HIGGS PAIR PRODUCTIONS AT N3LO+N3LL

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In collaboration with Hua-Sheng Shao
arXiv:2209.03914

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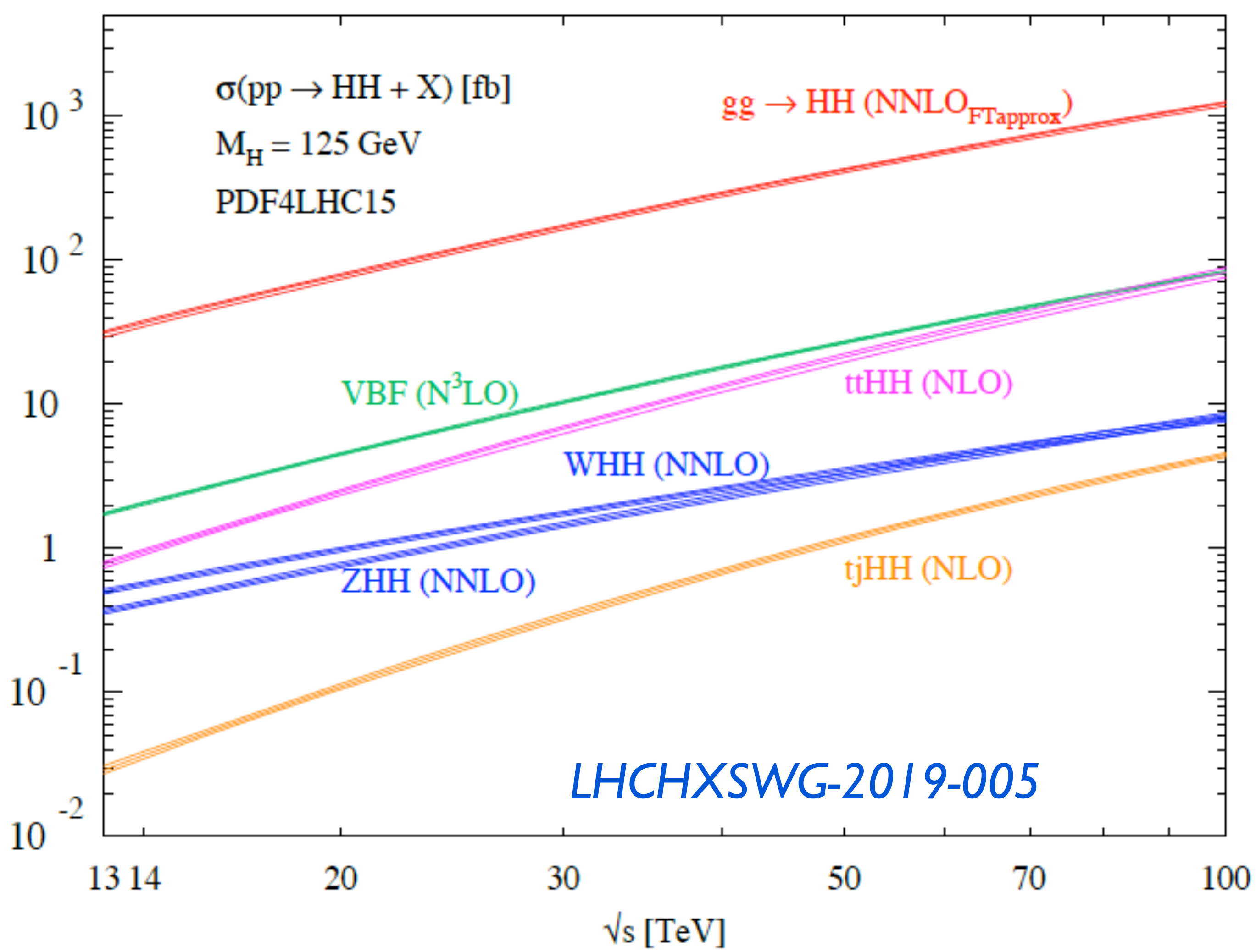
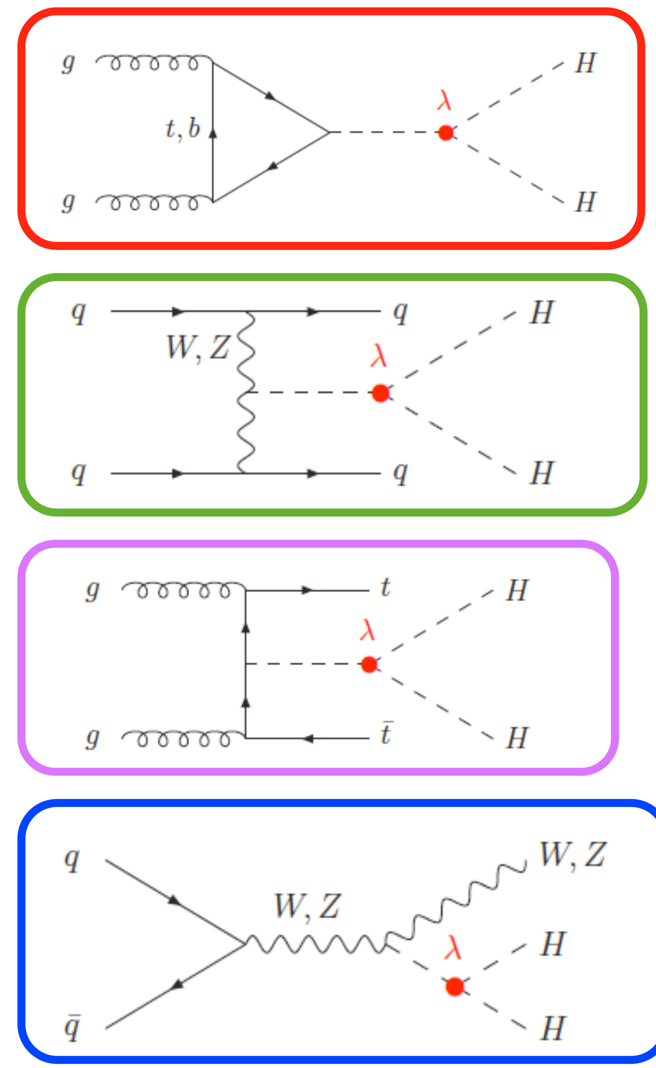
QUICK INTRODUCTION

- Why study Higgs pair productions?
  - directly probing trilinear coupling
  - Indirect constrain to quartic couping

} Constrain the Higgs field potential

Main production modes





$\sigma_h^{\mathrm{N^3LO}} = 54.72$ pb

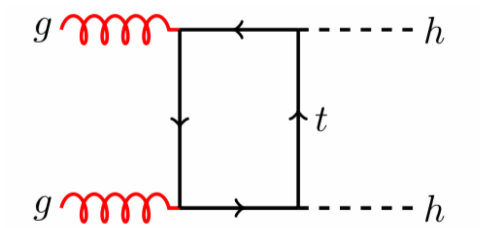
arXiv:1902.00134

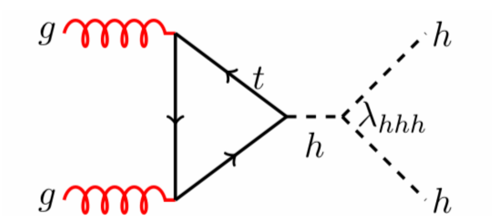
$\sigma_{hh}^{\mathrm{NNLO_{FTapprox}}} = 36.69$ fb

arXiv:1910.00012

$gg \to HH$

- Loop induced LO: destructively interfering box and triangle diagrams





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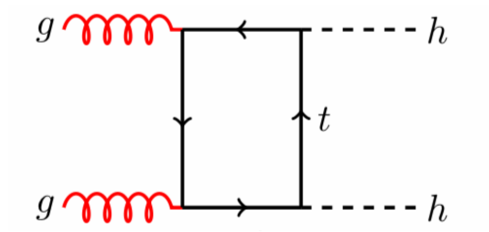
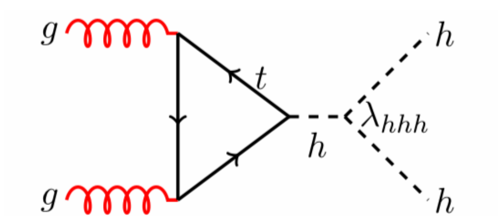




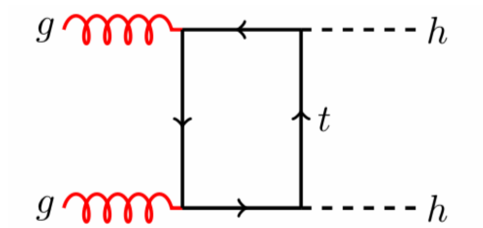
Higher order perturbative computations are quite challenging

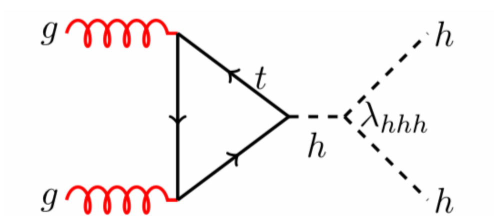
State-of-the-art : NLO

further improved by Soft gluon resummation/parton shower matching

$gg \to HH$

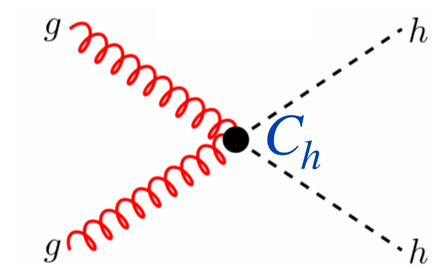
- **Loop induced LO: destructively interfering box and triangle diagrams**





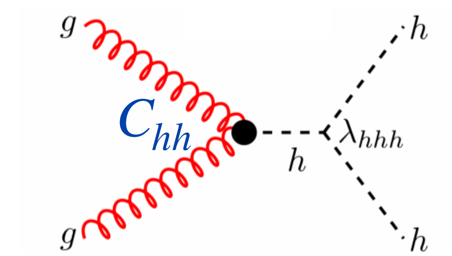
Infinite top quark approximation

$m_t \to \infty$



$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(C_h \frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right) G^a_{\mu\nu} G^{a\ \mu\nu}$$

$m_t \gg m_h$



C_h & C_{hh} : effective vertices

Higher order perturbative computations in this limit are more feasible

$gg \to HH$

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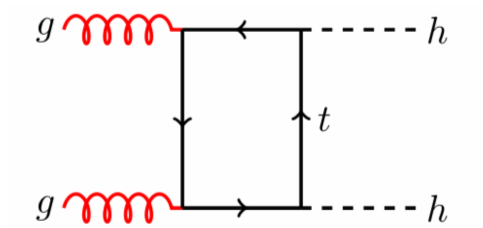

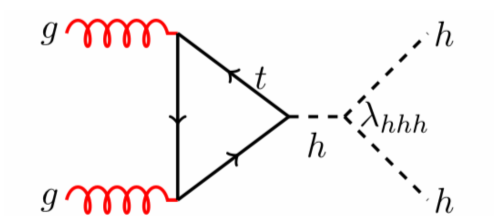

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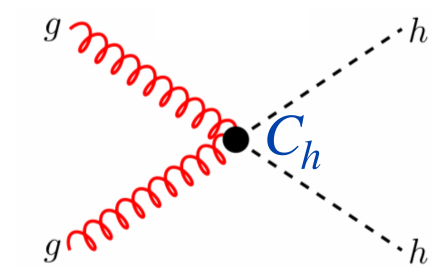

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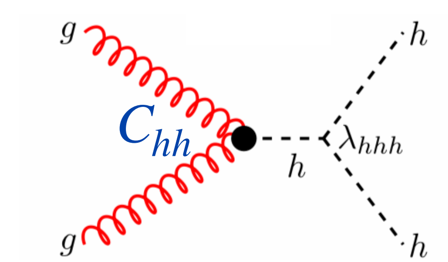
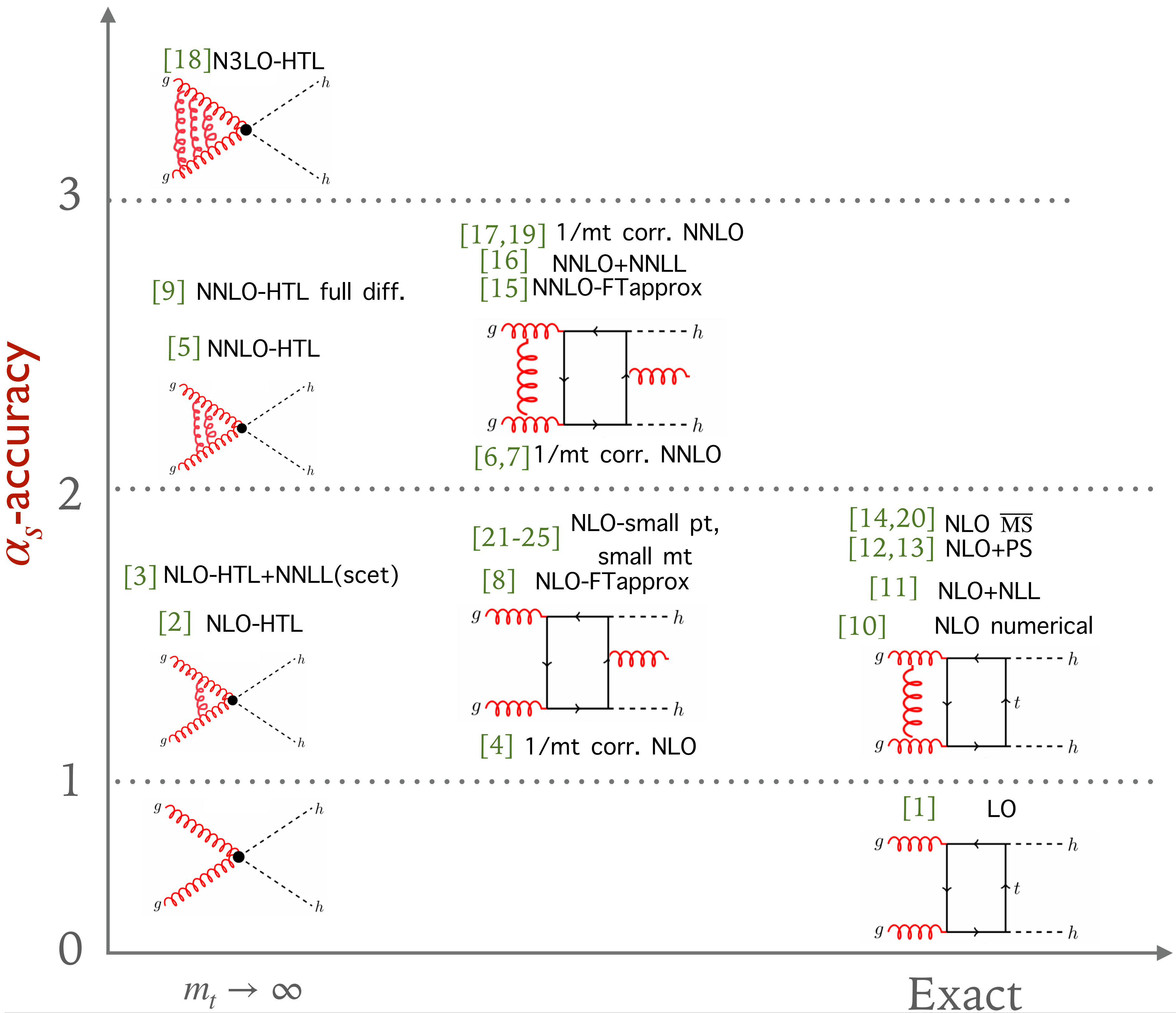


C_h & C_{hh} : effective vertices

Higher order perturbative computations in this limit are more feasible

Insufficient for phenomenological applications.
Numerous efforts to include the finite top mass corrections to this approximation

$gg \to HH$



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[2] Dawson,Dittmair, Spira '98
[3] Shao, Li, Li, Wang '13
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[5] Florian, Mazzitelli '13
[6,7] Grigo, et.al '14, '15
[8] Maltoni,Vryonidou,Zaro '14
[9] Florian, et.al '16
[10] Borowka, et.al '16, '16
[11] Ferreira, Pires '16
[12] Heinrich et.al '17
[13] Jones, Kuttimalai '17
[14] Baglio,et.al '18
[15] Grazzini,et.al '18
[16] Florian, Mazzitelli '18
[17] Davies, Steinhauser '19
[18] Chen, Li, Shao, Wang '19, '19
[19] Davies, et.al '19
[20] Baglio,et.al '21
[21] Davies et al '18, '18, '19
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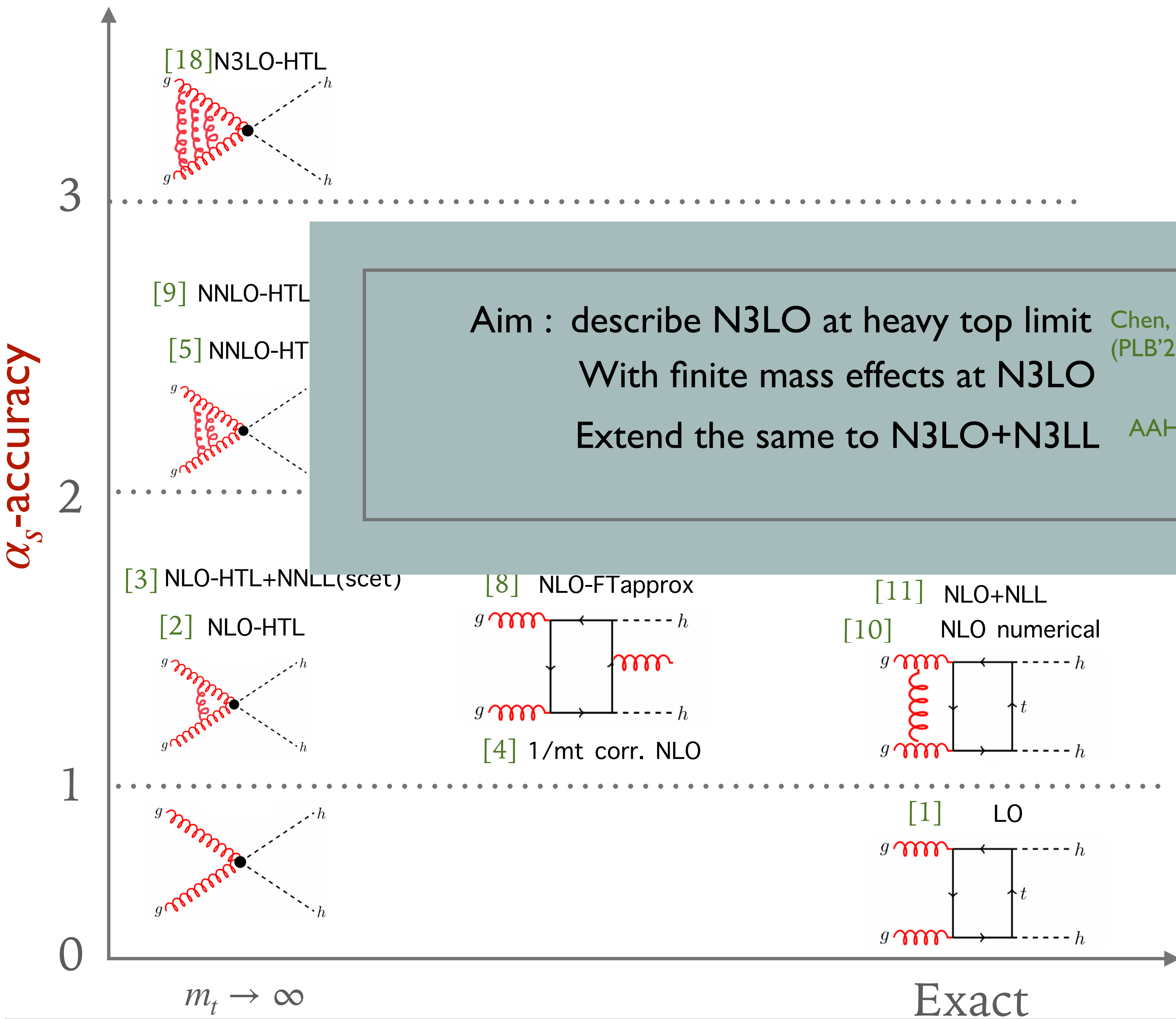

Aim : describe N3LO at heavy top limit — Chen, Li, Shao, Wang (PLB'20, JHEP'20)

