Top mass renormalization scheme uncertainties in Higgs cross sections

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Top mass uncertainties in Higgs XS

- Top quark crucial in Higgs phenomenology:

  ↓

  Largest coupling to Higgs → Main contribution in ggF loop

- (Di-)Higgs XS via ggF is a function of the top-quark mass

- Experimental uncertainties in the top mass are propagated to Higgs XS

- Theoretical uncertainties in the top-quark mass are relevant as well!

- Ambiguities in the mass definition have an impact (uncertainties) in Higgs observables

> The arbitrariness in scheme (and scale) choice for the renormalization of the top-quark mass leads to uncertainties in our theory predictions

Top mass renormalization schemes

- The top-quark mass is subject to renormalization, and therefore it suffers from a scheme (and in general a scale) ambiguity

- Most commonly used for the top-quark mass: **pole scheme**

Pole of the quark propagator is fixed to the same value, the **pole mass** M_t, at any order in perturbation theory

- 'Natural' choice when considering on-shell top quark production

- Alternatively, we can remove only the singular contributions in dim. reg.: **$\overline{\text{MS}}$ scheme**

Pole of the quark propagator receives corrections at any order
The **$\overline{\text{MS}}$ mass** $m_t(\mu_t)$ differs from M_t and depends on arbitrary scale μ_t

- The pole mass is affected by a non-perturbative ambiguity of $O(\Lambda_{QCD})$, absent in the $\overline{\text{MS}}$ mass

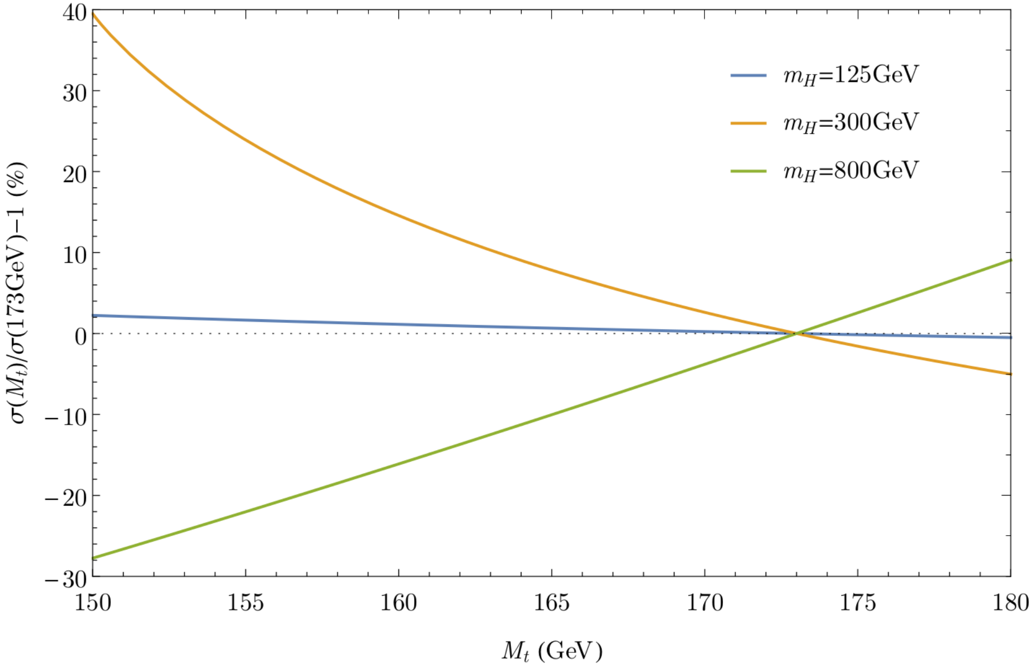
- The $\overline{\text{MS}}$ mass depends on an additional arbitrary scale, which leads to further uncertainties

A priori, no clear reason to prefer one scheme over the other for the tops inside the loop

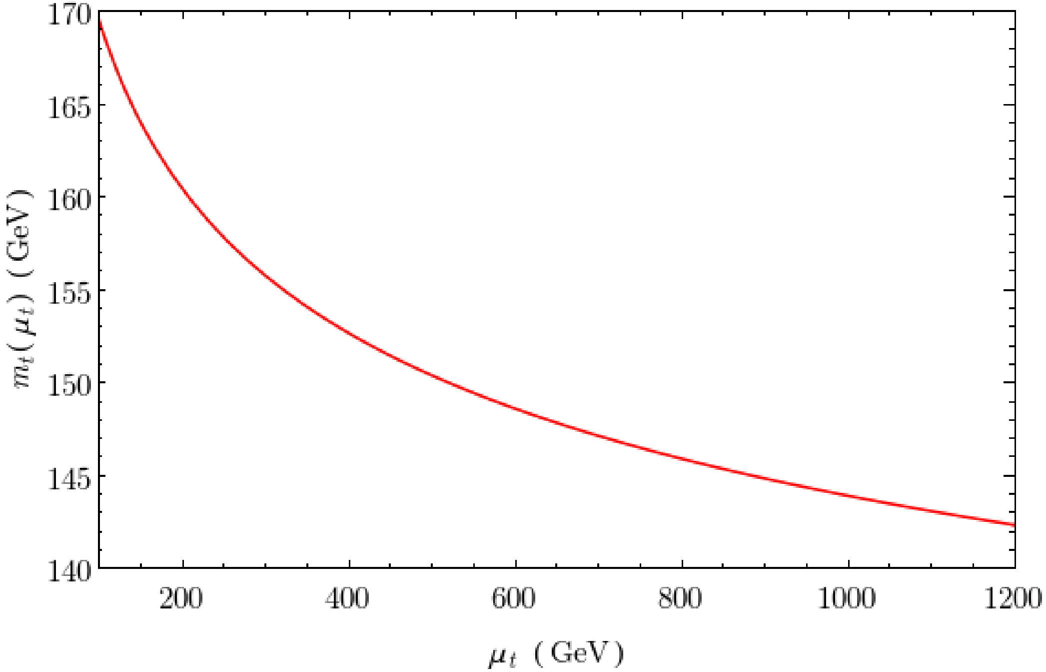
Top-mass-scheme uncertainties

Top-mass-scheme uncertainties at per-mille level for on-shell Higgs production

Very mild parametric dependence of the XS with M_t for m_h=125GeV



Numerical difference between M_t and $m_t(\mu_t)$ not 'enhanced' for μ_t of $O(m_h)$



(note that at LO the difference between OS and $\overline{\text{MS}}$ predictions is simply replacing $M_t \to m_t(\mu_t)$)

The situation will dramatically change if scales involved are larger!

