Theoretical frameworks for dark matter and the dark sector

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Overview

- The main purpose of this seminar is to review a selection of “theoretical frameworks” that are currently used in the analysis of DM search experiments
- I will divide these frameworks into two classes.

1. Extending the Standard Model (SM) matter/Higgs sector “only”:
   - Extended matter/Higgs sector ✓
   - Extended gauge sector ✗

2. Extending the SM matter/Higgs sector as well as the gauge sector:
   - Extended matter/Higgs sector ✓
   - Extended gauge sector ✓

- I will make examples of the interdisciplinary input the use of each framework requires

Outline

- Introduction: evidence for DM (overview)

- Extending the SM matter/Higgs sector only
  - WIMPs
  - Non relativistic effective theories

- Extending the SM matter/Higgs sector as well as the SM gauge sector
  - Scalar and fermionic DM coupled to a “dark photon”
  - Spin-1 DM models

- Summary and conclusion

Evidence for DM / Overview

- How do we know that DM actually exists? The evidence for DM is based on the gravitational pull it exerts on stars, galaxies and light from luminous sources

- Specifically, the evidence for DM is based on:
  - Velocity dispersion of **stars**
  - Rotation curve of spiral **galaxies**
  - Velocity dispersion of galaxies in **galaxy clusters**
  - Bending of light from distant luminous sources (lensing)
  - **Cosmological structure** formation
  - Cosmic Microwave Background (**CMB**) anisotropies

Small scales

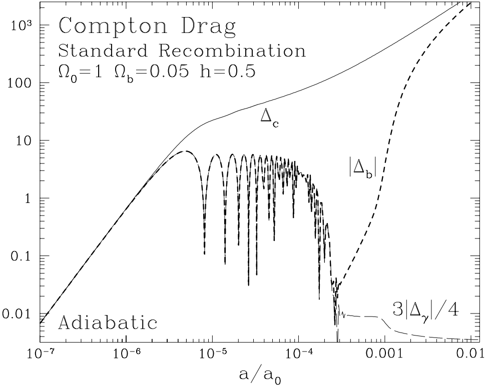
⋮

Large scales

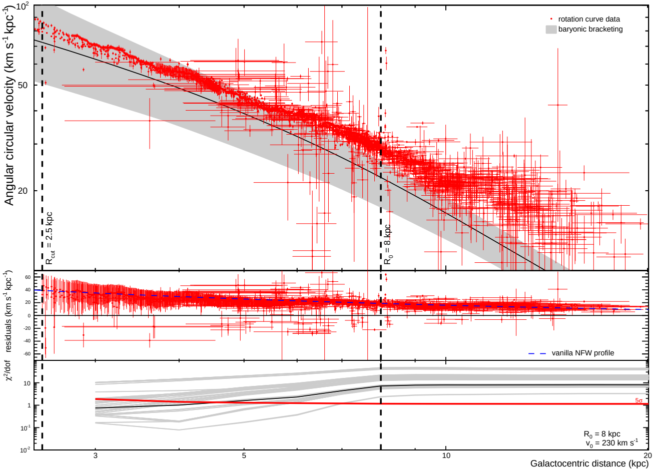
Evidence for DM / Large scale structures

Wayne T. Hu PhD thesis, "*Wandering in the Background: A CMB Explorer*";

astro-ph/9508126



Evidence for DM in the Milky Way



F. Iocco, M. Pato and G. Bertone, "Evidence for dark matter in the inner Milky Way," Nature Phys. **11**, 245 (2015)

Extending the SM matter/Higgs sector only

General considerations

- The extended framework should be capable of reproducing the present DM abundance, Y

- When the dominant number changing process is DM pair annihilation into SM particles

$$z\frac{H}{s}\frac{\mathrm{d}Y}{\mathrm{d}z}=\langle\sigma_{\mathrm{ann}}v\rangle\left(Y_{\mathrm{eq}}^2-Y^2\right)$$

- Freeze-out mechanism (i.e. *thermal production*): interplay of the two terms in the RHS
- Freeze-in (i.e. *non-thermal production*): second term not present in the RHS

- For weak-scale DM interactions, the freeze-out mechanism implies $m_\chi > \mathcal{O}(1)$ GeV

General considerations

- It should also agree with observations, e.g. evade DM direct detection constraints

