Spin polarization measurements in relativistic heavy-ion collisions

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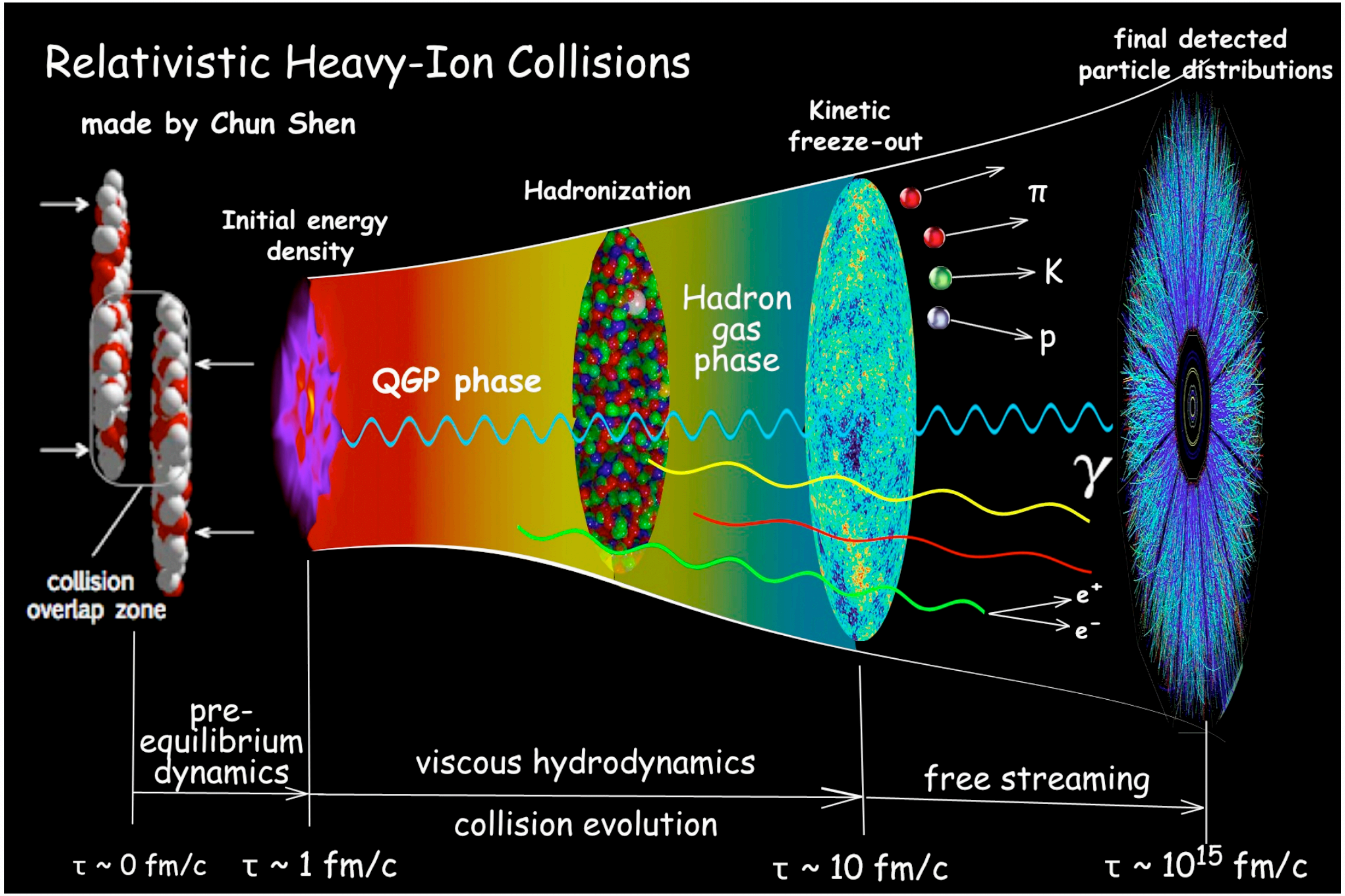
Relativistic heavy-ion collisions

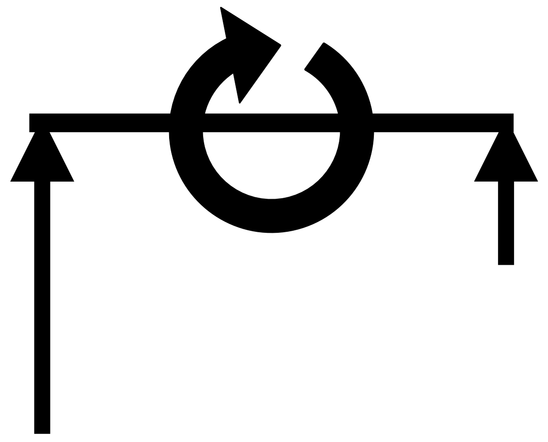


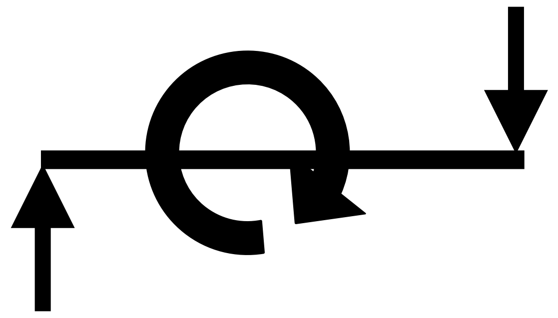
- Evolution of the medium is characterised by non-trivial velocity and vorticity fields.

Vorticity / Swirl / Rotation

vorticity ($\overrightarrow{\omega}$) – a measure of the “swirl” of the velocity flow field around any point

Non-relativistically: $\overrightarrow{\omega} = \frac{1}{2}\nabla \times \overrightarrow{v}$





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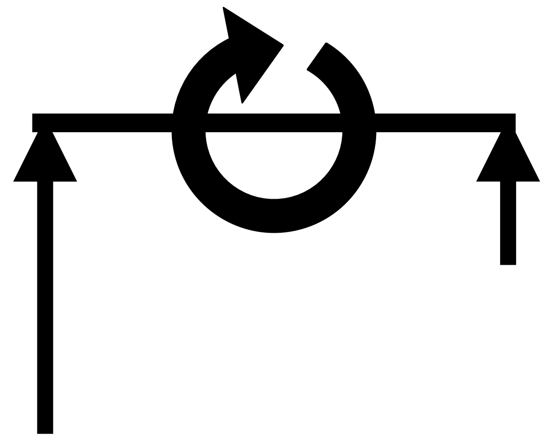
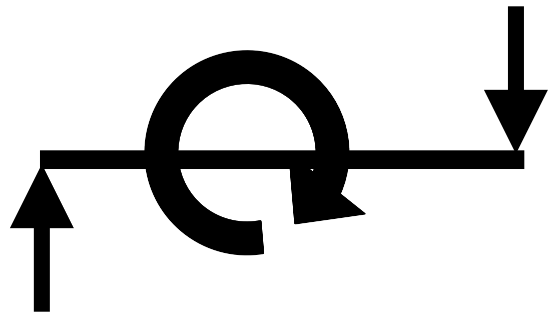
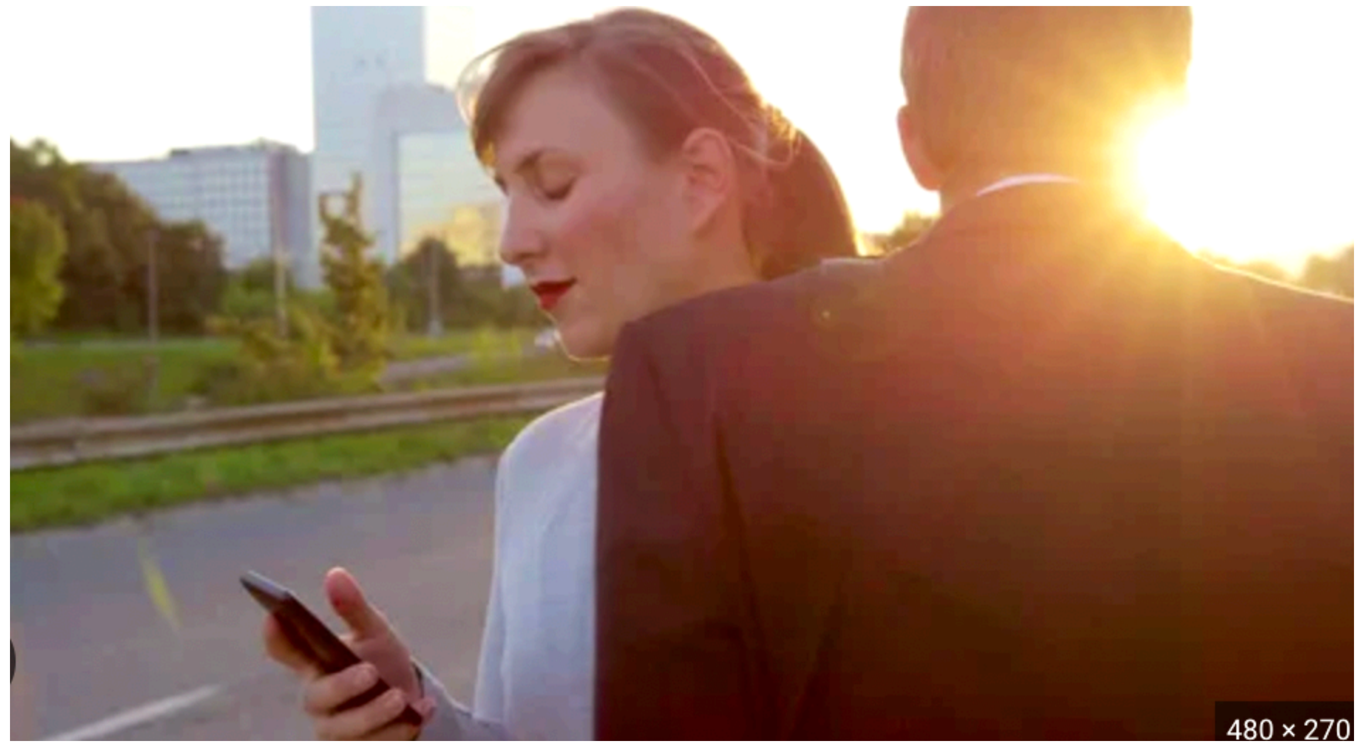
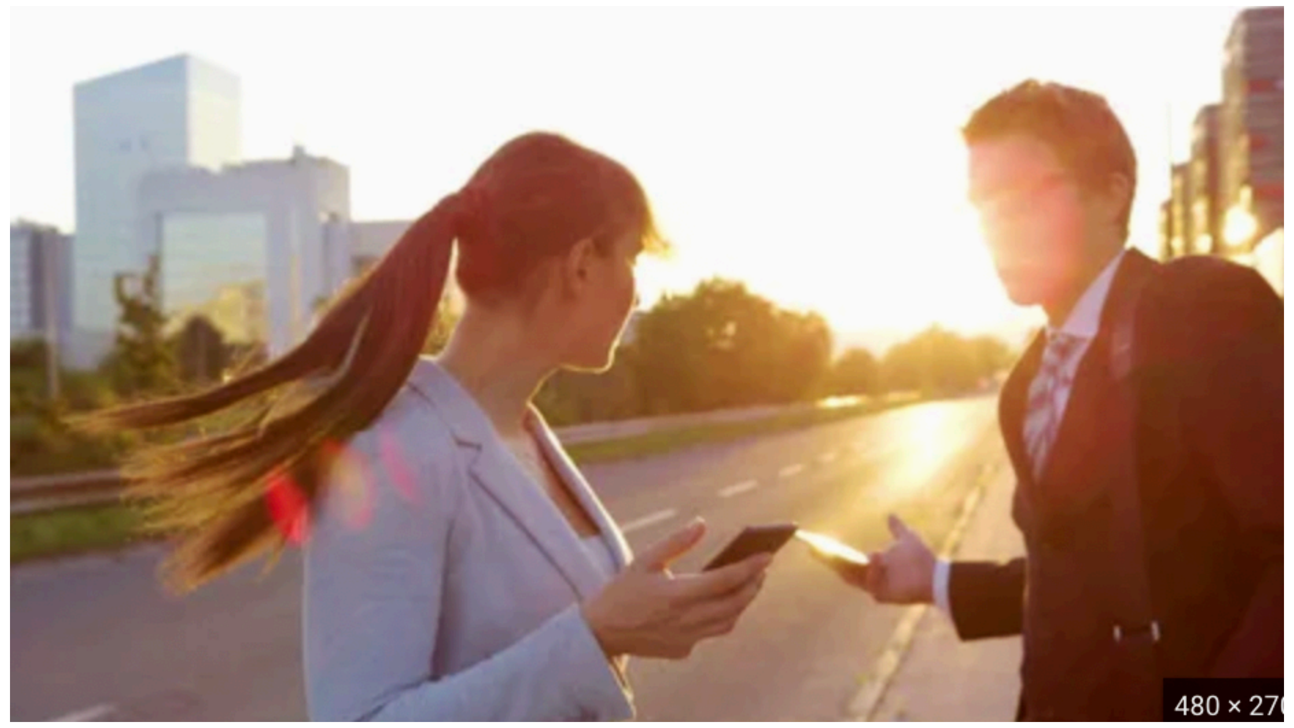




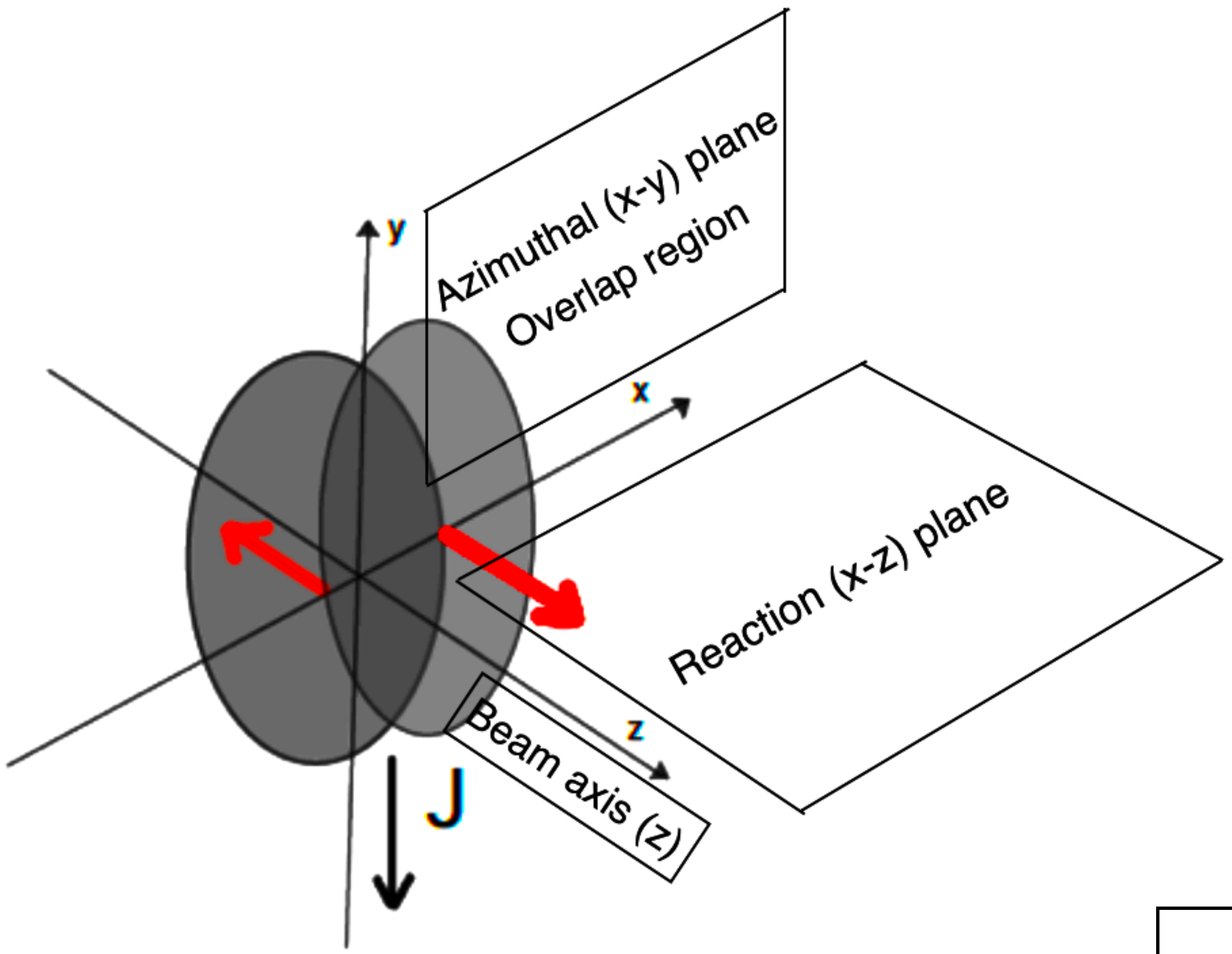
Want to experience vorticity? → Just bump into each other :)

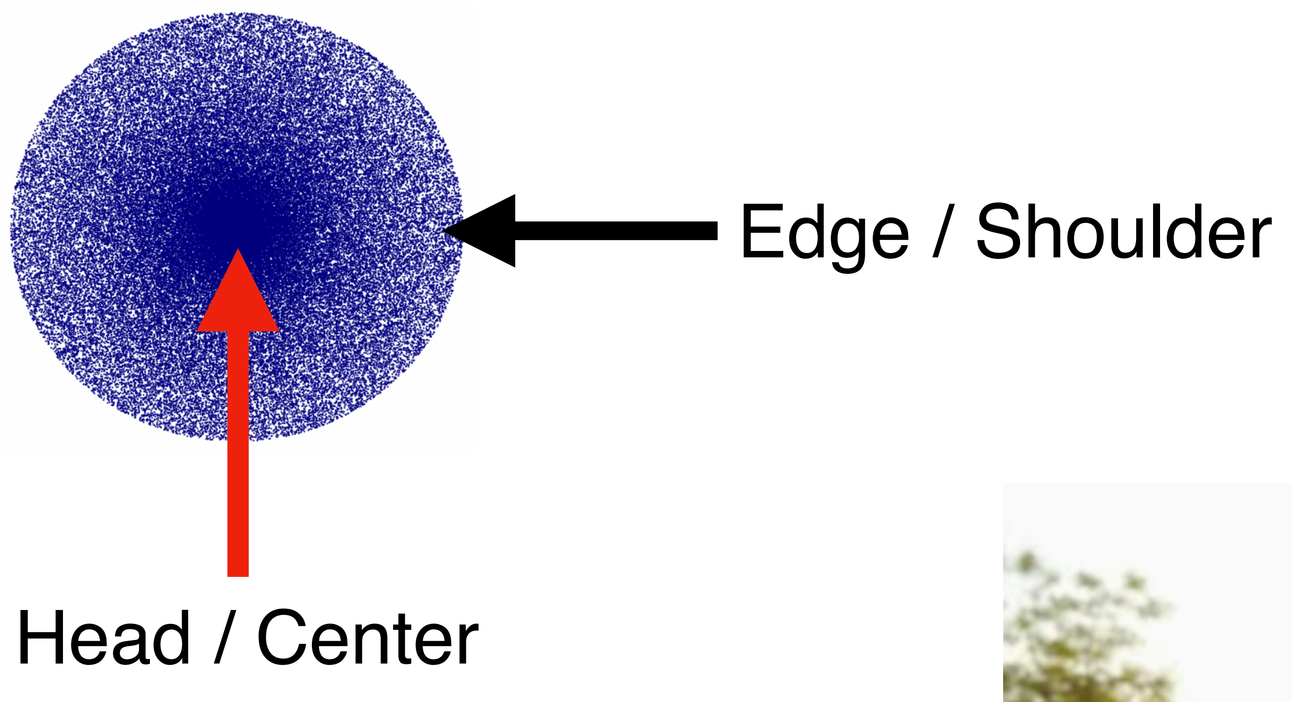


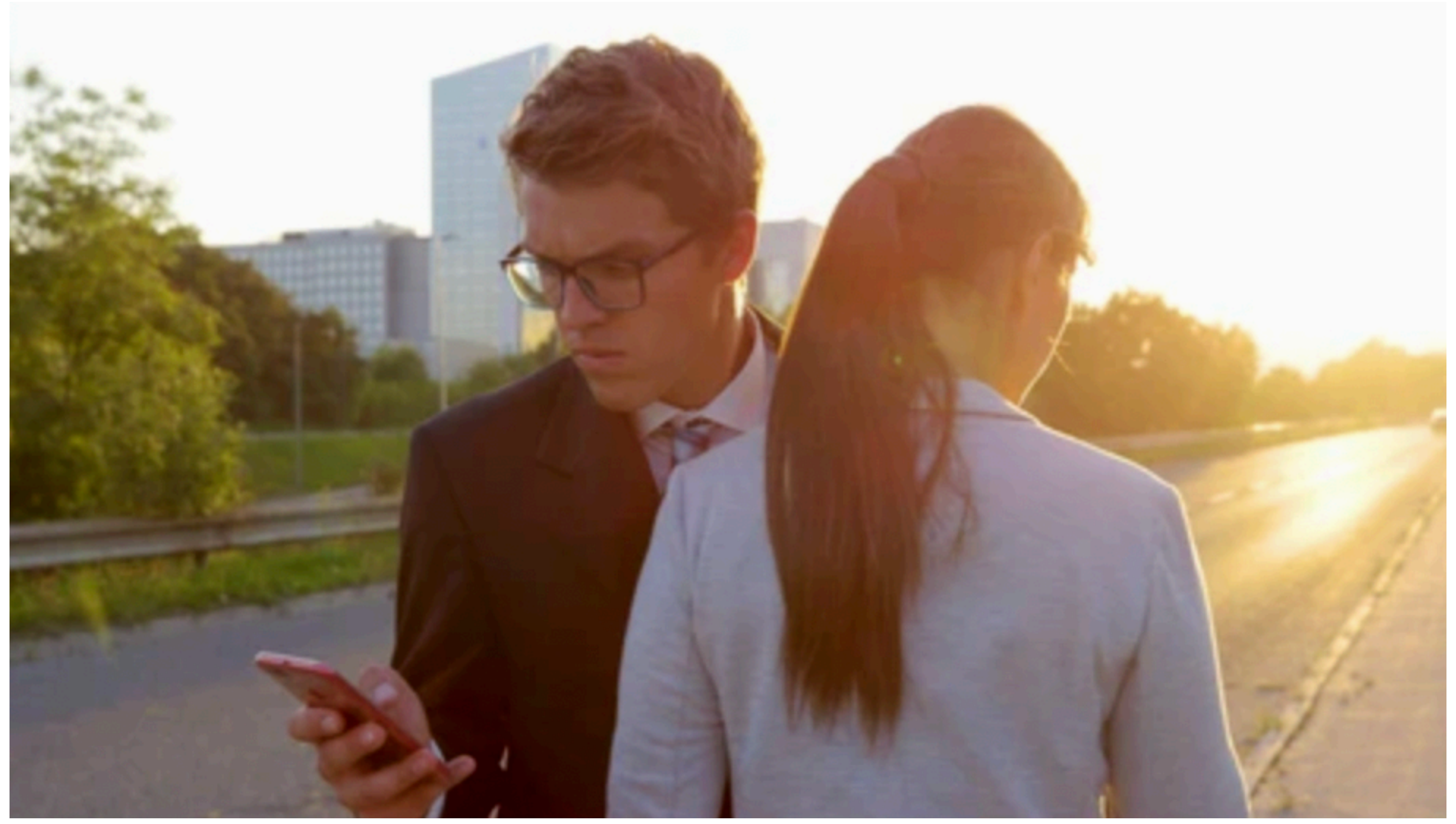


Global vorticity in heavy-ion collisions

- Relativistic Lorentz contracted nuclei bump into each other in the collider (LHC, RHIC):







$$\omega_y = \frac{1}{2}(\nabla \times v)_y \approx -\frac{1}{2}\frac{\partial v_z}{\partial x}$$

