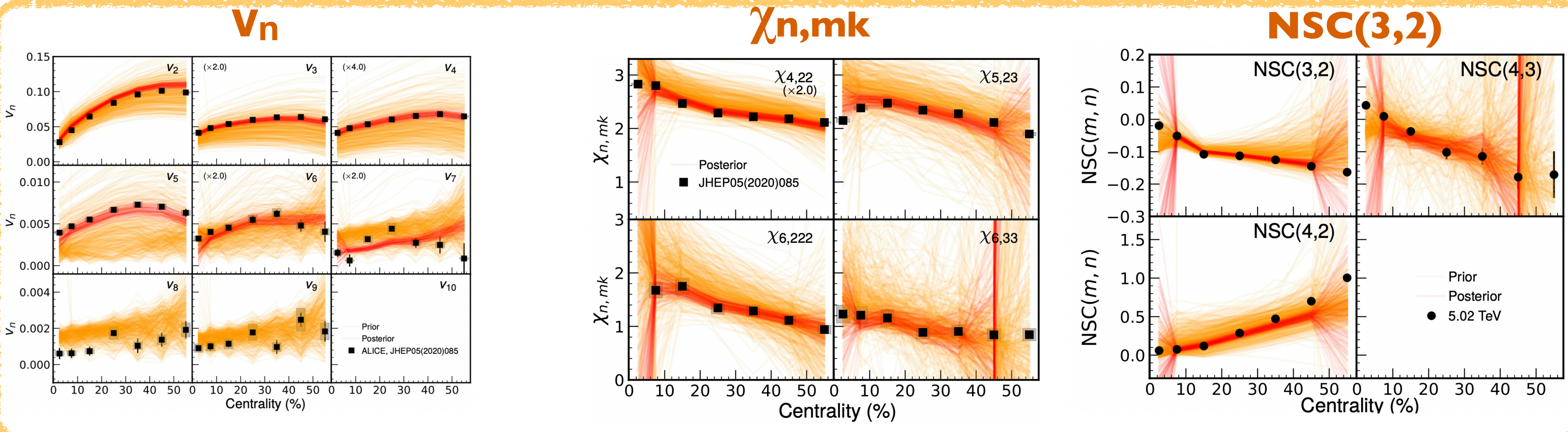


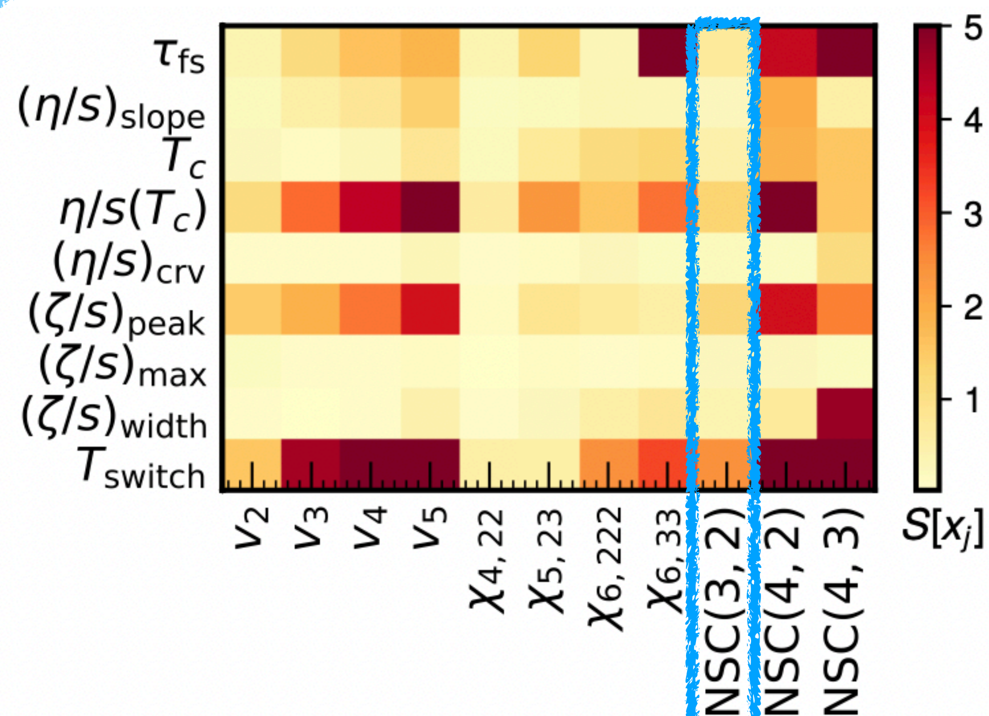


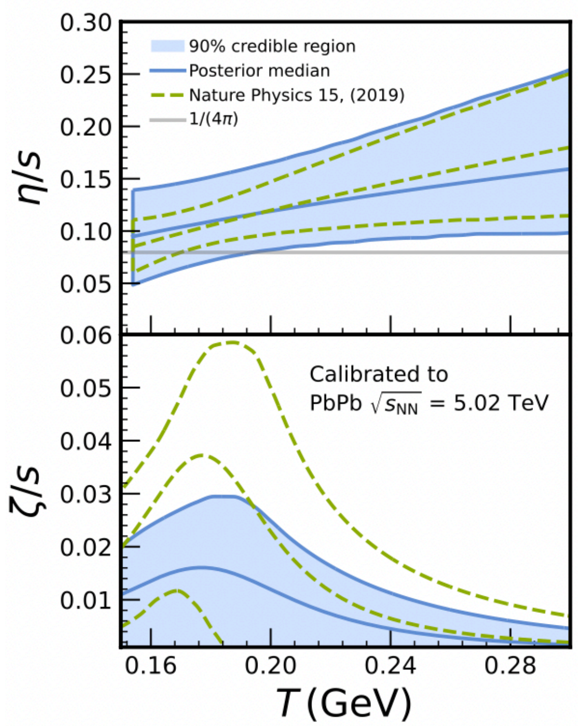
❖ First measurement of correlations between higher order moments of v_2 and v_3

- characteristic -, +, - signs observed for 4-, 6- and 8-particle cumulants of *mixed harmonic*
- Final state results quantitatively reproduced by the initial state correlations
- Experimental data provides direct constraints on the correlations of higher order moments of eccentricity coefficients from initial state models

