Improving Bayesian parameter estimation with the latest RHIC and LHC data including a new initial conditions model

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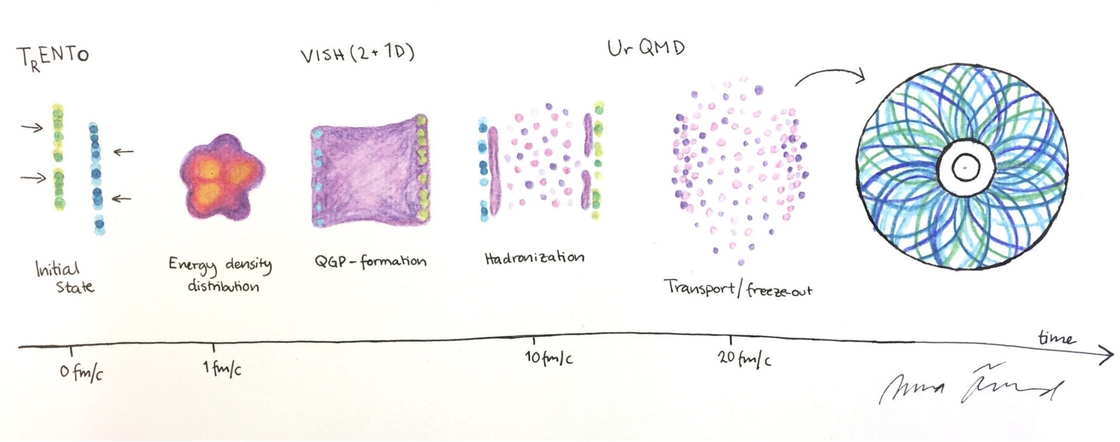
Spåtind 2023, Vinstra, Norway

The different stages of Heavy-Ion collisions



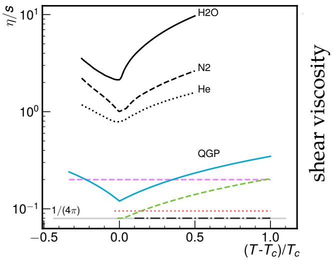
Credits to Anna Önnerstad

$$T^{\mu\nu} = e u^{\mu} u^{\nu} - (P + \Pi)\Delta_{\mu\nu} + \pi^{\mu\nu}, \qquad \delta_{\mu} T^{\mu\nu} = 0$$

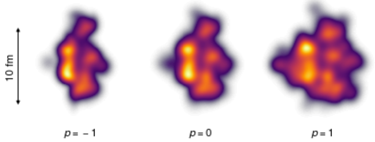
based on https://github.com/Duke-QCD

Collection of parameters

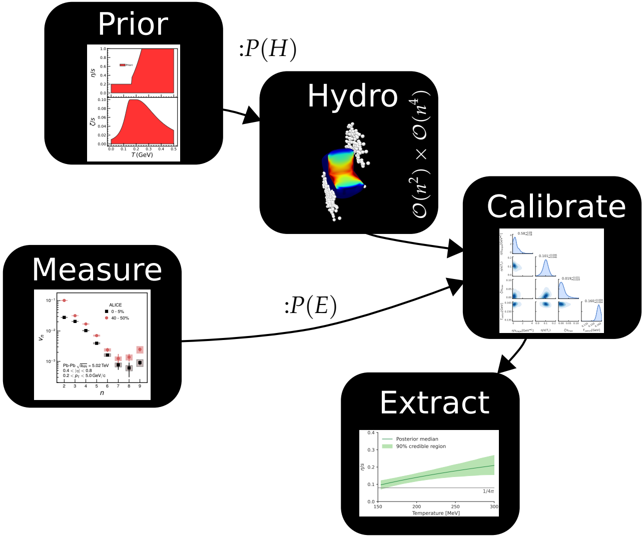
Parameter	Description
T_c	Temperature of const. $\eta/s(T)$, $T < T_c$
$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{\rm slope}$	Slope of $\eta/s(T)$ above T_c
$(\eta/s)_{\rm curve}$	Curvature of $\eta/s(T)$ above T_c
$(\zeta/s)_{\rm peak}$	Temperature of $\zeta/s(T)$ maximum
$(\zeta/s)_{\max}$	Maximum $\zeta/s(T)$
$(\zeta/s)_{\rm width}$	Width of $\zeta/s(T)$ peak
$T_{\rm switch}$	Switching / particlization temperature
N(2.76 TeV)	Overall normalization (2.76 TeV)
N(5.02 TeV)	Overall normalization (5.02 TeV)
p	Entropy deposition parameter
w	Nucleon width
σ_k	Std. dev. of nucleon multiplicity fluctuations
$d^3_{\min}$	Minimum volume per nucleon
$\tau_{\rm fs}$	Free-streaming time



