Au+Au collisions at RHIC

- Au+Au collision at STAR: longitudinal projection
- ~ 3000 produced particles streaming into the detector

Au+Au collisions at RHIC

- Au+Au collision at STAR: transverse projection
Pb+Pb collisions at the LHC: ALICE

- Pb+Pb collision at ALICE: ~ 1600 hadrons per unit rapidity
- How to describe/understand such a complex system?

Pb+Pb collisions at the LHC: ATLAS

- Traditional perturbative methods become inappropriate (collective phenomena, multiple scattering ...)

Pb+Pb collisions at the LHC: CMS

- The concept of particle is not so useful anymore ...
- One should rather speak about QCD matter

QCD matter: from hadrons ...

- At low energies, QCD matter exists only in the form of hadrons (mesons, baryons, nuclei)
At sufficiently high energies, the relevant degrees of freedom are partonic (quarks & gluons).

True for both p+p collisions and A+A collisions ...

New forms of QCD matter produced in HIC

Prior to the collision: 2 Lorentz–contracted nuclei (‘pancakes’)
  - 'Color Glass Condensate' (CGC)
Right after the collision: non–equilibrium partonic matter
  - 'Glasma' (from 'Glass' + 'Plasma')
At later stages ($\Delta t \gtrsim 1$ fm/c): local thermal equilibrium
  - 'Quark–Gluon Plasma' (QGP)
Final stage ($\Delta t \gtrsim 6$ fm/c): hadrons
  - 'final event', or 'particle production'

How to study these new forms of matter?

Standard perturbation theory in QCD (= expansion in powers of the coupling ‘constant’ $\alpha_s$) fails even at weak coupling, because of the high parton density.

High–density effects (multiple scattering, parton saturation, Debye screening etc) must be resummed to all orders in $\alpha_s$.

This results into effective theories.
The possibility of a strong coupling

- Besides, there is no guarantee that the coupling is weak!

**RHIC Scientists Serve Up "Perfect" Liquid**

New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a liquid.

- ‘Perfect fluid’ = $\alpha_s \to \infty$
- Interesting connection with string theory (‘AdS/CFT correspondence’).

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**Effective theories for Heavy Ion Collisions**

- A space–time picture of a heavy ion collision

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**Lecture 0: A QCD Primer**

**QCD: Quarks & Gluons**

- Electromagnetic interactions: Quantum Electrodynamics (QED)
  - matter: electron; interaction carrier: photon
  - interaction vertex:

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_f \bar{\psi}_f (i\gamma^\mu D_\mu - m) \psi_f
\]

- Strong interactions: Quantum Chromodynamics (QCD)
  - matter: quarks; interaction carriers: gluons
  - interaction vertices:

\[
\begin{align*}
  a_i & \sim g (t^a)_{ij} & a_i & \sim g (T^a)_{bc} \\
  a_i & \sim g (t^a)_{ij} & a_i & \sim g (T^a)_{bc}
\end{align*}
\]

- $i, j$: color indices of the quarks ($N_c = 3$ possible values)
- $a, b, c$: color indices of the gluons ($N_c^2 - 1 = 8$ possible values)
Running coupling: QED

- An electric charge polarizes the surrounding medium:
  
  \[ V(R) = \frac{e_{\text{eff}}(R)}{R} \]

- The effective charge depends upon the distance \( R \) from the bare one.
- Normally this leads to screening: \( e_{\text{eff}}(R) \) decreases with \( R \).

Running coupling: from QED to QCD

- The vacuum itself is a polarisable ‘medium’!

\[ QED: \quad \alpha_{\text{eff}}(R) = \frac{\alpha}{1 - \frac{2\alpha}{3\pi} \ln(1/mR)}; \quad \alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137} \]

- In QCD, the (longitudinal) gluons yield antiscreening!

\[ QCD: \quad \alpha_s(R) = \frac{g^2(R)}{4\pi} = \frac{2\pi N_c}{(11N_c - 2N_f) \ln(1/\Lambda_{QCD} R)} \]

Asymptotic freedom

- The coupling is weak at short distances, or large transferred momenta:
  \[ Q \sim 1/R \gg \Lambda_{QCD} \approx 200 \text{ MeV} \]

Confinement

- The quark–antiquark potential increases linearly with the distance.
- Quarks (and gluons) are confined into colorless hadrons
Quark–antiquark potential at finite $T$

- With increasing the temperature $T$, the potential flattens at shorter and shorter distances.

