Current Topics in Neutrino Physics Theory

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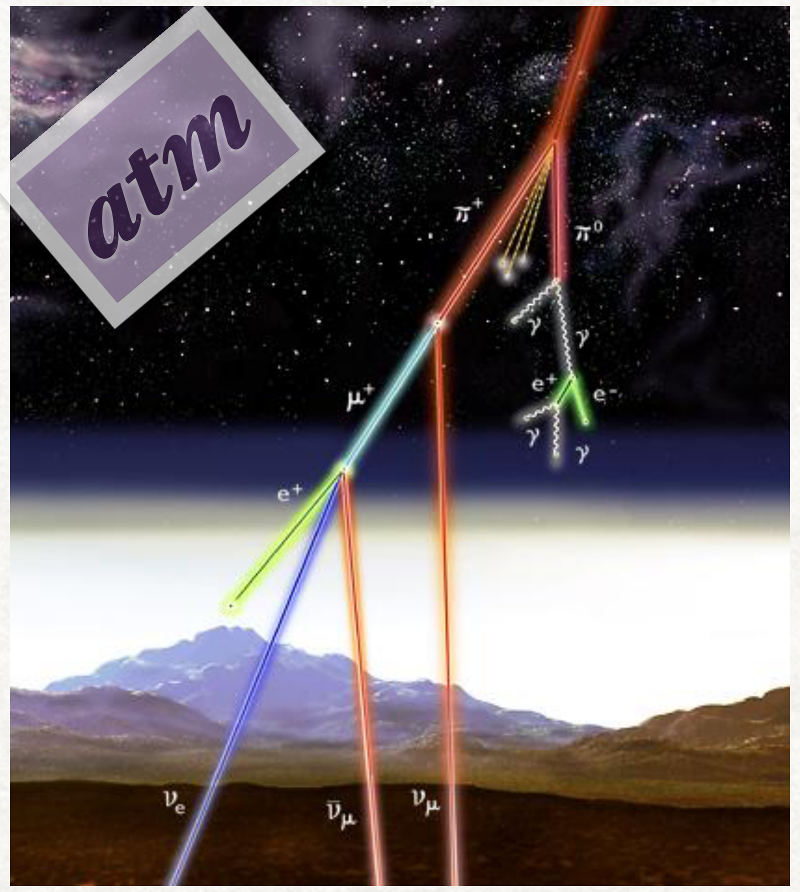
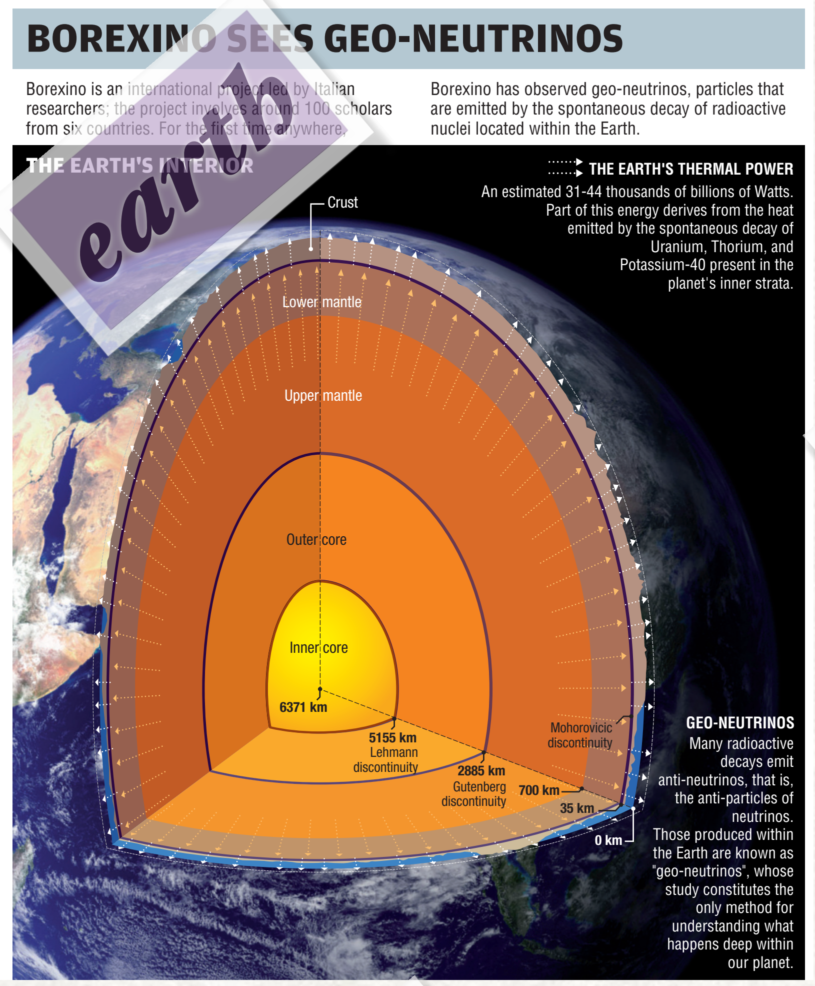
Spåtind 2023 - Nordic Conference in Particle Physics

Neutrinos are Ubiquitous

sun



atm



earth

BOREXINO SEES GEO-NEUTRINOS

Borexino is an international project led by Italian researchers; the project involves around 100 scholars from six countries. For the first time anywhere,

Borexino has observed geo-neutrinos, particles that are emitted by the spontaneous decay of radioactive nuclei located within the Earth.

THE EARTH'S THERMAL POWER

An estimated 31-44 thousands of billions of Watts. Part of this energy derives from the heat emitted by the spontaneous decay of Uranium, Thorium, and Potassium-40 present in the planet's inner strata.

GEO-NEUTRINOS

Many radioactive decays emit anti-neutrinos, that is, the anti-particles of neutrinos. Those produced within the Earth are known as "geo-neutrinos", whose study constitutes the only method for understanding what happens deep within our planet.

reactor

supernova

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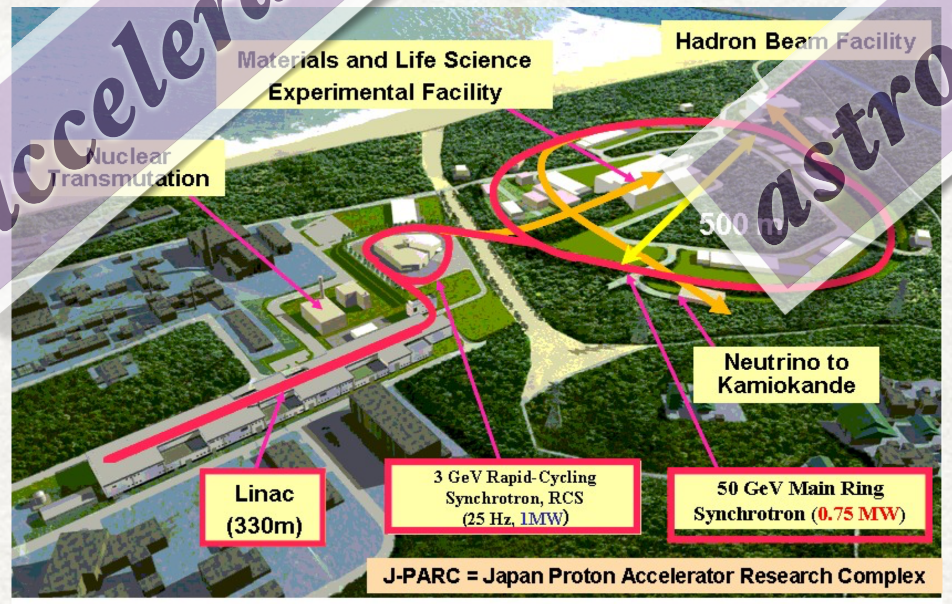
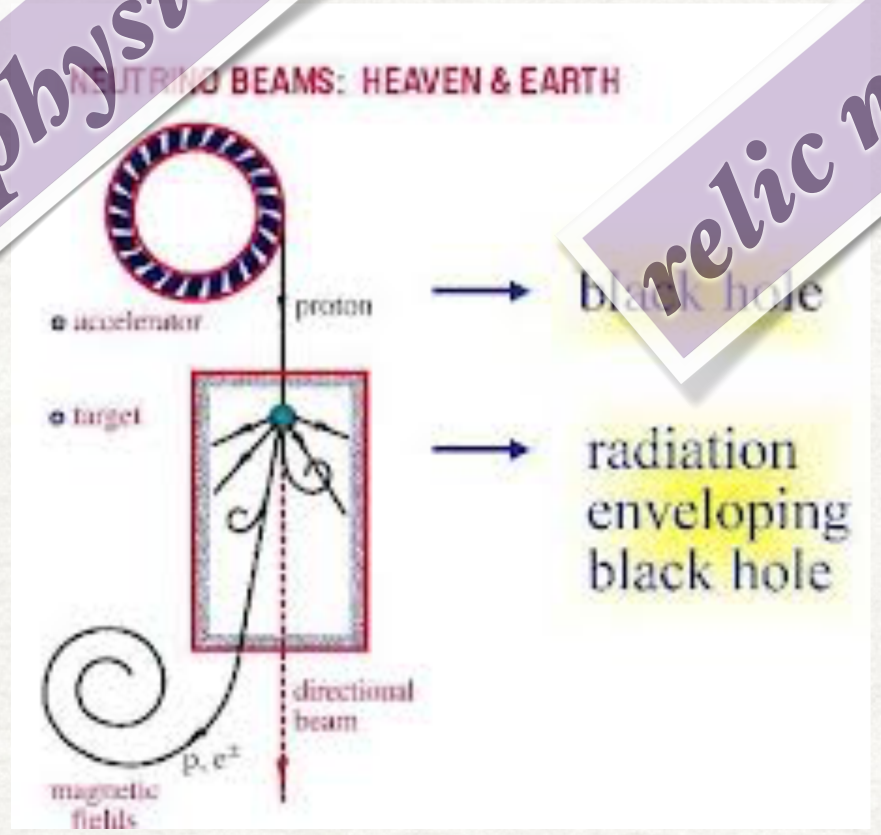


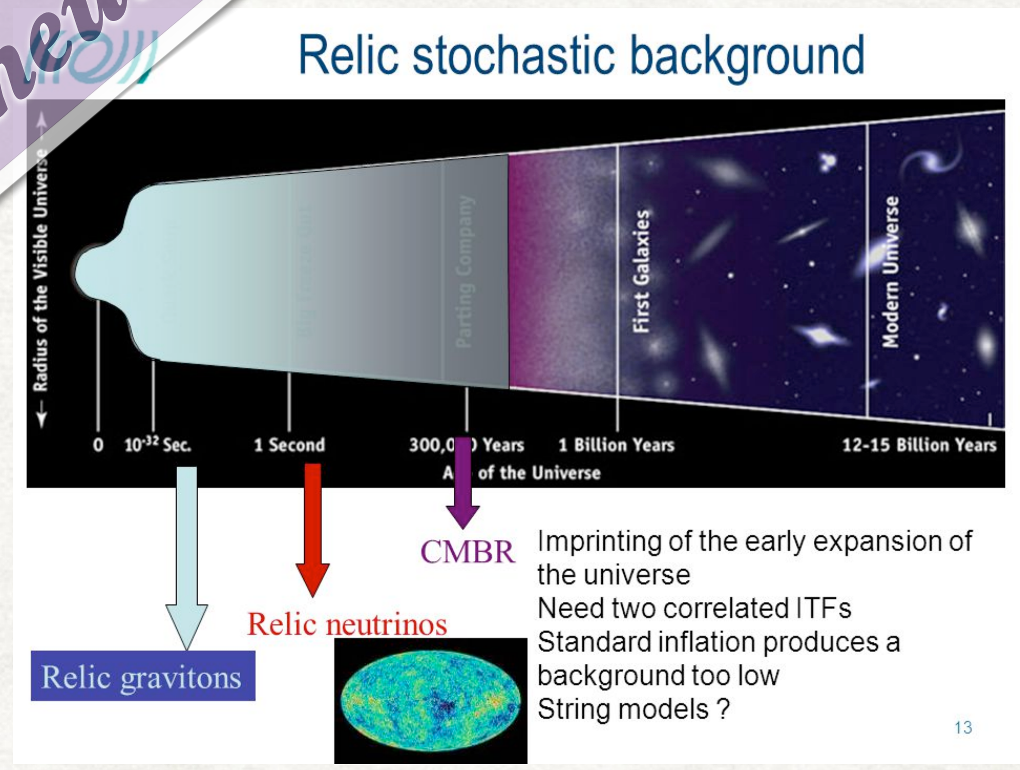
accelerator

J-PARC = Japan Proton Accelerator Research Complex



astrophysical

NEUTRINO BEAMS: HEAVEN & EARTH

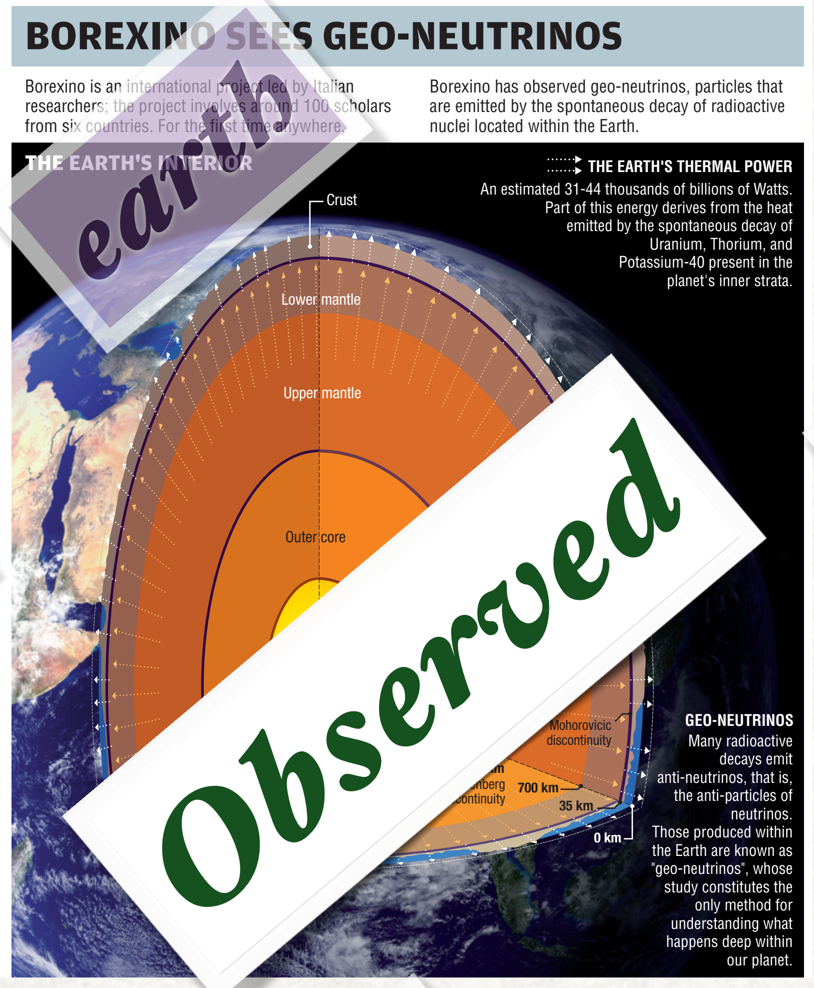


relic neutrinos

Relic stochastic background

Imprinting of the early expansion of the universe
Need two correlated ITFs
Standard inflation produces a background too low
String models ?

Neutrinos are Ubiquitous



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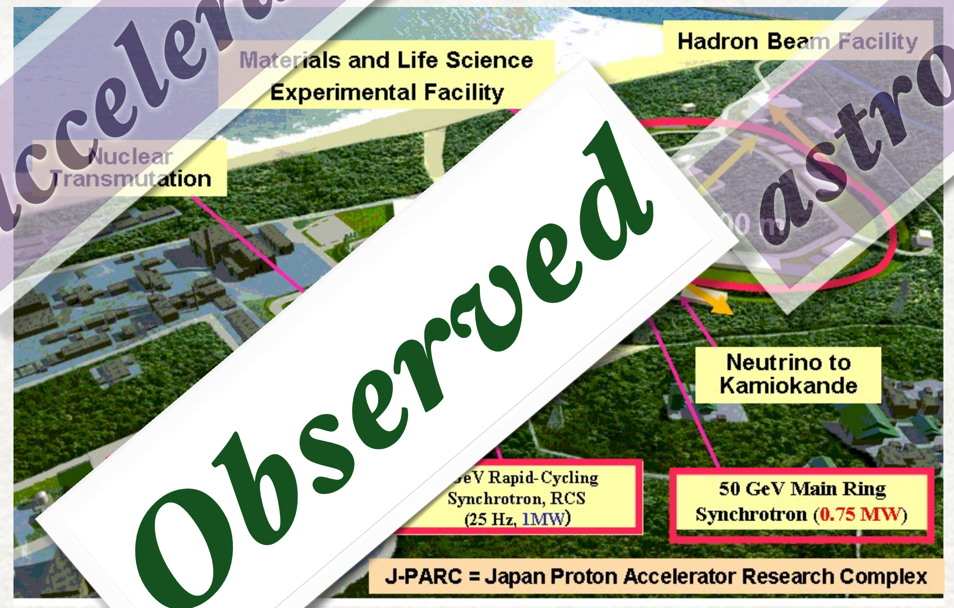

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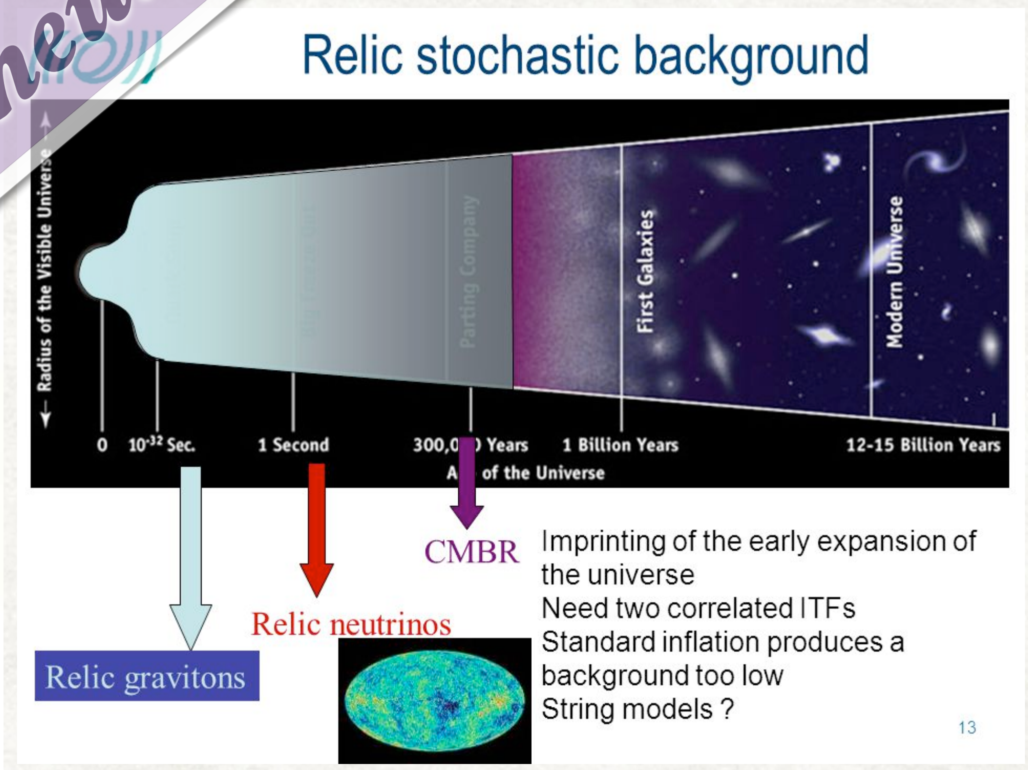

