

# Effective Field Theory descriptions of Higgs boson pair production

The 19th Workshop of the LHC Higgs Working Group

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based on [WG note \(to be published on CDS very soon\)](#) | November 28, 2022

INSTITUTE FOR THEORETICAL PHYSICS



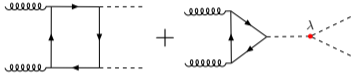
# Outline

- 1 Motivation
- 2 SMEFT and HEFT
- 3 MC tools
- 4 Benchmark study
- 5 Uncertainties
- 6 Reweighting
- 7 Summary

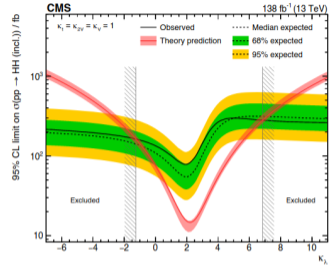
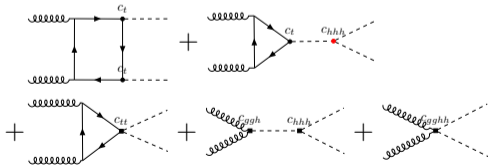
# Why study EFT phenomenology in $hh$ production?

- Is Higgs potential SM-like?  $V_{\text{SM}} \sim \frac{m_h^2}{2} h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$

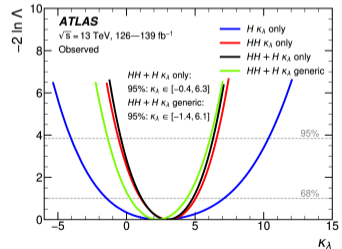
⇒ Trilinear Higgs coupling accessible in  $hh$  production



- However, to maintain some generality BSM deviations should enter in systematic way!



[2207.00043]



[2211.01216]

# Two bottom-up EFT systematics: SMEFT vs. HEFT

**Bottom-up EFT:** systematic parameterisation for unknown new physics above energy scale  $\Lambda$

- SMEFT:**
- SM fields + symmetries as building blocks of higher order operators
  - Light Higgs contained in EW doublet field  $\phi(x)$
  - Canonical counting ( $\Rightarrow$  expansion in  $\frac{1}{\Lambda}$ ):

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_{n=1} \sum_i \frac{C_i}{\Lambda^{2n}} \mathcal{O}_i^{(4+2n)}$$

- Truncate series at  $\frac{1}{\Lambda^2}$ , collecting all non-redundant (CP-even) operators (EFT basis)

$$\begin{aligned} \mathcal{L}_{SMEFT}^{(Warsaw)} \supset & \frac{C_{H\Box}}{\Lambda^2} (\phi^\dagger \phi) \Box (\phi^\dagger \phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu \phi) (\phi^\dagger D^\mu \phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger \phi)^3 \\ & + \frac{C_{uH}}{\Lambda^2} ((\phi^\dagger \phi) \bar{q}_L \phi^c t_R + h.c.) + \frac{C_{HG}}{\Lambda^2} (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu} + \frac{C_{uG}}{\Lambda^2} (\bar{q}_L \sigma^{\mu\nu} T^a G_{\mu\nu}^a \tilde{\phi} t_R + h.c.) \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{SMEFT}^{(SILH)} \supset & \frac{\bar{C}_H}{2v^2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi) - \frac{\bar{C}_6}{2v^2} \frac{m_h^2}{v^2} (\phi^\dagger \phi)^3 \\ & + \frac{\bar{C}_U}{v^2} y_t ((\phi^\dagger \phi) \bar{q}_L \tilde{\phi} t_R + h.c.) + \frac{4\bar{C}_g}{v^2} g_s^2 (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu} + \frac{\bar{C}_{ug}}{v^2} g_s (\bar{q}_L \sigma^{\mu\nu} T^a G_{\mu\nu}^a \tilde{\phi} t_R + h.c.) \end{aligned}$$

# Two bottom-up EFT systematics: SMEFT vs. HEFT

- HEFT:**
- Motivation as analogue to chiral pert. theory
  - Chiral dimension of operators  $d_\chi(\partial, \bar{\psi}\psi, g, y) = 1$
  - Light Higgs is EW gauge singlet  $h(x)$
  - Expansion in  $\frac{f^2}{\Lambda^2} \sim \frac{1}{16\pi^2}$  ( $\Rightarrow$  loop counting)

$$\mathcal{L}_{HEFT} \supset -m_t \left( c_t \frac{h}{v} + c_{tt} \frac{h^2}{v^2} \right) \bar{t}t - c_{hhh} \frac{m_h^2}{2v} h^3 + \frac{\alpha_s}{8\pi} \left( c_{ggh} \frac{h}{v} + c_{gggh} \frac{h^2}{v^2} \right) G_{\mu\nu}^a G^{a\mu\nu}$$

Naive translation SMEFT  $\leftrightarrow$  HEFT after field redefinition up to  $\mathcal{O}\left(\frac{1}{\Lambda^2}\right)$  in Lagrangian ( $C_{H,kin} = C_{H\Box} - 4C_{HD}$ )

However, formally:

$$c_i \sim \mathcal{O}(1) \text{ possible} \quad \leftrightarrow \quad \frac{E^2}{\Lambda^2} C_i \ll 1$$

$\Rightarrow$  Not generally applicable in practical calculations (fits, bounds, ...)

HEFT	SILH	Warsaw
$C_{hhh}$	$1 - \frac{3}{2}\bar{C}_H + \bar{C}_6$	$1 - 2\frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3\frac{v^2}{\Lambda^2} C_{H,kin}$
$c_t$	$1 - \frac{\bar{C}_H}{2} - \bar{C}_U$	$1 + \frac{v^2}{\Lambda^2} C_{H,kin} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{uH}$
$C_{tt}$	$-\frac{\bar{C}_H + 3\bar{C}_U}{4}$	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,kin}$
$C_{ggh}$	$128\pi^2 \bar{C}_g$	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s(\mu)} C_{HG}$
$C_{gggh}$	$64\pi^2 \bar{C}_g$	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s(\mu)} C_{HG}$

# SMEFT truncation

Dimension 6 operators in amplitude  $\left(\frac{C'_i}{\Lambda^2} = c_i - c_{i,sm}\right)$ :

$$\begin{aligned}
 \mathcal{M} = & \text{[Diagram 1: Box diagram with } 1 + \frac{C'_1}{\Lambda^2} \text{]} + \text{[Diagram 2: Triangle diagram with } 1 + \frac{C'_2}{\Lambda^2} \text{]} + \text{[Diagram 3: Triangle diagram with } \frac{C'_3}{\Lambda^2} \text{]} + \text{[Diagram 4: Triangle diagram with } \frac{C'_4}{\Lambda^2} \text{]} + \text{[Diagram 5: Triangle diagram with } \frac{C'_5}{\Lambda^2} \text{]} + \dots \\
 = & \mathcal{M}_{\text{SM}} + \underbrace{\frac{1}{\Lambda^2} \mathcal{M}_{\text{si}}}_{\text{dim6}} \left( + \underbrace{\frac{1}{\Lambda^4} \mathcal{M}_{\text{di}}}_{\text{dim6}^2} \right)
 \end{aligned}$$

⇒ Double operator insertion same order as (neglected) dimension 8 operators (and field redefinition)!