Top-mass-scheme uncertainties: H*

- Issue pointed out a few years ago in the context of di-Higgs production, [Baglio et al., 1811.05692] but also affecting off-shell Higgs (production and decay) and H+jet

- NLO (LO) studies have been performed for H* and HH (H+jet) [Baglio et al., 1811.05692, 2003.03227] [Jones and Spira, 2003.01700]

  ($t\bar{t}$H cross section also has been studied using the $\overline{\text{MS}}$ scheme [Aldaya Martin, Moch, Saibel])

- NLO cross section for off-shell Higgs production: [Jones and Spira, 2003.01700]

$$\sigma(gg \to H^*)\Big|_{Q=125\text{ GeV}} = 42.17^{+0.4\%}_{-0.5\%}\,\text{pb}, \qquad \sigma(gg \to H^*)\Big|_{Q=300\text{ GeV}} = 9.85^{+7.5\%}_{-0.3\%}\,\text{pb}$$

$$\sigma(gg \to H^*)\Big|_{Q=400\text{ GeV}} = 9.43^{+0.1\%}_{-0.9\%}\,\text{pb}, \qquad \sigma(gg \to H^*)\Big|_{Q=600\text{ GeV}} = 1.97^{+0.0\%}_{-15.9\%}\,\text{pb}$$

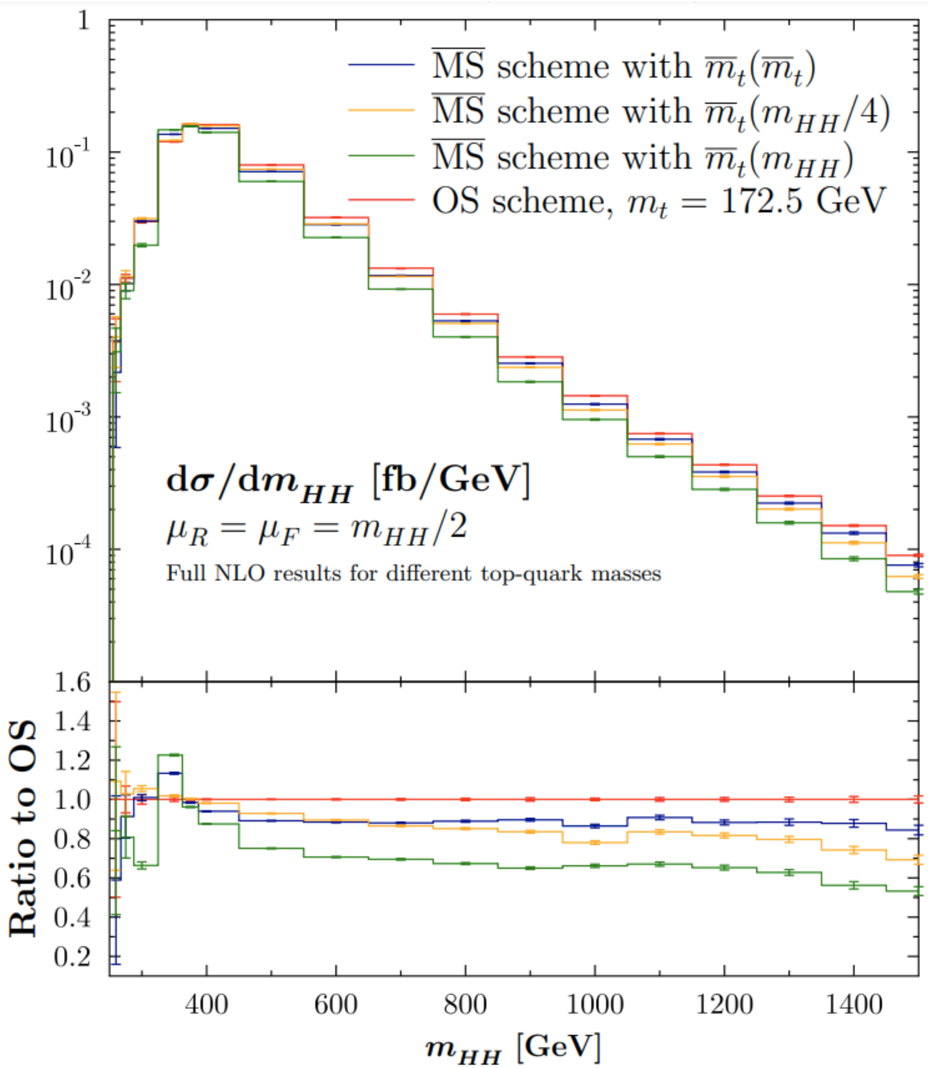
$$\sigma(gg \to H^*)\Big|_{Q=900\text{ GeV}} = 0.230^{+0.0\%}_{-22.3\%}\,\text{pb}, \qquad \sigma(gg \to H^*)\Big|_{Q=1200\text{ GeV}} = 0.0402^{+0.0\%}_{-26.0\%}\,\text{pb}$$

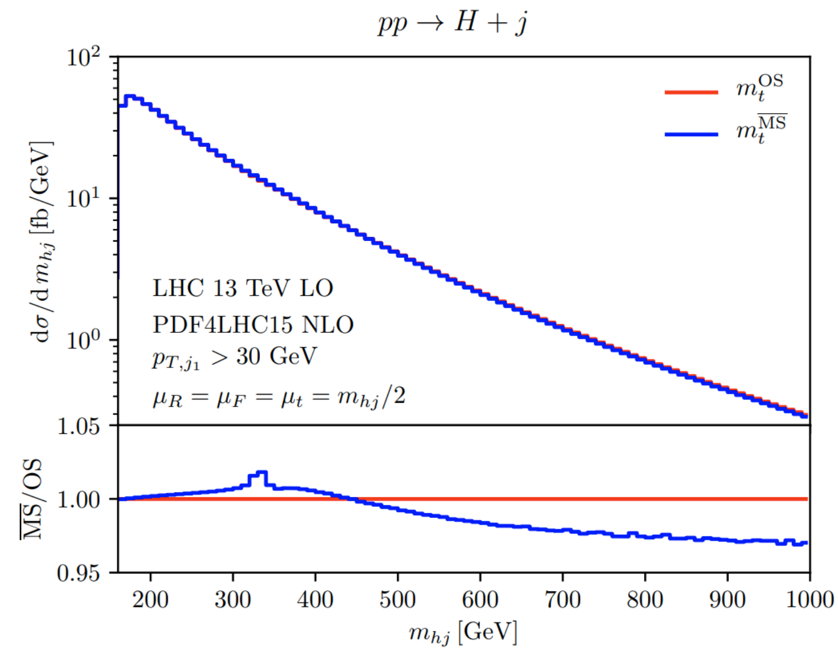
Central value: OS scheme

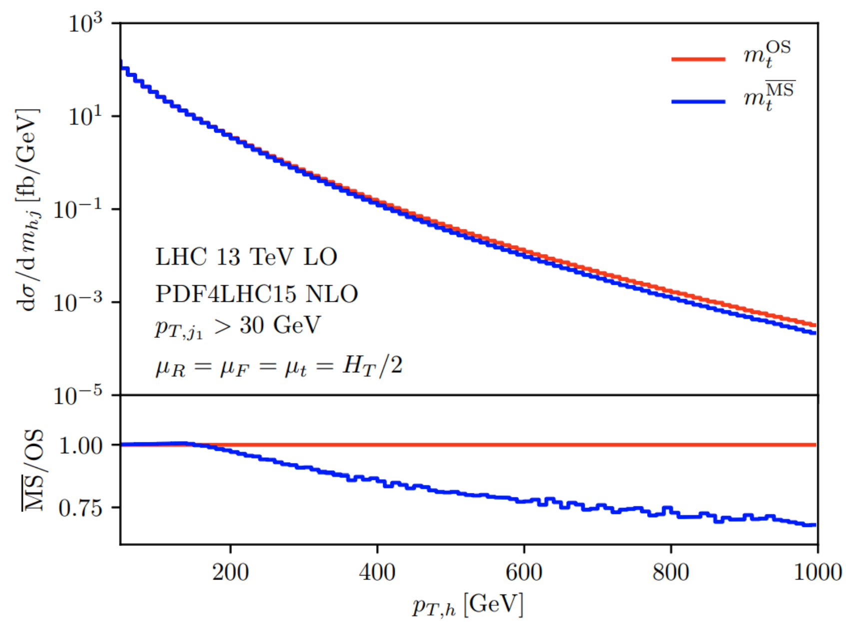
Uncertainty: envelope of $\overline{\text{MS}}$ calculation with μ_t={Q/4,Q/2,Q,$m_t(m_t)$}

Top-scheme uncertainties are dominant for large invariant masses!