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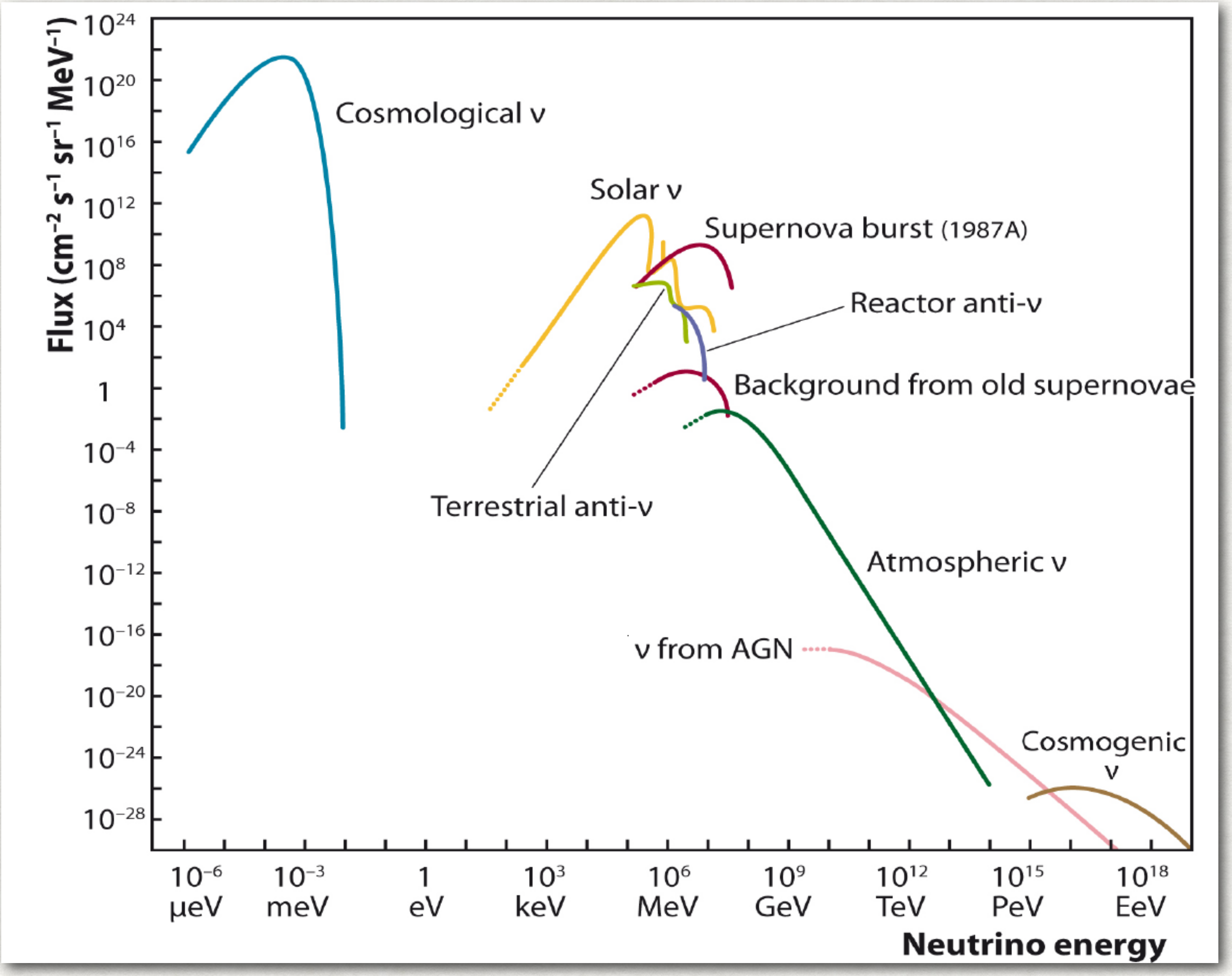
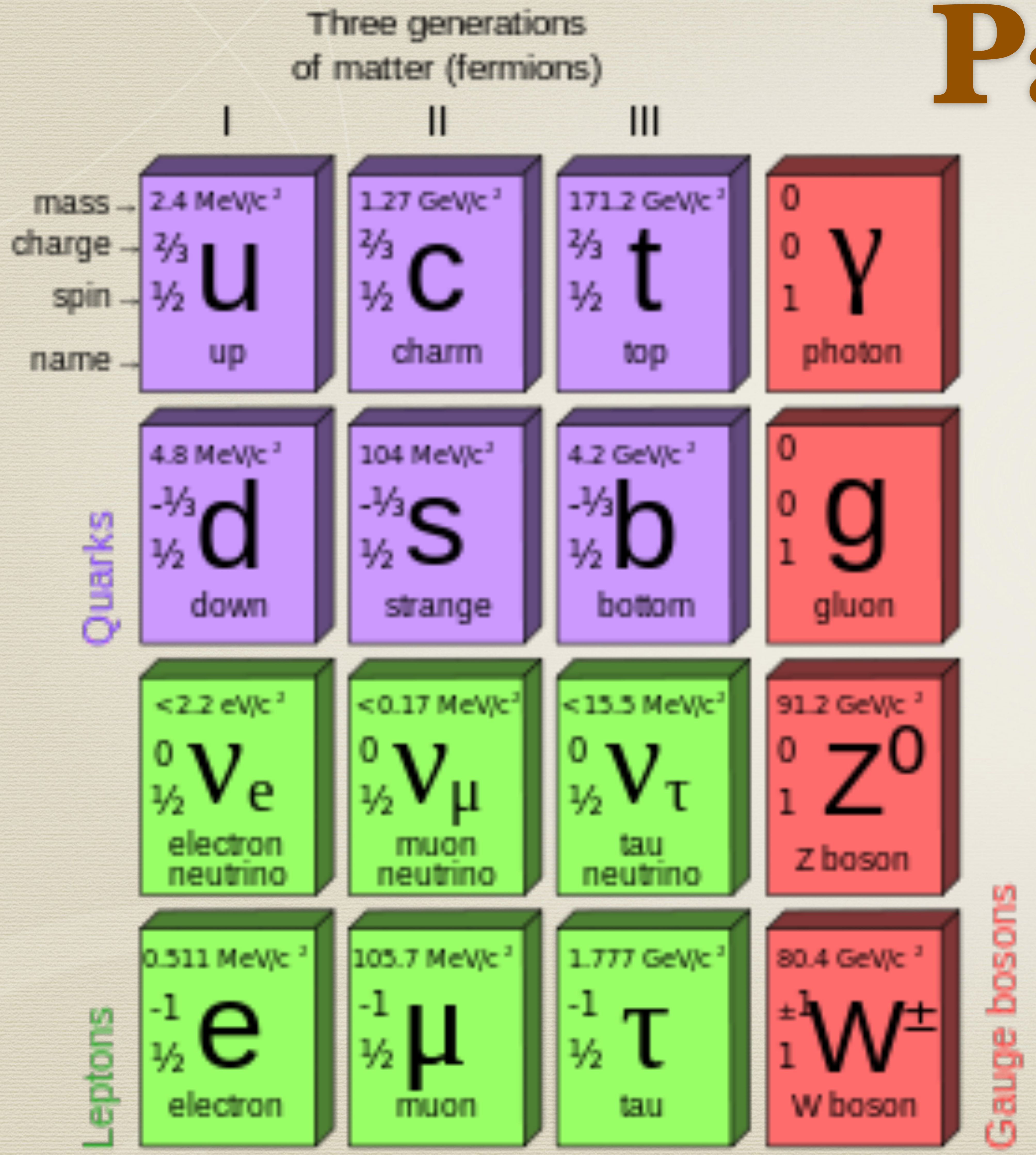

Flux (cm⁻² s⁻¹ sr⁻¹ MeV⁻¹)
10^24
10^20
10^16
10^12
10^8
10^4
1
10^−4
10^−8
10^−12
10^−16
10^−20
10^−24
10^−28
Cosmological ν
Solar ν
Supernova burst (1987A)
Reactor anti-ν
Background from old supernovae
Terrestrial anti-ν
Atmospheric ν
ν from AGN
Cosmogenic ν
10^−6 μeV
10^−3 meV
1 eV
10^3 keV
10^6 MeV
10^9 GeV
10^12 TeV
10^15 PeV
10^18 EeV
Neutrino energy

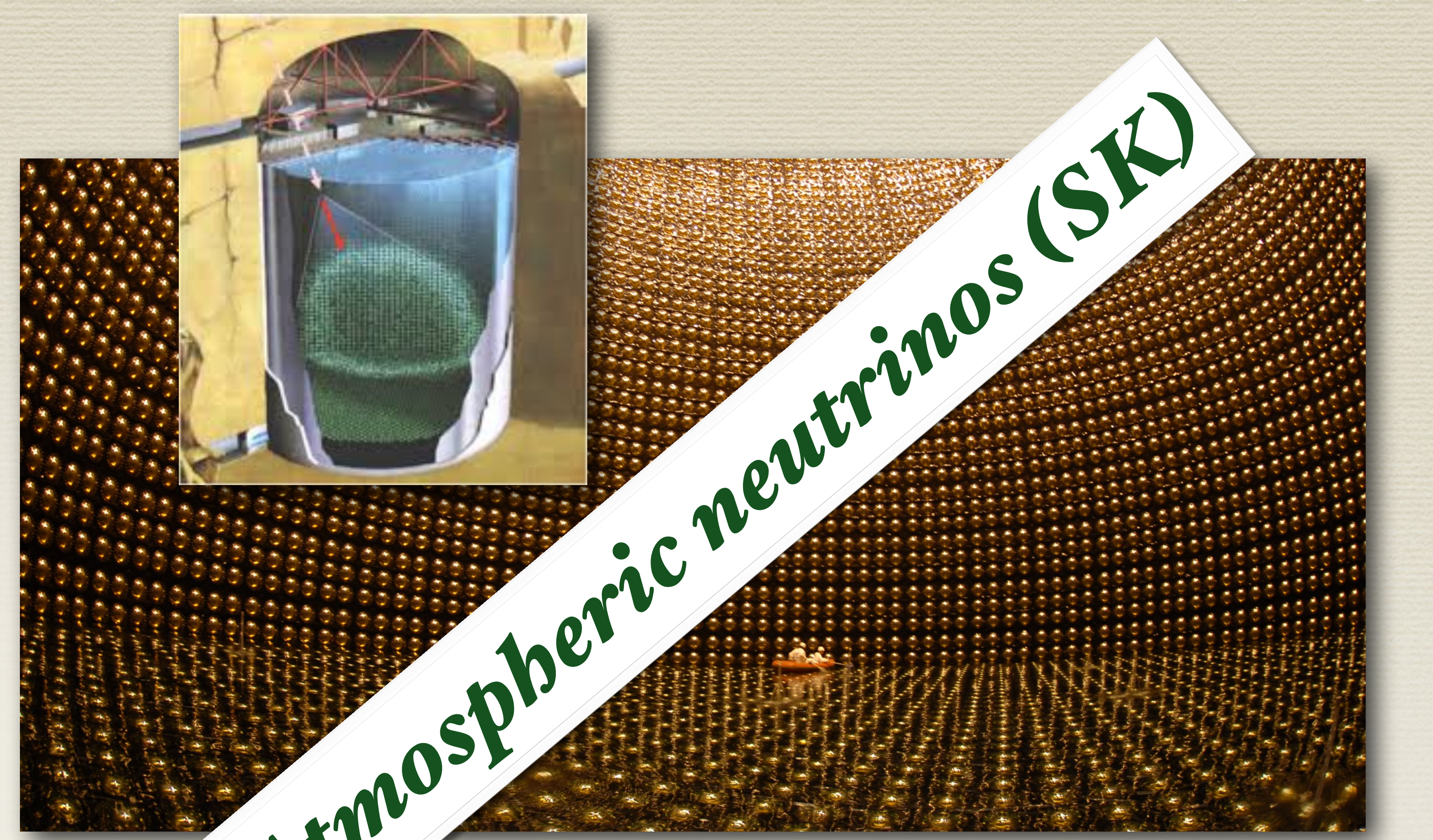
Standard Model of Elementary Particles



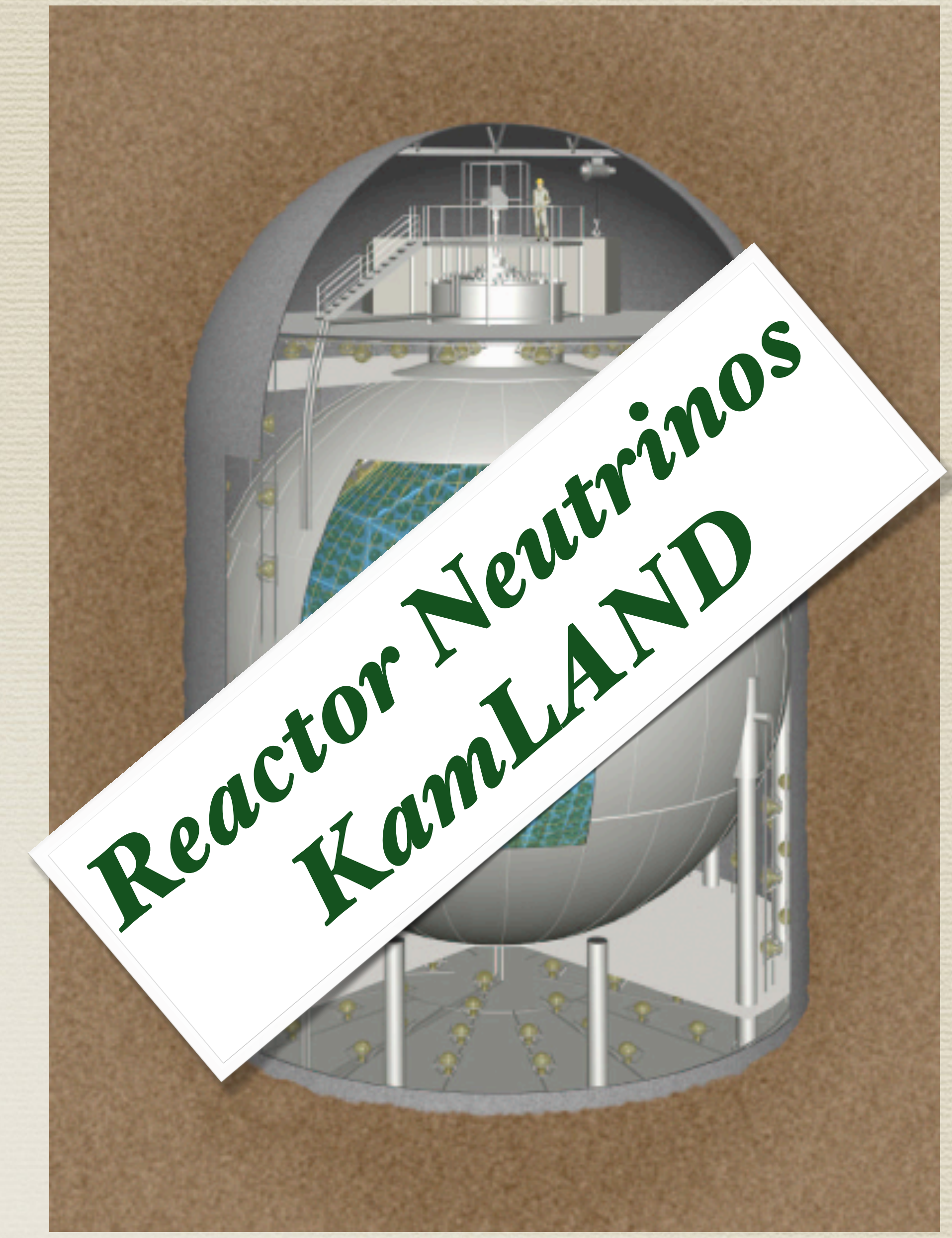
- *No right-handed neutrinos*
- $B \times L_e \times L_\mu \times L_\tau$

Neutrinos were postulated to be massless in the SM

Solar neutrinos (Homestake, SAGE
Gallex, SK, SNO, Borexino)



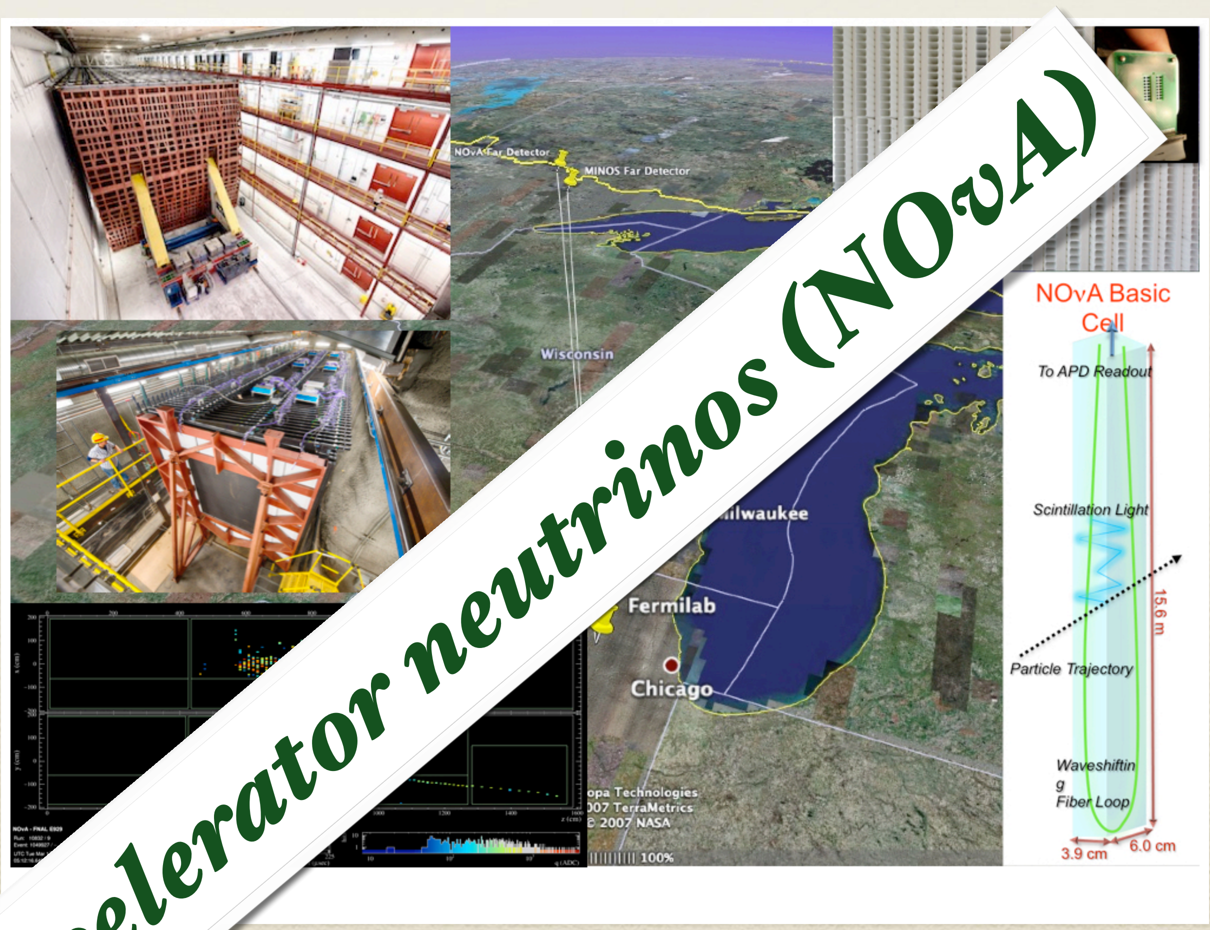
Atmospheric neutrinos (SK)