Bayesian analysis with more flow observables



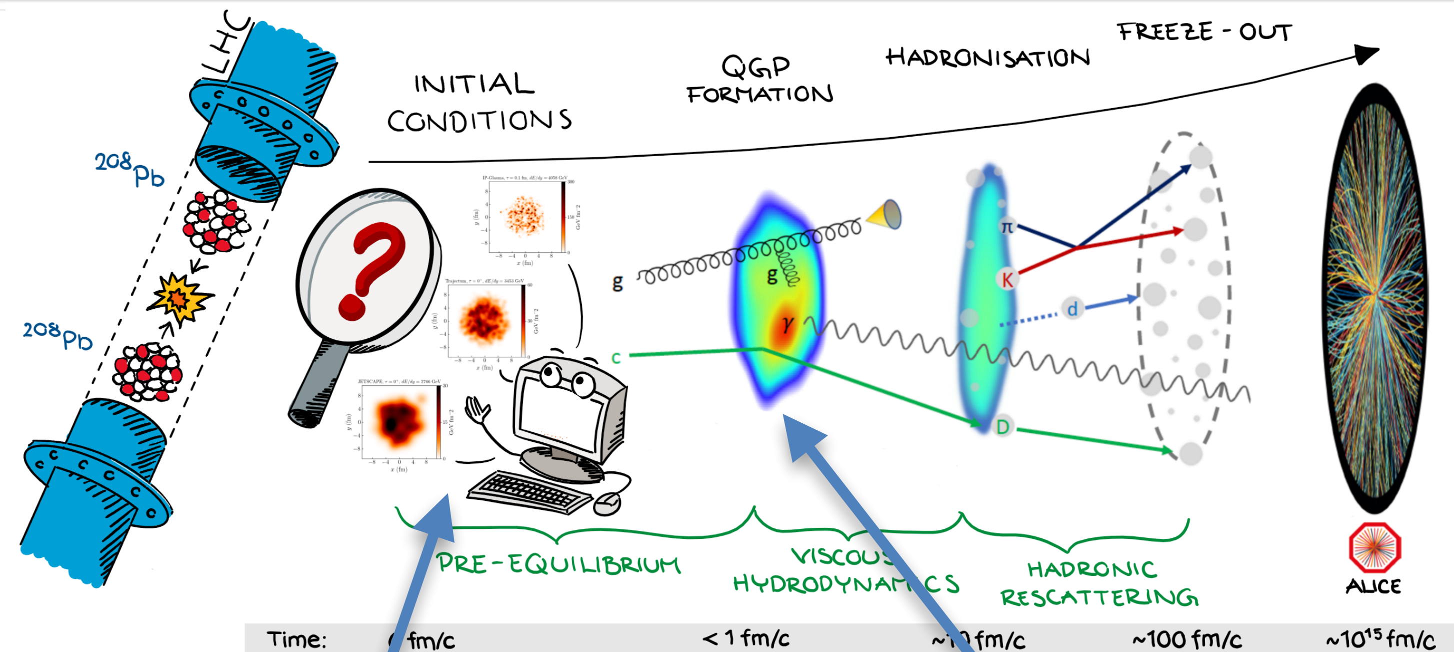




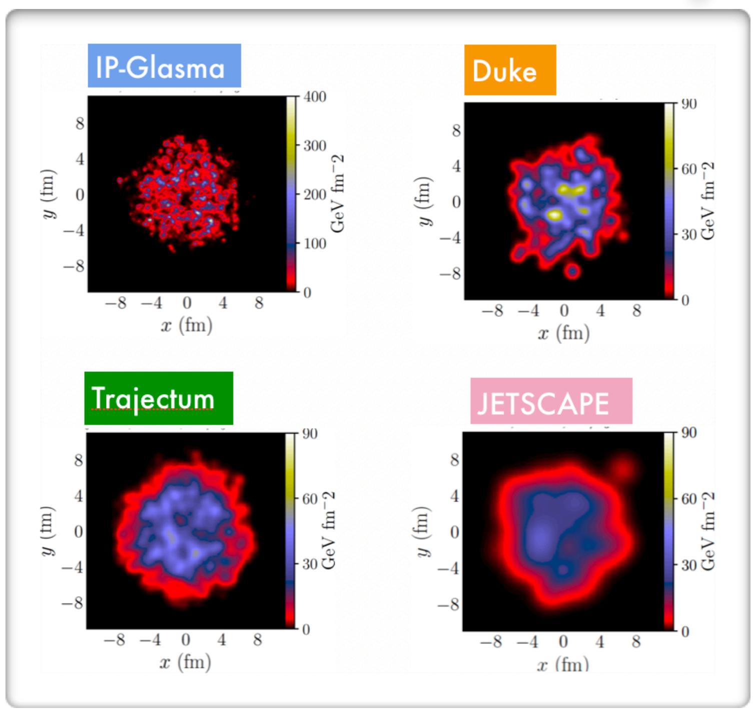
Initial conditions	
Parameter	MAP value
Norm	21.06
p	0.0077
σ_k	0.881
d^3_{min}	0.975
τ_{fs}	0.901
T_c	0.147
T_{switch}	0.160

Details, see talk later by D. J. Kim

QGP study: state-of-the-art



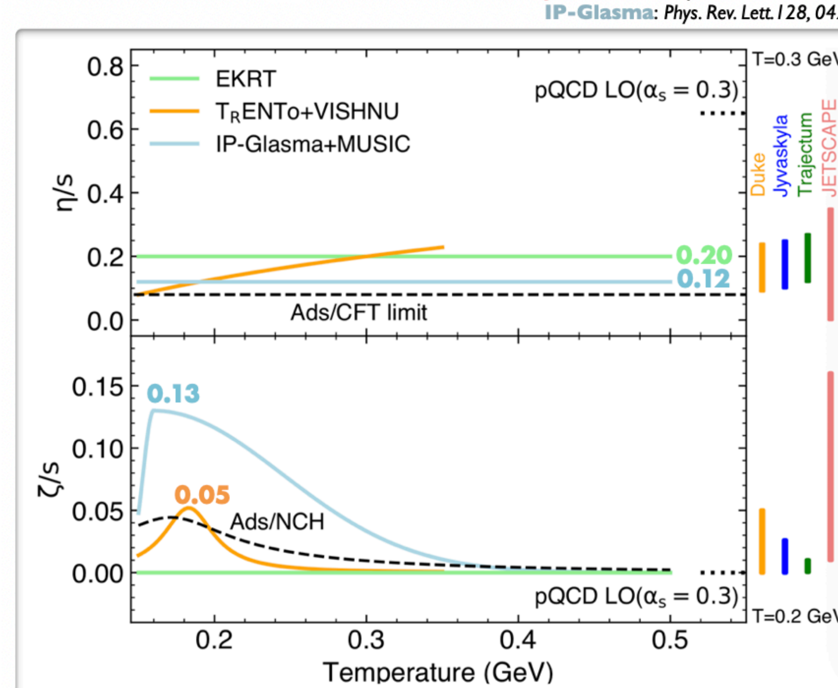
Initial State models
(initial energy density)



Extracted viscosities of QGP

- shear viscosity η/s
- bulk viscosity ζ/s

Duke: *Nature Phys.* 15 (2019) 11, 1113
Jyväskylä: *Phys. Rev. C 104, 054904 (2021)*
Trajectum: *Phys. Rev. Lett. 126, 202301 (2021)*
JETSCAPE: *Phys. Rev. Lett. 126, 242301 (2021)*
IP-Glasma: *Phys. Rev. Lett. 128, 042301 (2022)*



Huge uncertainties of the extracted *QGP properties*, due to poorly known *initial conditions*

Correlations for the next decade

Higher-order transverse momentum fluctuations in heavy-ion collisions

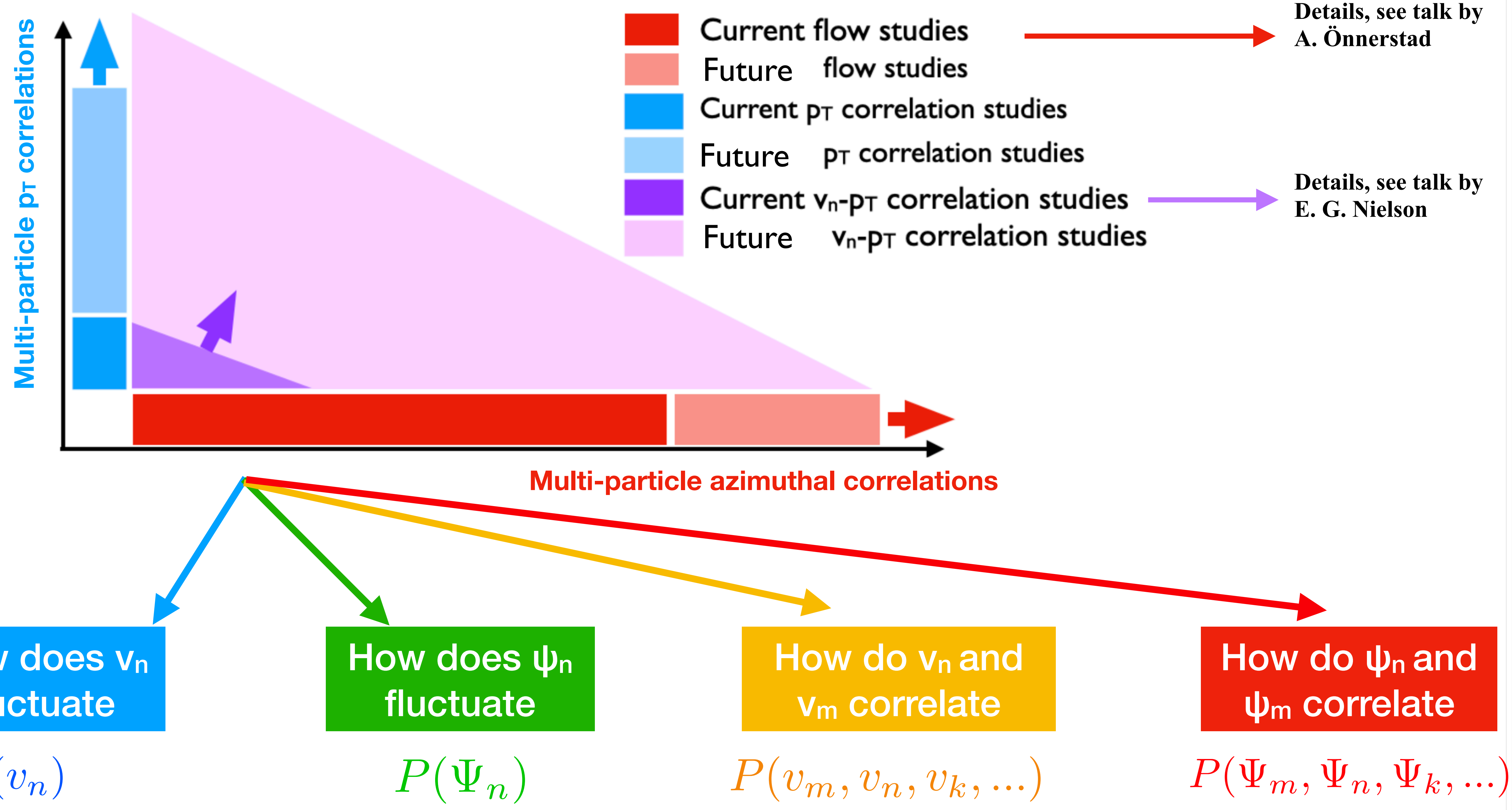
Somadutta Bhatta, Chunjian Zhang, and Jiangyong Jia
Phys. Rev. C **105**, 024904 – Published 7 February 2022

p_T-p_T-p_T correlations

Higher order cumulants of transverse momentum and harmonic flow in relativistic heavy ion collisions

Piotr Bożek and Rupam Samanta
Phys. Rev. C **104**, 014905 – Published 28 July 2021