With finite mass effects at N3LO

Extend the same to N3LO+N3LL — AAH, Shao 2209.03914

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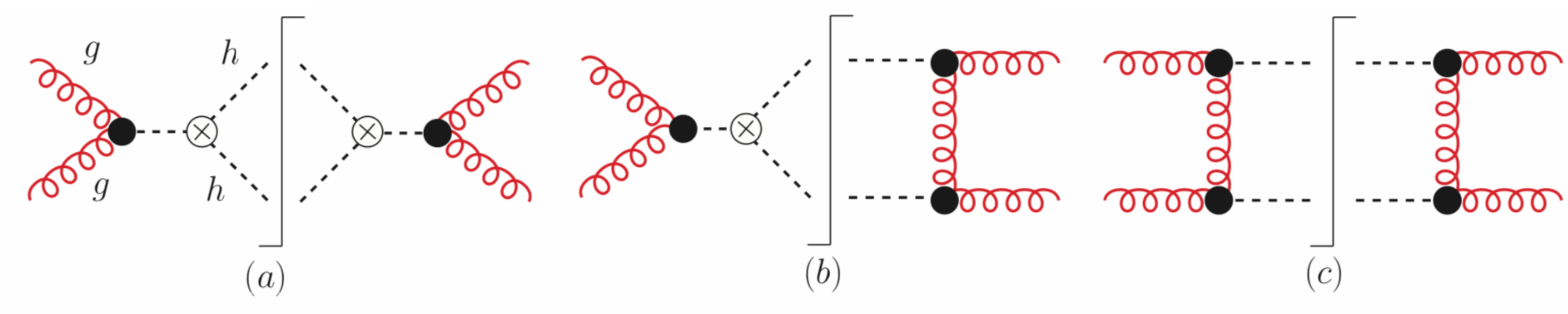
INFINITE TOP QUARK MASS LIMIT : OVERVIEW

- Breakdown to 3 channels : depending on number of effective vertices

$$d\sigma_{hh} = d\sigma^a_{hh} + d\sigma^b_{hh} + d\sigma^c_{hh}.$$



	LO	NLO	NNLO	N^{3}LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
class-a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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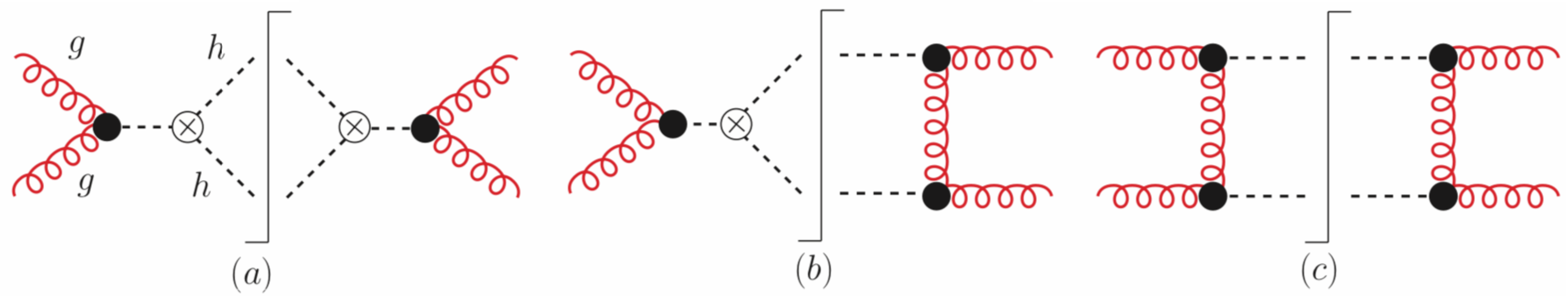

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at N^{3}LO

Class-a $\rightarrow$ N^{3}LO

Class-b $\rightarrow$ NNLO

Class-c $\rightarrow$ NLO

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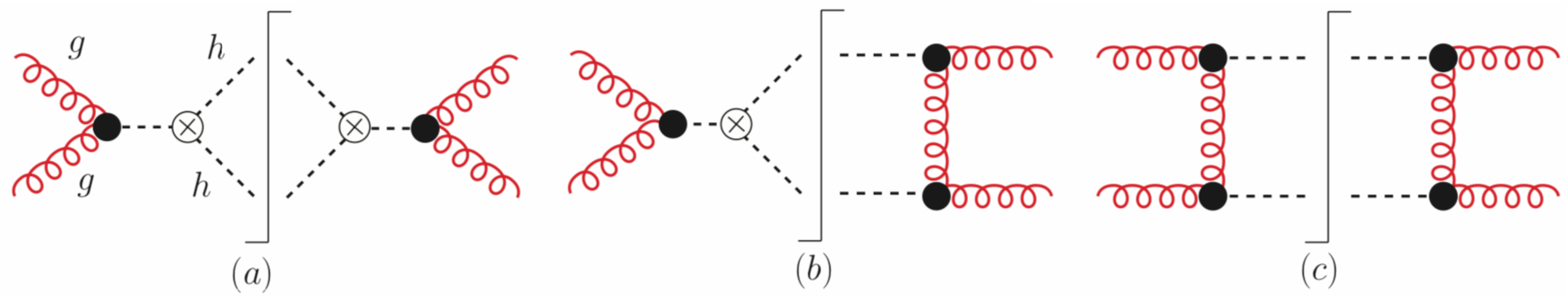

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at N^{3}LO

Class-a $\rightarrow$ N^{3}LO

Class-b $\rightarrow$ NNLO

Class-c $\rightarrow$ NLO

- Class-a : share same topology as ggH at the large mass limit

$$\frac{d\sigma^a_{hh}}{dm_{hh}} = f_{h\to hh}\left(\frac{C_{hh}}{C_h} - \frac{6\lambda_{hhh}v^2}{m^2_{hh}-m^2_h}\right)^2 \times \left(\sigma_h\big|_{m_h\to m_{hh}}\right)$$

Phase space factor mapping from ggH $\longrightarrow$ ggHH

$$f_{h\to hh} = \frac{\sqrt{m^2_{hh}-4m^2_h}}{16\pi^2 v^2}$$

at N^{3}LO

From iHixs2

Dulat et al. CPC'18

$m_{hh} \longrightarrow$ Higgs pair invariant mass

INFINITE TOP QUARK MASS LIMIT : OVERVIEW

- **Class-b : to NNLO accuracy - use q_t-subtraction method** Catani & Grazzini PRL'07

$$d\sigma^b_{hh} = d\sigma^b_{hh}\Big|_{p_T^{hh} < p_T^{\rm veto}} + d\sigma^b_{hh}\Big|_{p_T^{hh} > p_T^{\rm veto}}$$

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q_t-resummation formalism

$$\frac{d\sigma^b_{hh}}{dp_T^{hh}} = H^b \otimes B_g \otimes B_g \otimes S \times \left(1 + \mathcal{O}\left(\frac{(p_T^{hh})^2}{Q^2}\right)\right)$$

Hard function

Banerjee et al., JHEP'18

TMD beam function

Soft functions

Process independent

Gehrmann et al.PRL'12, JHEP'14; Luebbert et al., JHEP'16; Echevarria et al. JHEP'16; Luo et al., '19

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HH + jet at NLO for class-b

MG5_aMC@NLO

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HH + jet at NLO for class-b — MG5_aMC@NLO

- Class-c : to NLO accuracy - MG5_aMC@NLO

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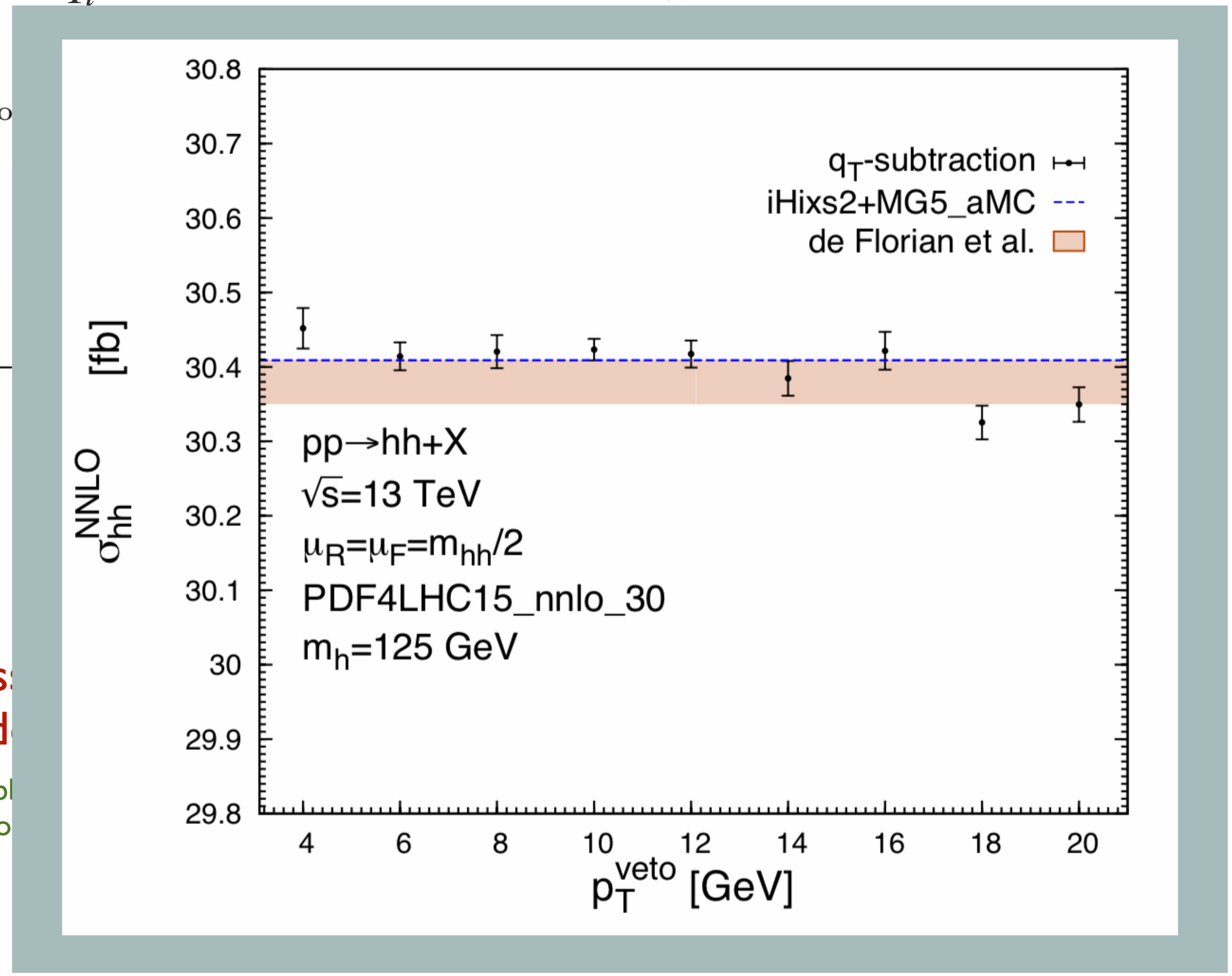

- Class-c : to NLO accuracy - MG5_aMC@NLO

THRESHOLD RESUMMATION : OVERVIEW

- Resummation is relevant at production threshold, defined as $z = \frac{m_{hh}^2}{\hat{s}} \to 1$,
- Required due to threshold enhanced logarithms $\log(1-z)/(1-z)_+$, arising from soft gluon emissions

$m_{hh} \longrightarrow$ Higgs pair invariant mass

$\hat{s} \longrightarrow$ partonic center of mass energy square

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- At production threshold, the partonic cross section shows an exponential behaviour

Naively : the partonic cross section,

Convoluted exponential

$$\hat{\sigma}_{hh}(m_{hh}^2, z) = C_0\left(m_{hh}^2\right)\delta(1-z) \quad \mathscr{C}\exp\left(\mathcal{Q}\left(m_{hh}^2, z\right)\right)$$

3-loop corrections
Universal

Virtual +
non-logarithmic soft contributions

Soft logarithms
Universal, depends only on initial partons

THRESHOLD RESUMMATION : OVERVIEW

- Resummation is convenient to perform in Mellin-N space, where the convolutions become normal products

$$\Delta_{hh}^{res}(N) = \int_0^1 dz\ z^{N-1} \hat{\sigma}_{hh}(z, m_{hh}^2)$$

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$z \to 1 \longrightarrow N \to \infty$

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \to \frac{\ln^2(\overline{N}) + \zeta_2}{2} + \mathcal{O}\left(\frac{1}{\overline{N}}\right) \qquad \overline{N} = N \exp(\gamma_E)$$

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- In the N-space, the master formula :

$$\bar{\omega} = \frac{\alpha_s}{2\pi}\beta_0 \log \overline{N}$$

$$\Delta^{\rm res}(N, m^2_{hh}, \mu^2_R, \mu^2_F)\Big|_{\rm N^kLL} = \left(\bar{g}_{0,0} + \alpha_s \bar{g}_{0,1} + \alpha_s^2 \bar{g}_{0,2} + \cdots\right)\Big|_{\rm N^kLO} \exp\Big(\tilde{C}_{0,\zeta_2}(\alpha_s) + \underbrace{g_1(\bar{\omega})\ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s\ g_3(\bar{\omega}) + \cdots}_{f(\log N)}\Big)$$

N-independent
Process dependent

N-independent

$f(\log N)$

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[Sterman]

[Catani, Trentedue]

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Leading logarithm

LL : resum terms $\alpha_s^n \log^{n+1} N$ highest logarithms at each α_s order

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$$= \left(\bar{\bar{g}}_{0,0} + \alpha_s \bar{\bar{g}}_{0,1} + \alpha_s^2 \bar{\bar{g}}_{0,2} + \cdots\right)\Big|_{\mathrm{N^kLO}} \exp\left(g_1(\bar{\omega}) \; \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s \; g_3(\bar{\omega}) + \cdots\right)$$

[Sterman]

[Catani, Trentedue]

Next-to-leading logarithm

NLL : resum terms $\alpha_s^n \log^n N$

Next-to-highest logarithms at each α_s order

THRESHOLD RESUMMATION : OVERVIEW

- Resummation is convenient to perform in Mellin-N space, where the convolutions become normal products

$$\Delta^{res}_{hh}(N) = \int_0^1 dz\ z^{N-1}\hat{\sigma}_{hh}(z, m_{hh}^2)$$

$z \to 1 \longrightarrow \mathrm{N} \to \infty$

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \to \frac{\ln^2(\overline{N}) + \zeta_2}{2} + \mathcal{O}\left(\frac{1}{\overline{N}}\right) \qquad \overline{N} = N\exp(\gamma_E)$$

- In the N-space, the master formula :

$$\bar{\omega} = \frac{\alpha_s}{2\pi}\beta_0 \log \overline{N}$$

$$\Delta^{\rm res}(N, m_{hh}^2, \mu_R^2, \mu_F^2)\Big|_{\rm N^kLL} = \left(\bar{g}_{0,0} + \alpha_s \bar{g}_{0,1} + \alpha_s^2 \bar{g}_{0,2} + \cdots\right)\Big|_{\rm N^kLO} \exp\left(\tilde{C}_{0,\zeta_2}(\alpha_s) + g_1(\bar{\omega})\ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s\ g_3(\bar{\omega}) + \cdots\right)$$

$$= \left(\bar{\bar{g}}_{0,0} + \alpha_s \bar{\bar{g}}_{0,1} + \alpha_s^2 \bar{\bar{g}}_{0,2} + \cdots\right)\Big|_{\rm N^kLO} \exp\left(g_1(\bar{\omega})\ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s\ g_3(\bar{\omega}) + \cdots\right)$$

[Sterman]

[Catani, Trentedue]

Next-to-next-to-leading logarithm

NNLL : resum terms $\alpha_s^n \log^{n-1} N$

Next-to-next-to-highest logarithms at each α_s order

and so on…

THRESHOLD RESUMMATION : OVERVIEW

- Resummation schemes : freedom to choose some N-independent part inside/outside the exponent, up to a given logarithmic accuracy
- Depending on that we propose 4 schemes :

THRESHOLD RESUMMATION : OVERVIEW

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    - $\overline{\mathrm{N}}_1$ scheme :

N-independent term outside exponent

$$\Delta^{\mathrm{res}}_{\overline{N}_1}(N, m_{hh}^2, \mu_F^2)\Big|_{\mathrm{N^kLL}} = \left(\bar{\bar{g}}_{0,0} + \alpha_s \bar{\bar{g}}_{0,1} + \alpha_s^2 \bar{\bar{g}}_{0,2} + \cdots\right)\Big|_{\mathrm{N^kLO}} \exp\left(g_1(\bar{\omega}) \ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s \ g_3(\bar{\omega}) + \cdots\right)$$

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- $\overline{\mathrm{N}}_2$ scheme :

Part of N-independent term, coming from Mellin transformation, is kept within the exponent

$$\Delta^{\text{res}}_{\overline{N}_2}(N, m_{hh}^2, \mu_F^2)\Big|_{\text{N}^k\text{LL}} = \left(\bar{g}_{0,0} + \alpha_s \bar{g}_{0,1} + \alpha_s^2 \bar{g}_{0,2} + \cdots\right)\Big|_{\text{N}^k\text{LO}} \exp\left(\tilde{C}_{0,\zeta_2}(\alpha_s) + g_1(\bar{\omega})\ \ln \overline{N} + g_2(\bar{\omega}) + \alpha_s\ g_3(\bar{\omega}) + \cdots\right)$$

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  - N_1 scheme :

Resum $\log N$ terms instead of $\log \overline{N}$, with $\overline{N} = N \exp(\gamma_E)$

$$\Delta^{\rm res}_{N_1}(N, m_{hh}^2, \mu_F^2)\Big|_{\rm N^kLL} = \left(\tilde{g}_{0,0} + \alpha_s \tilde{g}_{0,1} + \alpha_s^2 \tilde{g}_{0,2} + \cdots\right)\Big|_{\rm N^kLO} \exp\left(g_1(\omega)\;\ln N + g_2(\omega) + \alpha_s\; g_3(\omega) + \cdots\right)$$

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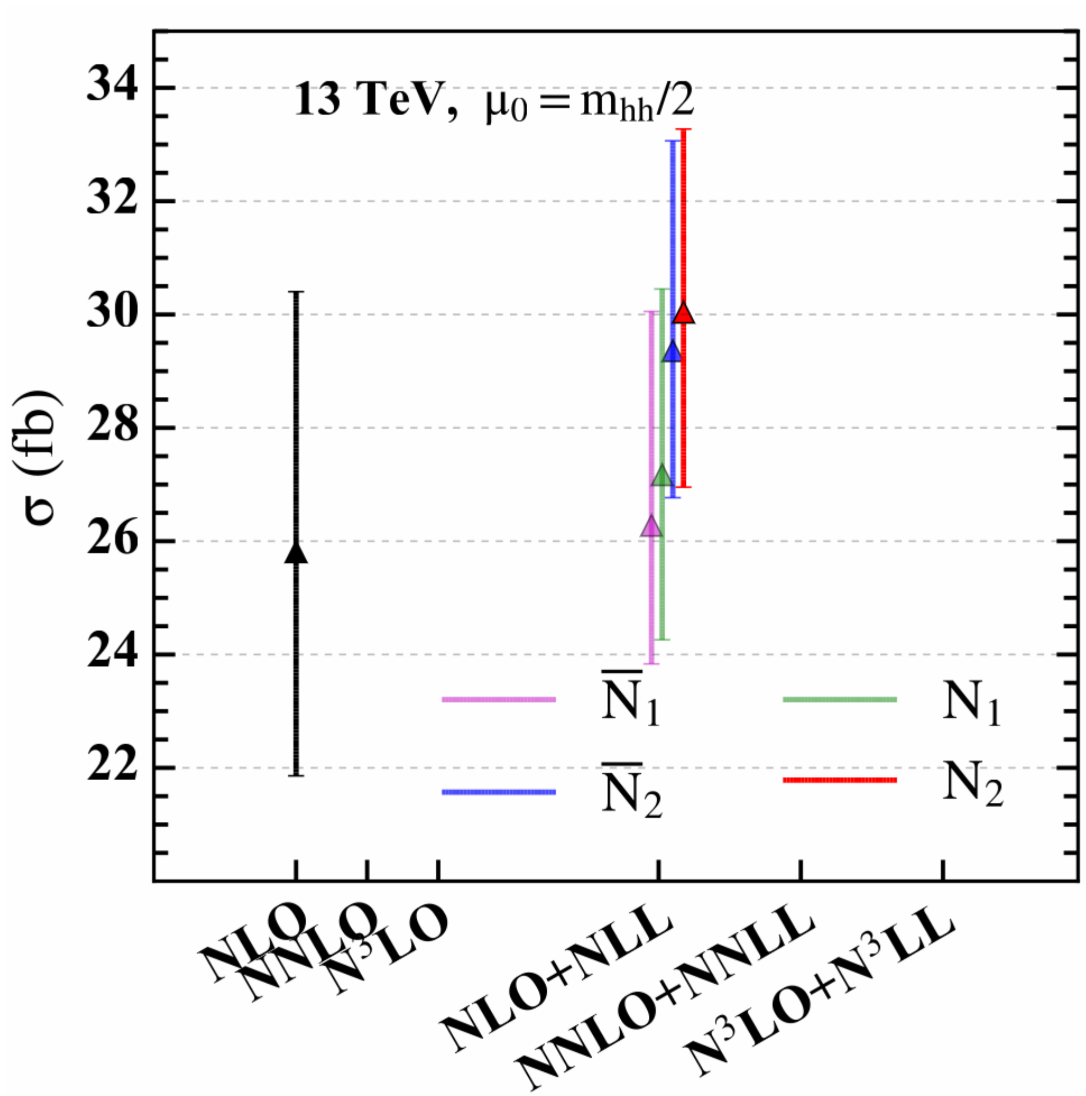
  - N_2 scheme :