Top-mass-scheme uncertainties: HH and H+jet







- Large uncertainties, especially in the tail
- Impact also in the total cross section:

$$\sigma_{\text{NLO}}(14\text{TeV}) = 32.81^{+4\%}_{-18\%}\text{fb}$$

Uncertainties very important when large scales are involved, especially in the $p_{T,h}$ tail

The way forward

How do we reduce this uncertainty?

Convince yourself one scheme is preferred over the other

Gain insight about what is a smart scale choice for μ_t

Compute higher orders

NNLO study of top scheme uncertainties for off-shell Higgs production

[JM, 2206.14667]

Reaching NNLO for H*

- Difficult task: heavy top limit cannot be used for these studies!

- Recently Higgs production with full top mass dependence computed at NNLO
  [Czakon, Harlander, Klappert, Niggetiedt]

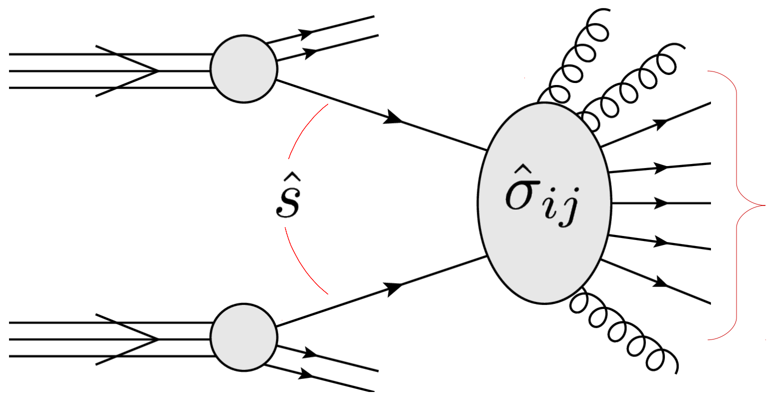
- Results only for on-shell case, but NNLO virtuals for arbitrary m_h, m_t are public
  [Czakon, Niggetiedt]

We can use them to compute NNLOsv with full m_t dependence and any value of m_h

- We can obtain NNLOsv results for H* production in both OS and $\overline{\text{MS}}$ schemes

$$\bar{\sigma}^{(2)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F) = \Big[\sigma^{(2)}(m;\mu_R,\mu_F) + m\Big(d^{(1)}(\mu_m)\,\partial_m\sigma^{(1)}(m;\mu_R,\mu_F) + \frac{1}{2}\left(d^{(1)}(\mu_m)\right)^2 m\,\partial_m^2\sigma^{(0)}(m;\mu_F) + d^{(2)}(\mu_m)\,\partial_m\sigma^{(0)}(m;\mu_F) + \beta_0\,d^{(1)}(\mu_m)\ln\left(\frac{\mu_R^2}{\mu_m^2}\right)\partial_m\sigma^{(0)}(m;\mu_F)\Big)\Big]_{m=m_t(\mu_m)}$$

Soft-virtual approximation



Final state F={H,H*,HH} with invariant mass Q

We consider the variable

$$z = \frac{Q^2}{\hat{s}}$$

When additional radiation is **soft**, we have z~1

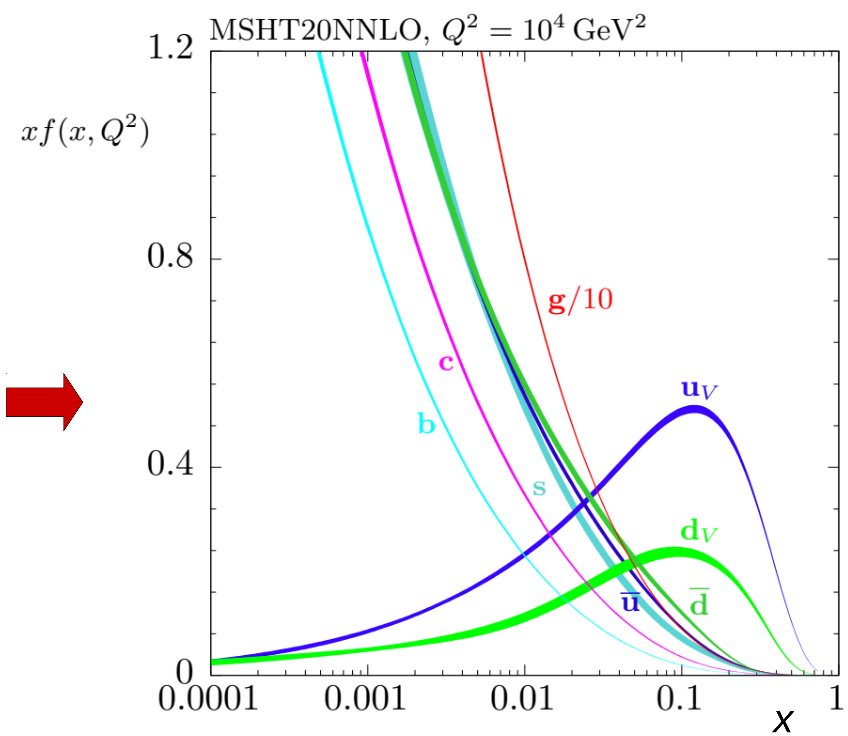
Logarithmically enhanced contributions in this limit
(more specifically on the conjugate variable of z in Mellin space, N)

- Calculation of total cross section much simpler in the soft limit!

- Universal structure: only process-dependent piece is encoded in the virtual corrections
  [de Florian, JM], [Catani, Cieri, de Florian, Ferrera, Grazzini]

- Why is it a good approximation?