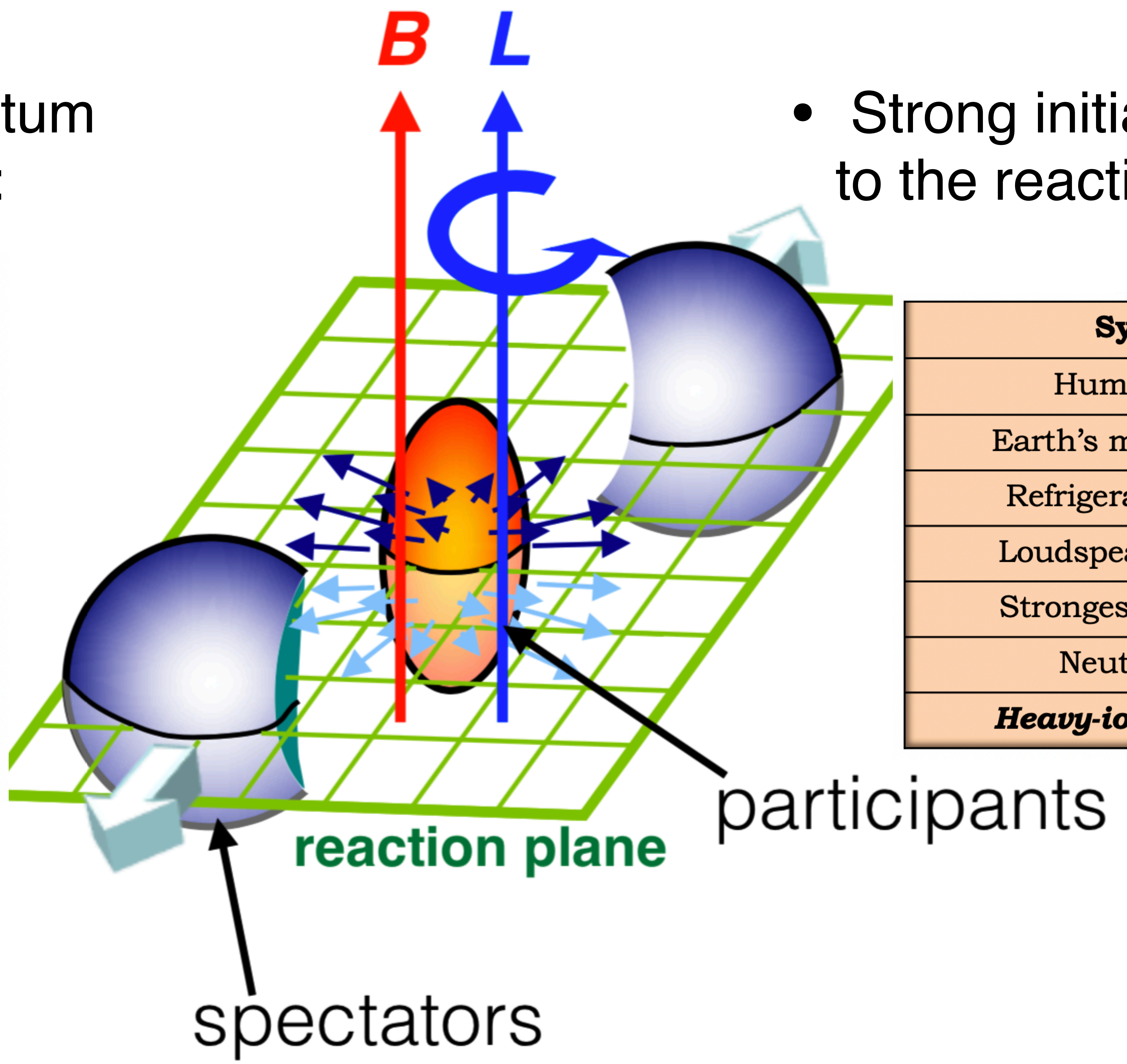
- Vorticity along the system orbital angular momentum due to initial longitudinal flow velocity gradients.

Global vorticity and polarization in heavy-ion collisions

- In relativistic non-central nuclear collisions:

- Large initial orbital angular momentum perpendicular to the reaction plane:

System	Vorticity (s^{-1})
Solar sub-surface	10^{-7}
Terrestrial atmosphere	10^{-5}
Great red spot of Jupiter	10^{-4}
Tornado core	10^{-1}
Heated soap bubbles	100
Turbulent flow in superfluid He	150
Heavy-ion collisions *STAR: Nature 548 (2017) 62*	**10^{7} - 10^{21}**



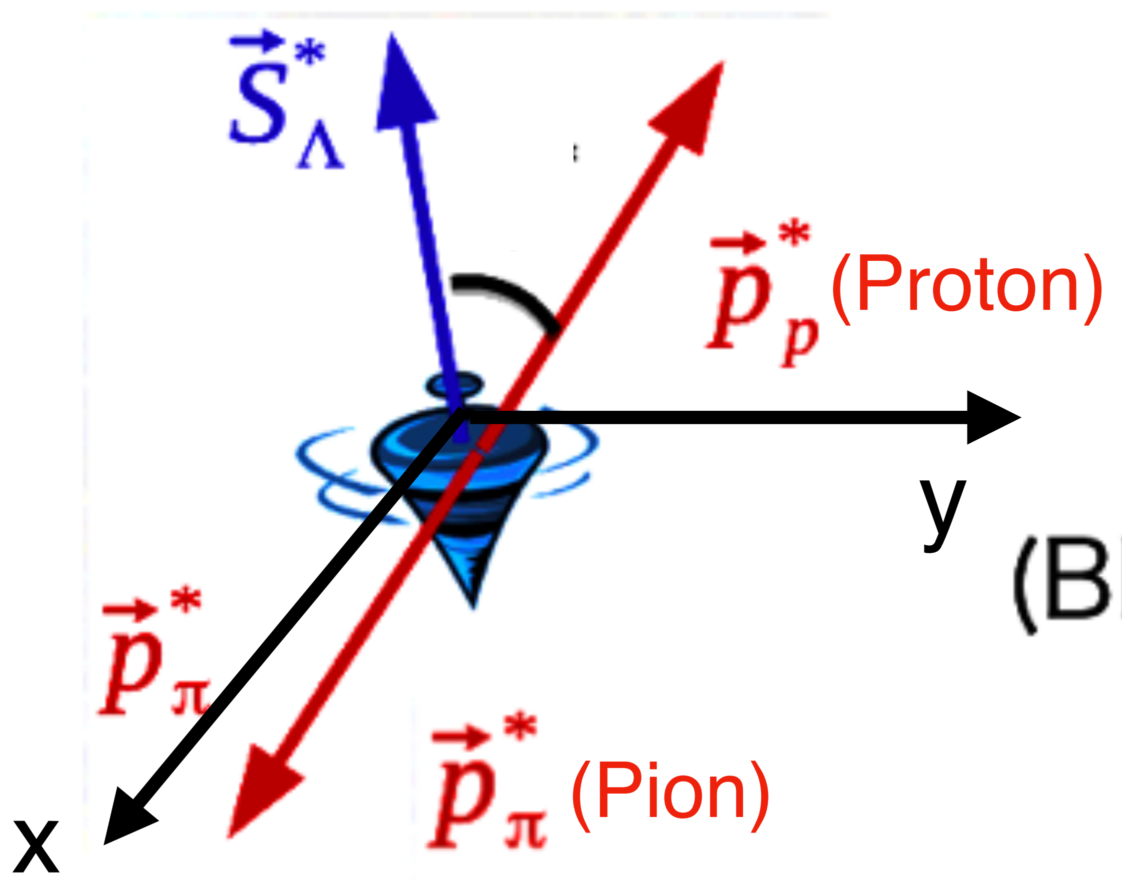
- Strong initial magnetic field perpendicular to the reaction plane:

System	Magnetic Field in Tesla
Human brain	10^{-12}
Earth's magnetic field	10^{-5}
Refrigerator magnet	10^{-3}
Loudspeaker magnet	1
Strongest field in lab	10^{3}
Neutron star	10^{6}
Heavy-ion collisions	***10^{15} - 10^{16}***

- Polarization due to vorticity ($\vec{L}$) $\longrightarrow$ $\vec{L}.\vec{S}$ (same for particle and anti-particle)
- Polarization due to Magnetic field ($\vec{B}$) $\longrightarrow$ $\vec{\mu}\,.\,\vec{B}$ (opposite for particle and anti-particle)

Hyperon polarization estimation

Λ ($\bar{\Lambda}$) hyperons ⟶ Parity violating weak decay



$$\Lambda \rightarrow p + \pi^-$$

(BR: 63.9%, cτ~7.9 cm)

- Daughter baryon is preferentially emitted in the direction of hyperon spin (opposite for antiparticle).

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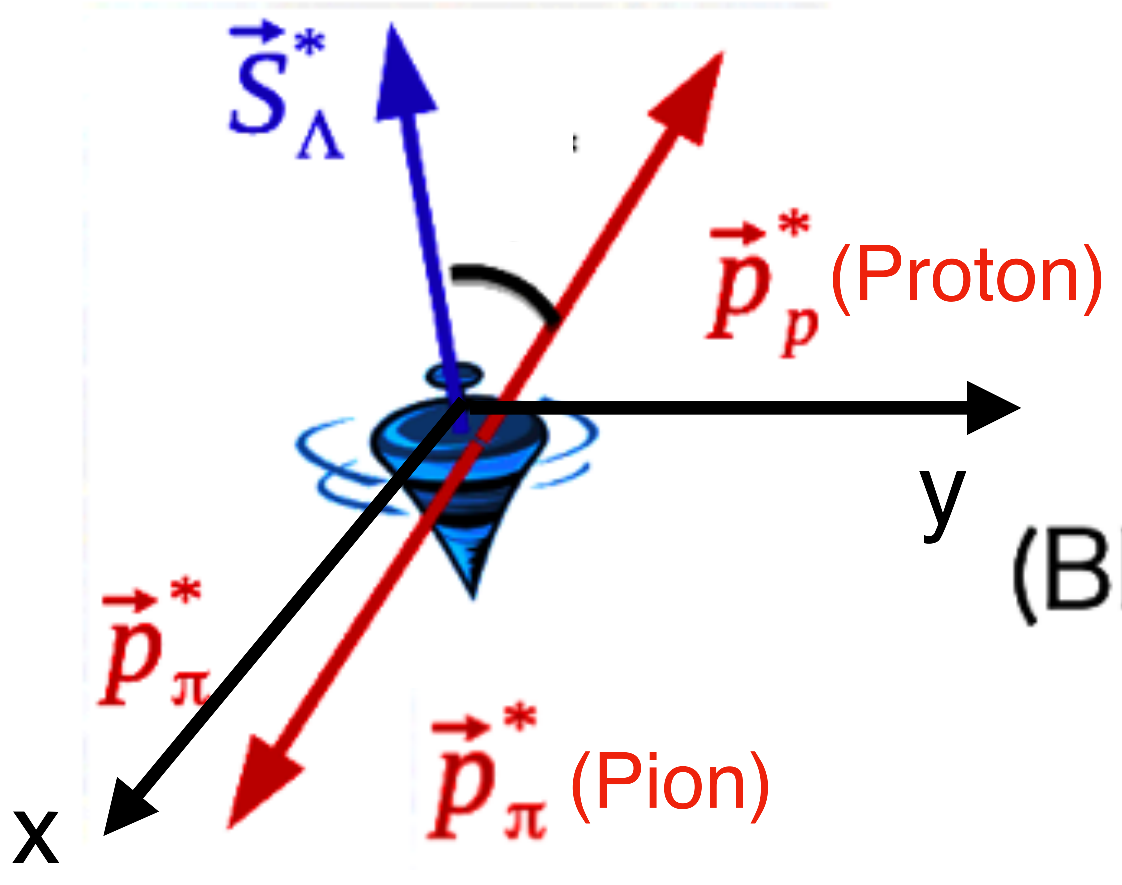

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- Daughter baryon is preferentially emitted in the direction of hyperon spin (opposite for antiparticle).

- Polarization estimation procedure:

a) Project the daughter proton's momentum direction on the vorticity axis.

b) Average over all hyperons.

$\hat{\mathbf{p}}^*_\mathrm{p}$ = unit vector along daughter momentum

Hyperon polarization along $\hat{L}$:

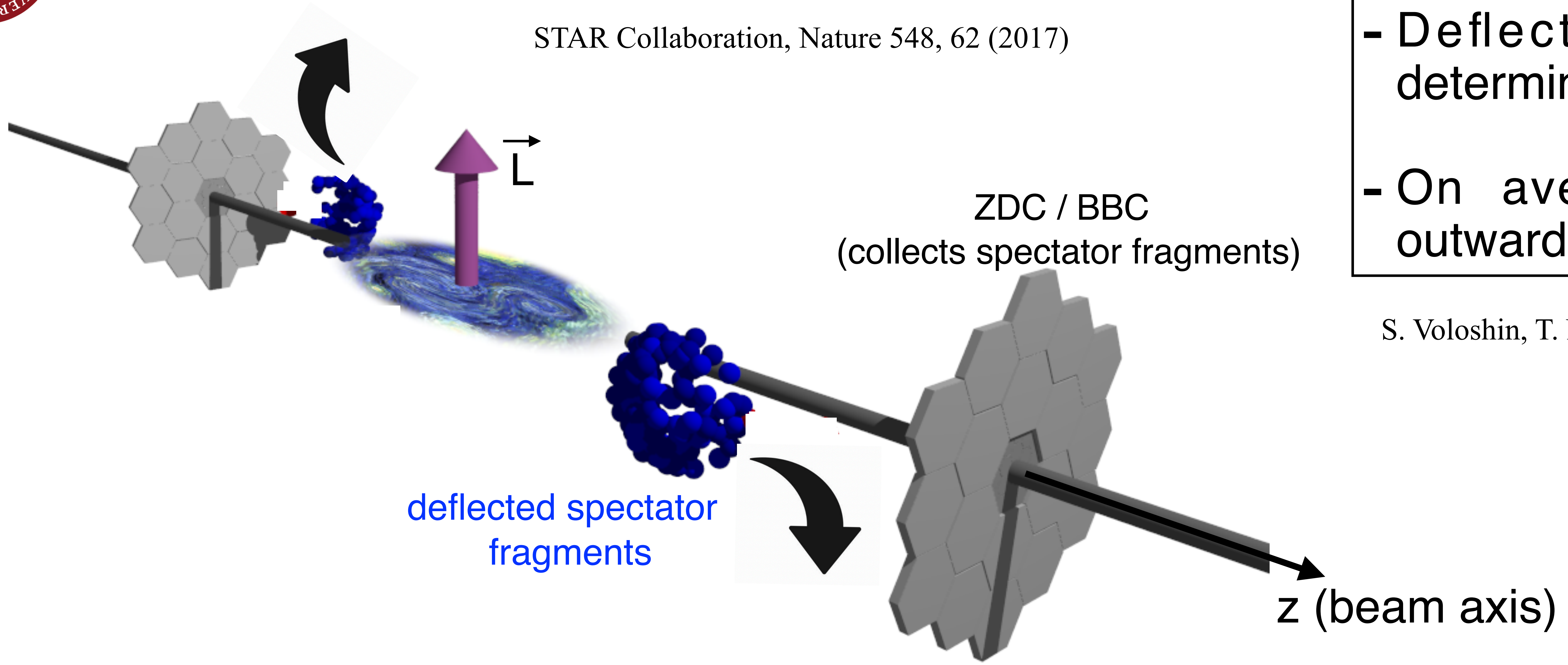
$$P_H = \frac{3}{\alpha_H}\langle(\hat{L}\cdot\hat{p}^*_p)\rangle \approx \langle\hat{L}\,.\,\hat{S}\rangle$$

α_H = hyperon decay parameter

Global polarization ⟶ one polarization direction (along $\hat{L}$) for the entire system.

Global hyperon polarization measurement in heavy-ion collisions

STAR Collaboration, Nature 548, 62 (2017)



- Deflection of the spectators determines the direction of $\vec{L}$
- On average spectators deflect outwards.