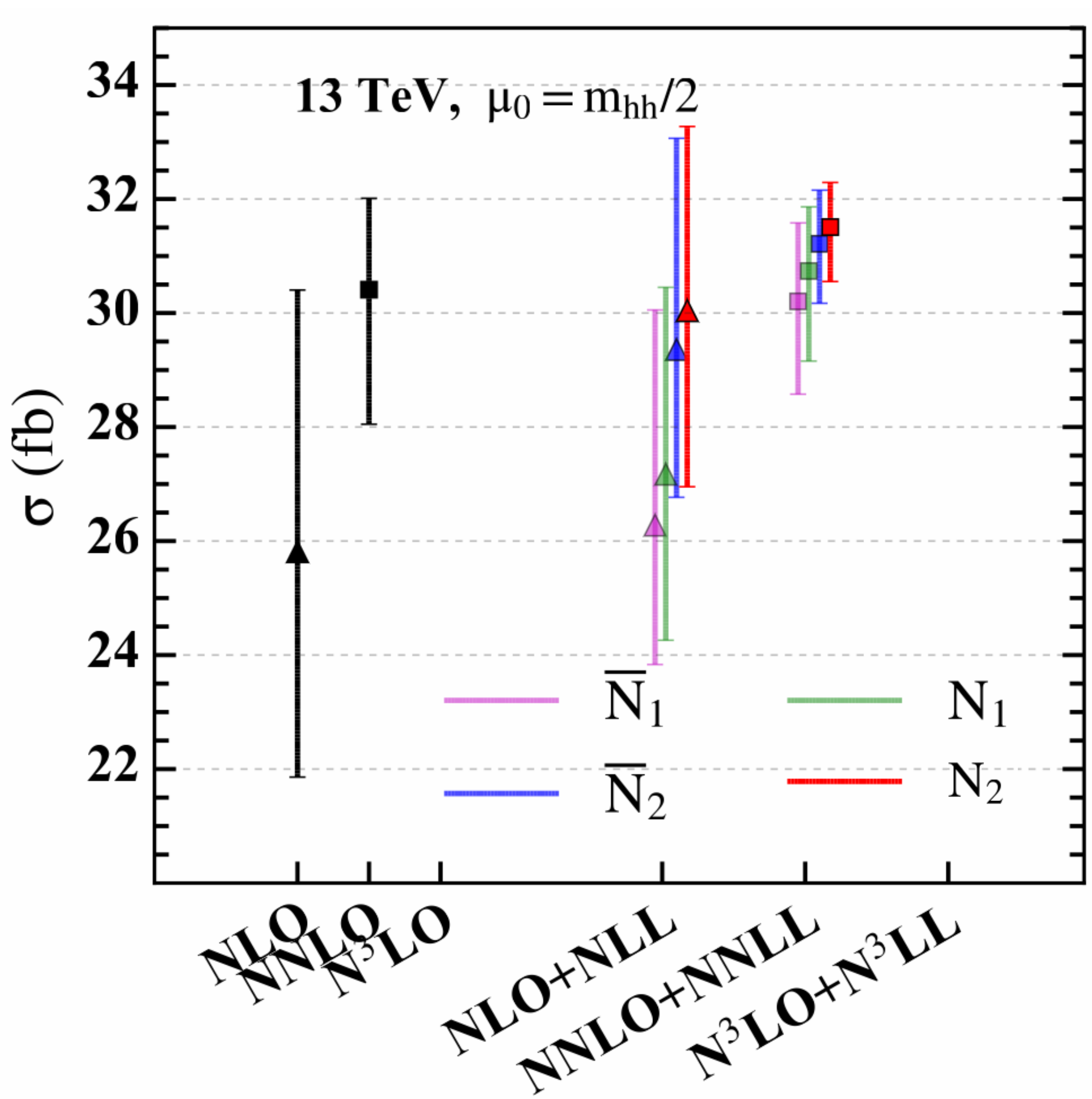
N_2- scheme with resuming $\log N$ terms

$$\Delta^{\mathrm{res}}_{N_2}(N, m_{hh}^2, \mu_F^2)\Big|_{\mathrm{N^kLL}} = \left(g_{0,0} + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \cdots\right)\Big|_{\mathrm{N^kLO}} \exp\left(\tilde{C}_{0,\zeta_2}(\alpha_s) + g_1(\omega)\ \ln N + g_2(\omega) + \alpha_s\ g_3(\omega) + \cdots\right)$$

RESUMMATION - SCHEME AMBIGUITIES



RESUMMATION - SCHEME AMBIGUITIES



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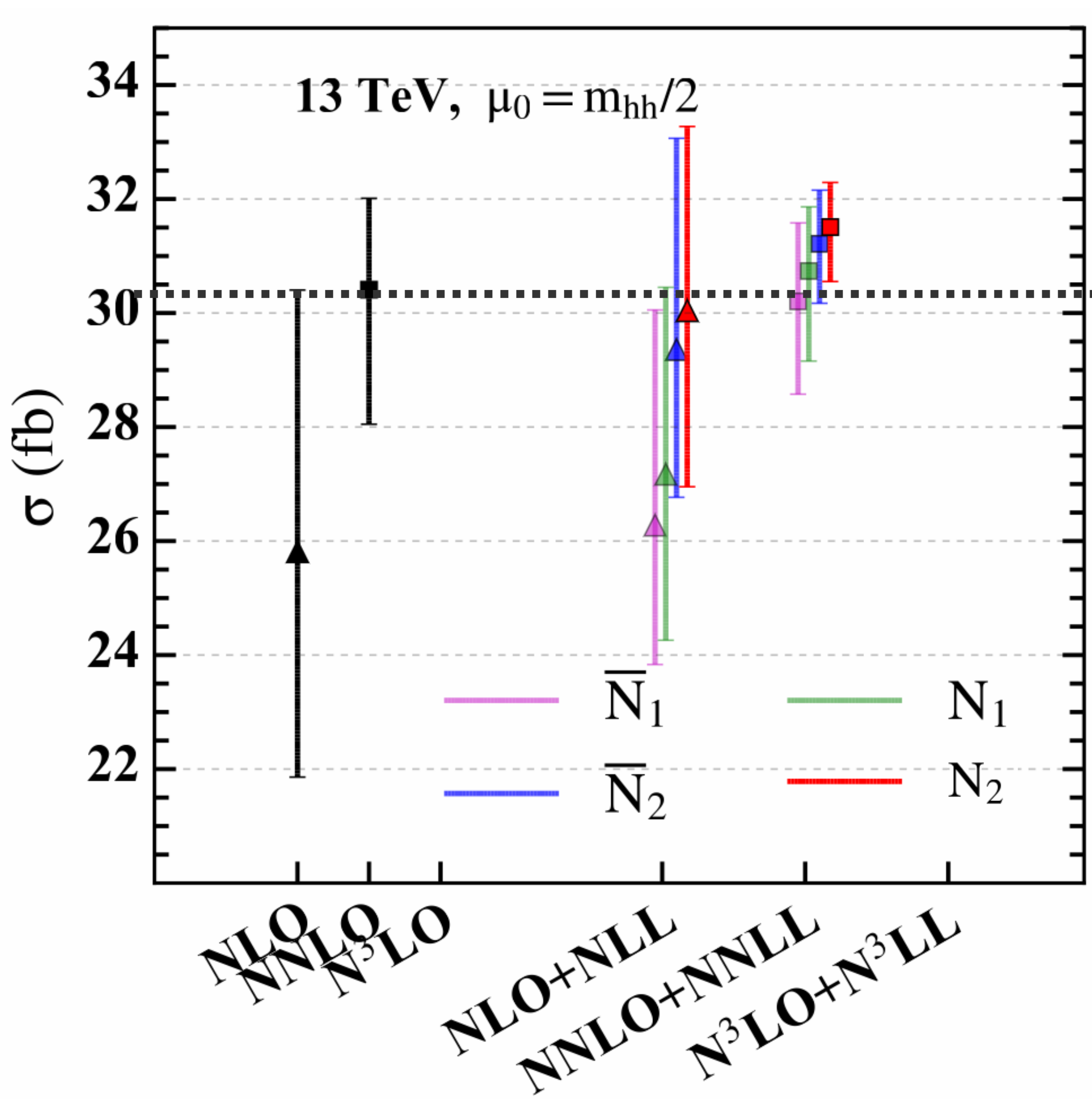
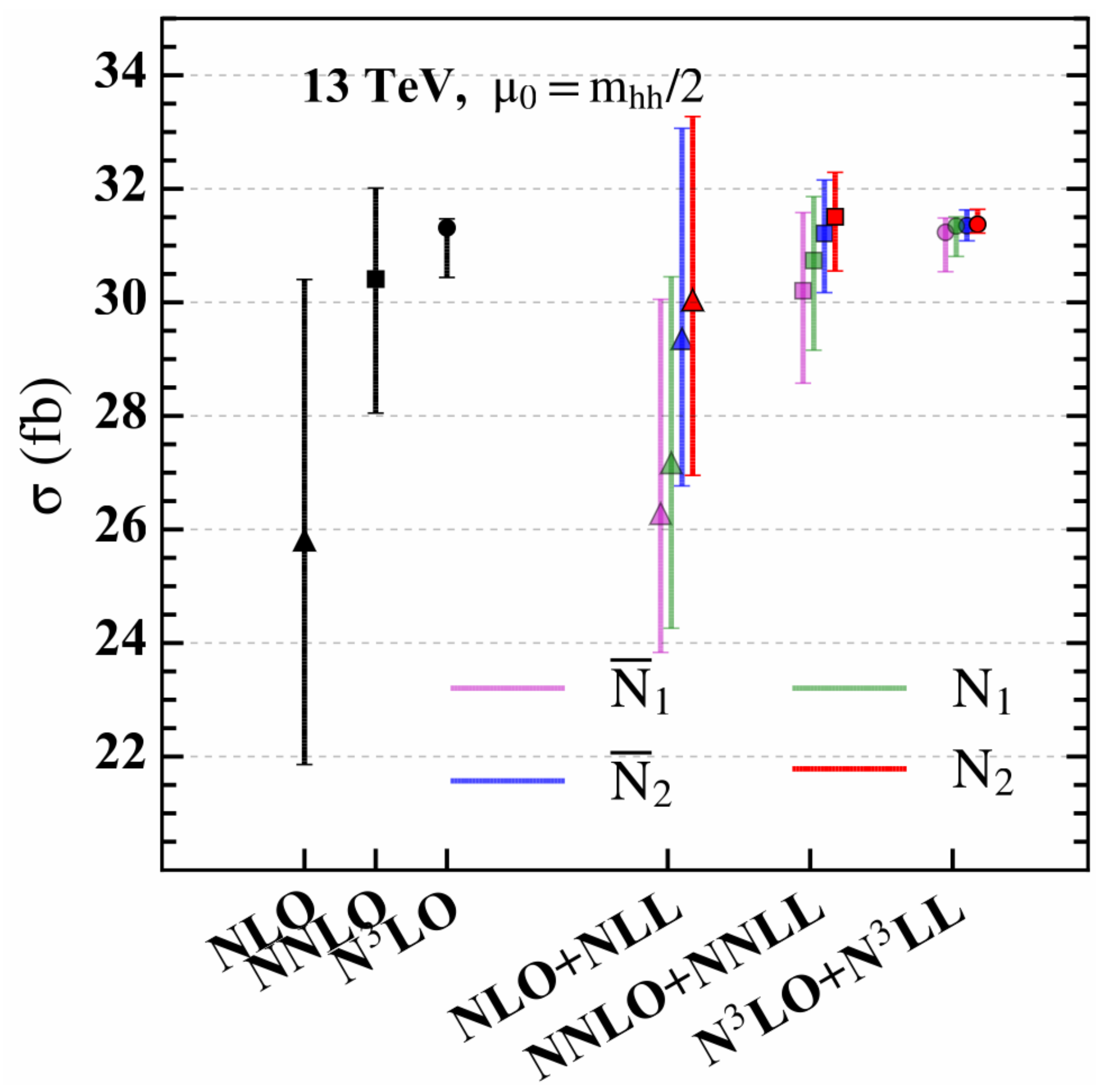
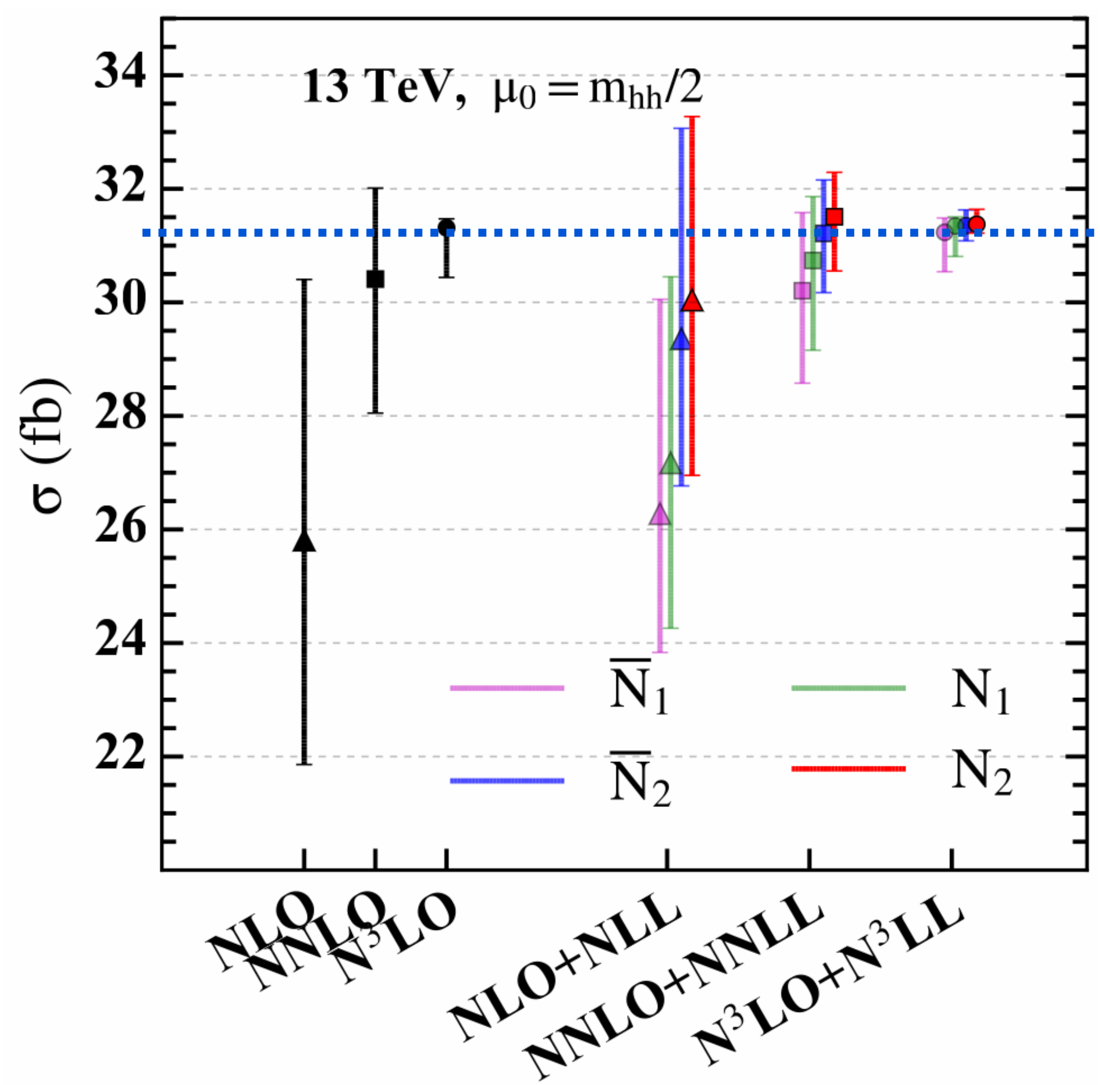