  PDFs (especially gluon) prefer low values of x



- Partonic energy tends to be close to the minimum:

$$\hat{s} = x_1 x_2 S \simeq Q^2$$

  predominantly only allowing for soft radiation

- SV-approx defined in Mellin space by dropping terms vanishing in large-N limit

Setup of the calculation

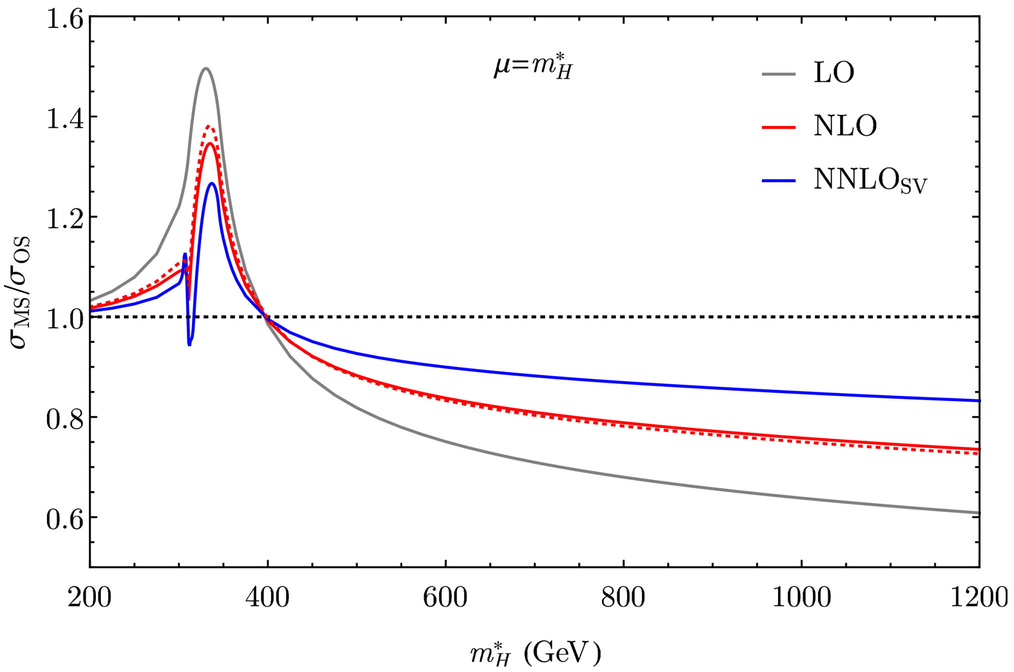
- Off-shell Higgs boson production in 13TeV pp collisions, pp $\rightarrow$ H*
- Higgs virtuality m_H* in range 200GeV – 1200GeV
- Top mass: M_t=172.GeV and $m_t(m_t)$=162.9GeV (note we use a dynamic μ_t scale)
- PDF4LHC15_nnlo at every order
- Central scales set to $\mu_0=m_H^*/2$
- μ_R, μ_F and μ_t varied by factor of 2, avoiding ratios larger than 2 (15-point variation)
- NNLO-SV defined in the following way:

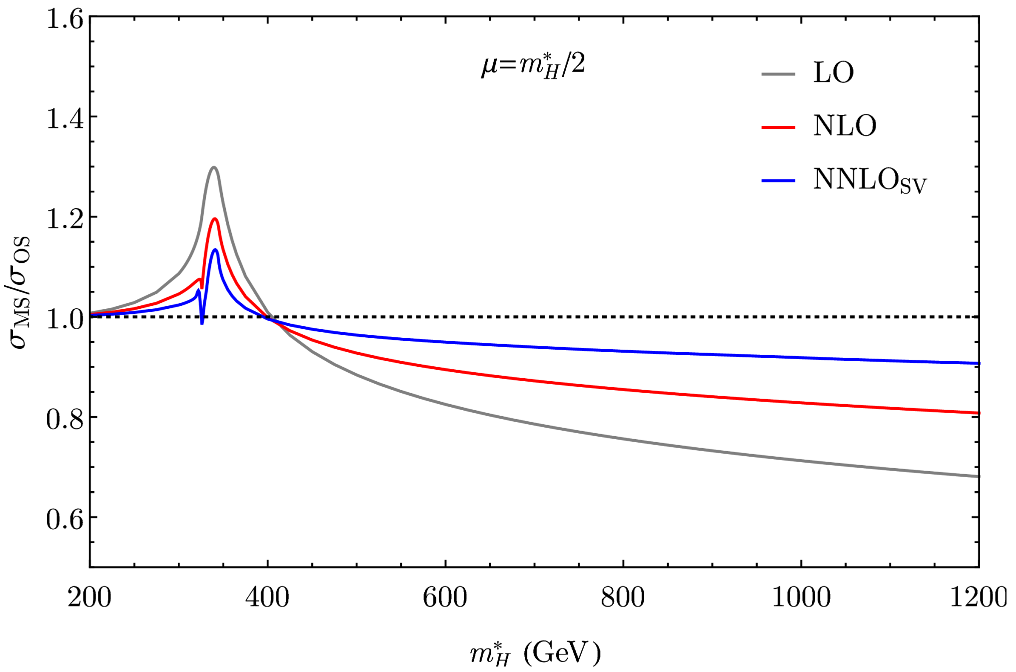
$$\sigma(\mathrm{NNLO_{SV}}) = \sigma(\mathrm{NLO}) + \Delta\sigma(\mathrm{NNLO_{SV}})$$

- NLO computed using iHixs, NNLO piece with dedicated code performing SV approx

$\overline{\text{MS}}$ vs OS scheme

- We compute the ratio of $\overline{\text{MS}}$ and OS cross sections vs m_H^* at each perturbative order
- For clarity, no scale variations included in these plots
- Validation: excellent agreement between NLO and NLO_{SV} (red dashed)





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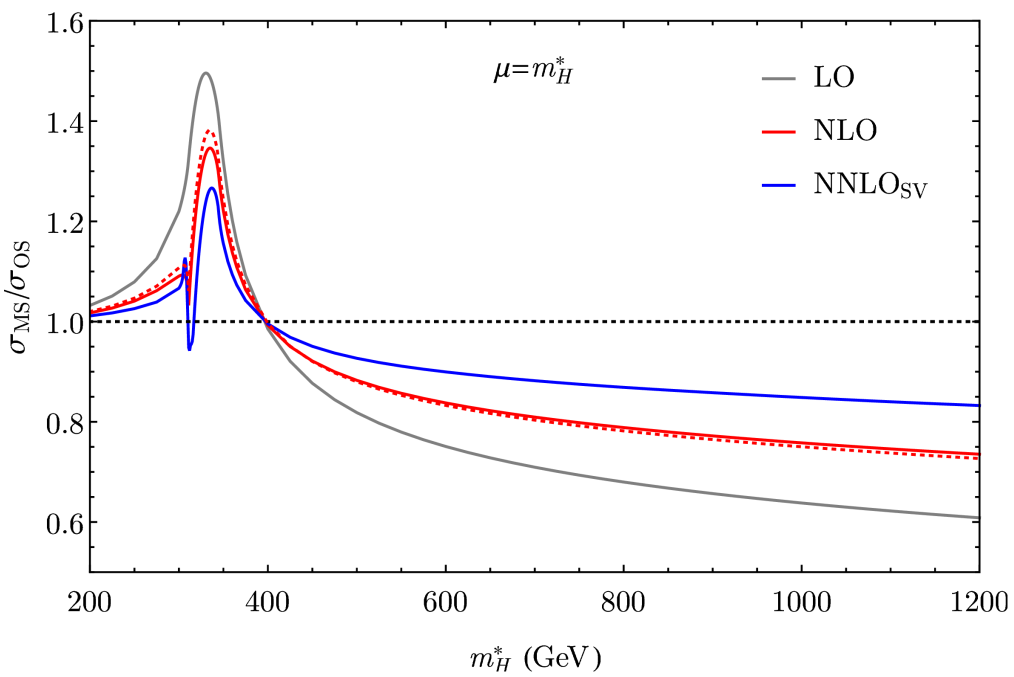