![Potential Flattening Graph](image)

- This eventually leads to a phase transition at some critical temperature $T_c$.

Quark–Gluon Plasma

- Lattice calculations of the pressure in QCD at finite $T$

![Lattice Pressure Graph](image)

- Rapid increase of the pressure
  - at $T \approx 270$ MeV with gluons only
  - at $T \approx 150$ to 180 MeV with light quarks

- Interpreted as a rise in the number of active degrees of freedom due to the liberation of quarks and gluons.

Debye screening

- Quark–Gluon Plasma (QGP) : a system of quarks and gluons which got free of confinement!

- How is that possible ???

![Debye Screening Equation](image)

- In a dense medium, color charges are screened by their neighbors

- The interaction potential decreases exponentially beyond the Debye radius $r_{Debye} = 1/m_{Debye}$

- Hadrons whose sizes are larger than $r_{Debye}$ cannot bind anymore.
Deconfinement phase transition

- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors.
- This phenomenon extends to the whole volume when the phase transition ends.
- Note: if the transition was first-order, it would go through a mixed phase containing a mixture of nucleons and plasma.

The actual scenario is a ‘cross-over’

Possible first-order scenario with critical bubbles

- ... but this is not what really happens!
- ... as explored by the expansion of the Early Universe...

This was firmly established by the Wuppertal–Budapest lattice group (Aoki et al., Nature, 443 (2006) 675)
**The Big Bang**

- formation of atoms
- nucleosynthesis
- confinement
- Quark Gluon Plasma
- EW transition
- end of inflation
- big bang

**Phase-diagram for QCD**

- ... as explored by the expansion of the Early Universe ...
- ... and in the ultrarelativistic heavy ion collisions.

**The Little Bang**

- The subject of these lectures

- $z = -ct$
- $z = ct$
- $\tau < 0$: hadronic wavefunctions prior to the collision
  - high-energy evolution & the Color Glass Condensate
  - it applies to any highly energetic hadron (proton or nucleus)
Lecture I: Initial conditions

- \( \tau < 0 \) : hadronic wavefunctions prior to the collision
- \( \tau \sim 0 \) fm/c : the hard scattering
  - production of hard particles: jets, direct photons, heavy quarks
  - calculable within (standard) perturbative QCD (‘leading twist’)

- \( \tau < 0 \) : hadronic wavefunctions prior to the collision
- \( \tau \sim 0 \) fm/c : the hard scattering
- \( \tau \sim 0.2 \) fm/c : strong color fields (or ‘glasma’)
  - semi–hard quanta \( (p_\perp \lesssim 2 \) GeV\): gluons, light quarks
  - make up for most of the multiplicity
  - sensitive to the physics of saturation (‘higher twist’)

Color Glass Condensate

**Parton picture**

- When an energetic hadron is probed on a hard resolution scale (momentum transfer \( Q^2 \gg \Lambda_{QCD}^2 \)), one sees a bunch of partons ...
  - with transverse area \( \sim 1/Q^2 \) ...
  - and longitudinal momentum fraction \( x = k_\perp/P \)
    - fixed by the kinematics
  - E.g. : in Deep Inelastic Scattering (DIS)
    \[ x = \frac{Q^2}{s} \]
    - \( s \) = center-of-mass energy squared
  - N.B.: high energy \( \iff \) small \( x \)
The partons relevant for the process under consideration carry the longitudinal momentum fractions:

\[ x_1 = \frac{p_a^+}{\sqrt{s}} e^{Y_a} + \frac{p_b^+}{\sqrt{s}} e^{Y_b}, \quad x_2 = \frac{p_a^-}{\sqrt{s}} e^{-Y_a} + \frac{p_b^-}{\sqrt{s}} e^{-Y_b} \]

- \( p_\perp \): transverse momenta of the produced particles
- \( Y \): their rapidities
- \( \sqrt{s} \): collision energy

99% of the total multiplicity lies below \( p_\perp = 2 \) GeV

- \( x \sim 10^{-2} \) at RHIC (\( \sqrt{s} = 200 \) GeV)
- \( x \sim 4 \times 10^{-4} \) at the LHC (\( \sqrt{s} = 5.5 \) TeV)