NLO+NLL is close to NNLO for N_2 & $\overline{N}_2$ schemes

RESUMMATION - SCHEME AMBIGUITIES

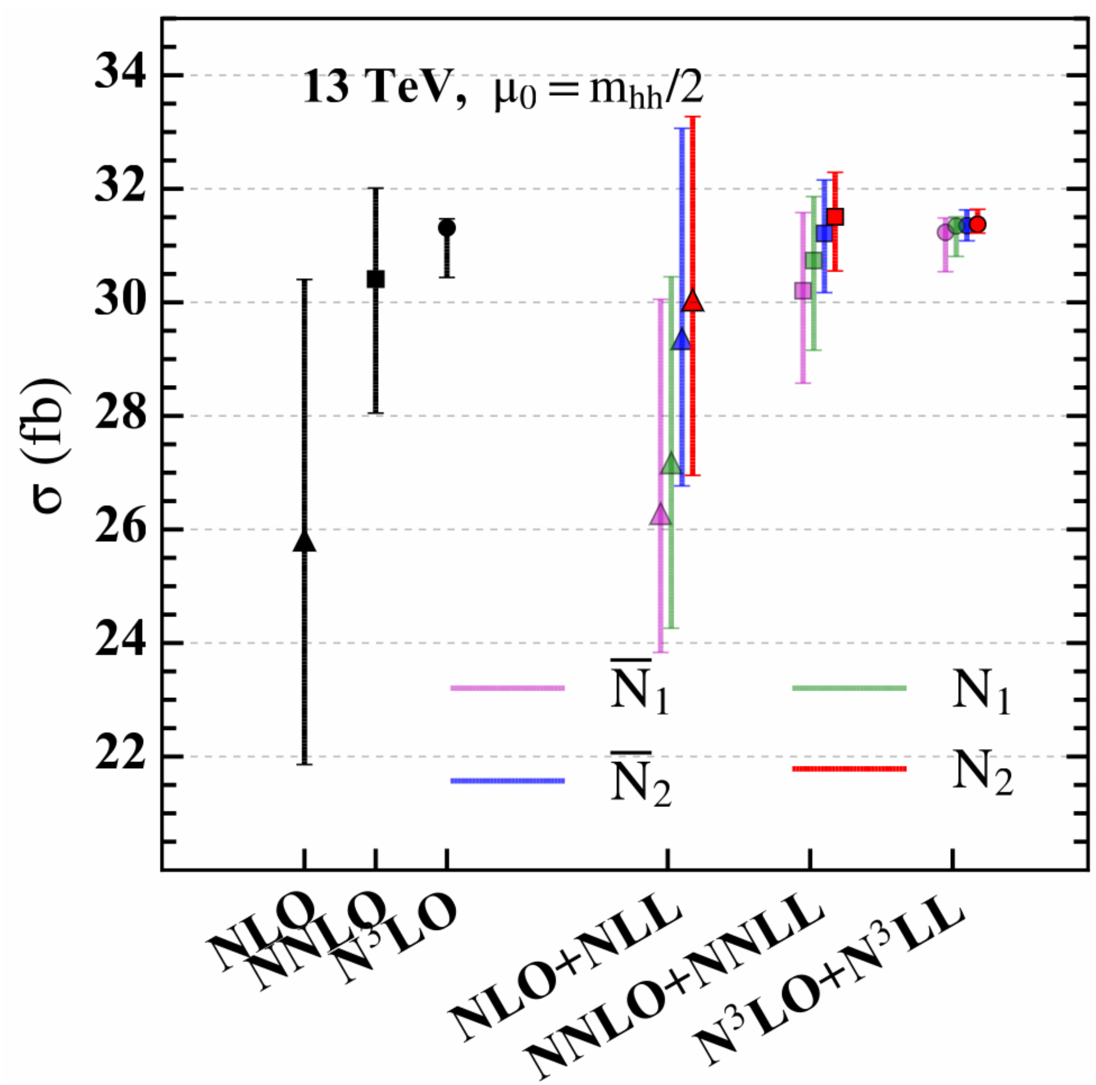


RESUMMATION – SCHEME AMBIGUITIES



N2LO+N2LL very close to N3LO in N_2 & $\overline{N}_2$ schemes

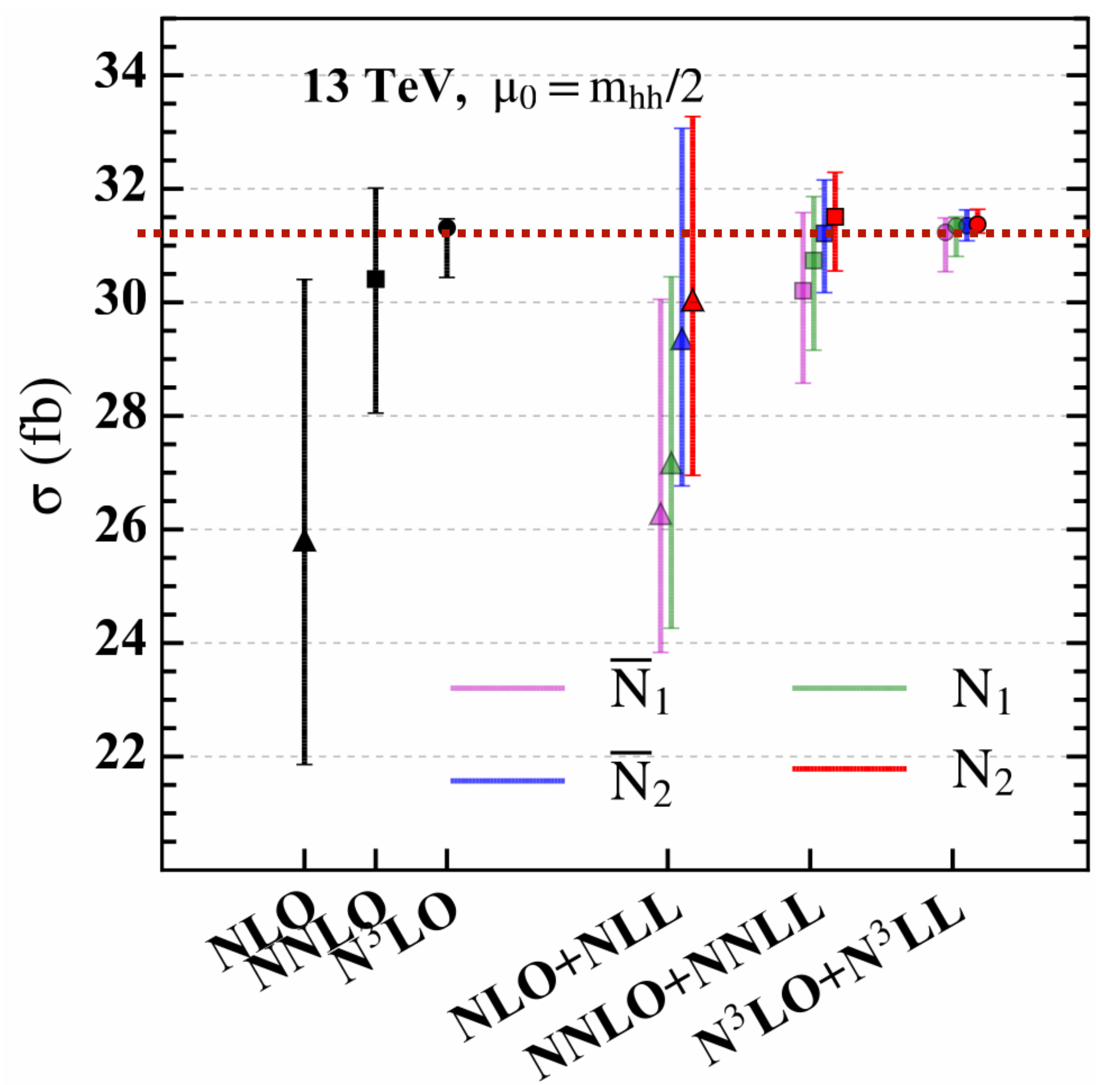
RESUMMATION – SCHEME AMBIGUITIES



N_2 & $\overline{N}_2$ are on equal footing.
We choose to work with $\overline{N}_2$ scheme
Bonus : symmetric error bar

$N^3LO + N^3LL$ *in* N_2 or N_2 *schemes could be extrapolated to predict* N^4LO

RESUMMATION - SCHEME AMBIGUITIES



Good perturbative convergence at N3LO

N_2 & $\overline{N}_2$ *are on equal footing.*
We choose to work with $\overline{N}_2$ *scheme*
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$N^3LO + N^3LL$ *in* N_2 or N_2 *schemes could be extrapolated to predict* N^4LO

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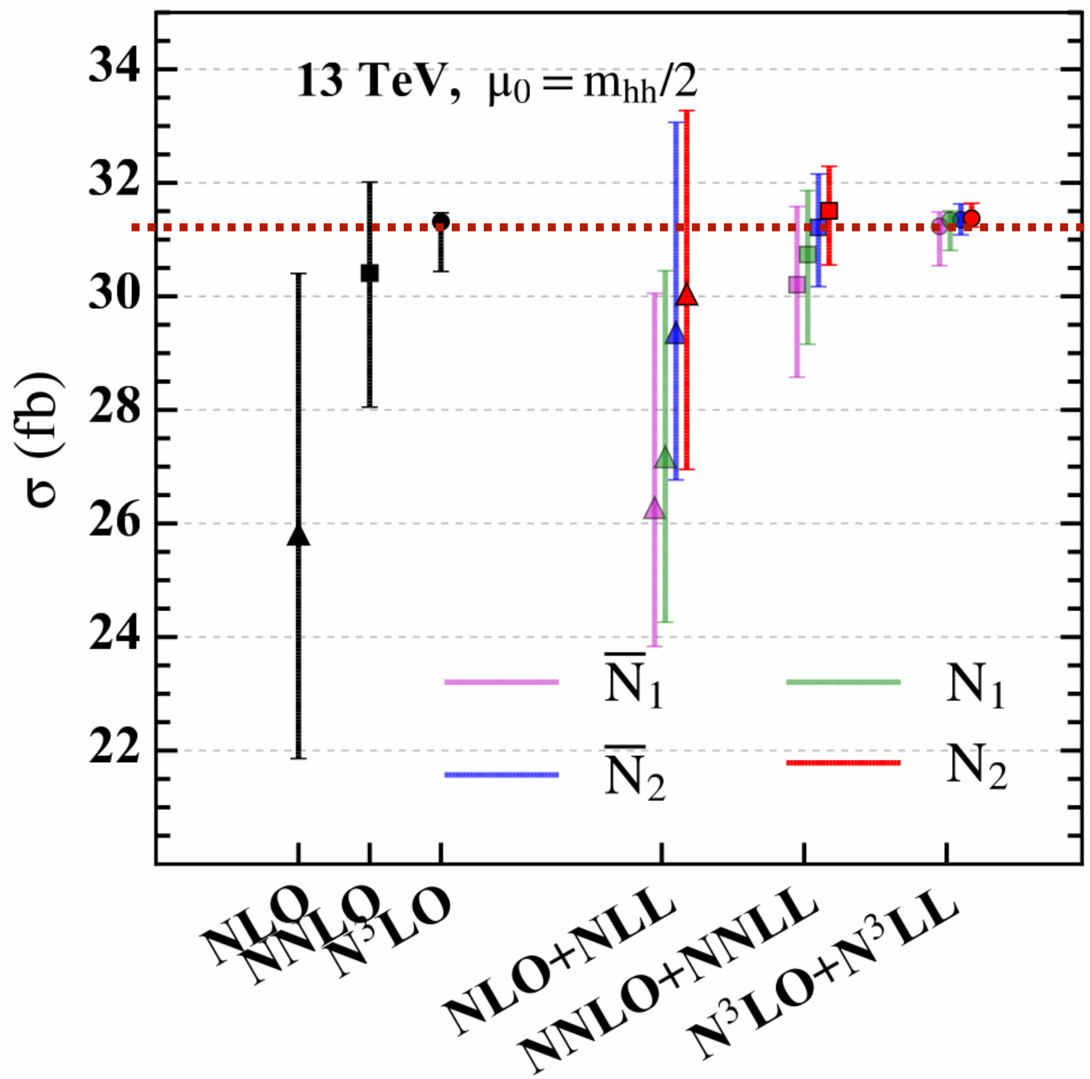

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$N^3LO + N^3LL$ *in* N_2 or N_2 *schemes could be extrapolated to predict* N^4LO

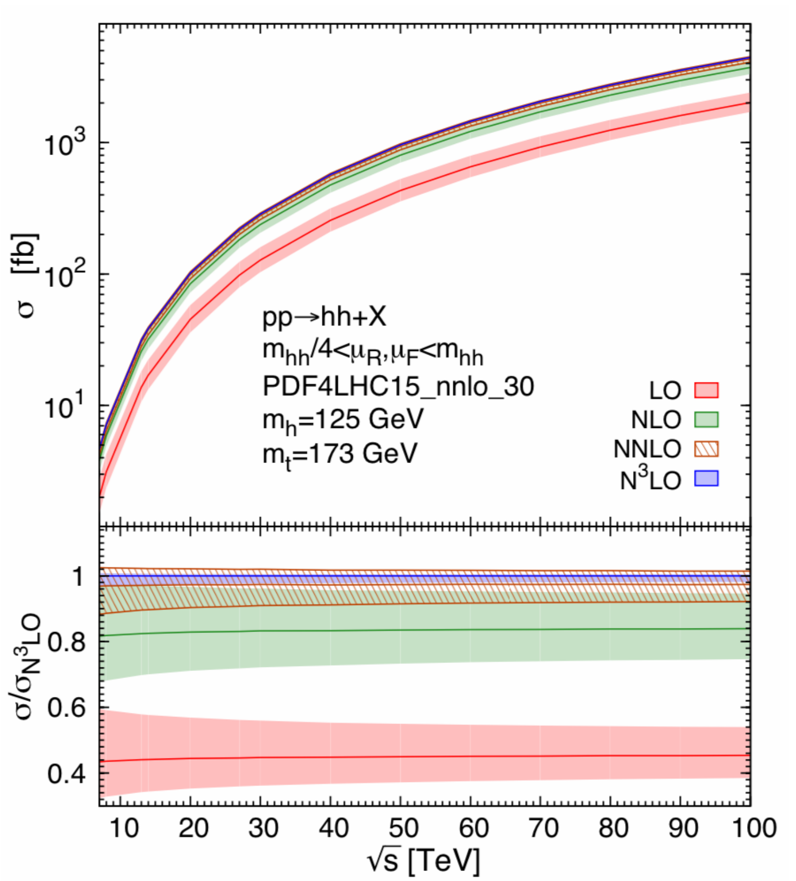
- Our results at NNLL in N_1 scheme are verified with 1505.07122 Florian, Mazzitelli JHEP'15
- With same numerical setup, the resum results for $gg \to H$ are verified with 1603.08000 Bonvini, Marzani, Muselli, Rottoli JHEP'16

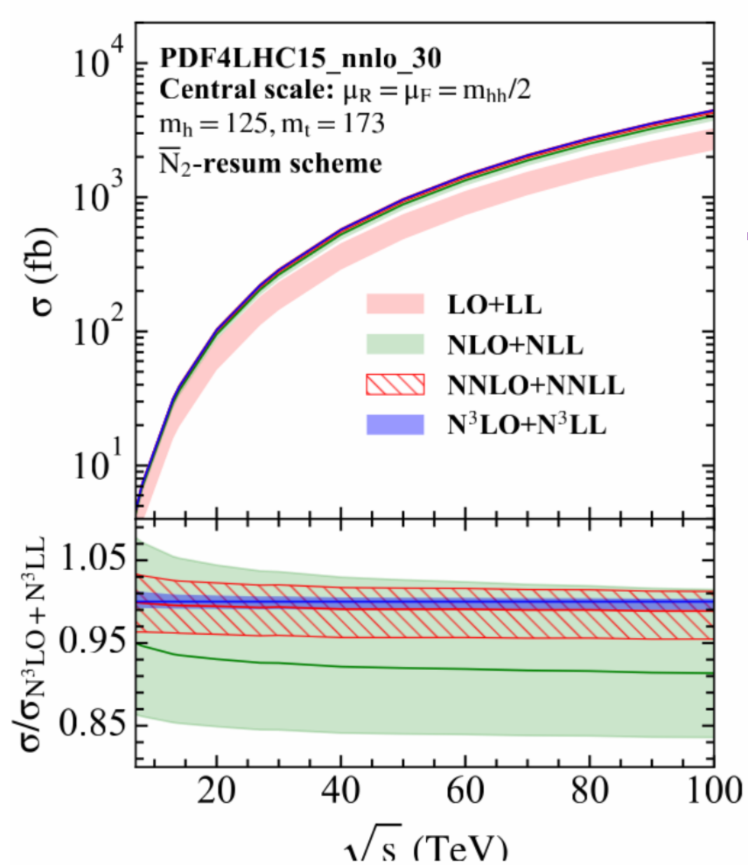
INFINITE TOP QUARK MASS LIMIT : RESULTS

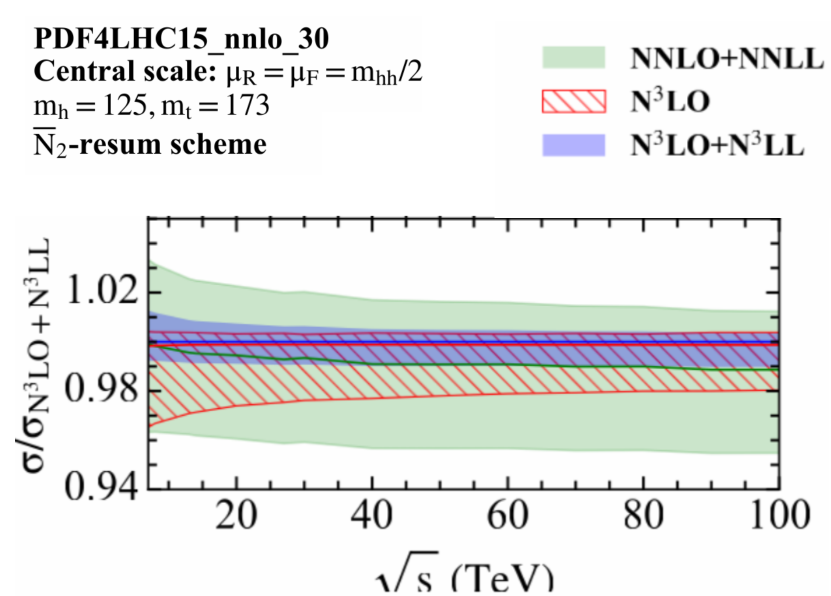
Inclusive cross section

in unit of fb central scale $\mu_0 = \frac{m_{hh}}{2}$

$\sqrt{s}$ [TeV]	Order k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$	
			N_2 scheme	$\overline{\mathrm{N}}_2$ scheme
13	0	$13.80^{+31\%}_{-22\%}$	$16.01^{+32\%}_{-23\%}$	$21.02^{+36\%}_{-24\%}$
	1	$25.81^{+18\%}_{-15\%}$	$30.04^{+10.8\%}_{-10.3\%}$	$29.36^{+12.6\%}_{-8.8\%}$
	2	$30.41^{+5.3\%}_{-7.8\%}$	$31.51^{+2.5\%}_{-3.0\%}$	$31.21^{+3.0\%}_{-3.3\%}$
	3	$31.31^{+0.50\%}_{-2.8\%}$	$31.37^{+0.84\%}_{-0.49\%}$	$31.35^{+0.88\%}_{-0.85\%}$







INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section

in unit of fb central scale $\mu_0 = \frac{m_{hh}}{2}$

$\sqrt{s}$ [TeV]	Order k	N^{k}LO	N^{k}LO+N^{k}LL	
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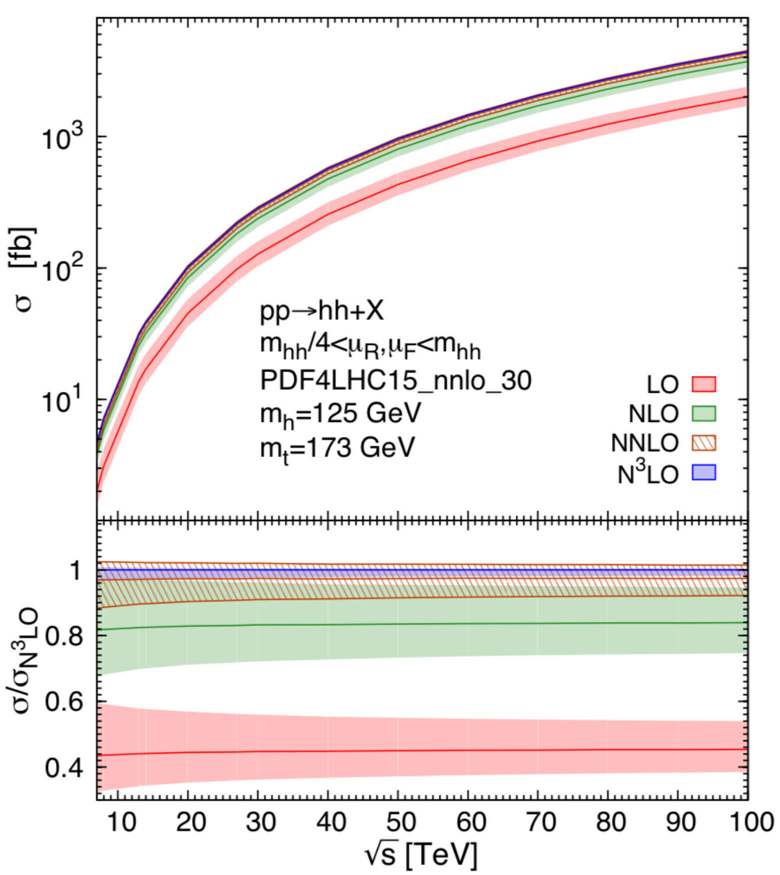
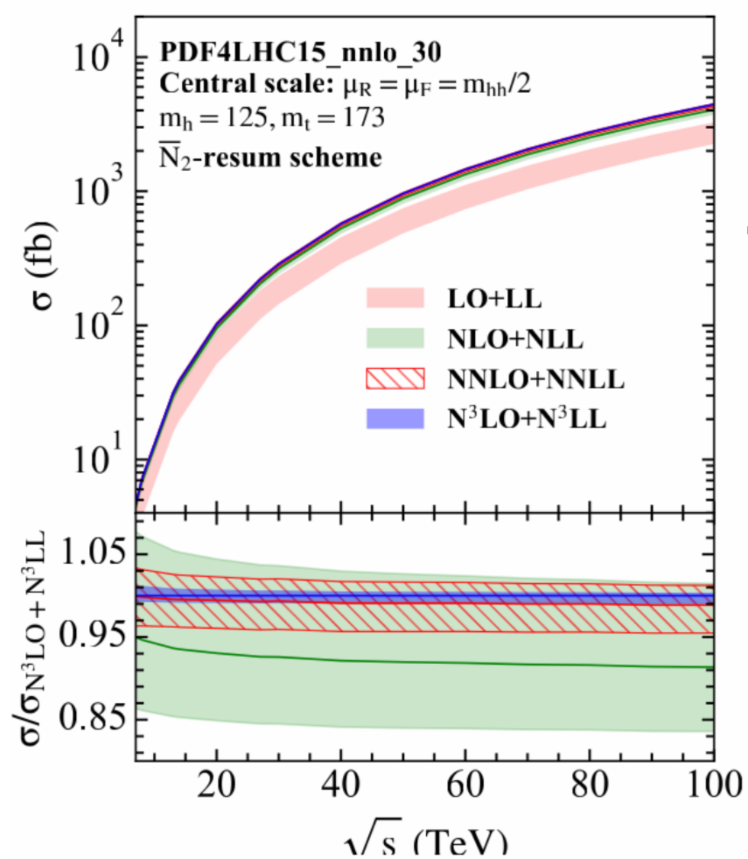
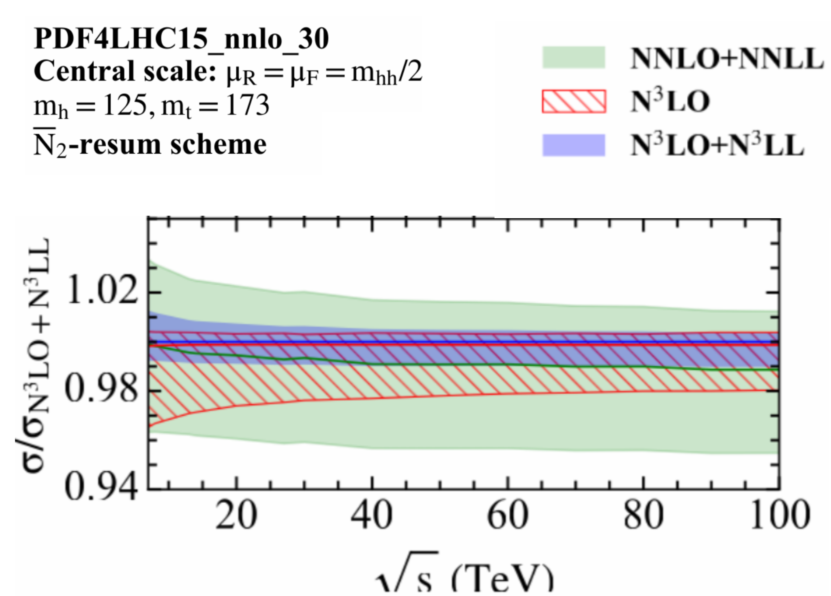
➤ Prominent QCD corrections

