

# Precise description of medium-induced emissions

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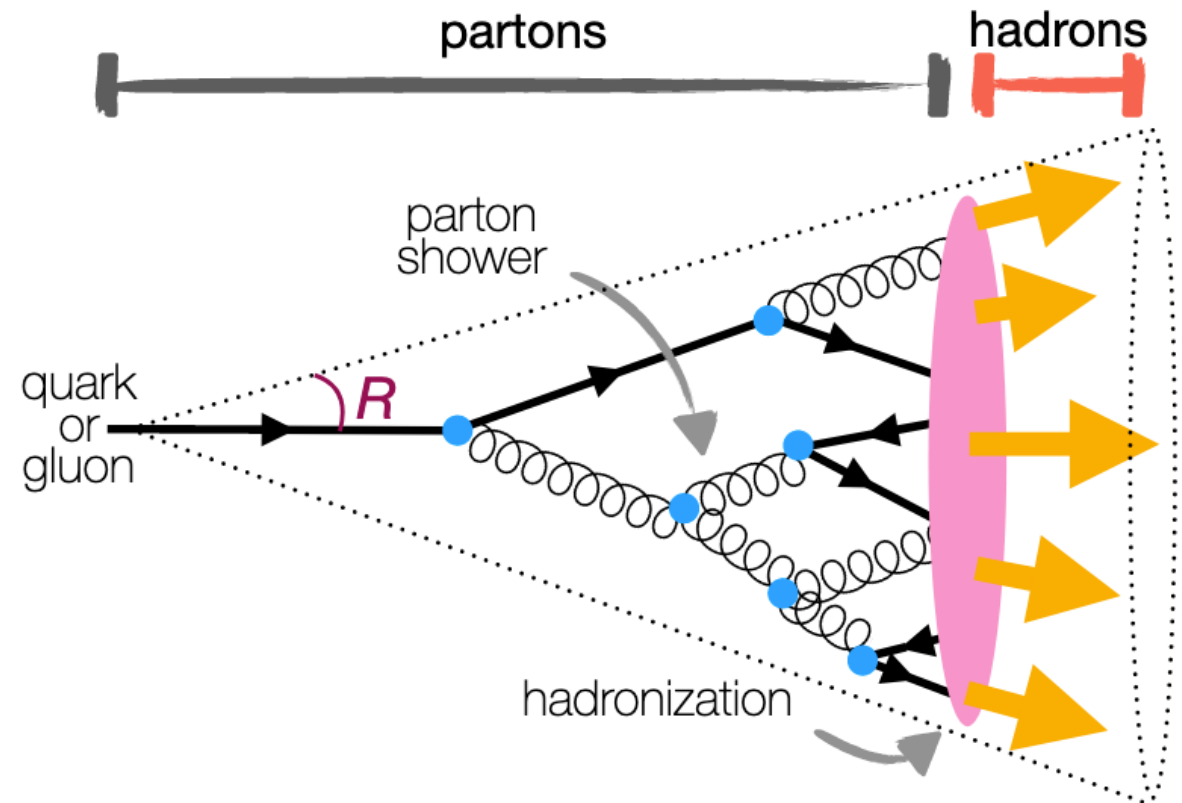
In collaboration with Konrad Tywoniuk  
Based on [2107.02542](#) and ongoing work



# Jets

## In particle collider events

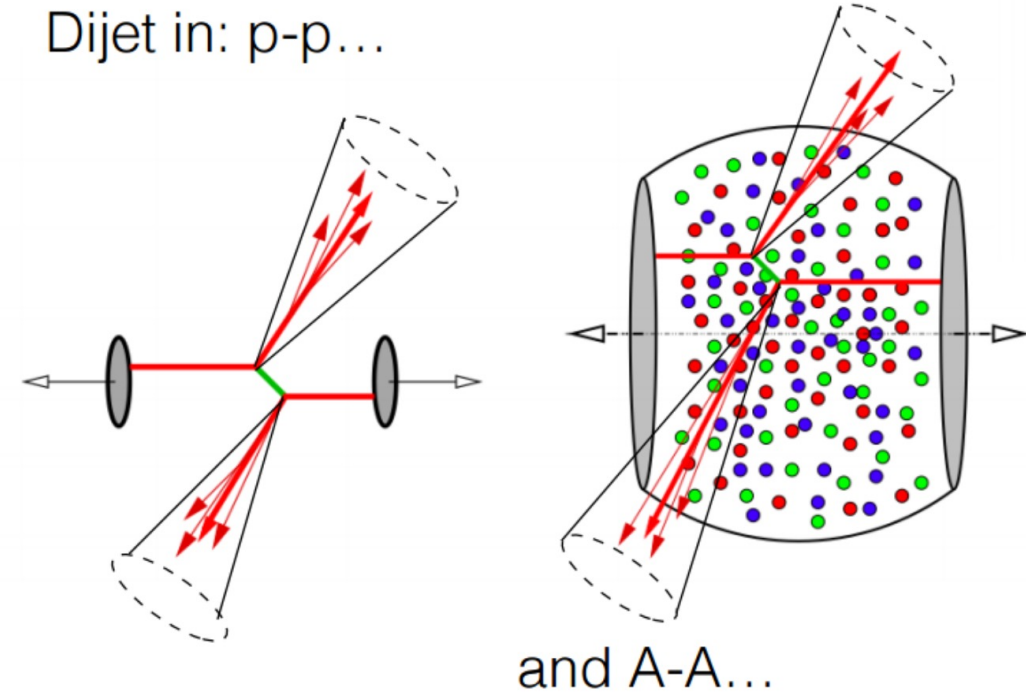
- Hard process makes highly virtual particle
- Particle radiates and creates collimated spray of hadrons
- This is called a **jet**
- Calculable to high precision in proton-proton collisions



[R. Cruz-Torres (2022)]

# Jets in heavy-ion collisions

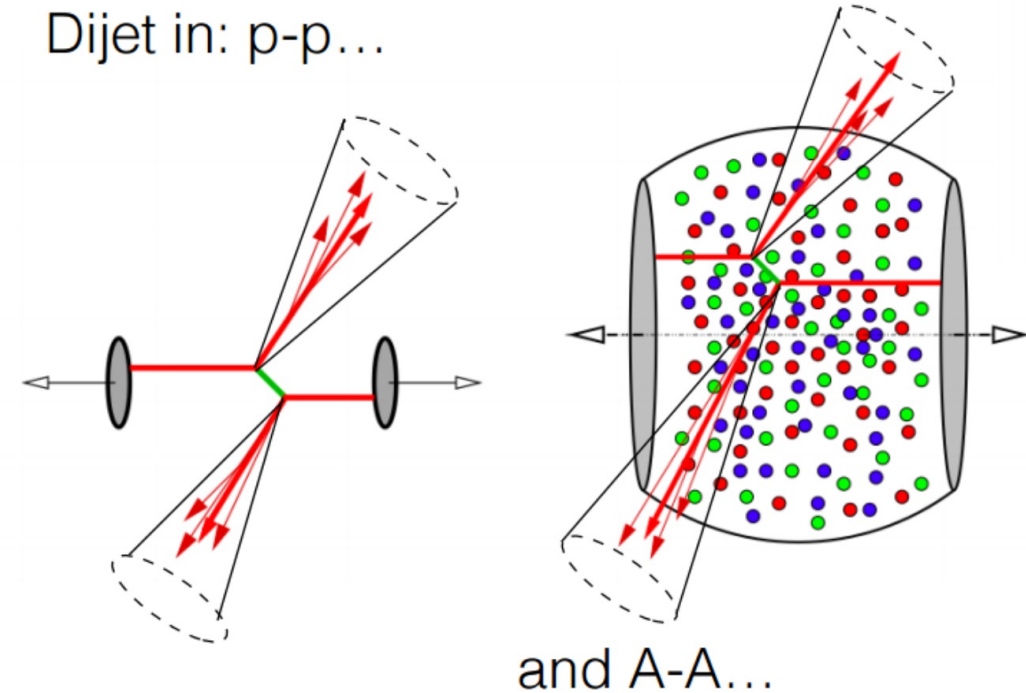
- Colliding two heavy nuclei creates quark-gluon plasma
- Jet must go through the medium (QGP) to reach the detector
- Medium interacts with jet and modifies it
- This is called **jet quenching**



[C. Andres (2022)]

# Jets in heavy-ion collisions

- Colliding two heavy nuclei creates quark-gluon plasma
- Jet must go through the medium (QGP) to reach the detector
- Medium interacts with jet and modifies it
- This is called **jet quenching**
  
- Modified in several ways:
  - Substructure modification
  - Energy loss
  - Change of direction



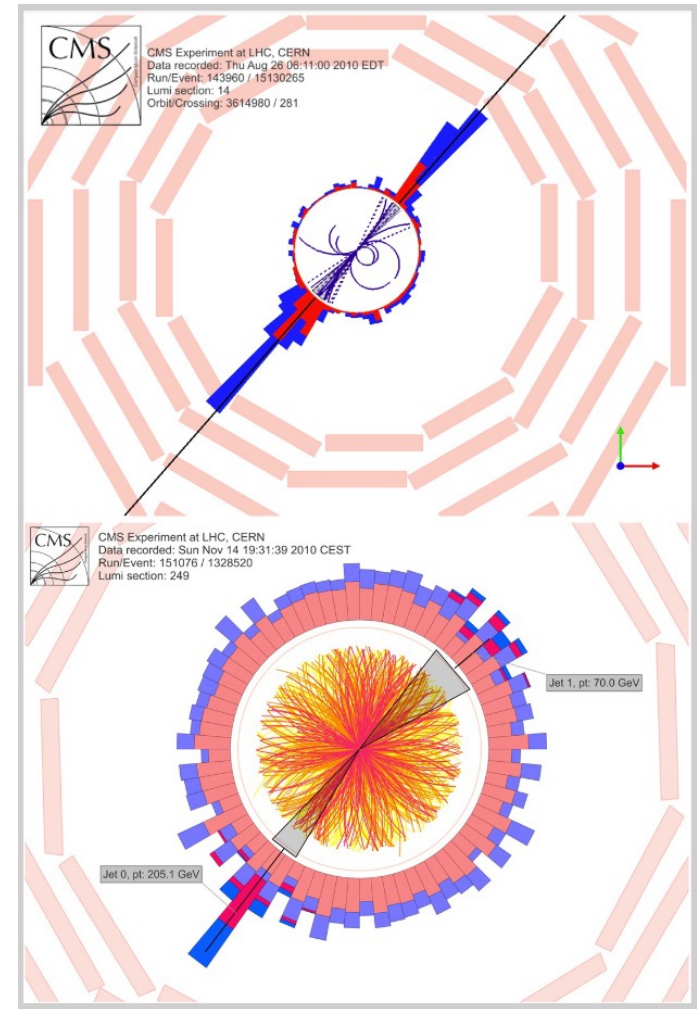
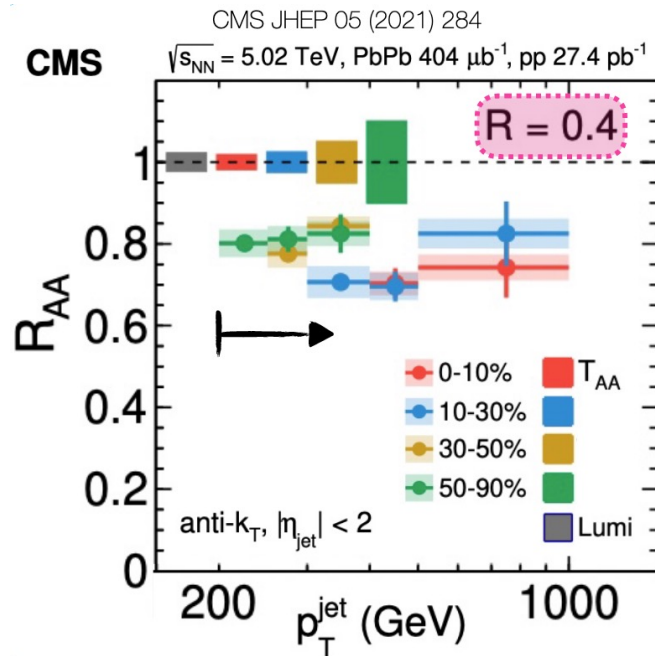
[C. Andres (2022)]

# Signatures of jet quenching

- There are several experimental observables of jet quenching
- Most prominent is the nuclear modification factor

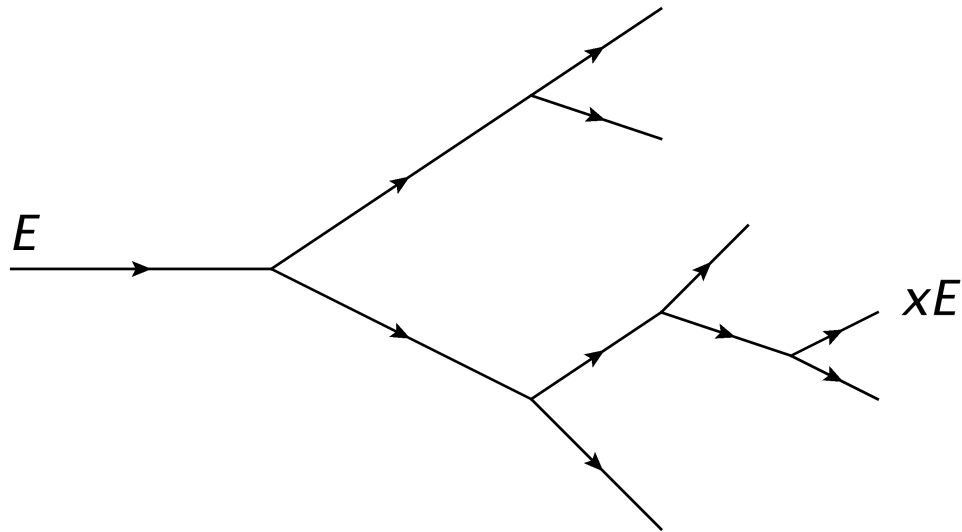
$$R_{AA} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}$$