Reactor Neutrinos
KamLAND

Neutrino Flavor Oscillations

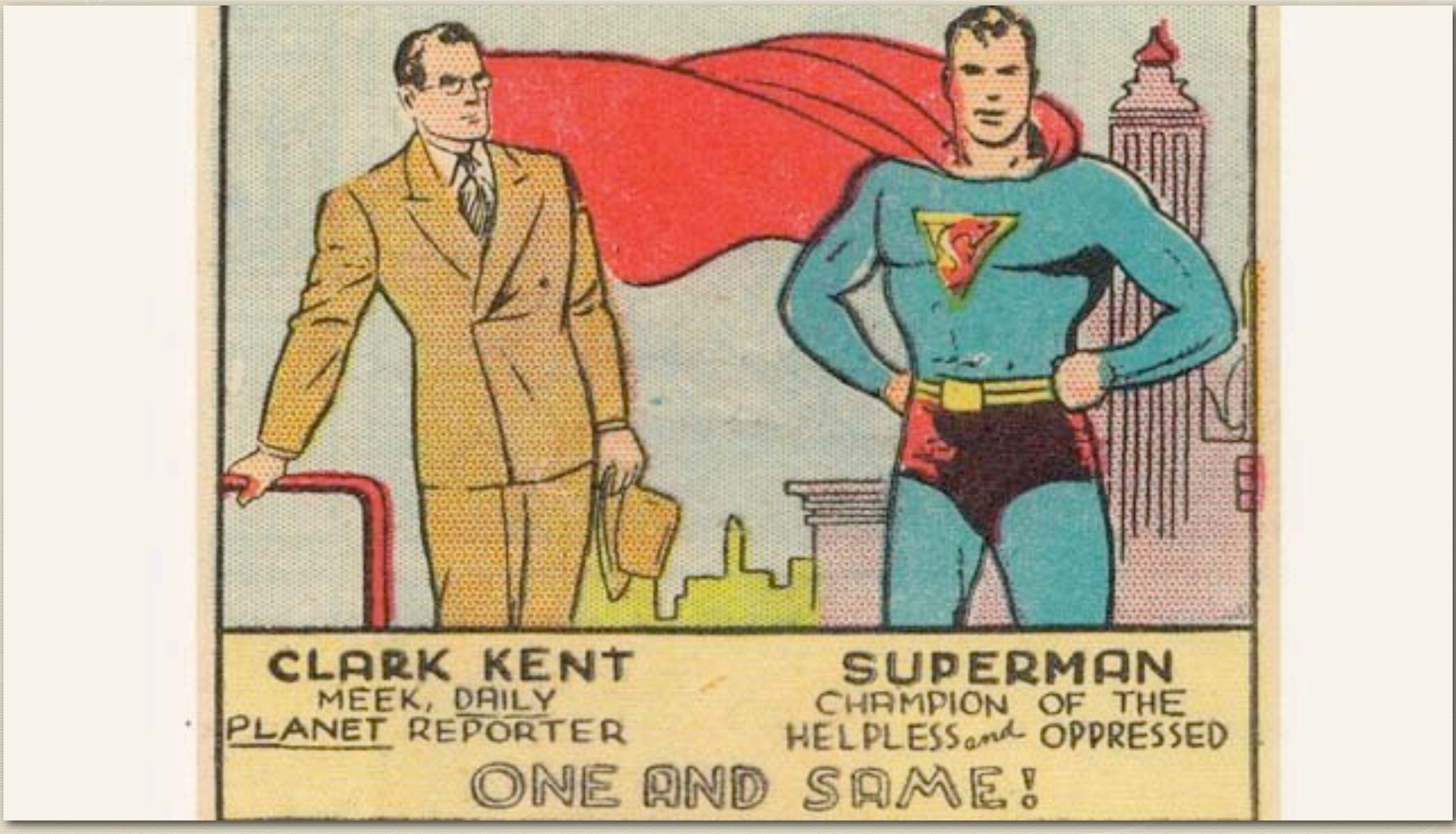
Accelerator neutrinos (T2K)



Accelerator neutrinos (NOvA)

Reactor neutrinos
Daya Bay, Reno, Double Chooz

Same guy, two identities



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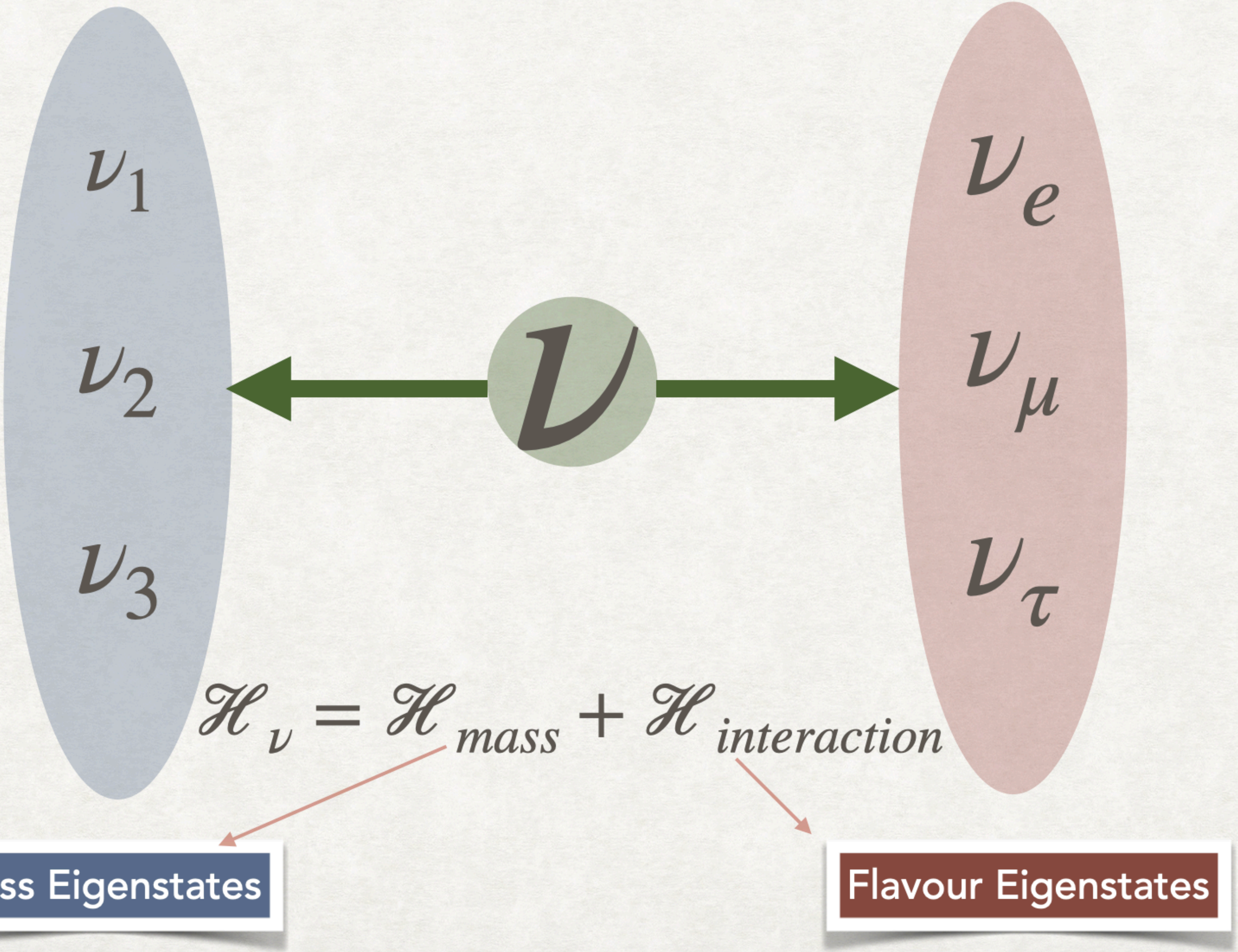
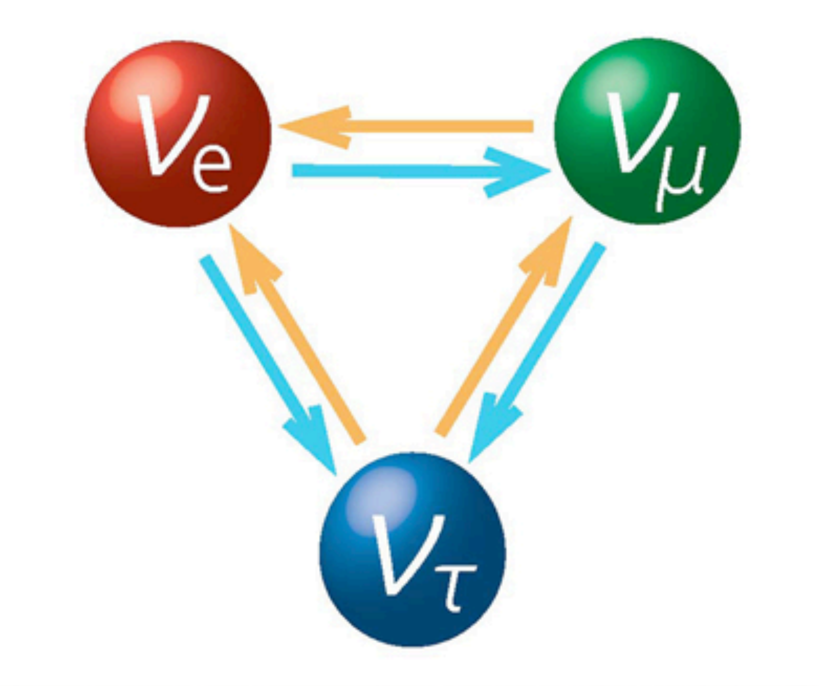


$$\mathcal{H}_\nu = \mathcal{H}_{mass} + \mathcal{H}_{interaction}$$

Mass Eigenstates

Flavour Eigenstates

Neutrino Oscillations



Nus are produced and detected by weak CC interactions

For example: $\pi^+ \rightarrow \mu^+ + \nu_\mu$ **flavor eigenstates**

Their propagation is defined in terms of **mass eigenstate**

The ***flavor eigenstates*** *can be written as a linear combination of the* ***mass eigenstates***

$$\langle \nu_k | \nu_j \rangle = \delta_{kj}$$

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i} |\nu_i\rangle$$

$$\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta}$$

$$|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle$$

Neutrino Oscillations

After time t or distance L, the state evolves to

$$|\nu(t)\rangle = \sum_{i=1}^{n} U_{\alpha i} e^{-iE_i t} |\nu_i\rangle$$

Neutrinos are detected by weak charged current interaction

For example: $\nu_\mu + N \to \mu^- + N'$ *as flavor eigenstates*

Probability that the detected flavor is ν_β

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu(t) \rangle|^2$$

Neutrino Oscillations

Assume that there are two generations of massive neutrinos

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad 0 \leq \theta \leq \frac{\pi}{2}$$

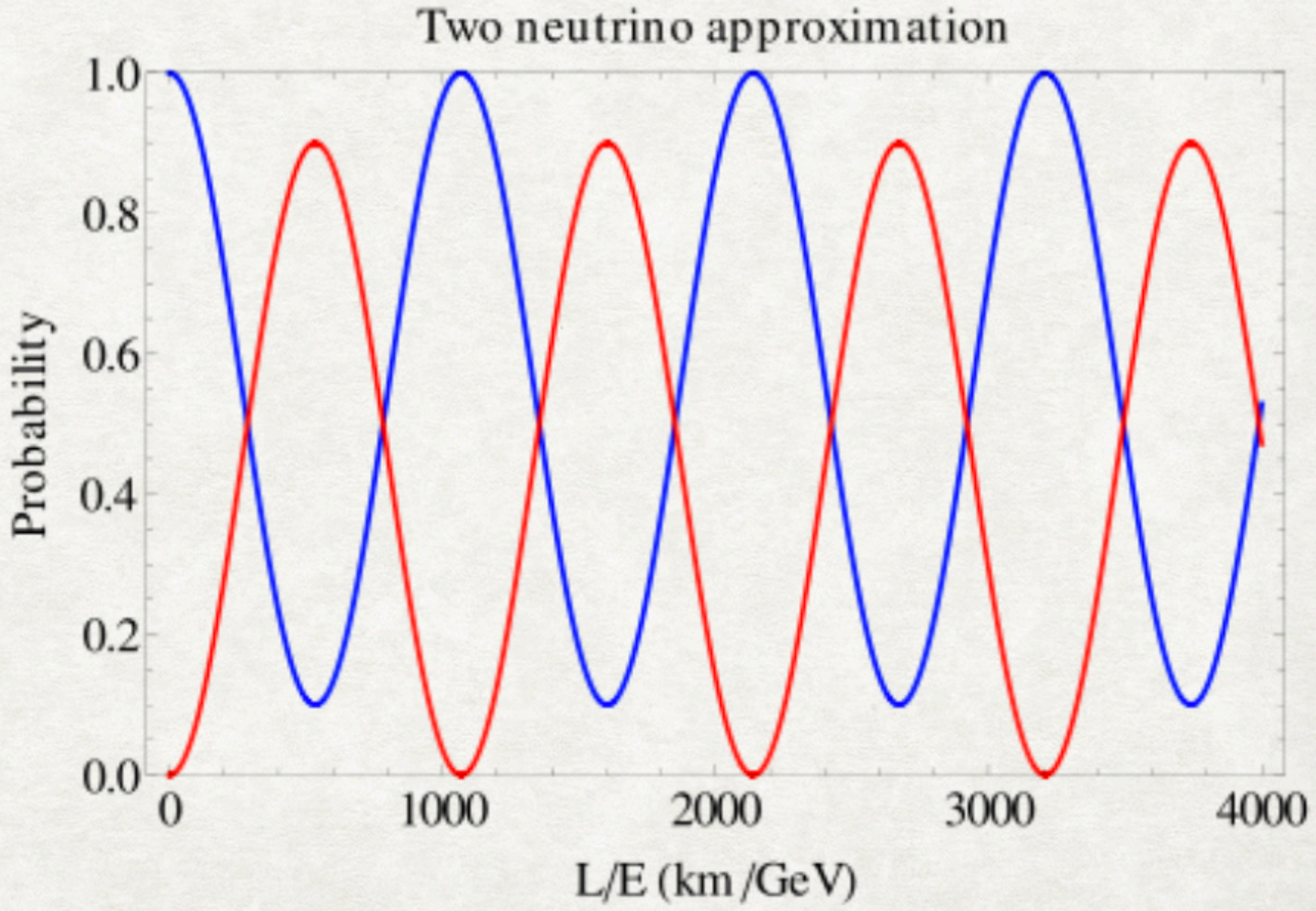
Neutrino Oscillations in Two Generations

- Flavor Eigenstates $\neq$ Mass Eigenstates

$$\nu_\mu = \cos\theta\,\nu_2 + \sin\theta\,\nu_3$$

$$\nu_\mu(t) = \cos\theta\, e^{-iE_2 t}\nu_2 + \sin\theta\, e^{-iE_3 t}\nu_3$$

$$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2\, L}{4E}\right)$$



Depends on both L and E

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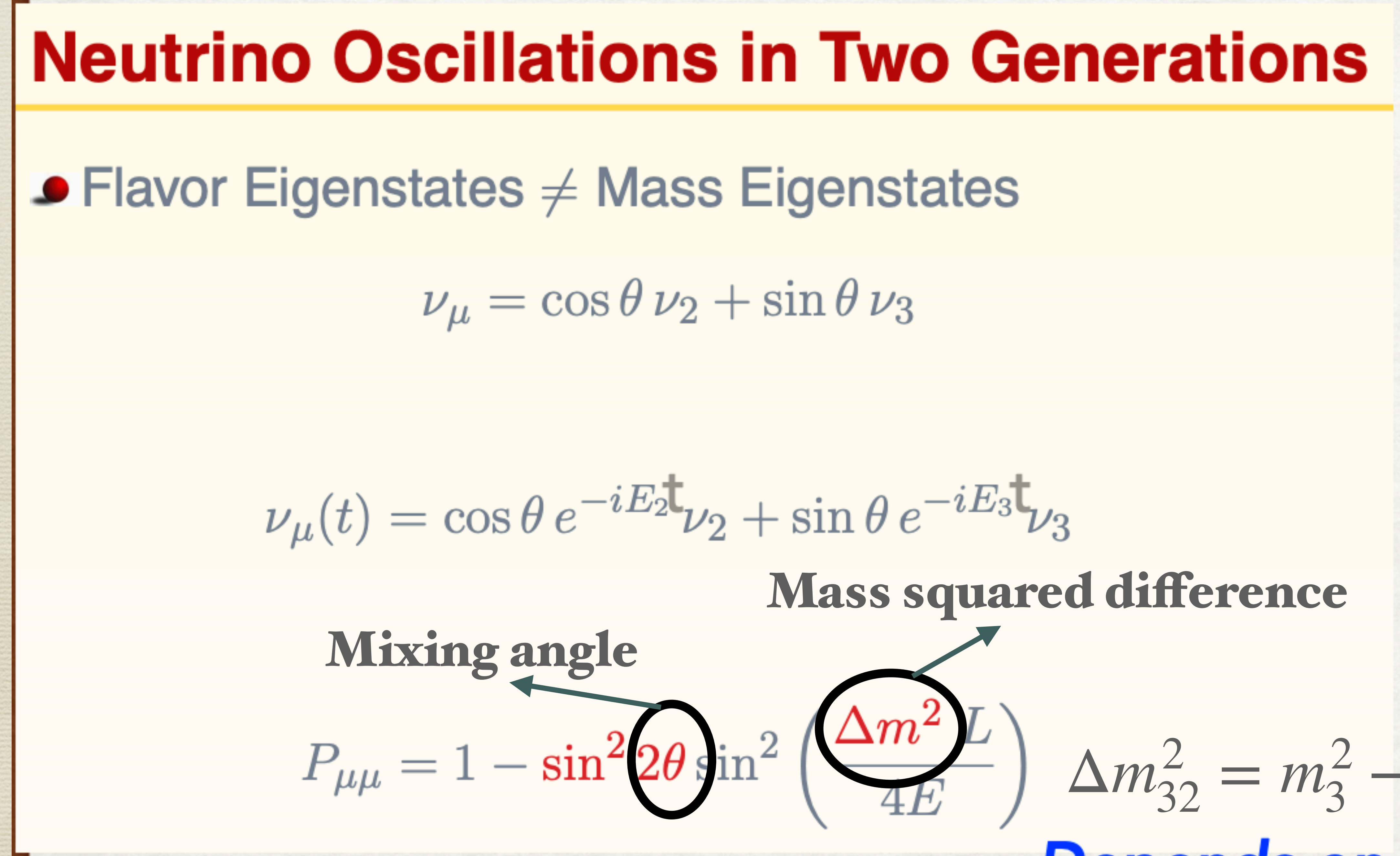

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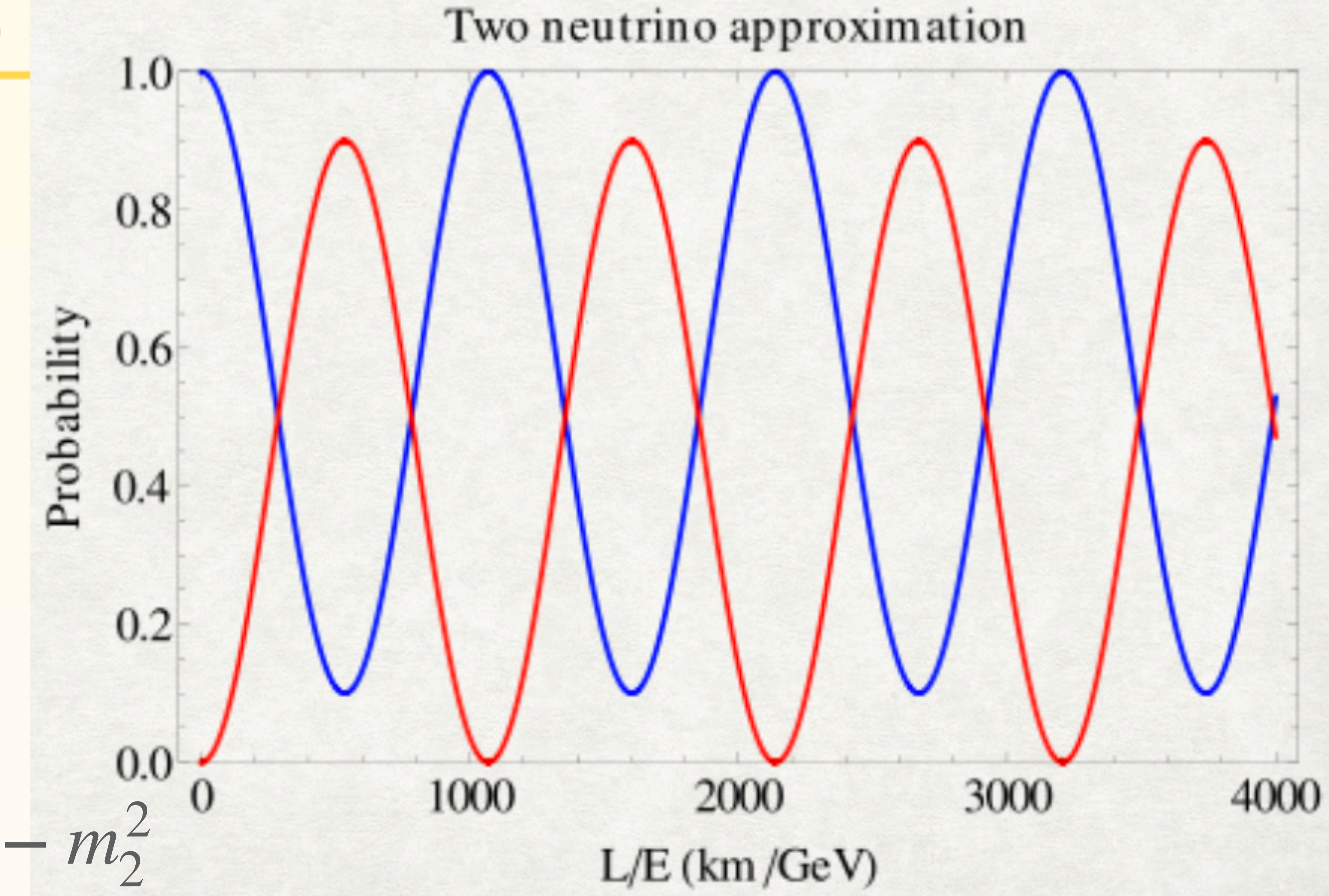
Mass squared difference