⇒ In HEFT the complete anomalous coupling enters at each vertex with no additional truncation

# SMEFT truncation of cross section

$$\sigma \simeq \left\{ \begin{array}{l} \sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim6}} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} + \sigma_{\text{SM} \times \text{dim6}^2} \\ \sigma_{(\text{SM} + \text{dim6} + \text{dim6}^2) \times (\text{SM} + \text{dim6} + \text{dim6}^2)} \end{array} \right.$$

(a) Truncation at leading order of expansion of powers in  $1/\Lambda^2$  of cross section  $\Rightarrow$  “most consistent” choice

(b) Truncation at leading order of expansion of powers in  $1/\Lambda^2$  of cross section  $\Rightarrow$  investigate uncertainty

(c) Truncate cross section at  $\mathcal{O}(1/\Lambda^4)$  from all dim6 operator insertions (ambiguous definition)

(d) Complete insertion, naive translation SMEFT  $\leftrightarrow$  HEFT

- Truncation (a) theoretically best suited for central value fit, however, negative (differential) cross section can appear, since Wilson coefficients not yet restricted close enough to SM  $\Rightarrow$  Perform analysis for truncation (a) and (b) separately!

## HEFT

HTL = Heavy top limit ( $m_t \rightarrow \infty$ )

- LO and NLO QCD HTL HPAIR [Gröber,Mühlleitner,Spira,Streicher '15]  
[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zirke '16]
- Full  $m_t$  NLO QCD POWHEG-BOX-V2/ggHH [Heinrich,Jones,Kerner,Luisoni,Vryonidou '17]  
[Heinrich,Jones,Kerner,Luisoni,Scyboz '18]  
[Heinrich,Jones,Kerner,Scyboz '20]

## SMEFT

- LO and NLO QCD HTL HPAIR [Gröber,Mühlleitner,Spira,Streicher '15]
- LO (1-loop) including chromo-magnetic operator  
SMEFT@NLO + MG5\_aMC@NLO [Degrande,Durieux,Maltoni,Mimasu,Vryonidou,Zhang '20]
- LO including chromo-magnetic operator [Brivio,Jiang,Trott '17]  
SMEFTsim + MG5\_aMC@NLO [Brivio '20]
- Full  $m_t$  NLO QCD POWHEG-BOX-V2/ggHH\_SMEFT  
with truncation options [Heinrich,JL,Scyboz '22]



## Combinations of results

- Full  $m_t$  NLO QCD

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke '16]  
 [Heinrich, Jones, Kerner, Luisoni, Vryonidou '17]  
 [Heinrich, Jones, Kerner, Luisoni, Scyboz '18]  
 [Heinrich, Jones, Kerner, Scyboz '20]

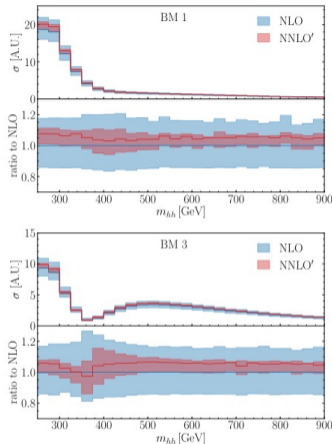
- NNLO QCD in HTL

[de Florian, Fabre, Mazitelli '16]

$$\frac{\sigma_{BSM}}{\sigma_{SM}} = a_1 \cdot c_t^4 + a_2 \cdot c_{tt}^2 + a_3 \cdot c_t^2 c_{hhh}^2 + a_4 \cdot c_{ggh}^2 c_{hhh}^2 + a_5 \cdot c_{gggh}^2 + a_6 \cdot c_{tt} c_t^2 + a_7 \cdot c_t^3 c_{hhh} + a_8 \cdot c_{tt} c_t c_{hhh} + a_9 \cdot c_{tt} c_{ggh} c_{hhh} + a_{10} \cdot c_{tt} c_{gggh} + a_{11} \cdot c_t^2 c_{ggh} c_{hhh} + a_{12} \cdot c_t^2 c_{gggh} + a_{13} \cdot c_t c_{hhh}^2 c_{ggh} + a_{14} \cdot c_t c_{hhh} c_{gggh} + a_{15} \cdot c_{ggh} c_{hhh} c_{gggh} + a_{16} \cdot c_t^3 c_{ggh} + a_{17} \cdot c_t c_{tt} c_{ggh} + a_{18} \cdot c_t c_{ggh}^2 c_{hhh} + a_{19} \cdot c_t c_{ggh} c_{gggh} + a_{20} \cdot c_t^2 c_{ggh}^2 + a_{21} \cdot c_{tt} c_{ggh}^2 + a_{22} \cdot c_{ggh}^3 c_{hhh} + a_{23} \cdot c_{ggh}^2 c_{gggh} + a_{24} \cdot c_{ggh}^4 + a_{25} \cdot c_{ggh}^3 c_t$$

- $a_i$  for inclusive cross section published

⇒ Reduction of scale uncertainty by factor  $\sim 2$



## Usage of code:

- Built on previous NLO SM calculation with full  $m_t$  dependence

[Borowka,Greiner,Heinrich,Jones,Kerner,et al. '16]

[Heinrich,Jones,Kerner,Luisoni,Vryonidou '17]

[Heinrich,Jones,Kerner,Luisoni,Scyboz '19]

- $mtdep$   $\left\{ \begin{array}{l} 0-2: \text{HTL approximations} \\ 3: \text{full } m_t \text{ dependence} \end{array} \right.$

- $m_h = 125$  GeV and  $m_t = 173$  GeV fixed for grids encoding virtual (2-loop) corrections!