$R_{AA} > 1 \rightarrow$  enhancement  
 $R_{AA} = 1 \rightarrow$  no medium modification  
 $R_{AA} < 1 \rightarrow$  suppression



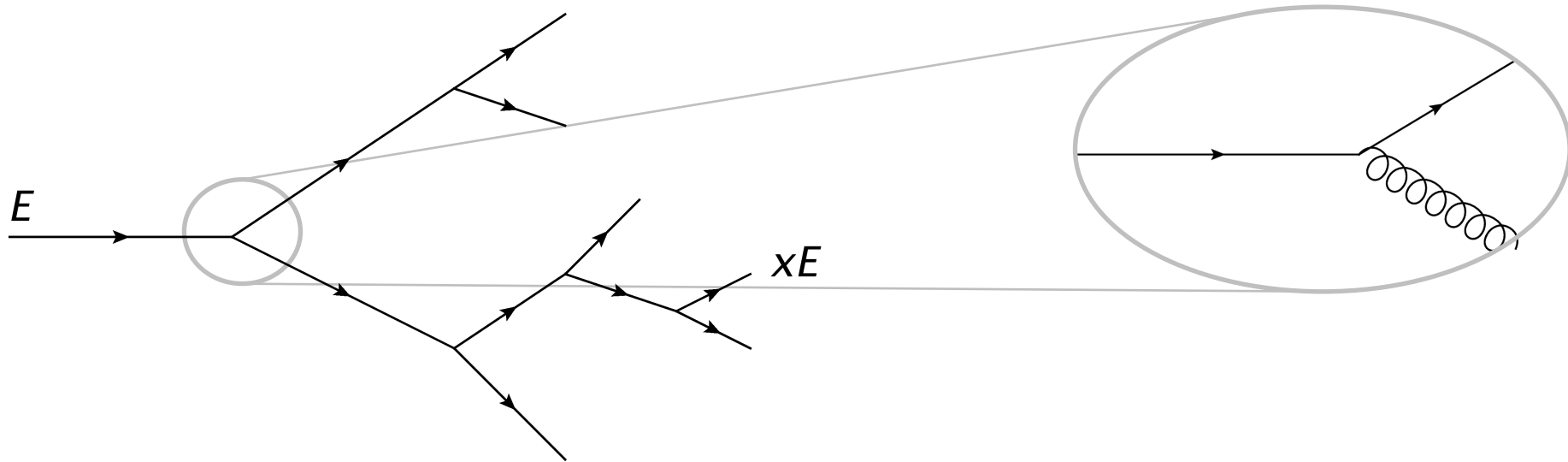
# Jets in heavy-ion collisions

- Partons going through the medium **scatter** with medium constituents
- Scatterings induce **emissions**
  - More emissions compared to vacuum jets
  - Emissions outside of the jet cone  $\rightarrow$  energy loss



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- To calculate many emissions we need to be able to calculate just one emission!

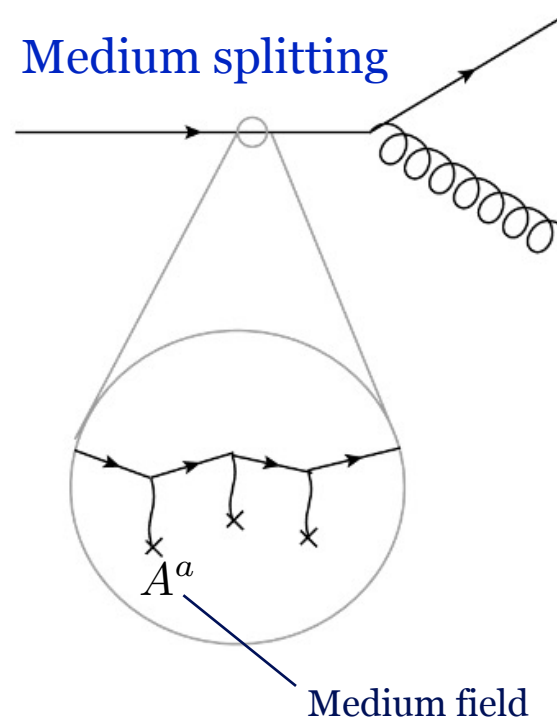
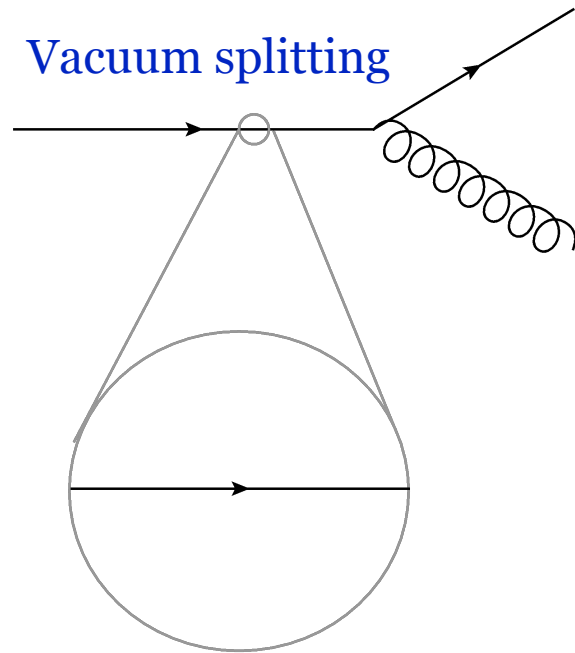
# Calculating jet quenching

- To make predictions we need a theory of how partons interact with the medium
- QCD with an external field  $A^a$
- This field represents the medium interaction
- Can make medium Feynman rules, and calculate medium Feynman diagrams



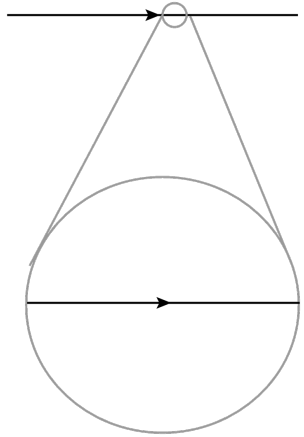
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# Calculating jet quenching

Vacuum propagator

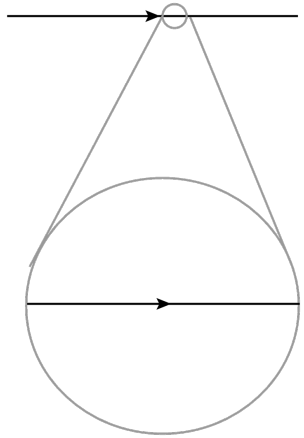


||

$$\frac{i\delta^{ij}}{\not{p} - m + i\epsilon}$$

# Calculating jet quenching

Vacuum propagator



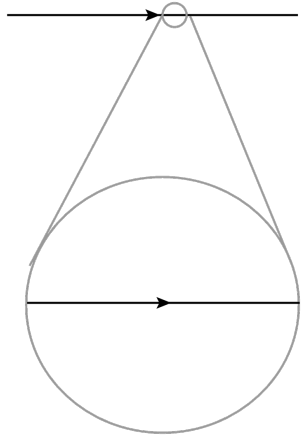
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✓  
NICE  
EASY

# Calculating jet quenching

Vacuum propagator

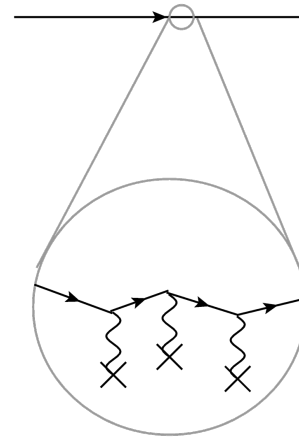


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Medium propagator

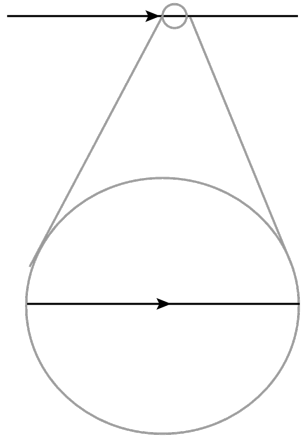


||

$$\mathcal{G}(\mathbf{x}, t; \mathbf{x}_0, t_0) = \int_{\mathbf{x}_0}^{\mathbf{x}} \mathcal{D}\mathbf{r} \exp \left[ i \frac{E}{2} \int_{t_0}^t ds \dot{\mathbf{r}}^2(s) \right] V_R(t, t_0; \mathbf{r}(t))$$

# Calculating jet quenching

Vacuum propagator

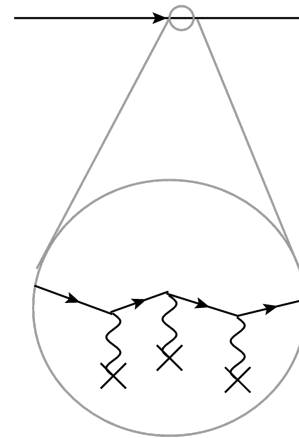


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Medium propagator



||

Varying path leads to  
path integral

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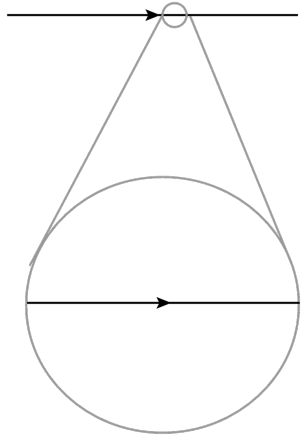
Color rotation leads to:

Wilson line

$$V_R(t, t_0; \mathbf{r}(t)) = \mathcal{P} \exp \left[ ig \int_{t_0}^t ds A^a(s, \mathbf{r}(s)) T_R^a \right]$$

# Calculating jet quenching

Vacuum propagator



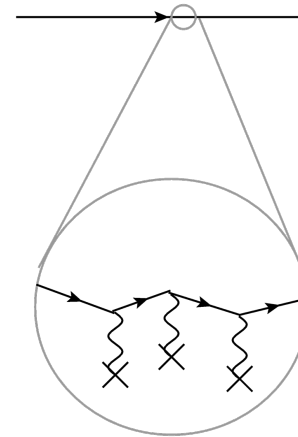
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NICE  
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Medium propagator



||

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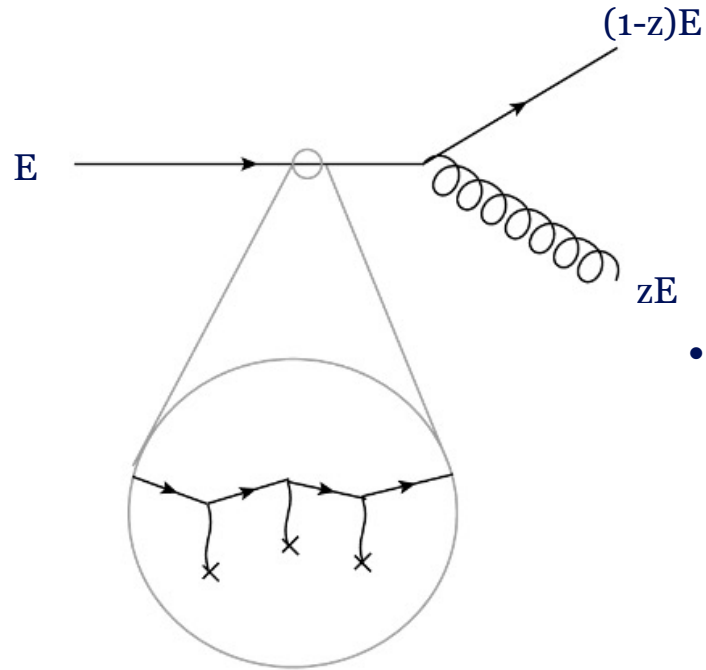
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NOT NICE  
NOT EASY