Mixing angle

$$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad \Delta m^2_{32} = m^2_3 - m^2_2$$



Depends on both L and E

Three Flavor Oscillations in Vacuum

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$P_{\beta\gamma}(L) = \delta_{\beta\gamma} \quad -4\sum_{j>1} Re\left(U_{\beta i}U^\star_{\gamma i}U^\star_{\beta j}U_{\gamma j}\right)\frac{\sin^2 \Delta m^2_{ij} L}{4E}$$

$$\pm 2\sum_{j>1} Im\left(U_{\beta i}U^\star_{\gamma i}U^\star_{\beta j}U_{\gamma j}\right)\frac{\sin \Delta m^2_{ij} L}{2E}.$$

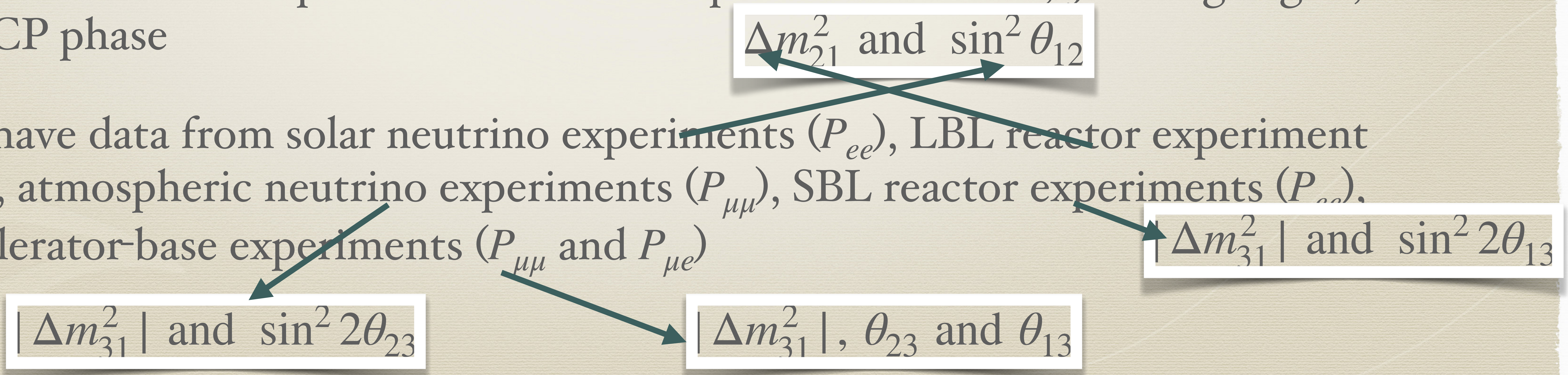
3 mixing angles

1 CP Phase

2 mass-squared diff

Oscillation Channels

- For $\beta = \gamma$ we get the "survival probability" => disappearance channel (say, $P_{\mu\mu}$)
- For $\beta \neq \gamma$, we get "transition probability" => "appearance channel" (say, $P_{\mu e}$)
- Oscillation experiments use either one or both to give information about neutrino oscillation parameters - 2 mass squared differences, 3 mixing angles, and the CP phase
- We have data from solar neutrino experiments (P_{ee}), LBL reactor experiment (P_{ee}), atmospheric neutrino experiments ($P_{\mu\mu}$), SBL reactor experiments (P_{ee}), accelerator-base experiments ($P_{\mu\mu}$ and $P_{\mu e}$)



Δm^2_{21} and $\sin^2\theta_{12}$

$|\Delta m^2_{31}|$ and $\sin^2 2\theta_{13}$

$|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$

$|\Delta m^2_{31}|$, θ_{23} and θ_{13}

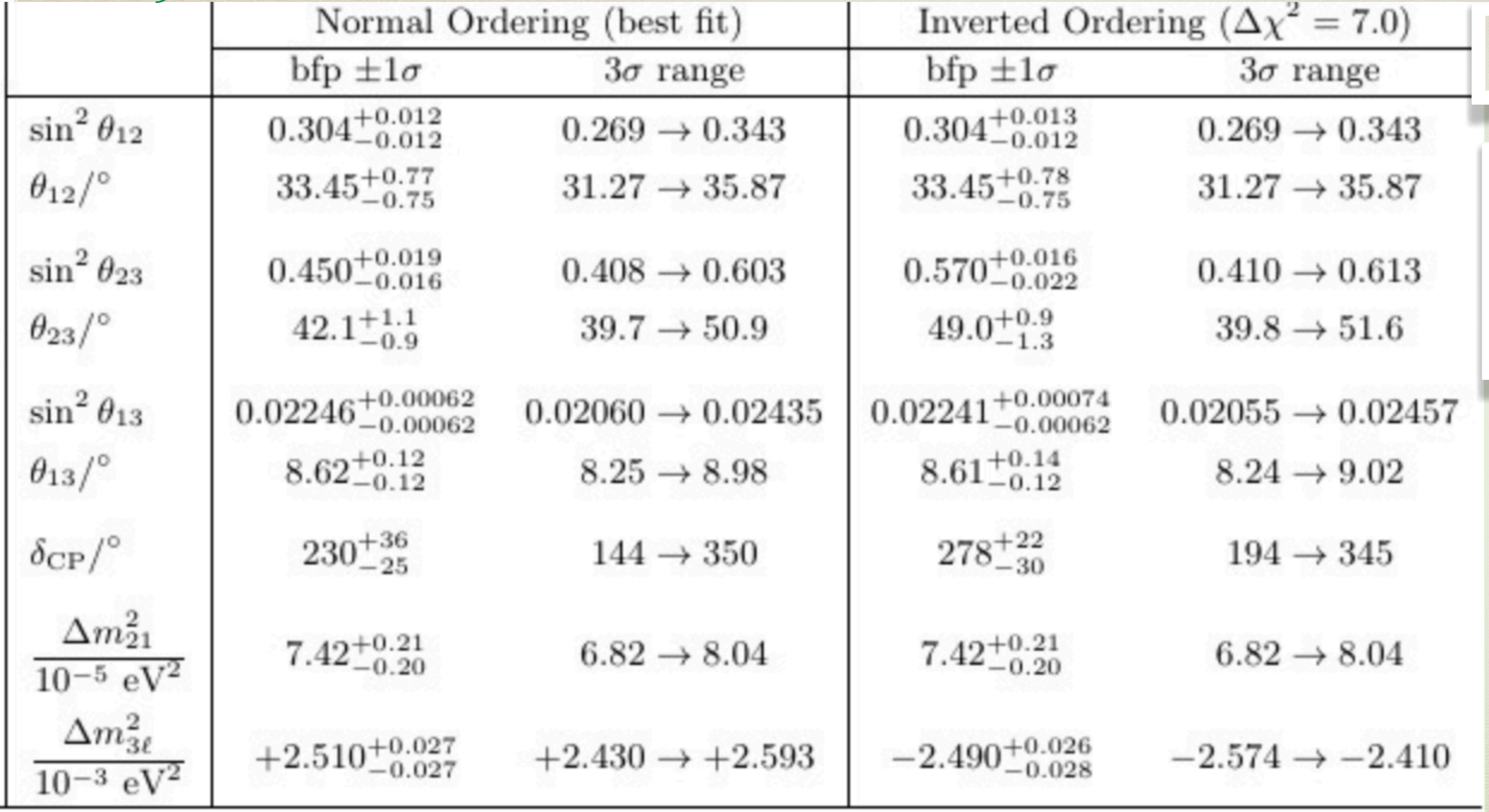
Current Status of Neutrino Oscillation Parameters NuFit5.1 2021

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
	$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
	$\delta_{\mathrm{CP}}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

Current Status of Neutrino Oscillation Parameters

"The Knowns"

NuFit5.1 2021



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Neutrino masses are tiny

Neutrino mixing angles are different from Quark mixing angles

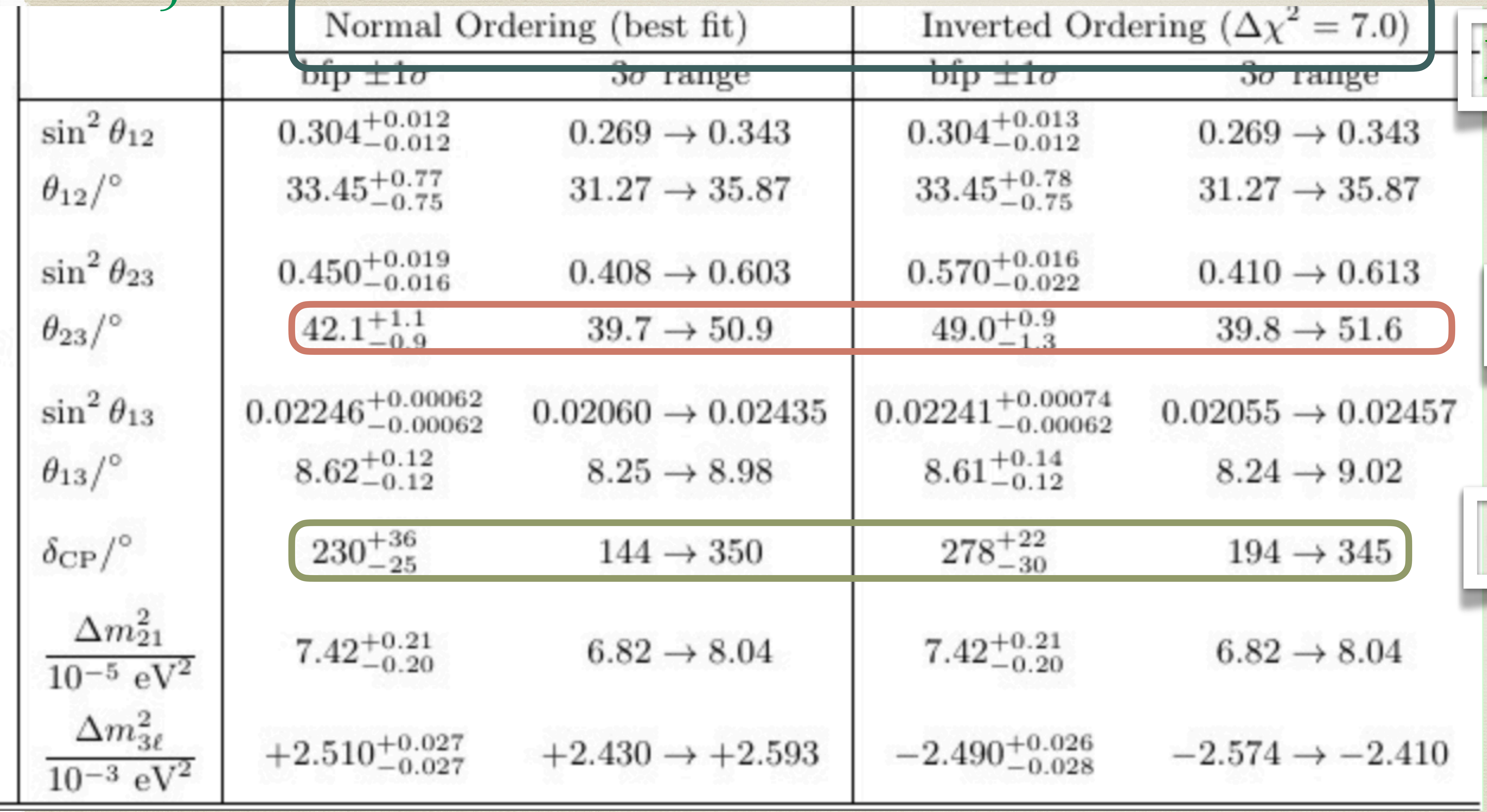
$\theta_{13} = $ small
$\theta_{23} \sim $ maximal
$\theta_{12} \sim $ large

Is this expected?

Current Status of Neutrino Oscillation Parameters

"The Unknowns"

NuFit5.1 2021



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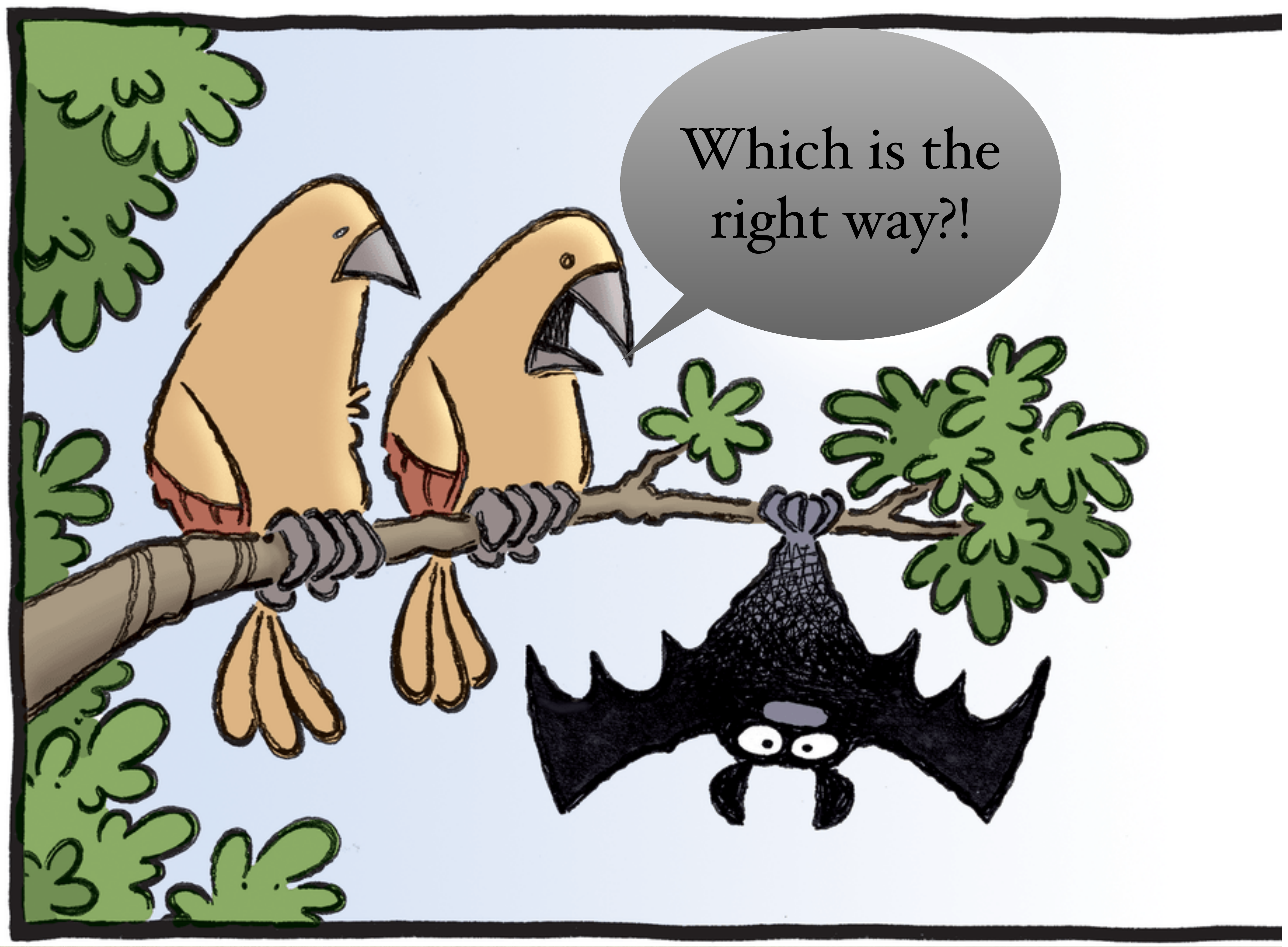
Neutrino Mass Ordering ?

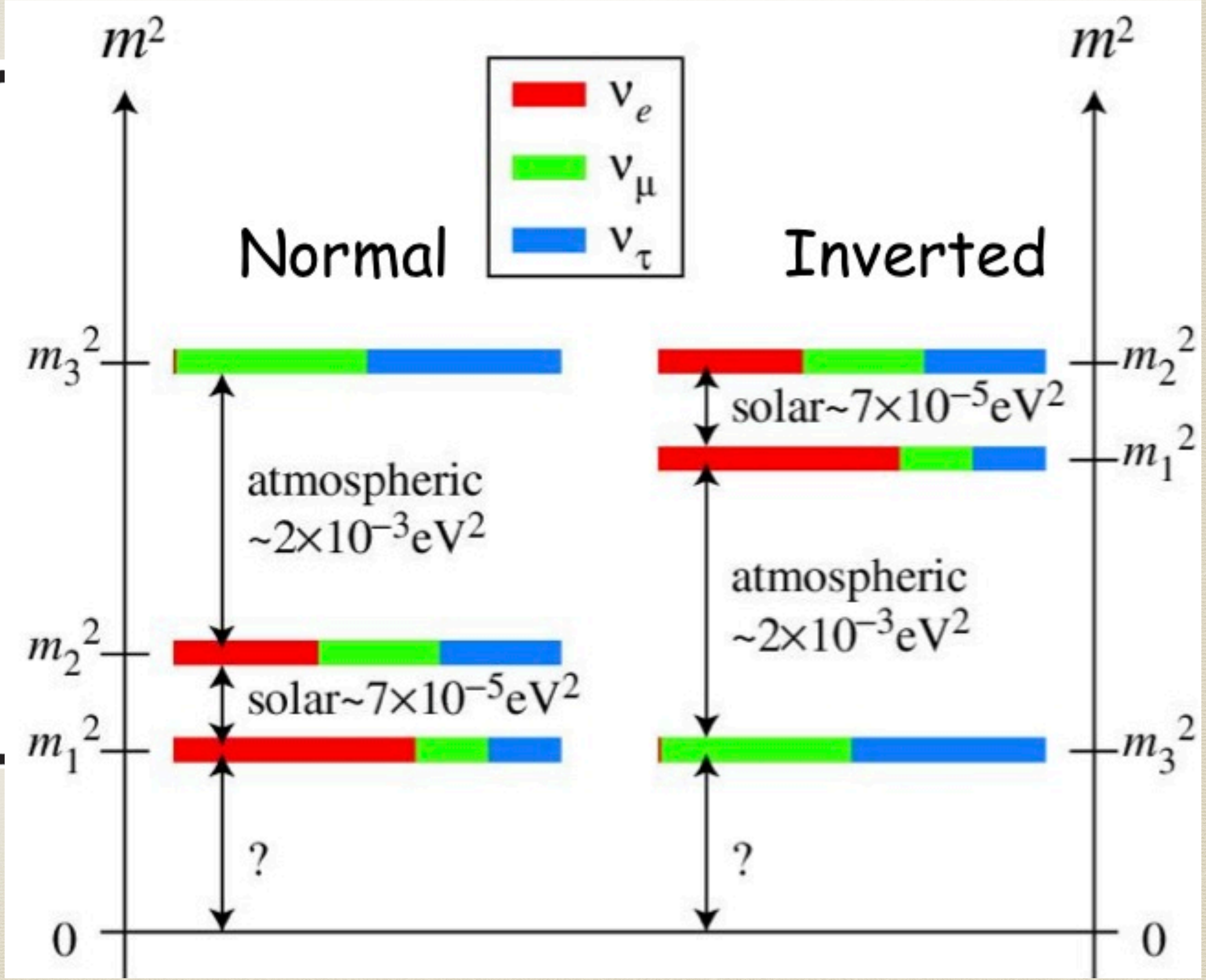
Octant of theta23 ?

CP Violation ?