- Matching to parton shower (Pythia or Herwig) available

⇒ Available at <http://powhegbox.mib.infn.it>

```
! ggHH production parameters:
mtdep 3          ! 0: Higgs effective field theory (HEFT)
!              ! 1: Born improved HEFT
!              ! 2: approximated full theory (FTapprox)
!              ! 3: full theory

hmass 125       ! Higgs boson mass
topmass 173     ! top quark mass (THIS VALUE IS HARD CODED IN THE VIRTUAL
!              ! MATRIX ELEMENT AND FOR CONSISTENCY HAS NOT TO BE CHANGED WHEN
!              ! RUNNING FULL THEORY PREDICTIONS - i.e. mtdep=3)

hdecaymode -1  ! PDG code for Higgs boson decay products (it affects only the SMC)
!              ! allowed values are:
!              ! 0 all decay channels open
!              ! 1-6 d dbar, u ubar,..., t tbar (as in HERWIG)
!              ! 7-9 e+ e-, mu+ mu-, tau+ tau-
!              ! 10 W+W-
!              ! 11 ZZ
!              ! 12 gamma gamma
!              ! -1 all decay channels closed

! Values of the Higgs couplings w.r.t SM
chhh 1.0        ! Trilinear Higgs self-coupling
ct 1.0          ! Top-Higgs Yukawa coupling
ctt 0.0        ! Two-top-two-Higgs (tthh) coupling
cggh 0.0       ! Effective gluon-gluon-Higgs coupling
cgghh 0.0      ! Effective two-gluon-two-Higgses coupling
```

## Usage of code:

- Built on previous NLO SM calculation with full

$m_t$  de

[Borowka,  
[Heinrich,  
[Heinrich,

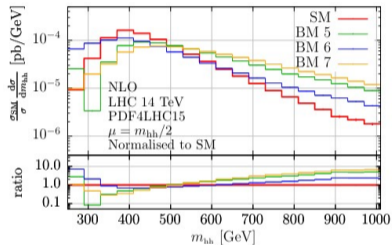
- mtdep

- $m_h =$

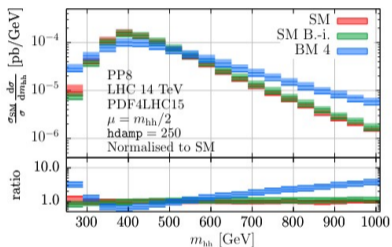
grids

- Match

available



Fixed order



Matched to PYTHIA-8

cgghh 0.0 | Effective two-gluon-two-Higgses coupling

⇒ Available at <http://powhegbox.mib.infn.it>

Usage of code (only new part of input file shown):

- Built on NLO HEFT ggHH

- $\text{usesmeft} \begin{cases} 0: \text{HEFT operators} \\ 1: \text{SMEFT operators} \end{cases}$

- $\text{multiple-insertion} \quad 0, \dots, 3$   
                                   $\updownarrow$   
                                  truncation option (a), ..., (d)

- No RGE effects of Wilson coefficients

⇒ Available at <http://powhegbox.mib.infn.it>

```
! Choose EFT parametrization
usesmeft 1 ! 0: use HEFT parametrization and ignore CHbox, CH, CuH, CHG (no truncat
! 1: use SMEFT (Warsaw) parametrization and ignore chhh, ct, ctt, cggh,
! 2: use HEFT parametrization and ignore CHbox, CH, CuH, CHG (with truncat

! Values of the Higgs couplings w.r.t SM: HEFT parametrization
chhh 1.0 ! Trilinear Higgs self-coupling
ct 1.0 ! Top-Higgs Yukawa coupling
ctt 0.0 ! Two-top-two-Higgs (tthh) coupling
cggh 0.0 ! Effective gluon-gluon-Higgs coupling
cgghh 0.0 ! Effective two-gluon-two-Higgses coupling

! Values of the Higgs couplings using SMEFT (Warsaw) parametrization (Wilson coefficients ent
Lambda 1.0 ! EFT counting mass Scale (in TeV)
CHbox 0.0 ! Kinetic term of SU(2)_L singlet (with d'Alembert operator)
CHD 0.0 ! second Kinetic term
CH 0.0 ! Additional term to Higgs potential
CuH 0.0 ! Modified Yukawa term
CHG 0.0 ! Higgs-Glue-Glue operator

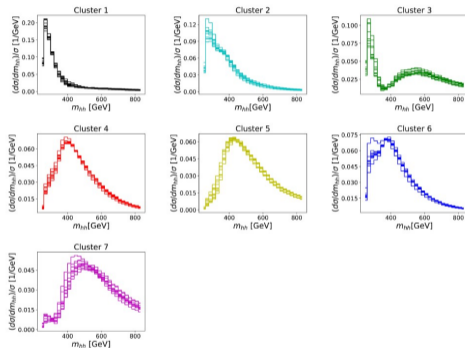
! Truncation options:
! 3: cross section based on |A_SM+A_dim6+A_dbldim6|^2
! 2: cross section based on |A_SM+A_dim6|^2+2*Re(A_SM x conj(A_dbldim6))
! 1: cross section based on |A_SM+A_dim6|^2
! 0: cross section based on |A_SM|^2+2*Re(A_SM*conj(A_dim6))
multiple-insertion 1
```

# Updated HEFT benchmarks

Published in WG note

benchmark	$C_{hhh}$	$C_t$	$C_{tt}$	$C_{ggh}$	$C_{gggh}$
SM	1	1	0	0	0
1*	5.105	1.1	0	0	0
2*	6.842	1.033	$\frac{1}{6}$	$-\frac{1}{3}$	0
3	2.21	1.05	$-\frac{1}{3}$	0.5	0.5
4*	2.79	0.9	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$
5	3.95	1.17	$-\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{2}$
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25
7	-0.10	0.94	1	$\frac{1}{6}$	$-\frac{1}{6}$

- Shape clusters defined using unsupervised ML
- Benchmarks chosen with clear shape features and satisfying experimental constraints
- \* denotes updated benchmark point, new constraints:  
 $0.83 \leq c_t \leq 1.17$  (and  $|c_{tt}| \leq 0.05$  for 1\*)



[Capozi, Heinrich '19]

Consider HEFT benchmark points with following characteristic  $m_{hh}$  shapes

- Benchmark 1\*: enhanced low  $m_{hh}$  region
- Benchmark 6\*: close-by double peaks or shoulder left

benchmark (* = modified)	$C_{hhh}$	$C_t$	$C_{tt}$	$C_{ggh}$	$C_{gggh}$	$C_{H,kin}$	$C_H$	$C_{uH}$	$C_{HG}$	$\Lambda$
SM	1	1	0	0	0	0	0	0	0	1 TeV
1*	5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV

$\Rightarrow$  SMEFT expansion based on  $E^2 \frac{C_i}{\Lambda^2} \ll 1$  justified?