- LO → NLO : (87% , 84%)
- NLO → NNLO : (18% , 16%)
- NNLO → N^{3}LO : (3% , 2.7%)
- NNLO → NNLO + NNLL : (3% , 1.7%)
- N^{3}LO → N^{3}LO + N^{3}LL : (0.13% , 0.11%)
- NNLO + NNLL → N^{3}LO + N^{3}LL :(0.5% , 1.14%)

at (13, 100) TeV







INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section

in unit of fb central scale $\mu_0 = \frac{m_{hh}}{2}$

$\sqrt{s}$ [TeV]	Order k	N^{k}LO	N^{k}LO+N^{k}LL	
			N_2 scheme	$\overline{N}_2$ scheme
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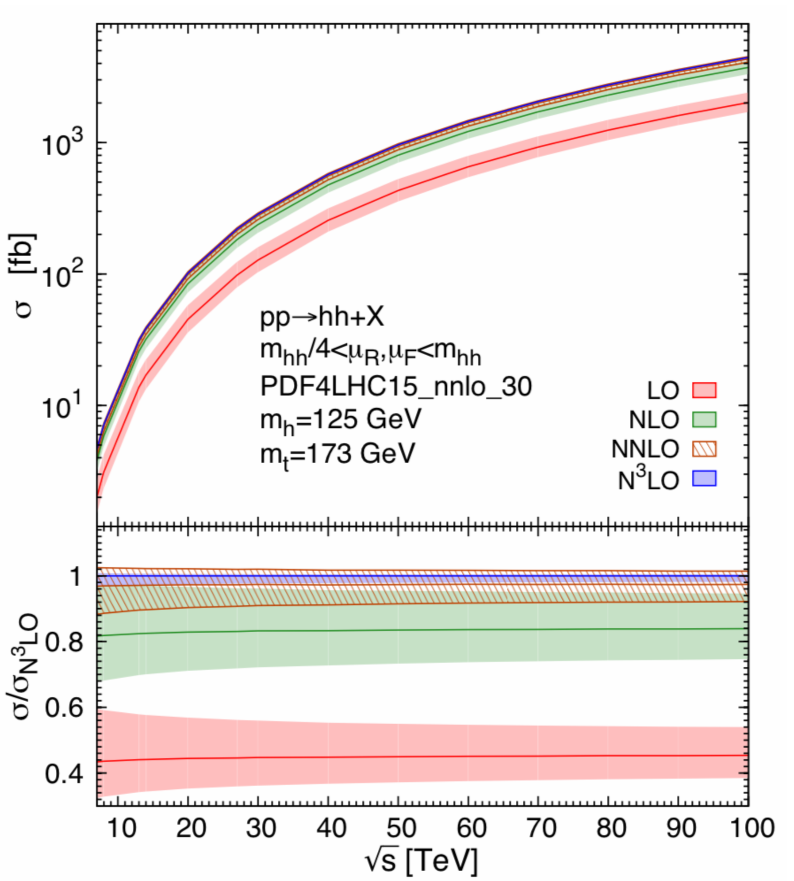
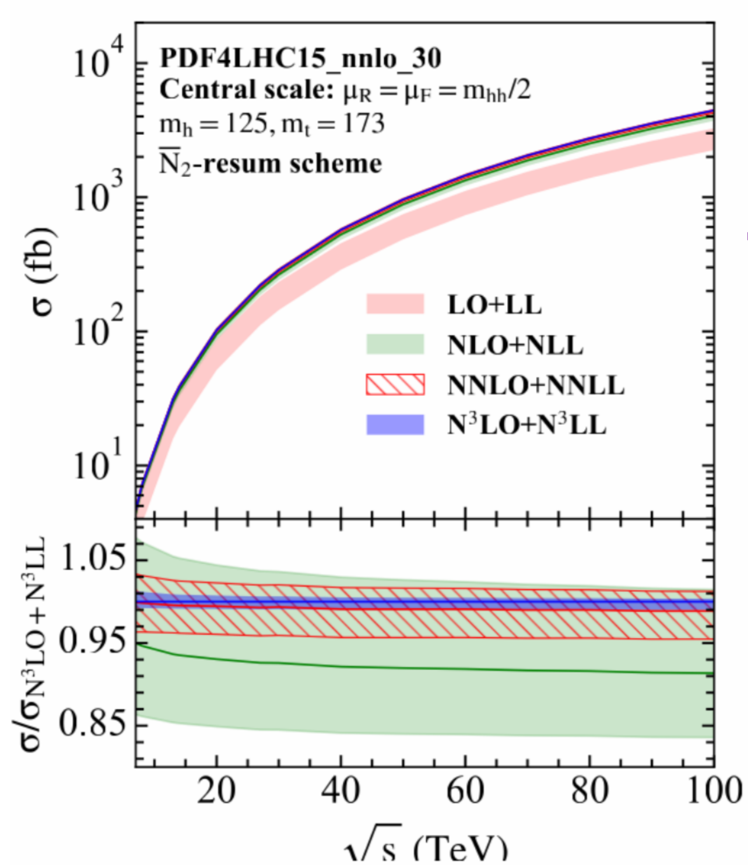
➤ Scale reduction to percent-level

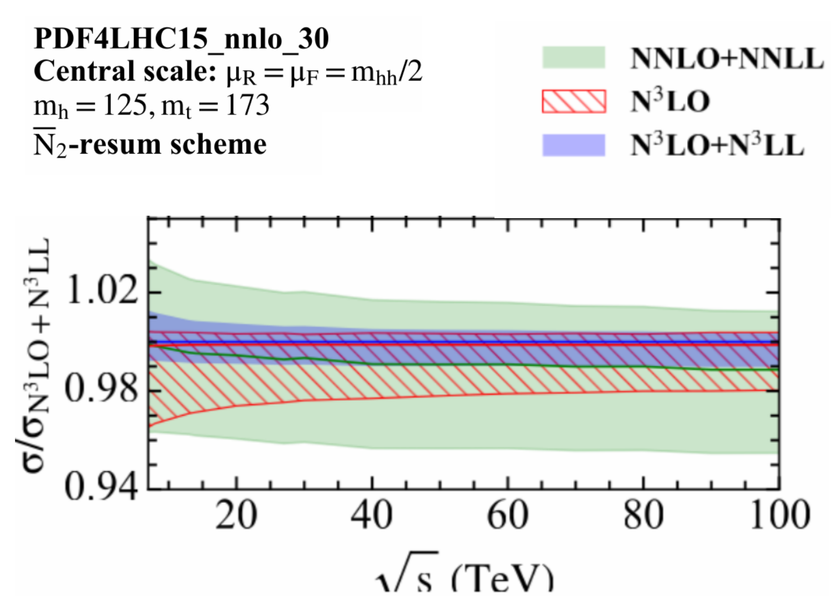
- $N^3LO \to N^3LO + N^3LL$: factor 2 reduction
- NNLO $\to N^3LO$: factor 4 reduction

Scale uncertainty: sub-percent level

➤ PDF uncertainty (3%) > scale uncertainty

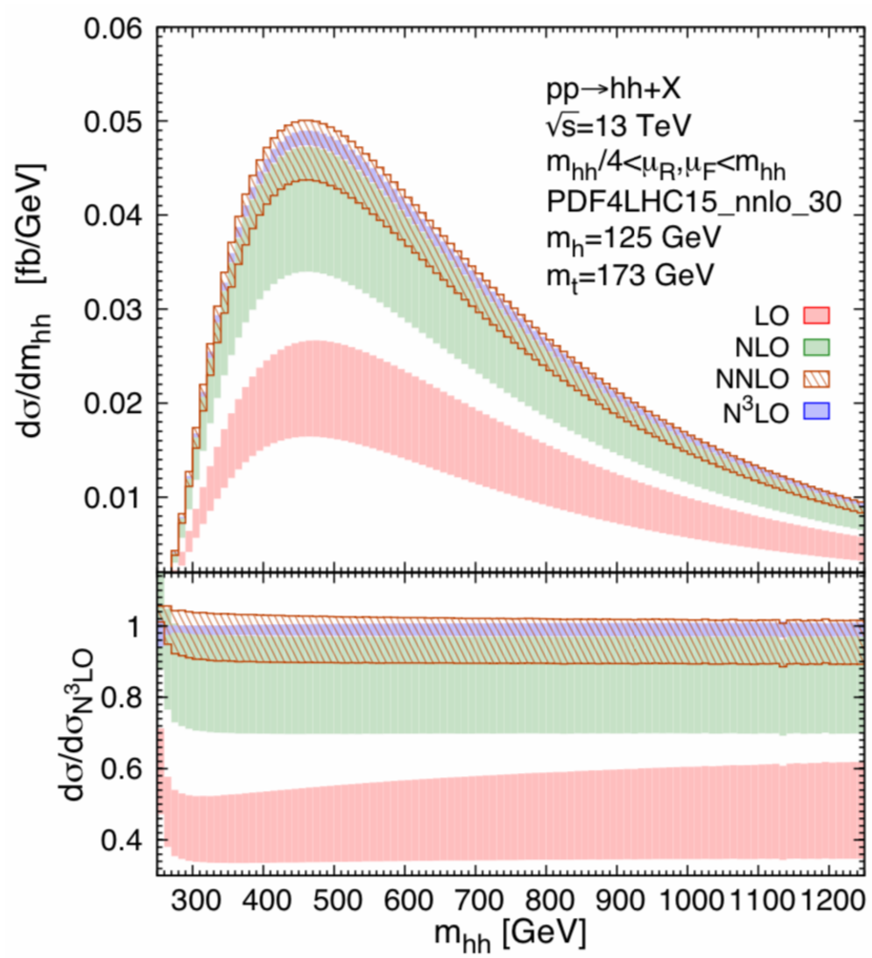






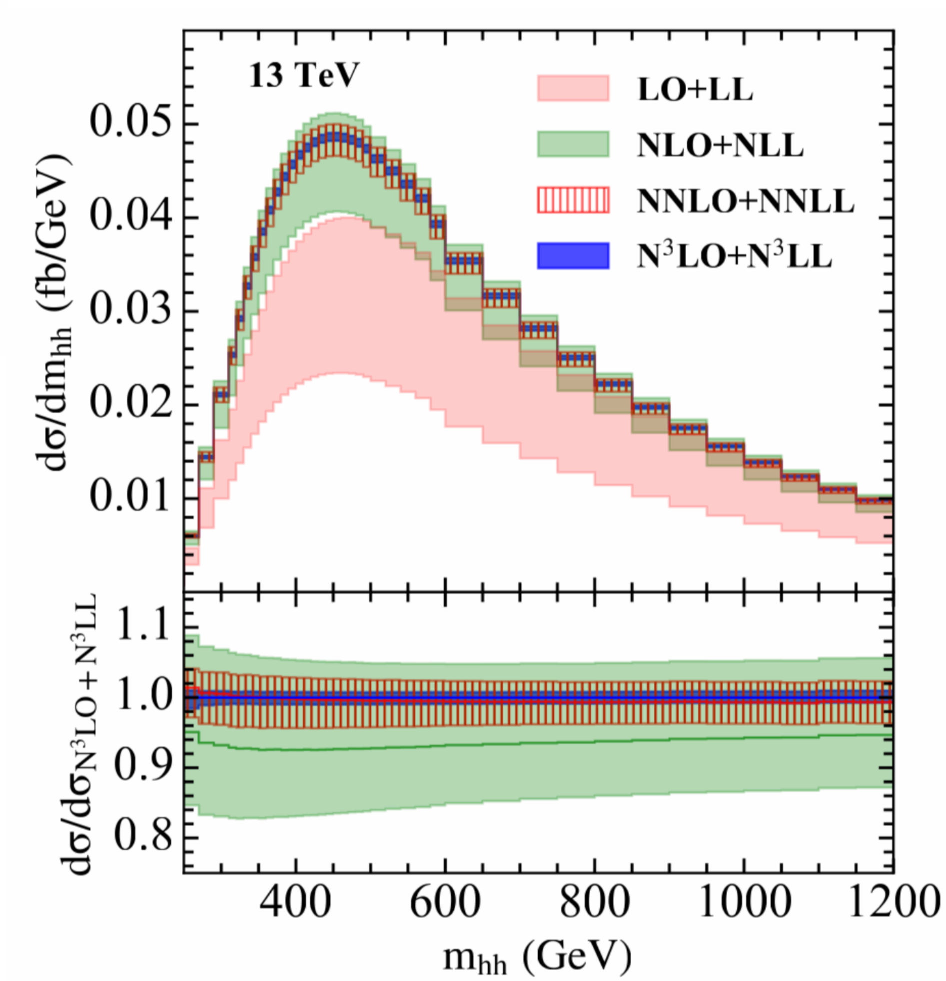
INFINITE TOP QUARK MASS LIMIT : RESULTS

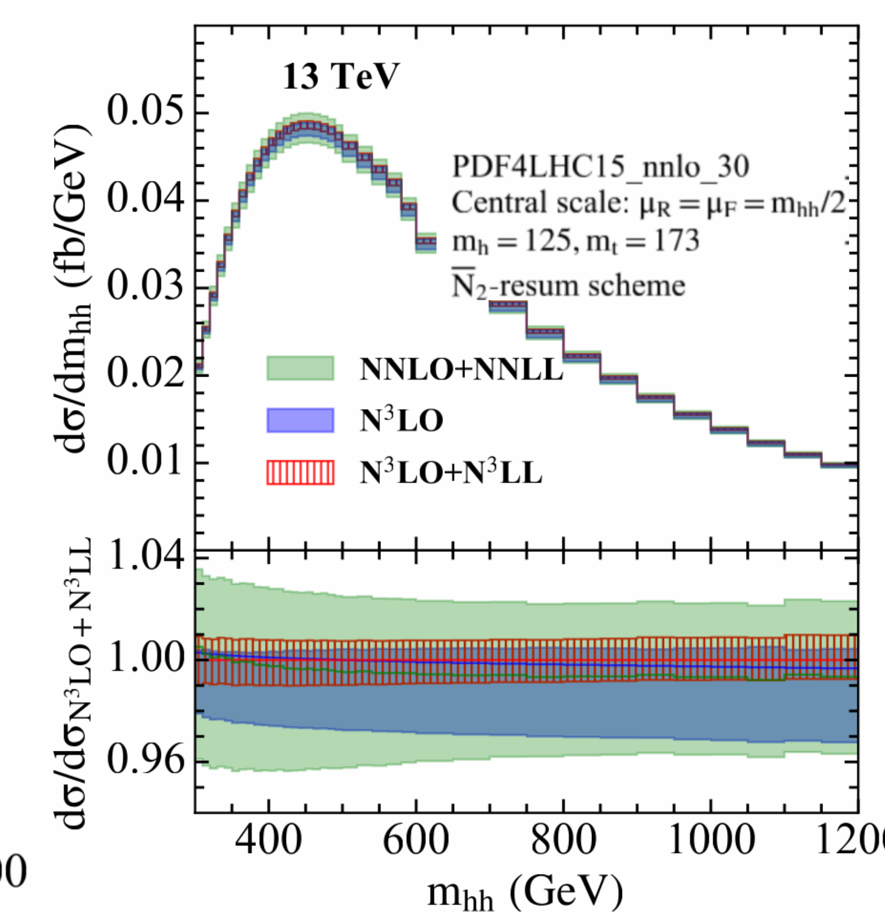
Invariant Mass distributions



- Shape of the distribution is almost unchanged
- Significant scale reductions with good perturbative convergence

Inclusion of higher order stabilises the invariant mass distribution





WITH TOP QUARK MASS EFFECTS

- To improve the results at large m_t-limit, they are combined with the finite top quark mass effects.

Not unique! *Different approximations*

- For N^3LO following approximations are considered

With $\left\{ \begin{array}{ll} N^kLO & \text{infinite top-quark mass limit} \\ N^lLO & \text{full top-quark mass dependence} \end{array} \right.$ $k > l$

Chen, Li, Shao, Wang (JHEP'20)

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Chen, Li, Shao, Wang (JHEP'20)

Only improve leading mt expansion term

- $\mathbf{N^kLO \oplus N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO \oplus N^lLO_{m_t}}} = d\sigma^{\mathrm{N^lLO}}_{m_t} + d\sigma^{\mathrm{N^kLO}}_{m_t\to\infty} - d\sigma^{\mathrm{N^lLO}}_{m_t\to\infty}$ *missing top mass in correction*

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missing top mass in correction

- $\mathbf{N^kLO_{B-i}{\oplus}N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO_{B-i} \oplus N^lLO_{m_t}}} = d\sigma^{\mathrm{N}^l\mathrm{LO}}_{m_t} + \left(d\sigma^{\mathrm{N^kLO}}_{m_t=\infty} - d\sigma^{\mathrm{N^lLO}}_{m_t=\infty}\right) \frac{d\sigma^{\mathrm{LO}}_{m_t}}{d\sigma^{\mathrm{LO}}_{m_t\to\infty}}$

Born mass improved for correction

WITH TOP QUARK MASS EFFECTS

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Not unique! *Different approximations*

- For N^3LO following approximations are considered

With { N^kLO infinite top-quark mass limit; N^lLO full top-quark mass dependence } $k > l$

Chen, Li, Shao, Wang (JHEP'20)

Only improve leading mt expansion term
missing top mass in correction

- $\mathbf{N^kLO \oplus N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO \oplus N^lLO_{m_t}}} = d\sigma_{m_t}^{\mathrm{N}^l\mathrm{LO}} + d\sigma_{m_t \to \infty}^{\mathrm{N}^k\mathrm{LO}} - d\sigma_{m_t \to \infty}^{\mathrm{N}^l\mathrm{LO}}$

- $\mathbf{N^kLO_{B-i} \oplus N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO_{B-i} \oplus N^lLO_{m_t}}} = d\sigma_{m_t}^{\mathrm{N}^l\mathrm{LO}} + \left(d\sigma_{m_t=\infty}^{\mathrm{N^kLO}} - d\sigma_{m_t=\infty}^{\mathrm{N^lLO}}\right) \frac{d\sigma_{m_t}^{\mathrm{LO}}}{d\sigma_{m_t \to \infty}^{\mathrm{LO}}}$

Born mass improved for correction

- $\mathbf{N^kLO \otimes N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO \otimes N^lLO_{m_t}}} = d\sigma_{m_t}^{\mathrm{N}^l\mathrm{LO}} \frac{d\sigma_{m_t \to \infty}^{\mathrm{N}^k\mathrm{LO}}}{d\sigma_{m_t \to \infty}^{\mathrm{N}^l\mathrm{LO}}} = d\sigma_{m_t}^{\mathrm{N}^l\mathrm{LO}} + \left(d\sigma_{m_t=\infty}^{\mathrm{N^kLO}} - d\sigma_{m_t=\infty}^{\mathrm{N^lLO}}\right) \frac{d\sigma_{m_t}^{\mathrm{N}^l\mathrm{LO}}}{d\sigma_{m_t \to \infty}^{\mathrm{N}^l\mathrm{LO}}}.$