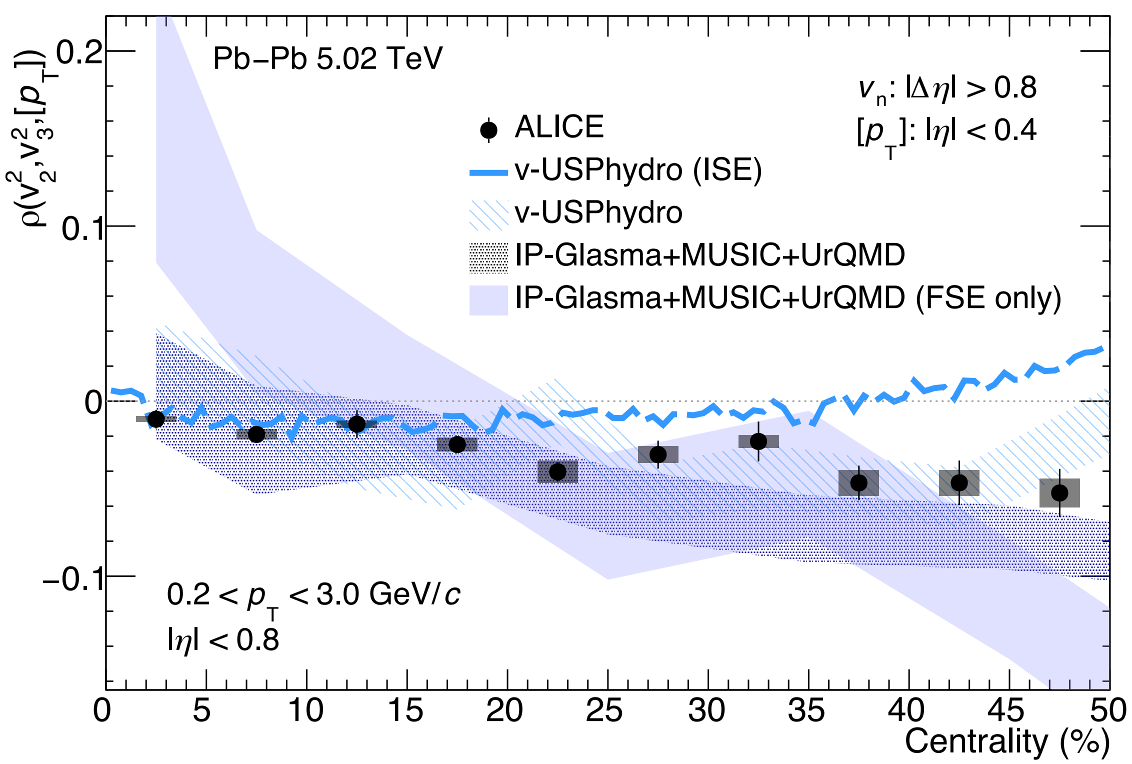
p_T-v_n^2-v_m^2 correlations



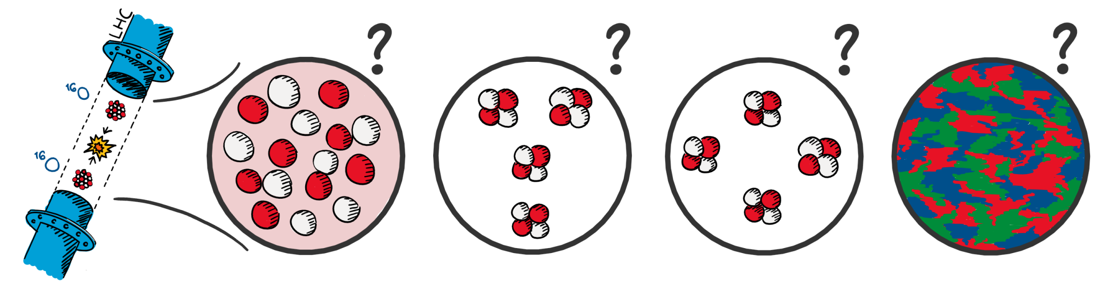
The first attempt

❖ The **first** measurement of higher-order [p_T], v_2 and v_3 correlations P. Bozek etc, PRC104 (2021) 1, 014905

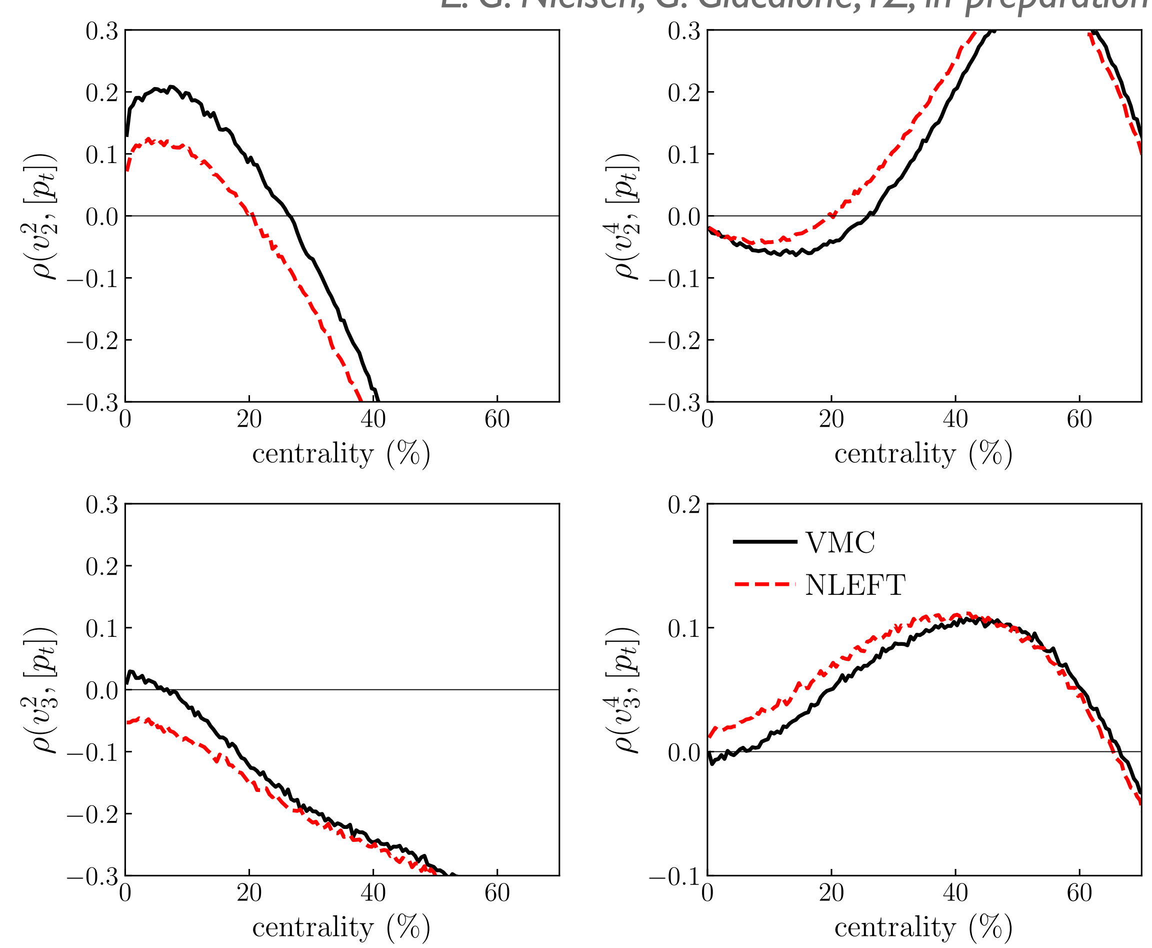
$$\rho(v_m^2, v_n^2, [p_T]) = \frac{Cor(v_m^2, v_n^2, [p_T])}{\sqrt{Var(v_m^2)}\sqrt{Var(v_n^2)}\sqrt{c_k}} - \frac{\langle v_m^2 \rangle}{\sqrt{Var(v_m^2)}} \cdot \rho_n - \frac{\langle v_n^2 \rangle}{\sqrt{Var(v_n^2)}} \cdot \rho_m - \frac{\langle [p_T] \rangle}{\sqrt{c_k}} \cdot \frac{SC(m,n)}{\sqrt{Var(v_m^2)}\sqrt{Var(v_n^2)}},$$



❖ The **first** calculation on higher-order [p_T], v_n^k correlations



E. G. Nielsen, G. Giacalone, YZ, in preparation



Summary

The developments of multi-particle correlations have been discussed

☆ **Multi-particle azimuthal angle correlations**

- More than two decades "flow study" to probe the QGP properties
- Allow to extract $\eta/s(T)$, $\zeta/s(T)$ via Bayesian analyses, representing the state-of-the-art understanding of QGP.

☆ **Multi-particle correlations between v_n and $[p_T]$**

- Direct access to the initial conditions -> pin down the uncertainty of QGP studies
- For the first time we see completely different flow behaviours using different initial state models
  - Size of nucleon
  - Extra information on IC?
  - Potential tool to study nuclear structure at the LHC

Thanks for your attention!