S. Voloshin, T. Niida; Phys. Rev. C 94, 021901(R) (2016)

Global hyperon polarization measurement in heavy-ion collisions

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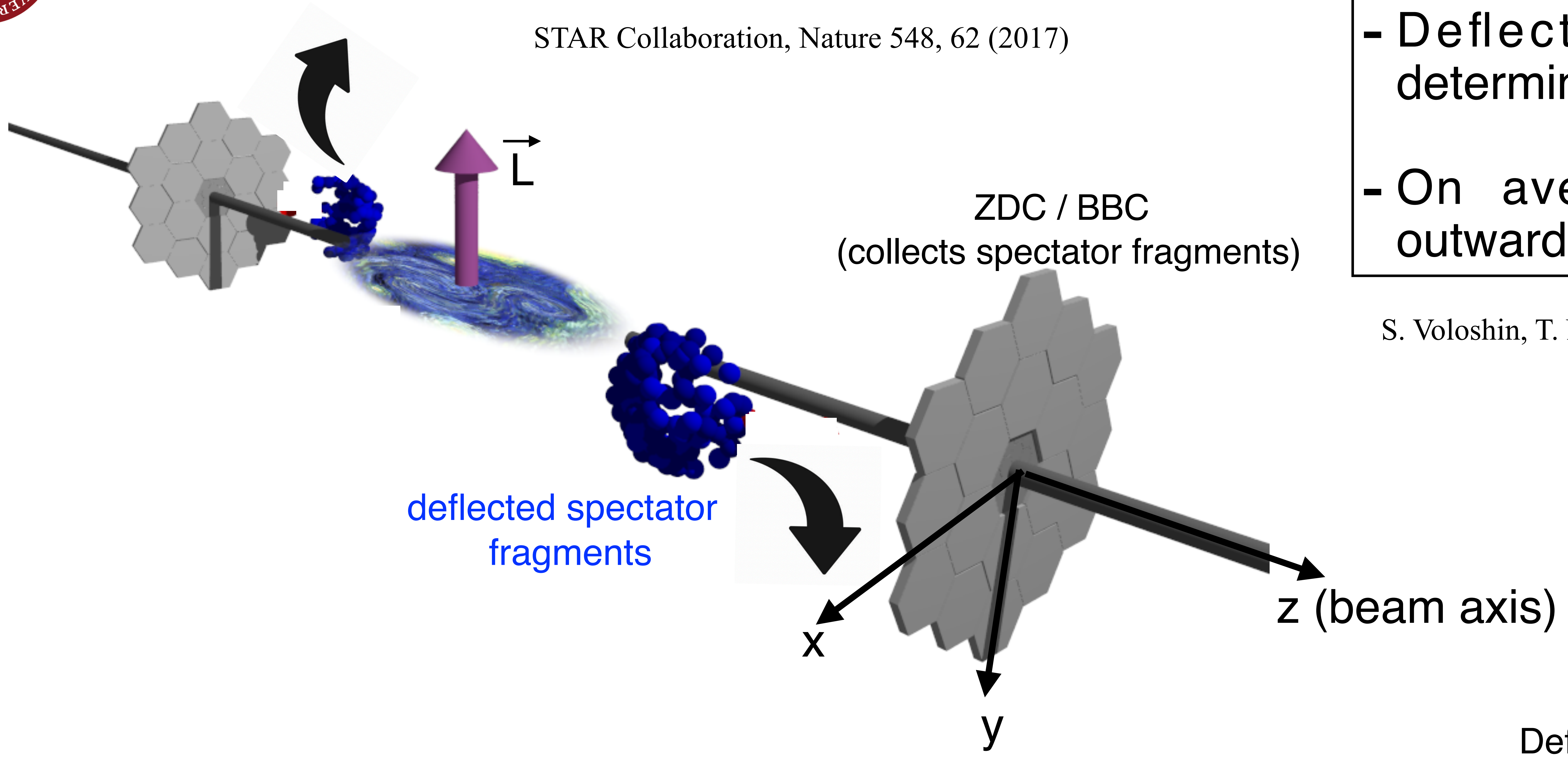

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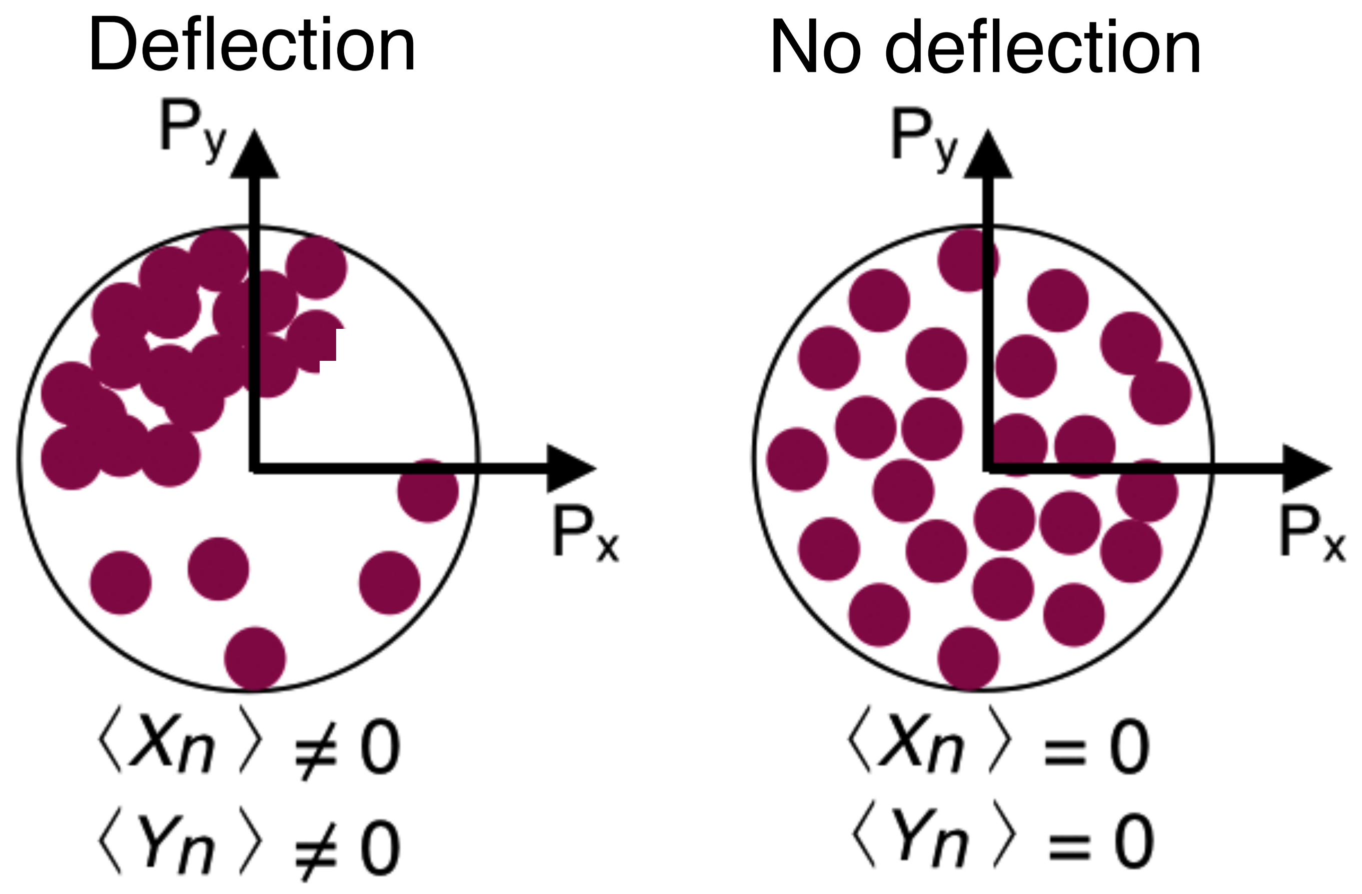
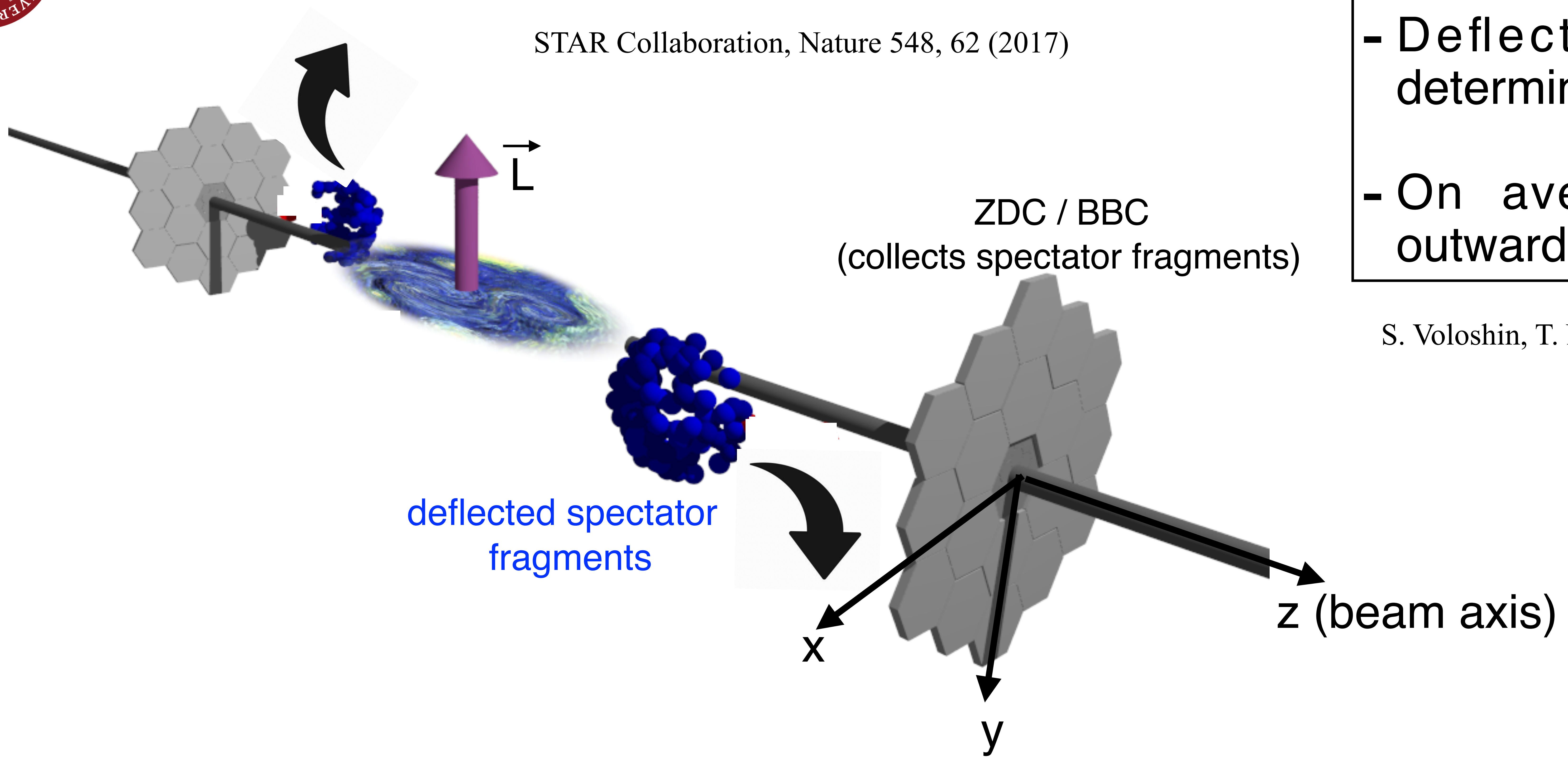


- Azimuthal distribution of the spectator fragments in the ZDC:

(Estimates spectator plane angle (Ψ_{SP}) ⟶ information about $\vec{L}$)

Global hyperon polarization measurement in heavy-ion collisions

STAR Collaboration, Nature 548, 62 (2017)



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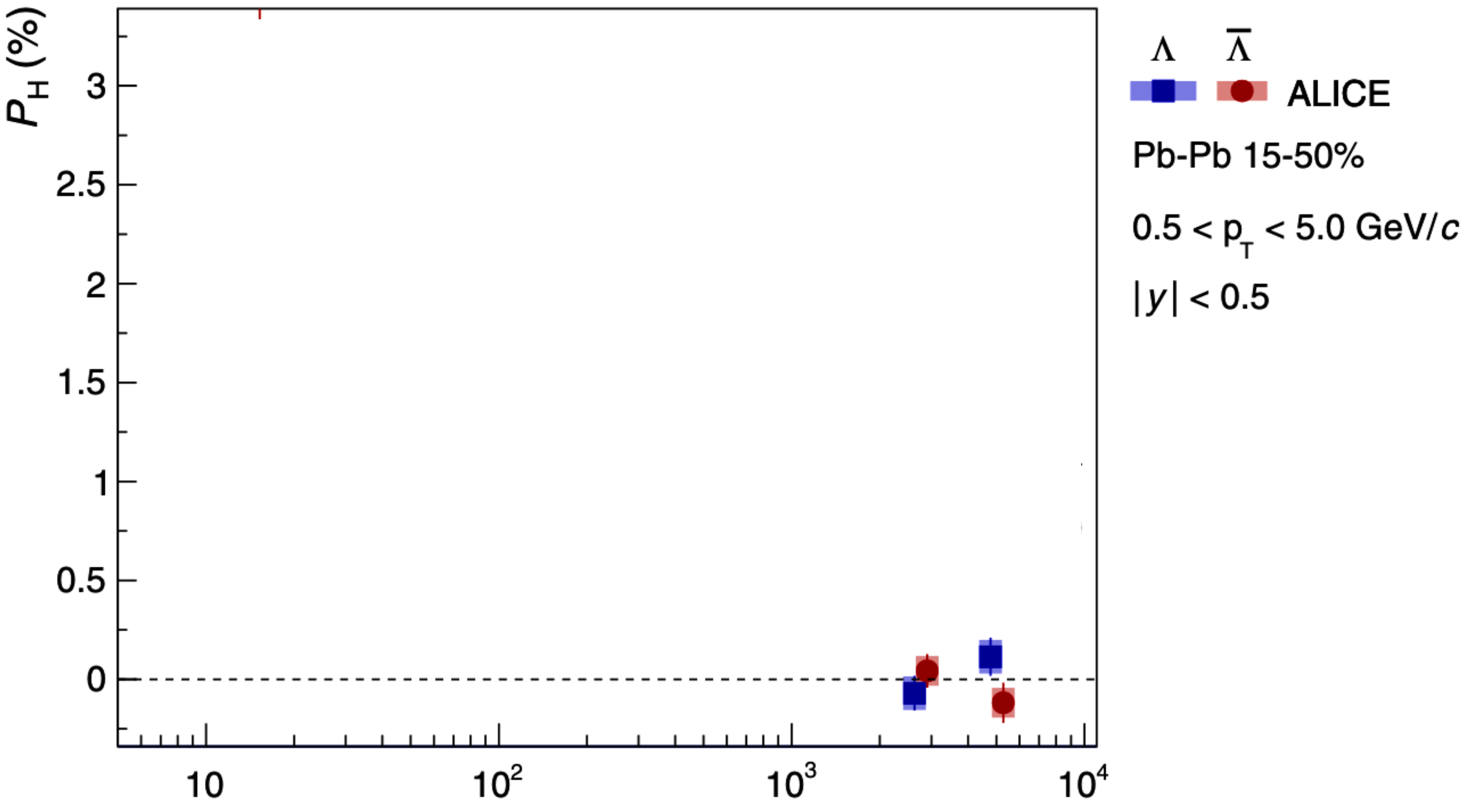
- Global hyperon polarization: $P_H = \frac{3}{\alpha_H}\langle(\hat{L}\cdot p_p^*)\rangle \approx \langle\hat{L}\,.\,\hat{S}\rangle$

ALICE, Phys. Rev. C 101, 044611 (2020)[erratum]

$$P_{\rm H} = -\frac{8}{\pi\alpha_{\rm H}}\frac{\langle\sin(\varphi_{\rm p}^* - \Psi_{\rm SP})\rangle}{R_{SP}^1}$$

- $\varphi_{\rm p}^*$ = azimuthal angle of daughter proton in $\Lambda(\bar{\Lambda})$ rest frame
- $R_{\rm SP}^1$ = Resolution of $\Psi_{\rm SP}$

$\Lambda(\bar{\Lambda})$ polarization in heavy-ion collisions



ALICE, Phys. Rev. C. 101, 044611 (2020)

PHYSICAL REVIEW C **101**, 044611 (2020)

Global polarization of Λ and $\overline{\Lambda}$ hyperons in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 and 5.02 TeV

S. Acharya *et al.**
(ALICE Collaboration)

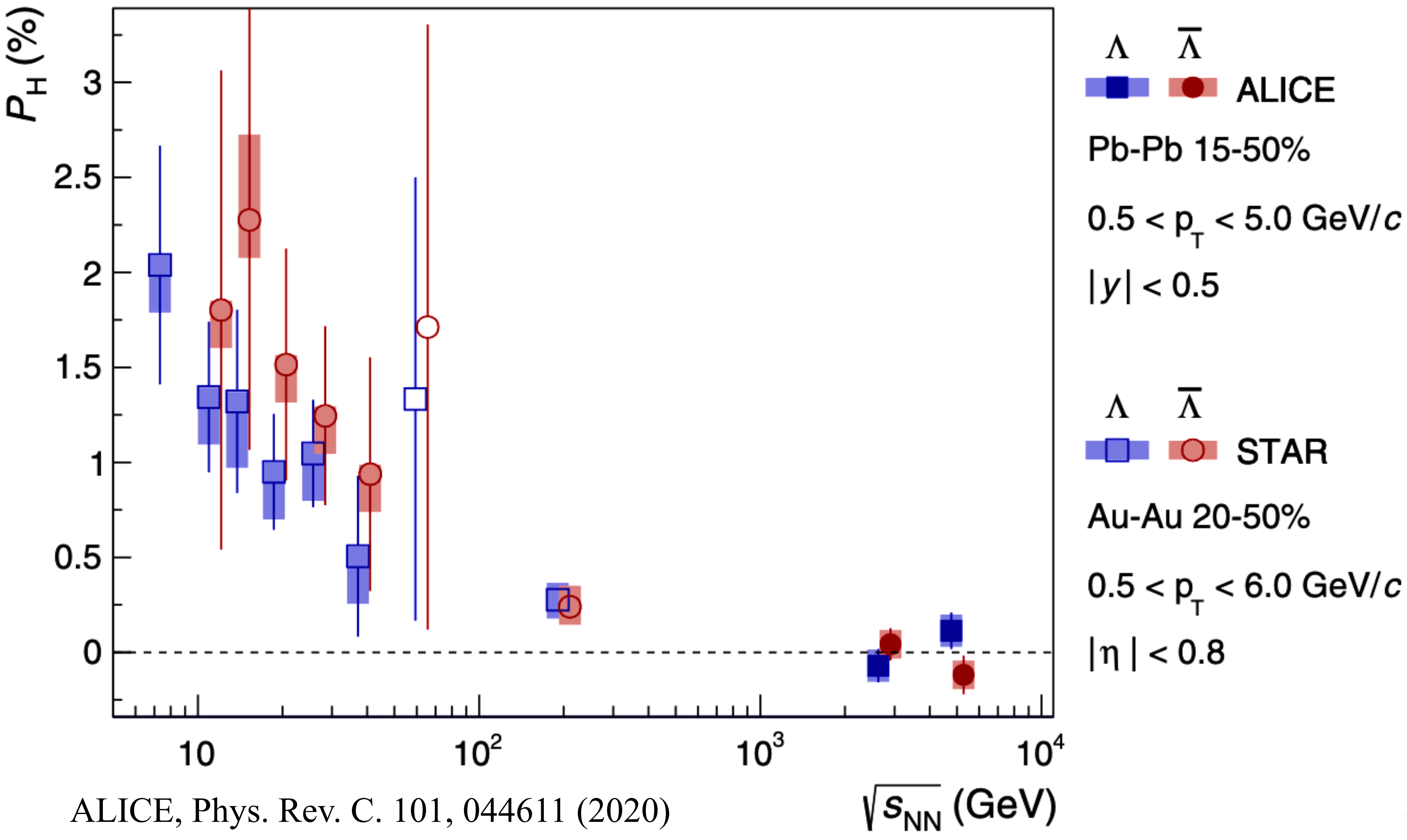
(Received 13 September 2019; accepted 24 February 2020; published 20 April 2020)

The global polarization of the Λ and $\overline{\Lambda}$ hyperons is measured for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV recorded with the ALICE at the Large Hadron Collider (LHC). The results are reported differentially as a function of collision centrality and hyperon's transverse momentum (p_T) for the range of centrality 5–50%, $0.5 < p_T < 5$ GeV/c, and rapidity $|y| < 0.5$. The hyperon global polarization averaged for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV is found to be consistent with zero, $\langle P_H \rangle (\%) \approx 0.01 \pm 0.06$ (stat.) ± 0.03 (syst.) in the collision centrality range 15–50%, where the largest signal is expected. The results are compatible with expectations based on an extrapolation from measurements at lower collision energies at the Relativistic Heavy Ion Collider, hydrodynamical model calculations, and empirical estimates based on collision energy dependence of directed flow, all of which predict the global polarization values at LHC energies of the order of 0.01%.

DOI: 10.1103/PhysRevC.101.044611

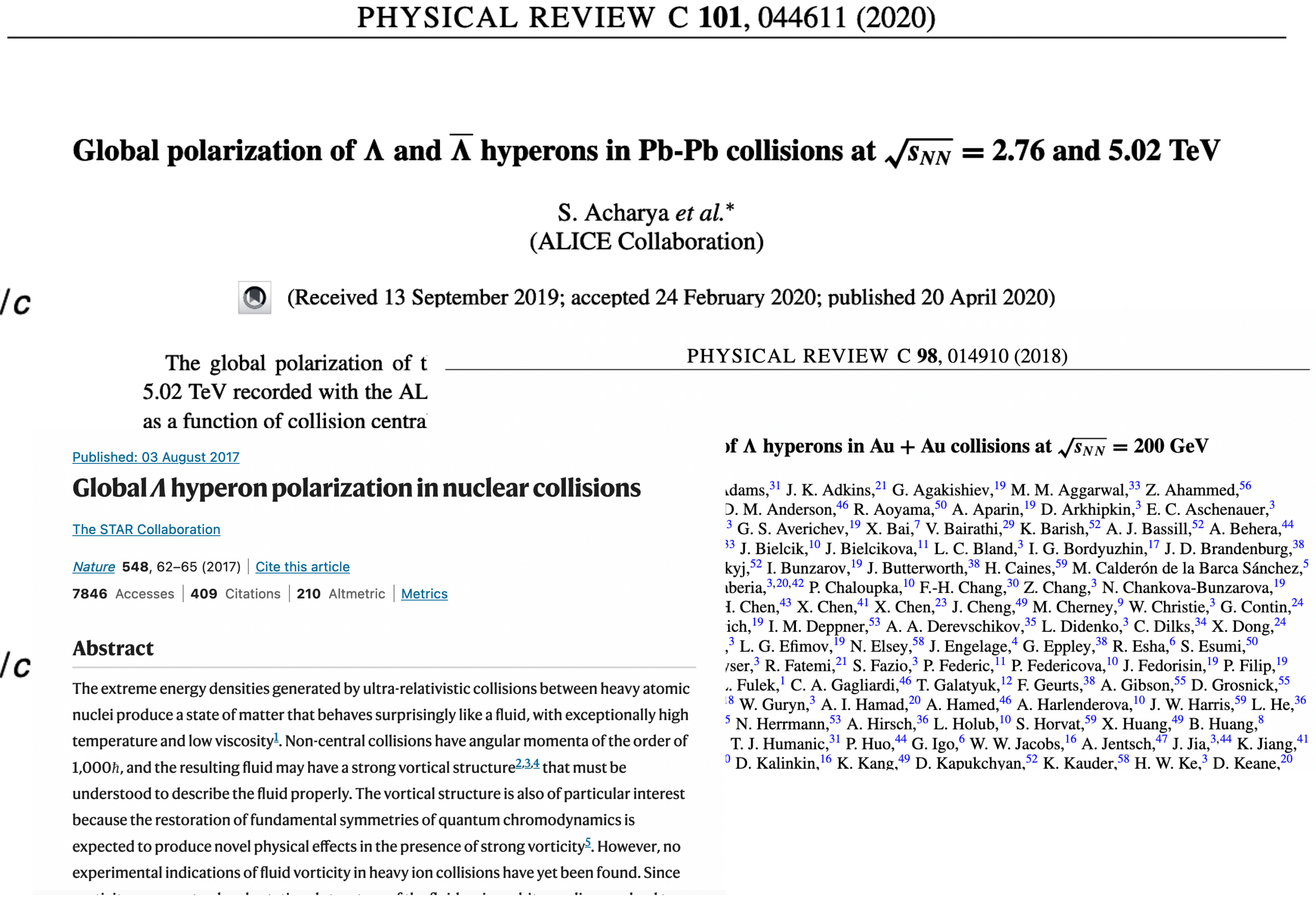
- Global hyperon polarization at the LHC (ALICE) is consistent with zero.