Same K factor for mass correction

WITH TOP QUARK MASS EFFECTS

- To improve the results at large m_t-limit, they are combined with the finite top quark mass effects. *Not unique!* *Different approximations*
- For $\mathrm{N^3LO}$ following approximations are considered

With $\begin{cases} \mathrm{N^kLO} & \text{infinite top-quark mass limit} \\ \mathrm{N^lLO} & \text{full top-quark mass dependence} \end{cases}$ $k > l$

Chen, Li, Shao, Wang (JHEP'20)

Only improve leading mt expansion term

missing top mass in correction

- $\mathbf{N^kLO \oplus N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO\oplus N^lLO_{m_t}}} = d\sigma^{\mathrm{N^lLO}}_{m_t} + d\sigma^{\mathrm{N^kLO}}_{m_t\to\infty} - d\sigma^{\mathrm{N^lLO}}_{m_t\to\infty}$

- $\mathbf{N^kLO_{B-i} \oplus N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO_{B-i}\oplus N^lLO_{m_t}}} = d\sigma^{\mathrm{N^lLO}}_{m_t} + \left(d\sigma^{\mathrm{N^kLO}}_{m_t=\infty} - d\sigma^{\mathrm{N^lLO}}_{m_t=\infty}\right)\dfrac{d\sigma^{\mathrm{LO}}_{m_t}}{d\sigma^{\mathrm{LO}}_{m_t\to\infty}}$

Born mass improved for correction

- $\mathbf{N^kLO \otimes N^lLO}_{m_t}$: $d\sigma^{\mathbf{N^kLO\otimes N^lLO_{m_t}}} = d\sigma^{\mathrm{N^lLO}}_{m_t}\dfrac{d\sigma^{\mathrm{N^kLO}}_{m_t\to\infty}}{d\sigma^{\mathrm{N^lLO}}_{m_t\to\infty}} = d\sigma^{\mathrm{N^lLO}}_{m_t} + \left(d\sigma^{\mathrm{N^kLO}}_{m_t=\infty} - d\sigma^{\mathrm{N^lLO}}_{m_t=\infty}\right)\dfrac{d\sigma^{\mathrm{N^lLO}}_{m_t}}{d\sigma^{\mathrm{N^lLO}}_{m_t\to\infty}}.$

Same K factor for mass correction

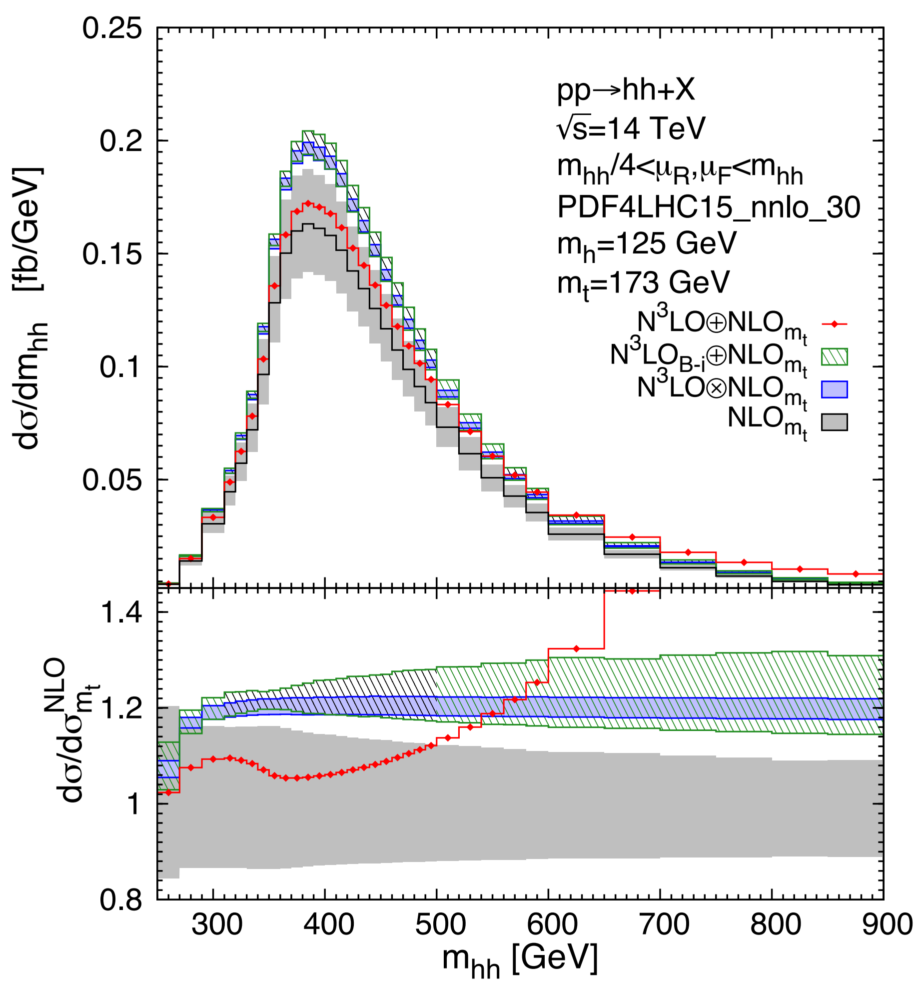
WITH TOP QUARK MASS EFFECTS : RESULTS

N3LO

NLO_{m_t} is 6.8% larger than $\mathrm{NLO}|_{m_t \to \infty}$ at 13TeV - POWHEG

Heinrich et al. JHEP'19

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO_{m_t}	$27.56^{+14\%}_{-13\%}$	$32.64^{+14\%}_{-12\%}$	$126.2^{+12\%}_{-10\%}$	$1119^{+13\%}_{-13\%}$
$\mathrm{NNLO}\oplus\mathrm{NLO}_{m_t}$	$32.16^{+5.9\%}_{-5.9\%}$	$38.29^{+5.6\%}_{-5.5\%}$	$157.3^{+3.0\%}_{-4.7\%}$	$1717^{+5.8\%}_{-12\%}$
$\mathrm{NNLO}_{\mathrm{B-i}}\oplus\mathrm{NLO}_{m_t}$	$33.08^{+5.0\%}_{-4.9\%}$	$39.16^{+4.9\%}_{-5.0\%}$	$150.8^{+4.6\%}_{-5.7\%}$	$1330^{+4.0\%}_{-7.2\%}$
$\mathrm{NNLO}\otimes\mathrm{NLO}_{m_t}$	$32.47^{+5.3\%}_{-7.8\%}$	$38.42^{+5.2\%}_{-7.6\%}$	$147.6^{+4.8\%}_{-6.7\%}$	$1298^{+4.2\%}_{-5.3\%}$
$\mathrm{N^3LO}\oplus\mathrm{NLO}_{m_t}$	$33.06^{+2.1\%}_{-2.9\%}$	$39.40^{+1.7\%}_{-2.8\%}$	$163.3^{+4.0\%}_{-8.3\%}$	$1833^{+14\%}_{-20\%}$
$\mathrm{N^3LO}_{\mathrm{B-i}}\oplus\mathrm{NLO}_{m_t}$	$34.17^{+1.9\%}_{-4.6\%}$	$40.44^{+1.9\%}_{-4.7\%}$	$155.5^{+2.3\%}_{-5.0\%}$	$1372^{+2.8\%}_{-5.0\%}$
$\mathrm{N^3LO}\otimes\mathrm{NLO}_{m_t}$	$33.43^{+0.66\%}_{-2.8\%}$	$39.56^{+0.64\%}_{-2.7\%}$	$151.7^{+0.53\%}_{-2.4\%}$	$1333^{+0.51\%}_{-1.8\%}$



$\mathrm{N^3LO} \otimes \mathrm{NLO}_{m_t}$ *: most accurate out of three above approximations*

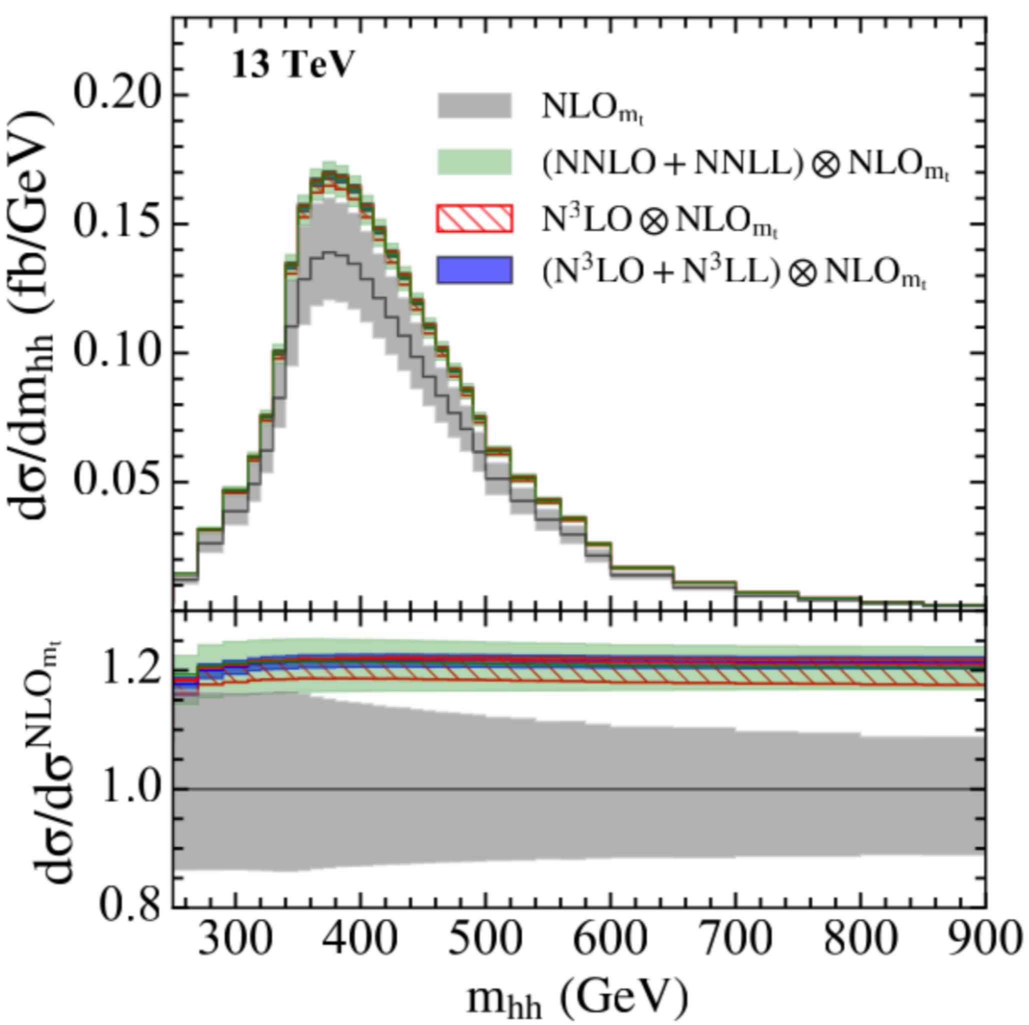
- Enhancement :
  - 20% for $\mathrm{NLO}_{m_t} \to \mathrm{NNLO} \otimes \mathrm{NLO}_{m_t}$ — At 13 TeV
  - 3% for $\to \mathrm{NNLO} \otimes \mathrm{NLO}_{m_t} \to \mathrm{N^3LO} \otimes \mathrm{NLO}_{m_t}$
- Scale uncertainty within 3%

WITH TOP QUARK MASS EFFECTS : RESULTS

N3LO+N3LL

For N3LO+N3LL, we consider only the NLO-improved approximation:

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO_{m_t}	$27.56^{+13.9\%}_{-12.7\%}$	$32.64^{+13.5\%}_{-12.47\%}$	$126.1^{+11.5\%}_{-10.4\%}$	$1119^{+10.7\%}_{-9.9\%}$
$(\mathrm{NNLO}+\mathrm{NNLL})\otimes\mathrm{NLO}_{m_t}$	$33.33^{+3.0\%}_{-3.3\%}$	$39.42^{+3.0\%}_{-3.4\%}$	$150.8^{+2.7\%}_{-3.4\%}$	$1320^{+2.4\%}_{-3.4\%}$
$\mathrm{N^3LO}\otimes\mathrm{NLO}_{m_t}$	$33.43^{+0.50\%}_{-2.8\%}$	$39.56^{+0.50\%}_{-2.7\%}$	$151.7^{+0.46\%}_{-2.3\%}$	$1333^{+0.51\%}_{-1.8\%}$
$(\mathrm{N^3LO}+\mathrm{N^3LL})\otimes\mathrm{NLO}_{m_t}$	$33.47^{+0.88\%}_{-0.85\%}$	$39.60^{+0.85\%}_{-0.87\%}$	$151.9^{+0.63\%}_{-0.94\%}$	$1335^{+0.35\%}_{-1.0\%}$



- **Enhancement :** At 13 TeV
  - 21% for $\mathrm{NLO}_{m_t} \rightarrow (\mathrm{NNLO}+\mathrm{NNLL})\otimes\mathrm{NLO}_{m_t}$
  - 0.4% for $\rightarrow (\mathrm{NNLO}+\mathrm{NNLL})\otimes\mathrm{NLO}_{m_t} \rightarrow (\mathrm{N^3LO}+\mathrm{N^3LL})\otimes\mathrm{NLO}_{m_t}$
- **Scale uncertainty :**
  - $(\mathrm{NNLO}+\mathrm{NNLL})\otimes\mathrm{NLO}_{m_t} \sim 3\%$
  - $(\mathrm{N^3LO}+\mathrm{N^3LL})\otimes\mathrm{NLO}_{m_t}$: sub-percent level

SUMMARY & OUTLOOK

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- For the Higgs pair productions through gluon fusion channel, in the large top mass limit, we have the N^3LO calculations Chen, Li, Shao, Wang (PLB'20, JHEP'20) , which further extended to $N^3LO + N^3LL$

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- To improve the finite top quark mass corrections, we reweight the NLO_{m_t} with higher order K-factors in three different ways: ($N^3LO \oplus NLO_{m_t}$, $N^3LO_{B-i} \oplus NLO_{m_t}$, $N^3LO \otimes NLO_{m_t}$)
  - $N^3LO \otimes NLO_{m_t}$ captures the best scale uncertainty, with 3% improvement in QCD corrections
  - $\left(N^3LO + N^3LL\right) \otimes NLO_{m_t}$ captures sub-percent level scale uncertainty

SUMMARY & OUTLOOK

- Mass scheme uncertainties : (-4%, 18%). Not expected to improve from our results
- Further improvement in the results could be achieved by combining our results with the most advanced finite top mass approximation - $NNLO_{FT}$

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At 13 TeV

$$\frac{\left(\mathrm{N^3LO + N^3LL}\right)}{\mathrm{NNLO}} \times \mathrm{NNLO_{FT}} = 31.79^{+0.88\%}_{-0.85\%}$$

Scale reduction by factor > 4

$$\mathrm{NNLO_{FT}} : 31.05^{+2.2\%}_{-5.0\%}$$

5% missing top mass corrections

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THANKS FOR THE ATTENTION !

Back up slides

INFINITE TOP QUARK MASS LIMIT : CROSS CHECKS

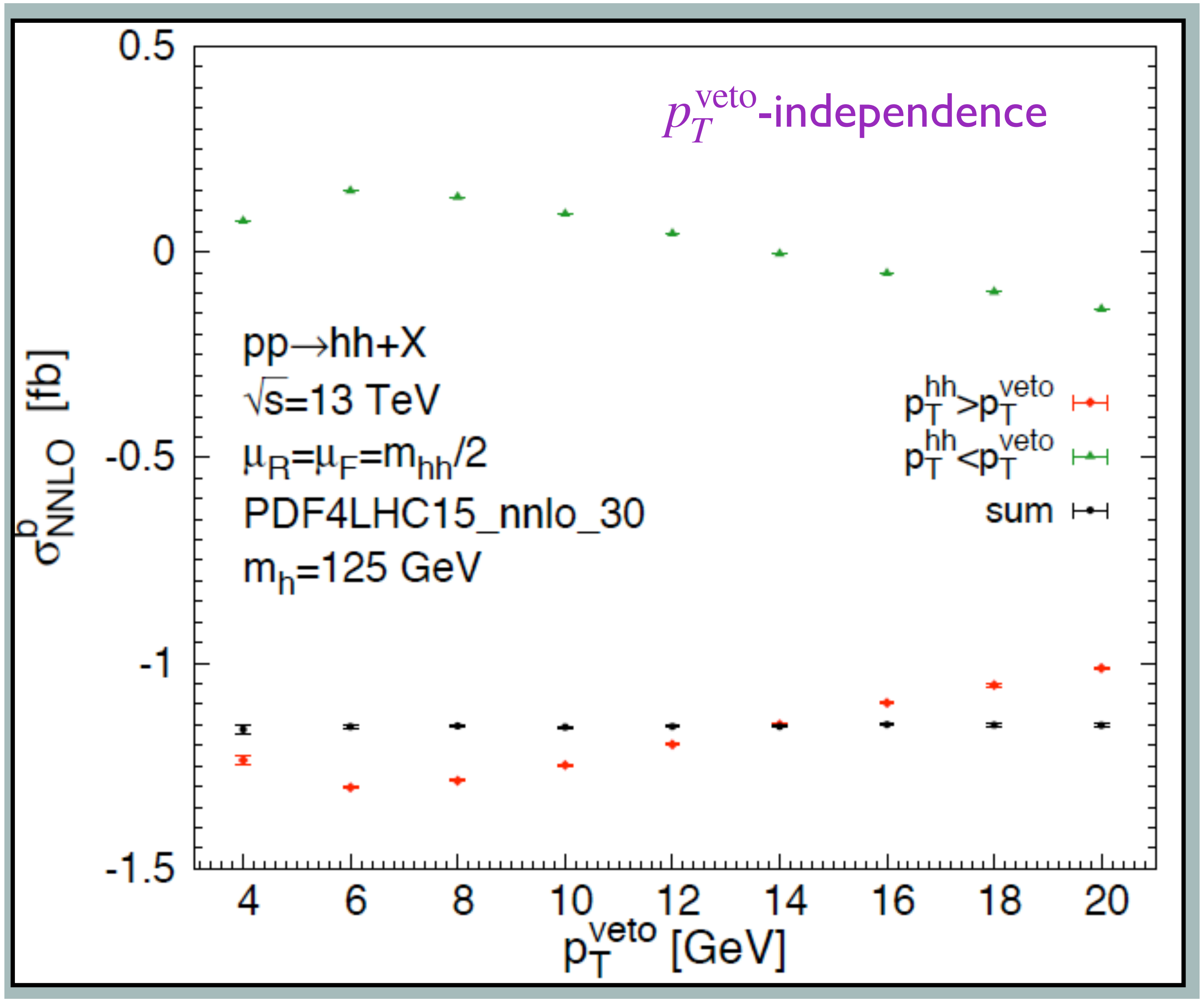
Chen, Li, Shao, Wang (PLB'20, JHEP'20)

