Beyond the Standard Model – Bogdan Dobrescu (Fermilab)
September 2011 - European School of HEP: Remnants from Lecture 1:

Comparison between solutions to the hierarchy problem:

1. Dynamically-broken supersymmetry
Susy breaking scale is exponentially suppressed compared to $M_{\text{Planck}}$
due to gauge dynamics.
Problem: $\mu$ term (the Higgsino mass) is supersymmetric.
Why $\mu \sim v$? (some solutions exist)

2. Technicolor
Exponential hierarchy between $M_{\text{Planck}}$ and the scale where the technicolor gauge interaction becomes strong.
Problem: fit to the electroweak data? (some solutions exist)

3. RS1
$1/M_{\text{Planck}}$ is exponentially suppressed compared to $1/v$.

1.28

Solution (?) #5: Large extra dimensions (ADD)
Graviton only in flat extra dimensions
Gravitational interactions measured at distances $\gtrsim 10^{-3}\text{cm}$:

$$F_N = \frac{m_1 m_2}{M_{\text{Planck}} r^2}$$

We may live on a wall in extra dimensions!
(Arkani-Hamed, Dimopoulos, Dvali, 1998)

Scale of quantum gravity may be as low as $\sim 10$ TeV:

$$M_s = \left( \frac{M_{\text{Planck}}^2}{L^n} \right)^{1/(2+n)}$$

1.25

Solution #4: Composite Higgs models

Higgs boson is a bound state of top quark with a new quark $\chi$.
"Top seesaw model" (Chivukula et al, hep-ph/9809470)

Binding may be due to some strongly-interacting heavy gauge bosons

Scale of Higgs compositeness may be as low as a few TeV.

Homework 1.5: What are the quantum numbers of $\chi$? How would you search for this hypothetical particle?

1.24

(Partial) Solution #6: Little Higgs

1-loop quadratic divergences cancelled by partners carrying
the same spin. (Arkani-Hamed et al, hep-ph/0206021)

Effective theory valid up to scales of order $\sim 5$ TeV, where some
unspecified new dynamics takes over.
1-loop quadratic divergences are cancelled if there is a parity which interchanges each SM particle with a new particle that transforms under a twin SM gauge group.

If the new particles are neutral under the SM gauge group, then these partners would be very hard to see at the LHC.

This is unlike all other known solutions, where a $\tilde{t}$ squark or a $\chi$ quark or something else is visible at the TeV scale.

Effective theory valid again only up to scales of order $\sim 5$ TeV...

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Elementary particles “observed” in experiments:

- **Leptons**
  \[
  \begin{pmatrix}
    \nu^e_L \\
    e_L \\
    \mu_L \\
    \tau_L
  \end{pmatrix},
  \begin{pmatrix}
    \nu^\mu_L \\
    \mu_R
  \end{pmatrix},
  \begin{pmatrix}
    \nu^\tau_L \\
    \tau_R
  \end{pmatrix}
  \]

- **Quarks**
  \[
  \begin{pmatrix}
    u_L \\
    d_L \\
    c_L \\
    s_L \\
    t_L \\
    b_L
  \end{pmatrix},
  \begin{pmatrix}
    c_R \\
    u_R \\
    s_R \\
    d_R \\
    t_R \\
    b_R
  \end{pmatrix}
  \]

- **SU(3) × SU(2) × U(1) gauge bosons** (spin 1)
- **8 gluons** + $W, Z, \gamma$

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**Beyond the Standard Model**

**Lecture 2**

**Bogdan Dobrescu** (Fermilab)

**Outline:**
- Electroweak symmetry breaking (Lecture 1)
- Quark and lepton masses; vectorlike quarks (Lecture 2)
- New gauge bosons (Lecture 3)
- WIMPs and cascade decays (Lecture 4)
- How to search for new phenomena (Lecture 5)

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**Fermion masses**

All Standard Model fermions are chiral.

- “left-handed” top (feels the weak interaction)
- “right-handed” top (no interaction with $W^\pm$)
All Standard Model fermions are chiral.

The two top quarks:
- “left-handed” top (feels the weak interaction)
- “right-handed” top (no interaction with $W^\pm$)

Top mass: $t_L$ turns into $t_R$ and vice-versa

Quark and lepton masses at the 1 TeV scale:

How is electroweak symmetry breaking communicated to fermions?

Many attempts at explaining the hierarchy of Yukawa couplings:
- discrete symmetries $\rightarrow (\langle \phi \rangle / M)^n$ suppressions.
- GUT relations.
- wave function overlaps in extra dimensions.
- ...
- loop suppressions:
  - Georgi, Glashow, 1972 – attempts to calculate the electron mass as a one loop contribution involving the muon mass.
  - Many papers in the 1980’s (e.g., Balakrishna, Kagan, Mohapatra, 1988)

Typical scheme: 3rd generation masses at tree level, 2nd generation masses at one loop, 1st generation masses at two loops.
Let us assume that only the top quark gets its mass at tree level, $y_t t_R Q^3_L H$, and introduce some interactions that communicate EWSB to the other quarks and leptons.