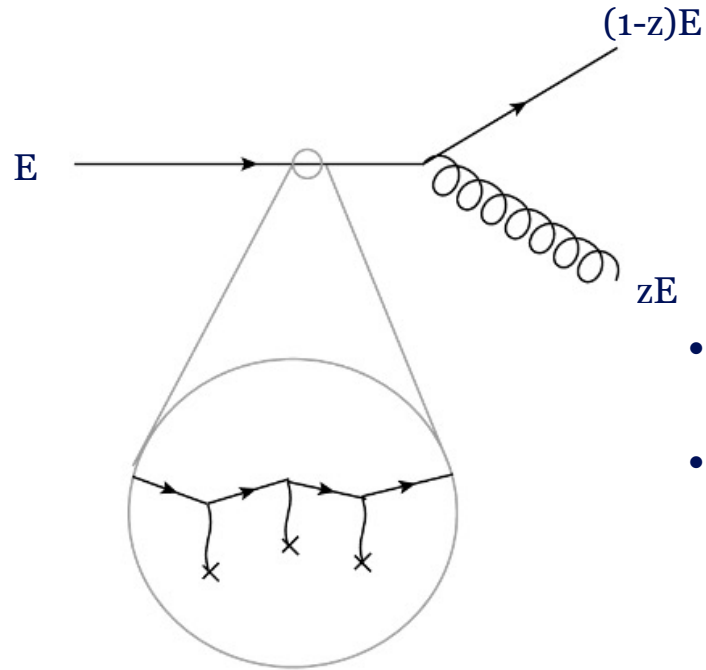
# Medium-induced emissions



$$= \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 \int_{\mathbf{p}_1 \mathbf{l}_1 \mathbf{l}_2 \bar{\mathbf{p}}_2 \bar{\mathbf{l}}_2} \mathbf{l}_1 \cdot \bar{\mathbf{l}}_2 \tilde{S}^4(\mathbf{p}, \mathbf{l}_2, \bar{\mathbf{l}}_2, \bar{\mathbf{p}}_2 - \mathbf{P} | t_{\infty}, t_2) \times \tilde{S}^3(\mathbf{l}_2, \mathbf{l}_1, \mathbf{p}_1 - \bar{\mathbf{p}}_2 | t_2, t_1) \mathcal{P}(\mathbf{p}_1 - \mathbf{p}_0 | t_1, t_0).$$

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# Medium-induced emissions



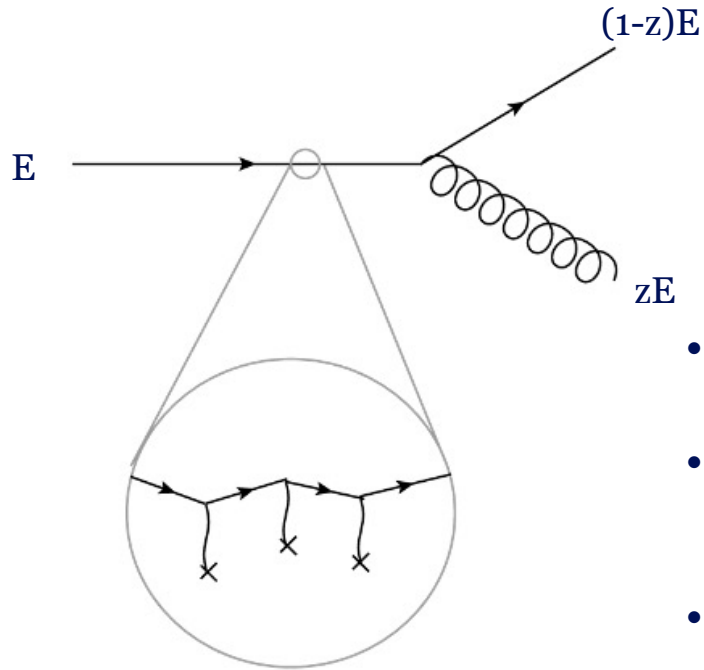
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- $\mathcal{P}(\mathbf{p}_1 - \mathbf{p}_0 | t_1, t_0) \sim \langle \mathcal{G} \mathcal{G}^\dagger \rangle$

✓ Can solve ☺



# Medium-induced emissions



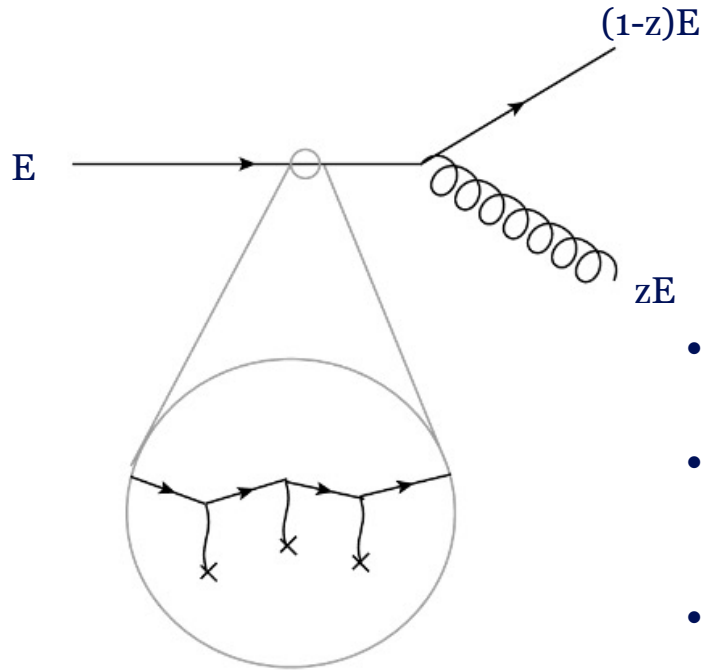
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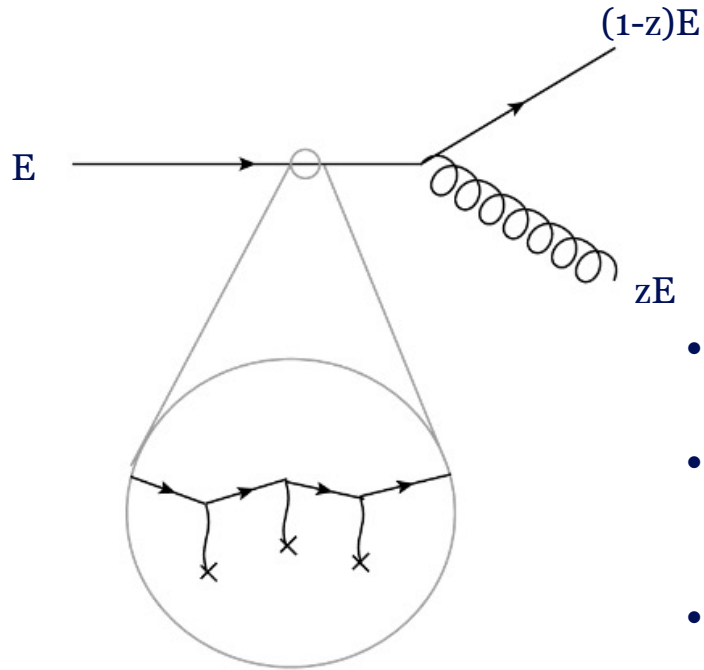
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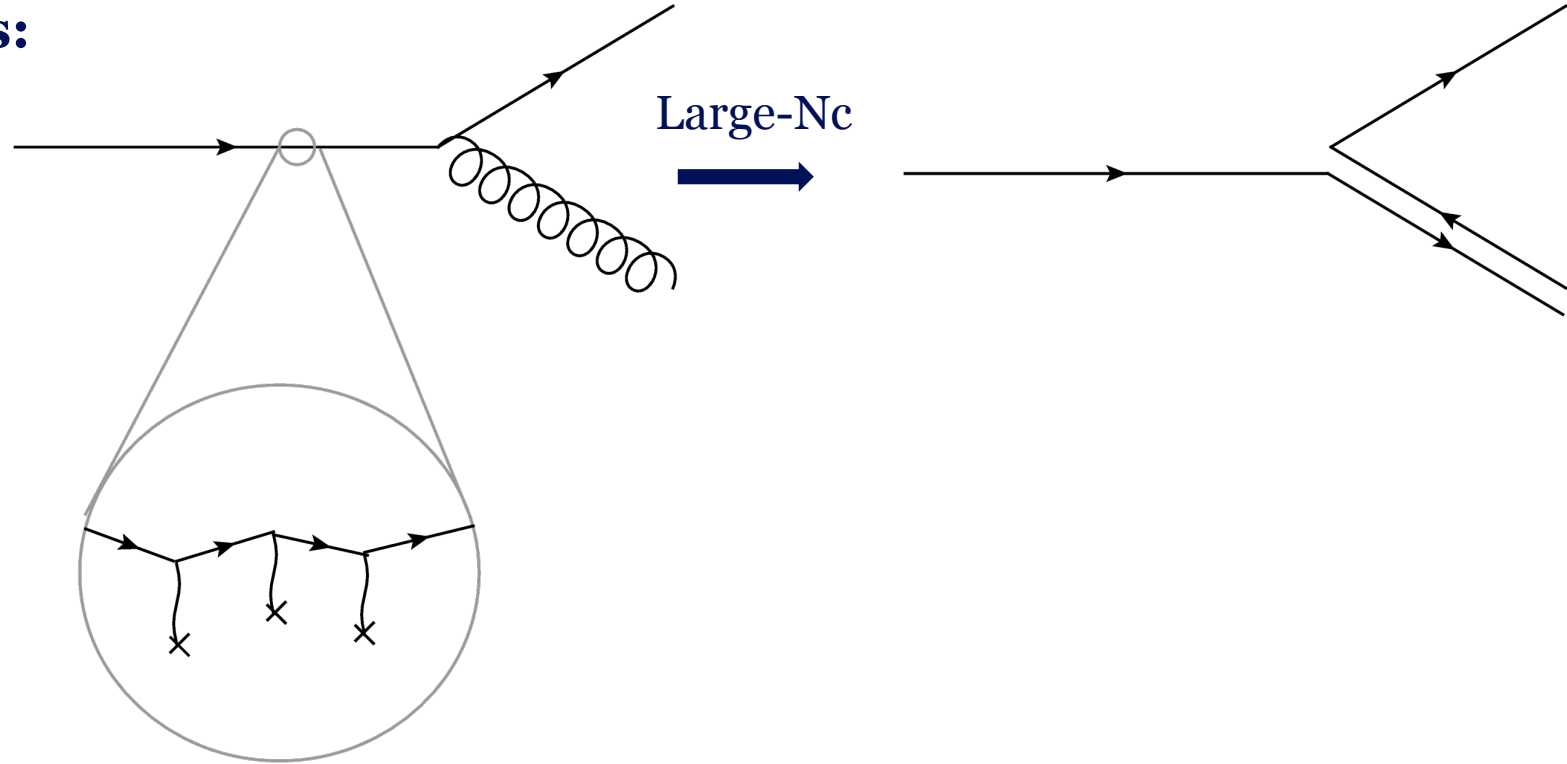
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- Conventionally use two approximations to deal with this:
  - Large- $N_c$  approximation
  - Eikonal approximation

# Medium-induced emissions

Two simplifying approximations:

1. Large- $N_c$  approximation

- Take number of colors ( $N_c$ ) to infinity



# Medium-induced emissions

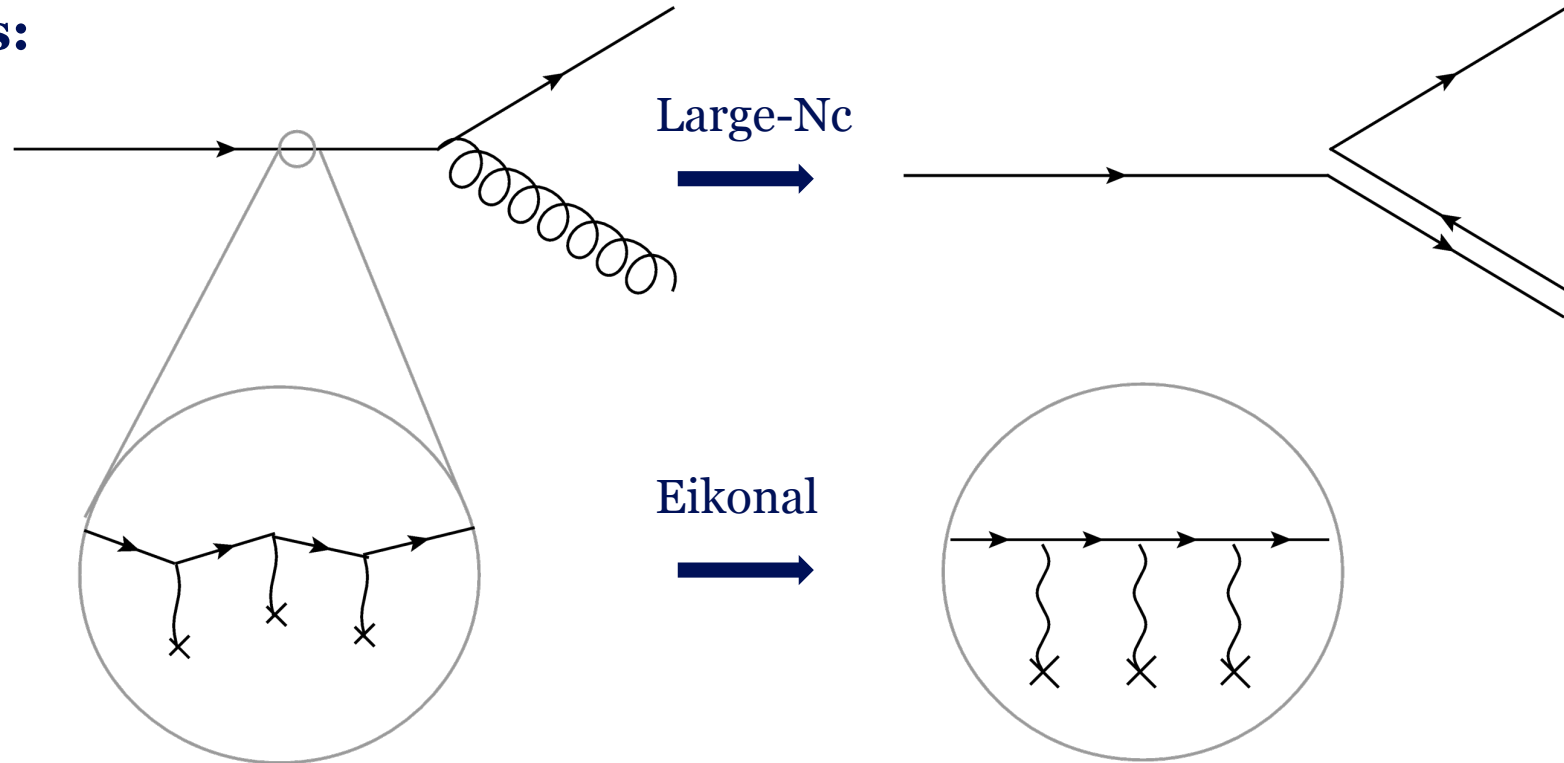
## Two simplifying approximations:

### 1. Large- $N_c$ approximation

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### 2. Eikonal approximation

- Partons travel on straight lines
- Good for high energy



# Medium-induced emissions

## Two simplifying approximations:

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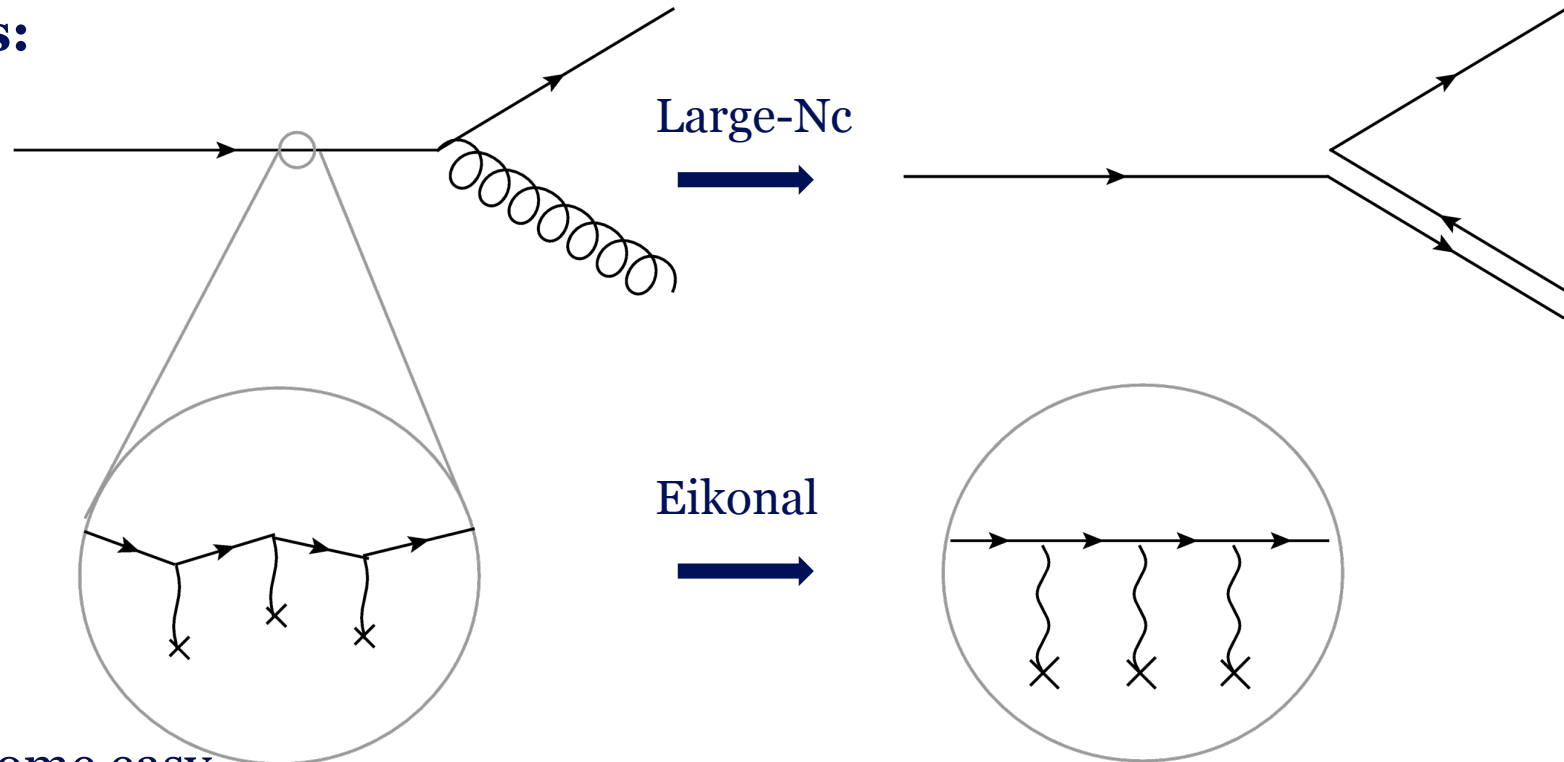
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- Using this the hard calculations become easy

- Our work: Do calculations without using these approximations

- Figure out the error



# Medium-induced emissions

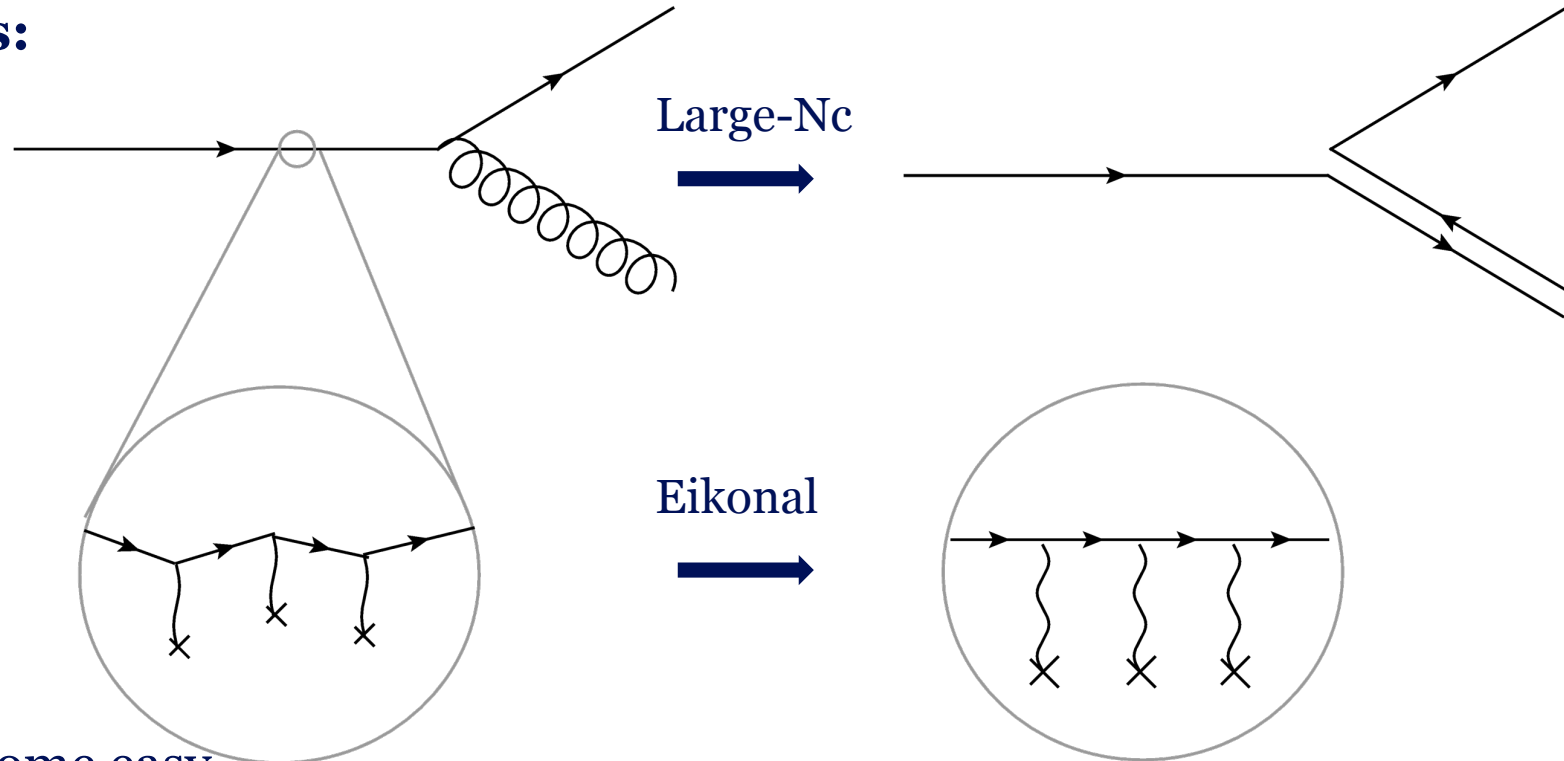
## Two simplifying approximations:

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- Using this the hard calculations become easy
- Our work: Do calculations without using these approximations
  - Figure out the error
  - (Someone has to do it)

# Plan

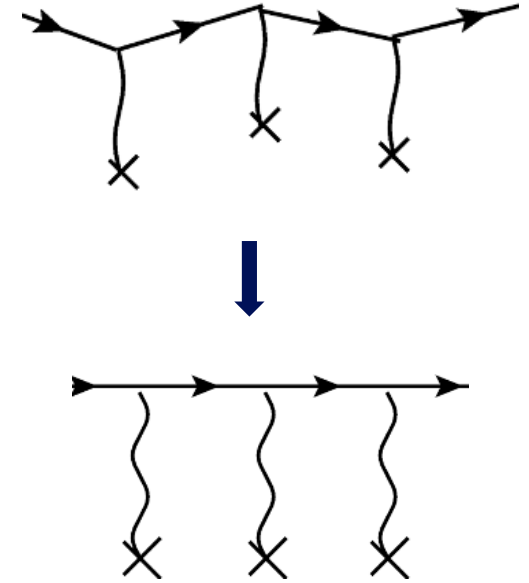
1. Calculate finite- $N_c$  corrections
2. Calculate non-eikonal corrections **and** finite- $N_c$  corrections



# Finite Nc corrections

- First let's use the eikonal approximation, and not the large-Nc
- Path integral turns to an exponential

$$\begin{aligned}\tilde{S}^4(\mathbf{p}, l_2, \bar{l}_2, \bar{\mathbf{p}}_2 - \mathbf{P} | t_\infty, t_2) &\sim \langle g g g^\dagger g^\dagger \rangle \\ &\sim \# e^{(\dots)} \langle V V V^\dagger V^\dagger \rangle\end{aligned}$$



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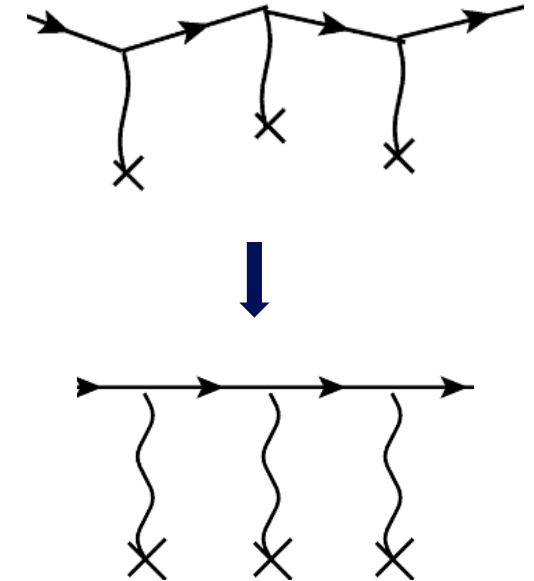
$$\begin{aligned} \tilde{S}^4(\mathbf{p}, \mathbf{l}_2, \bar{\mathbf{l}}_2, \bar{\mathbf{p}}_2 - \mathbf{P} | t_\infty, t_2) &\sim \langle \mathcal{G} \mathcal{G} \mathcal{G}^\dagger \mathcal{G}^\dagger \rangle \\ &\sim \# e^{(\dots)} \langle V V V^\dagger V^\dagger \rangle \end{aligned}$$

- Still have to deal with Wilson lines!