	LO	NLO	NNLO	N^{3}LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

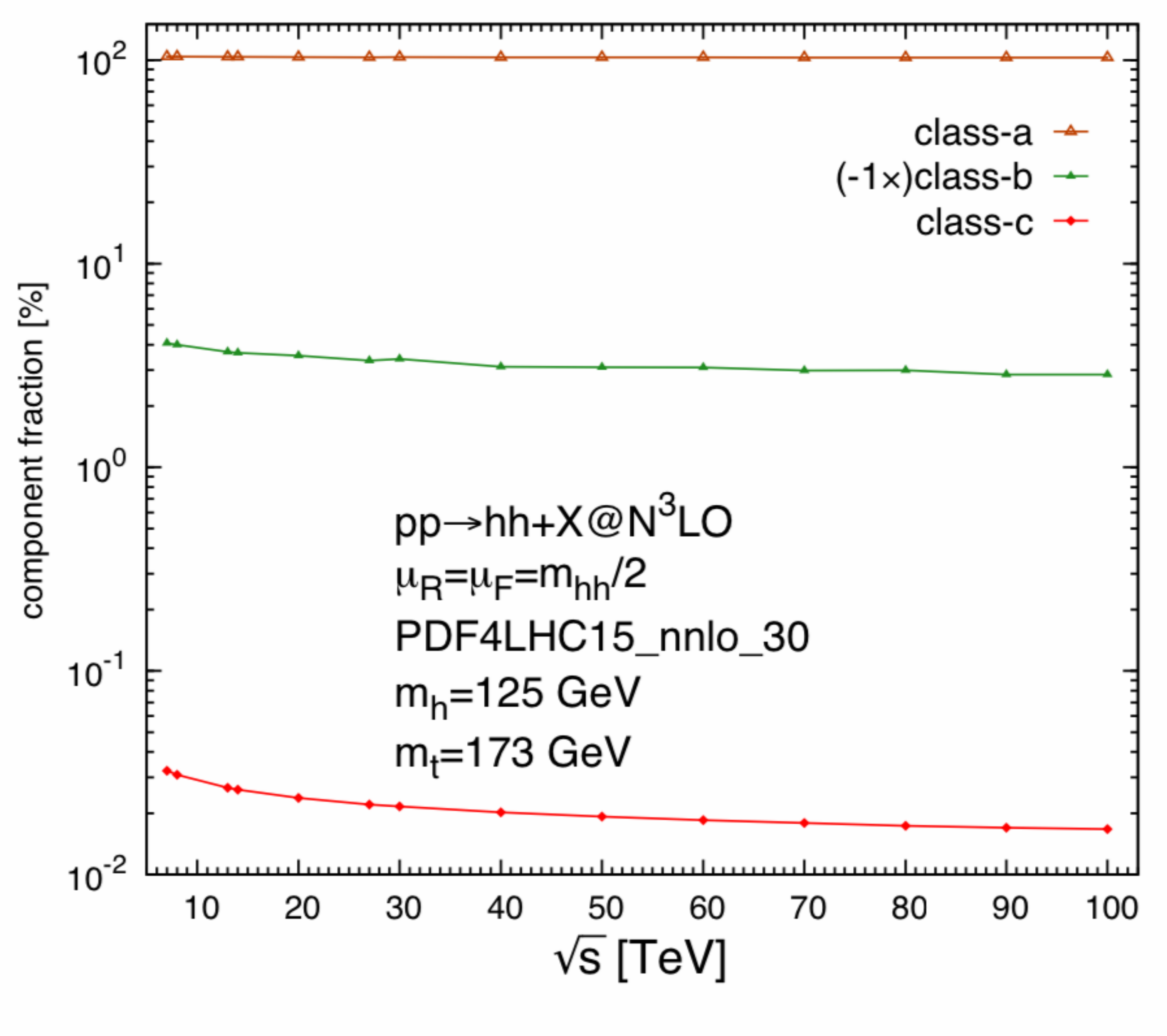
- ✓ At least two independent calculations
- ✓ Orthogonal check with NNLO ggHH
- ✓ Check piece-by-piece

	NLO	NNLO	N^{3}LO
order	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	iHixs2 q_T-subtraction MG5_aMC	iHixs2 q_T-subtraction	iHixs2
b	-	q_T-subtraction MG5_aMC	q_T-subtraction
c	-	-	q_T-subtraction MG5_aMC

CLASS B – p_T^{veto} INDEPENDENCE



CLASS A,B,C



class-a
(-1×)class-b
class-c
pp→hh+X@N3LO
μR=μF=mhh/2
PDF4LHC15_nnlo_30
mh=125 GeV
mt=173 GeV
component fraction [%]
√s [TeV]
10
20
30
40
50
60
70
80
90
100

THRESHOLD RESUMMATION : OVERVIEW

- Perform Inverse mellin transfrom numerically to get the real space cross section
- To avoid double counting, Matching procedure :

$$\sigma^{\mathrm{N^3LO+N^3LL}} = \left\{ \sigma^{\mathrm{N^3LL}} - \sigma^{\mathrm{N^3LL}} \Big|_{\mathcal{O}(\alpha_s^5)} \right\} + \sigma^{\mathrm{N^3LO}}$$

↓

Improves the predictions
with missing higher order
logarithmic terms.

INFINITE TOP QUARK MASS LIMIT : RESULTS

Other differential distributions

Chen, Li, Shao, Wang (PLB'20, JHEP'20)

- Approximated N^{3}LO differential distribution: $d\sigma_{hh}^{(a,1),\mathrm{N^3LO}}$ is not known

$$\frac{d\sigma_{hh}^{\mathrm{AN^3LO}}}{dO} = \frac{d\sigma_{hh}^{(\mathrm{a},1),\mathrm{NNLO}}}{dO}\frac{\sigma_{hh}^{(\mathrm{a},1),\mathrm{N^3LO}}}{\sigma_{hh}^{(\mathrm{a},1),\mathrm{NNLO}}} + \frac{d\sigma_{hh}^{(\mathrm{a},2),\mathrm{N^3LO}}}{dO} + \frac{d\sigma_{hh}^{\mathrm{b,NNLO}}}{dO} + \frac{d\sigma_{hh}^{\mathrm{c,NLO}}}{dO}.$$

Known

Using q_t-subtraction method

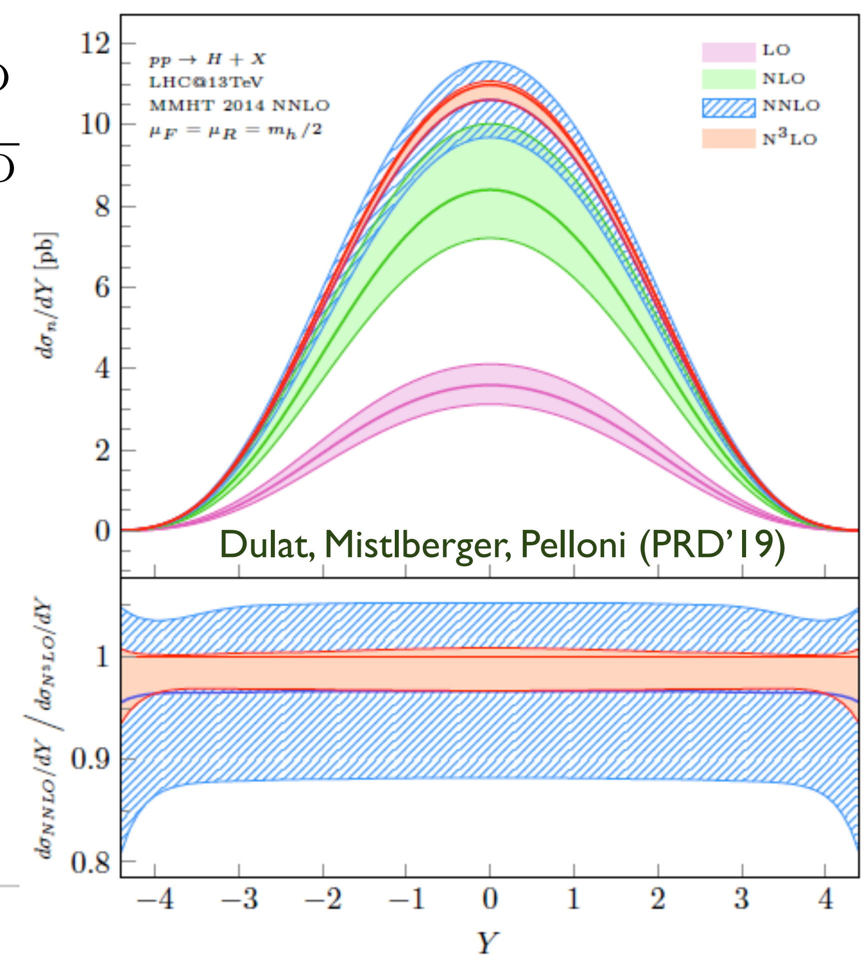
Need knowledge of fully-differential N^{3}LO ggH

Chen et al (PRL'21)

Assumption : $$\frac{d\sigma_{hh}^{(a,1),\mathrm{N^3LO}}}{dO} \overset{?}{\approx} \frac{d\sigma_{hh}^{(a,1),\mathrm{NNLO}}}{dO}\frac{\sigma_{hh}^{(a,1),\mathrm{N^3LO}}}{\sigma_{hh}^{(a,1),\mathrm{NNLO}}}$$

$$d\sigma_{hh}^{a} = d\sigma_{hh}^{(a,1)} + d\sigma_{hh}^{(a,2)}$$
$$d\sigma_{hh}^{(a,1)} \equiv d\sigma_{hh}^{a}\Big|_{C_{hh}\to C_h},$$
$$d\sigma_{hh}^{(a,2)} \equiv d\sigma_{hh}^{a} - d\sigma_{hh}^{(a,1)},$$

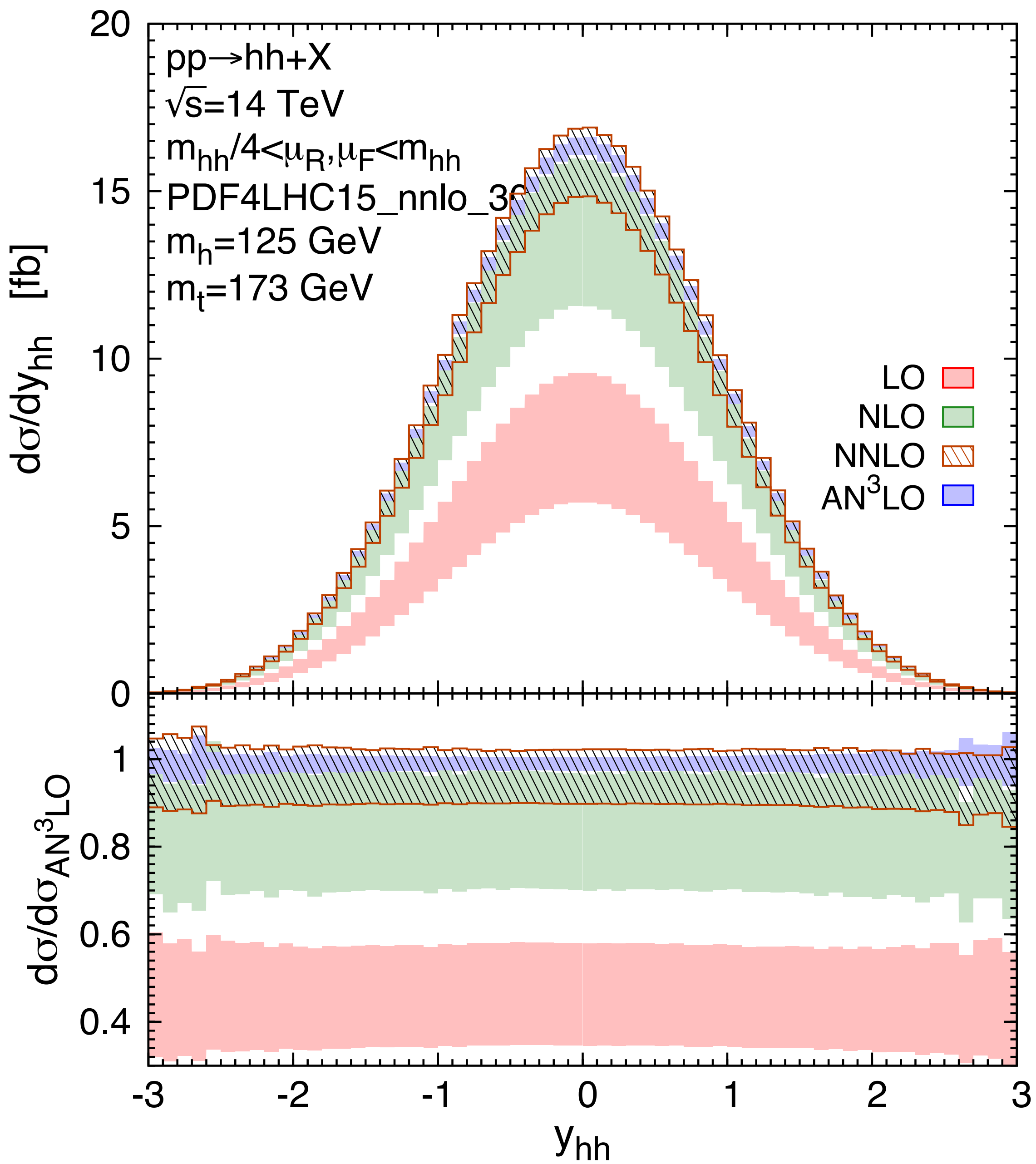
It is justified given the extremely fat K-factor for $\frac{d\sigma_h}{dY}$

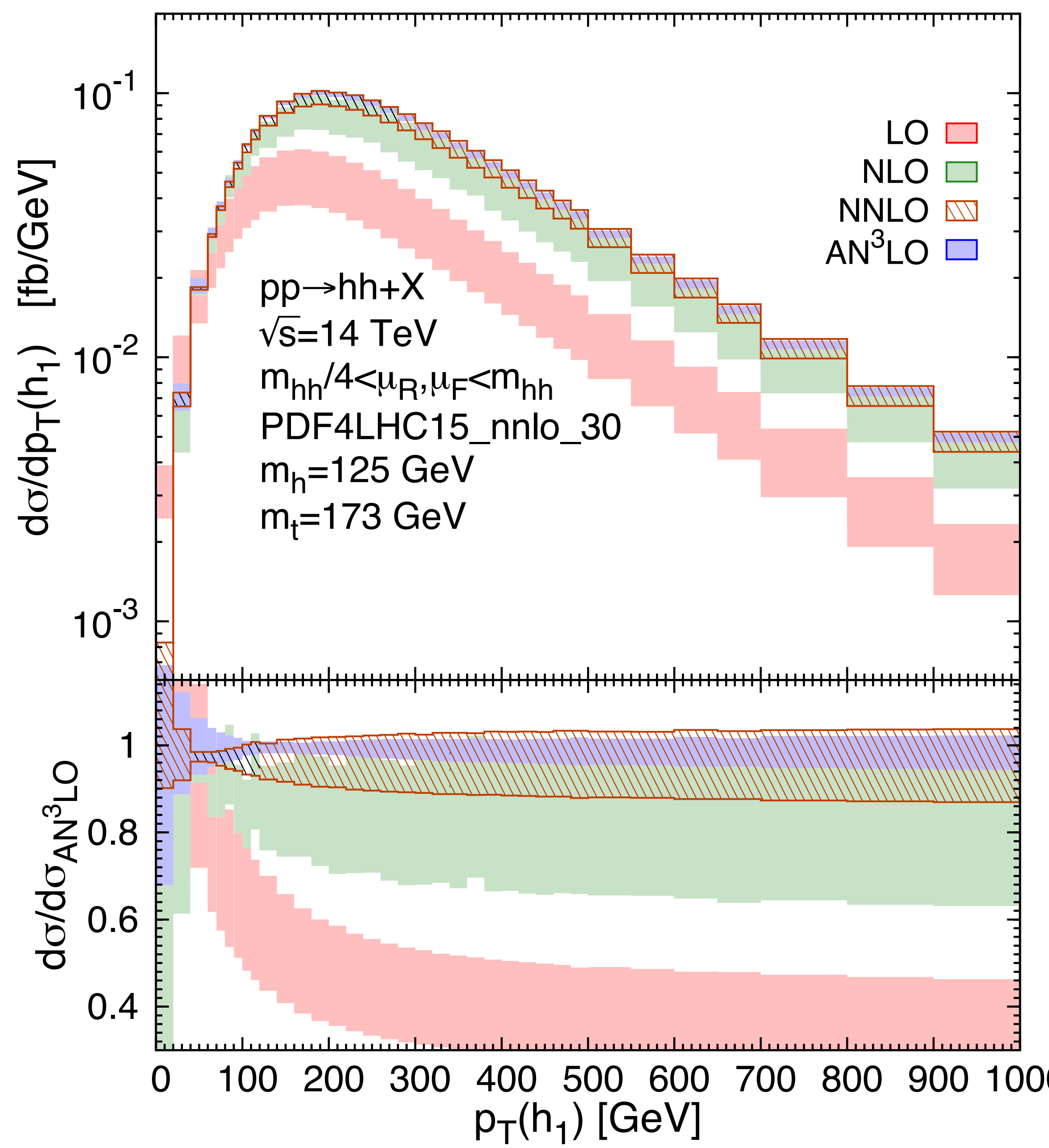


INFINITE TOP QUARK MASS LIMIT : RESULTS

Other differential distributions

Chen, Li, Shao, Wang
(PLB'20, JHEP'20)