Initial Stages 2023 conference, Copenhagen

Scientific Programme

- Partonic structure of protons and nuclei
- Physics at low-x and gluon saturation
- The initial stages and nuclear structure in heavy-ion collisions
- Collective dynamics from small to large systems
- New theoretical techniques at large and small coupling
- New facilities: DIS and hadronic experiments

Programme and Nordic Organisation Committee

- You Zhou 周铀 (NBI, Conference Chair)
- Jens Jørgen Gaardhøje (NBI, Conference co-chair)
- Ian Bearden (NBI, Conference co-chair)
- Jürgen Schukraft (NBI/CERN)
- Mateusz Ploskon (LBNL)
- Jiangyong Jia (Stony Brook/BNL)
- Aleksi Kurkela (Stavanger U.)
- Tuomas Lappi (Jyvaskyla U.)
- Christian Bierlich (Lund U.)
- Alice Ohlson (Lund U.)
- Vytautas Vislavicius (Lund U.)
- Larissa Bravina (Oslo U.)
- Katarína Křížková Gajdošová (CERN)
- Debojit Sarkar (NBI, Conference secretary)

Backup

Intense interactions between two communities

RIKEN BNL Research Center

Physics Opportunities from the RHIC Isobar Run

This workshop will be held virtually.
January 25–28, 2022

Jan. 2022 (BNL)

GSI Helmholtzzentrum für Schwerionenforschung GmbH

EMMI

EMMI Rapid Reaction Task Forces

2022

EMMI RRTF
Nuclear physics confronts relativistic collisions of isobars
part 1: May 30-Jun 03, 2022, U Heidelberg, Germany
part 2: Oct 12-14, 2022, U Heidelberg, Germany

May 2022 (Heidelberg)
Oct. 2022 (Heidelberg)

DE LA RECHERCHE À L'INDUSTRIE
cea
ESNT
Espace de Structure Nucléaire Théorique
DSM - DAM

Deciphering nuclear phenomenology across energy scales

Back to the ESNT page

20-23 September 2022

Sept. 2022 (Saclay)

PROGRAM

Deciphering nuclear phenomenology across energy scales
Organizers: G. Giacalone (Univ. Heidelberg, contact), J.-Y. Ollitrault (CEA IPhT), You Zhou (N. Bohr Institute)

The plan is to have 4 days of intense discussions among experts (both theorists and experimentalists) from the high and low-energy nuclear communities to achieve the following goals :

1. Reviewing the variety of deformation effects (static deformation versus shape fluctuations, shape coexistence, shape changes when exciting nuclei...) observed in low-energy experiments, their imprint on observable phenomena in low-energy nuclear structure and reactions, and the types of deformation that are deduced from the data (axial, non-axial, octupolarity, hexadecapolarity...).
2. Reviewing the status of the manifestations of nuclear structure at high-energy colliders, what phenomena are deduced from such data (e.g. deformations, neutron-skin...), and how nuclear structure effects are modeled in theoretical calculations.
3. Pointing out important conceptual issues centred around the question : are high-energy and low-energy experiments observing the manifestation of the same thing ?
4. Identifying information from potential new low-energy experiments that may help explain high-energy data (e.g. recent evidence of deformation in collisions of 96Zr and 96Ru nuclei), and, conversely, assessing whether the great flexibility of high-energy experiments may be used to probe phenomena difficult to access via spectroscopic experiments (e.g. the deformations of any stable odd-mass nucleus, imprint of octupole deformations...).
5. Identifying optimal choices of nuclei that, if collided at high energy, would help make progress in our understanding of nuclear phenomenology across energy scales.

PRELIMINARY PROGRAM

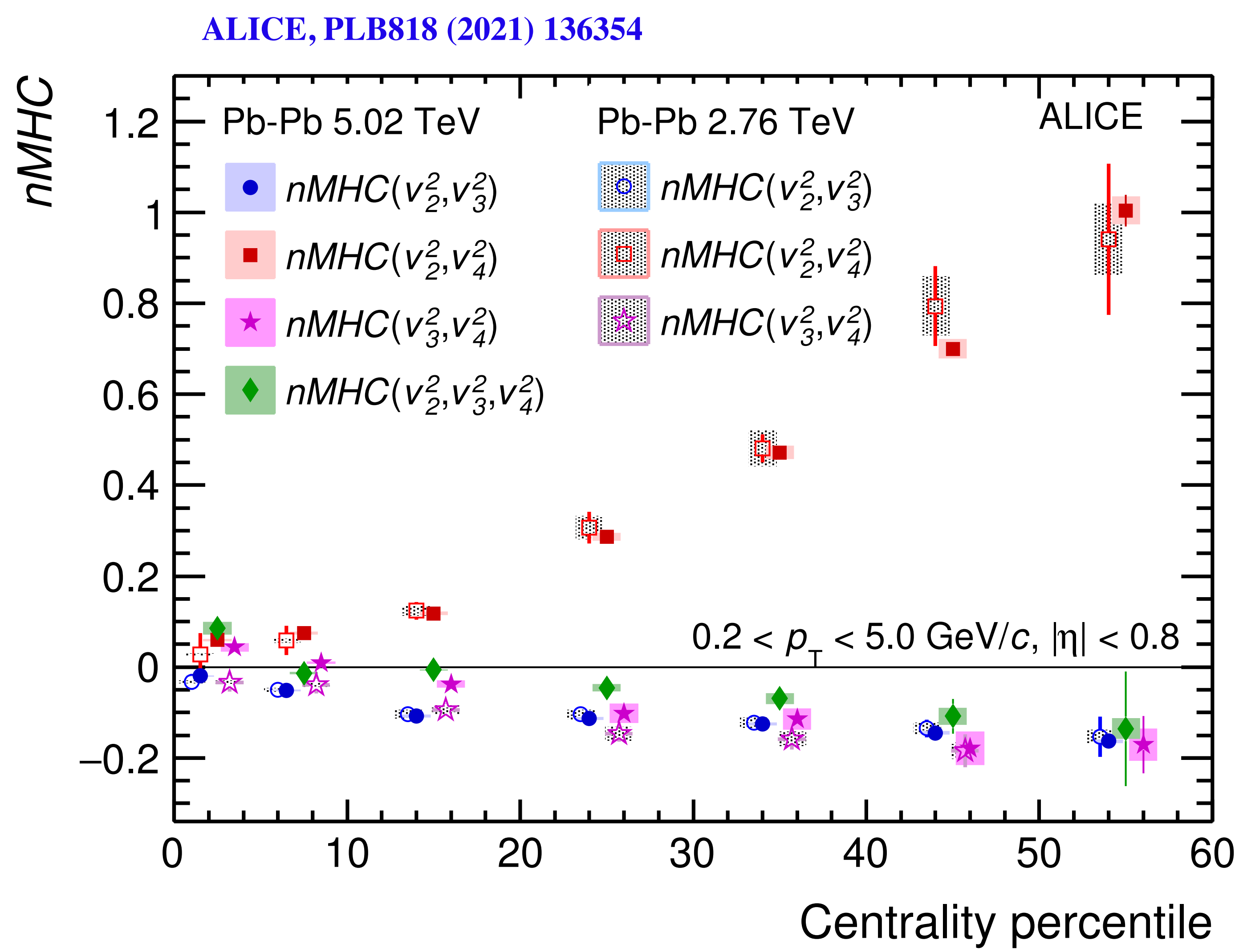
Day 1 – Tuesday 20/09/2022 – Topic : ***Nuclear deformation across energy scales***

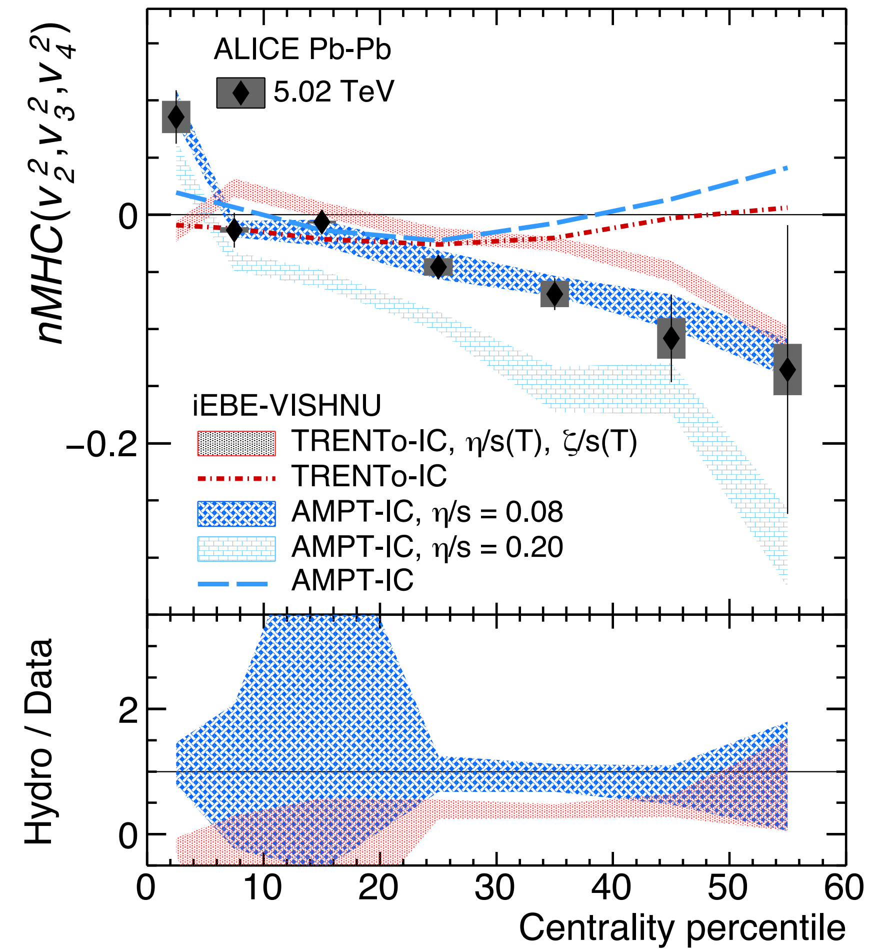
Day 2 – Wed. 21/09/2022 – Topic : ***Relativistic collisions of isobars : the 2021 breakthrough***