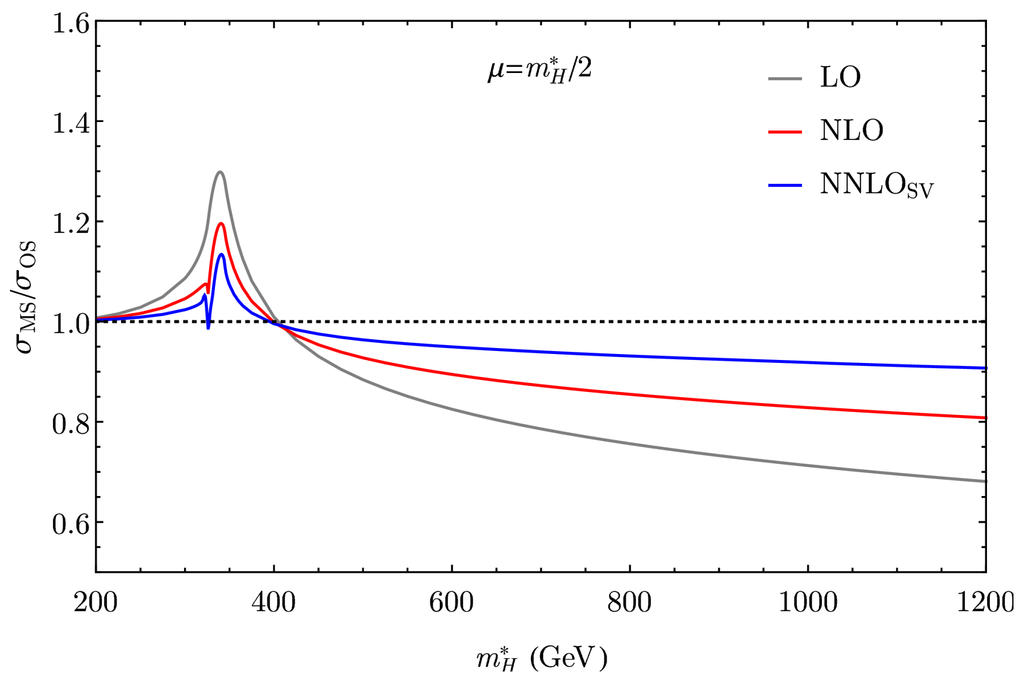

- Difference between the two schemes always reduced as we increase the order
- Larger differences for larger scale choice: higher scales means lower $m_t(\mu_t)$

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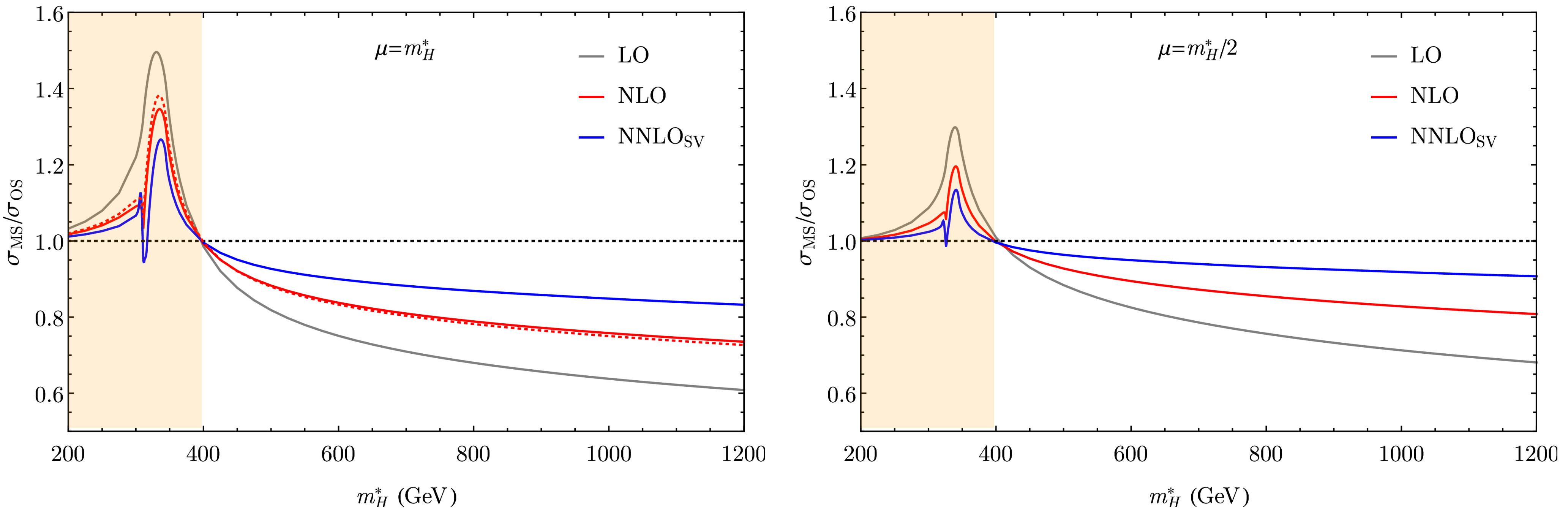

- $\overline{\text{MS}}$ scheme cross section larger below 400GeV
- Largest deviation: 50%, 35%, 27% at LO, NLO, NNLO for $\mu = m_H^*$,
  down to 30%, 20%, 13% at LO, NLO, NNLO for $\mu = m_H^*/2$

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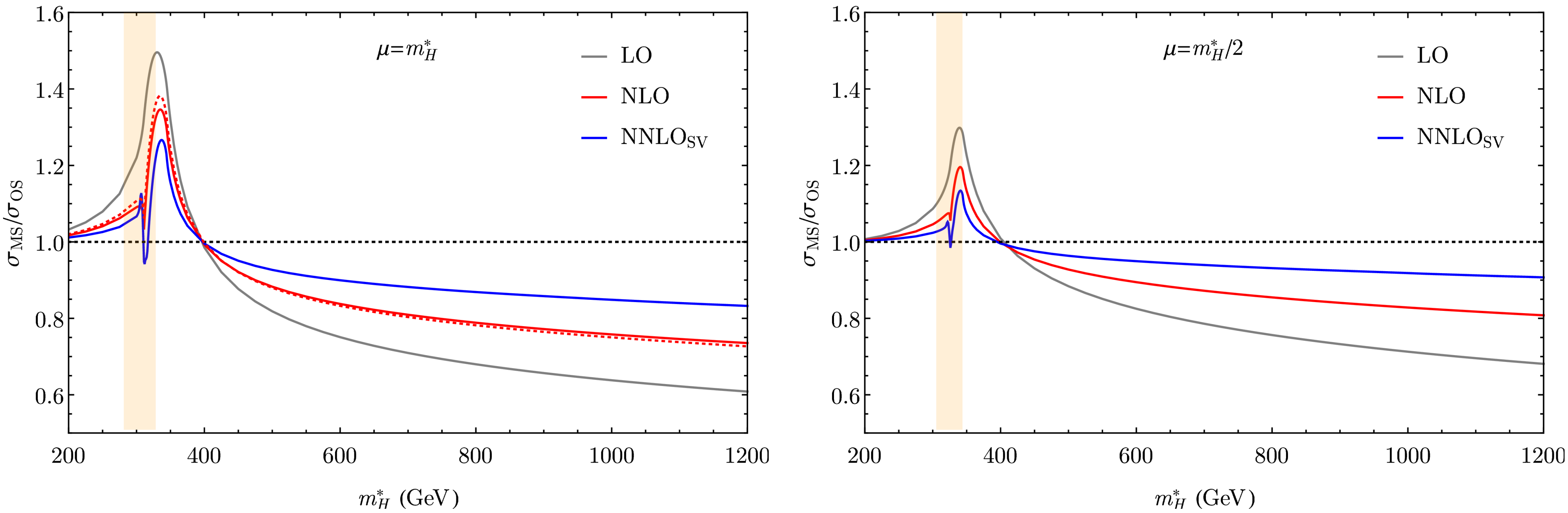