\( \Delta x_\perp \sim 1/Q \) and longitudinal momentum \( k_z = xP \)

The gluon distribution rises very fast with increasing energy:

- Gluon distribution \( xg(x, Q^2) \):
  # of gluons with transverse size \( \Delta x_\perp \sim 1/Q \) and longitudinal momentum \( k_z = xP \)
Phase–space enhancement for the emission of
- collinear \((k_\perp \to 0)\)
- and/or soft (low–energy) \((x \to 0)\) gluons

The parent parton can be either a quark or a gluon
\[ C_F = t^a t^a = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}, \quad C_A = T^a T^a = N_c = 3 \]

The daughter gluon can in turn radiate an even softer gluon!

Gluon cascades

- \(n\) gluons strictly ordered in \(x\)
- The \(n\)-gluon cascade contributes
  \[ \frac{1}{n!} (\alpha_s Y)^n \]
- The sum of all the cascades exponentiates:
  \[ x g(x, Q^2) \propto e^{\omega \alpha_s Y} \text{ BFKL evolution} \]
  \( (Balitsk\text{y}, Fadin, Kuraev, Lipatov, 75–78)\)

This evolution is linear:
the emitted gluons do not interact with each other

2 gluons

- The ‘cost’ of the addition gluon:
  \[ \alpha_s \int_x^{1} \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x} \]

Formally, a process of higher order in \(\alpha_s\), but which is enhanced by the available rapidity interval
- \(Y \equiv \ln(1/x)\): rapidity difference between the parent quark and the last emitted gluon
- When \(\alpha_s Y \gtrsim 1 \implies \text{need for resummation!} \)

Gluon recombination

- The gluon density rises with decreasing \(x\) (increasing energy)
- Eventually gluons start overlapping with each other and then they interact: \(2 \rightarrow 1\) gluon recombination
- These interactions stop the growth: saturation
Saturation momentum

- Number of gluons per unit area:
  \[ N \sim \frac{x g_A(x, Q^2)}{\pi R_A^2} \]
- Recombination cross-section
  \[ \sigma \sim \frac{\alpha_s}{Q^2} \]
- Recombination happens if \( N \sigma \gtrsim 1 \), i.e. \( Q^2 \lesssim Q_s^2 \), with
  \[ Q_s^2(x, A) \sim \alpha_s \frac{x g_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.25}} \]
- Low \( Q^2 \) \( \rightarrow \) large area \( \sim 1/Q^2 \) \( \rightarrow \) strong overlapping

Saturation scale as a function of \( x \) and \( A \)

- \( x \sim 10^{-5} \): \( Q_s \sim 1 \) GeV for proton and \( \sim 3 \) GeV for Pb or Au

Multiplicities at the LHC: p+p

- In a high–energy scattering, the saturated gluons are released in the final state
  - typical transverse momentum \( \langle p_T \rangle \sim Q_s(E) \)
  - average multiplicity \( dN/d\eta \sim Q_s^2(E) \)

Multiplicities in HIC: RHIC & LHC

- Logarithmic growth \( \ln(s) \) excluded by the LHC data
- Larger energy exponent \( (E^\lambda) \) for A+A than for p+p
  - this difference is theoretically understood
A very robust, qualitative, prediction of saturation:
DIS at HERA, Au+Au at RHIC, p+p at the LHC ...
(looking forward to the relevant Pb+Pb data at the LHC)

- The single–inclusive spectra for particle production depend...
  - ... upon the particle transverse momentum $p_T$
  - ... and the COM energy of the collision $\sqrt{s}$
  ... only via the ratio of $p_T$ to the saturation momentum $Q_s$:
  $$\frac{dN}{d\eta d^2 p_T} \simeq F(\tau) \quad \text{with} \quad \tau \equiv \frac{p_T^2}{Q_s^2(\sqrt{s}/\sqrt{s})}$$
- At high energy, $Q_s$ is the only intrinsic scale in the problem!

The need for an effective theory

- How to compute the saturation scale from first principles?

Relatively hard scale ($Q_s \gg \Lambda_{\text{QCD}}$) $\Rightarrow$ weak coupling!
- ... but high density $\Rightarrow$ strong non–linear effects
- Solution: a reorganization of perturbation theory!
  (McLerran and Venugopalan, 94; E.I., McLerran, and Leonidov, 00)
How to scatter 2 CGC’s?