Day 3 – Thursday 22/09/2022 – Topic : ***Nuclear models across energy scales***

Day 4 – Friday 23/09/2022 – Topic : ***Future perspectives on nuclear studies at high energy***

Correlations between $v_m^2, v_n^2, v_k^2, \ldots$





$$\mathrm{MHC}(v_2^2, v_3^2, v_4^2) = \langle\langle e^{i(2\varphi_1+3\varphi_2+4\varphi_3-2\varphi_4-3\varphi_5-4\varphi_6)}\rangle\rangle_c$$
$$= \langle v_2^2\, v_3^2\, v_4^2\rangle - \langle v_2^2\, v_3^2\rangle\langle v_4^2\rangle - \langle v_2^2\, v_4^2\rangle\langle v_3^2\rangle$$
$$-\langle v_3^2\, v_4^2\rangle\langle v_2^2\rangle + 2\langle v_2^2\rangle\langle v_3^2\rangle\langle v_4^2\rangle.$$

- Non-zero value of *nMHC*(v_2^2, v_3^2, v_4^2) in Pb-Pb collisions
  - Highly non-trivial correlations among three flow coefficients

O-O projection studies

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ALICE

CERN

ALICE-PUBLIC-2021-004

ALICE physics projections for a short oxygen-beam run at the LHC

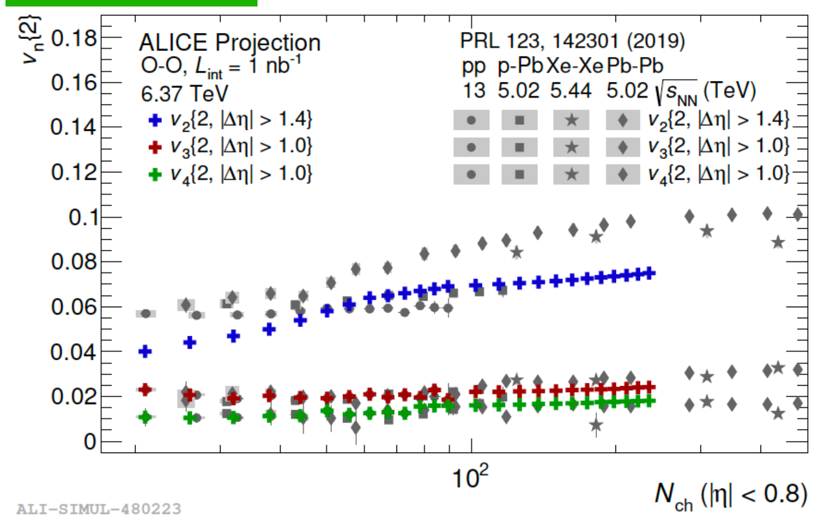
ALICE Collaboration[*]

Abstract

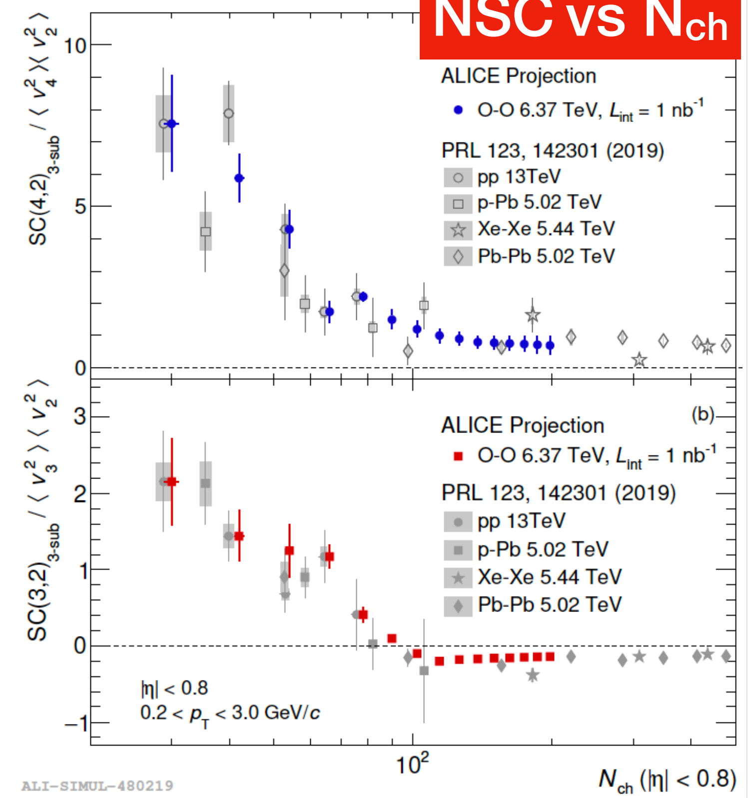
This document collects performance projections for a selection of measurements that can be carried out with a short O–O run during the LHC Run 3. The baseline centre-of-mass energy per nucleon–nucleon collision is $\sqrt{s_{NN}} = 6.37$ TeV and measurement uncertainties are given for the integrated luminosity $L_{int} = 1$ nb^{-1}. Some projections for p–O collisions are also included. These studies were presented at the CERN workshop on Opportunities of O–O and p–O collisions at the LHC [1, 2].

(did not consider the structure of ^{16}O)