- NLO and NNLO curves present sudden variations close to $t\bar{t}$ threshold
- Traced back to large mass derivatives in OS → $\overline{\text{MS}}$ conversion

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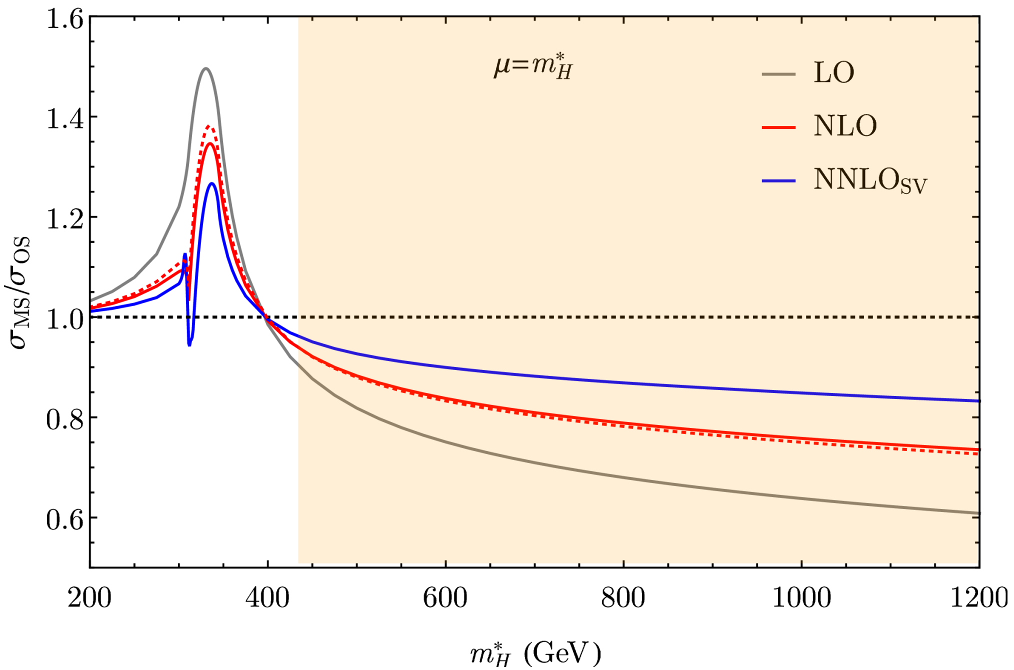
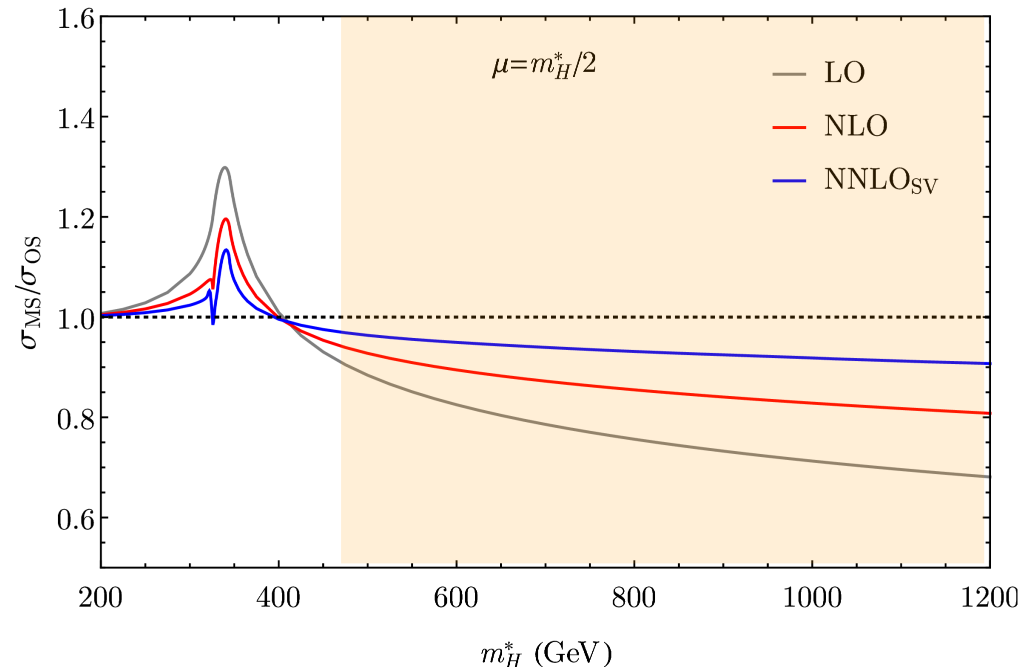
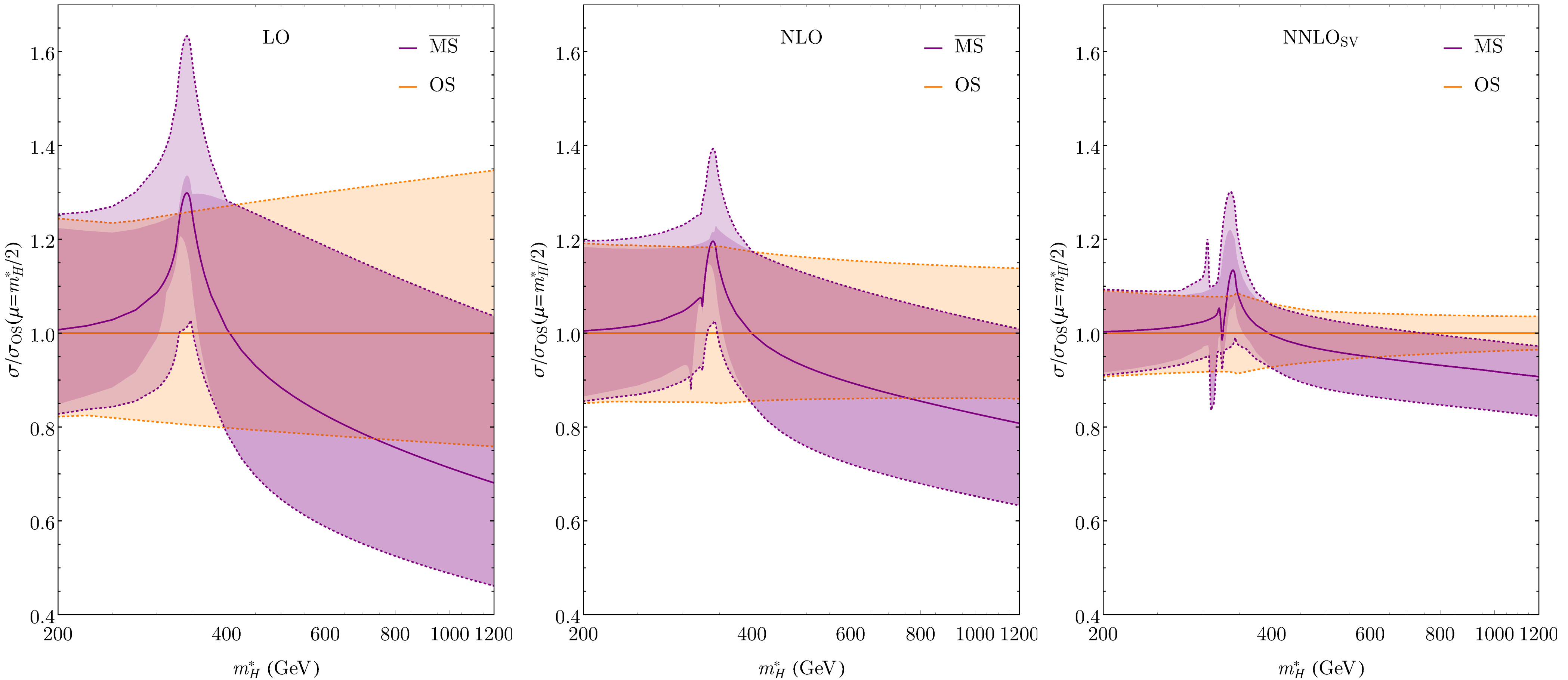