The next task is to answer these questions

Neutrino Mass Ordering





Sign of Δm^2_{21} is known

Sign of Δm^2_{31} is unknown

What depends on sign of Δm^2_{31}

- Neutrino oscillation probabilities could depend on the sign of Δm^2_{31}

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4 \sum_{k>j} \mathfrak{Re}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$
$$+ 2 \sum_{k>j} \mathfrak{Im}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right) .$$

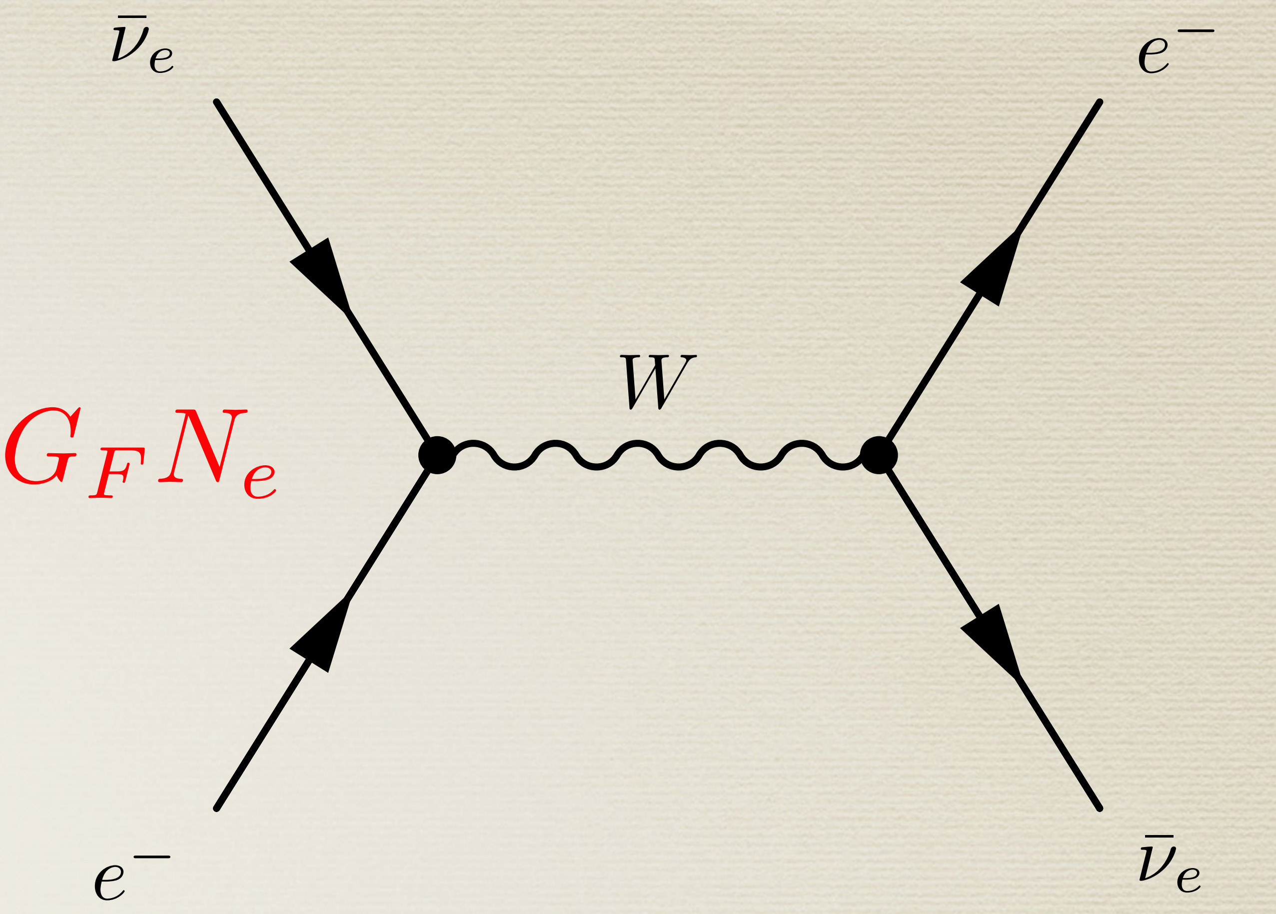
- However, this is not an effective way of determining the sign of Δm^2_{kj} due to the presence of "degeneracies"

Matter Effects

Forward scattering of ν_e and $\bar{\nu}_e$ with electrons in matter

Effective potential

$\sqrt{2}G_F N_e$



This effective potential modifies the neutrino mass and mixing in matter

$$(\Delta m^2)^m = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2)^2 \sin^2 2\theta}$$

$$\sin^2 2\theta_m = \frac{(\Delta m^2)^2 \sin^2 2\theta}{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2)^2 \sin^2 2\theta}$$

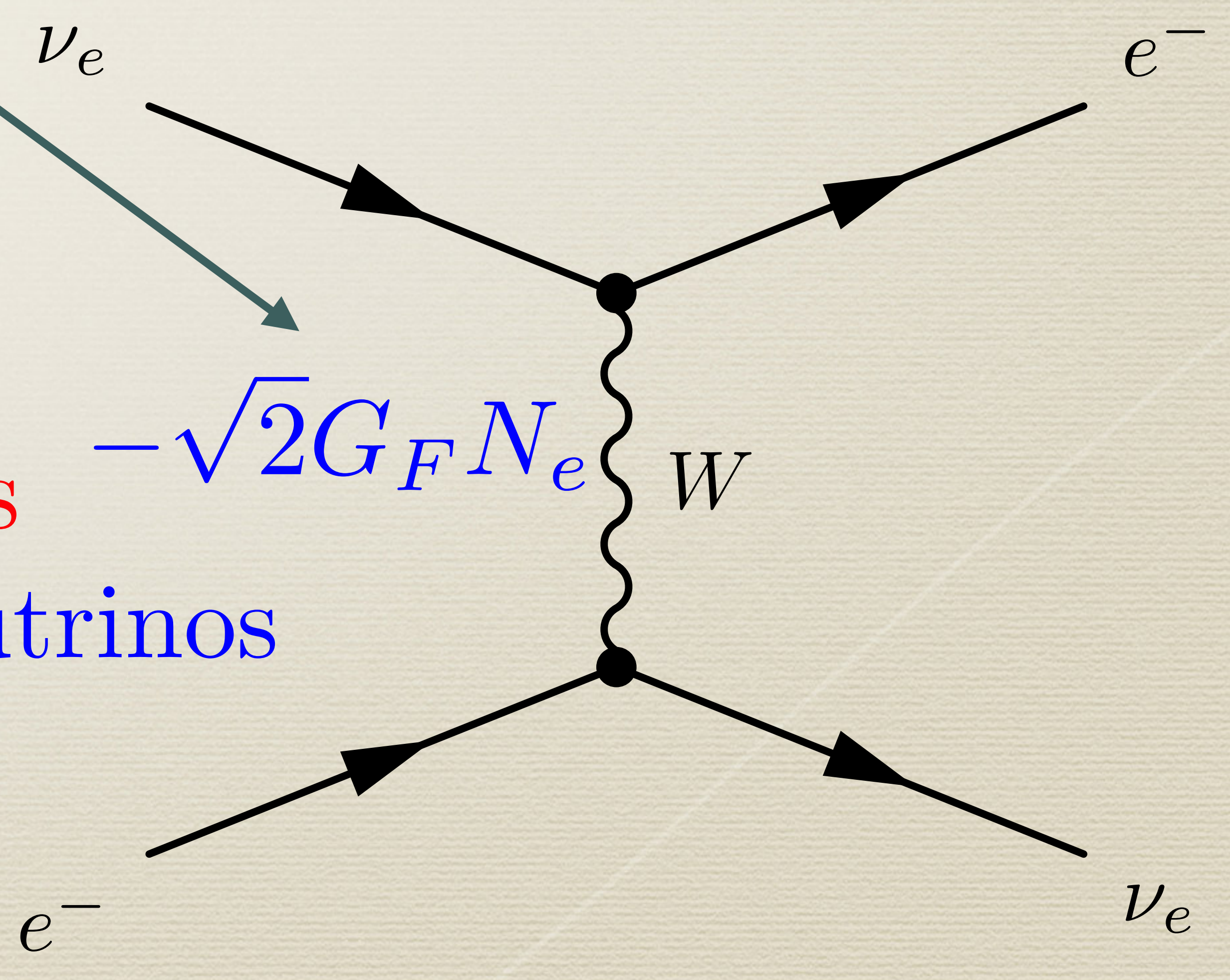
Relative sign

$A = 2E * (\text{effective potential})$

$A = 2\sqrt{2}G_F N_e E$ for neutrinos

$A = -2\sqrt{2}G_F N_e E$ for antineutrinos

$-\sqrt{2}G_F N_e$

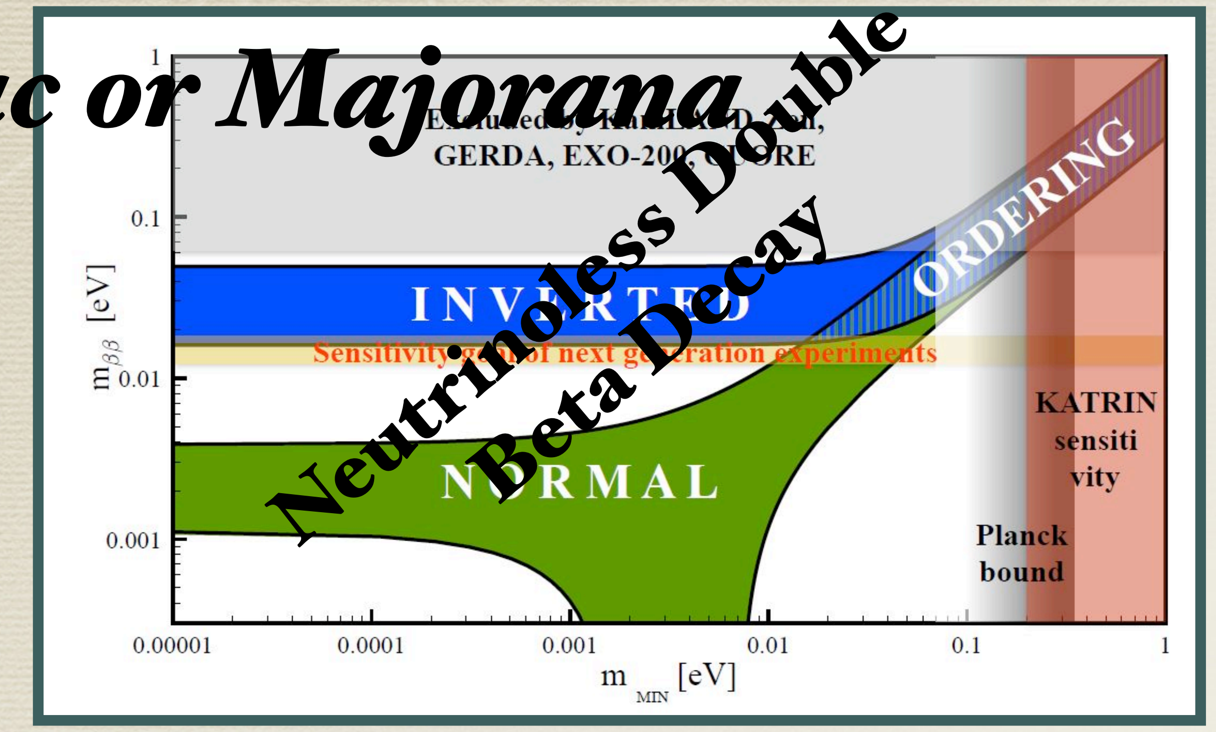


Normal ordering ➔ matter effects for neutrinos

Inverted ordering ➔ matter effects for antineutrinos

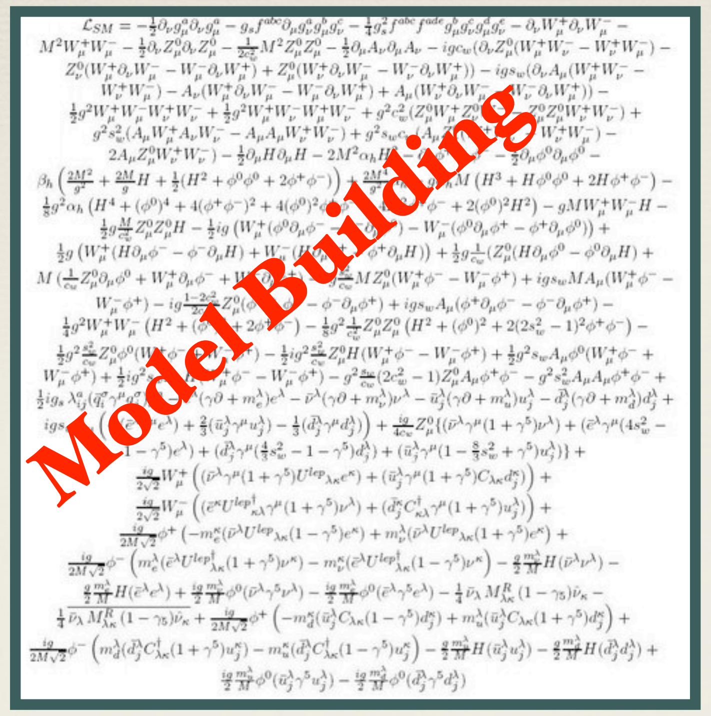
Mass Ordering

- Why do we need to know the mass ordering?

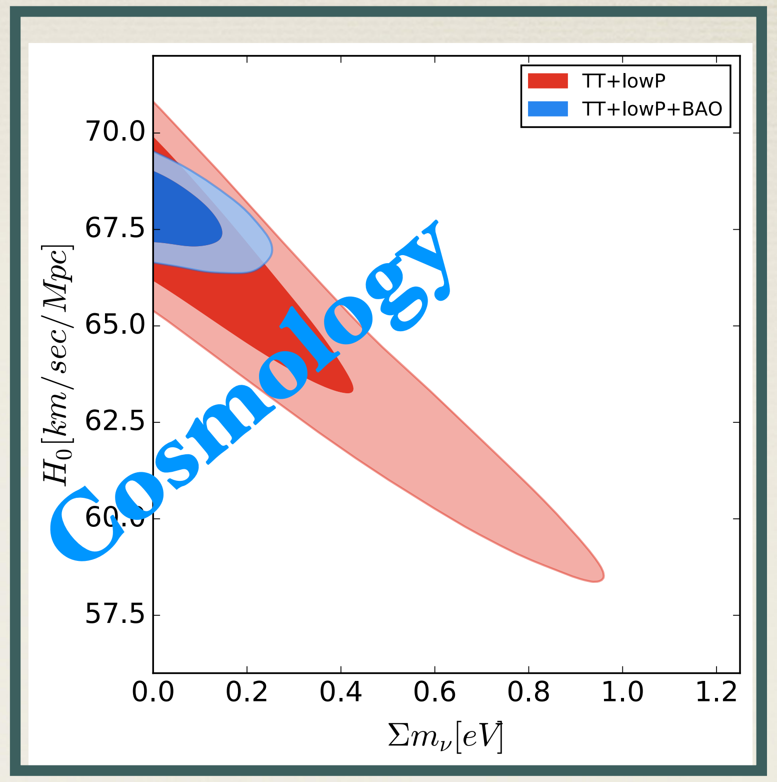


Dirac or Majorana

Neutrinoless Double Beta Decay



Model Building



Cosmology

- Why have the current experiments not been able to measure this?

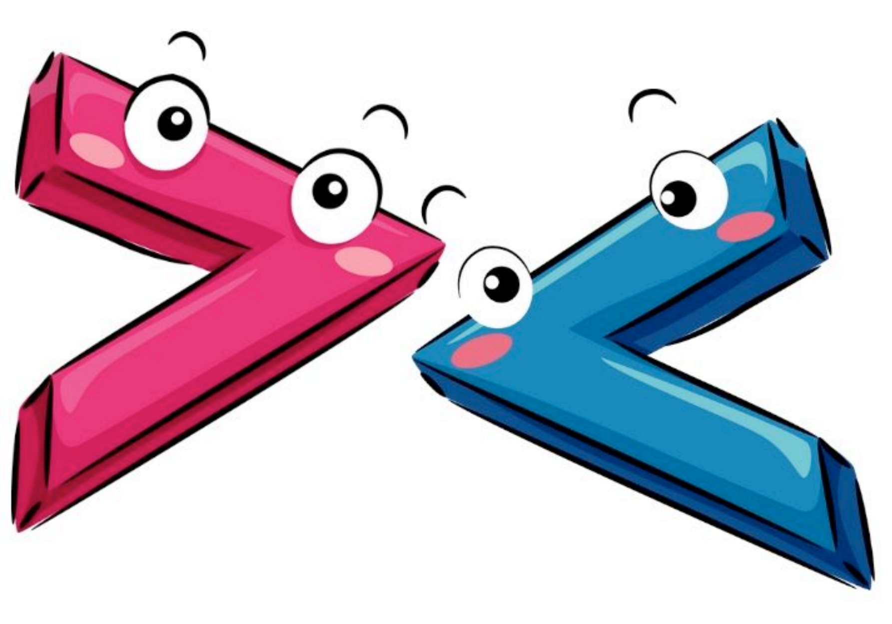
Have not been able to see Δm^2_{31} driven matter effects with statistical significance

- How will the future experiments determine it?

Δm^2_{31} driven matter effects will be observed at LBL experiment DUNE
Δm^2_{31} driven matter effects will be observed at atm experiments like IceCube, ORCA, HK, INO….
Δm^2_{31} and Δm^2_{21} interference effects will be observed at JUNO

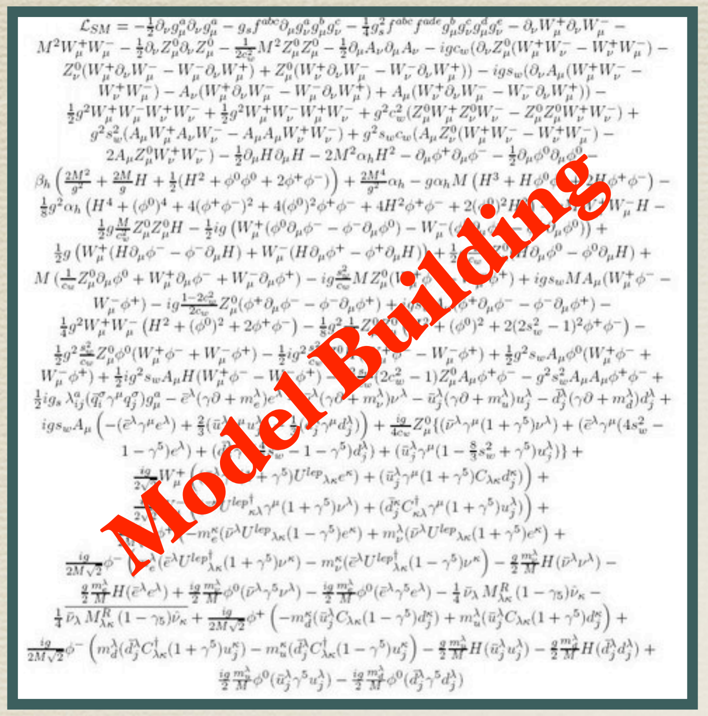
Octant of theta23

θ_{23}



$\pi/4$

- Why do we need to know the octant of theta23?



How much matter effects in MO experiments

Affects Osc Probabilities

- Why have the current experiments not been able to measure this?

Sensitivity mostly coming from experiments sensitive to $P_{\mu\mu}$ *which depends on* $\sin^2 2\theta_{23}$

- How will the future experiments determine it?

Long baseline experiments such as DUNE and T2HK will measure the octant via a combo of $P_{\mu e}$ *and* $p_{\mu\mu}$

CP Violation

- Why bother?