$\Lambda(\bar{\Lambda})$ polarization in heavy-ion collisions



ALICE, Phys. Rev. C. 101, 044611 (2020)

STAR, Phys. Rev. C 98, 014910 (2018)



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The global polarization of t
5.02 TeV recorded with the AL
as a function of collision centra

PHYSICAL REVIEW C **98**, 014910 (2018)

of Λ hyperons in Au + Au collisions at $\sqrt{s_{NN}}$ = 200 GeV

Published: 03 August 2017

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration

Nature **548**, 62–65 (2017) | Cite this article

7846 Accesses | **409** Citations | **210** Altmetric | Metrics

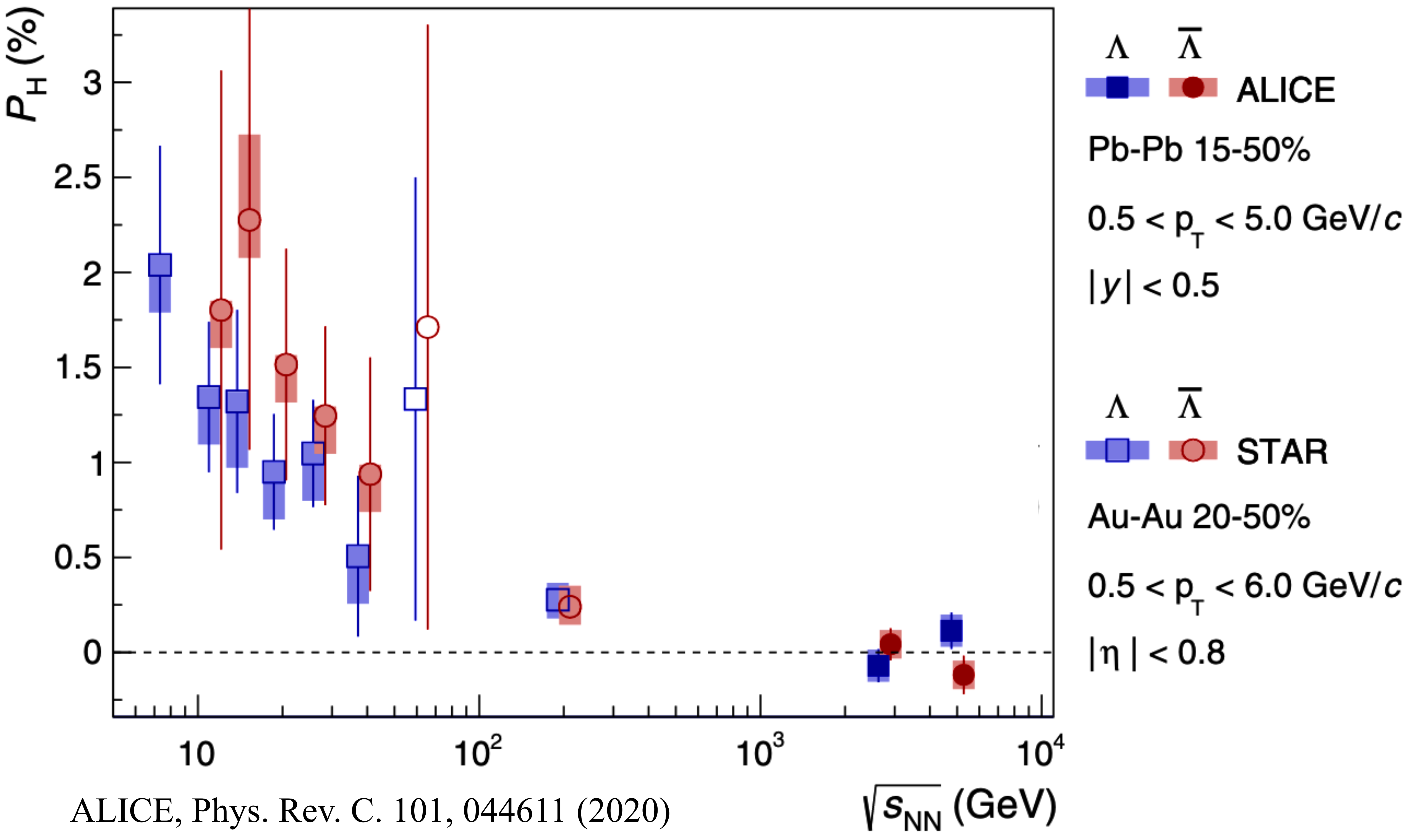
Abstract

The extreme energy densities generated by ultra-relativistic collisions between heavy atomic nuclei produce a state of matter that behaves surprisingly like a fluid, with exceptionally high temperature and low viscosity[1]. Non-central collisions have angular momenta of the order of 1,000ħ, and the resulting fluid may have a strong vortical structure[2,3,4] that must be understood to describe the fluid properly. The vortical structure is also of particular interest because the restoration of fundamental symmetries of quantum chromodynamics is expected to produce novel physical effects in the presence of strong vorticity[5]. However, no experimental indications of fluid vorticity in heavy ion collisions have yet been found. Since

- Global hyperon polarization at the LHC (ALICE) is consistent with zero.
- Polarization at mid-rapidity decreases with collision energy.

ALICE, Phys. Rev. C. 101, 044611 (2020)
STAR, Phys. Rev. C 98, 014910 (2018)
STAR, Nature, volume 548, pages 62–65 (2017)

$\Lambda(\bar{\Lambda})$ polarization in heavy-ion collisions



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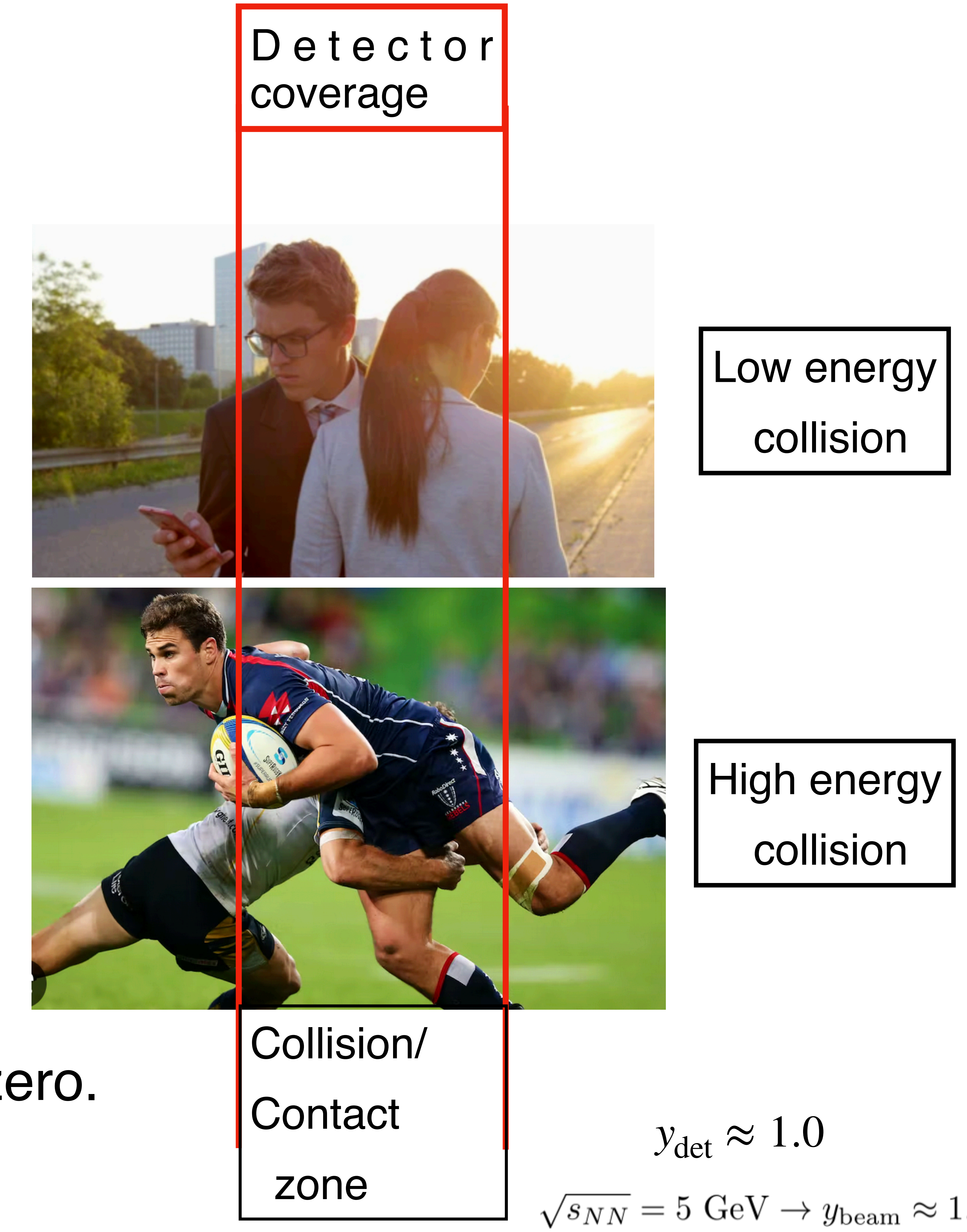


$y_{\text{det}} \approx 1.0$

$\sqrt{s_{NN}} = 5$ GeV $\rightarrow y_{\text{beam}} \approx 1.5$

$\sqrt{s_{NN}} = 39$ GeV $\rightarrow y_{\text{beam}} \approx 3.7$

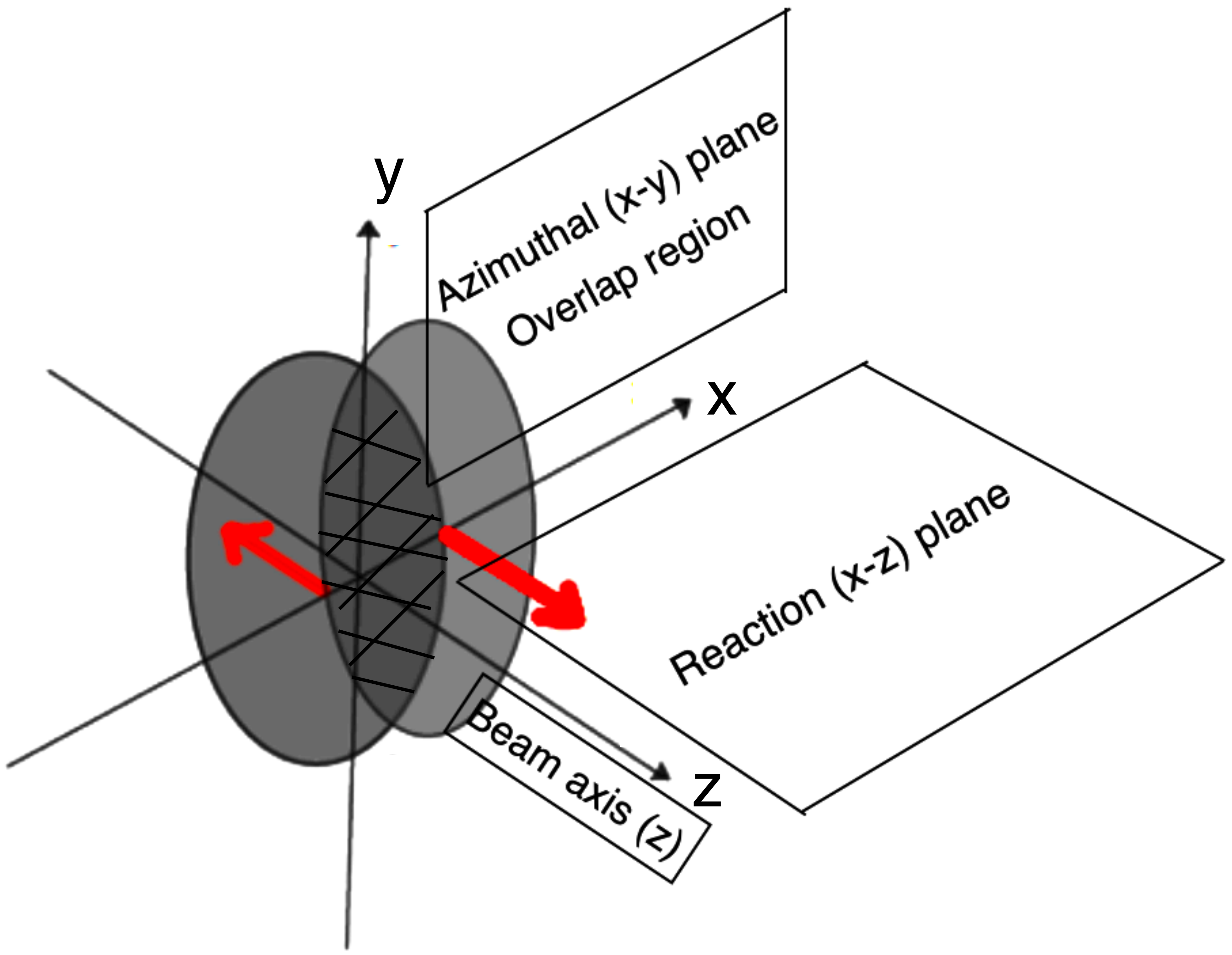
$\sqrt{s_{NN}} = 200$ GeV $\rightarrow y_{\text{beam}} \approx 5.4$

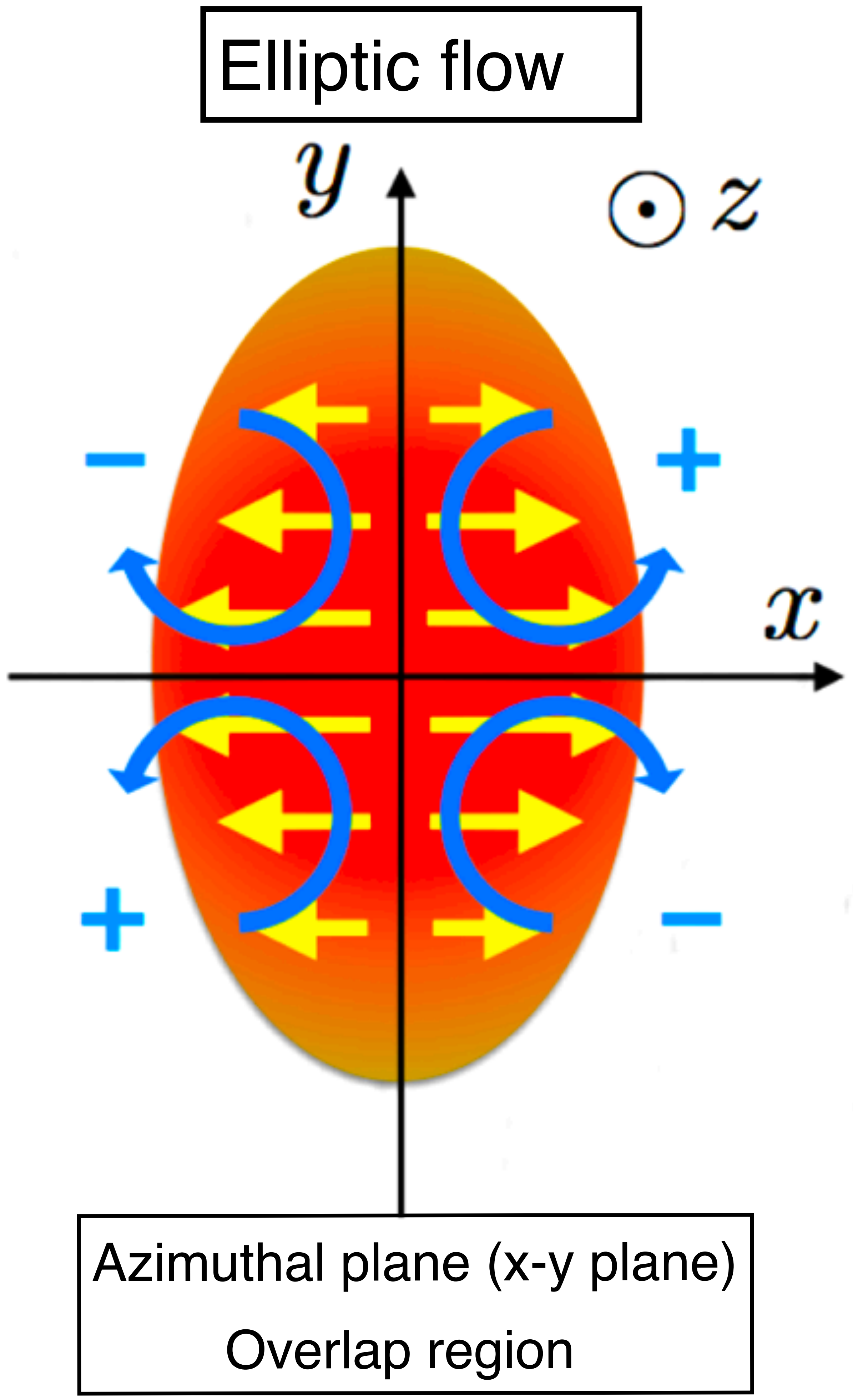
$\sqrt{s_{NN}} = 2700$ GeV $\rightarrow y_{\text{beam}} \approx 8$

- Global hyperon polarization at the LHC (ALICE) is consistent with zero.
- Polarization at mid-rapidity decreases with collision energy.
- Bjorken boost invariance at mid-rapidity? Vorticity migrates to forward rapidity?
- For higher $\sqrt{s_{NN}}$, P_H measurement at forward rapidity needs detector upgrade.