- $\overline{\text{MS}}$ cross section smaller in the tail
- Deviation at m_H^*=1.2TeV: -39%, -26%, -17% at LO, NLO, NNLO for $\mu = m_H^*$,
  down to -32%, -19%, -9% at LO, NLO, NNLO for $\mu = m_H^*/2$

$\overline{\text{MS}}$ vs OS scheme: scale uncertainties

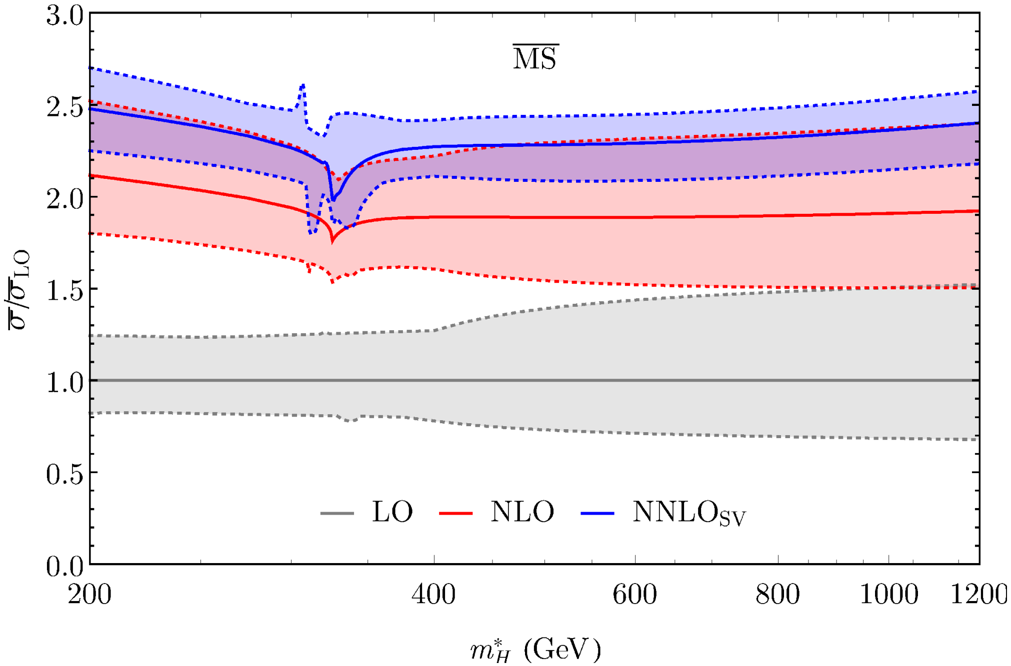
Central scale: $\mu_0 = m_H^*/2$, 15-point variation (darker purple band: 7-point variation with $\mu_t = \mu_R$)

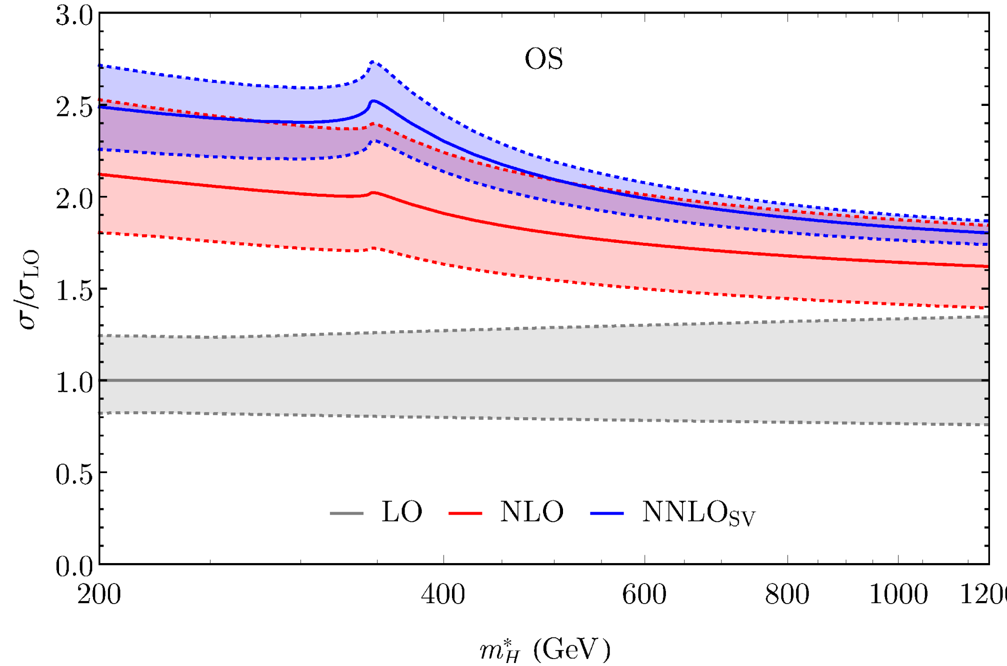


- Scale uncertainties largely reduced in both schemes when increasing order
- Sizeable overlap between $\overline{\text{MS}}$ and OS bands, central values grow closer with h.o. corrections
- Independent variations of μ_t crucial to capture true uncertainty close to $t\bar{t}$ threshold

$\overline{\text{MS}}$ vs OS scheme: K-factors

- We compare the K-factors to evaluate the quality of the perturbative convergence





- Up to $t\bar{t}$ threshold both schemes have similar-sized corrections
- For large invariant masses the OS scheme converges much faster
- OS K-fac: 1.62 (NLO) and 1.11 (NNLO) vs $\overline{\text{MS}}$ K-fac: 1.92 (NLO) and 1.25 (NNLO) for m_h*=1.2TeV
- Missing h.o. corrections expected to be larger in $\overline{\text{MS}}$ scheme, and bringing both schemes closer
- OS scheme seems to be preferable choice for large invariant masses