2.7

The one-loop diagram responsible for the tau mass:

Only one linear combination of $L^i_L$ couples to $t_R$

$\Rightarrow m_{\nu_\tau} = m_{\tau_e} = 0$ at 1-loop

$\lambda_{33} \lambda'_{33} \frac{m_t}{16\pi^2} \ln \left( \frac{\Lambda^2}{M^2_r} \right)$

Some new physics cuts off the loop integral at a scale $\Lambda$:
a superpartner of $r$, or some dynamics if $r$ is a composite particle,
or some particle integrated out to generate the Yukawa couplings of $r$.

$m_{\tau}$ depends on $\frac{\Lambda}{M_r}$ (only a lower limit on $M_r$ is set by phenomenology).

2.9

Case study 2: A scalar leptoquark

2.10

Case study 2: A scalar leptoquark

$r$: scalar field transforming as (3,2,+7/6) under $SU(3)_C \times SU(2)_W \times U(1)_Y$

$r = \begin{pmatrix} r_u \\ r_d \end{pmatrix}$ charge + 5/3 charge + 2/3

Most general renormalizable interactions with SM fermions

$\lambda_{ij} r \overline{t}_R L^i_L + \lambda'_{ij} r \overline{Q}_L e^j_R$  

($r$ is a leptoquark)

$i, j = 1, 2, 3$ label the fermion generations:

$L^1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$, $L^2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$, ...

In the absence of masses, the identity of fermion generations is ill defined.

2.8

Charm mass induced by a two-loop “rainbow” diagram:

Only one linear combination of $Q^1_L$ and $Q^2_L$
couples to $\tau_R$ $\Rightarrow m_{\tau_u} = 0$ at 1-loop

$\lambda_{23} \lambda'_{23} \frac{1}{16\pi^2} \ln \left( \frac{\Lambda^2}{M^2_r} \right)$

If there are no other contributions to $m_c$, 
the $m_c/m_\tau$ ratio at 1 TeV requires $\lambda_{23} \lambda'_{23} \approx (3.3)^2$ for $\Lambda \approx 10M_r$. 

2.10
Muon mass induced by 3-loop “rainbow” and nonplanar diagrams:

\[
\mu \simeq \lambda_{22} \lambda_{22} m_c \left[ 1 + O(1/N_c) \right] \frac{N_c}{16\pi^2} \ln \left( \frac{\Lambda^2}{M_r^2} \right)
\]

\( \frac{m_\mu}{m_c} \) ratio requires \( \lambda_{22} \lambda_{22}' [1 + O(1/N_c)] \approx (1.5)^2 \)

At 3-loops the electron is still massless!

2.11

Up-quark mass induced at 4-loops:

\[
u \simeq \lambda_{12} \lambda_{12} m_\mu \frac{1}{16\pi^2} \ln \left( \frac{\Lambda^2}{M_r^2} \right)
\]

Correct \( m_u/m_\mu \) ratio requires \( \lambda_{12} \lambda_{12}' \approx (0.6)^2 \)

2.12

“Domino” mechanism
Phenomenological constraints on the $r$ leptoquark:

Tree level flavor-changing processes induced by $r$ exchange:

$$\mu \rightarrow e \text{ conversion}$$

$$K^+ \rightarrow \pi^0 \mu^+ e^-, \ldots$$

$$\tau^+ \rightarrow K^0 e^+, \ldots$$

$$\pi^+ \rightarrow e^+ \nu \text{ versus } \pi^+ \rightarrow \mu^+ \nu$$

$$\Rightarrow \quad M_r > O(5 - 50) \text{ TeV.}$$

Homework 2.1: draw the Feynman diagrams for these processes. Estimate the coefficients of the 4-fermion effective operators induced by integrating $r$ out.

Origin of quark and lepton masses may be probed through low energy flavor-changing processes (and could be hidden in high $p_T$ processes).

Vectorlike fermions

A 4th generation of chiral fermions may exist, but there are strong constraints on it from electroweak measurements and also from flavor processes.

**Vectorlike (i.e. non-chiral) fermions may also exist.** Their masses are allowed by the $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge symmetry, $\Rightarrow$ may naturally be heavier than the $t$ quark.

Vectorlike fermions are the simplest spin-1/2 particles, and would be a novel form of matter.

The hierarchy problem, and the pattern of quark and lepton masses, suggest the existence of structures beyond the SM.

However, there might be novel phenomena that are not related to these.

**Let’s not search only under the lamppost ...**

What kind of new particles may exist?