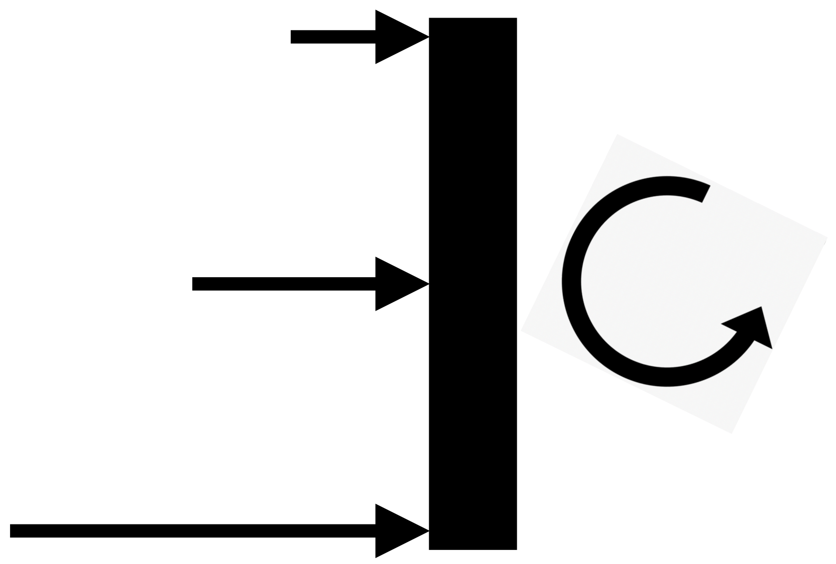
Elliptic flow induced polarization along the beam direction (P_z)

- Source of vorticity along the beam direction (z axis):



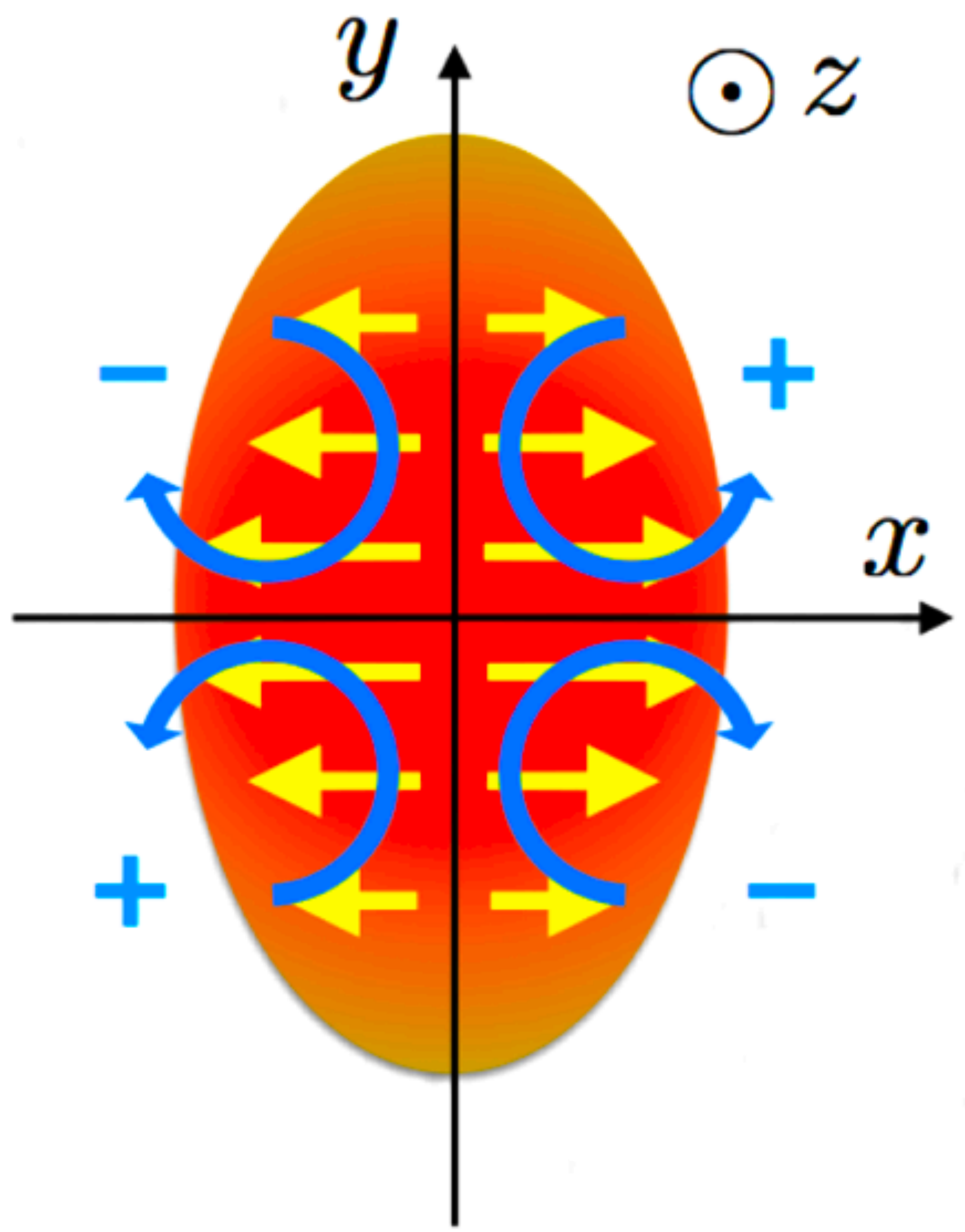


$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{v}$$



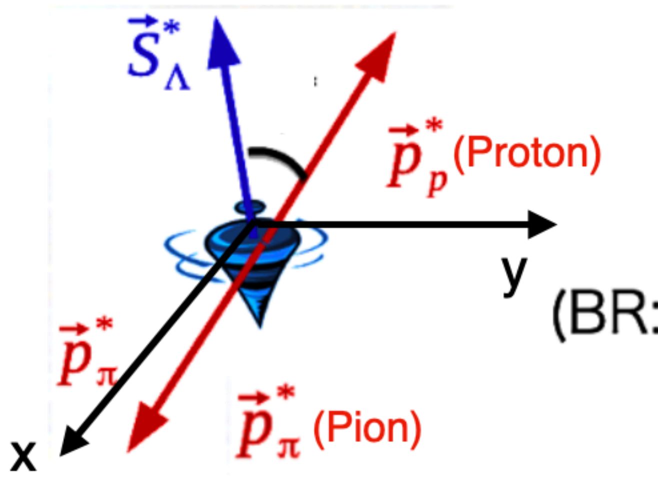
- Particle spin polarization along beam (z) axis has azimuthal angle dependence - local polarization.

Elliptic flow induced polarization along the beam direction (P_z)



S. Voloshin, EPJ Web Conf.171, 07002 (2018)

$$P_z(\phi) \approx sin(2\phi - 2\Psi_2)$$



$\Lambda \to p + \pi^-$

(BR: 63.9%, c τ ~7.9 cm)

- Local polarization (along z axis)-

$$P_z \approx \langle(\hat{p}^*_p \cdot \hat{z})\rangle$$

$$P_z = \frac{\langle \cos\theta^*_p \rangle}{\alpha_H \langle (\cos\theta^*_p)^2 \rangle} = \frac{3\langle \cos\theta^*_p \rangle}{\alpha_H}$$ (if perfect detector)

ALICE, Phys. Rev. Lett. 128, 172005. (2022)

$\langle(\cos\theta^*_p)^2\rangle$ = correction for finite acceptance along z

$$P_{z,s2} = \langle P_z \sin(2\varphi - 2\Psi_2)\rangle$$

$P_{z,s2}$ estimates magnitude and phase of P_z.

Elliptic flow induced polarization along the beam direction (P_z)

ALICE, Phys. Rev. Lett. 128, 172005. (2022)

PHYSICAL REVIEW LETTERS **128,** 172005 (2022)

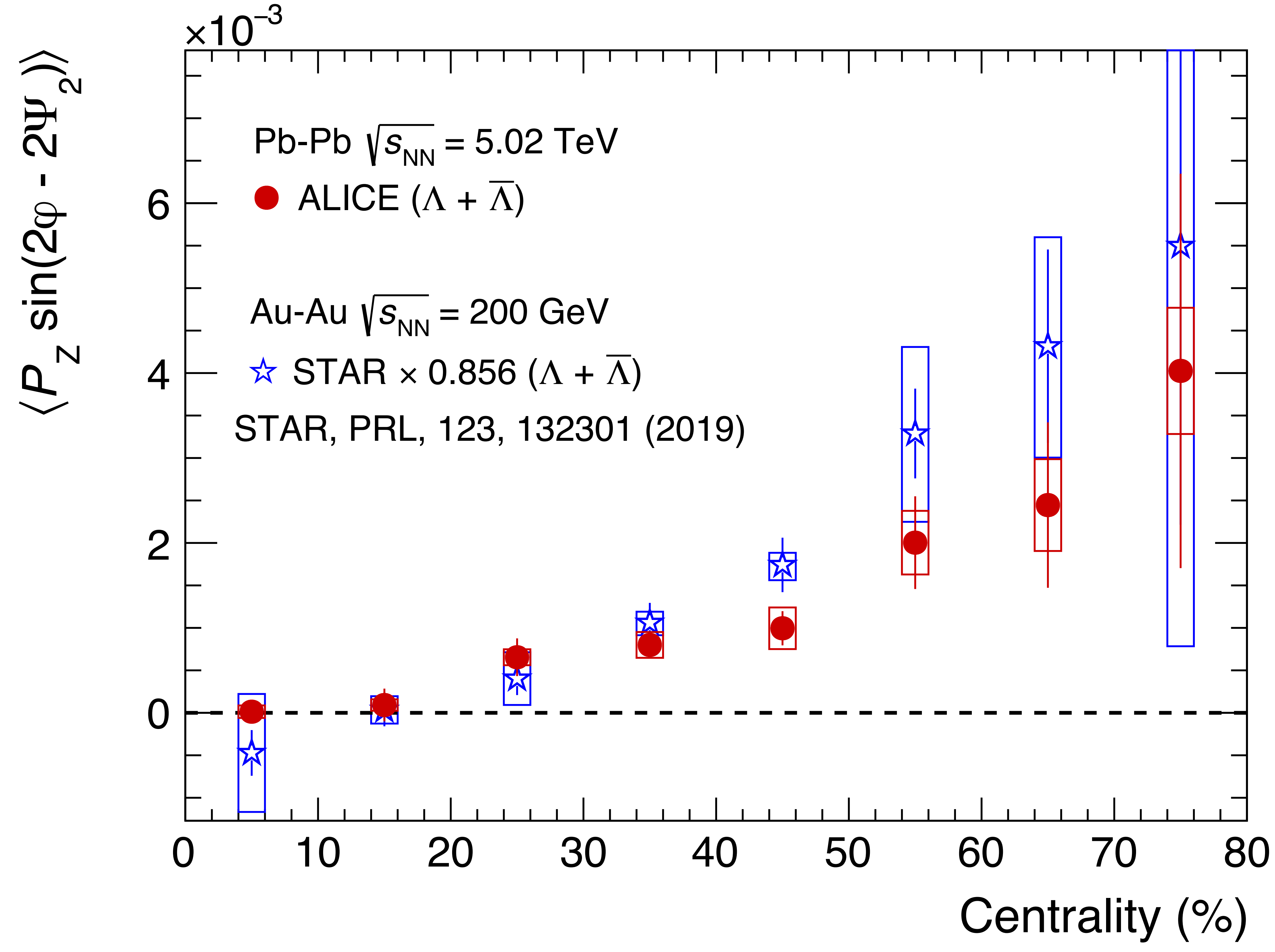
Polarization of Λ and $\bar{\Lambda}$ Hyperons along the Beam Direction in Pb-Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV

S. Acharya *et al.*[*]
(ALICE Collaboration)

(Received 3 September 2021; revised 4 January 2022; accepted 16 March 2022; published 29 April 2022)

The polarization of the Λ and $\bar{\Lambda}$ hyperons along the beam (z) direction, P_z, has been measured in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV recorded with ALICE at the Large Hadron Collider (LHC). The main contribution to P_z comes from elliptic flow-induced vorticity and can be characterized by the second Fourier sine coefficient $P_{z,s2} = \langle P_z \sin(2\varphi - 2\Psi_2)\rangle$, where φ is the hyperon azimuthal emission angle and Ψ_2 is the elliptic flow plane angle. We report the measurement of $P_{z,s2}$ for different collision centralities and in the 30%–50% centrality interval as a function of the hyperon transverse momentum and rapidity. The $P_{z,s2}$ is positive similarly as measured by the STAR Collaboration in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV, with somewhat smaller amplitude in the semicentral collisions. This is the first experimental evidence of a nonzero hyperon P_z in Pb-Pb collisions at the LHC. The comparison of the measured $P_{z,s2}$ with the hydrodynamic model calculations shows sensitivity to the competing contributions from thermal and the recently found shear-induced vorticity, as well as to whether the polarization is acquired at the quark-gluon plasma or the hadronic phase.

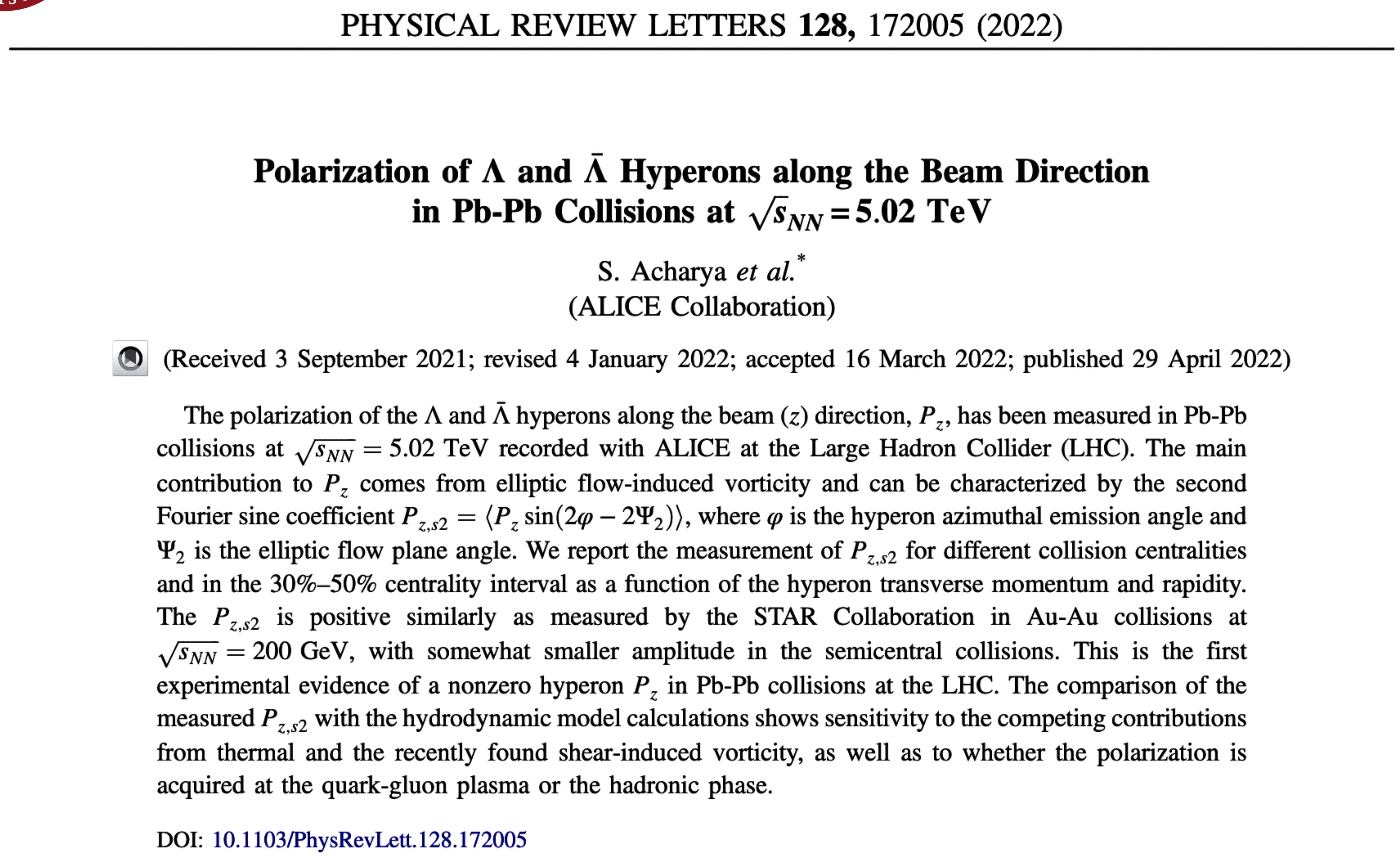
DOI: 10.1103/PhysRevLett.128.172005



- $P_{z,s2}$ at the LHC is similar in magnitude to top RHIC energy.

Elliptic flow induced polarization along the beam direction (P_z)

ALICE, Phys. Rev. Lett. 128, 172005. (2022)



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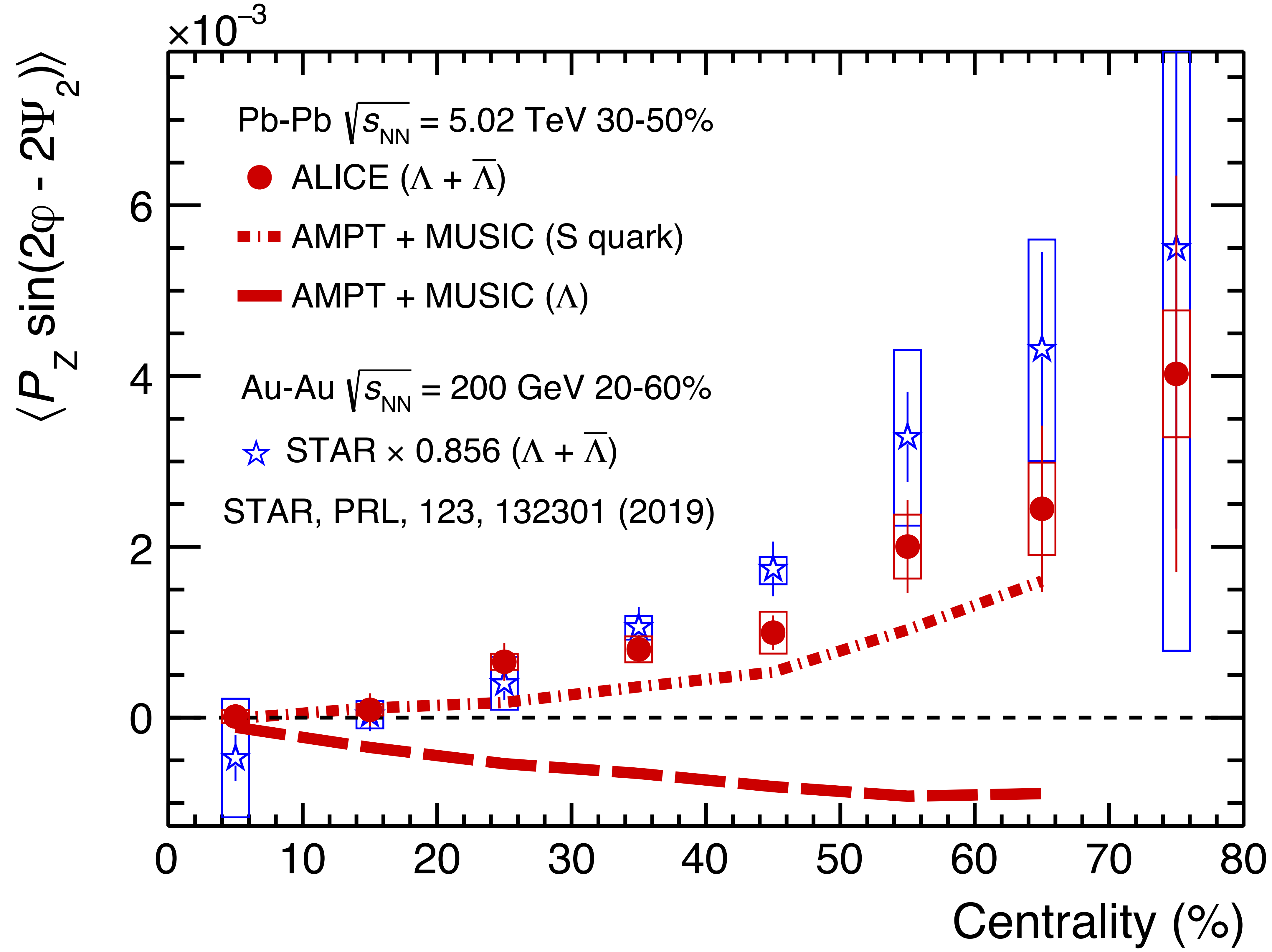


Model description: B. Fu et al.; PRL 127, 142301 (2021)

- $P_{z,s2}$ at the LHC is similar in magnitude to top RHIC energy.
- The data results are compared with the (fluid shear + thermal vorticity) based AMPT + MUSIC model.
- The model qualitatively explains the data with constituent strange quark mass as spin carrier mass! Spin-orbit coupling happens only at the partonic level?

Summary and Outlook

- Spin polarisation is sensitive to the gradients of velocity and temperature fields → probes the "fine structure" of the QGP.

- Magnitude and life time of the magnetic field can be investigated using the spin polarization in heavy-ion collisions.

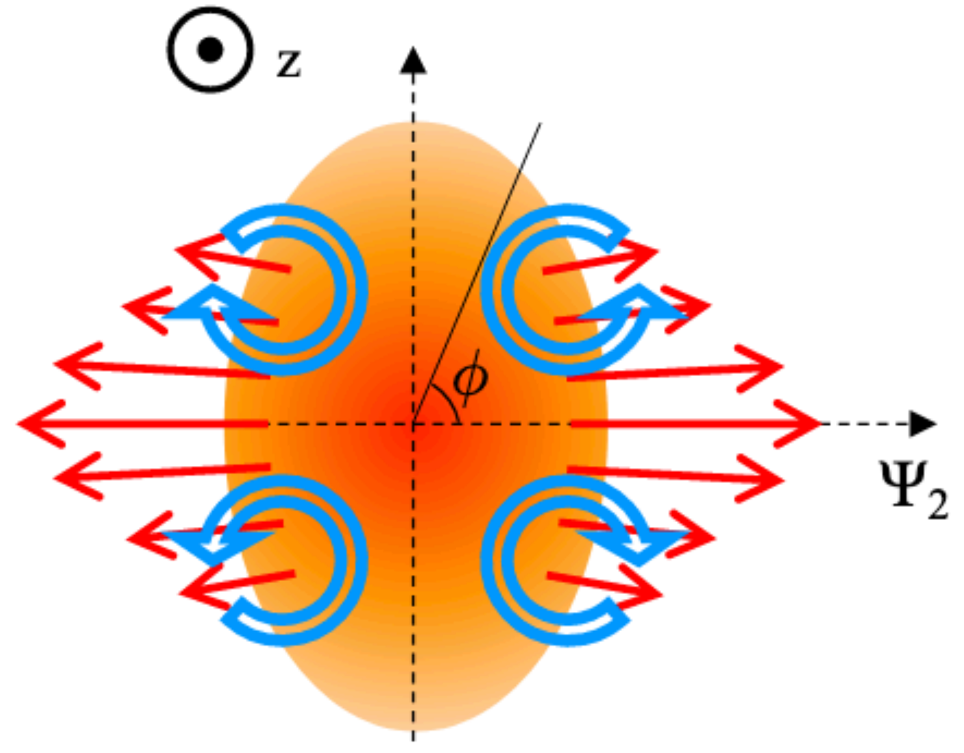
- Spin polarization results provide critical constraints for the development of theoretical framework involving spin degrees of freedom.

- New high statistics datasets at STAR (BES II) and Run 3 at the LHC will allow more differential and precision measurements of global and local polarization in heavy-ion collisions.