$C_{HG}$  obtained using  $\alpha_s(m_Z) = 0.118$

# Naive benchmark translation

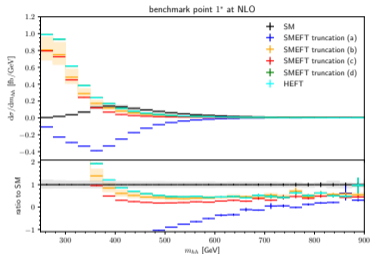
Total cross section generated at  $\sqrt{s} = 13$  TeV

benchmark	$\sigma_{\text{NLO}}$ [fb] option (b)	K-factor option (b)	ratio to SM option (b)	$\sigma_{\text{NLO}}$ [fb] option (a)	$\sigma_{\text{NLO}}$ [fb] HEFT
SM	$27.94^{+13.7\%}_{-12.8\%}$	1.67	1	-	-
$\Lambda = 1$ TeV					
1*	$74.29^{+19.8\%}_{-15.6\%}$	2.13	2.66	-61.17	94.32
6*	$72.51^{+20.6\%}_{-16.4\%}$	1.90	2.60	52.89	91.40
$\Lambda = 2$ TeV					
1*	$14.03^{+12.0\%}_{-11.9\%}$	1.56	0.502	5.58	-
6*	$35.39^{+17.5\%}_{-15.2\%}$	1.76	1.27	34.18	-

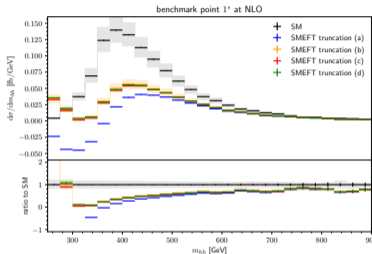
# Invariant mass distributions at NLO QCD ( $\sqrt{s} = 13 \text{ TeV}$ )

■ Benchmark 1\*:

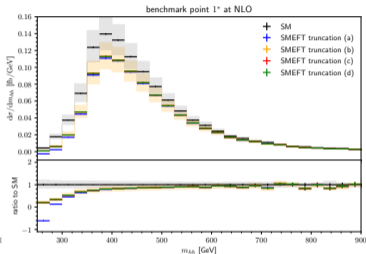
$C_{hh}$	$C_t$	$C_{tt}$	$C_{ggh}$	$C_{gggh}$	$C_{H,\text{kin}}$	$C_H$	$C_{UH}$	$C_{HG}$
5.105	1.1	0	0	0	4.95	-6.81	3.28	0



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$



$\Lambda = 4 \text{ TeV}$

■ Truncation (a): negative cross sections

■ Shape approaches SM for increasing  $\Lambda$

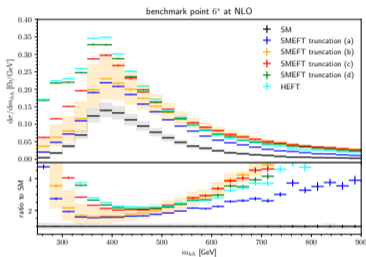
⇒ Valid HEFT point invalid in SMEFT after direct translation



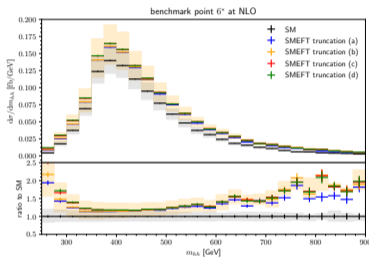
# Invariant mass distributions at NLO QCD ( $\sqrt{s} = 13 \text{ TeV}$ )

■ Benchmark 6\*:

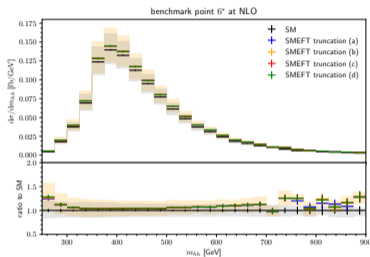
$C_{hhh}$	$C_t$	$C_{tt}$	$C_{ggh}$	$C_{gggh}$	$C_{H,kin}$	$C_H$	$C_{uH}$	$C_{HG}$
-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$



$\Lambda = 4 \text{ TeV}$

■ No negative cross section

■ No shoulder left (except for (d))

■ Shape indistinguishable from SM for  $\Lambda = 4 \text{ TeV}$  within scale uncertainties

■ Difference between HEFT and (d) only due to  $\alpha_s$  scale dependence

$$\Delta\sigma \sim \begin{matrix} +\Delta_{\text{scale}+} \\ -\Delta_{\text{scale}-} \end{matrix} + \begin{matrix} +\Delta_{m_t \text{ scheme}+} \\ -\Delta_{m_t \text{ scheme}-} \end{matrix} \pm \Delta_{\text{num. grid}} \left( \pm \Delta_{\text{EFT trunc.}} \right) \pm \Delta_{\text{PDF}+\alpha_s} \pm \Delta_{\text{EW}}$$

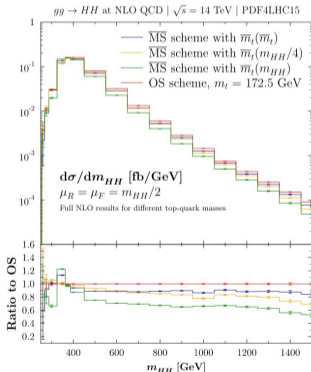
- $\Delta_{\text{EW}}$ : Full NLO EW unknown, only partial results of top Yukawa [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]  
[Mühlleitner, Schlenk, Spira '22]
- $\Delta_{\text{PDF}+\alpha_s} \approx 3\%$  ( $\sqrt{s} = 13 \text{ TeV}$ ): B.I. NNLO HTL and employing PDF4LHCNNLO [twiki *hh* cross group]  
stable for  $c_{hhh}$  variation, but might rise if tail enhanced
- $\Delta_{\text{EFT trunc.}}$ : No quantitative prescription, qualitative observation of truncation options
- $\Delta_{\text{scale} \pm}$ : Determined by 7-point variation of  $\mu_R, \mu_F = \{0.5, 1, 2\} \cdot \mu_0$   
 $\mathcal{O}(15\%)$  for NLO QCD SM, 15 - 20% for NLO QCD SMEFT truncation (b) benchmark 1\* & 6\*
- $\Delta_{m_t \text{ scheme} \pm}$ : In principle needs determination for each point in EFT parameter space! (not yet available)
- $\Delta_{\text{num. grid}}$ : Numerical uncertainty of grids for virtual contribution, not covered by Monte Carlo statistical uncertainty of POWHEG!