Case study 3: A vectorlike $t'$ quark

A vectorlike quark $\chi$ which transforms as $(3,1,+2/3)$ under $SU(3)_c \times SU(2)_W \times U(1)_Y$ would mix with the top quark:

$$\mathcal{L} = -\left(\bar{\nu}_L, \chi_L\right) \begin{pmatrix} \lambda t v_H & 0 \\ M_0 & M_\chi \end{pmatrix} \left(\nu_R^3 \chi_R\right)$$

$v_H \simeq 174$ GeV is the Higgs vacuum expectation value

$\lambda_t$ is the top Yukawa coupling

$M_0$ and $M_\chi$ are mass parameters

$\chi$ is present in: Top-quark seesaw theory, Little Higgs models, ...
Transform the gauge eigenstates $u^3$ and $\chi$ to the physical states $t$ (discovered at the Tevatron) and $t'$ (remains to be discovered):

$\begin{pmatrix} t_L \\ t'_L \end{pmatrix} = \begin{pmatrix} \cos \theta_L & -\sin \theta_L \\ \sin \theta_L & \cos \theta_L \end{pmatrix} \begin{pmatrix} u^3 \\ \chi \end{pmatrix}$

The three initial parameters $\lambda_t, M_0, M_\chi$ are replaced by physical parameters: $m_t$ (measured!), $m_{t'}$ and $s_L \equiv \sin \theta_L$.

**Homework 2.2:** Express $s_L$ in terms of $\lambda_t$ and $m_{t'}$.

Show that for $m_{t'} \to \infty$, the mixing vanishes ($s_L \to 0$), so that the new physics decouples from the standard model.

Decay widths of $t'$:

$\Gamma(t' \to W^+ b) = \frac{s_L^2 m_t^3}{32 \pi v_H^2} \left( 1 + \frac{M_0^2}{m_{t'}^2} \right) \left( 1 + \frac{2M_0^2}{m_{t'}^2} \right)$

$\Gamma(t' \to Z t) = \frac{s_L^2 c_L^2 m_t^3}{64 \pi v_H^2} \left[ 1 + O \left( \frac{M_0^2}{m_{t'}^2} \right) \right]$  

If $m_\tau > M_0 + m_t$:

$\Gamma(t' \to h t) = \frac{s_L^2 c_L^2 m_t^3}{64 \pi v_H^2} \left[ 1 + \frac{m_\tau^2 - M_0^2}{m_{t'}^2} \right] \left( 1 + \frac{m_\tau^2}{m_{t'}^2} \right) + 4m_t^2 \left[ 1 - \frac{m_{t'}^2 + M_0^2}{m_{t'}^2} \right] - 4m_t^2)^{1/2}$

Branching fractions of $t'$:

![](image1.png)

**Interactions of left-handed quarks with $W$ and $Z$:**

$t_L - b_L - W^+_\mu : i \frac{g}{\sqrt{2}} s_L \gamma_\mu \Rightarrow m_t < 0.07$ (from single top production)

$t'_L - b_L - W^+_\mu : i \frac{g}{\sqrt{2}} s_L \gamma_\mu$

$t_L - W_L - Z : i \frac{g}{\cos \theta_W} \left( \frac{1}{2} Z^2 - \frac{2}{3} \sin^2 \theta_W \right) \gamma_\mu$

$t'_L - W_L - Z : i \frac{g}{\cos \theta_W} \left( \frac{1}{2} Z^2 - \frac{2}{3} \sin^2 \theta_W \right) \gamma_\mu$

Interactions of $t_R, t'_R$ with the $Z$ are identical with those of the SM $t_R$. Interactions with the Higgs boson:

\[
\frac{-1}{v_H^2} h^0 (c_L^3 m_t t'_R + s_L^2 m_t \bar{t}_R t'_R + c_L s_L m_t \bar{t}_L t'_L + c_L s_L m_t \bar{t}_L t_R) + \text{H.c.}
\]

**Homework 2.3:**

Compute the branching fractions of $t'$ in the $m_{t'} \gg M_0 + m_t$ limit.

**Homework 2.4:**

Analyze a similar theory where there is a vectorlike quark transforming as $(3,2,1/6)$ under $SU(3)_c \times SU(2)_W \times U(1)_Y$.  

![](image2.png)
QCD production of $t't'$ pairs, followed by $t'$ decays, leads to various final states:

$$(W^+b)(W^-\bar{b})$$

$$(Zt)(W^-\bar{b}) \text{ or } (Z\bar{t})(W^+b)$$

$$(ht)(W^-\bar{b}) \text{ or } (h\bar{t})(W^+b)$$, with $h \rightarrow b\bar{b}$ or $h \rightarrow W^+W^-$

Example:

The Higgs boson could be discovered in the $W^+W^- + 4b$ sample...