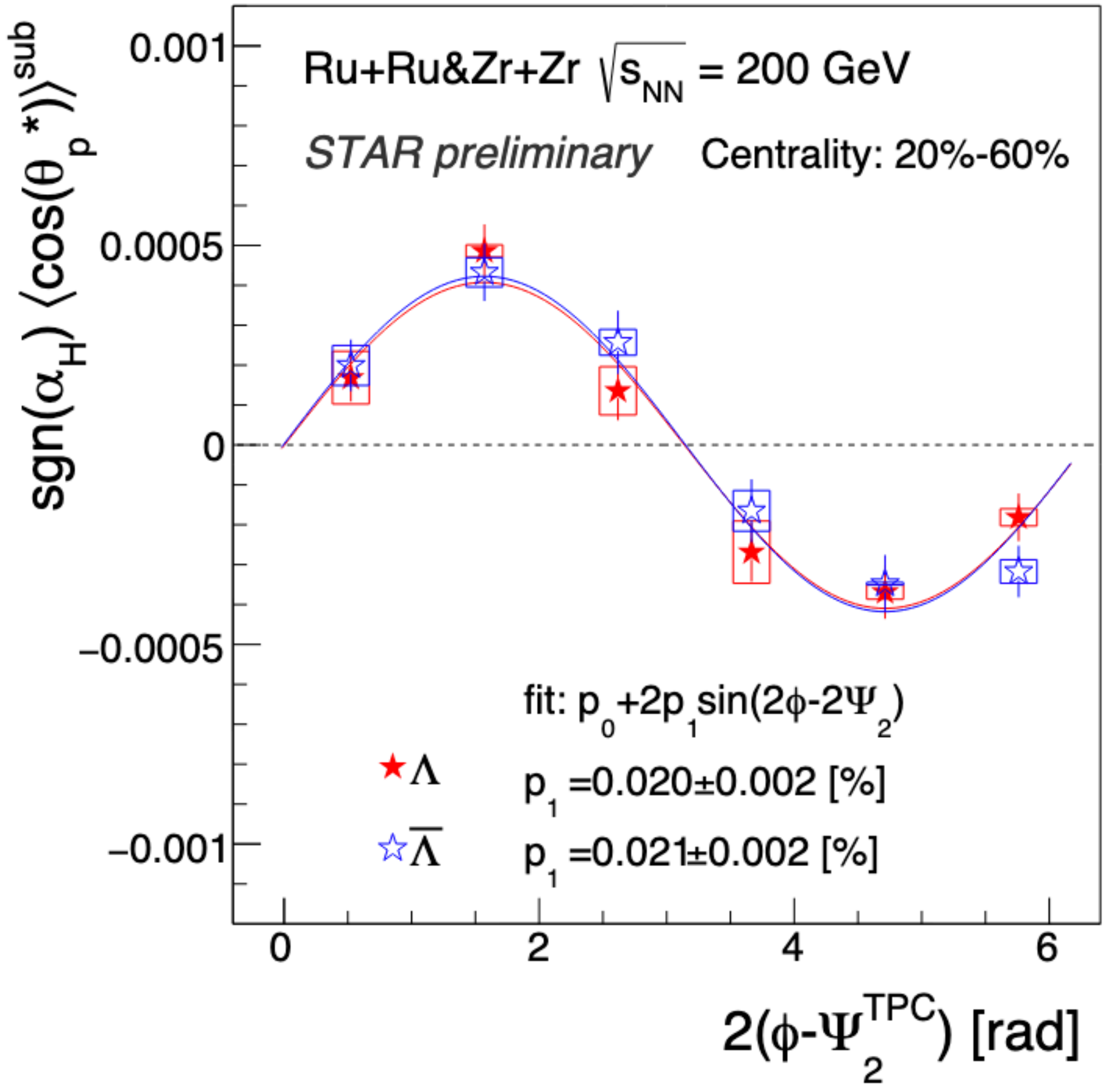
Thank you

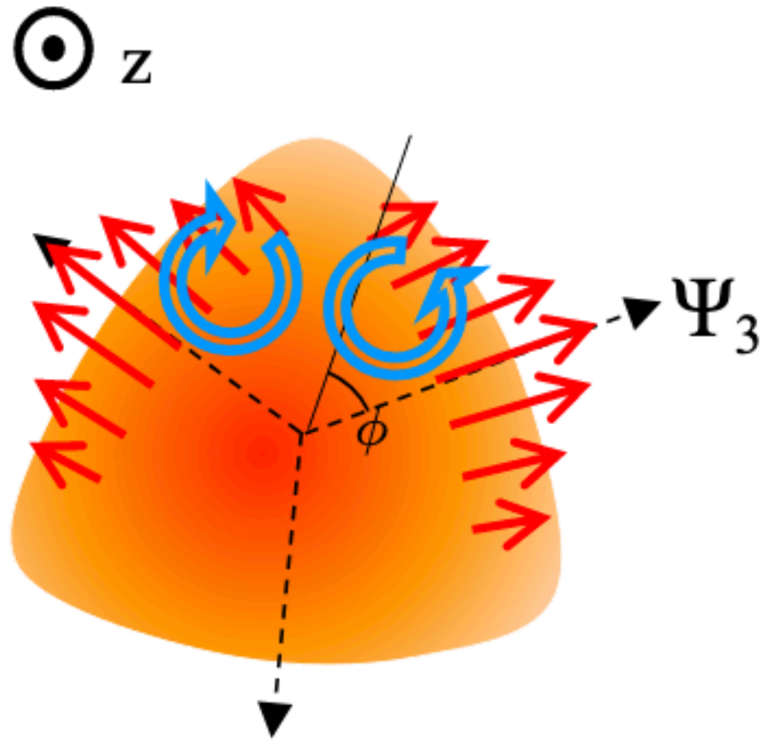
Back Up

Anisotropic flow induced polarization along the beam direction (P_z)



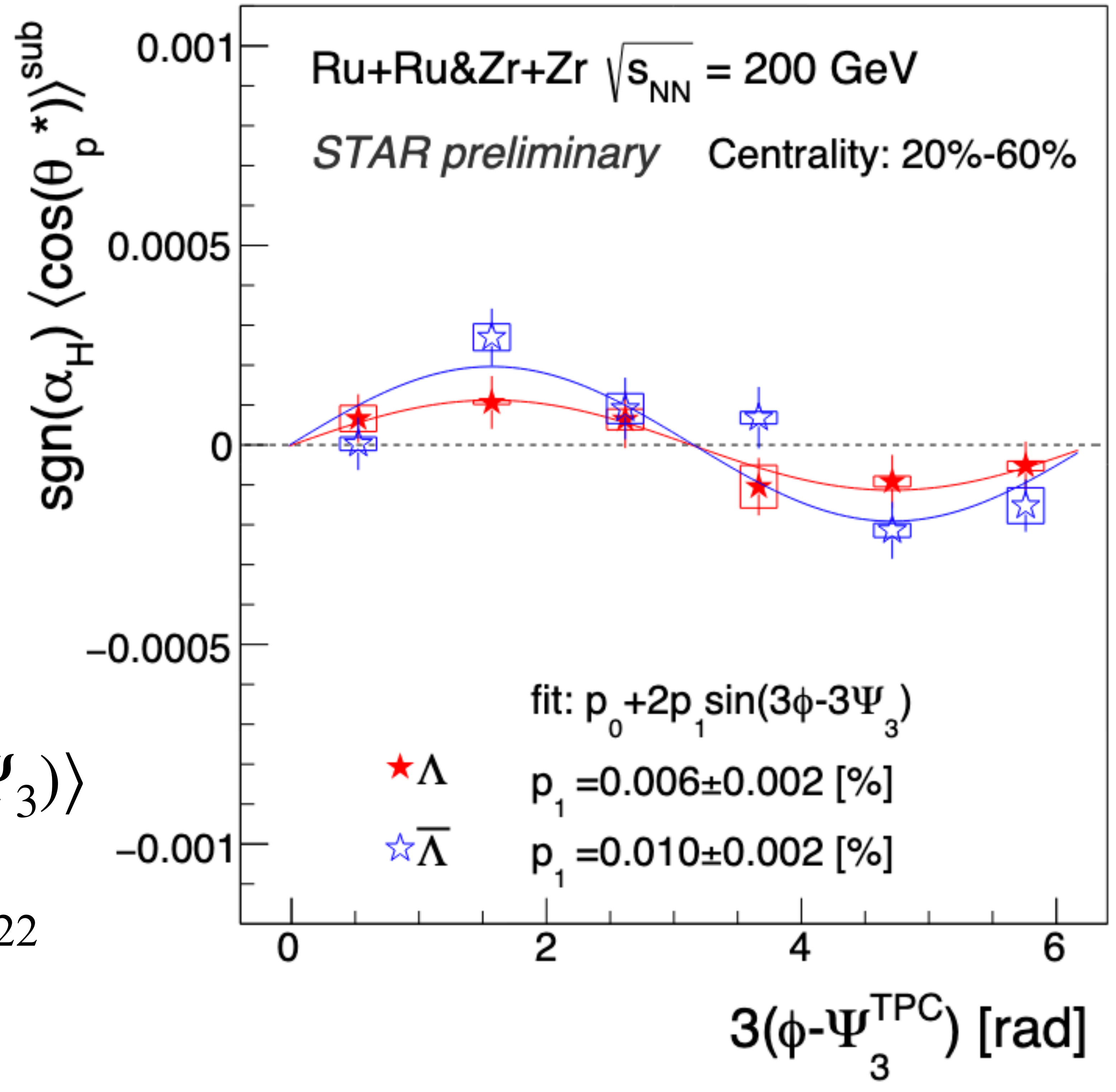
$P_{z,s2} = \langle P_z \sin(2\varphi - 2\Psi_2) \rangle$





$P_{z,s3} = \langle P_z \sin(3\varphi - 3\Psi_3) \rangle$

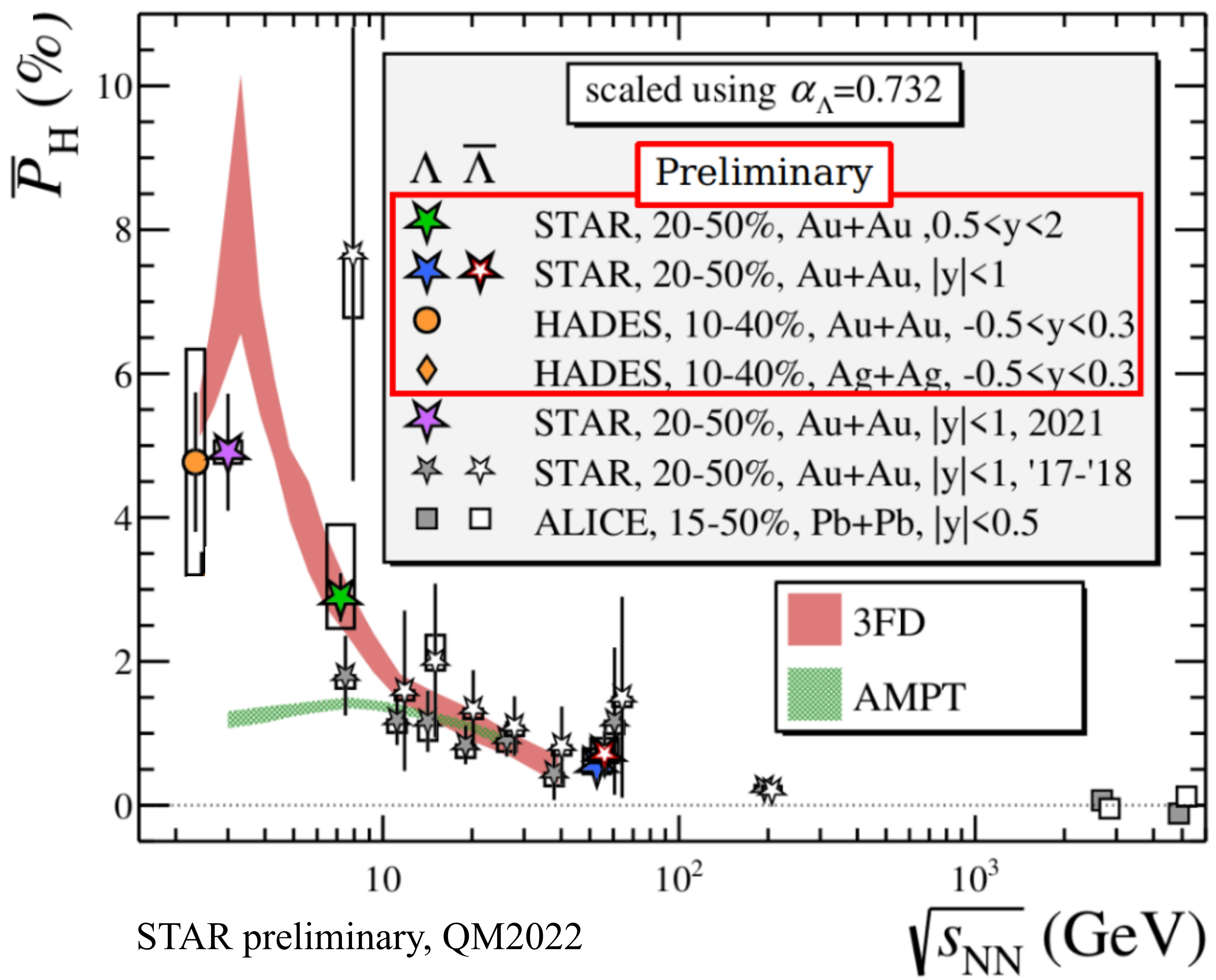
STAR preliminary, QM2022



- Local polarization (along z axis)- $P_z \approx \langle (\hat{p}_p^* \cdot \hat{z}) \rangle$ $\quad P_z = \dfrac{\langle \cos\theta_p^* \rangle}{\alpha_H \langle (\cos\theta_p^*)^2 \rangle}$

- Elliptic flow induced polarization along beam axis: $\sin(2\varphi - 2\Psi_2)$ dependence
- Triangular flow induced polarization along beam axis: $\sin(3\varphi - 3\Psi_3)$ dependence
- No difference between Λ and $\bar{\Lambda}$ polarization.

$\Lambda(\bar{\Lambda})$ polarization in heavy-ion collisions ($\sqrt{s_{NN}}$ dependence)



STAR preliminary, QM2022

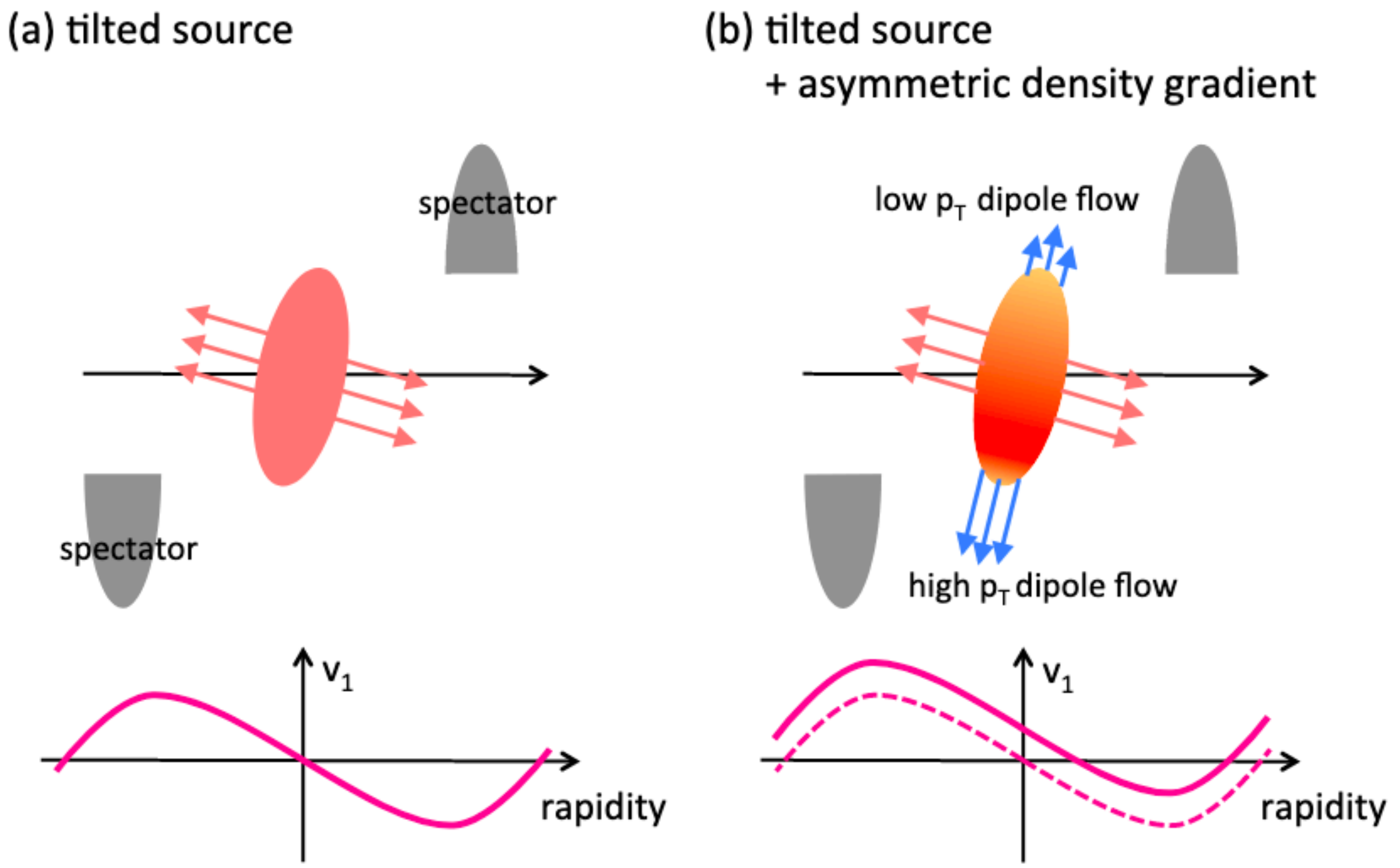
Task: Locate $\sqrt{s_{NN}}$ with maximum P_H

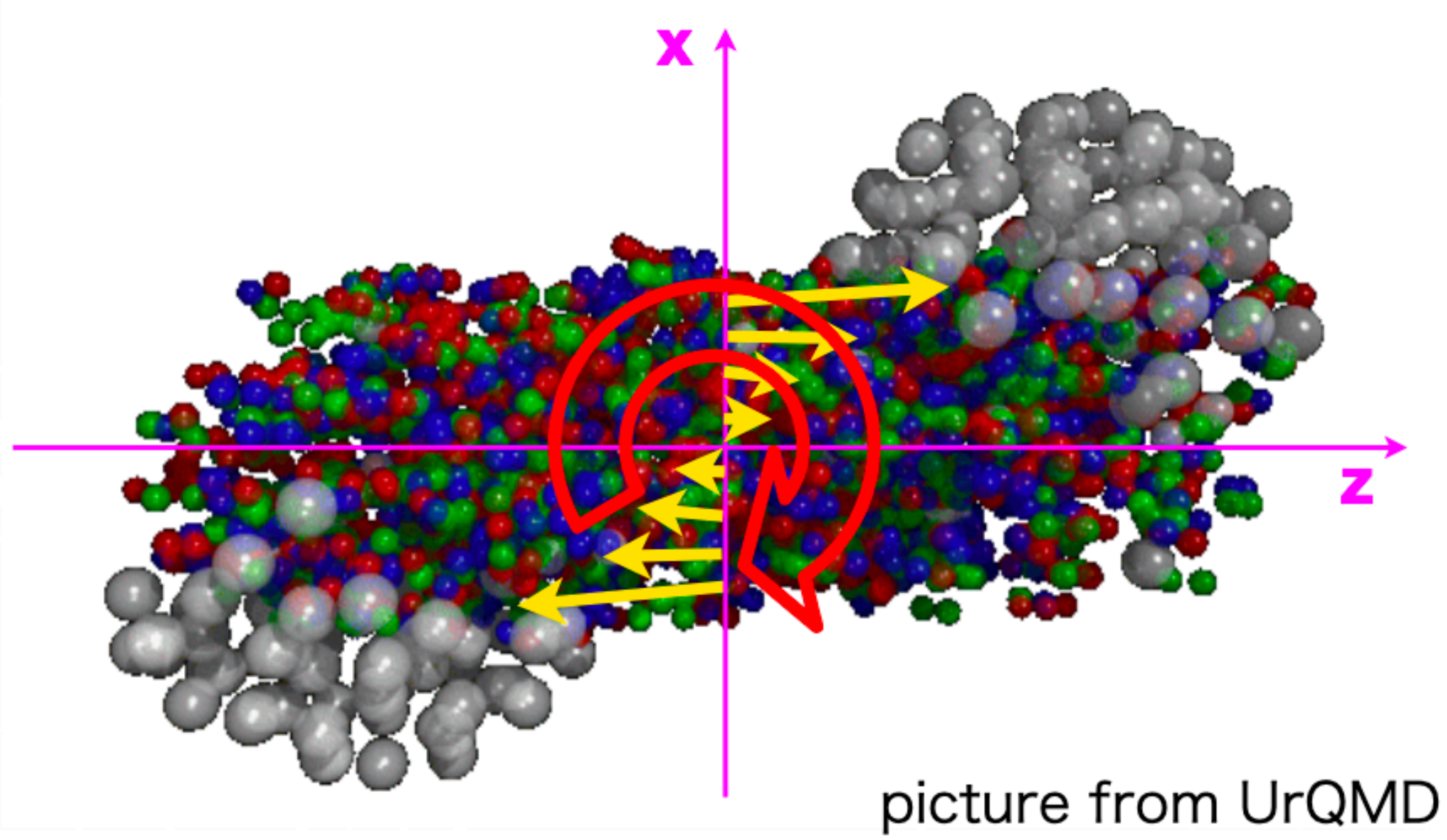
STAR, Phys. Rev. C 98, 014910 (2018)
STAR, Phys. Rev. C 104 (2021) L061901 (2021)
ALICE, Phys. Rev. C 101, 044611 (2020)[erratum]

- The P_H is expected to vanish at $\sqrt{s_{NN}} \approx 2m_N \approx 1.9$ GeV.
- A peak of P_H is expected in the range $1.9 < \sqrt{s_{NN}} < 7.7$ GeV.
- Model predicts P_H(max) at $\sqrt{s_{NN}} \approx 3$ GeV. (X.-G. Deng et al., Phys. Rev. C 101, 064908 (2020))
- The P_H at $\sqrt{s_{NN}}$ = 2.42 GeV (HADES) is consistent with STAR 3 GeV result within exp. uncertainty.
- More high precision data points at low $\sqrt{s_{NN}}$ needed to locate P_H(max).