# $m_t$ renormalisation scheme uncertainty

[Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher '18]  
 [Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira,Streicher '20]  
 [Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira '20]

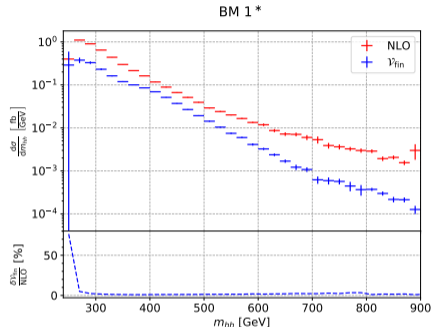
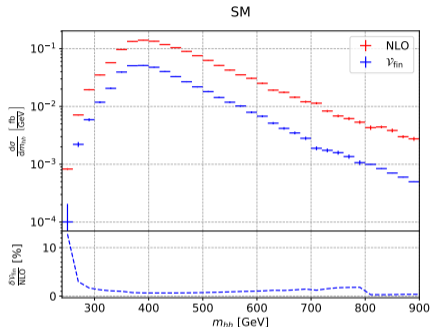
$$\bar{m}_t(m_t) = \frac{m_t}{1 + \frac{4}{3} \frac{\alpha_s(m_t)}{\pi} + K_2 \left( \frac{\alpha_s(m_t)}{\pi} \right)^2 + K_3 \left( \frac{\alpha_s(m_t)}{\pi} \right)^3}$$

- Prediction depends on  $m_t$  scheme (on-shell vs.  $\overline{MS}$  with varying scale)
- Uncertainty sensitive to choice of  $C_{hhh} = \kappa_\lambda$
- Sensitivity to variations of  $C_{tt}$  expected



$\kappa_\lambda = -10$ :	$\sigma_{tot} = 1438(1)_{-6}^{+10\%}$ fb,
$\kappa_\lambda = -5$ :	$\sigma_{tot} = 512.8(3)_{-7}^{+10\%}$ fb,
$\kappa_\lambda = -1$ :	$\sigma_{tot} = 113.66(7)_{-9}^{+8\%}$ fb,
$\kappa_\lambda = 0$ :	$\sigma_{tot} = 61.22(6)_{-12}^{+6\%}$ fb,
$\kappa_\lambda = 1$ :	$\sigma_{tot} = 27.73(7)_{-18}^{+4\%}$ fb,
$\kappa_\lambda = 2$ :	$\sigma_{tot} = 13.2(1)_{-23}^{+1\%}$ fb,
$\kappa_\lambda = 2.4$ :	$\sigma_{tot} = 12.7(1)_{-22}^{+4\%}$ fb,
$\kappa_\lambda = 3$ :	$\sigma_{tot} = 17.6(1)_{-15}^{+9\%}$ fb,
$\kappa_\lambda = 5$ :	$\sigma_{tot} = 83.2(3)_{-4}^{+13\%}$ fb,
$\kappa_\lambda = 10$ :	$\sigma_{tot} = 579(1)_{-4}^{+12\%}$ fb

# Numerical grids uncertainty



- Low (and high)  $m_{hh}$  region very sparsely populated in virtual grids, due to small contribution in SM
- ⇒  $\mathcal{O}(12\%)$  uncertainty for SM in first bin not represented by Monte Carlo statistical uncertainty in POWHEG
- ⇒ Uncertainty much worse for scenarios with enhanced low  $m_{hh}$  region

# Reweighting of NLO HEFT

- NLO MC programs are nice, **but** computationally expensive
- ⇒ Reweighting using set of MC samples!

- Expansion of inclusive and differential cross section:

$$\sigma_{hh}^{\text{NLO}} = \text{Poly}(\mathbf{c}, \mathbf{A}) = \mathbf{c}^T \cdot \mathbf{A}$$

$$\frac{d\sigma_{hh}}{dm_{hh}} = \text{Poly}(\mathbf{c}, d\mathbf{A}|m_{hh}) = \mathbf{c}^T \cdot d\mathbf{A}$$

- $\mathbf{A}$  and  $d\mathbf{A}$  with respective covariance matrix  $\Sigma_{(d)\mathbf{A}}$  derived using least square fit of 63 MC samples
- $d\mathbf{A}$  available for  $m_{hh} \in [250, 1050]$  GeV in 20 GeV bins and two broader bins  $[1050, 1200]$  GeV and  $[1200, 1400]$  GeV
- 3 sets for scale variation  $\mu_R = \mu_F = \{\frac{1}{2}, 1, 2\} \cdot \mu_0$  with  $\mu_0 = \frac{m_{hh}}{2}$

$$\begin{aligned} \sigma_{hh}^{\text{NLO}} = & A_1 c_t^4 + A_2 c_{tt}^2 + (A_3 c_t^2 + A_4 c_{ggh}^2) c_{hhh}^2 \\ & + A_5 c_{gghh}^2 + (A_6 c_{tt} + A_7 c_t c_{hhh}) c_t^2 \\ & + (A_8 c_t c_{hhh} + A_9 c_{ggh} c_{hhh}) c_{tt} + A_{10} c_{tt} c_{gghh} \\ & + (A_{11} c_{ggh} c_{hhh} + A_{12} c_{gghh}) c_t^2 \\ & + (A_{13} c_{hhh} c_{ggh} + A_{14} c_{gghh}) c_t c_{hhh} \\ & + A_{15} c_{ggh} c_{gghh} c_{hhh} + A_{16} c_t^3 c_{ggh} \\ & + A_{17} c_t c_{tt} c_{ggh} + A_{18} c_t c_{ggh} c_{hhh} \\ & + A_{19} c_t c_{ggh} c_{gghh} + A_{20} c_t^2 c_{ggh}^2 \\ & + A_{21} c_{tt} c_{ggh}^2 + A_{22} c_{ggh}^3 c_{hhh} \\ & + A_{23} c_{ggh}^2 c_{gghh} \end{aligned}$$

Values provided along with WG note!  
(to be found in HEPdata)

# Reweighting of NLO HEFT and statistical uncertainties

- Weights obtained according to

$$w_{\text{HEFT}} = \frac{\text{Poly}(\mathbf{c}, d\mathbf{A}|m_{hh})}{\text{Poly}(\mathbf{c}_{\text{SM}}, d\mathbf{A}|m_{hh})}$$

- Corresponding uncertainty calculated using  $\delta_{w_{\text{HEFT}}} = \sqrt{\mathbf{J}_w \Sigma_{d\mathbf{A}} \mathbf{J}_w^T}$  with

$$\mathbf{J}_w = \frac{\mathbf{c}^T}{\text{Poly}(\mathbf{c}_{\text{SM}}, d\mathbf{A}|m_{hh})} - \frac{\text{Poly}(\mathbf{c}, d\mathbf{A}|m_{hh}) \cdot \mathbf{c}_{\text{SM}}^T}{\text{Poly}(\mathbf{c}_{\text{SM}}, d\mathbf{A}|m_{hh})^2}.$$

- Final statistical uncertainty in reweighted bin  $j$

$$\delta^j = N^j \sqrt{\left(\frac{\delta_{w_{\text{HEFT}}}^j}{w_{\text{HEFT}}^j}\right)^2 + \left(\frac{\delta_{\text{SM}}^j}{N_{\text{SM}}^j}\right)^2}, \quad \text{with}$$