Trento p-value, http://qcd.phy.duke.edu/trento/



Bayesian parameter estimation



Bayes' theorem:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

$$P(E) = \sum_{i=1}^{n} P(E|H_i)P(H_i)$$

- Find optimal set of model parameters that best reproduce the experimental data.
- Utilize constraints, such as flow observables, to help narrow down the $\eta/s(T)$ and such.

Testing a single set of parameters requires $\mathcal{O}(10^4)$ hydro events, and evaluating eight different parameters five times each requires $5^8 \times 10^4 \approx 10^9$ hydro events.
That's roughly 10^5 CPU years!

Our arsenal of observables - stochastic approach

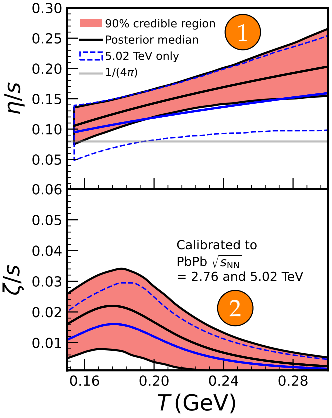
- Together, various flow observables cover the sensitivity for all components of transport properties.

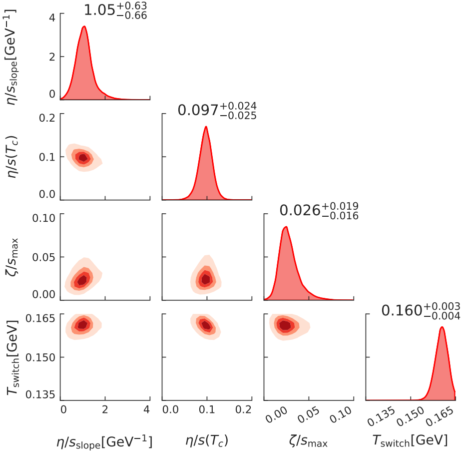
Name	Symbol	Measure	Sensitivity-stochastic approach
Flow coefficients	v_n	System expansion and anisotropy of the flow	Average $\langle \eta/s \rangle$ and $\zeta/s(T)$ peak
(Normalized) Symmetric cumulants	(N)SC(k, l, m)	Correlation between magnitudes of flow harmonics	$\eta/s(T)$ temperature dependence
Non-linear flow mode coefficients	$\chi_{n,mk}$	Quantification of the non-linear response	$\eta/s(T)$ at the freeze-out
Symmetry-plane correlations	$\rho_{n,mk}$	Correlations between the directions of flow harmonics	$\eta/s(T)$

Thanks to excellent ALICE papers over years:

- Phys.Rev.Lett. 117 (2016) 182301, Phys.Lett. B773 (2017) 68, Phys.Rev. C 97 (2018) 024906, JHEP05 (2020) 085, Phys.Lett. B818 (2021) 136354, Phys.Rev.Lett. 127 (2021) 092302 - flow
- Phys.Rev.Lett. 106 (2011) 032301, Phys.Rev.C 88 (2013) 044910, Phys.Lett. B772 (2017) 567-577, Phys.Rev.C 101, 044907 (2020) - $N_{\rm ch}$ and $\langle p_{\rm T} \rangle$

Results: Jyvaskyla (2022) – combined collision energy analysis (2.76 + 5.02 TeV)