Important parameter in the neutrino mixing matrix

Key player in model of neutrino mass - pointer at the correct BSM theory

CPV in nu osc

↓

Pointer to leptogenesis

↑

- ***Seesaw Models***

CP Violation

- If we observe a difference between flavor oscillations of neutrinos and antineutrino → CP violation
- CP dependence in neutrino oscillations comes from the phase δ_{CP} in the neutrino mixing matrix … this phase is mostly referred to as the "Dirac CP phase"

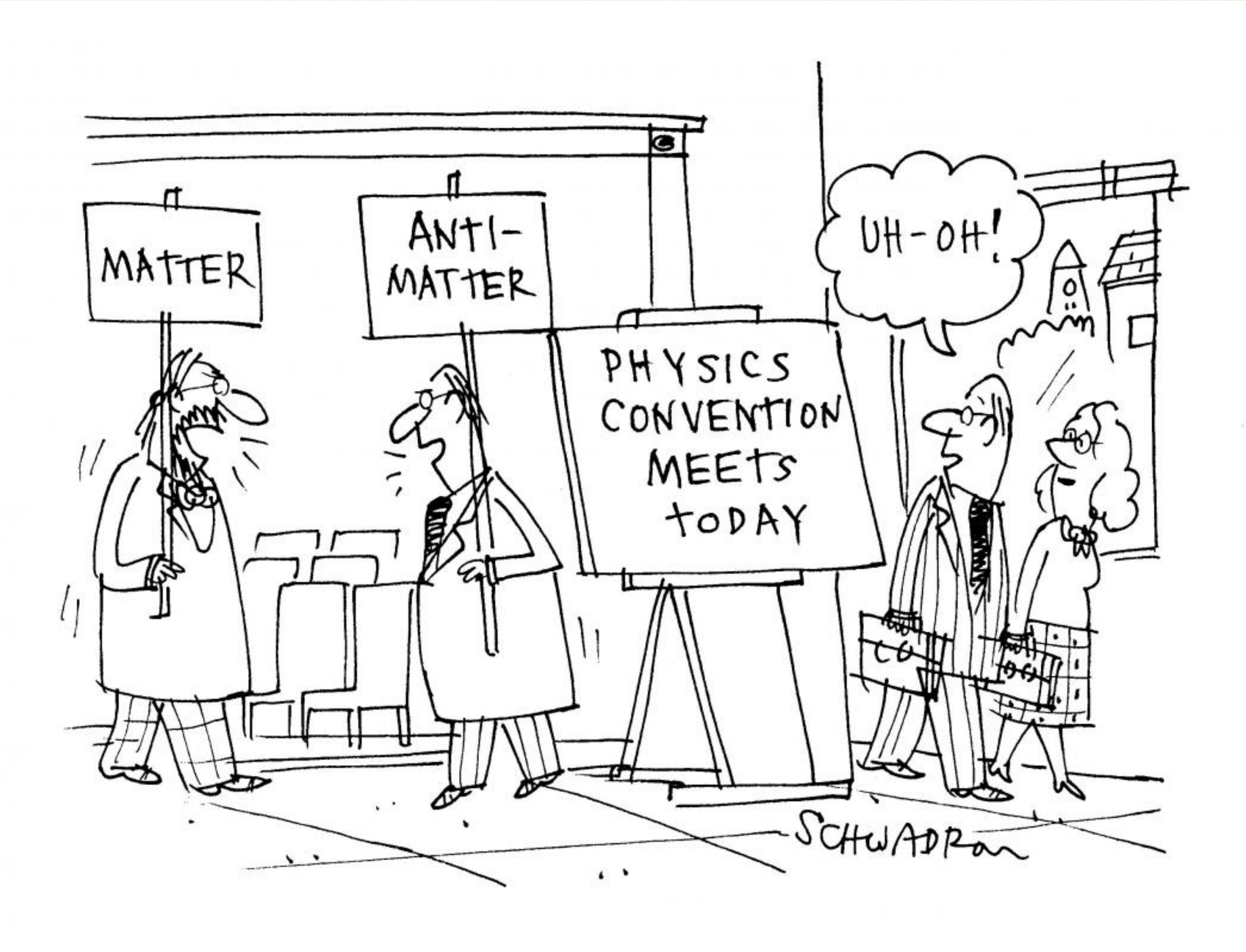
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric Accelerator

Reactor Accelerator

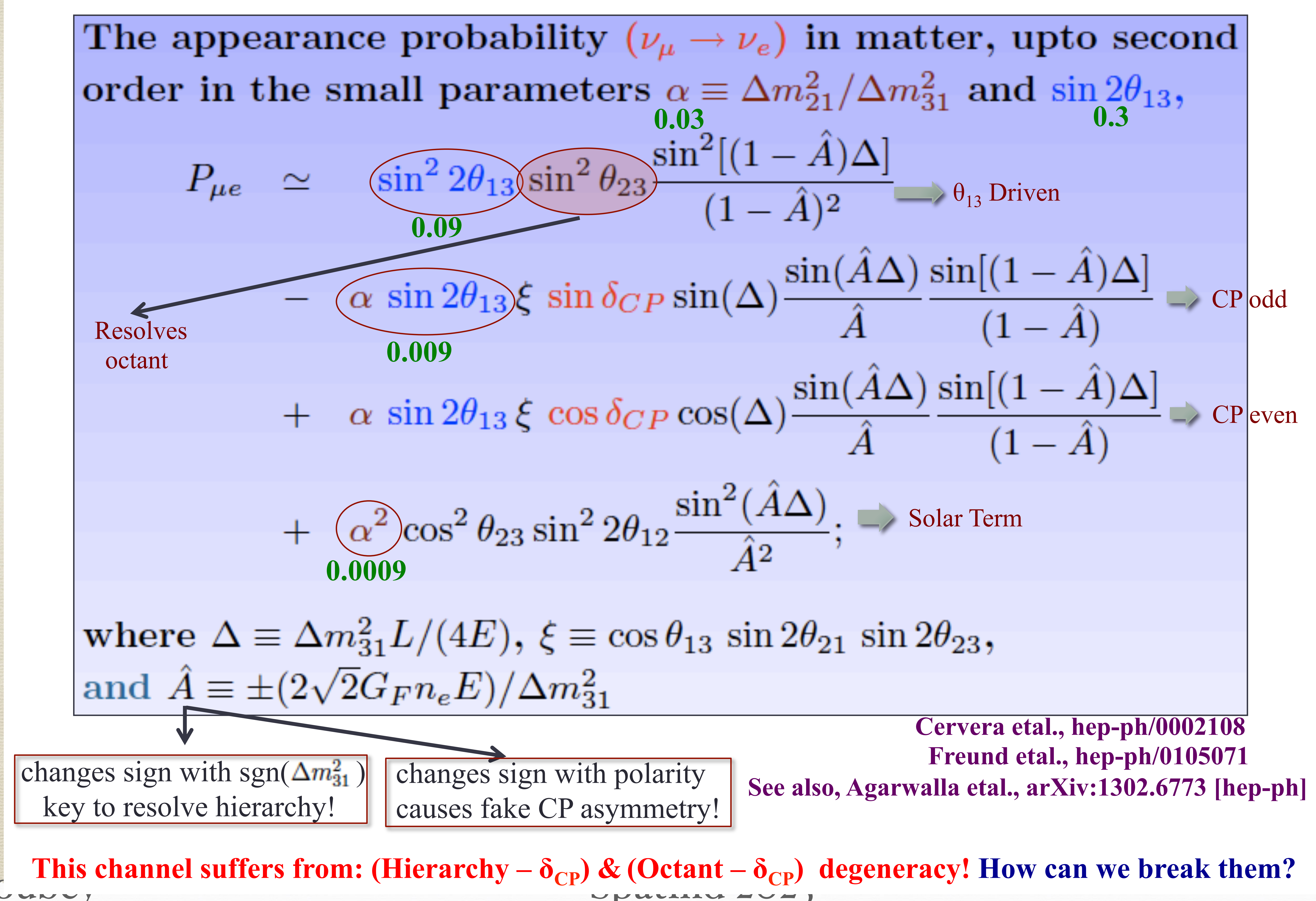
Solar Reactor

Neutrinoless double beta dk



CP Violation

The best method to see CP violation is to measure the oscillation probability



The appearance probability $(\nu_\mu \to \nu_e)$ in matter, upto second order in the small parameters $\underset{0.03}{\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31}}$ and $\underset{0.3}{\sin 2\theta_{13}}$,

$$
\begin{aligned}
P_{\mu e} \;\simeq\;& \underset{0.09}{\boxed{\sin^2 2\theta_{13}}}\,\boxed{\sin^2\theta_{23}}\,\frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \rightarrow \theta_{13}\text{ Driven}\\
&- \underset{0.009}{\boxed{\alpha\,\sin 2\theta_{13}}}\,\xi\,\sin\delta_{CP}\,\sin(\Delta)\,\frac{\sin(\hat{A}\Delta)}{\hat{A}}\,\frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \rightarrow \text{CP odd}\\
&+ \alpha\,\sin 2\theta_{13}\,\xi\,\cos\delta_{CP}\,\cos(\Delta)\,\frac{\sin(\hat{A}\Delta)}{\hat{A}}\,\frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \rightarrow \text{CP even}\\
&+ \underset{0.0009}{\boxed{\alpha^2}}\cos^2\theta_{23}\,\sin^2 2\theta_{12}\,\frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}; \rightarrow \text{Solar Term}
\end{aligned}
$$

Resolves
octant

where $\Delta \equiv \Delta m^2_{31} L/(4E)$, $\xi \equiv \cos\theta_{13}\,\sin 2\theta_{21}\,\sin 2\theta_{23}$,
and $\hat{A} \equiv \pm(2\sqrt{2}G_F n_e E)/\Delta m^2_{31}$

changes sign with sgn(Δm^2_{31})
key to resolve hierarchy!

changes sign with polarity
causes fake CP asymmetry!

Cervera etal., hep-ph/0002108
Freund etal., hep-ph/0105071
See also, Agarwalla etal., arXiv:1302.6773 [hep-ph]

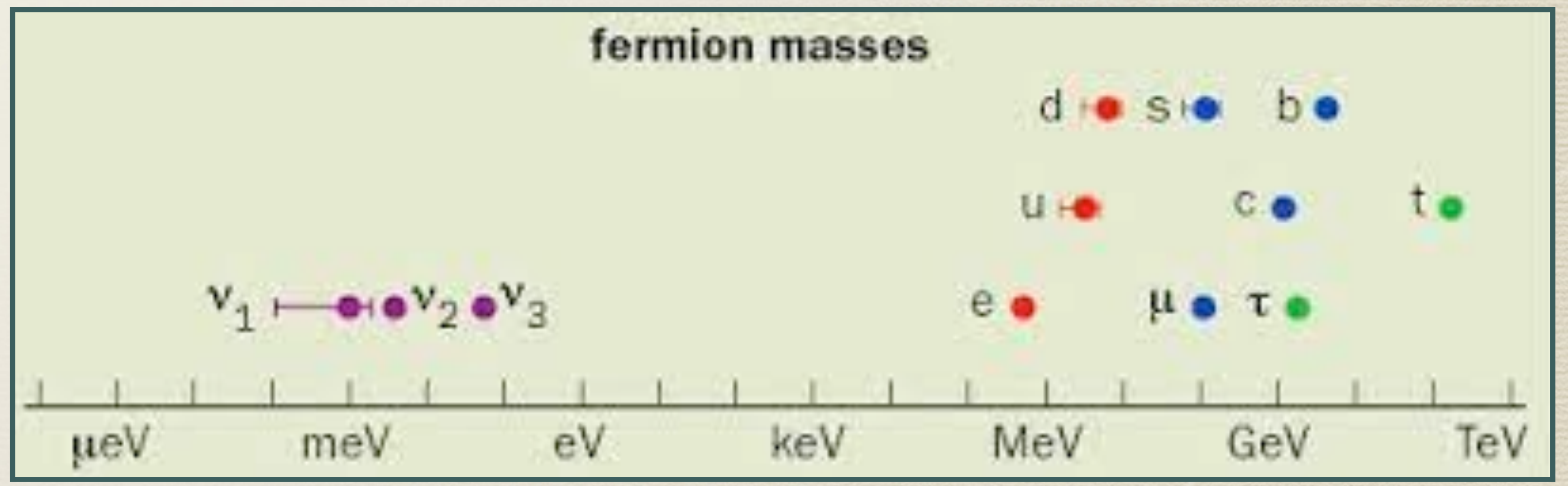
This channel suffers from: (Hierarchy – δ_{CP}) & (Octant – δ_{CP}) degeneracy! How can we break them?

Neutrino Mass (Dirac)

Add to the SM ν_R *which can give a Yukawa term*

$$\mathcal{L}_Y^\nu = -Y\bar{\psi}_L \nu_R \tilde{\phi} + h.c. \quad (\tilde{\phi} = i\tau_2\phi^*) \qquad \bar{\psi}_L = (\bar{\nu}_L\ \bar{e}_L)$$

–After spontaneous symmetry breaking we get a Dirac mass term for the neutrinos



$$\mathcal{L}^\nu_{Mass} = \bar{\nu}_L M_D \nu_R + h.c.$$

Dirac Neutrinos

$$M_D = Y v_{SM} \quad (Y \sim 10^{-12})$$

Conserves lepton number

- We wish to explain the smallness of neutrino masses
- We wish to explain also the peculiarity of the mixing angles
- Smallness of neutrino masses can be explained naturally if the masses were generated either via -
- Higher loops - radiative neutrino mass models
- Higher dimensional operators - seesaw models
- The mixing pattern could come from some symmetry related to flavours
- We also wish to relate nu masses to baryogengesis and dark matter

Neutrino Mass (Majorana)

Allow for lepton number violation in your effective theory

- A natural way to obtain small Majorana masses is to write down a 5-dimensional operator

Weinberg'79

$$-\mathcal{L}_\nu^Y = C_\nu^5 \frac{1}{\Lambda} LLHH + h.c.$$

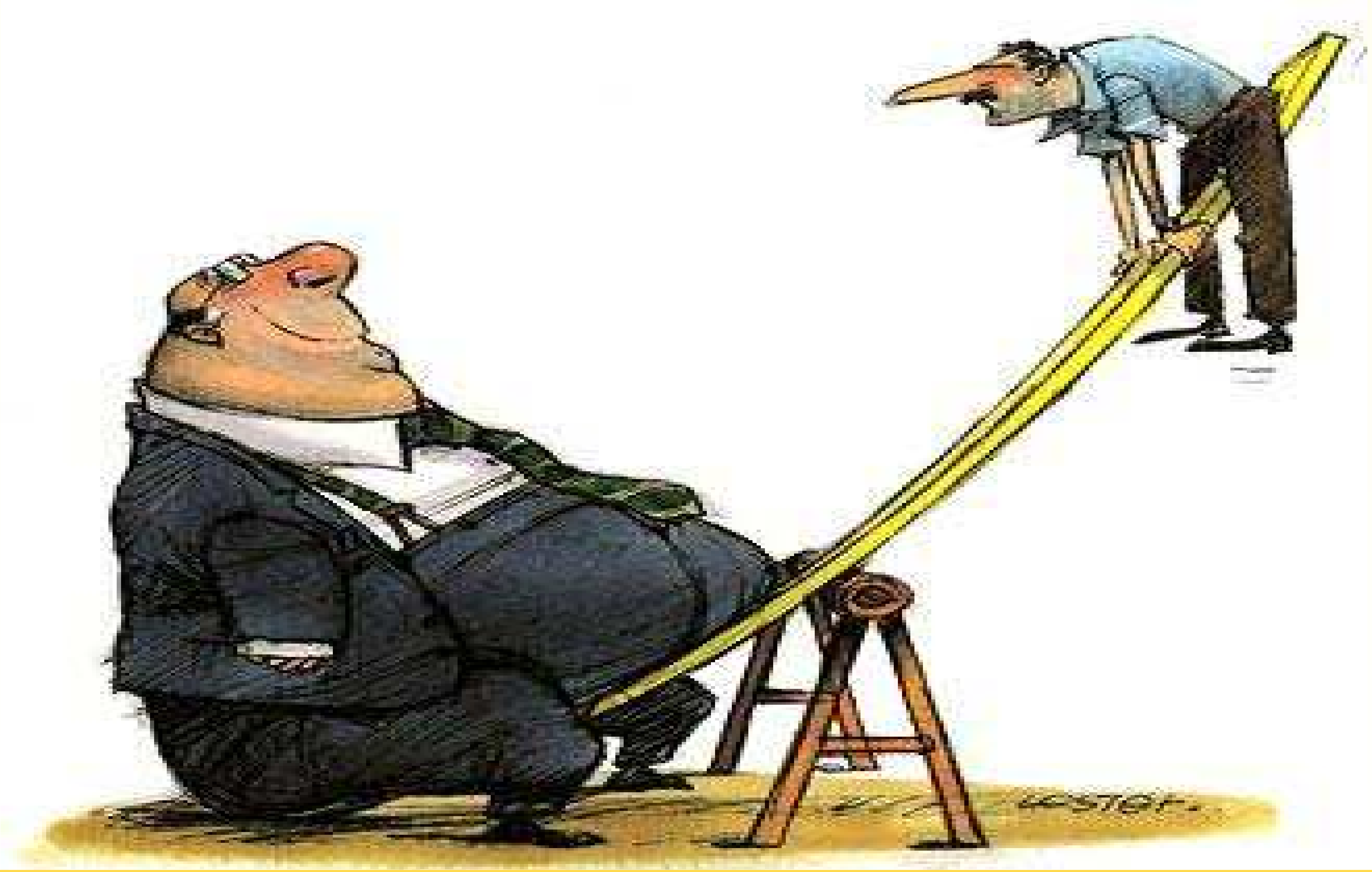
lepton number broken by 2 units

$$\Rightarrow m_\nu = \frac{C_\nu^5 v^2}{\Lambda}$$

- For $C_\nu^5 \sim 1$, one gets $m_\nu \sim 0.1$ eV when $\Lambda \sim 10^{15}$ GeV
- Is this an indication of new physics at a higher scale?

The Seesaw Mechanism

- High scale corresponds to a heavy particle which gets integrated out ⇒ 5-dim effective operator suppressed by the mass of the heavy particle



Go BSM

So the aim is to introduce (a) new heavy particle(s) and write down a UV complete theory

The SM gauge group can also also be extended in these BSM models

The terms in this extended theory need to be invariant under the SM gauge group and any added gauge groups

The Seesaw Mechanism

- Ways to generate a gauge invariant term from the SM doublets L and H: $2 \otimes 2 = 3 \oplus 1$

- Type I: L and H form a SU(2) singlet
  - Mediated by a heavy SU(2) singlet fermion with Y=0

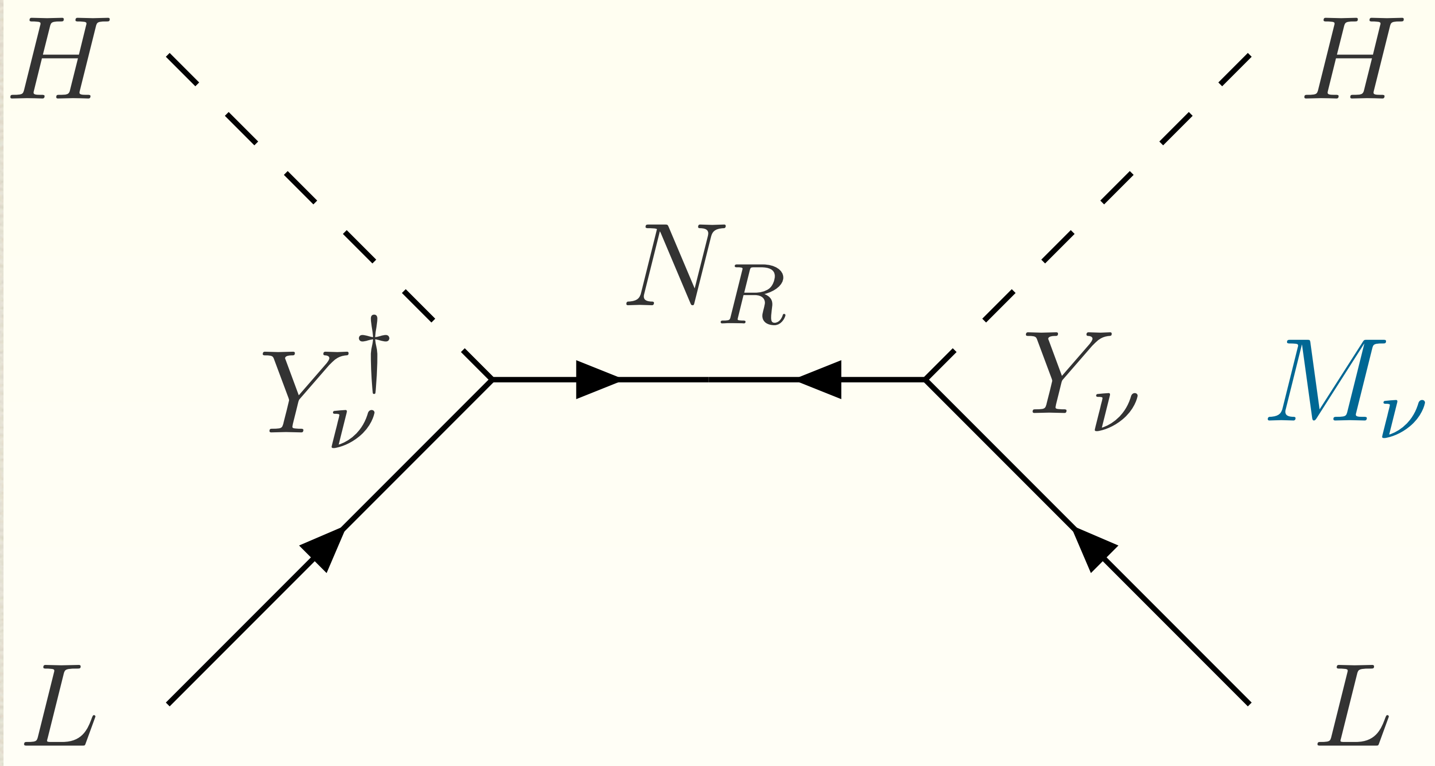
- Type II: L and L form a SU(2) triplet
  - Mediated by a heavy SU(2) triplet scalar with Y=1

- Type III: L and H form a SU(2) triplet
  - Mediated by a heavy SU(2) triplet fermion with Y=0