- Inspection of the expected rate of, e.g., DM-induced nuclear recoils or electronic transitions in direct detection experiments shows different ways to do so:

$$\mathrm{d}\mathscr{R} = \frac{\rho_\chi}{m_\chi}\int_{|\boldsymbol{v}|>v_{\mathrm{min}}} \mathrm{d}\boldsymbol{v}\,|\boldsymbol{v}| f_\chi(\boldsymbol{v}+\boldsymbol{v}_\oplus)\,\mathrm{d}\sigma$$

1) m_χ **is too small. No particles with velocity above** v_{min}
2) **Inelastic scattering**

3) **Matrix element is momentum and/or velocity suppressed**

SUSY WIMPs

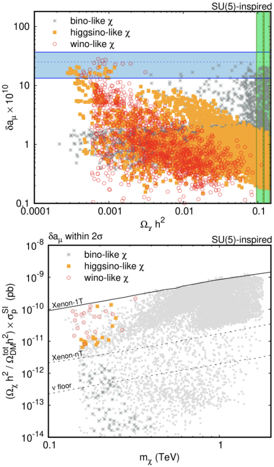
- Prime candidate: neutralino. Linear combination of neutral components of SU(2) singlet (bino), doublets (higgsinos) and triplet (wino)

- WIMP-quark interactions are given by

$$\mathscr{L}_{\rm SI} = f_q \bar{\chi}\chi\bar{q}q + g_q \bar{\chi}\gamma^\mu\partial^\nu\chi(\bar{q}\gamma_\mu\partial_\nu q - \partial_\nu\bar{q}\gamma_\mu q)$$

$$\mathscr{L}_{\rm SD} = f_q\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu(c_q + d_q\gamma_5)q$$

- Effective couplings non-trivially depend on SUSY parameters: enhancements and cancellations are possible



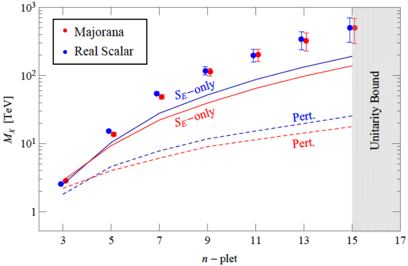
M. Chakraborti, L. Roszkowski, S. Trojanowski, "GUT-constrained supersymmetry and dark matter in light of the new $(g-2)_\mu$ determination," JHEP **05** (2021), 252

Minimal DM

- The framework relies on a systematic classification and characterisation of all SU(2) representations, without a restriction to SUSY-induced combinations

- An odd representation of dimension n (n-uplet) and zero hypercharge (Y=0) includes a neutral component: $Q = Y + T_3$, $T_3 = \mathrm{diag}\,[(n+1)/2 - i]$, $i = 1, \ldots, n$

- Only two free parameters, n and M_χ:



S. Bottaro *et al.* "Closing the window on WIMP Dark Matter", Eur. Phys. J. C **82** (2022) no.1, 31

Non-relativistic effective theories

- In general, bounds from DM-induced nuclear recoil searches can be evaded if the amplitude for non-relativistic DM-nucleon scattering, $\mathscr{M}_{\chi N}$, is "suppressed"

- It is often *assumed* to be a function of the momentum transfer only, i.e. $\mathscr{M}_{\chi N} = \mathscr{M}_{\chi N}(\boldsymbol{q})$

- Is this the most general form $\mathscr{M}_{\chi N}$ can have?

- More pragmatically, is this the "typical" form for $\mathscr{M}_{\chi N}$?

Non-relativistic effective theories

- In the non-relativistic limit, $\mathscr{M}_{\chi N}$ is constrained by Galilean invariance and momentum conservation. Consequently it can at most depend on 2 3D momenta:

  $$\mathscr{M}_{\chi N} = \mathscr{M}_{\chi N}(\boldsymbol{q}, \boldsymbol{v}^{\perp})$$

  where $\boldsymbol{v}^{\perp} = \boldsymbol{v} + \boldsymbol{q}/(2\mu_{\chi N})$

- Furthermore, in the non-relativistic limit $|\boldsymbol{q}|/m_N \ll 1$ and $|\boldsymbol{v}| \ll 1$, which implies

  $$\mathscr{M}_{\chi N} = \sum_{i<\infty} c_i^N \langle \mathscr{O}_i \rangle$$

  where $\mathscr{O}_i$ are quantum mechanical operators: $\mathscr{O}_4 = \mathbf{S}_\chi \cdot \mathbf{S}_N$, $\mathscr{O}_7 = \mathbf{S}_\chi \cdot \boldsymbol{v}^{\perp}$, etc...