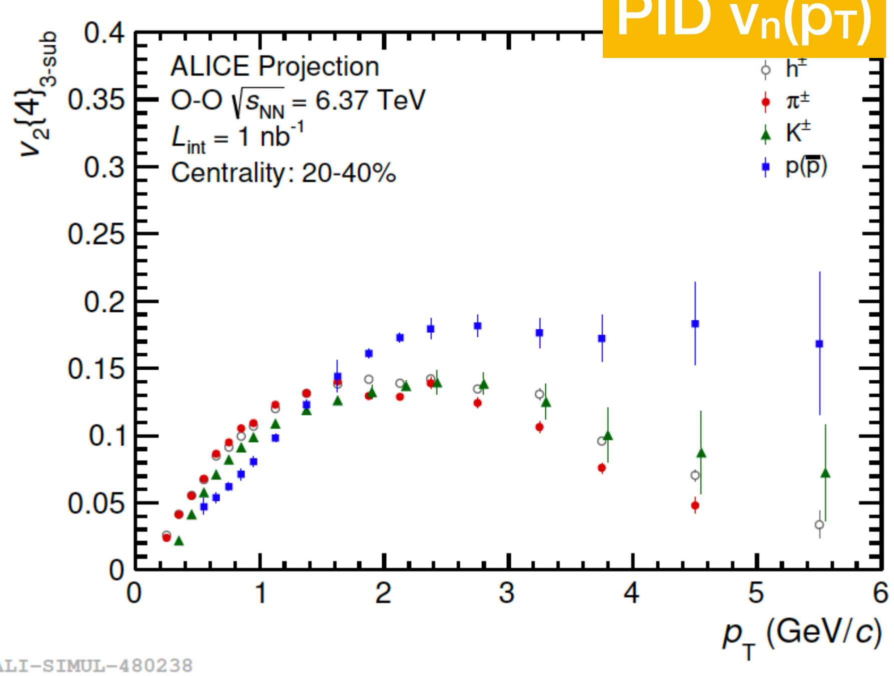
v_n vs N_{ch}



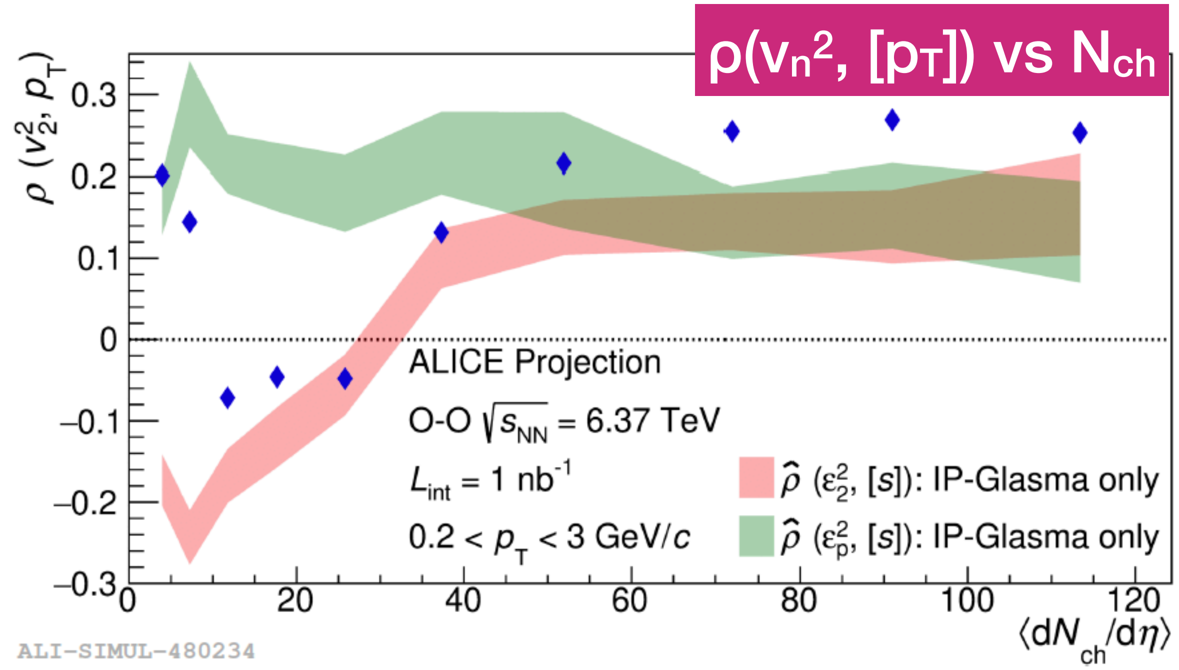
NSC vs N_{ch}



PID $v_n(p_T)$

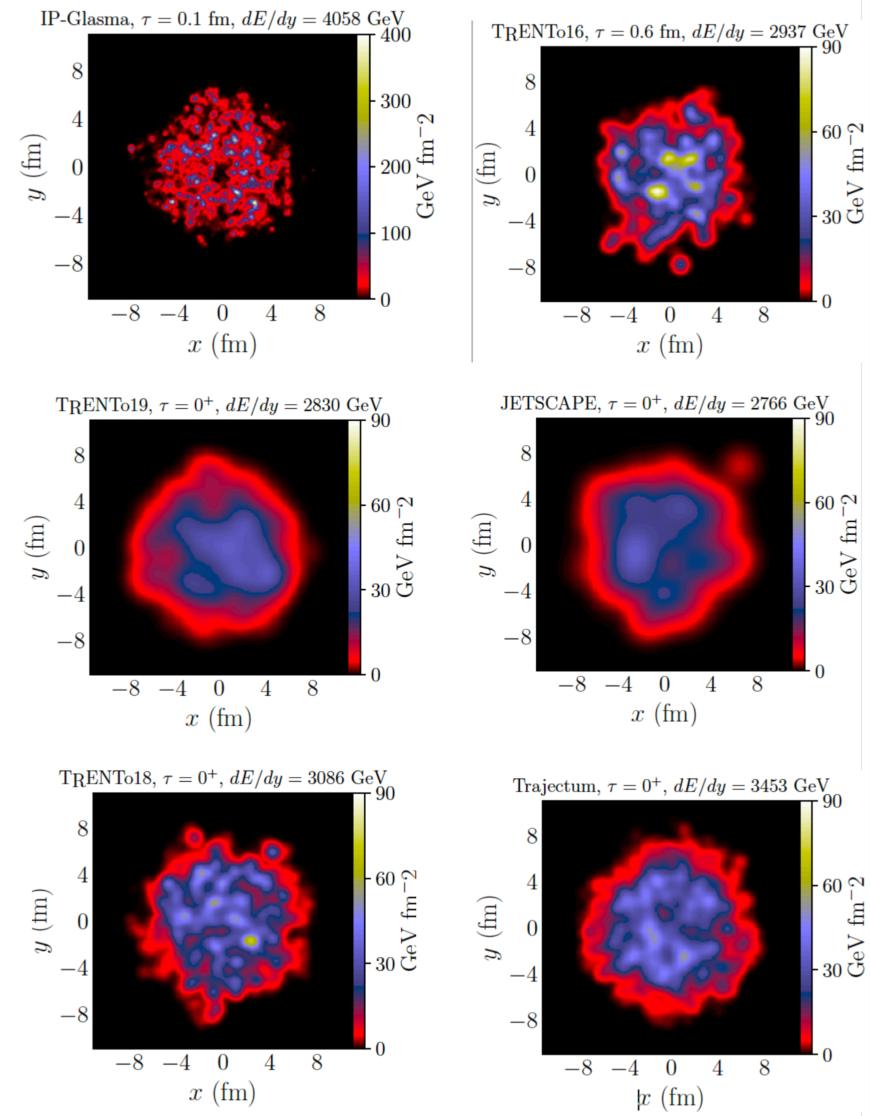


$\rho(v_n^2, [p_T])$ vs N_{ch}



Current status of initial state models

Credits: G. Giacalone



– **"sharp" models: IP-GLASMA and TRENTo 2016**

[Schenke, Shen, Tribedy **2005.14682**]

[Bass, Bernhard, Moreland **1605.03954**]

Nucleons have a width of ~0.5fm (trento), 3 sub-nucleons with size ~0.1fm (IP-Glasma). Trento is used for the entropy density at the beginning of hydro.

– **"fat" models: TRENTo 2019 and JETSCAPE**

[Bass, Bernhard, Moreland **Nature Phys. 15 (2019)**]

[JETSCAPE Collaboration **2011.01430**, **2010.03928**]

[Parkkila, Onnerstad, Kim **2106.05019**]

The Trento parametrization is now used for the energy density at tau=0+. There is no substructure. The nucleon width is now ~1fm. Very smooth profiles.

– **"bumpy" models: TRENTo 2018 and Trajectum**

[Bass, Bernhard, Moreland **1808.02106**]

[Nijs, van der Schee, Gürsoy, Snellings **2010.15130**, **2010.15134**]

The Trento parametrization is the energy density at tau=0+. Substructure is included: 4-6 constituents with width ~0.4fm. Profiles with some lumpiness.

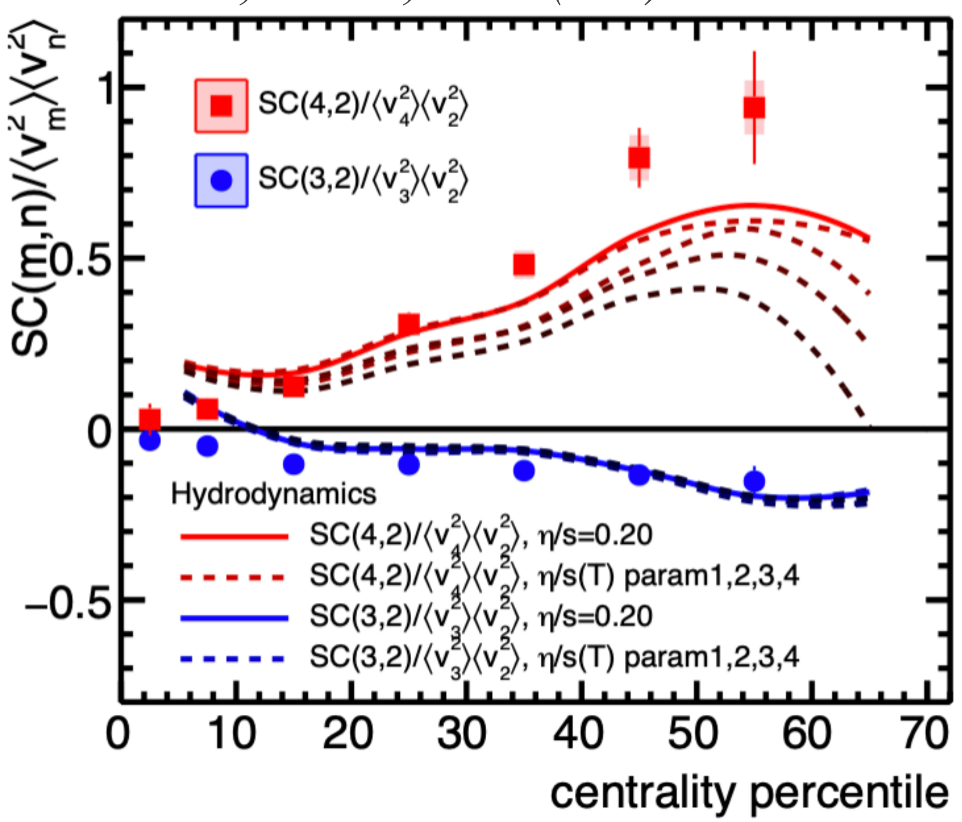
How can we access the initial conditions in EXP ?

(Normalized) Symmetric Cumulant

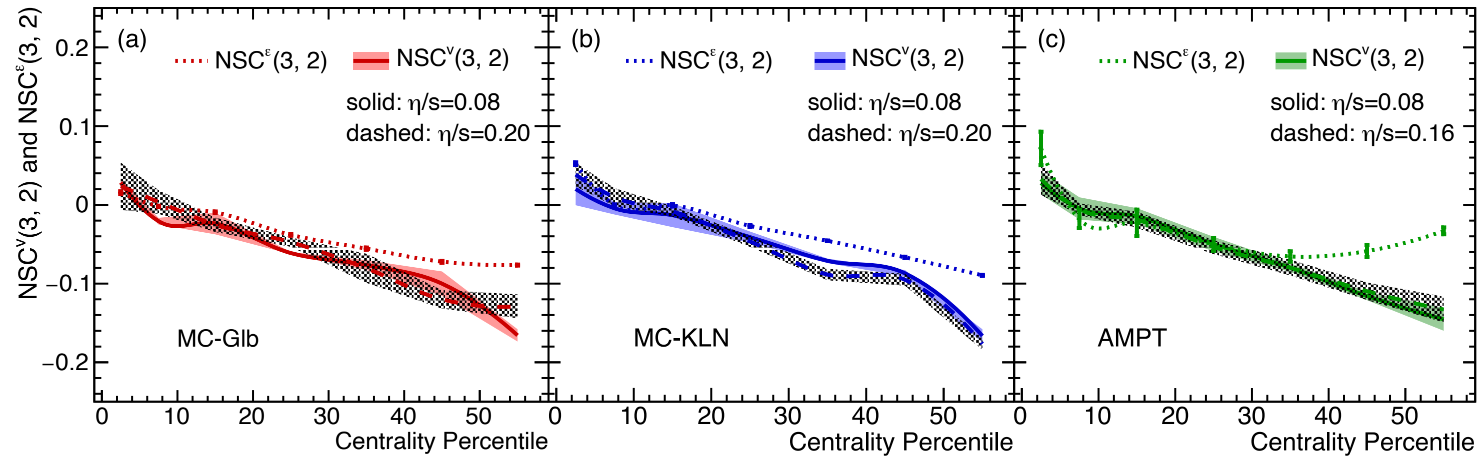
Symmetric cumulants:

$$SC(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

ALICE, PRL117, 182301 (2016)