Combination of uncertainties

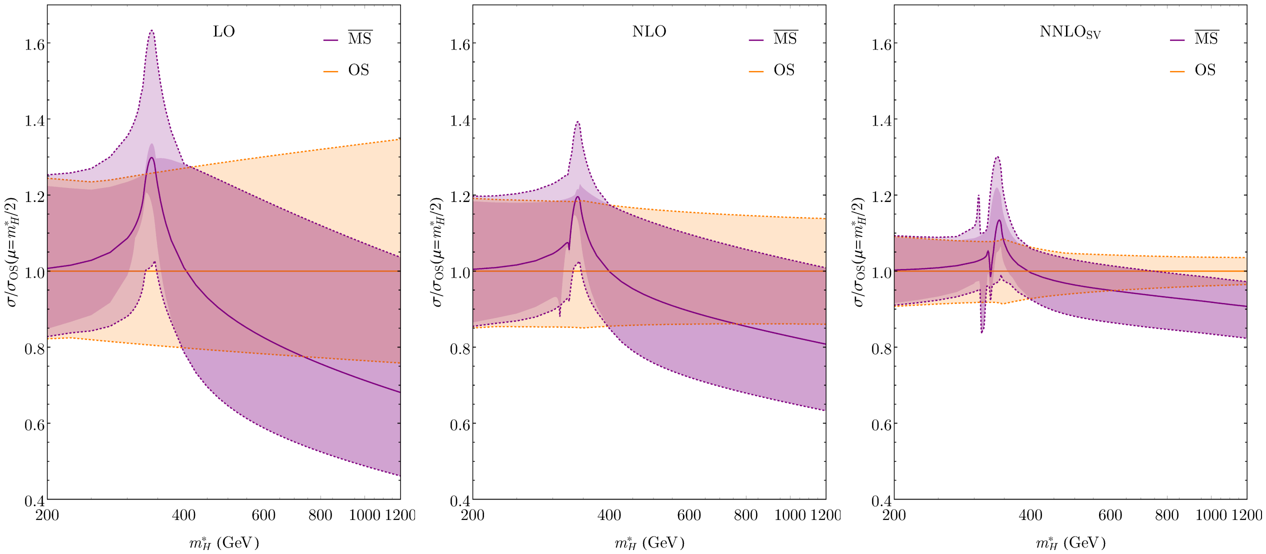
- Most conservative approach: envelope of $\overline{\text{MS}}$ (15-point) and OS (7-point) bands

Combined ‘usual’ μ_R and μ_F uncertainty with top mass scheme and scale uncertainty

- Alternative procedure: take 7-point OS prediction and add linearly μ_t-only variation

- Both approaches lead to quantitatively similar results

- Combined uncertainty significantly reduced at NNLO_{SV}



For instance for m_h*=800GeV

$$\delta\sigma_{\text{LO}} = ^{+32\%}_{-48\%}$$

$$\delta\sigma_{\text{NLO}} = ^{+15\%}_{-32\%}$$

$$\delta\sigma_{\text{NNLO}_{\text{SV}}} = ^{+3.8\%}_{-15\%}$$

- However they can still be overly conservative, e.g. in the m_h* tail

What about di-Higgs?

- Full top-quark mass dependence at NNLO(SV) currently out of reach
- Up to NLO, qualitative features similar to off-shell Higgs production ($m_h{}^* \to m_{hh}$)
- We can consider the large self-coupling limit:

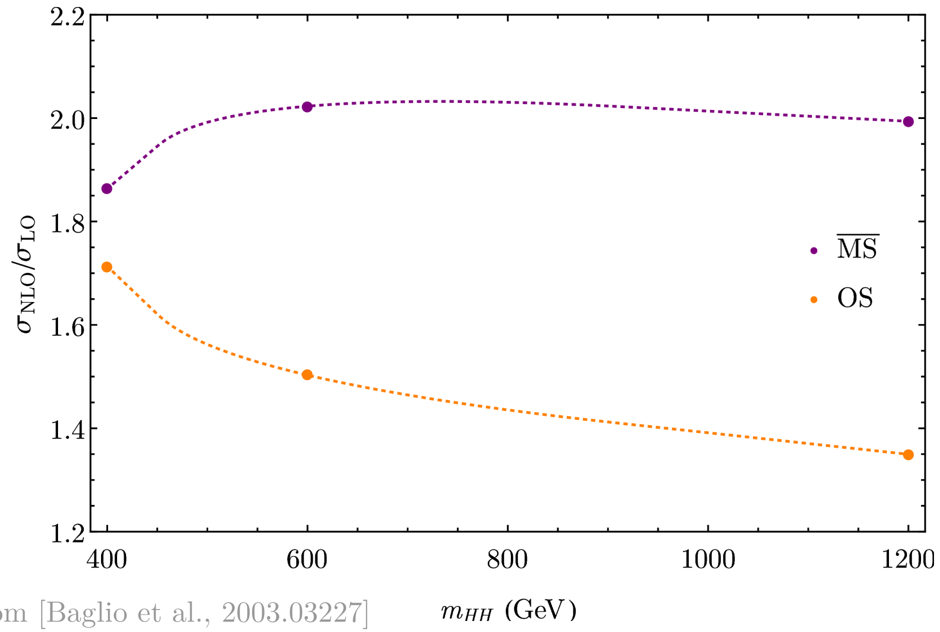
Uncertainty in total XS from envelope in m_{hh} distribution

$$\Delta\sigma^{HH}_{\mathrm{LO}}(\lambda_{hhh}\to\infty) = +36\% - 22\%\ (+17\% - 3.6\%)$$
$$\Delta\sigma^{HH}_{\mathrm{NLO}}(\lambda_{hhh}\to\infty) = +23\% - 16\%\ (+8.3\% - 1.8\%)$$
$$\Delta\sigma^{HH}_{\mathrm{NNLO_{SV}}}(\lambda_{hhh}\to\infty) = +12\% - 9.0\%\ (+4.9\% - 1.3\%)$$

Combined uncertainty — μ_t variation

Significant reduction of the uncertainties when increasing the order

- NLO K-factors in SM di-Higgs also seem to indicate better convergence of OS scheme



Data from [Baglio et al., 2003.03227]

Analytic structure in high energy limit seems to lead to opposite conclusion...

Logs in QCD corrections to form factors:

$$\log\frac{m_t^2}{\hat{s}} \to \log\frac{\mu_t^2}{\hat{s}} + \frac{4}{3}$$

(OS) $(\overline{\mathrm{MS}})$

[Baglio et al., 2003.03227]

Summary and Outlook

- Uncertainties arising from top-mass renormalization are relevant in Higgs observables
- Can become a dominant source if large scales are involved

  → Off-shell Higgs → Di-Higgs → Higgs p_T tail

- First NNLO-accurate study of these uncertainties, for off-shell Higgs production
- Based on construction of $\mathrm{NNLO_{SV}}$ cross section with full top mass dependence
- Significant differences between schemes, though compatible within uncertainties
- Higher-order corrections bring OS and $\overline{\mathrm{MS}}$ predictions closer to each other
- Substantial reduction of scheme and scale uncertainties at $\mathrm{NNLO_{SV}}$
- At large values of m_h* the OS scheme presents smaller perturbative corrections → Preferred scheme in this region
- Similar indications for HH, though further studies are needed

Thanks!