J. Fan, M. Reece and L. T. Wang, "Non-relativistic effective theory of dark matter direct detection," JCAP **11** (2010), 042
A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers and Y. Xu, "The Effective Field Theory of Dark Matter Direct Detection," JCAP **02** (2013), 004

Spin-independent and -dependent interactions

- For spin-independent interactions, $\mathscr{M}_{\chi N} = c_1^N \langle \mathbb{1}_{\chi N} \rangle$

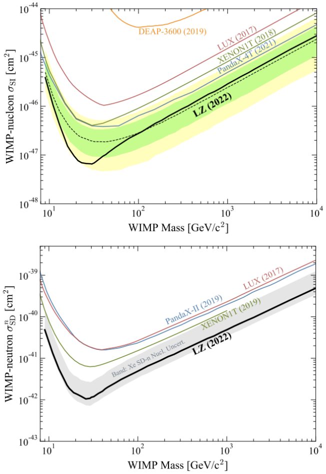
  Experimental results presented in terms of,

$$\sigma_{\mathrm{N}}^{SI} = \frac{\mu_{\chi N}^2}{\pi}(c_1^N)^2$$

- For spin-dependent interactions, $\mathscr{M}_{\chi N} = c_4^N \langle \mathbf{S}_N \cdot \mathbf{S}_\chi \rangle$

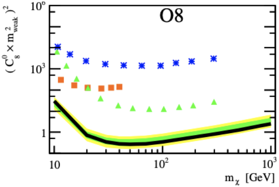
  Experimental results presented in terms of,

$$\sigma_N^{\mathrm{SD}} = \frac{\mu_{\chi N}^2 j_\chi (j_\chi + 1)}{4\pi}(c_4^N)^2$$

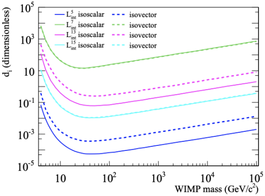


J. Aalbers *et al.*, "First Dark Matter Search Results from the LUX-ZEPLIN (LZ) Experiment," arXiv:2207.03764

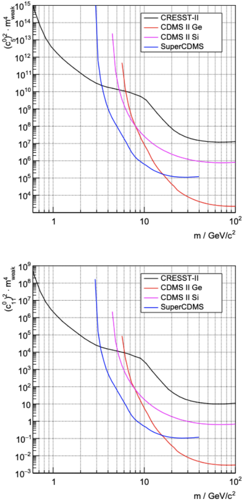
Non-standard effective operators



E. Aprile *et al.*, "Effective field theory search for high-energy nuclear recoils using the XENON100 dark matter detector," Phys. Rev. D **96** (2017) no.4, 042004



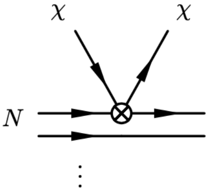
J. Xia *et al.*, "PandaX-II Constraints on Spin-Dependent WIMP-Nucleon Effective Interactions," Phys. Lett. B **792** (2019), 193-198

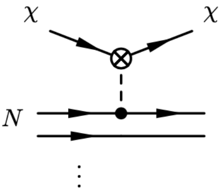


G. Angloher *et al.*, "Limits on Dark Matter Effective Field Theory Parameters with CRESST-II," Eur. Phys. J. C **79** (2019) no.1, 43

Including mesons: Chiral Effective Field Theory

- Non-relativistic effective theories do not account for meson exchange effects in DM-nucleus scattering:





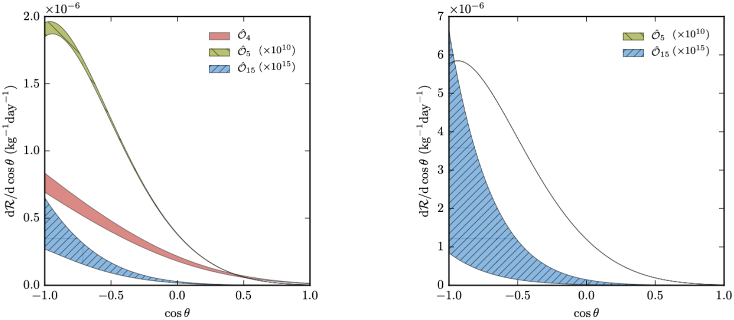
- Meson exchange effects induce a $\boldsymbol{q}$-dependence in the c_i^N coupling constants,

$$c_i^N = c_i^N(\boldsymbol{q})$$

- This $\boldsymbol{q}$-dependence can be computed in Chiral Effective Field Theory

  F. Bishara, J. Brod, B. Grinstein and J. Zupan, "Chiral Effective Theory of Dark Matter Direct Detection," JCAP **02** (2017), 009

Uncertainty quantification in Chiral Effective Field Theory



D. Gazda, R. Catena and C. Forssén, "Ab initio nuclear response functions for dark matter searches," Phys. Rev. D **95** (2017) no.10, 103011

Extending the SM matter/Higgs sector
and
the SM gauge sector

General considerations

- The lack of WIMP discovery motivated the exploration of "alternative frameworks"
- Exploration driven by "detectability" and the question:"why did DM escape detection?"
- Possible answer: DM has a sub-GeV mass and is too light to induce a nuclear recoil

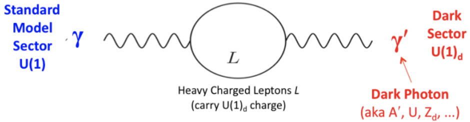
- Insisting on the thermal production mechanism, naturally leads to the idea of dark sector:
  - For $m_{\mathrm{DM}} < \mathcal{O}(1)$ GeV, WIMPs are overproduced by the freeze-out mechanism
  - Introducing a new, "light" mediator reconciles thermal production with CMB data

- Below, by "dark sector" I will denote a model with sub-GeV DM and a light gauge mediator

Dark sectors including a “dark photon”

- Perhaps the simplest way to add a new mediator to the SM is by a $\mathrm{U}(1)_d$ gauge group

- A “kinetic mixing” between the $\mathrm{U}(1)_d$ gauge boson, i.e. the “dark photon”, and the ordinary photon is compatible with gauge symmetries. It arises via loops



- The “dark sector” Lagrangian is given by

$$\mathscr{L}_{\mathrm{DS}} = -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} m^2_{A'} A'_{\mu} A'^{\mu} + \frac{1}{2} \varepsilon F_{\mu\nu} F'^{\mu\nu} + \mathscr{L}_{\mathrm{DM}}$$

- $\mathscr{L}_{\mathrm{DM}}$ is the DM Lagrangian
- The model’s parameter are $m_{A'}$, m_{DM}, ε, and the new gauge coupling g_d

Dark sectors with scalar and fermionic DM

- Standard benchmarks for $\mathscr{L}_{\mathrm{DM}}$ when DM has spin 0 or spin 1/2 (thermal targets):

- Majorana DM:

$$\mathscr{L}_{\mathrm{DM}} = \mathscr{L}_{\mathrm{kin}} - \frac{1}{2} g_d A'_\mu \, \bar{\chi} \gamma^\mu \gamma^5 \chi$$

- Pseudo-Dirac DM:

$$\mathscr{L}_{\mathrm{DM}} = \mathscr{L}_{\mathrm{kin}} - g_d A'_\mu \, i \left(\bar{\chi}_1 \gamma^\mu \chi_2 - \bar{\chi}_2 \gamma^\mu \chi_1 \right)$$

- Scalar Elastic DM:

$$\mathscr{L}_{\mathrm{DM}} = \mathscr{L}_{\mathrm{kin}} - g_d A'_\mu \, i \left(\chi^* \partial^\mu \chi - \chi \partial^\mu \chi^* \right)$$

- Scalar Inelastic DM:

$$\mathscr{L}_{\mathrm{DM}} = \mathscr{L}_{\mathrm{kin}} - g_d A'_\mu \, i \left(\chi_1^* \partial^\mu \chi_2 - \chi_2^* \partial^\mu \chi_1 \right)$$