X. Zhu etc, PRC95, 044902 (2017)



$$v_2 \propto \varepsilon_2 \qquad v_3 \propto \varepsilon_3$$

$$\frac{\langle v_2^2\, v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle}{\langle v_2^2 \rangle \langle v_3^2 \rangle} = \frac{\langle \varepsilon_2^2\, \varepsilon_3^2 \rangle - \langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}{\langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}$$

Or: $NSC^v(3,2) = NSC^\varepsilon(3,2)$

Final state   Initial state

❖ The very first direct measurement of correlations between v_n and v_m

- NSC(4,2) is sensitive to η/s(T), good probe of QGP properties.
- NSC(3,2) is insensitive to η/s(T), provides a direct access into the initial conditions (despite details of systems evolution)

$\langle p_T \rangle$ - v_n correlations

- Shape of the fireball: Anisotropic flow
- Size of the fireball: radial flow, $[p_{\rm T}]$
- Initial geometry and fluctuations of shape and size
- Final state: correlation between v_n and $p_{\rm T}$

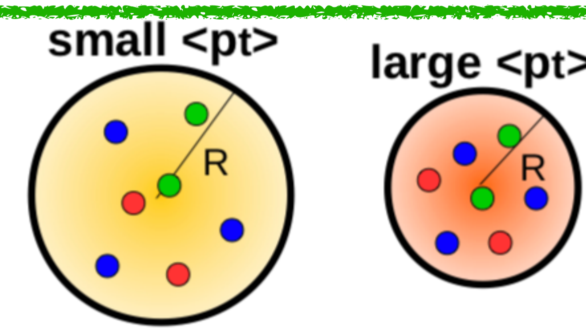
$$\rho(v_n^2, [p_{\rm T}]) = \frac{cov(v_n^2, [p_{\rm T}])}{\sqrt{var(v_n^2)}\sqrt{var([p_{\rm T}])}}$$

P. Bozek etc, PRC96 (2017) 014904

$$V_n \propto \mathcal{E}_n$$

$$\frac{d\langle p_t\rangle}{\langle p_t\rangle} \propto \frac{dE}{E}$$

$$\frac{d\langle p_t\rangle}{\langle p_t\rangle} \propto -\frac{dR}{R}$$



- Assuming $v_n \propto \varepsilon_n$, $[p_T] \propto E_0$

$$\rho(v_n^2, [p_{\rm T}]) = \rho(\varepsilon_n^2, [E_0])$$

final-state model calculation

Initial-state model estimation

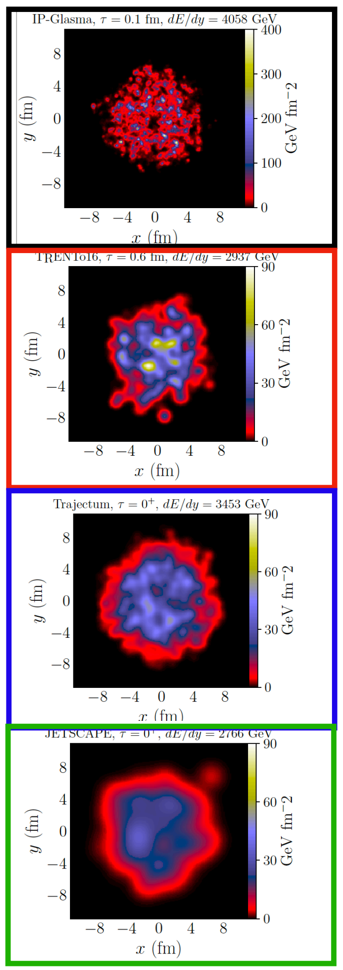
- One can compare $\rho(v_n^2, [p_{\rm T}])$ measurements to $\rho(\varepsilon_n^2, [E_0])$ calculations, to constrain the initial state model

Details, see earlier talk by E. G. Nielsen

Difference in IP-Glasma and TRENTo: potential explanations

❖ Sensitive to the nucleon width parameter (size of nucleon)

- IP-Glasma ~ 0.4; v-USPhydro ~ 0.5; Trajectum~0.7; JETSCAPE (TRENTo) ~ 1.1
- w(IP-Glasma) < w(v-USPhydro) < w(Trajectum) < w(JETSCAPE)
- New constraints on the ***nucleon size***

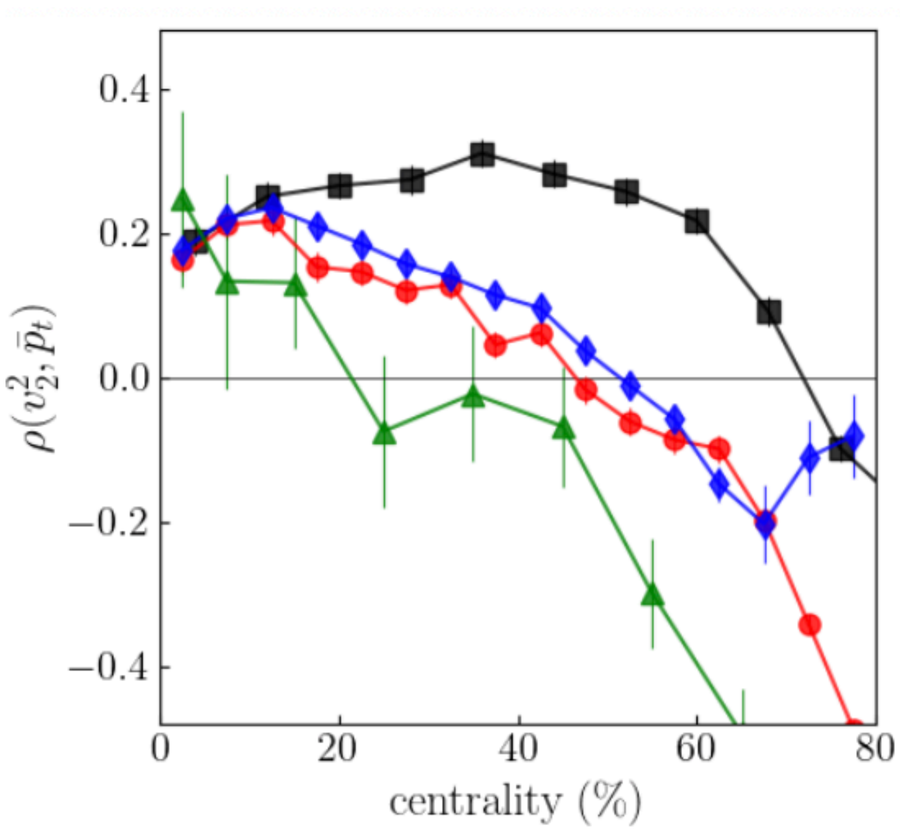


w ~ 0.4

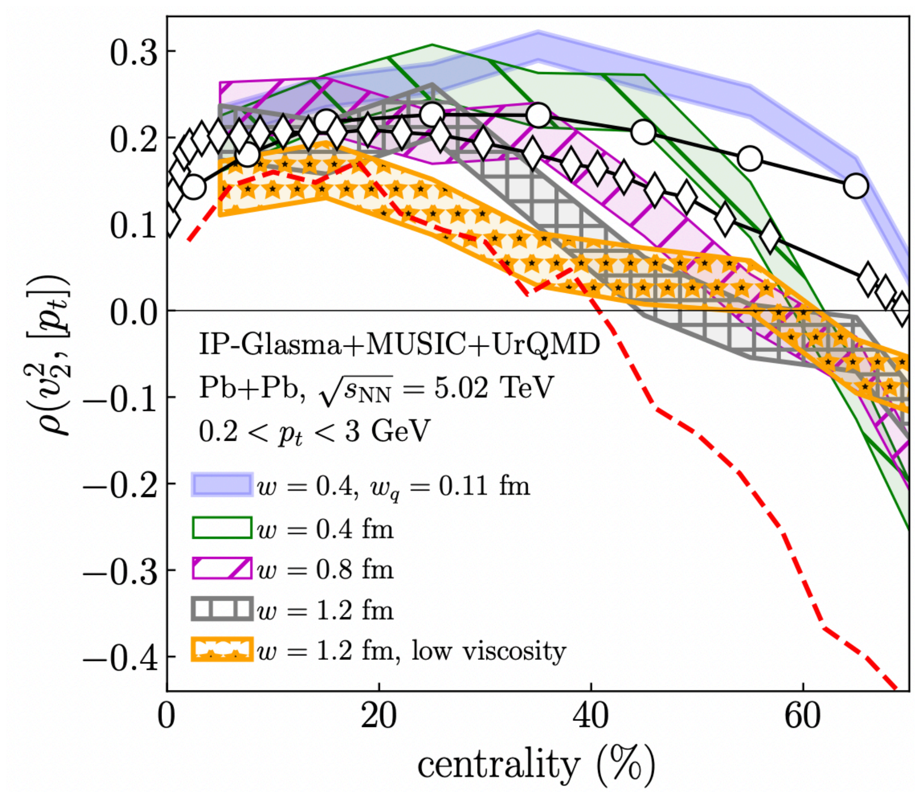
w ~ 0.5

w ~ 0.7

w ~ 1.1



Giacalone, Schenke, Shen, PRL128 042301 (2022)