Global vorticity and directed flow from tilted source

STAR Collaboration, Phys. Rev. C 98, 014915





picture from UrQMD

Directed flow:

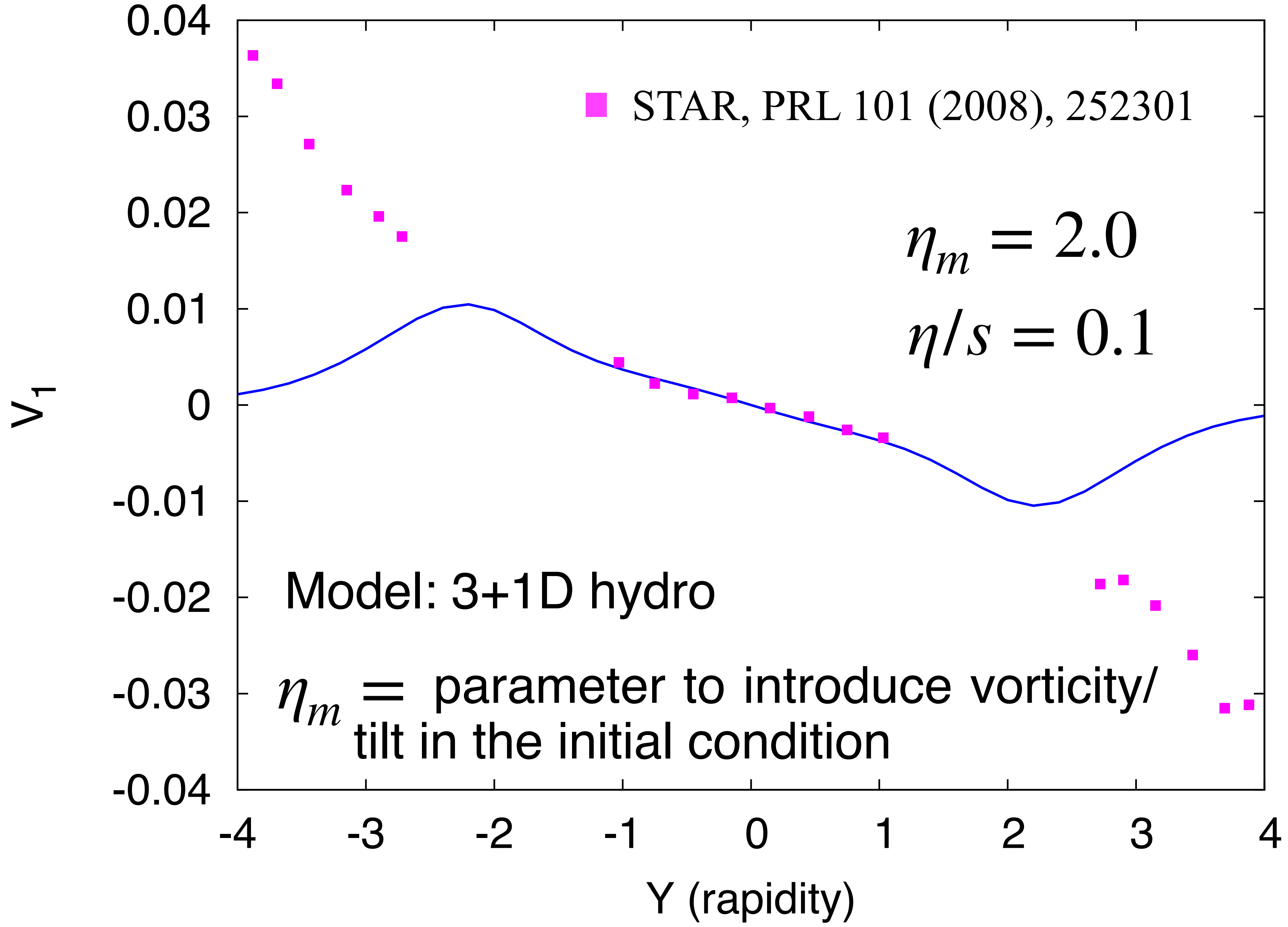
$$v_1 = \cos(\phi - \Psi_{RP})$$

- Asymmetries in the initial velocity field generate vorticity (tilt) in the system → generates directed flow (V_1).

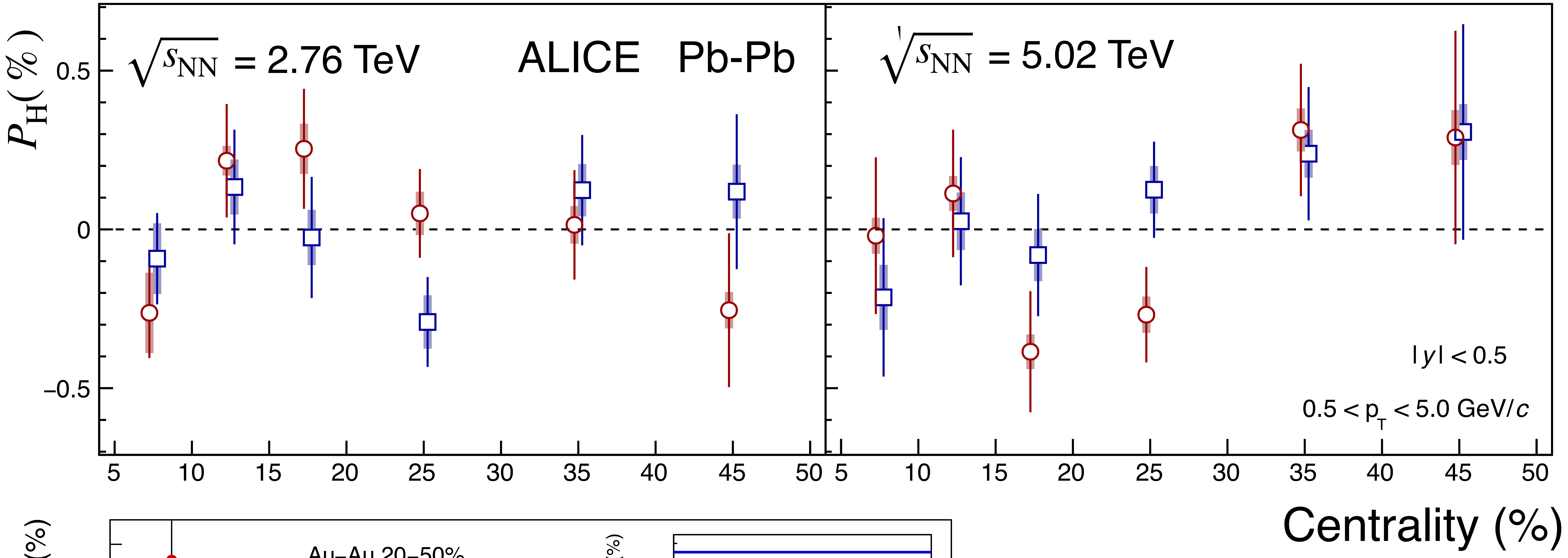
S. A. Voloshin, EPJ Web Conf., 171 (2018) 07002

- To describe the v_1 in heavy-ion collisions → vorticity (tilt) has to be taken into account.

Becattini et al, Eur. Phys. J. C **78,** 354 (2018) (arXiv:1501.04468 [nucl-th] v3)



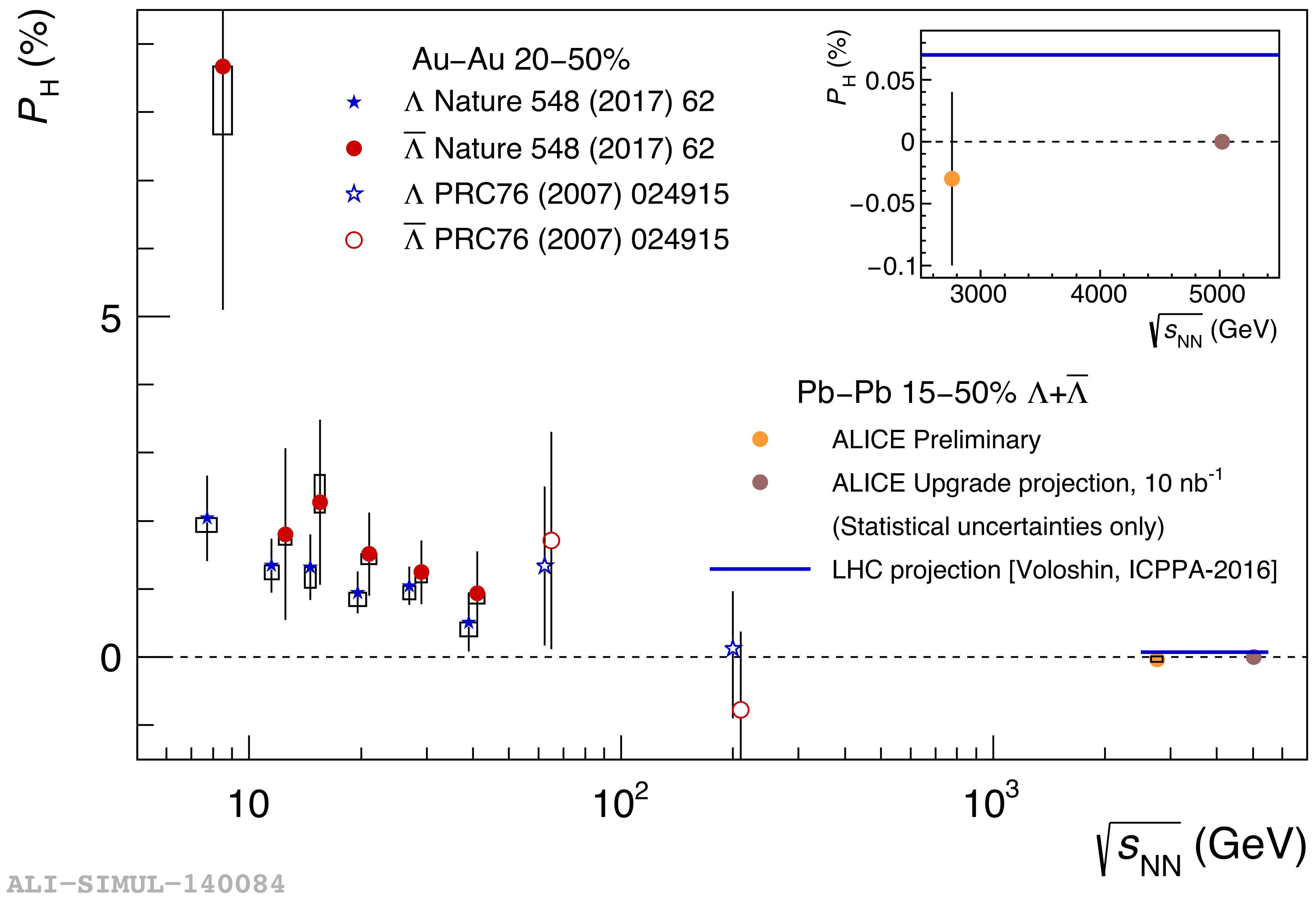
Global polarization (P_H) in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV in ALICE



ALICE, Phys. Rev. C 101, 044611 (2020)[erratum]
STAR, Phys. Rev. C 98, 014910 (2018)

$\Lambda = p + \pi^-$

$\overline{\Lambda} = \overline{p} + \pi^+$

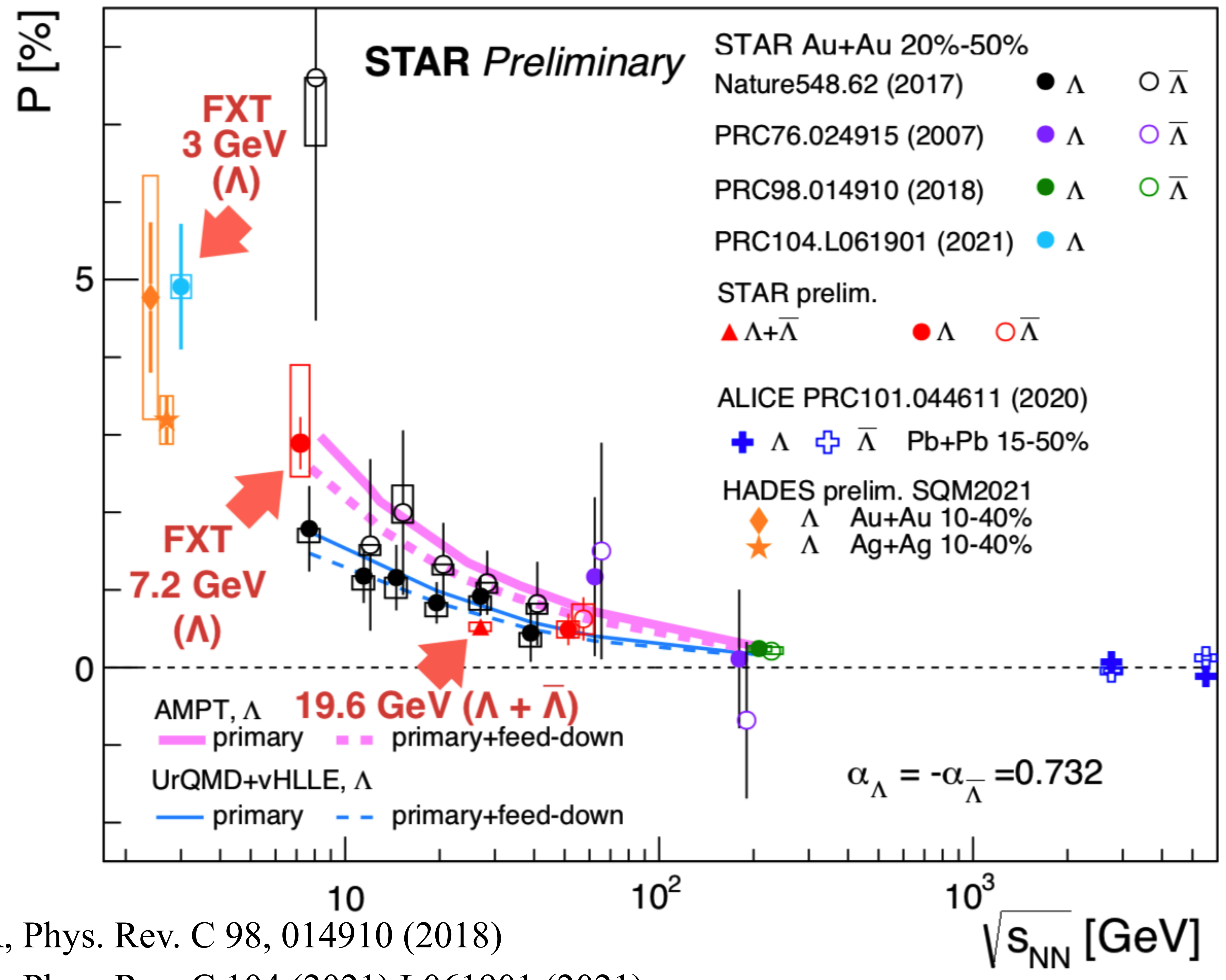


- P_H consistent with zero within experimental uncertainties.
- No visible difference between Λ and $\bar{\Lambda}$ polarization.
- P_H decreases with collision energy (due to higher baryon transparency at higher collision energies).

ALICE-PUBLIC-2019-001, https://cds.cern.ch/record/2661798

S. A. Voloshin, ICPPA - 2016

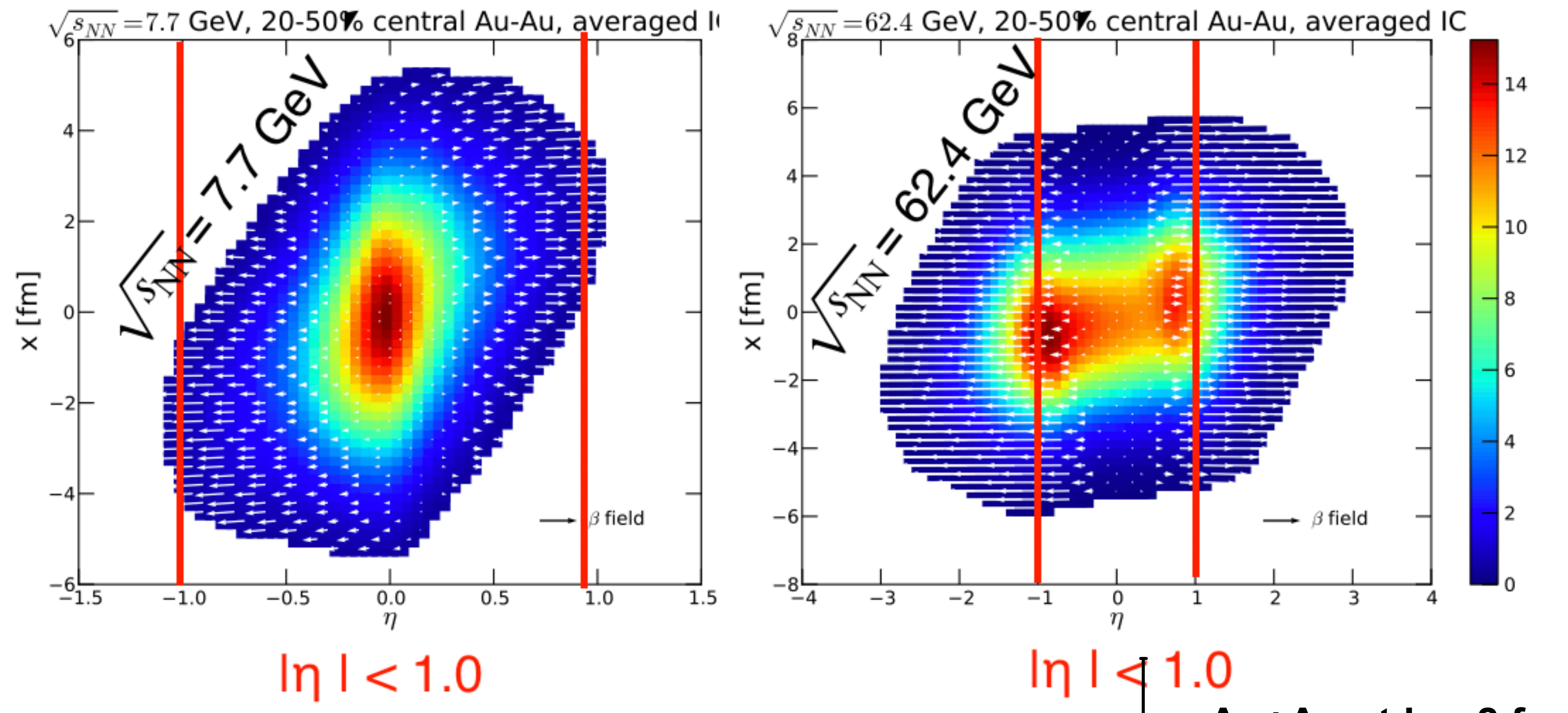
$\Lambda(\bar{\Lambda})$ polarization in heavy-ion collisions (rapidity dependence)