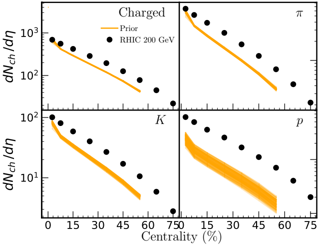


1. Significantly improved $\eta/s(T)$ uncertainty
2. Non-zero $\zeta/s(T)$
3. Overall better convergence for parameter components

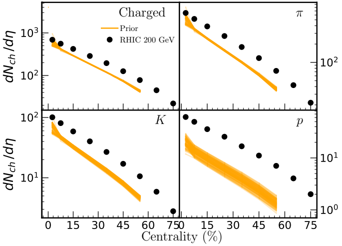
- Together with two collision energies and added observables, the uncertainty has reduced!

J.E. Parkkila, A. Onnerstad, M. Virta, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, Phys. Lett. B 835 (2022) 137485

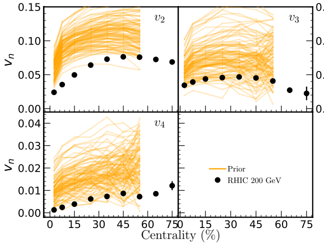
Inclusion of RHIC data

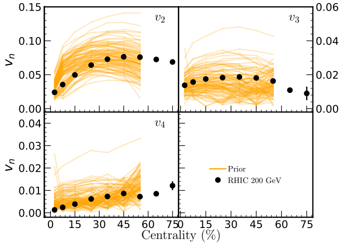


Fixed $w_{nucl} = 0.5$

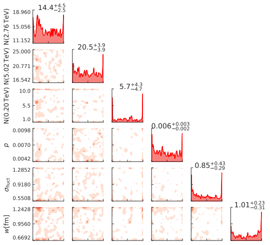


w_{nucl} =[0.67-1.24]



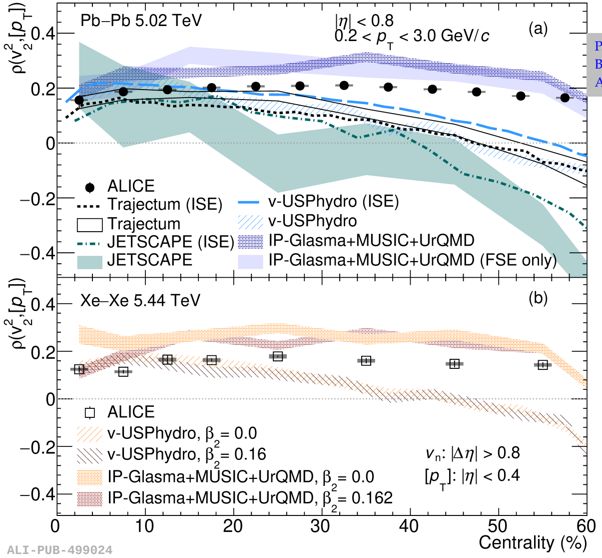


- Included observables for RHIC 200 GeV data: v_2 - v_4, $\langle p_T \rangle$ and N_{ch} for charged particles and PID
- Fixed nuclen width $w = 0.5$ and now relaxed [0.67-1.24]
- small improvements for N_{ch} and better for v_n
- Posterior distributions don't converge



Preliminary

v_n, $[p_T]$ correlation - shortage of TRENTo model

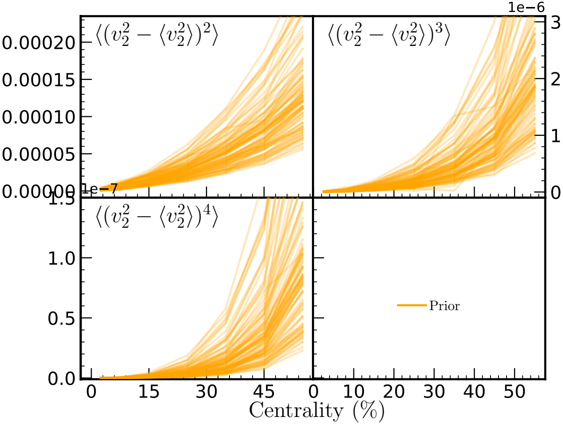


P. Bozek, R. Samanta, Phys. Rev. C 102, 034905
B. Schenke, C. Shen, D. Teaney, Phys. Rev. C 102, 034905
ALICE, arXiv:2111.06106