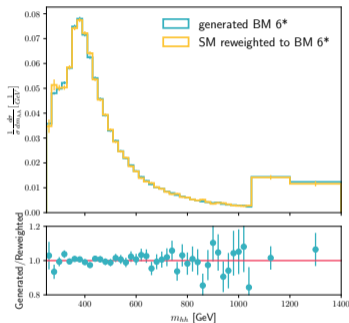
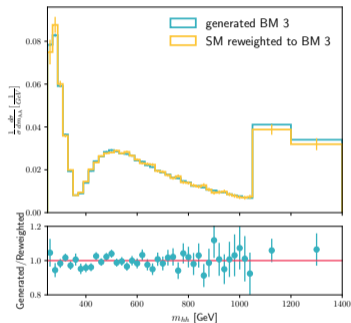
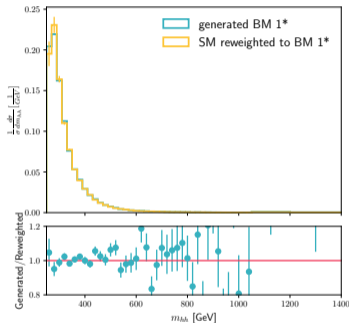
$N^j$ : sum of weighted events

$N_{\text{SM}}^j$ : sum of weighted SM events

$w_{\text{HEFT}}^j$ : weight

$\delta_{\text{SM}}^j$ : weighted statistical uncertainty for SM events

# Validation of reweighting method



⇒ Very good agreement for  $m_{hh}$  distribution!

- SMEFT and HEFT both valid EFT approaches based on different assumptions
  - Public implementations listed & usage of full NLO QCD implementations in POWHEG shown
  - List of HEFT benchmarks updated
  - BM study: Naive translation from HEFT  $\rightarrow$  SMEFT can lead out of validity of  $\frac{1}{\Lambda^2}$  expansion  
 $\Rightarrow$  We advocate to study both EFT representations separately
  - Discussion of uncertainties
  - Reweighting procedure and validation presented, coefficients published with WG note
- $\Rightarrow$  More details in (upcoming) WG note!



# Virtual grids for ggHH\_SMEFT

Split matrix in kinematic part times coupling coefficient for HEFT and SMEFT

$$\begin{aligned}\mathcal{M}_{LO} &:= m_1 \cdot c_t^2 + m_2 \cdot c_t c_{hhh} + m_3 \cdot c_{tt} + m_4 \cdot c_g c_{hhh} + m_5 \cdot c_{gg} \\ &= m_1 + m_2 + \frac{1}{\Lambda^2} \left( 2m_1 \cdot C_t' + m_2 \cdot (C_t' + C_{hhh}') + m_3 \cdot C_{tt}' + m_4 \cdot C_g' + m_5 \cdot C_{gg}' \right) + \frac{1}{\Lambda^4} \left( m_1 \cdot C_t'^2 + m_2 \cdot C_t' C_{hhh}' \right) \\ \mathcal{M}_{NLO} &:= M_1 \cdot c_t^2 + M_2 \cdot c_t c_{hhh} + M_3 \cdot c_{tt} + M_4 \cdot c_g c_{hhh} + M_5 \cdot c_{gg} + M_6 \cdot c_g^2 + M_7 \cdot c_g c_t \\ &= M_1 + M_2 + \frac{1}{\Lambda^2} \left( 2M_1 \cdot C_t' + M_2 \cdot (C_t' + C_{hhh}') + M_3 \cdot C_{tt}' + M_4 \cdot C_g' + M_5 \cdot C_{gg}' + M_7 \cdot C_g' \right) \\ &\quad + \frac{1}{\Lambda^4} \left( M_1 \cdot C_t'^2 + M_2 \cdot C_t' C_{hhh}' + M_6 \cdot C_g'^2 + M_7 \cdot C_g' C_t' \right)\end{aligned}$$

The virtual grids, given as kinematic coefficients  $a_i$  of the squared matrix element

$$\begin{aligned}|\mathcal{M}_{NLO}|^2 &= a_1 \cdot c_t^4 + a_2 \cdot c_{tt}^2 + a_3 \cdot c_t^2 c_{hhh}^2 + a_4 \cdot c_{ggh}^2 c_{hhh}^2 + a_5 \cdot c_{ggh}^2 + a_6 \cdot c_{tt} c_t^2 + a_7 \cdot c_t^3 c_{hhh} + a_8 \cdot c_{tt} c_t c_{hhh} + a_9 \cdot c_{tt} c_{ggh} c_{hhh} \\ &\quad + a_{10} \cdot c_{tt} c_{ggh} + a_{11} \cdot c_t^2 c_{ggh} c_{hhh} + a_{12} \cdot c_t^2 c_{ggh}^2 + a_{13} \cdot c_t c_{hhh}^2 c_{ggh} + a_{14} \cdot c_t c_{hhh} c_{ggh} + a_{15} \cdot c_{ggh} c_{hhh} c_{ggh} + a_{16} \cdot c_t^3 c_{ggh} \\ &\quad + a_{17} \cdot c_t c_{tt} c_{ggh} + a_{18} \cdot c_t c_{ggh} c_{hhh} + a_{19} \cdot c_t c_{ggh} c_{ggh} + a_{20} \cdot c_t^2 c_{ggh}^2 + a_{21} \cdot c_{tt} c_{ggh}^2 + a_{22} \cdot c_{ggh}^3 c_{hhh} + a_{23} \cdot c_{ggh}^2 c_{ggh}^2,\end{aligned}$$

can be understood as combinations of  $m_i \times M_j$  obtained from  $\mathcal{M}_{LO} \times \mathcal{M}_{NLO}$ . After rearrangement, the squared matrix elements entering the truncated cross sections in SMEFT (slide 7) are expressed in terms of the same  $a_i$ , except for truncation (b), where

$$\Delta\sigma_{(b)} = m_2 \times M_4 \cdot \frac{C_{ggh}'(C_{hhh}' - C_t')}{\Lambda^4} + m_4 \times M_7 \frac{C_{ggh}'^2}{\Lambda^4}$$

needs to be added.

# NLO cross section

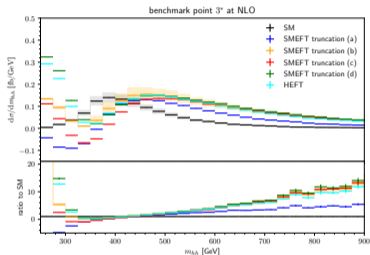
Generated at  $\sqrt{s} = 13$  TeV

benchmark	$\sigma_{\text{NLO}}$ [fb] option (b)	K-factor option (b)	ratio to SM option (b)	$\sigma_{\text{NLO}}$ [fb] option (a)	$\sigma_{\text{NLO}}$ [fb] HEFT
SM	27.94 <sup>+13.7%</sup> <sub>-12.8%</sub>	1.67	1	-	-
$\Lambda = 1$ TeV					
1*	74.29 <sup>+19.8%</sup> <sub>-15.6%</sub>	2.13	2.66	-61.17	94.32
3*	69.20 <sup>+11.7%</sup> <sub>-10.3%</sub>	1.82	2.47	29.64	72.43
6*	72.51 <sup>+20.6%</sup> <sub>-16.4%</sub>	1.90	2.60	52.89	91.40
$\Lambda = 2$ TeV					
1*	14.03 <sup>+12.0%</sup> <sub>-11.9%</sub>	1.56	0.502	5.58	-
3*	30.81 <sup>+16.0%</sup> <sub>-14.4%</sub>	1.71	1.10	28.35	-
6*	35.39 <sup>+17.5%</sup> <sub>-15.2%</sub>	1.76	1.27	34.18	-