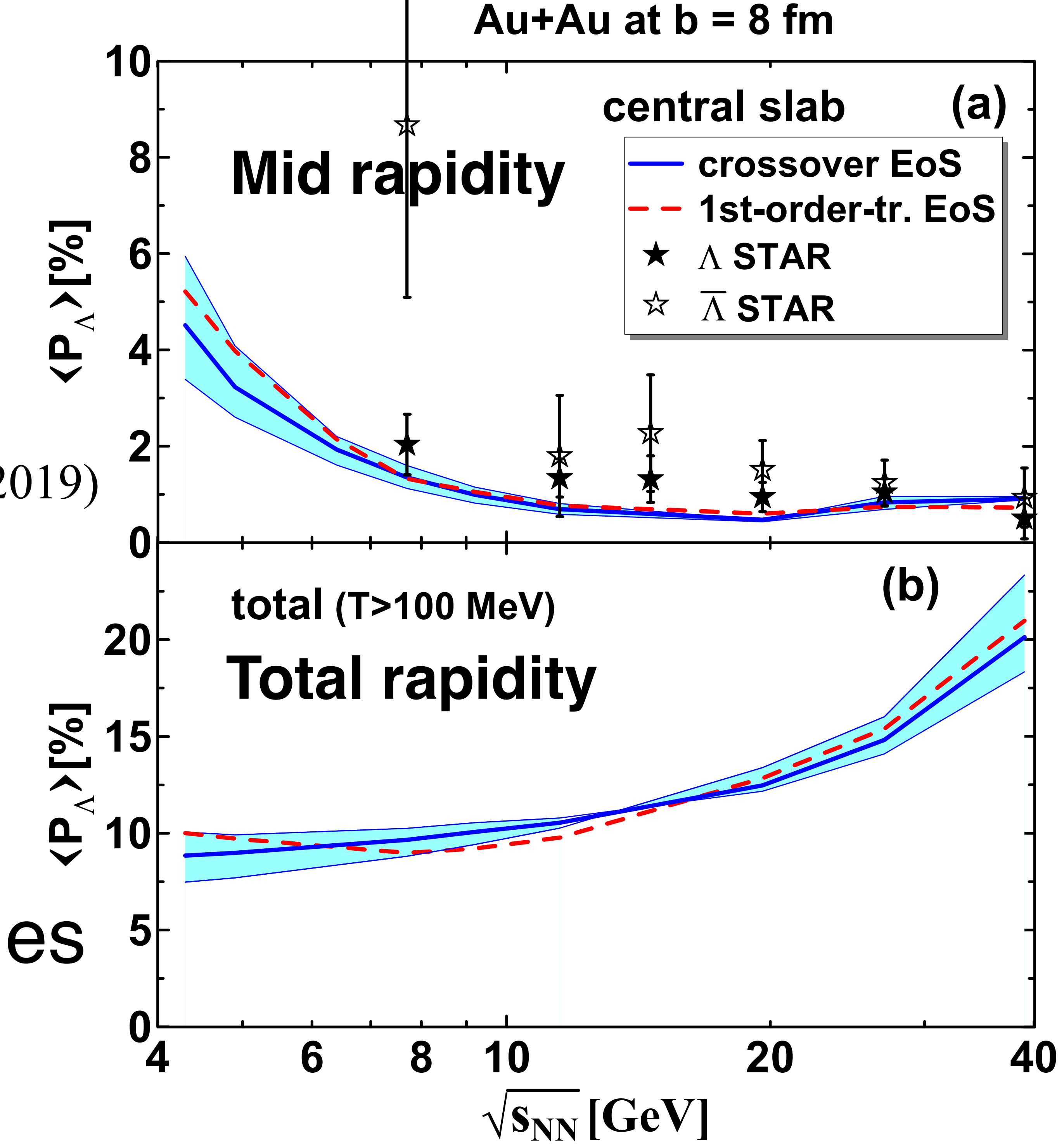


STAR, Phys. Rev. C 98, 014910 (2018)

STAR, Phys. Rev. C 104 (2021) L061901 (2021)

Iu. Karpenko, F. Becattini Eur. Phys. J. C **77,** 213 (2017)



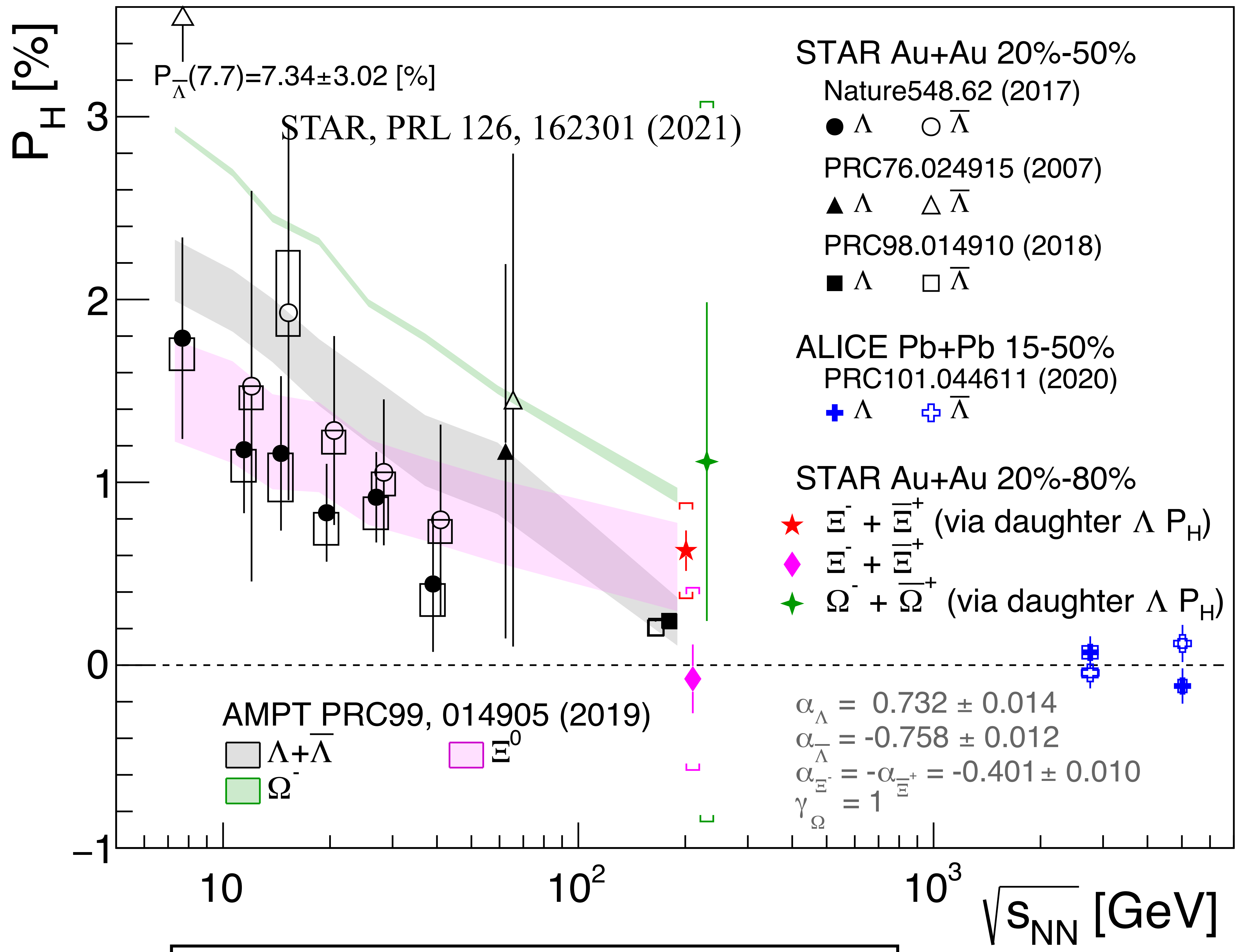


Ivanov, Toneev, Soldatov
Phys. Rev. C 100, 014908 (2019)

- Polarization at mid-rapidity decreases with collision energy.
- Bjorken boost invariance at mid-rapidity? vorticity migrates to forward rapidity?
- For higher $\sqrt{s_{NN}}$ (i.e. higher y_{beam}), not possible to measure P_H at forward rapidities at STAR/ALICE (needs detector upgrade).

Polarization of multi-strange hyperons

$$\Xi = \Lambda + \pi, \;\; \Lambda = \mathrm{p} + \pi$$
$$\Omega = \Lambda + \mathrm{K}, \;\; \Lambda = \mathrm{p} + \pi$$



$$\mathbf{P} = \frac{\langle \mathbf{s} \rangle}{s} \approx \frac{(s+1)}{3}\frac{\boldsymbol{\omega}}{T},$$

Becattini et al., PRC 95, 054902 (2017)

- General method: Estimation of mother polarization from daughter momentum distribution in parent rest frame:

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\Xi,\Omega}\mathbf{P}_{\Xi,\Omega} \cdot \hat{\mathbf{p}}^*_{\Lambda}),$$

$$\alpha_{\Xi} = -0.401 \qquad \alpha_{\Omega} = 0.0157,$$

Zyla et al. (PDG), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

- Issue: when $\alpha \approx 0 \longrightarrow \frac{\mathrm{d}N}{\mathrm{d}\Omega^*}$ isotropic

  Example: ($\Omega = \Lambda + \mathrm{K}, \quad \alpha_{\Omega} = 0.0157$)

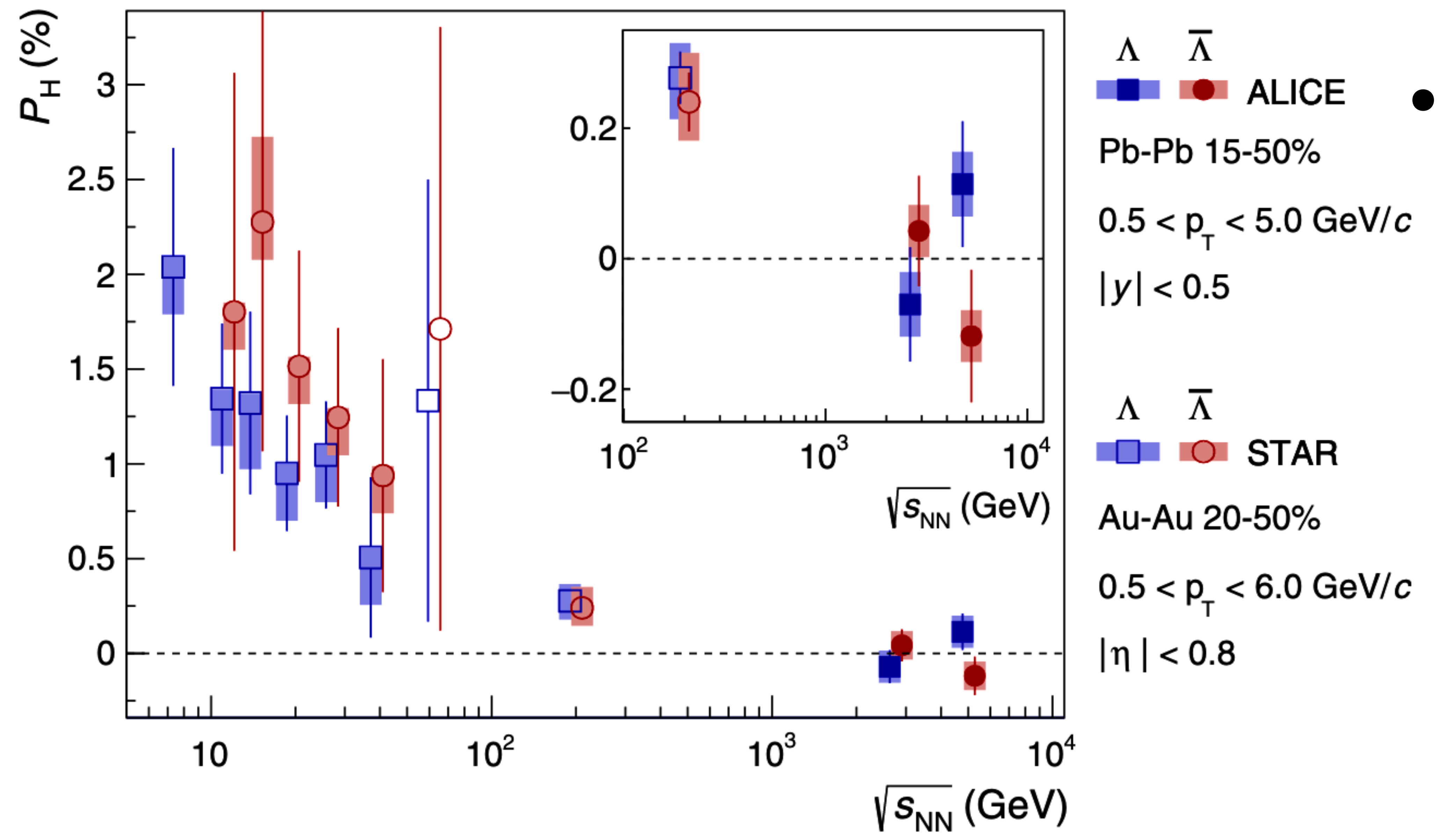
- Solution: $\mathbf{P}_{\Xi,\Omega}$ estimation from $\mathbf{P}_{\Lambda}$ $(\Lambda = \mathrm{p} + \pi, \alpha_{\Lambda} = 0.732)$ (assumption: polarization transfer from parent to daughter)

$\mathbf{P}_{\Omega}(s = 3/2) > \mathbf{P}_{\Lambda}(s = 1/2)$ Spin effect only?.

$\mathbf{P}_{\Xi}(s = 1/2) > \mathbf{P}_{\Lambda}(s = 1/2)$

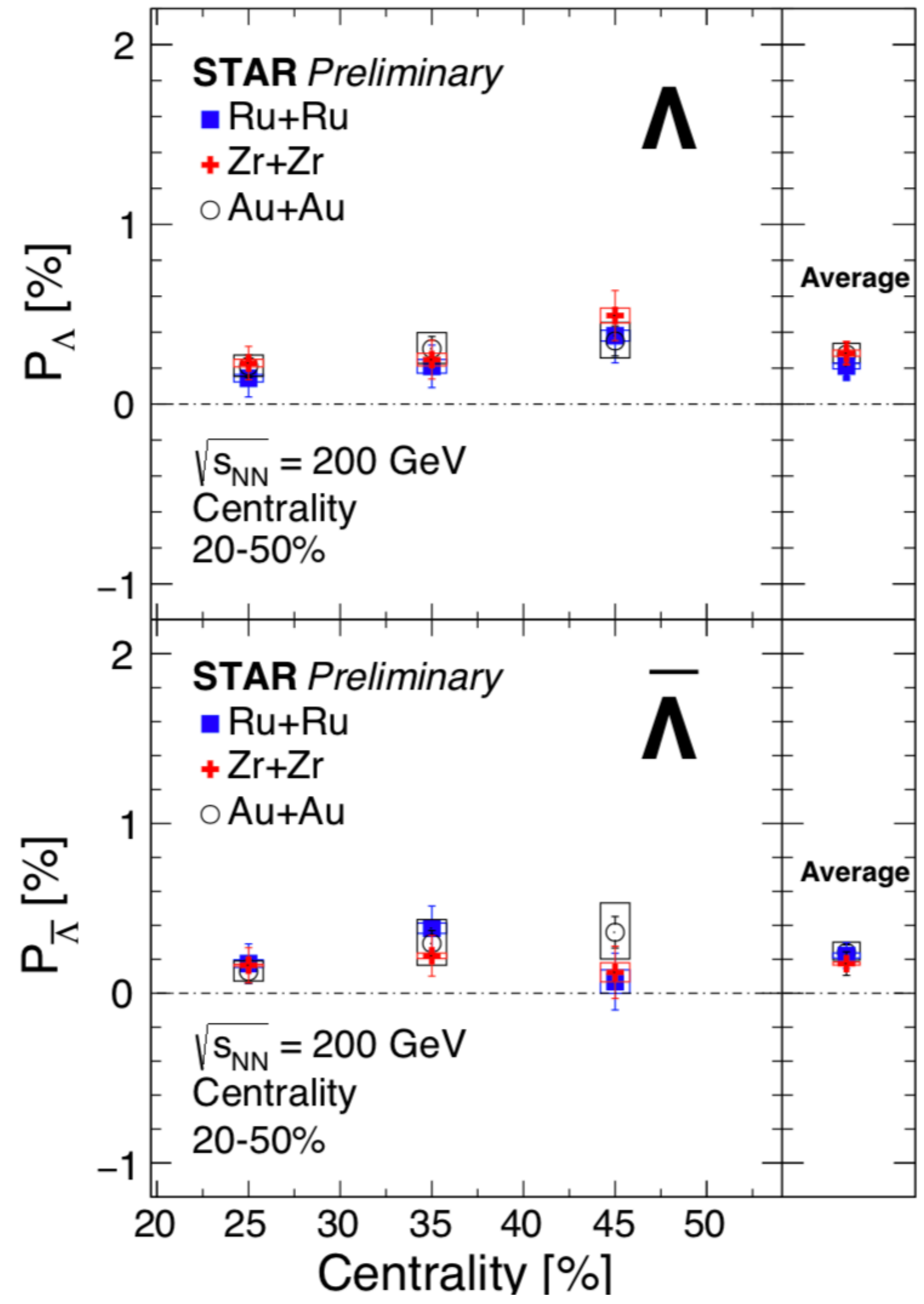
Different freeze-out scenario? Mass effects? Localised effects?? More precision measurements needed.

$\Lambda(\bar{\Lambda})$ polarization in heavy-ion collisions (B field effect)



ALICE, Phys. Rev. C. 101, 044611 (2020)
STAR, Phys. Rev. C 98, 014910 (2018)

- $P_{(\bar{\Lambda})}$ is consistent with P_{Λ} within measurement uncertainty (however, $P_{(\bar{\Lambda})}$ is systematically higher than P_{Λ}).



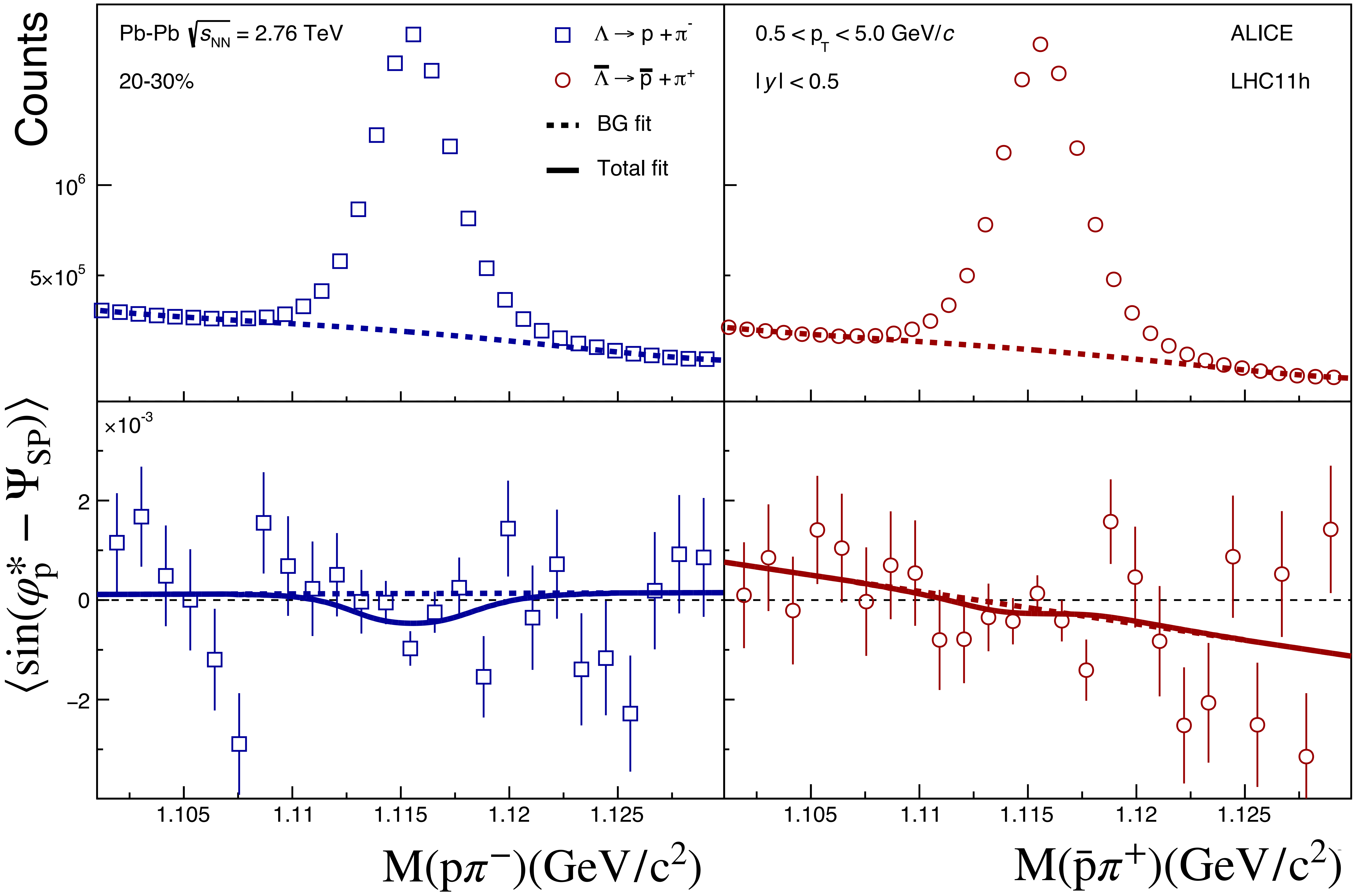
- No observed effect/splitting of $P_{\Lambda}/P_{(\bar{\Lambda})}$ due to different magnetic (B) fields in Ru+Ru and Zr+Zr collisions.

STAR preliminary, QM2022

Measuring global polarization (P_H) in ALICE using invariant mass method

ALICE, Phys. Rev. C 101, 044611 (2020)[erratum]

$\Lambda = p + \pi^-$ $\quad\quad$ $\overline{\Lambda} = \overline{p} + \pi^+$



$$P_H = -\frac{8}{\pi \alpha_H} \frac{\langle \sin(\varphi_p^* - \Psi_{SP}) \rangle}{R_{SP}^1}$$

- P_H measured from the fit to Q ($\langle \sin(\varphi_p^* - \Psi_{SP}) \rangle$) -

$$Q(M_{inv}) = f^S(M_{inv})Q^S + f^{BG}(M_{inv})Q^{BG}(M_{inv})$$

f^S , f^{BG} signal, background fraction of Λ ($\bar{\Lambda}$)

Q^S ⟶polarization signal,

$Q^{BG}(M_{inv})$⟶Λ ($\bar{\Lambda}$) background contribution.