Type I Seesaw

- Introduce heavy right handed neutrinos $N_R \sim (1,1,0)$

$$-\mathcal{L}_Y = Y_\nu \bar{L}\tilde{H}N_R + \frac{1}{2}M_N\overline{N_R^c}N_R + h.c.$$



$$M_\nu = \begin{pmatrix} 0 & vY_\nu \\ vY_\nu & M_R \end{pmatrix}$$

$$\boxed{m_\nu = -v^2 Y_\nu M_N^{-1} Y_\nu^T}$$

Minkowski'77

Yanagida'79, Gelmann, Ramond, Slansky'79

Glashow'80, Mohapatra, Senjanovic'80

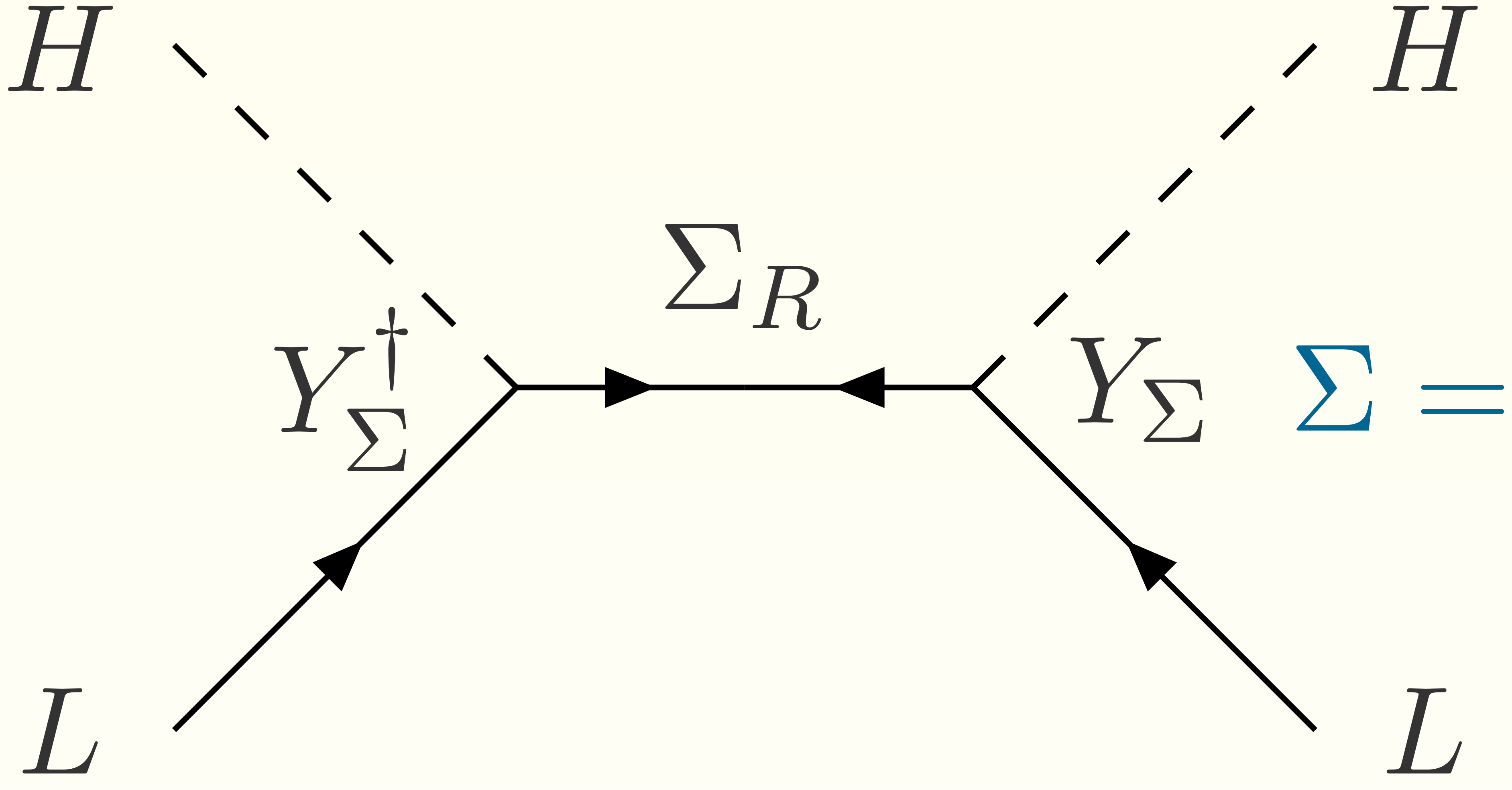
- Fits naturally in $\underline{16}$ of SO(10) GUTS
- Scale of $M_N \sim 10^{15}$ GeV very naturally
- At least 2 RH neutrino required

Type III Seesaw

- Introduce heavy fermion triplets $\Sigma_R \sim (1,3,0)$

$$-\mathcal{L}_Y = Y_\Sigma \bar{L}\tilde{H}\Sigma_R + \frac{1}{2}M_\Sigma Tr(\overline{\Sigma_R^c}\Sigma_R) + h.c.$$

Couples to the W boson => strong limits



$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

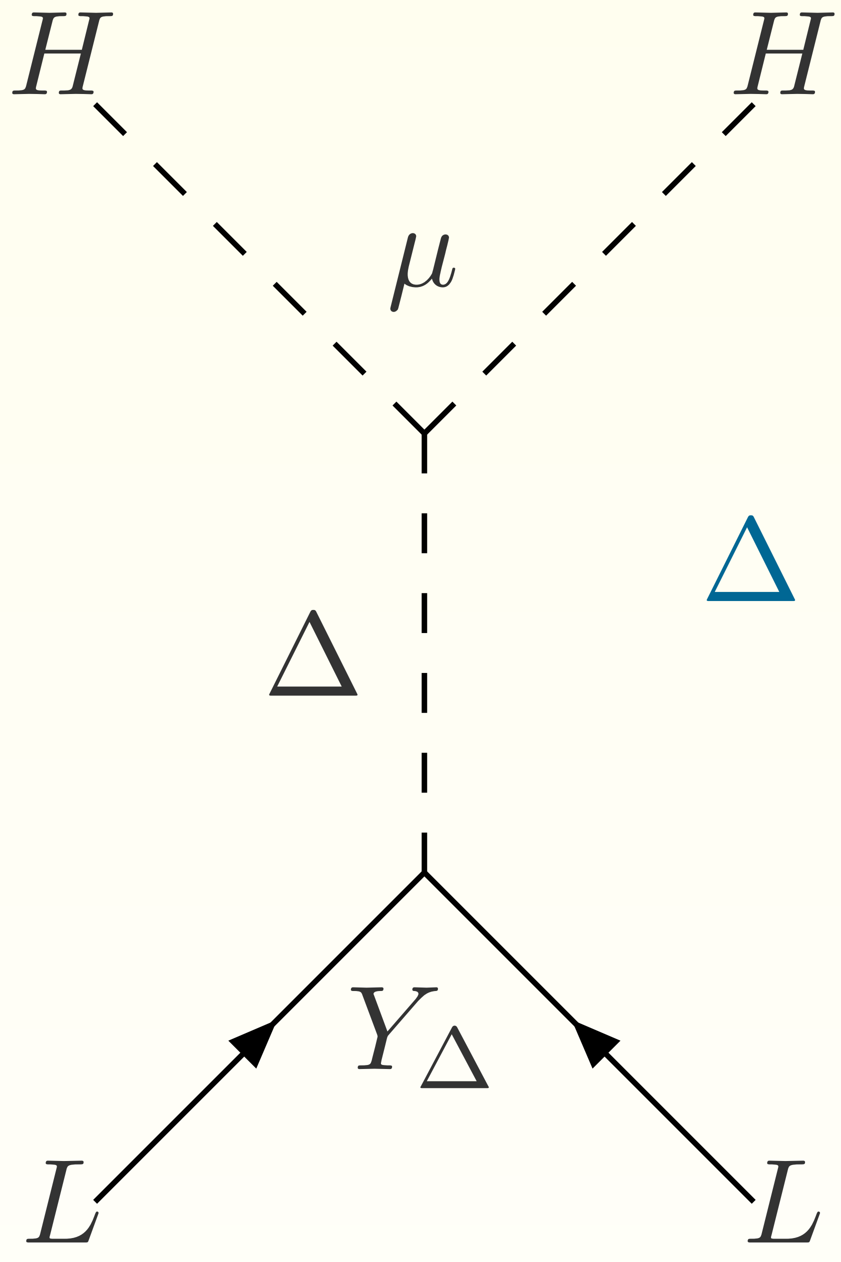
$$\boxed{m_\nu = -v^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T}$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

Type II Seesaw

- Introduce heavy scalar triplet $\Delta \sim (1,3,1)$

$$-\mathcal{L}_{II} = Y_\Delta L^T C^{-1} i\sigma_2 \Delta L + M_\Delta^2 Tr(\Delta^\dagger \Delta) + \mu H^T i\sigma_2 \Delta^\dagger H + ...$$



Couples to the W boson => strong limits

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

$$m_\nu = Y_\Delta \frac{\mu\, v^2}{M_\Delta^2}$$

Konetschny,Kummer '77, Chen,Li '80, Magg, Wetterich '80
Schecter,Valle '80, Lazarides,Shafi,Wetterich '81,
Mohapatra,Senjanovic '81

- Basically Seesaw of the Δ VEV v_Δ
- Fits naturally in L-R symmetric models

Neutrino Masses
+
Baryon Asymmetry
+
Dark Matter

Leptogenesis

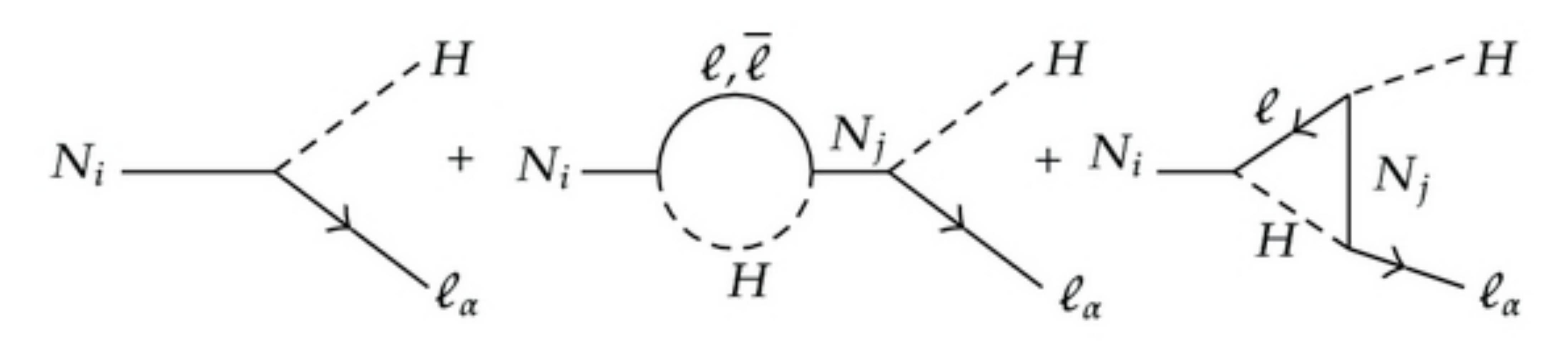
SEESAW MECHANISM → LEPTOGENESIS

Heavy RH neutrinos in Type I seesaw

- CP violating out of equilibrium decays of RH neutrinos

$$-\mathcal{L}_Y = Y_\nu \bar{L}\tilde{H}N_R + \frac{1}{2}M_N\overline{N^c_R}N_R + h.c.$$

$$N_i \xrightarrow{\Gamma} l\,H^\dagger \qquad N_i \xrightarrow{\bar{\Gamma}} \bar{l}\,H$$



CPV

CP asymmetry: $\epsilon \equiv -\dfrac{\Gamma-\bar{\Gamma}}{\Gamma+\bar{\Gamma}}$

Fukugita and Yanagida, 1986

- Observing CP violation in neutrino expts crucial

AND

- Majorana neutrinos crucial

CP asymmetry → *Lepton asymmetry* → *Baryon asymmetry*

RH Neutrinos as Dark Matter (an example)

Biswas, SC, Covi, Khan JCAP 1802 (2018)

new particles

Gauge	Baryon Fields			Lepton Fields			Scalar Fields		
Group	$Q_L^i = (u_L^i, d_L^i)^T$	u_R^i	d_R^i	$L_L^i = (\nu_L^i, e_L^i)^T$	e_R^i	$\boxed{N_R^i}$	ϕ_h	$\boxed{\phi_H}$	$\boxed{\eta}$
$\mathrm{SU(2)_L}$	2	1	1	2	1	1	2	1	2
$\mathrm{U(1)_Y}$	1/6	2/3	−1/3	−1/2	−1	0	1/2	0	1/2
$\mathbb{Z}_2$	+	+	+	+	+	−	+	+	−

inert doublet

Gauge	Baryonic Fields	Lepton Fields			Scalar Fields		
Group	(Q_L^i, u_R^i, d_R^i)	(L_L^e, e_R, N_R^e)	$(L_L^\mu, \mu_R, N_R^\mu)$	$(L_L^\tau, \tau_R, N_R^\tau)$	ϕ_h	ϕ_H	η
$\mathrm{U(1)}_{L_\mu - L_\tau}$	0	0	1	−1	0	1	0

Leads to mu-tau symmetry => maximal θ_{23} and zero θ_{13}

$$\begin{aligned}\mathcal{L} \;=\; & \mathcal{L}_{SM} + \mathcal{L}_N + (D_\mu \phi_H)^\dagger (D^\mu \phi_H) + (D_\mu \eta)^\dagger (D^\mu \eta) + \sum_{j=\mu,\,\tau} Q^j \,\bar{L_j} \gamma_\rho L_j Z^\rho_{\mu\tau} \\ & -\frac{1}{4} F_{\mu\tau\,\rho\sigma} F_{\mu\tau}{}^{\rho\sigma} - V(\phi_h, \phi_H, \eta)\,,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_N \;=\; & \sum_{i=e,\,\mu,\,\tau} \frac{i}{2} \bar{N_i} \gamma^\mu D_\mu N_i - \frac{1}{2}\, M_{ee}\, \bar{N^c_e} N_e - \frac{1}{2}\, M_{\mu\tau}\,(\bar{N^c_\mu} N_\tau + \bar{N^c_\tau} N_\mu) \\ & -\frac{1}{2}\, h_{e\mu} (\bar{N^c_e} N_\mu + \bar{N^c_\mu} N_e) \phi_H^\dagger - \frac{1}{2}\, h_{e\tau} (\bar{N^c_e} N_\tau + \bar{N^c_\tau} N_e) \phi_H \\ & - \sum_{\alpha=e,\,\mu,\,\tau} h_\alpha \bar{L_\alpha} \tilde{\eta} N_\alpha + h.c.\,,\end{aligned}$$

$$\begin{aligned}V(\phi_h, \phi_H, \eta) \;=\; & -\mu_H^2 \phi_H^\dagger \phi_H - \mu_h^2 \phi_h^\dagger \phi_h + \mu_\eta^2 \eta^\dagger \eta + \lambda_1 (\phi_h^\dagger \phi_h)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\phi_H^\dagger \phi_H)^2 \\ & + \lambda_{12} (\phi_h^\dagger \phi_h)(\eta^\dagger \eta) + \lambda_{13} (\phi_h^\dagger \phi_h)(\phi_H^\dagger \phi_H) + \lambda_{23} (\phi_H^\dagger \phi_H)(\eta^\dagger \eta) + \lambda_4 (\phi_h^\dagger \eta)(\eta^\dagger \phi_h) \\ & + \frac{1}{2} \lambda_5 \left((\phi_h^\dagger \eta)^2 + h.c. \right)\,.\end{aligned}$$

The RH Neutrino Masses

Lmu-Ltau Symmetric

$$\mathcal{M}_R = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & 0 & M_{\mu\tau}\, e^{i\xi} \\ 0 & M_{\mu\tau}\, e^{i\xi} & 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_N \;=\; & \sum_{i=e,\,\mu,\,\tau} \frac{i}{2}\bar{N}_i\gamma^\mu D_\mu N_i - \frac{1}{2}\,M_{ee}\,\bar{N}^c_e N_e - \frac{1}{2}\,M_{\mu\tau}\,(\bar{N}^c_\mu N_\tau + \bar{N}^c_\tau N_\mu) \\ & -\frac{1}{2}\,h_{e\mu}(\bar{N}^c_e N_\mu + \bar{N}^c_\mu N_e)\phi^\dagger_H - \frac{1}{2}\,h_{e\tau}(\bar{N}^c_e N_\tau + \bar{N}^c_\tau N_e)\phi_H \\ & - \sum_{\alpha=e,\,\mu,\,\tau} h_\alpha \bar{L_\alpha}\tilde{\eta}N_\alpha + h.c.\,, \end{aligned}$$

Dark Matter

$$\begin{aligned} M'_{2/3} \;&=\; \pm M_{\mu\tau}e^{i\xi} \\ M'_1 \;&=\; M_{ee}\,, \end{aligned}$$

N₂ and N₃ are exactly degenerate and serve as a two-component DM of the Universe

The RH Neutrino Masses

$$\begin{aligned}\mathcal{L}_N \;=\; & \sum_{i=e,\,\mu,\,\tau} \frac{i}{2}\bar{N}_i\gamma^\mu D_\mu N_i - \frac{1}{2}\,M_{ee}\,\bar{N}_e^c N_e - \frac{1}{2}\,M_{\mu\tau}\,(\bar{N}_\mu^c N_\tau + \bar{N}_\tau^c N_\mu) \\ & -\frac{1}{2}\,h_{e\mu}(\bar{N}_e^c N_\mu + \bar{N}_\mu^c N_e)\phi_H^\dagger - \frac{1}{2}\,h_{e\tau}(\bar{N}_e^c N_\tau + \bar{N}_\tau^c N_e)\phi_H \\ & - \sum_{\alpha=e,\,\mu,\,\tau} h_\alpha \bar{L_\alpha}\tilde{\eta} N_\alpha + h.c.\,,\end{aligned}$$

Lmu-Ltau Symmetric

$$\mathcal{M}_R = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & 0 & M_{\mu\tau}\,e^{i\xi} \\ 0 & M_{\mu\tau}\,e^{i\xi} & 0 \end{pmatrix}$$

Dark Matter ↓

$$\begin{aligned} M'_{2/3} \;&=\; \pm M_{\mu\tau}e^{i\xi} \\ M'_1 \;&=\; M_{ee}\,, \end{aligned}$$

$\theta_{23} = \pi/4$

$\theta_{13} = 0$

Lmu-Ltau Broken spontaneously

$$\mathcal{M}_R = \begin{pmatrix} M_{ee} & \frac{v_{\mu\tau}}{\sqrt{2}}h_{e\mu} & \frac{v_{\mu\tau}}{\sqrt{2}}h_{e\tau} \\ \frac{v_{\mu\tau}}{\sqrt{2}}h_{e\mu} & 0 & M_{\mu\tau}\,e^{i\xi} \\ \frac{v_{\mu\tau}}{\sqrt{2}}h_{e\tau} & M_{\mu\tau}\,e^{i\xi} & 0 \end{pmatrix}$$

The mass splitting between them is given at first order for $M_{ee} \gg M_{\mu\tau}$ by

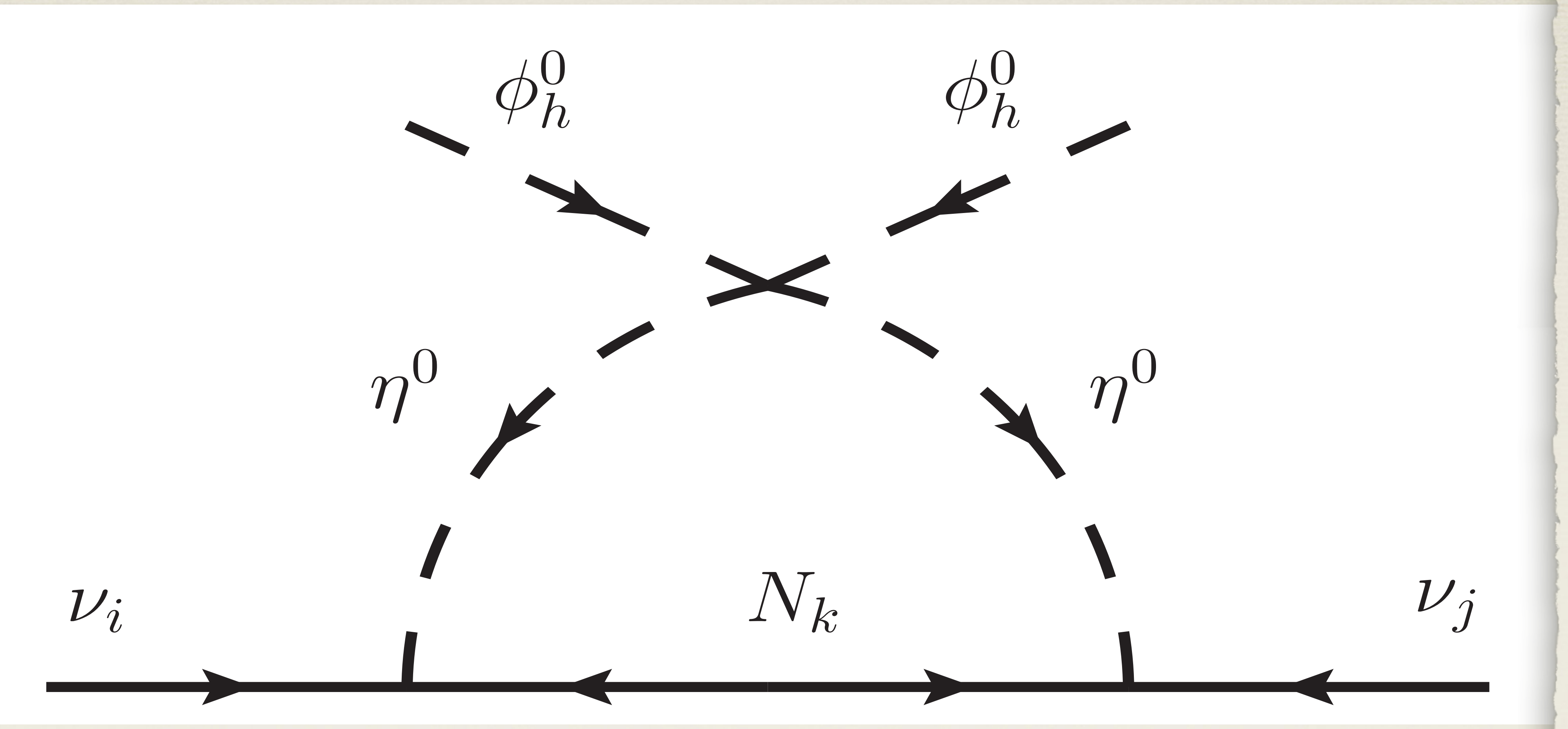
$$\Delta M_{23} = \frac{(h_{e\mu} + h_{e\tau})^2 v_{\mu\tau}^2}{2M_{ee}}$$