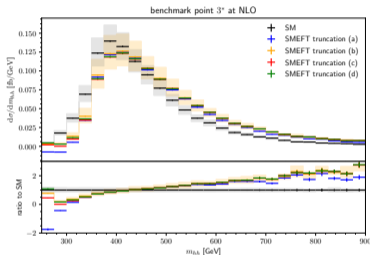
# Invariant mass distributions at NLO QCD ( $\sqrt{s} = 13 \text{ TeV}$ )

■ Benchmark 3\*:

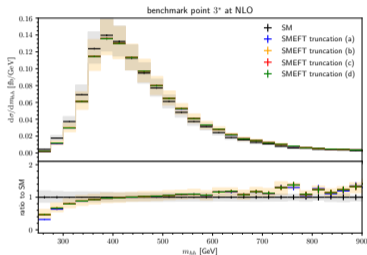
$C_{hhh}$	$C_t$	$C_{tt}$	$C_{ggh}$	$C_{gggh}$	$C_{H,\text{kin}}$	$C_H$	$C_{uH}$	$C_{HG}$
2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$

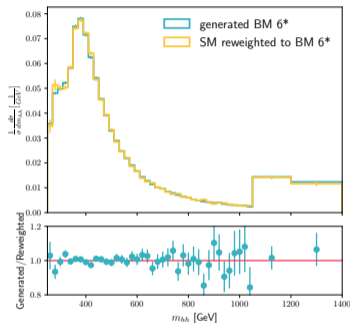
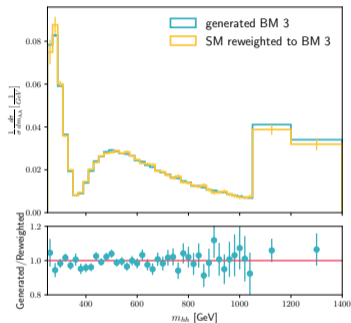
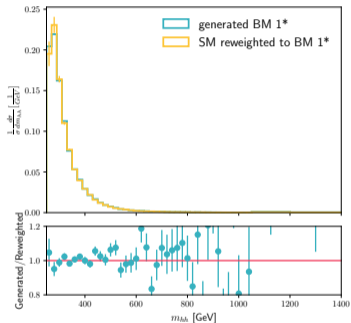


$\Lambda = 4 \text{ TeV}$

■ Truncation (c): double operator insertion quite substantial

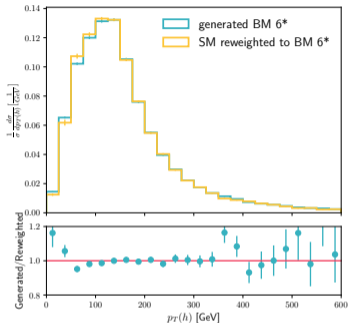
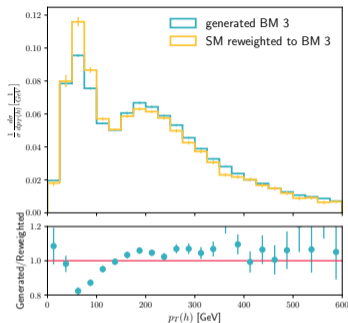
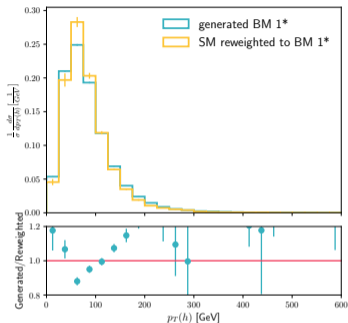
■ Shape close but distinguishable from SM for increasing  $\Lambda$

# Validation of reweighting method



⇒ Very good agreement for  $m_{hh}$  distribution!

# Validation of reweighting method and caveats



- Shape features reconstructed, but clearly not optimized for  $p_t(h)$  distributions!
- Reweighting of SM events according to  $m_{hh}$  does account which diagram type in HEFT benchmarks has dominant bin contribution  $\Rightarrow$  insensitive to additional jet radiation
- For benchmarks with enhanced low  $m_{hh}$  especially weaker prediction, since sparsely populated by SM events

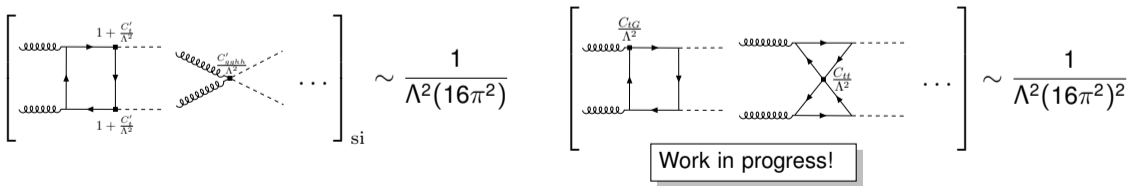
# Loop counting in SMEFT (“weak” UV assumption)

Considering couplings of general renormalisable UV physics [Arzt, Einhorn, Wudka '94] or using chiral dimensions leads to:

$$\begin{aligned} \mathcal{O}_H \sim [\kappa^4] (\phi^\dagger \phi)^3 &\Rightarrow \frac{C_H}{\Lambda^2} \sim \frac{1}{\Lambda^2} & \mathcal{O}_{HG} \sim [\kappa^4] (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu} &\Rightarrow \frac{C_{HG}}{\Lambda^2} \sim \frac{1}{\Lambda^2 (16\pi^2)} \\ \mathcal{O}_{tt} \sim [\kappa^2] \bar{t}_R \gamma_\mu t_R \bar{t}_R \gamma^\mu t_R &\Rightarrow \frac{C_{tt}}{\Lambda^2} \sim \frac{1}{\Lambda^2}, & \mathcal{O}_{tG} \sim [\kappa^4] (\bar{q}_L \sigma^{\mu\nu} T^a t_R) \tilde{\phi} G_{\mu\nu}^a &\Rightarrow \frac{C_{tG}}{\Lambda^2} \sim \frac{1}{\Lambda^2 (16\pi^2)} \end{aligned}$$

with  $\kappa$  generic weak coupling and  $\frac{C(d_\chi)}{\Lambda^2} \sim \frac{1}{\Lambda^2} \left(\frac{1}{16\pi^2}\right)^{(d_\chi-4)/2}$  [Buchalla, Heinrich, Müller-Salutti, Pandler '22]