- Wilson line correlators can be calculated through a system of differential equations!

$$\frac{d}{dt} \begin{bmatrix} \langle \text{tr}[VV] \text{tr}[VV] \rangle \\ \langle \text{tr}[VVVV] \rangle \end{bmatrix} = \mathbb{V}(t) \begin{bmatrix} \langle \text{tr}[VV] \text{tr}[VV] \rangle \\ \langle \text{tr}[VVVV] \rangle \end{bmatrix}$$

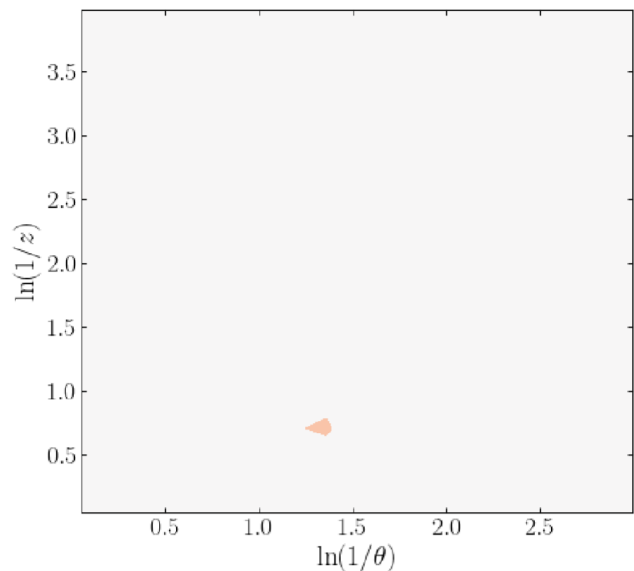
- The potential matrix  $\mathbb{V}(t)$  simplifies greatly in the large-Nc limit
- We solved finite Nc and large-Nc and compared the result
- Expect correction around  $\sim 1/N_c^2 \simeq 10\%$



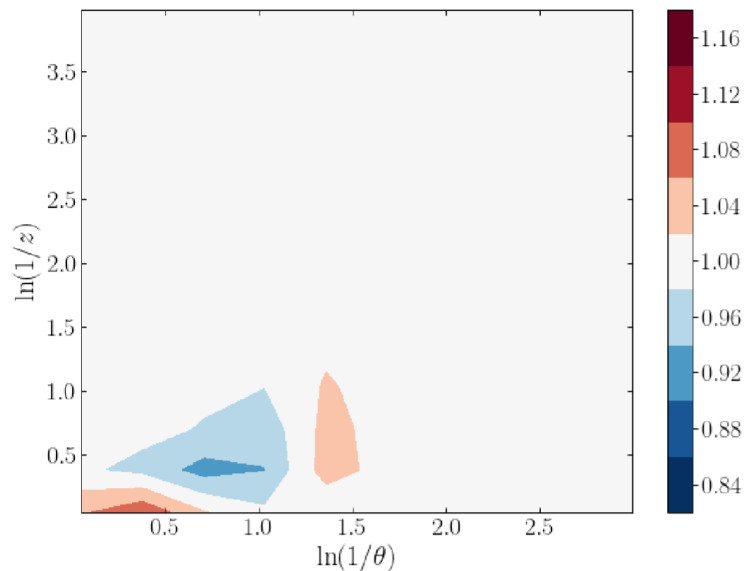
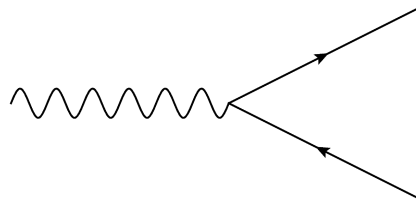
# Finite $N_c$ corrections

Ratio of emission spectrum for large- $N_c$ /finite  $N_c$

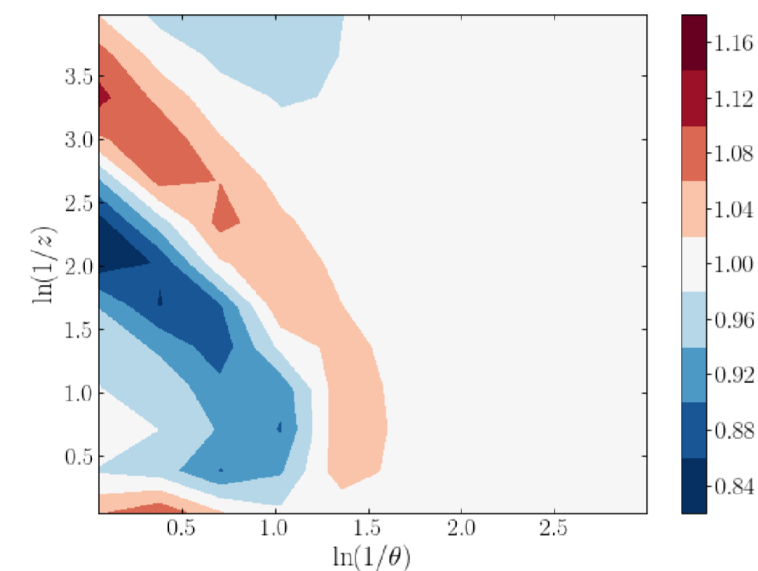
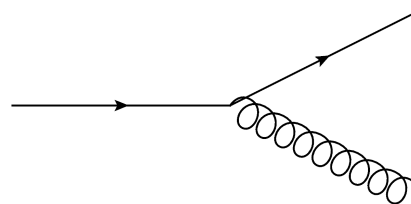
Up to 16%



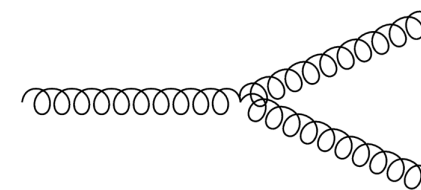
(a) Photon splitting.



(b) Quark-gluon splitting.

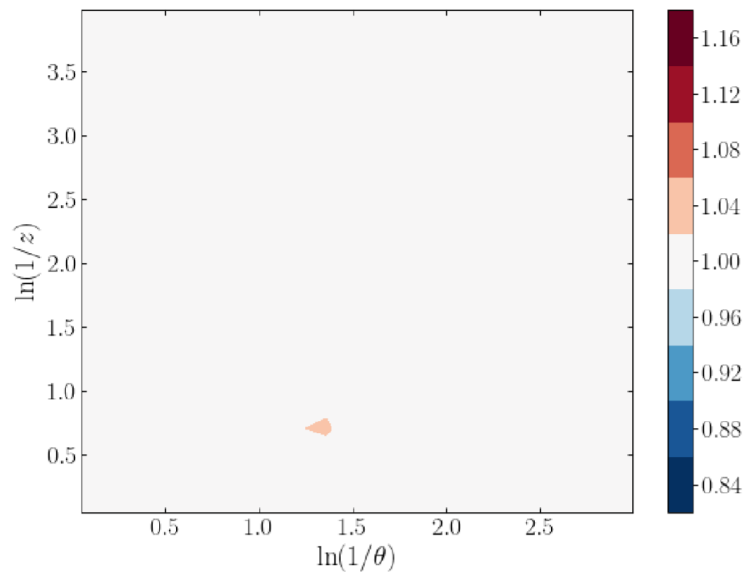


(c) Gluon-gluon splitting.

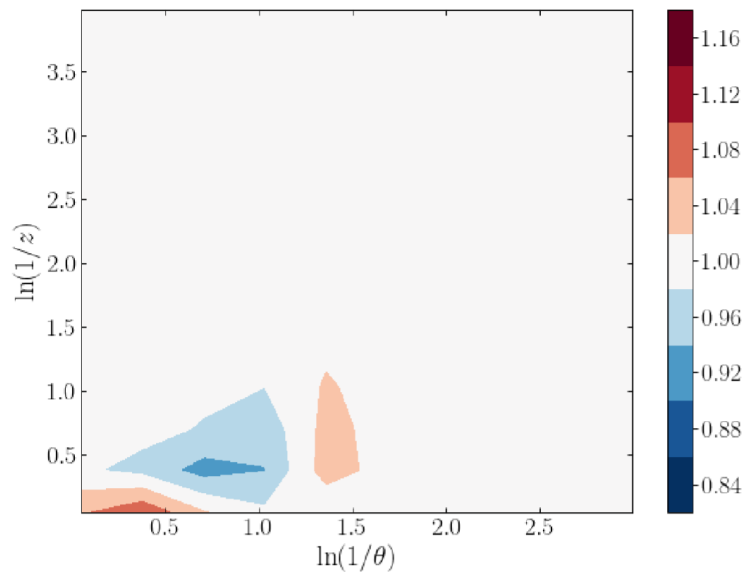
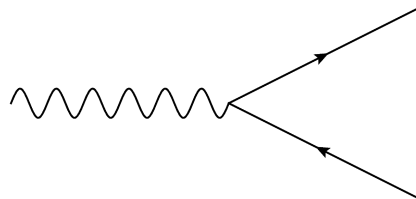


# Finite $N_c$ corrections

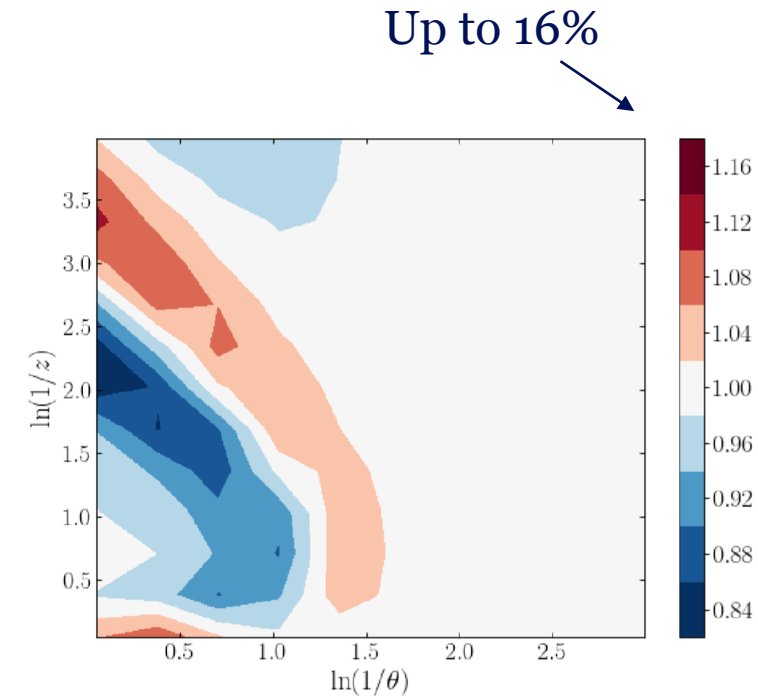
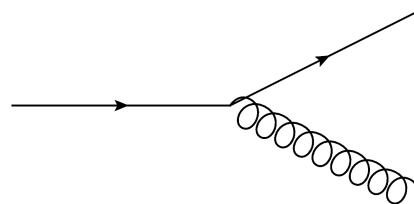
Ratio of emission spectrum for large- $N_c$ /finite  $N_c$



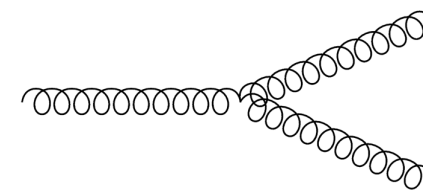
(a) Photon splitting.



(b) Quark-gluon splitting.



(c) Gluon-gluon splitting.



**More complex color structure leads to bigger correction!**

# Plan

1. Calculate finite- $N_c$  corrections
2. Calculate non-eikonal corrections **and** finite- $N_c$  corrections

# Plan

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# Non-eikonal corrections

## Partons get kicked around by medium

- Path through the medium is not straight
- Must keep the full path integral description

$$\begin{aligned}\tilde{S}^4(\mathbf{u}, \bar{\mathbf{u}}, \mathbf{u}_2, \bar{\mathbf{u}}_2 | t, t_2) &\sim \langle \mathcal{G} \mathcal{G} \mathcal{G}^\dagger \mathcal{G}^\dagger \rangle \\ &\sim \# \int_{\mathbf{u}_2}^{\mathbf{u}} \mathcal{D}\mathbf{u} \int_{\bar{\mathbf{u}}_2}^{\bar{\mathbf{u}}} \mathcal{D}\bar{\mathbf{u}} e^{i\frac{\omega}{2} \int_{t_2}^t ds (\dot{\mathbf{u}}^2 - \dot{\bar{\mathbf{u}}}^2)} \langle V V V^\dagger V^\dagger \rangle\end{aligned}$$

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- Path through the medium is not straight
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$$\begin{aligned}\tilde{S}^4(\mathbf{u}, \bar{\mathbf{u}}, \mathbf{u}_2, \bar{\mathbf{u}}_2 | t, t_2) &\sim \langle \mathcal{G} \mathcal{G} \mathcal{G}^\dagger \mathcal{G}^\dagger \rangle \\ &\sim \# \int_{\mathbf{u}_2}^{\mathbf{u}} \mathcal{D}\mathbf{u} \int_{\bar{\mathbf{u}}_2}^{\bar{\mathbf{u}}} \mathcal{D}\bar{\mathbf{u}} e^{i\frac{\omega}{2} \int_{t_2}^t ds (\dot{\mathbf{u}}^2 - \dot{\bar{\mathbf{u}}}^2)} \langle V V V^\dagger V^\dagger \rangle\end{aligned}$$

- Again, this can be turned into a system of differential equations
- Now it is a more complicated Schrödinger equation

$$\left( i \frac{\partial}{\partial t} + \frac{\partial_{\mathbf{u}}^2 - \partial_{\mathbf{v}}^2}{2\omega} \right) \begin{bmatrix} \tilde{S}_1^4 \\ \tilde{S}_2^4 \end{bmatrix} = i\mathbb{V}(t, \mathbf{u}, \mathbf{v}) \begin{bmatrix} \tilde{S}_1^4 \\ \tilde{S}_2^4 \end{bmatrix}$$



# Non-eikonal corrections

## Partons get kicked around by medium

- Path through the medium is not straight
- Must keep the full path integral description

$$\begin{aligned} \tilde{S}^4(\mathbf{u}, \bar{\mathbf{u}}, \mathbf{u}_2, \bar{\mathbf{u}}_2 | t, t_2) &\sim \langle \mathcal{G} \mathcal{G} \mathcal{G}^\dagger \mathcal{G}^\dagger \rangle \\ &\sim \# \int_{\mathbf{u}_2}^{\mathbf{u}} \mathcal{D}\mathbf{u} \int_{\bar{\mathbf{u}}_2}^{\bar{\mathbf{u}}} \mathcal{D}\bar{\mathbf{u}} e^{i\frac{\omega}{2} \int_{t_2}^t ds (\dot{\mathbf{u}}^2 - \dot{\bar{\mathbf{u}}}^2)} \langle V V V^\dagger V^\dagger \rangle \end{aligned}$$