$$\rho(v_2^2, [p_T]) = \frac{\langle \delta v_2^2 \, \delta[p_T] \rangle}{\sqrt{\langle (\delta v_2^2)^2 \rangle \langle (\delta [p_T])^2 \rangle}} \quad (1)$$

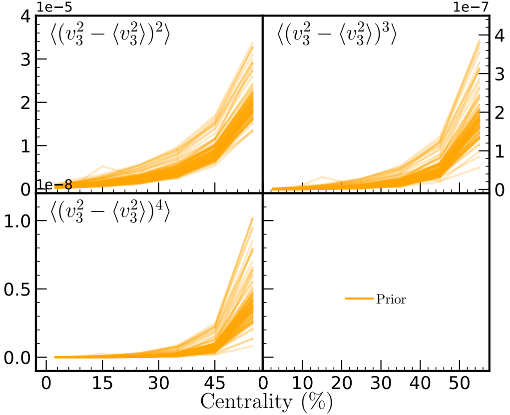
- Correlation between $[p_T]$ and v_2:
  - can be used to differentiate initial state models
  - More peripheral → best described by models with IP-Glasma
  - strong centrality dependence on the models with Trento
- Ongoing progress:
  - Calculate sensitivity
  - Adapt it to the Bayesian Analysis

Moments of δv_n



Moments of a distribution

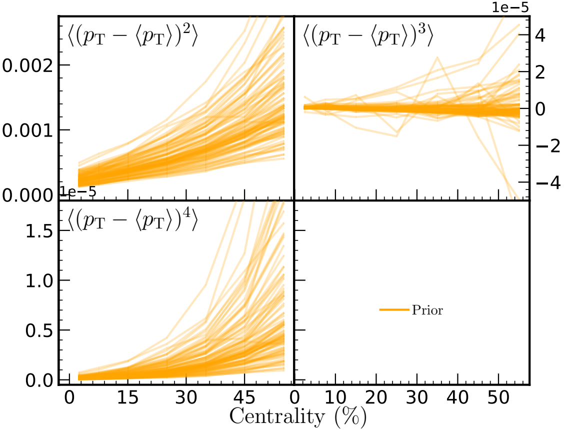
Variance: $(X - \mu)^2$, **skewness:** $(X - \mu)^3$ and **kurtosis:** $(X - \mu)^4$



- Characterizes the fluctuation of different order
- $|\eta| < 0.8$ and $0.2\,\mathrm{GeV} < p_T < 5.0\,\mathrm{GeV}$

Prior distribution of $\rho(v_n^2, [p_T])$

Moments of $[p_T]$ distribution

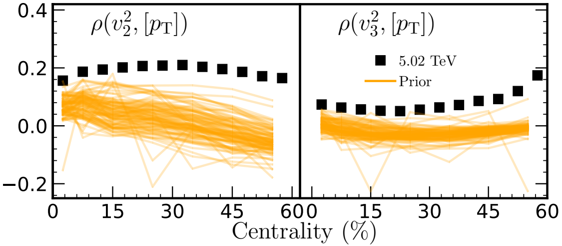


- $|\eta| < 0.8$ and $0.2\,\text{GeV} < p_T < 5.0\,\text{GeV}$

v_n^2, $[p_T]$ correlation

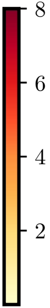
$$\rho(v_2^2, [p_T]) = \frac{\langle (v_2^2 - \langle v_2^2\rangle)([p_T] - \langle [p_T]\rangle)\rangle}{\sqrt{\langle (v_2^2 - \langle v_2^2\rangle)^2\rangle \langle ([p_T] - \langle [p_T]\rangle)^2\rangle}}$$