Dark sectors with spin-1 DM

- Proposed benchmarks for $\mathscr{L}_{\rm DM}$ when DM has spin 1 (thermal targets):
  - b_5 coupling:

$$\mathscr{L}_{\rm DM} = \mathscr{L}_{\rm kin} - i b_5 A'^{\mu} \left(X^{\dagger}_{\nu} \partial_{\mu} X^{\nu} - (\partial_{\mu} X^{\dagger\nu}) X_{\nu} \right)$$

  - $\Re(b_6)$ coupling:

$$\mathscr{L}_{\rm DM} = \mathscr{L}_{\rm kin} - \Re(b_6) A'^{\mu}\, \partial_{\nu} \left(X^{\dagger\nu} X_{\mu} + X^{\dagger}_{\mu} X^{\nu} \right)$$

  - $\Im(b_6)$ coupling:

$$\mathscr{L}_{\rm DM} = \mathscr{L}_{\rm kin} - i\Im(b_6) A'^{\mu}\, \partial_{\nu} \left(X^{\dagger\nu} X_{\mu} - X^{\dagger}_{\mu} X^{\nu} \right)$$

  - $\Re(b_7)$ coupling:

$$\mathscr{L}_{\rm DM} = \mathscr{L}_{\rm kin} - \Re(b_7) A'^{\sigma}\, \epsilon_{\mu\nu\rho\sigma} \left(X^{\dagger\mu} \partial^{\nu} X^{\rho} + X^{\mu} \partial^{\nu} X^{\dagger\rho} \right)$$

  - $\Im(b_7)$ coupling:

$$\mathscr{L}_{\rm DM} = \mathscr{L}_{\rm kin} - i\Im(b_7) A'^{\sigma}\, \epsilon_{\mu\nu\rho\sigma} \left(X^{\dagger\mu} \partial^{\nu} X^{\rho} - X^{\mu} \partial^{\nu} X^{\dagger\rho} \right)$$

Non-relativistic vs relativistic tests of dark sectors

- Comparing these models with observations, one is interested in two search strategies:
- Non-relativistic tests, i.e. DM direct detection via *electronic transitions*
- Relativistic tests, i.e. beam dump and fixed target experiments

- In the non-relativistic case, I use an effective theory framework for sub-GeV DM
- combined with solid state physics methods, e.g. density functional theory (DFT)
- The above models for spin-0, -1/2 and -1 DM can be matched on this framework

- In the relativistic case, I present an overview of constraints on DM thermal targets
- focusing on spin-0 and spin-1/2 (standard), and spin-1 DM (new) separately

Non-relativistic DM-electron scattering in detector materials

- In any theoretical framework for Sub-GeV DM-electron scattering, the amplitude for non-relativistic DM scattering by free electrons, $\mathscr{M}_{\chi e}$, plays a key role
- It is often *assumed* to be a function of the momentum transfer only, i.e. $\mathscr{M}_{\chi e} = \mathscr{M}_{\chi e}(\boldsymbol{q})$

- From $\mathscr{M}_{\chi e}$, one calculates the rate of DM-induced electronic transitions in detectors

$$\mathrm{d}\mathscr{R} = \frac{\rho_\chi}{m_\chi} \int \mathrm{d}\boldsymbol{v}\, |\boldsymbol{v}| f(\boldsymbol{v} + \boldsymbol{v}_\oplus)\, \mathrm{d}\sigma_{1\rightarrow 2}$$

where

$$\mathrm{d}\sigma_{1\rightarrow 2} = \frac{1}{16 m_\chi^2 m_e^2 |\boldsymbol{v}|} \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} \Big| \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3} \psi_2^*(\boldsymbol{k}+\boldsymbol{q}) \mathscr{M}_{\chi e}(\dots) \psi_1(\boldsymbol{k}) \Big|^2 \delta(E_2 - E_1)$$

Non-relativistic effective theories reloaded

- In the non-relativistic limit, i.e. $|\boldsymbol{q}|/m_e \ll 1$ and $|\boldsymbol{v}| \ll 1$, $\mathscr{M}_{\chi e}$ admits an effective theory expansion (analogous to the one in the case of nuclear recoils)

$$\mathscr{M}_{\chi e} = \sum_{i<\infty} c_i \langle \mathscr{O}_i \rangle$$

where $\mathscr{O}_i$ are quantum mechanical operators: $\mathscr{O}_4 = \mathbf{S}_\chi \cdot \mathbf{S}_N$, $\mathscr{O}_7 = \mathbf{S}_\chi \cdot \boldsymbol{v}^\perp$, etc...