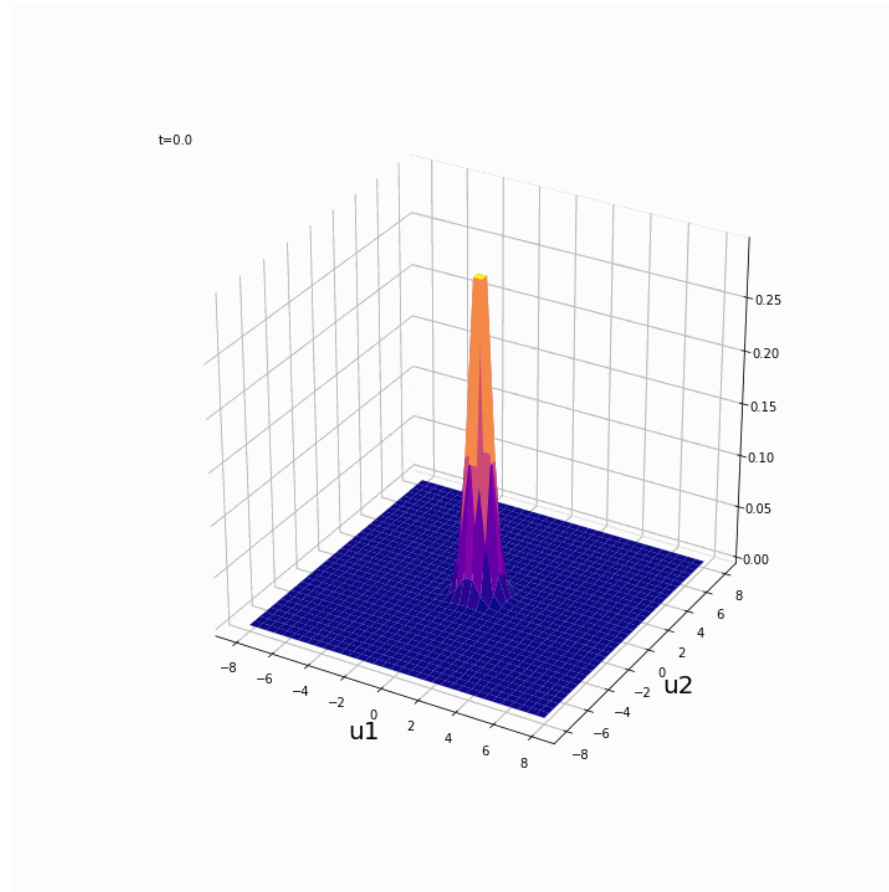
- Again, this can be turned into a system of differential equations
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$$\left( i \frac{\partial}{\partial t} + \frac{\partial_{\mathbf{u}}^2 - \partial_{\mathbf{v}}^2}{2\omega} \right) \begin{bmatrix} \tilde{S}_1^4 \\ \tilde{S}_2^4 \end{bmatrix} = i\mathbb{V}(t, \mathbf{u}, \mathbf{v}) \begin{bmatrix} \tilde{S}_1^4 \\ \tilde{S}_2^4 \end{bmatrix} \xrightarrow{\text{Eikonal limit}} \frac{d}{dt} \begin{bmatrix} \langle \text{tr}[VV] \text{tr}[VV] \rangle \\ \langle \text{tr}[VVVV] \rangle \end{bmatrix} \stackrel{\text{Same as before}}{=} \mathbb{V}(t) \begin{bmatrix} \langle \text{tr}[VV] \text{tr}[VV] \rangle \\ \langle \text{tr}[VVVV] \rangle \end{bmatrix}$$