- Clear centrality dependence
- Gains negative values



Observable sensitivities

	$dN_{ch}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{T,\pi^\pm}\rangle$	$\langle p_{T,K^\pm}\rangle$	$\langle p_{T,p^\pm}\rangle$	v_2	v_3	v_4	v_5	SPC(4,2)	NSC(3,2)	NSC(4,2)	NSC(4,3)	NSC(5,2)	NSC(5,3)	NSC(6,2)	NSC(6,3)	NSC(6,4)
τ_{fs}	0.70	0.70	0.03	0.10	0.13	0.56	0.71	0.50	4.69	2.00	11.13	4.93	8.34	9.61	3.20	17.69	5.61	5.55
$(\eta/s)_{slope}$	0.18	0.21	0.04	0.05	0.22	0.74	1.55	1.81	1.51	1.42	12.04	3.94	1.60	3.64	5.38	11.75	5.73	4.07
T_c	0.73	0.51	0.18	0.33	1.57	0.66	1.26	0.88	2.96	0.63	5.45	7.11	15.07	11.11	9.17	14.10	13.66	9.53
$\eta/s(T_c)$	0.02	0.09	0.04	0.02	0.46	2.32	4.69	5.56	5.08	0.85	16.77	13.61	5.78	41.75	10.31	13.92	8.18	23.98
$(\eta/s)_{crv}$	0.02	0.04	0.00	0.00	0.03	0.01	0.04	0.08	0.35	0.41	1.84	1.01	2.49	2.27	2.13	1.84	1.61	3.99
$(\zeta/s)_{peak}$	0.33	0.16	0.10	0.22	1.26	0.44	1.06	1.72	3.56	2.68	7.60	3.90	5.61	6.61	5.03	4.38	8.46	13.19
$(\zeta/s)_{max}$	0.64	0.65	0.08	0.13	0.14	0.09	0.33	0.51	0.66	0.35	1.14	3.35	3.19	4.03	5.67	8.24	1.00	13.93
$(\zeta/s)_{width}$	0.20	0.19	0.03	0.08	0.09	0.07	0.03	0.27	1.09	0.28	10.74	8.72	6.54	10.90	4.82	7.09	2.14	11.12
T_{switch}	0.51	0.33	0.21	0.23	1.42	0.62	1.07	0.82	2.57	0.73	20.31	7.92	10.46	7.67	3.68	7.35	8.81	14.93



Sensitivity

$S[x_j] = \Delta/\delta.$, where $\Delta = \frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{|\hat{O}(\vec{x})|}$

- NSCs most sensitive to multiple different parameters
- v_ns show sensitivity to specific shear viscosity

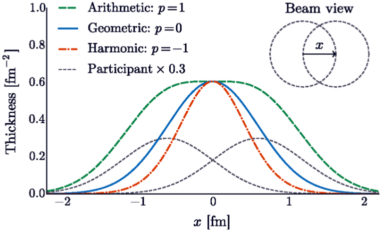
$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{slope}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{curve}}$$

$$(\zeta/s)(T) = \frac{(\zeta/s)_{max}}{1 + \left(\frac{T - (\zeta/s)_{T_{peak}}}{(\zeta/s)_{width}}\right)^2}$$

TRENTo vs. EKRT: Ongoing

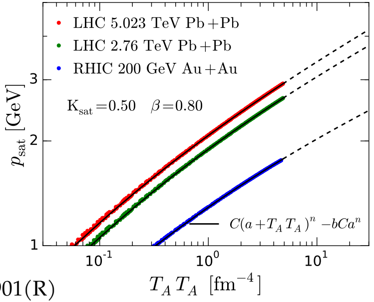
TRENTo[1]

- Flexibility to produce some other models
- Unable to predict ($\sqrt{s_{NN}}$ - Cent) dependence
- Has six free parameters



EKRT [2]

- Only two free parameters, K_{sat} and β_{sat}
- ($\sqrt{S_{NN}}$ - Cent) dependence comes automatically from the gluon saturation and mini-jet production
- Computationally a bit heavier → much improved(H. Hirvonen's talk)