- The rate of DM-induced electron transitions in materials can now be written as)

$$\frac{\mathrm{d}\mathscr{R}}{\mathrm{d}\ln \Delta E} = \frac{n_\chi}{128\pi m_\chi^2 m_e^2} \int \mathrm{d}q\, q\, \hat{\eta}(q, \Delta E) \sum_{l=1}^{r} \mathfrak{R}\left[\mathscr{R}_l^*(q, v) \overline{\mathscr{W}}_l(q, \Delta E) \right]$$

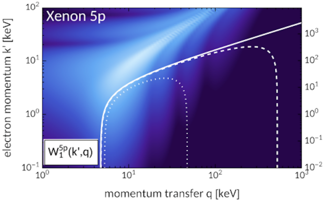
Particle physics input **Response functions from solid state physics**

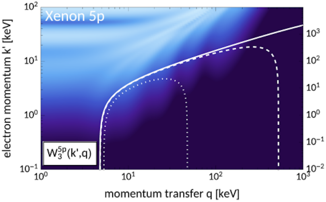
R. Catena, T. Emken, N. A. Spaldin and W. Tarantino, "Atomic responses to general dark matter-electron interactions," Phys. Rev. Res. **2** (2020) no.3, 033195
R. Catena, T. Emken, M. Matas, N. A. Spaldin and E. Urdshals, "Crystal responses to general dark matter-electron interactions," Phys. Rev. Res. **3** (2021) no.3, 033149

Response function formalism

- Selected response functions for xenon

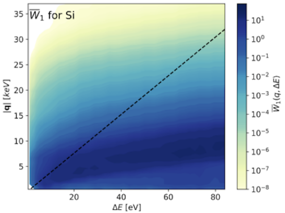
  R. Catena, T. Emken, N. A. Spaldin and W. Tarantino, ” Phys. Rev. Res. **2** (2020) no.3, 033195

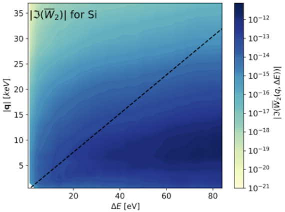




- Selected response functions for silicon crystals

  R. Catena, T. Emken, M. Matas, N. A. Spaldin and E. Urdshals, Phys. Rev. Res. **3** (2021) no.3, 033149



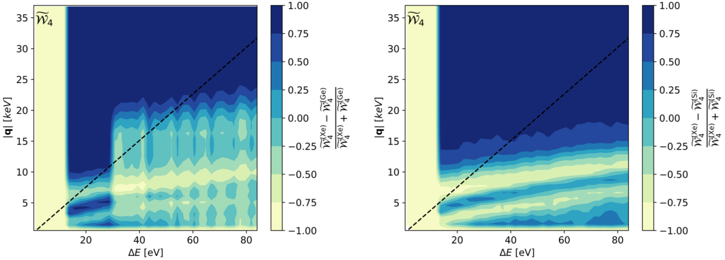


Response function formalism

- The response functions $\mathscr{W}_l(q, \Delta E)$ admit the following compact representation

$$\mathscr{W}_l(\boldsymbol{q}, \Delta E) = \frac{2}{\pi} \Delta E \sum_{\{1\},\{2\}} \mathscr{B}_l \, \delta(\Delta E - E_2 + E_1)$$

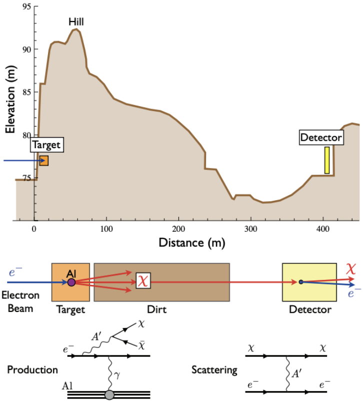
where the $\mathscr{B}_l$'s are (up to 5) material-specific electron wave function overlap integrals