- This can be solved numerically

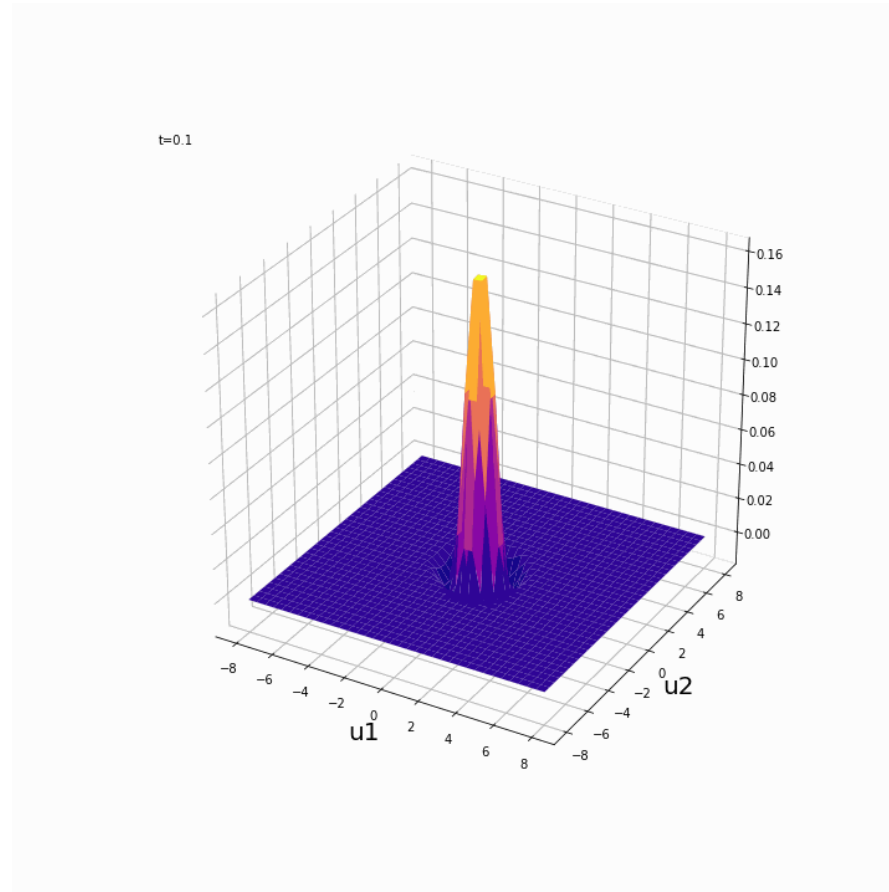
# Non-eikonal corrections

- Solution of Schrödinger equation



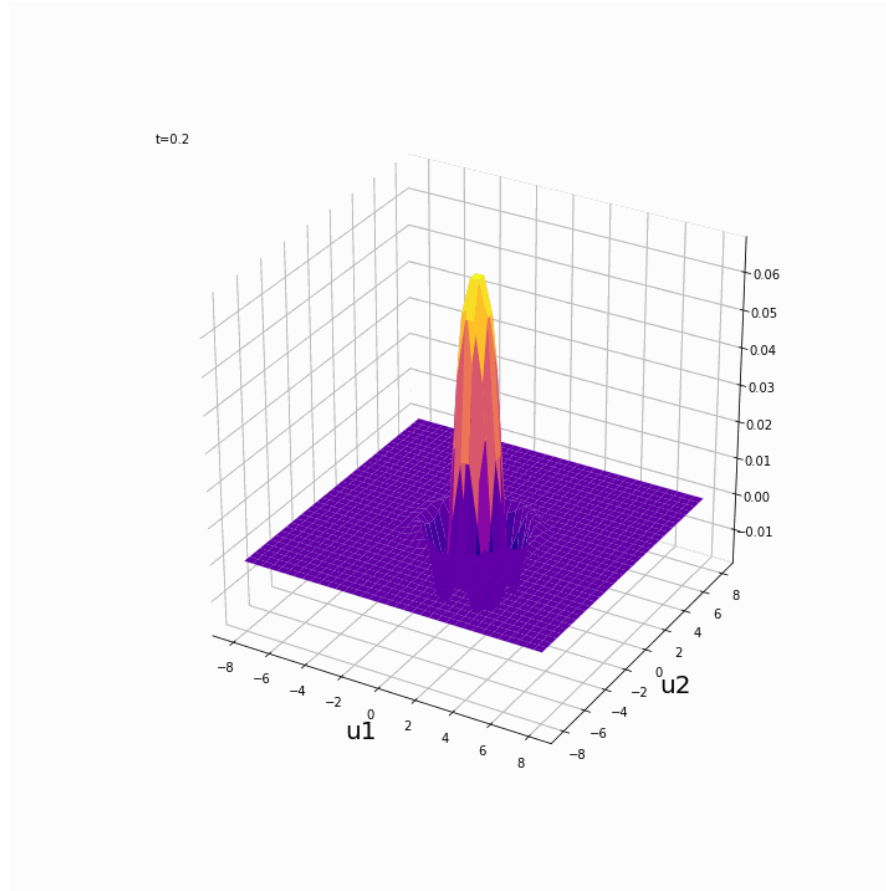
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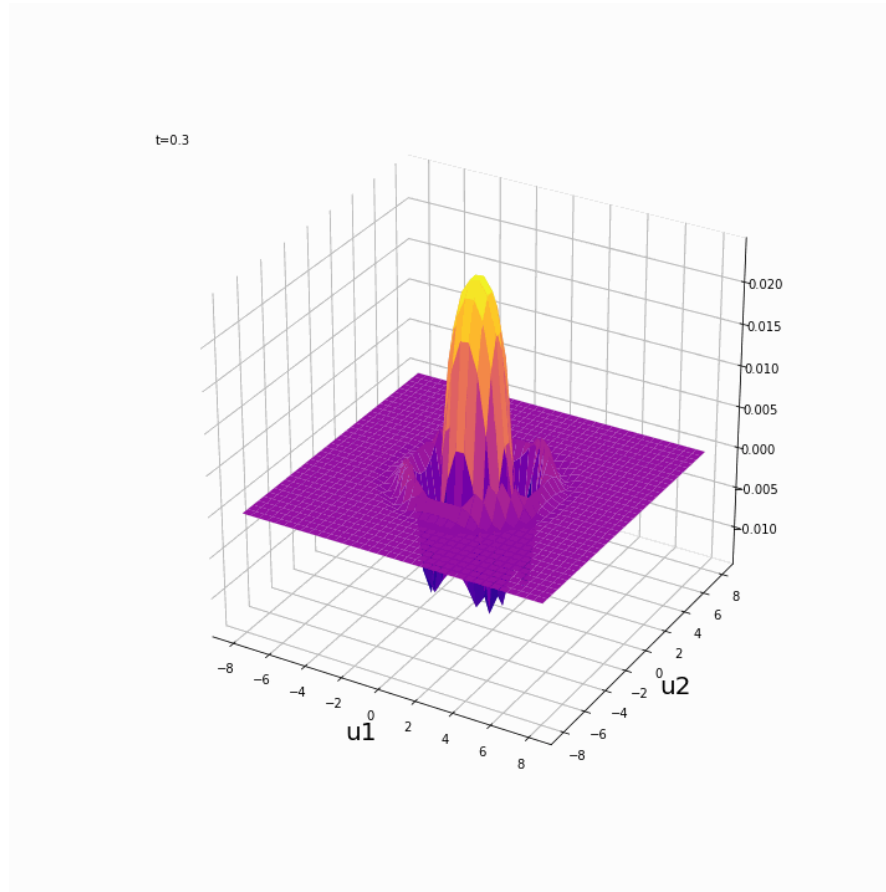
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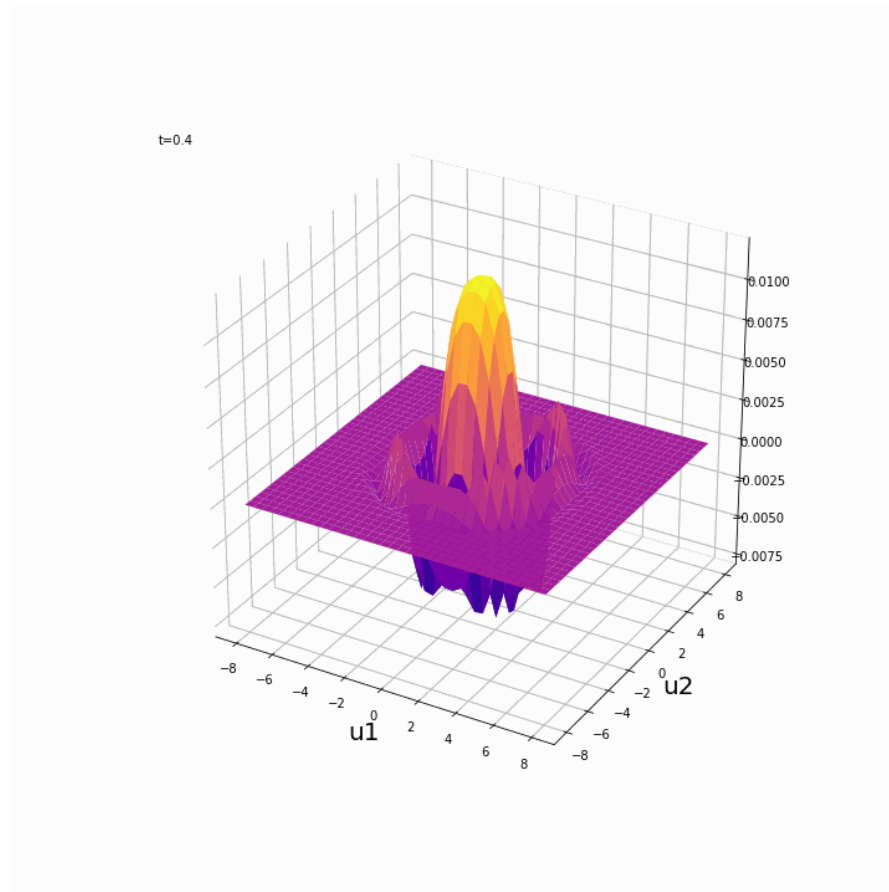
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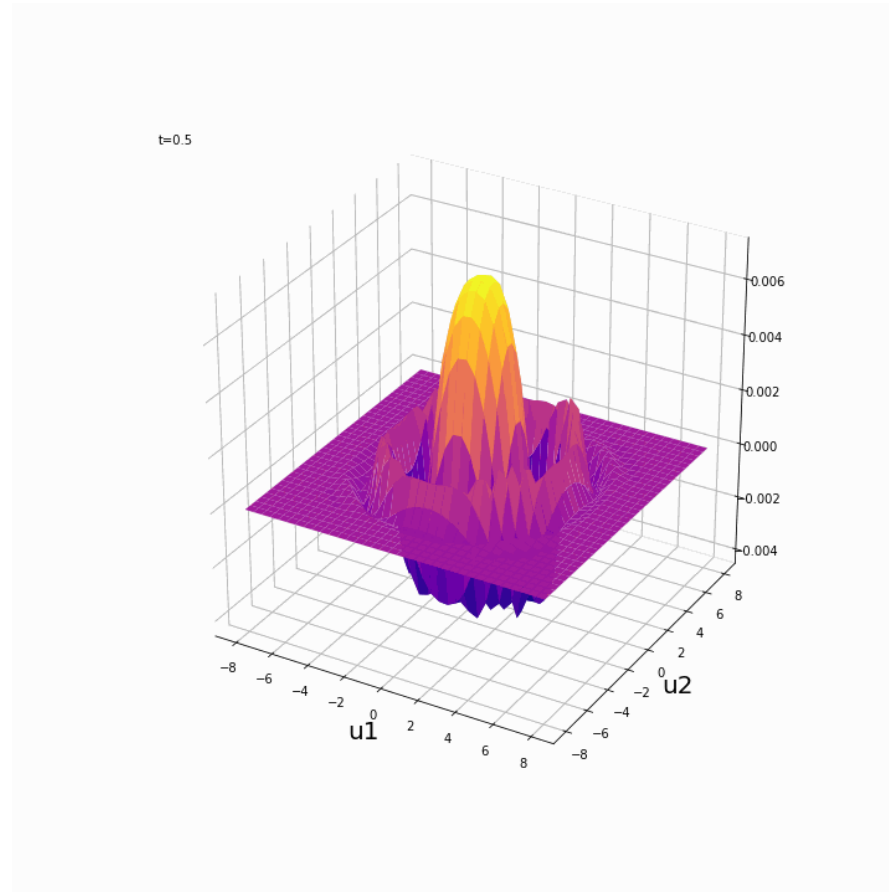
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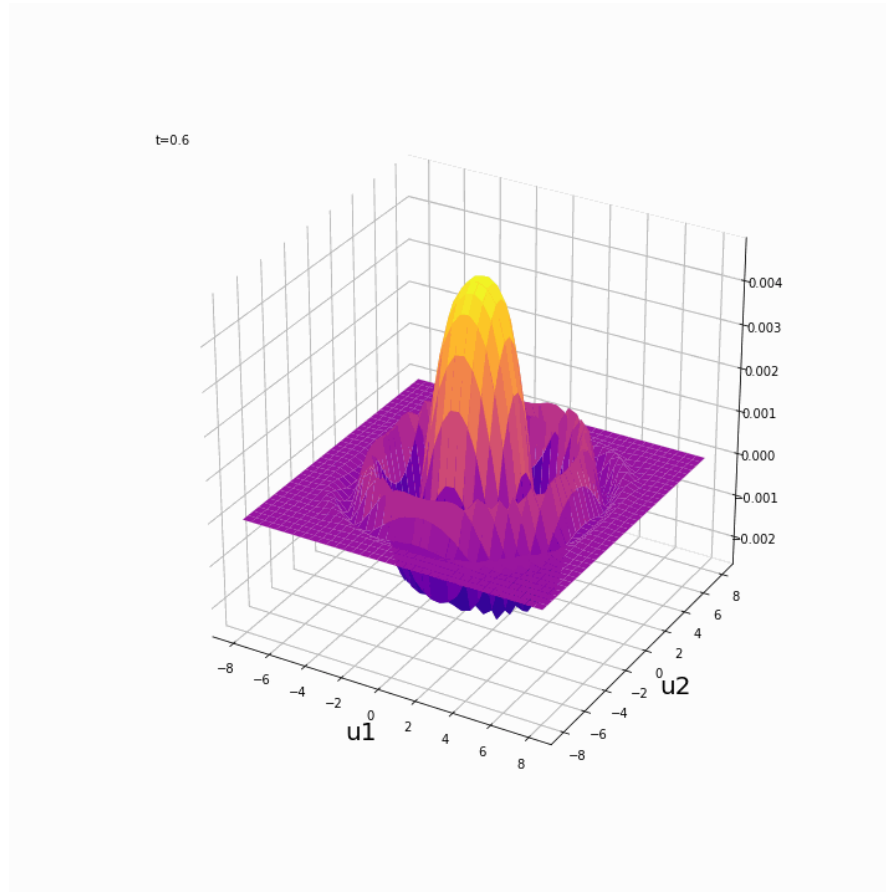
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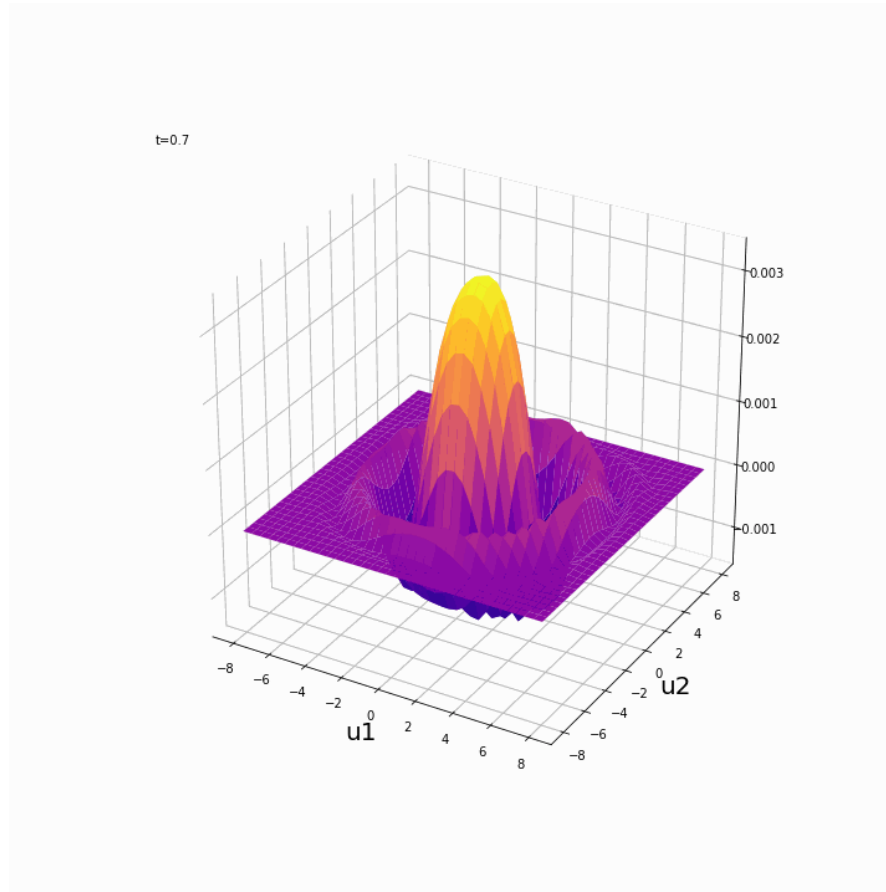
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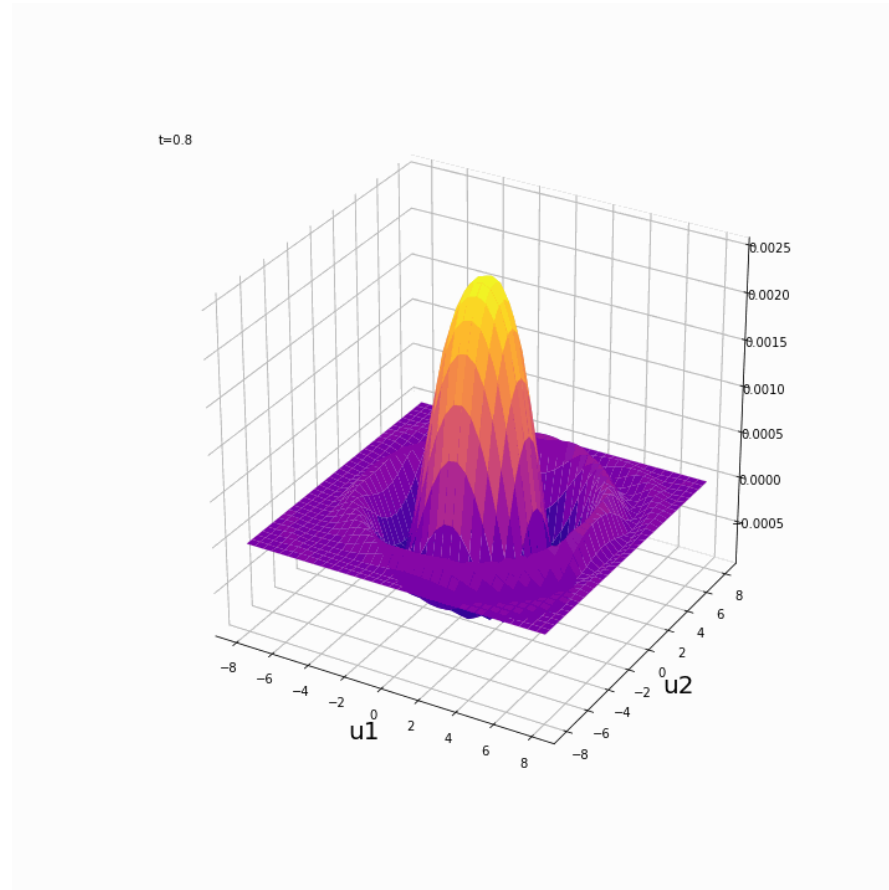
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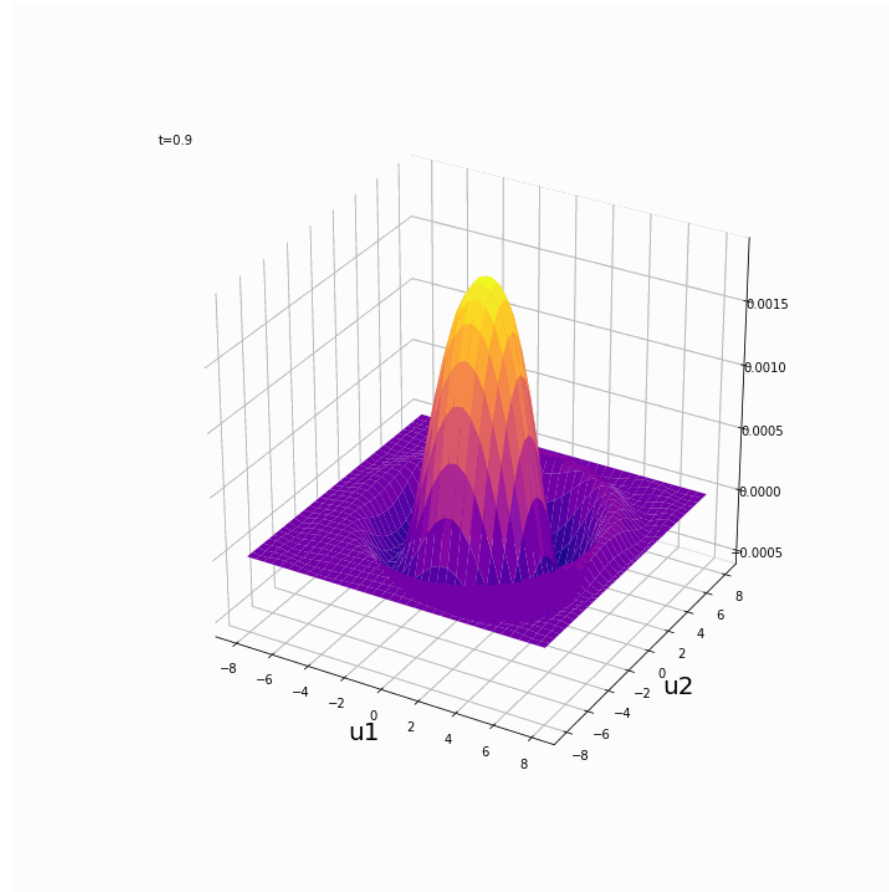
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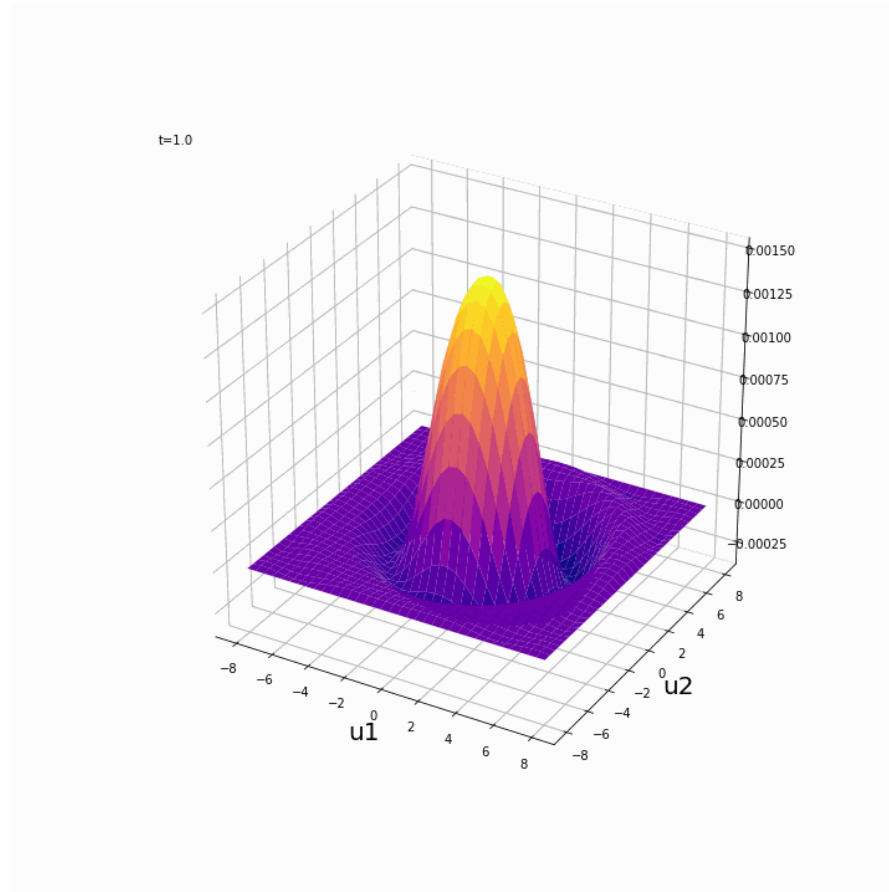
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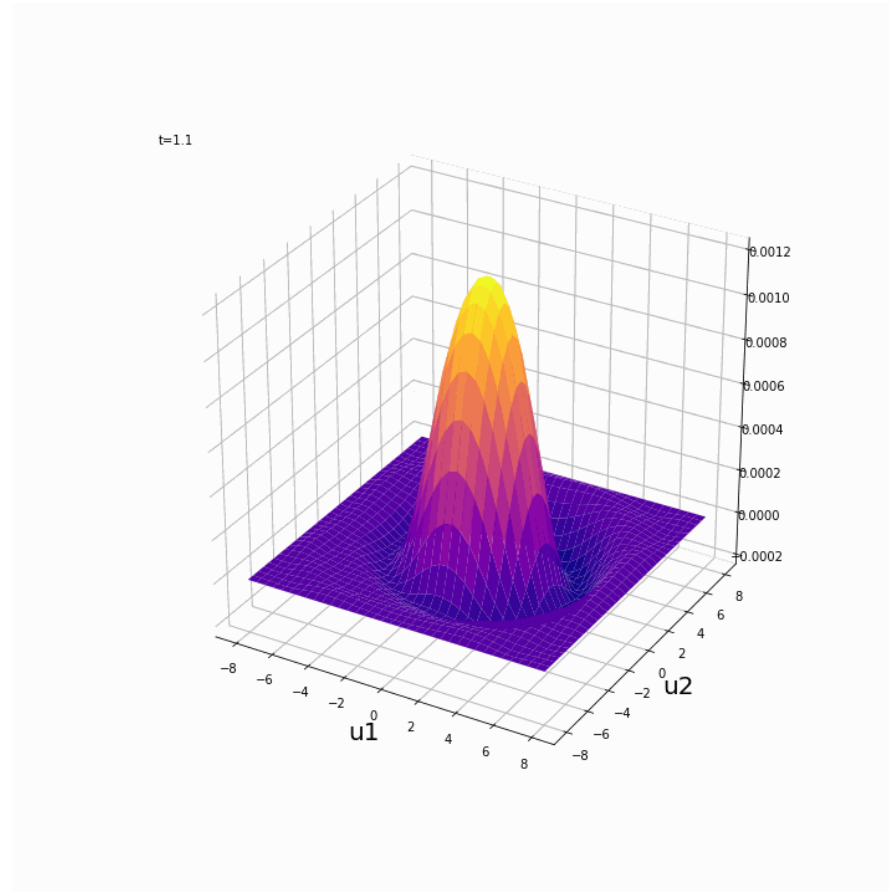
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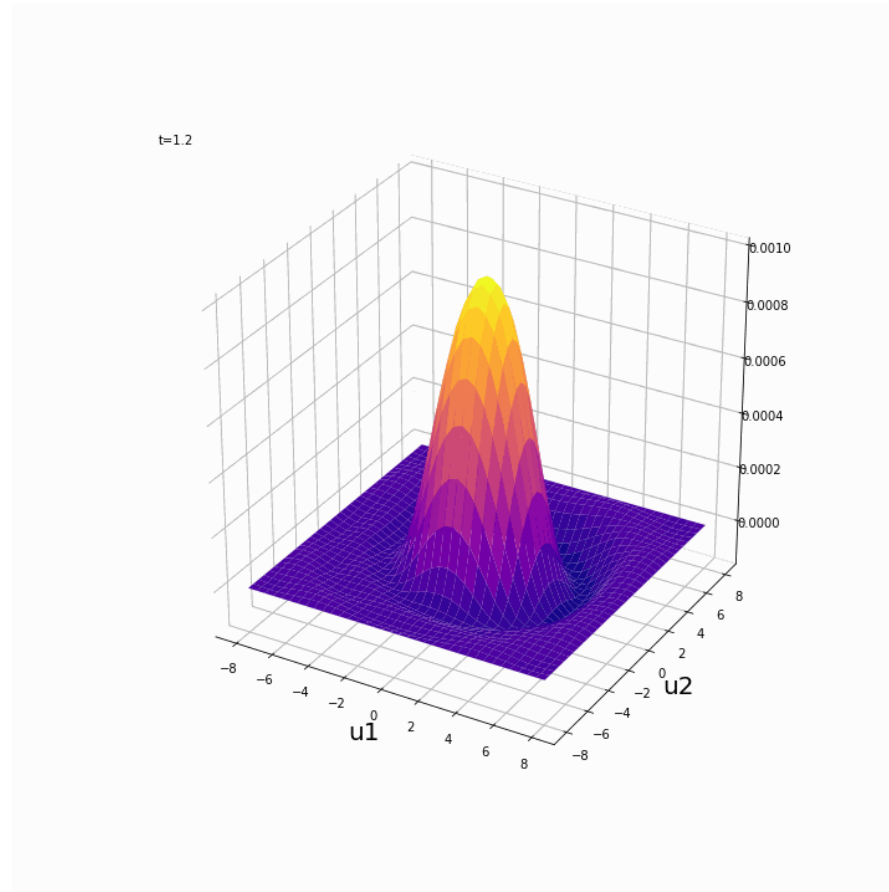
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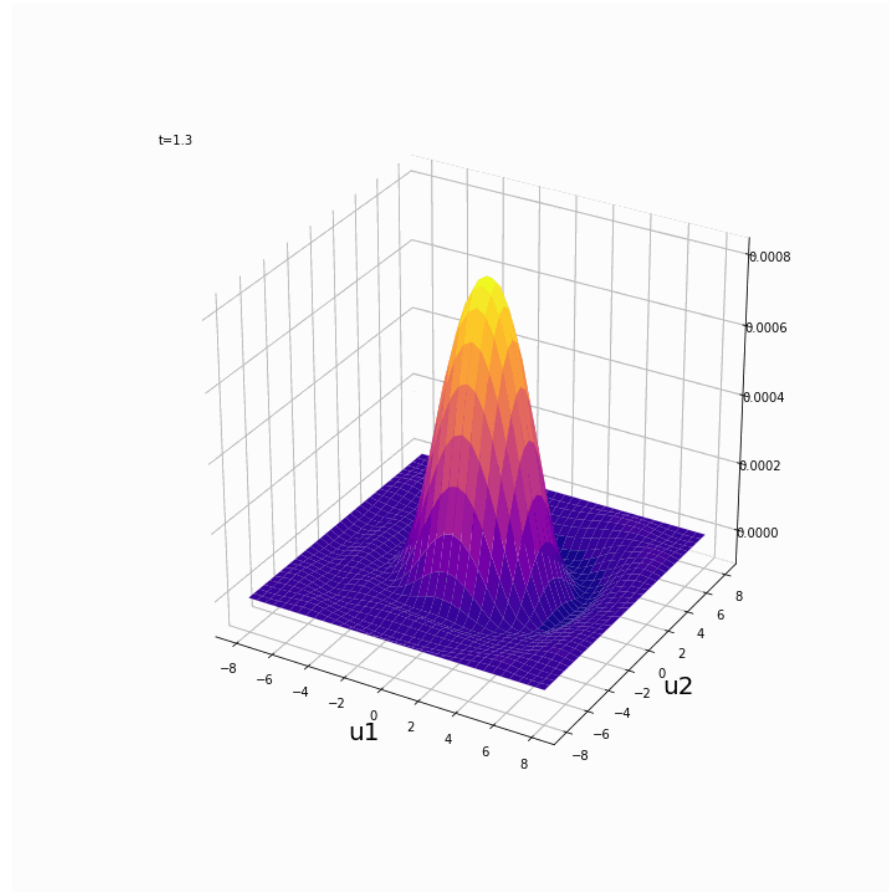