⇒ Chromomagnetic operator enters with overall loop factor suppression  $\frac{1}{16\pi^2}$  compared to born:



# Reweighting of MC samples within SMEFT (SKETCH)

Parametrisation of cross section works in principle the same. Expansion in similar kinematic structures to HEFT leads to:

$$\frac{\sigma_{BSM}^{\text{SMEFT (a)}}}{\sigma_{SM}} = A_1 \cdot \left(1 + 4 \frac{C'_t}{\Lambda^2}\right) + A_3 \cdot \left(1 + 2 \frac{C'_t}{\Lambda^2} + 2 \frac{C'_{hhh}}{\Lambda^2}\right) + (A_6 + A_8) \cdot \frac{C'_{tt}}{\Lambda^2} + A_7 \cdot \left(1 + \frac{C'_{hhh}}{\Lambda^2} + 3 \frac{C'_t}{\Lambda^2}\right) + (A'_{11} + A'_{13} + A'_{16}) \cdot \frac{C'_{ggh}}{\Lambda^2} + \dots$$

$$= B_0^{SM} + B_1^{(a)} \cdot \frac{C_{H,kin}}{\Lambda^2} + B_2^{(a)} \cdot \frac{C_H}{\Lambda^2} + B_3^{(a)} \cdot \frac{C_{uH}}{\Lambda^2} + B_4^{(a)} \cdot \frac{C_{HG}}{\Lambda^2}$$

$$\frac{\sigma_{BSM}^{\text{SMEFT (b)}}}{\sigma_{SM}} = A_1 \cdot \left(1 + 4 \frac{C'_t}{\Lambda^2} + 4 \frac{C_t'^2}{\Lambda^4}\right) + A_2 \cdot \frac{C_{tt}'^2}{\Lambda^4} + A_3 \cdot \left(1 + 2 \left(\frac{C'_t}{\Lambda^2} + \frac{C'_{hhh}}{\Lambda^2}\right) + \left(\frac{C'_t}{\Lambda^2} + \frac{C'_{hhh}}{\Lambda^2}\right)^2\right) + A'_4 \cdot \frac{C_{ggh}'^2}{\Lambda^4} + A'_5 \cdot \frac{C_{gghh}'^2}{\Lambda^4}$$

$$+ A_6 \cdot \frac{C'_{tt}}{\Lambda^2} \left(1 + 2 \frac{C'_t}{\Lambda^2}\right) + A_7 \cdot \left(1 + 3 \frac{C'_t}{\Lambda^2} + \frac{C'_{hhh}}{\Lambda^2} + 2 \frac{C'_t}{\Lambda^2} \left(\frac{C'_t}{\Lambda^2} + \frac{C'_{hhh}}{\Lambda^2}\right)\right) + \dots + \Delta A'_1 \cdot \frac{C'_{ggh}}{\Lambda^2} \left(\frac{C'_{hhh}}{\Lambda^2} - \frac{C'_t}{\Lambda^2}\right) + \Delta A'_2 \cdot \frac{C_{ggh}'^2}{\Lambda^4}$$

$$= B_0^{SM} + B_1^{(a)} \cdot \frac{C_{H,kin}}{\Lambda^2} + B_2^{(a)} \cdot \frac{C_H}{\Lambda^2} + B_3^{(a)} \cdot \frac{C_{uH}}{\Lambda^2} + B_4^{(a)} \cdot \frac{C_{HG}}{\Lambda^2} + B_5^{(b)} \cdot \frac{C_{H,kin}^2}{\Lambda^4} + B_6^{(b)} \cdot \frac{C_H^2}{\Lambda^4} + B_7^{(b)} \cdot \frac{C_{uH}^2}{\Lambda^4} + B_8^{(b)} \cdot \frac{C_{HG}^2}{\Lambda^4}$$

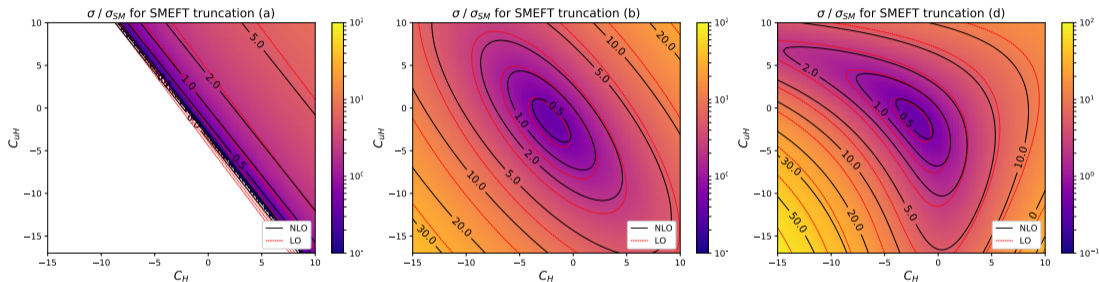
$$+ B_9^{(b)} \cdot \frac{C_{H,kin}}{\Lambda^2} \frac{C_H}{\Lambda^2} + B_{10}^{(b)} \cdot \frac{C_{H,kin}}{\Lambda^2} \frac{C_{uH}}{\Lambda^2} + B_{11}^{(b)} \cdot \frac{C_{H,kin}}{\Lambda^2} \frac{C_{HG}}{\Lambda^2} + B_{12}^{(b)} \cdot \frac{C_H}{\Lambda^2} \frac{C_{uH}}{\Lambda^2} + B_{13}^{(b)} \cdot \frac{C_H}{\Lambda^2} \frac{C_{HG}}{\Lambda^2} + B_{14}^{(b)} \cdot \frac{C_{uH}}{\Lambda^2} \frac{C_{HG}}{\Lambda^2}$$

- **CAUTION:** At least  $A'_i$  need to be reevaluated for Warsaw basis, since different factors of scale dependent  $\alpha_s(\mu)$  enter the calculation!  $\Delta A'_i$  do not appear in HEFT (see backup on virtual grids)
- **RECOMMENDATION:** Evaluate new and separate MC samples for truncation option (a) and (b), respectively, in order to project on new expansion coefficients  $B_i^{(a)}$  and  $B_i^{(b)}$

$C_{ggh}$	$\frac{1}{\alpha_s(\mu)} \frac{C'_{ggh}}{\Lambda^2}$	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s(\mu)} C_{HG}$
$C_{gghh}$	$\frac{1}{\alpha_s(\mu)} \frac{C'_{gghh}}{\Lambda^2}$	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s(\mu)} C_{HG}$

# NLO cross section

Generated at  $\sqrt{s} = 13$  TeV with  $\Lambda = 1$  TeV



■ Large area of negative cross section for truncation (a)

■ Flat directions differ substantially

■ Non-trivial shape for HEFT-like option (d)