$\theta_{23} \neq \pi/4$

$\theta_{13} \neq 0$

The Light Neutrino Mass

Type-I seesaw is forbidden by the Z2 symmetry



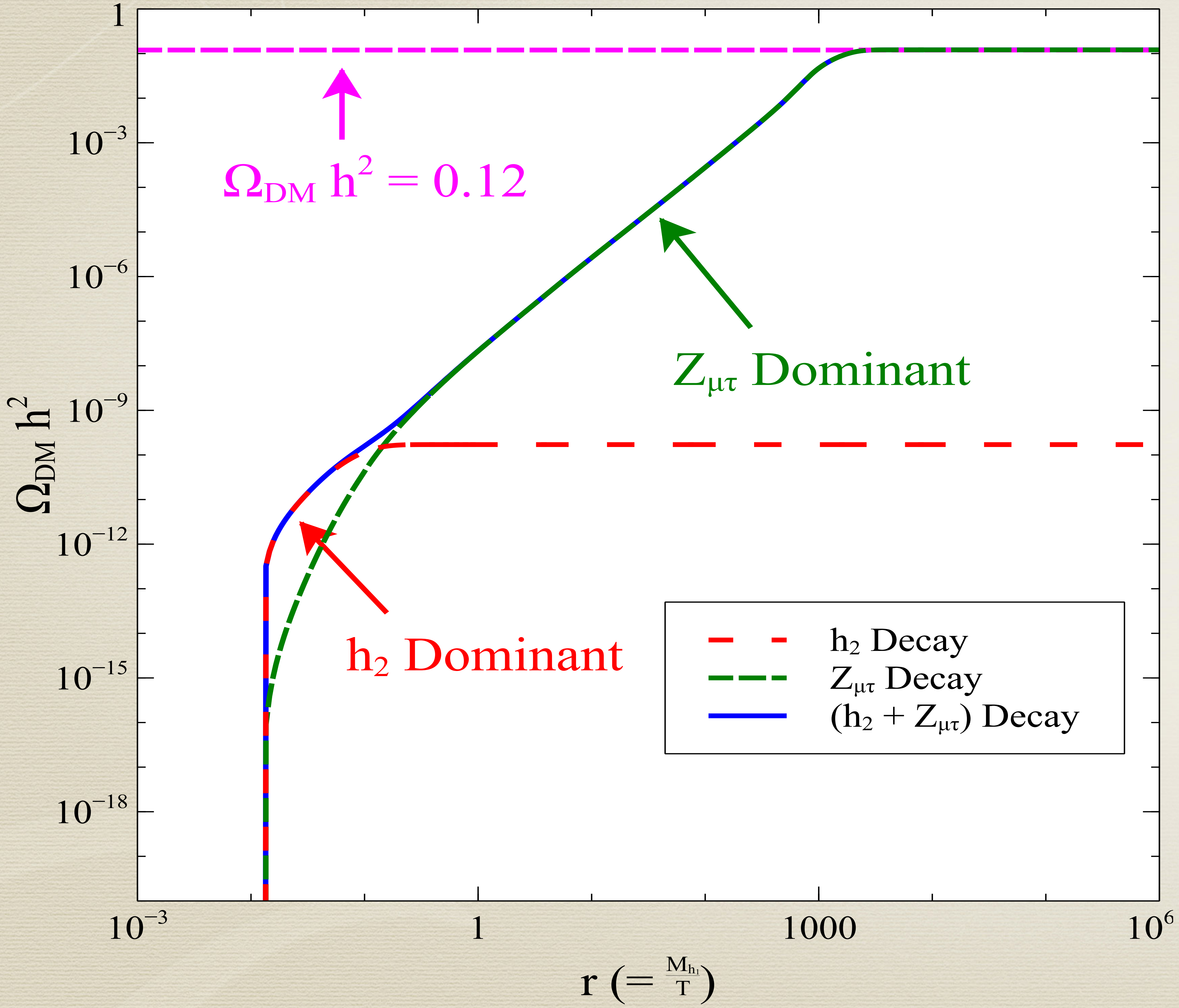
$$M^{\nu}_{ij} = \sum_k \frac{y_{ik}\, y_{jk}\, M_k}{16\,\pi^2} \left[\frac{M^2_{\eta^0_R}}{M^2_{\eta^0_R} - M_k^2} \ln \frac{M^2_{\eta^0_R}}{M_k^2} - \frac{M^2_{\eta^0_I}}{M^2_{\eta^0_I} - M_k^2} \ln \frac{M^2_{\eta^0_I}}{M_k^2} \right]$$

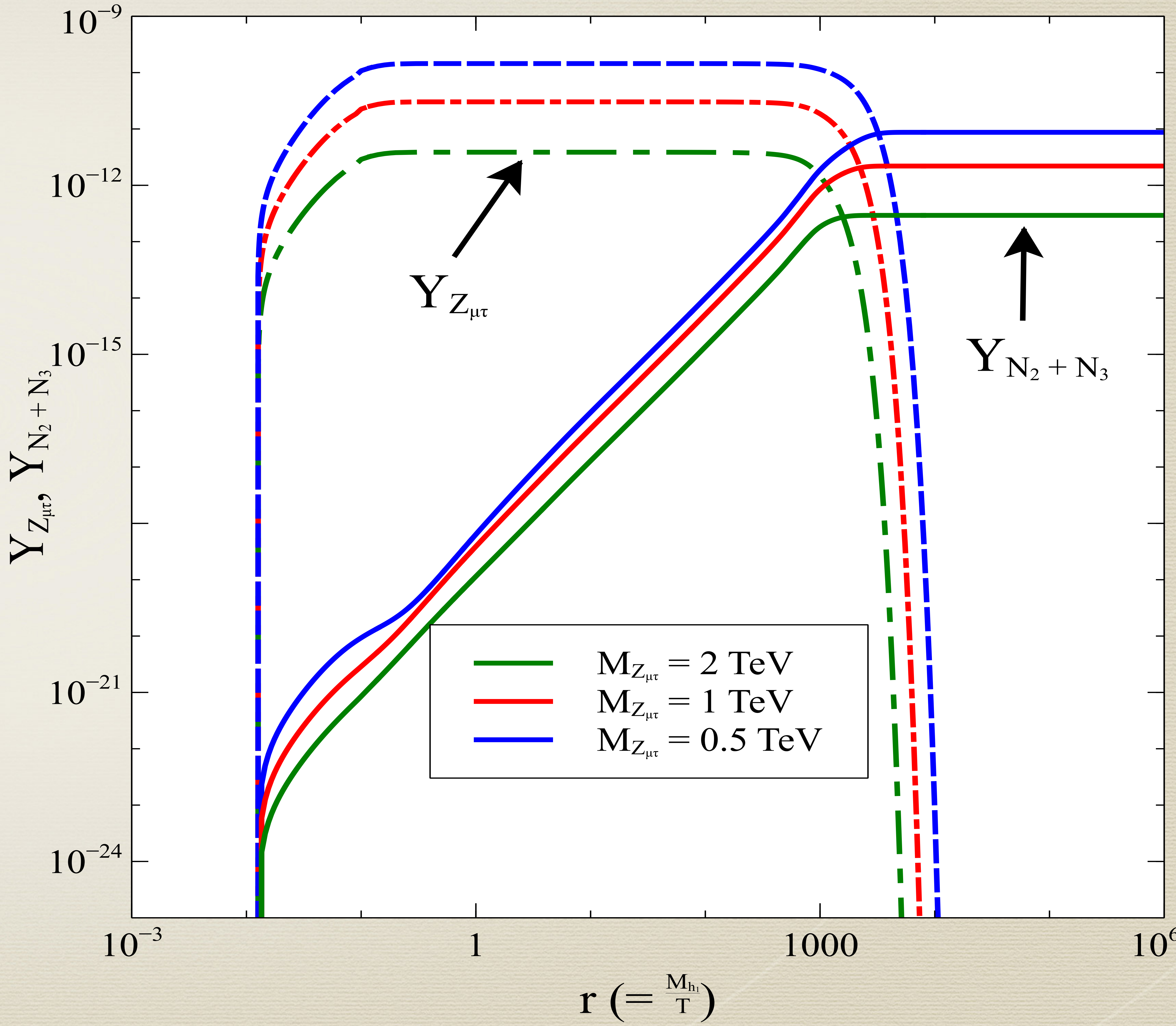
Mixing matrix of the RH neutrinos

$$y_{ji} \;=\; h_j U_{ji}$$

$M_{\eta^0_{R,I}} \;\sim\; 10^6\;\; \mathrm{GeV}$ $\lambda_5 \sim 10^{-3}$ $y^2_{ji} \sim 10^{-1}$ → $M_\nu \sim 10^{-11}\;\mathrm{GeV}$

Creating N_2/N_3 by Freeze-in





Common Origin of Neutrino Masses, Dark Matter and Baryon Asymmetry

(An example case)

Gauge Group	Fermion Fields							Scalar Fields		
	$\Psi_{1L}=(\psi_1,\psi_2)_L^T$	ψ_{1R}	ψ_{2R}	$\Psi_{2L}=(\psi_3,\psi_4)_L^T$	ψ_{3R}	ψ_{4R}	N_i	ϕ_h	ϕ_D	η_D
$\mathrm{SU}(3)_c$	1	1	1	1	1	1	1	1	1	1
$\mathrm{SU}(2)_L$	1	1	1	1	1	1	1	2	1	1
$\mathrm{SU}(2)_D$	2	1	1	2	1	1	1	1	2	2
$\mathbb{Z}_3\times\mathbb{Z}_2$	$(\omega,1)$	$(\omega,1)$	$(\omega,1)$	$(\omega^2,-1)$	$(\omega^2,-1)$	$(\omega^2,-1)$	$(1,1)$	$(1,1)$	$(1,1)$	$(\omega,1)$

Biswas, SC, Covi, Khan JHEP 05 (2019)

Falkowski,Ruderman,Volansky JHEP (2011)

$$
\begin{aligned}
\mathcal{L} \;=\; & \mathcal{L}_{SM} + i\,\overline{\Psi_k}\gamma^{\mu} D^{k}_{\mu}\Psi_k + (D^{D}_{\mu}\phi_D)^{\dagger}(D^{D\mu}\phi_D) + (D^{D}_{\mu}\eta_D)^{\dagger}(D^{D\mu}\eta_D) + \left(y_{ij}\bar{L}_i\tilde{\phi}_h N_{jR} + h.c.\right) \\
& + \left(\lambda_1\overline{\Psi_{1L}}\,\tilde{\phi}_D\,\psi_{1R} + \lambda_2\overline{\Psi_{1L}}\,\phi_D\,\psi_{2R} + \lambda_3\overline{\Psi_{2L}}\,\tilde{\phi}_D\,\psi_{3R} + \lambda_4\overline{\Psi_{2L}}\,\phi_D\,\psi_{4R} + h.c.\right) \\
& - \alpha_j\overline{\Psi_{1L}}\eta_D N_{jR} + i\,\overline{N_{jR}}\,\not{\partial}\,N_{jR} - M_j\overline{N^{c}_{jR}}N_{jR} - \mathcal{V}(\phi_h, \phi_D, \eta_D)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}(\phi_h, \phi_D, \eta_D) \;=\; & -\mu_h^2(\phi_h^{\dagger}\phi_h) + \lambda_h(\phi_h^{\dagger}\phi_h)^2 - \mu_D^2(\phi_D^{\dagger}\phi_D) + \lambda_D(\phi_D^{\dagger}\phi_D)^2 \\
& + \mu_\eta^2(\eta_D^{\dagger}\eta_D) + \lambda_\eta(\eta_D^{\dagger}\eta_D)^2 + \lambda_{hD}(\phi_h^{\dagger}\phi_h)(\phi_D^{\dagger}\phi_D) + \lambda_{h\eta}(\phi_h^{\dagger}\phi_h)(\eta_D^{\dagger}\eta_D) \\
& + \lambda_{D1}(\phi_D^{\dagger}\phi_D)(\eta_D^{\dagger}\eta_D) + \lambda_{D2}(\phi_D^{\dagger}\eta_D)(\eta_D^{\dagger}\phi_D) + \lambda_{D3}(\phi_D\eta_D^3 + h.c.)\,.
\end{aligned}
$$

$$
\phi_h = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}, \quad \phi_D = \begin{pmatrix} 0 \\ \frac{v_D+H}{\sqrt{2}} \end{pmatrix}
$$

$$
\begin{aligned}
\mathcal{L} \;=\; & \mathcal{L}_{SM} + i\,\overline{\Psi_k}\gamma^\mu D^k_\mu \Psi_k + (D^D_\mu \phi_D)^\dagger (D^{D\mu}\phi_D) + (D^D_\mu \eta_D)^\dagger (D^{D\mu}\eta_D) + \left(y_{ij}\bar{L}_i\tilde{\phi}_h N_{j\,R} + h.c.\right) \\
& + \left(\lambda_1 \overline{\Psi_{1L}}\, \tilde{\phi_D}\, \psi_{1\,R} + \lambda_2 \overline{\Psi_{1L}}\, \phi_D\, \psi_{2\,R} + \lambda_3 \overline{\Psi_{2L}}\, \tilde{\phi_D}\, \psi_{3\,R} + \lambda_4 \overline{\Psi_{2L}}\, \phi_D\, \psi_{4\,R} + h.c.\right) \\
& - \alpha_j \overline{\Psi_{1L}} \eta_D N_{j\,R} + i\,\overline{N_{j\,R}}\, \partial\!\!\!/\, N_{j\,R} - M_j \overline{N^c_{j\,R}} N_{j\,R} - \mathcal{V}(\phi_h, \phi_D, \eta_D)\,,
\end{aligned}
$$

Neutrino Mass

Type-I Seesaw

$$m_\nu = -M_D M_R^{-1} M_D^T \qquad M_D = \frac{y_{i\,j}\, v}{\sqrt{2}}$$

$$\begin{aligned}\mathcal{L} \;=\;& \mathcal{L}_{SM} + i\,\overline{\Psi_k}\gamma^\mu D^k_\mu\,\Psi_k + (D^D_\mu \phi_D)^\dagger (D^{D\mu}\phi_D) + (D^D_\mu \eta_D)^\dagger (D^{D\mu}\eta_D) + \left(y_{ij}\bar{L}_i\tilde{\phi}_h N_{j\,R} + h.c.\right) \\ & + \left(\lambda_1 \overline{\Psi_{1L}}\,\tilde{\phi_D}\,\psi_{1\,R} + \lambda_2\overline{\Psi_{1L}}\,\phi_D\,\psi_{2\,R} + \lambda_3\overline{\Psi_{2L}}\,\tilde{\phi_D}\,\psi_{3\,R} + \lambda_4\overline{\Psi_{2L}}\,\phi_D\,\psi_{4\,R} + h.c.\right) \\ & -\alpha_j\overline{\Psi_{1L}}\eta_D N_{j\,R} + i\,\overline{N_{j\,R}}\,\partial\!\!\!/\,N_{j\,R} - M_j\overline{N^c_{j\,R}}N_{j\,R} - \mathcal{V}(\phi_h,\phi_D,\eta_D)\,,\end{aligned}$$

Neutrino Mass

Type-I Seesaw

$$m_\nu = -M_D M_R^{-1} M_D^T \qquad M_D = \frac{y_{i\,j}\,v}{\sqrt{2}}$$

Leptogenesis

CP violating out-of-equilibrium decay of N1 results in a lepton asymmetry

$$\begin{aligned}\mathcal{L} \; = \; & \mathcal{L}_{SM} + i\,\overline{\Psi_k}\gamma^\mu D^k_\mu\,\Psi_k + (D^D_\mu \phi_D)^\dagger (D^{D\mu}\phi_D) + (D^D_\mu \eta_D)^\dagger (D^{D\mu}\eta_D) + \left(y_{ij}\bar{L}_i\tilde{\phi}_h N_{j\,R} + h.c.\right) \\ & + \left(\lambda_1\overline{\Psi_{1L}}\,\tilde{\phi_D}\,\psi_{1\,R} + \lambda_2\overline{\Psi_{1L}}\,\phi_D\,\psi_{2\,R} + \lambda_3\overline{\Psi_{2L}}\,\tilde{\phi_D}\,\psi_{3\,R} + \lambda_4\overline{\Psi_{2L}}\,\phi_D\,\psi_{4\,R} + h.c.\right) \\ & -\alpha_j\overline{\Psi_{1L}}\eta_D N_{j\,R} + i\,\overline{N_{j\,R}}\,\partial\!\!\!/\,N_{j\,R} - M_j\overline{N^c_{j\,R}}N_{j\,R} - \mathcal{V}(\phi_h,\phi_D,\eta_D)\,,\end{aligned}$$

Neutrino Mass

Type-I Seesaw

$$m_\nu = -M_D M_R^{-1} M_D^T \qquad M_D = \frac{y_{i\,j}\,v}{\sqrt{2}}$$

Leptogenesis

CP violating out-of-equilibrium decay of N1 results in a lepton asymmetry

Dark Matter (Asymmetric)

Decay of N1 results in producing the dark matter

Nu Mass, Lepton asymmetry, DM density are all related via the Yukawa coupling

$$m_\nu = -M_D M_R^{-1} M_D^T \qquad M_D = \frac{y_{i\,j}\, v}{\sqrt{2}}$$

Neutrino Masses

$$\epsilon_l = \frac{\Gamma(N_1 \to L\phi_h) - \Gamma(N_1 \to \bar{L}\phi_h^\dagger)}{\Gamma_{N_1}},$$

Leptogenesis

$$= \frac{M_{N_1}}{16\pi M_{N_2}} \frac{\mathrm{Im}\left[3\left((y^\dagger y)^\star_{12}\right)^2 + 2\alpha_1^\star \alpha_2 (y^\dagger y)^\star_{12}\right]}{\left[(y^\dagger y)_{11} + \alpha_1 \alpha_1^\star\right]}$$

$$\epsilon_D = \frac{\Gamma(N_1 \to \psi_{1L}\,\eta_D) - \Gamma(N_1 \to \overline{\psi_{1L}}\,\eta_D^\dagger)}{\Gamma_{N_1}},$$

Dark Matter

$$= \frac{M_{N_1}}{16\pi M_{N_2}} \frac{\mathrm{Im}\left[2\alpha_1^\star \alpha_2 (y^\dagger y)^\star_{12} + 3(\alpha_1^\star \alpha_2)^2\right]}{\left[(y^\dagger y)_{11} + \alpha_1 \alpha_1^\star\right]}$$

Conclusions

- Neutrino masses and mixing, dark matter and baryon asymmetry are all observational evidences for physics beyond the standard model
- Neutrino oscillation parameters are expected to be well determined in the next generation LBL, reactor and atmospheric neutrino experiments
- CP violation is of particular interest
- Tiny neutrino masses and peculiar mixing indicate new physics and new symmetries
- One needs a common theoretical framework to explain all the above mentioned observational evidences of BSM

Thank You!