❖ Different types of thickness functions

- TRENTo $\left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$ with p≈0 $\sqrt{T_A T_B}$ IP-Glasma $T_A T_B$ type

❖ Different contributions from pre-hydrodynamic phase (free streaming) and sub-nucleon structure

Higher-order correlations

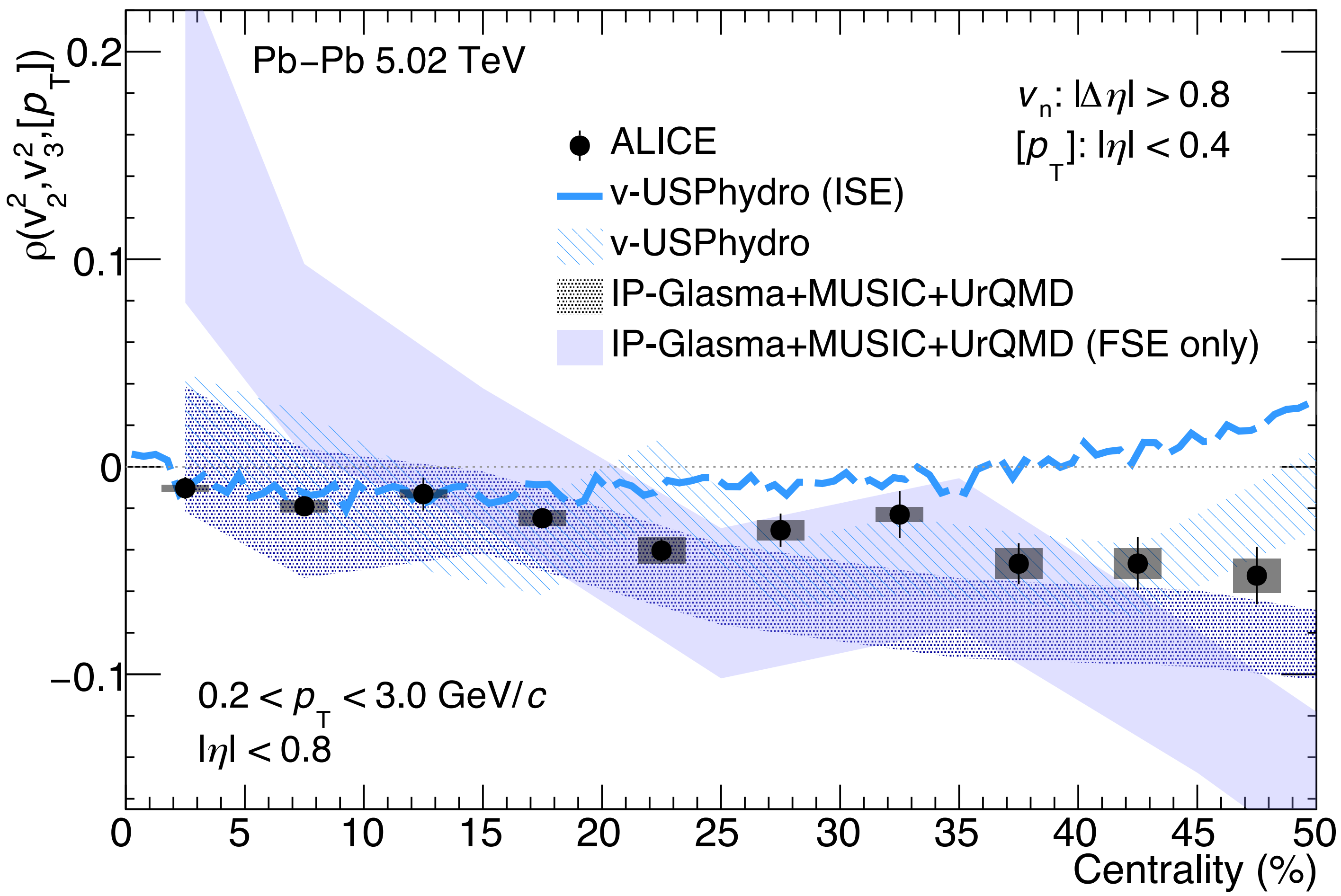
❖ The **first** measurement of higher-order $[p_T]$, v_2 and v_3 correlations **P. Bozek etc, PRC104 (2021) 1, 014905**

$$\rho(v_m^2, v_n^2, [p_T]) = \frac{C(v_m^2, v_n^2, [p_T])}{\sqrt{\mathrm{Var}(v_m^2)}\sqrt{\mathrm{Var}(v_n^2)}\sqrt{c_k}} - \frac{\langle v_m^2 \rangle}{\sqrt{\mathrm{Var}(v_m^2)}} \cdot \rho_n - \frac{\langle v_n^2 \rangle}{\sqrt{\mathrm{Var}(v_n^2)}} \cdot \rho_m - \frac{\langle [p_T] \rangle}{\sqrt{c_k}} \cdot \frac{SC(m,n)}{\sqrt{\mathrm{Var}(v_m^2)}\sqrt{\mathrm{Var}(v_n^2)}}$$

❖ the first ρ_{23} measurement is non-zero
- negative for the presented centrality
- anti-correlations between two flow coefficients and $[p_T]$

❖ ρ_{23} from IP-Glasma and v-USPhydro are different for centrality >40%
- Weaker centrality dependence of full IP-Glasma while strong dependence for FSE only, indication?
- More simulations are needed

❖ Not conclusive on which model works better due to sizeable uncertainties from model calculations



$P(v_m, v_n, v_k, \ldots)$

Mixed harmonic cumulants with 4-particles

$$\mathrm{MHC}(v_m^2, v_n^2) = \mathrm{SC}(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \, \langle v_n^2 \rangle$$

Mixed harmonic cumulants with 6-particles

$$\begin{aligned}\mathrm{MHC}(v_2^4, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3-2\varphi_4-2\varphi_5-3\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^4\, v_3^2\rangle - 4\langle v_2^2\, v_3^2\rangle\langle v_2^2\rangle - \langle v_2^4\rangle\langle v_3^2\rangle \\ &\quad + 4\langle v_2^2\rangle^2\langle v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^4) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3-2\varphi_4-3\varphi_5-3\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^4\rangle - 4\langle v_2^2\, v_3^2\rangle\langle v_3^2\rangle - \langle v_2^2\rangle\langle v_3^4\rangle \\ &\quad + 4\langle v_2^2\rangle\langle v_3^2\rangle^2.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^2, v_4^2) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+4\varphi_3-2\varphi_4-3\varphi_5-4\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^2\, v_4^2\rangle - \langle v_2^2\, v_3^2\rangle\langle v_4^2\rangle - \langle v_2^2\, v_4^2\rangle\langle v_3^2\rangle \\ &\quad - \langle v_3^2\, v_4^2\rangle\langle v_2^2\rangle + 2\langle v_2^2\rangle\langle v_3^2\rangle\langle v_4^2\rangle.\end{aligned}$$

Mixed harmonic cumulants with 8-particles

$$\begin{aligned}\mathrm{MHC}(v_2^6, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+2\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-2\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^6\, v_3^2\rangle - 9\langle v_2^4\, v_3^2\rangle\langle v_2^2\rangle - \langle v_2^6\rangle\langle v_3^2\rangle \\ &\quad - 9\langle v_2^4\rangle\langle v_2^2\, v_3^2\rangle - 36\langle v_2^2\rangle^3\langle v_3^2\rangle \\ &\quad + 18\langle v_2^2\rangle\langle v_3^2\rangle\langle v_2^4\rangle + 36\langle v_2^2\rangle^2\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^4, v_3^4) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-3\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^4\, v_3^4\rangle - 4\langle v_2^4\, v_3^2\rangle\langle v_3^2\rangle \\ &\quad - 4\langle v_2^2\, v_3^4\rangle\langle v_2^2\rangle - \langle v_2^4\rangle\langle v_3^4\rangle \\ &\quad - 8\langle v_2^2\, v_3^2\rangle^2 - 24\langle v_2^2\rangle^2\langle v_3^2\rangle^2 \\ &\quad + 4\langle v_2^2\rangle^2\langle v_3^4\rangle + 4\langle v_2^4\rangle\langle v_3^2\rangle^2 \\ &\quad + 32\langle v_2^2\rangle\langle v_3^2\rangle\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^6) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-3\varphi_6-3\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^6\rangle - 9\langle v_2^2\, v_3^4\rangle\langle v_3^2\rangle - \langle v_3^6\rangle\langle v_2^2\rangle \\ &\quad - 9\langle v_3^4\rangle\langle v_2^2\, v_3^2\rangle - 36\langle v_2^2\rangle\langle v_3^2\rangle^3 \\ &\quad + 18\langle v_2^2\rangle\langle v_3^2\rangle\langle v_3^4\rangle + 36\langle v_3^2\rangle^2\langle v_2^2\, v_3^2\rangle.\end{aligned}$$