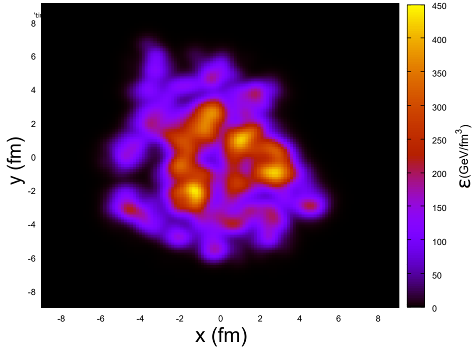


Phys. Rev. C 93, 014912

[1]. J. S. Moreland, J. E. Bernhard, and S. A. Bass, PRC **92** 011901(R)
[2]. H. Niemi, K. J. Eskola, and R. Paatelainen, PRC **93** 024907

EKRT

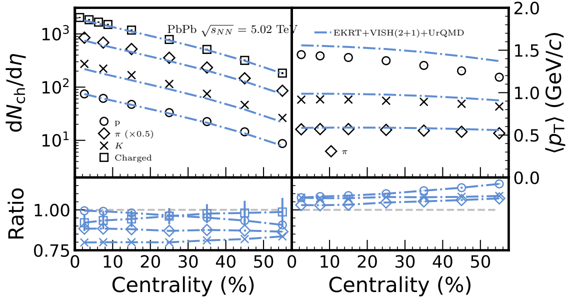
- Low-p_T particles are produced by saturation
- Transverse energy profile is
  $\frac{dE_T}{d^2r}(p_{sat}, \sqrt{s_{NN}}, A, r, b) = \frac{K_{sat}}{\pi} p_{sat}^3 \Delta y$
- The resulting energy density
  $e(r, \tau_s(r)) = \frac{dE_T}{dV} = \frac{dE_T}{d^2r} \frac{1}{\tau_s(r)\Delta y} = \frac{K_{sat}}{\pi} p_{sat}^4$
- Variable β describes how are soft particles from $3 \rightarrow 2$ processes included
- K_{sat} gives momentum range when saturation is considered to take place

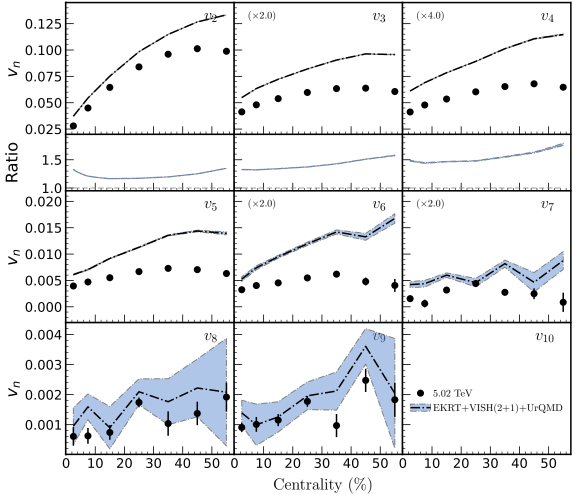


Courtesy of Henry Hirvonen

EKRT PbPb 5.02 TeV results

- Particle distributions relatively well described
- $\langle p_T \rangle$ a bit overestimated
- Large overestimation in flow harmonics





Outlook

Experiments

- RHIC data (AuAu collisions) - Energy and system size dependence (ongoing)
- LHC pPb and pp data - System size dependence
- Use new observables in BA
  - Higher order ($n > 5$) Symmetric cumulants (Anna)
  - Improved Symmetric Plane Correlation (SPC) : independent from flow magnitude correlations (Maxim)
  - Asymmetric Cumulants (AC) (Cindy)
  - $\rho(v_n, p_\mathrm{T}) = \langle \delta v_2^2 \, \delta[p_T] \rangle / \sqrt{\langle (\delta v_2^2)^2 \rangle \langle (\delta[p_T])^2 \rangle}$