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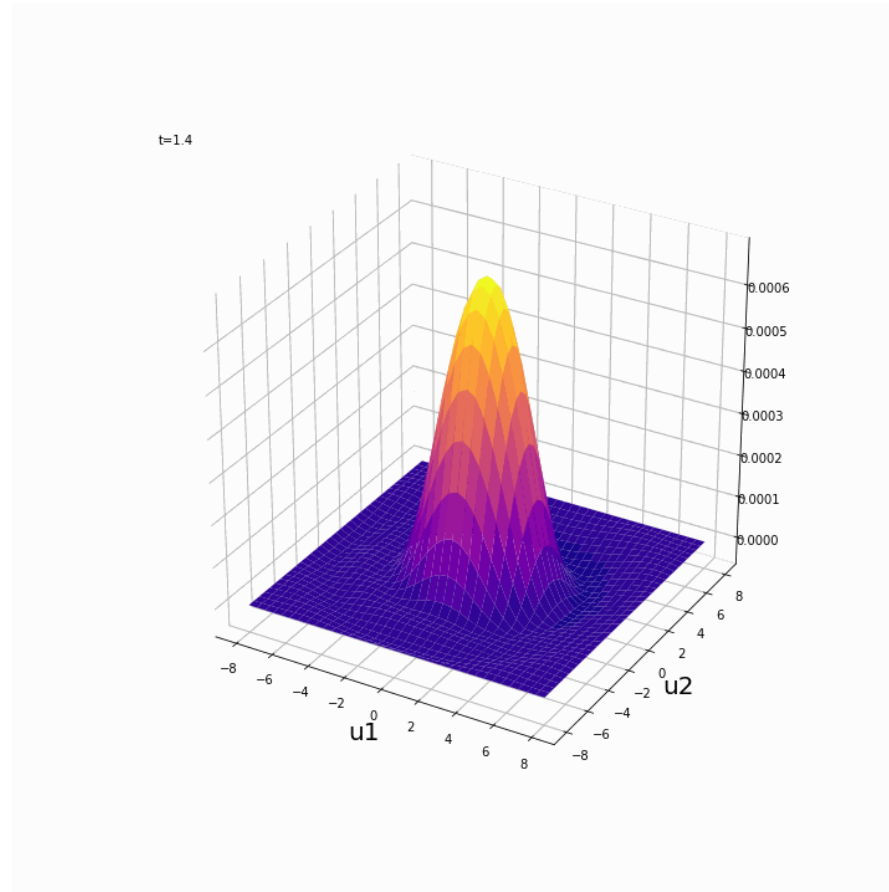
# Non-eikonal corrections

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# Non-eikonal corrections

- Solution of Schrödinger equation



- Still some work left on numerics



# Plan

1. Calculate finite-Nc corrections ✓
2. Calculate non-eikonal corrections **and** finite-Nc corrections
  - Theory
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# Conclusion and outlook

- People often use approximations to calculate medium-induced emissions
  - The large- $N_c$  approximation
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  - The large- $N_c$  approximation
  - The Eikonal approximation
- We showed how to calculate emissions without these approximations
  
- At finite  $N_c$  one must calculate **Wilson line correlators**
  - This can be transformed to a system of differential equations
  - Corrections are usually small, up to 16%
  
- For **non-eikonal** corrections one must calculate **path integrals of Wilson line correlators**
  - This can be transformed to a system of Schrödinger equations
  - Some numerical work remains to know the size of the corrections

**Thank you for your attention!**