R. Catena, *et al.* "Dark matter - electron interactions in materials beyond the dark photon model" arXiv:2210.07305

Relativistic dark sector searches at beam dump experiments

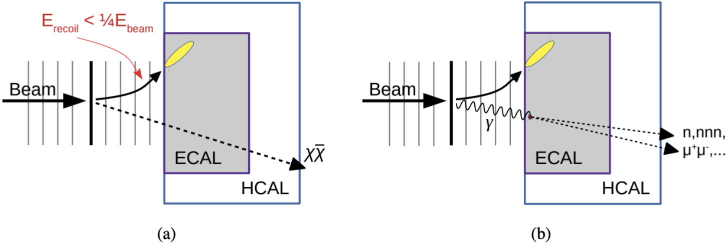
- Example of a beam dump experiment for dark sector searches (layout of E137)



J. D. Bjorken *et al.*, "Search for neutral metastable penetrating particles produced in the SLAC beam dump", Phys. Rev. D **38**, 3375 (1988)

Relativistic dark sector searches at LDMX

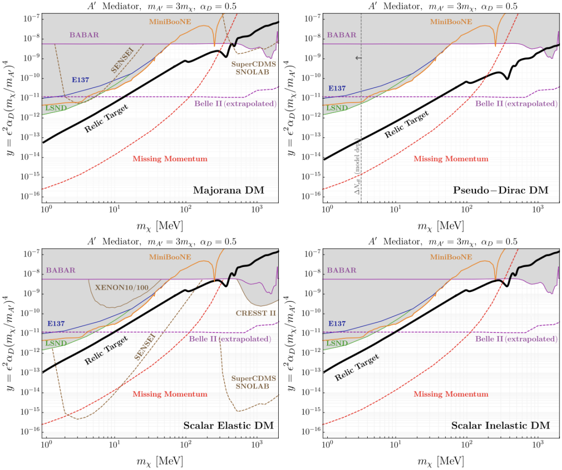
- Conceptual schematic of an expected signal (a) and background (b) process at LDMX.



T. Åkeson *et al.* FERMILAB-PUB-18-324-A, SLAC-PUB-17303

Prospects for dark sector searches at beam dump experiments and LDMX

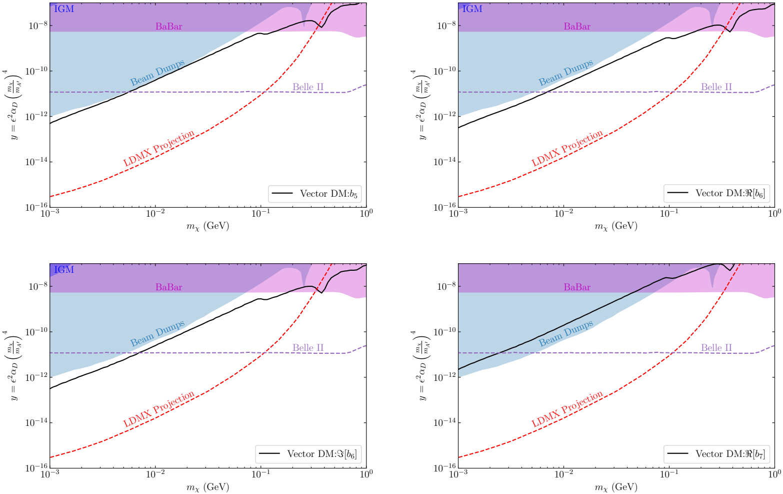
- Spin 0 and spin 1/2 thermal targets:



T. Åkeson *et al.* FERMILAB-PUB-18-324-A, SLAC-PUB-17303

Prospects for dark sector searches at beam dump experiments and LDMX

- Spin 1 thermal targets (preliminary):



R. Catena and T. Gray, "New thermal targets for spin-1 DM searches at beam dump and fixed target experiments", in progress

Summary and conclusions

- I reviewed a selection of theoretical frameworks used in the analysis of DM search experiments

- Focusing on models extending the SM matter/Higgs sector only, I discussed aspects of:
  - SUSY DM,
  - Minimal DM
  - Non-relativistic effective theories

- Focusing on frameworks including a DM candidate and a particle mediator, I reviewed dark sector models for spin-0 and spin-1/2 as well as spin-1 (new) DM

- The highly interdisciplinary character of the DM/dark sectors field calls for an increased collaboration between particle, nuclear and solid state physicists