Theory

- Improving the initial conditions with
  - EKRT (ongoing)
  - IP+Glasma
- Testing hydro limit with small systems?
- Study the substructure

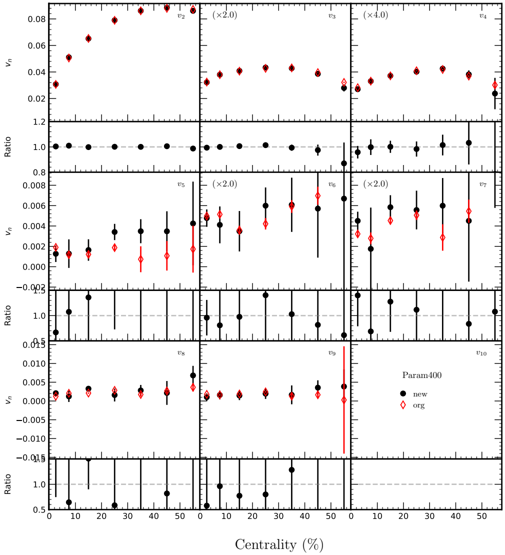
Thanks

Thank you for your attention!

Acknowledgments:

- CSC for providing the $\sim$24 million CPU hours
- Harri Niemi, Kari Eskola, Jonah E. Bernhard, J. Scott Moreland and Steffen A. Bass for their useful comments

Requirement of $\langle p_T \rangle$ event by event



Full equation

$$\rho(v_2^2, [p_T]) = \frac{\langle (v_2^2 - \langle v_2^2 \rangle)([p_T] - \langle [p_T] \rangle) \rangle}{\sqrt{\langle (v_2^2 - \langle v_2^2 \rangle)^2 \rangle \langle ([p_T] - \langle [p_T] \rangle)^2 \rangle}}$$

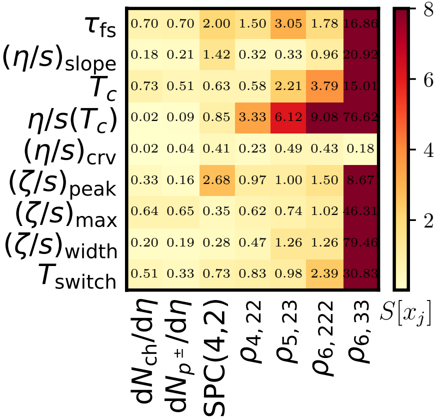
- With one set of hydro runs?
  - Store all particles from hydro
  - → Uses too much space
- Use mean values from previous runs!

SPC sensitivity comparison

- Previous biased scalar product method
  $\langle \cos(a_1 n_1 \Psi_1 + \cdots + a_k n_k \Psi_k) \rangle_{SP} = \frac{\langle v_{n_1}^{a_1} \ldots v_{n_k}^{a_k} \cos(a_1 n_1 \Psi_1 + \cdots + a_k n_k \Psi_k) \rangle}{\sqrt{\langle v_{n_1}^{2a_1} \rangle \ldots \langle v_{n_k}^{2a_k} \rangle}}$
- New Gaussian distribution method

$$\langle \cos(a_1 n_1 \Psi_1 + \cdots + a_k n_k \Psi_k) \rangle_{GE} = \int d\Theta N_\Theta(\Theta) \cos\Theta \approx \sqrt{\frac{\pi}{4}} \frac{\langle v_{n_1}^{a_1} \ldots v_{n_k}^{a_k} \cos(a_1 n_1 \Psi_1 + \cdots + a_k n_k \Psi_k) \rangle}{\sqrt{\langle v_{n_1}^{2a_1} \ldots v_{n_k}^{2a_k} \rangle}}$$

- Way to probe the non-linear response, e.g.
  $v_2^2 v_4 e^{i4(\Psi_4 - \Psi_2)} = \omega_2 \omega_4 c_2^2 c_4 e^{i4(\Phi_4 - \Phi_2)} + \omega_{422} \omega_2^2 c_2^2$



- The new SPC shows higher dependence on the specific bulk viscosity rather than the specific shear viscosity.