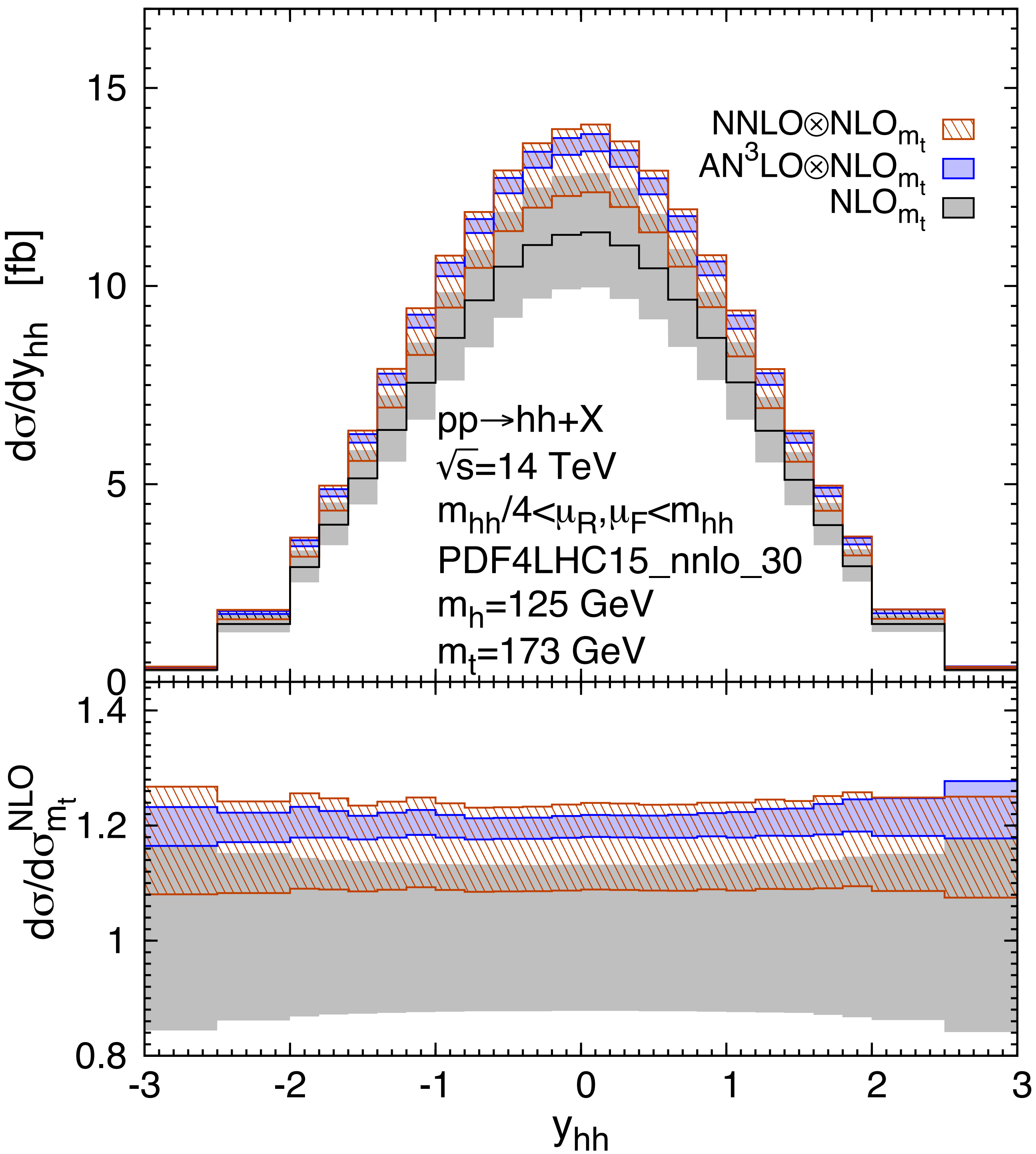


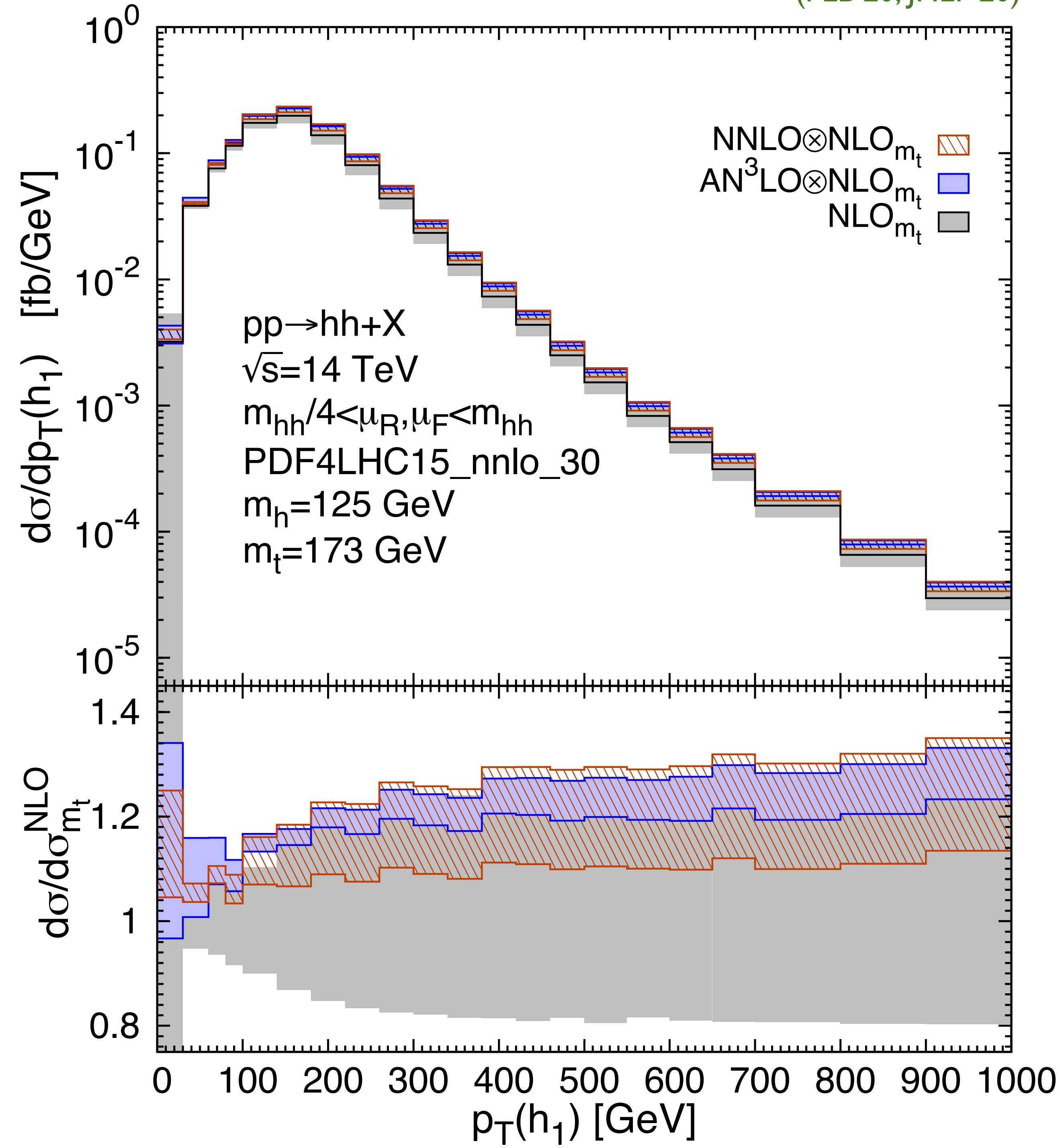
INFINITE TOP QUARK MASS LIMIT : RESULTS

Other differential distributions

NLO_{mt} from Powheg, Heinrich et al. JHEP'19

Chen, Li, Shao, Wang (PLB'20, JHEP'20)





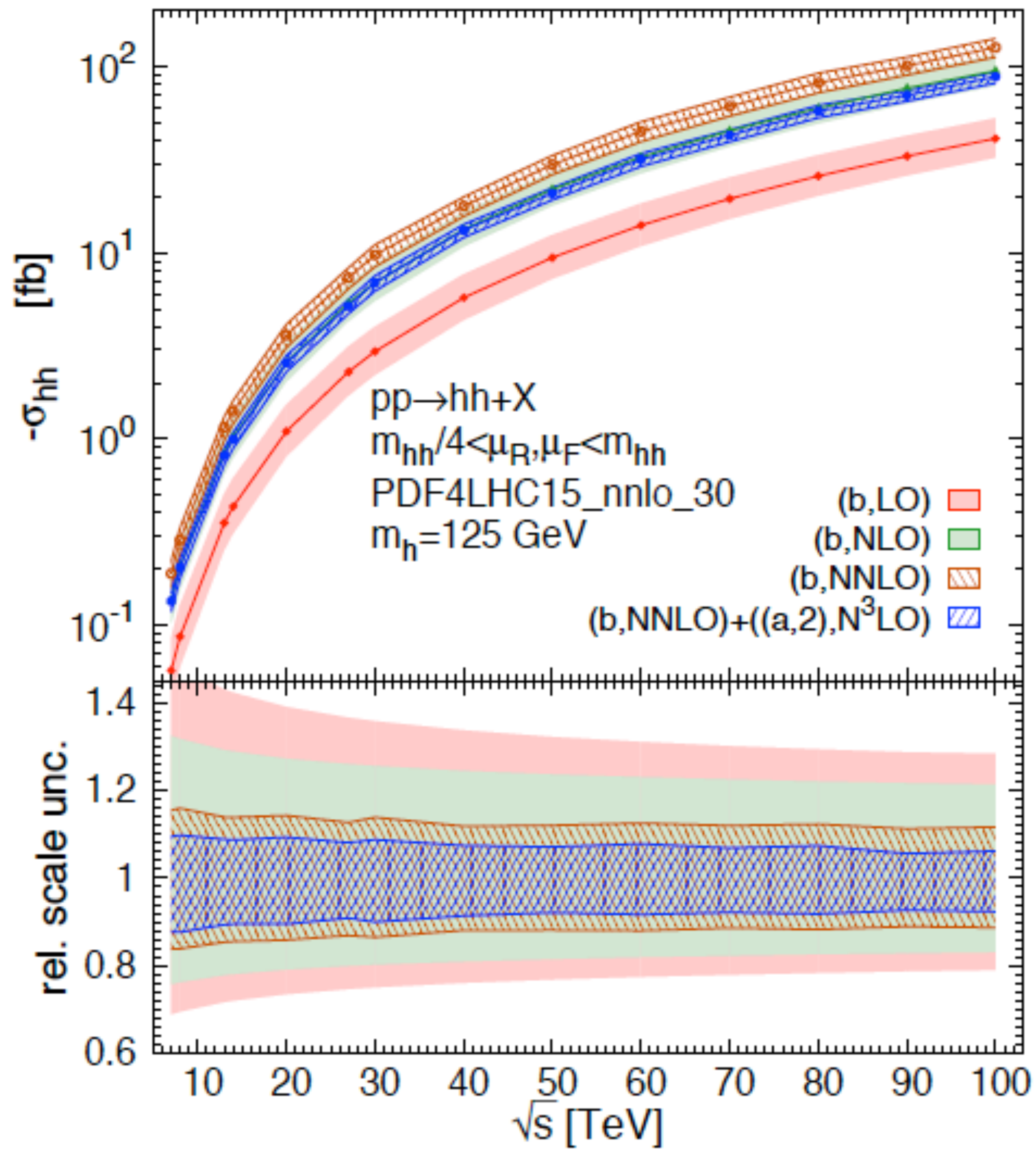
INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section

Chen, Li, Shao, Wang (PLB'20, JHEP'20)

➤ Non trivial scale cancellation occurs due to operator scale mixing

Zoller (JHEP'16)

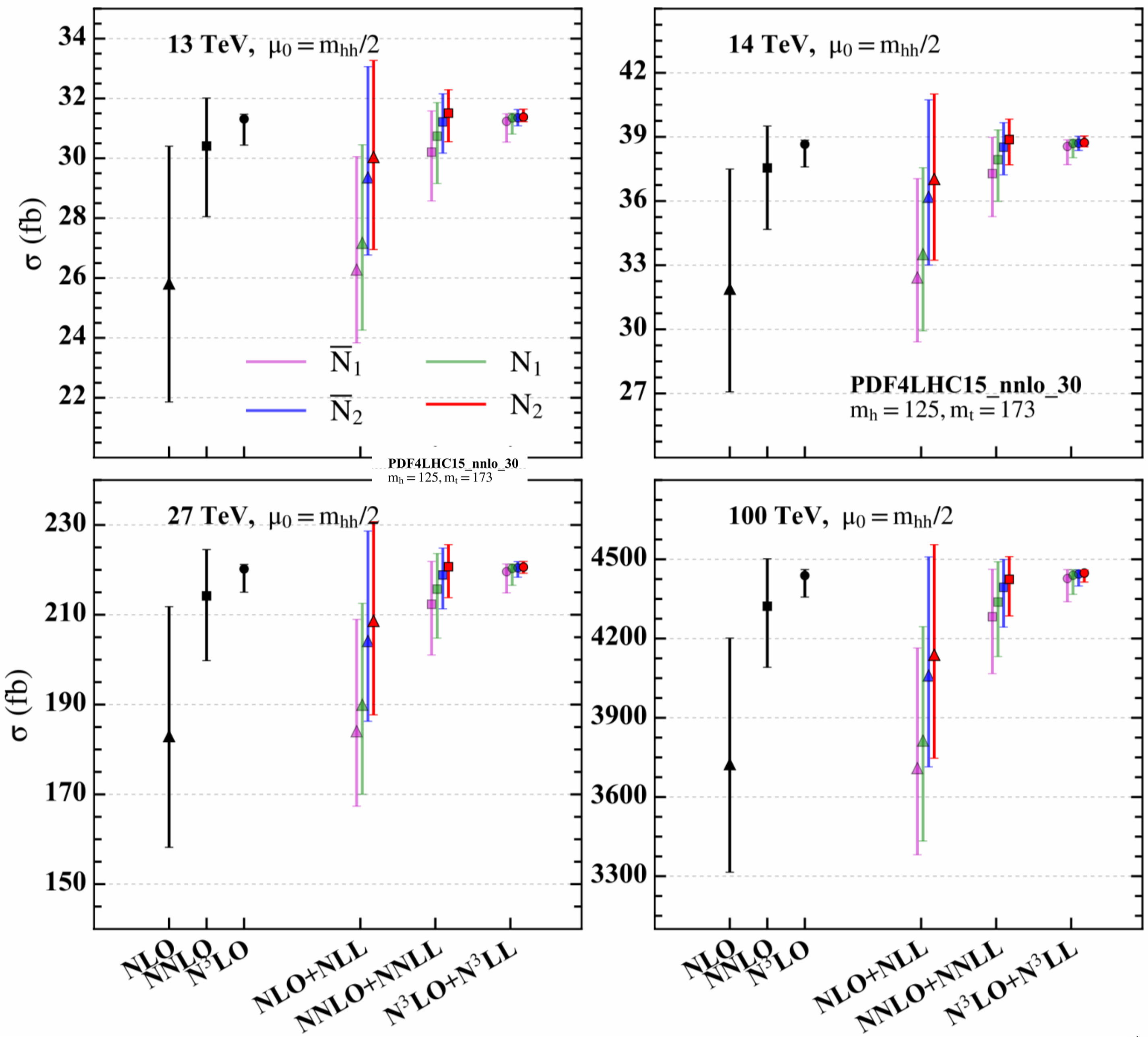


Class-b with including class-(a,2) further reduce the scale uncertainty

$$d\sigma^a_{hh} = d\sigma^{(a,1)}_{hh} + d\sigma^{(a,2)}_{hh}$$
$$d\sigma^{(a,1)}_{hh} \equiv d\sigma^a_{hh}\Big|_{C_{hh}\to C_h},$$
$$d\sigma^{(a,2)}_{hh} \equiv d\sigma^a_{hh} - d\sigma^{(a,1)}_{hh},$$

INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section



at 14 TeV

$$\sigma_{N^3LO+N^3LL} = 38.70 \begin{pmatrix} +0.85\% \\ -0.87\% \end{pmatrix}_{\text{scale}} \begin{pmatrix} +0.08\% \\ -0.39\% \end{pmatrix}_{\text{scheme}} \text{ fb}$$

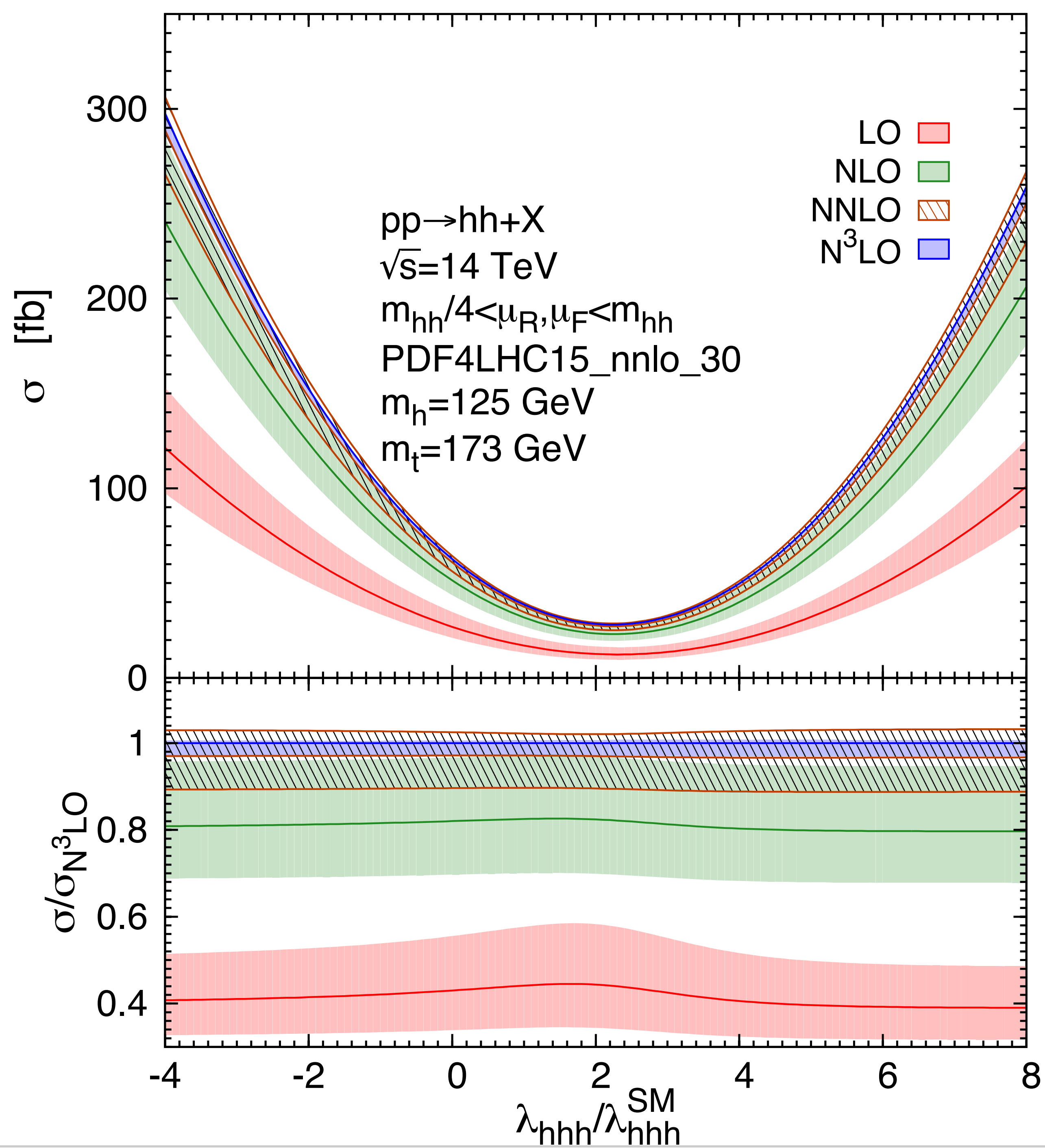
$\sqrt{s}$ [TeV]	Order k	N^kLO	N^kLO+N^kLL			
			N_1 scheme	N_2 scheme	$\overline{N}_1$ scheme	$\overline{N}_2$ scheme
13	0	$13.80^{+31\%}_{-22\%}$	$16.01^{+32\%}_{-23\%}$	$16.01^{+32\%}_{-23\%}$	$21.02^{+36\%}_{-24\%}$	$21.02^{+36\%}_{-24\%}$
	1	$25.81^{+18\%}_{-15\%}$	$27.17^{+12.1\%}_{-10.7\%}$	$30.04^{+10.8\%}_{-10.3\%}$	$26.30^{+14.4\%}_{-9.3\%}$	$29.36^{+12.6\%}_{-8.8\%}$
	2	$30.41^{+5.3\%}_{-7.8\%}$	$30.74^{+3.7\%}_{-5.1\%}$	$31.51^{+2.5\%}_{-3.0\%}$	$30.20^{+4.6\%}_{-5.4\%}$	$31.21^{+3.0\%}_{-3.3\%}$
	3	$31.31^{+0.50\%}_{-2.8\%}$	$31.34^{+0.51\%}_{-1.7\%}$	$31.37^{+0.84\%}_{-0.49\%}$	$31.23^{+0.81\%}_{-2.2\%}$	$31.35^{+0.88\%}_{-0.85\%}$
14	0	$17.06^{+31\%}_{-22\%}$	$19.72^{+32\%}_{-23\%}$	$19.72^{+32\%}_{-23\%}$	$25.80^{+35\%}_{-24\%}$	$25.80^{+35\%}_{-24\%}$
	1	$31.89^{+18\%}_{-15\%}$	$33.52^{+12\%}_{-10.7\%}$	$37.03^{+10.7\%}_{-10.3\%}$	$32.42^{+14.3\%}_{-9.3\%}$	$36.19^{+12.5\%}_{-8.8\%}$
	2	$37.55^{+5.2\%}_{-7.6\%}$	$37.93^{+3.7\%}_{-5.1\%}$	$38.88^{+2.4\%}_{-3.0\%}$	$37.28^{+4.5\%}_{-5.4\%}$	$38.52^{+3.0\%}_{-3.4\%}$
	3	$38.65^{+0.50\%}_{-2.7\%}$	$38.69^{+0.50\%}_{-1.7\%}$	$38.73^{+0.81\%}_{-0.51\%}$	$38.55^{+0.80\%}_{-2.2\%}$	$38.70^{+0.85\%}_{-0.87\%}$
27	0	$98.22^{+26\%}_{-19\%}$	$110.6^{+27\%}_{-20\%}$	$110.6^{+27\%}_{-20\%}$	$141.1^{+29\%}_{-21\%}$	$141.1^{+29\%}_{-21\%}$
	1	$183.0^{+16\%}_{-14\%}$	$190.0^{+11.9\%}_{-10.5\%}$	$208.6^{+10.6\%}_{-10\%}$	$18.41^{+13.5\%}_{-9.1\%}$	$204.2^{+12.0\%}_{-8.8\%}$
	2	$214.2^{+4.8\%}_{-6.7\%}$	$215.7^{+3.7\%}_{-5.1\%}$	$220.7^{+2.2\%}_{-3.1\%}$	$212.3^{+4.5\%}_{-5.3\%}$	$218.9^{+2.7\%}_{-3.4\%}$
	3	$220.2^{+0.46\%}_{-2.3\%}$	$220.3^{+0.44\%}_{-1.7\%}$	$220.6^{+0.57\%}_{-0.62\%}$	$219.6^{+0.77\%}_{-2.2\%}$	$220.4^{+0.63\%}_{-0.94\%}$
100	0	$2015^{+19\%}_{-15\%}$	$2195^{+19\%}_{-15\%}$	$2195^{+19\%}_{-15\%}$	$2697^{+21\%}_{-17\%}$	$2697^{+21\%}_{-17\%}$
	1	$3724^{+13\%}_{-11\%}$	$3813^{+11.3\%}_{-10\%}$	$4138^{+10.1\%}_{-9.5\%}$	$3709^{+12.3\%}_{-8.8\%}$	$4060^{+11.1\%}_{-8.5\%}$
	2	$4322^{+4.2\%}_{-5.3\%}$	$4338^{+3.5\%}_{-4.8\%}$	$4424^{+1.9\%}_{-3.1\%}$	$4283^{+4.2\%}_{-5.0\%}$	$4394^{+2.4\%}_{-3.4\%}$
	3	$4439^{+0.51\%}_{-1.8\%}$	$4440^{+0.47\%}_{-1.6\%}$	$4448^{+0.26\%}_{-0.77\%}$	$4427^{+0.76\%}_{-2.0\%}$	$4444^{+0.35\%}_{-1.02\%}$

INFINITE TOP QUARK MASS LIMIT : RESULTS

Inclusive cross section

Chen, Li, Shao, Wang (PLB'20, JHEP'20)

- Variation with respect to λ_{hhh}



- Largest deconstruction between two LO diagrams at $\frac{\lambda_{hhh}}{\lambda_{hhh}^{SM}} \to 2$
- N3LO corrections only marginally distort the NNLO predictions