- A heavy ion collision at high energy

- Main difficulty: How to treat collisions involving a large number of partons?

Proton–proton collisions

- Dilute–Dilute: one parton from each projectile interact

- Collinear factorization scheme of perturbative QCD
  - usual pdf’s + DGLAP evolution
  - partonic cross-sections
  - Caution: forward rapidity ($Y \gg 1$) & not too hard $p_\perp \Rightarrow x_2 \ll 1$

Proton–nucleus collisions (1)

- Most interesting situation: forward particle production ($Y \gtrsim 3$) at 'semi–hard' momenta ($p_\perp \sim 1 \div 5$ GeV)
  - very small $x_2 \ll 1$ in the nucleus
  - $p_\perp$ comparable to $Q_s(A, x_2)$

- Dilute–Dense: new factorization scheme needed
  - similar to deep inelastic scattering at small $x$

Proton–nucleus collisions (2)

- How to include both multiple scattering and saturation?
  - proton = collinear factorization (large $x_1$)
  - nucleus = described as a CGC
  - parton—CGC cross-section to all orders in the gluon density
CGC factorization for ‘dilute–dense’

- The color charges in the target ($\rho_a$) are ‘frozen’ during the collision (by Lorentz time dilation)
  - compute the scattering between the parton and a fixed configuration of color charges
  - average over all the configurations by integrating over $\rho_a$ with the CGC weight function

$$\left\langle \frac{dN}{dY \, d^2 p_\perp} \right\rangle_Y = \int [D\rho] \ W_Y[\rho] \ \frac{dN}{dY \, d^2 p_\perp} [\rho]$$

- The target color field $A_{\mu}^a$ (as generated by $\rho_a$) is strong and must be resummed to all orders

Eikonal approximation

- A very energetic particle is **not deflected** by its interactions

$$\Psi_i(x_\perp) \rightarrow U_{ij}(x_\perp) \Psi_j(x_\perp), \quad U(x_\perp) = T \exp \left\{ i \int dx^+ A_a^+(x^-, x_\perp) t^a \right\}$$

- The sum of all the interactions simply **exponentiates**
- The single–particle state gets multiplied by a complex exponential known as Wilson line

Jets

- Two back–to–back jets in the transverse plane: visible via 2–particle azimuthal correlations

Nuclear modification factor in d+Au at RHIC

$$R_{d+Au} \equiv \frac{1}{2A} \frac{dN_{d+Au}/d^2 p_\perp d\eta}{dN_{pp}/d^2 p_\perp d\eta}$$

- $R_{d+Au}$ would be one in the absence of nuclear effects

- $R_{d+Au}$ decreases with increasing rapidity
- Strong suppression ($R \sim 0.5$) for $\eta = 3$: coherent scattering
The **CGC factorization**

- **Gluon production in the scattering between 2 CGC’s**:
  \[
  \frac{dN}{dY d^2p_{\perp}} = \int [D_1 \rho_1 D_2 \rho_2] W_{Y_{\text{beam}}-y} \rho_1 W_{Y_{\text{beam}}+y} \rho_2 \frac{dN}{dY d^2p_{\perp}} \text{class}
  \]

- The classical solution is **non-linear** to all orders in \( \rho_1 \) and \( \rho_2 \):
  \[
  D_\mu F^{\mu\nu}(x) = \delta^{\mu+} \rho_1(x) + \delta^{\mu-} \rho_2(x)
  \]

- All the leading logs of \( 1/x_{1,2} \) are absorbed in the \( W’\)s.

- **d+Au**: the ‘away jet’ gets smeared out = saturation in Au

**Multiplicity in HIC at the LHC**

- Non–linear effects in the wavefunctions: gluon saturation
  - 2 CGC weight functions: \( W_{Y_1} [\rho_1], W_{Y_2} [\rho_2] \)
  - generalized pdf’s: multi–parton correlations
  - ... and in the scattering: multiple interactions
  - classical Yang–Mills equations with 2 sources

- Excellent fit by the CGC approach
  - All the models include some form of saturation
  - HIJING : energy dependent low–p\(_T\) cutoff
The geometry of a HIC

Non-central collision

“peripheral” collision ($b \sim b_{\text{max}}$)

“central” collision ($b \sim 0$)

Re enactment plane

Number of participants ($N_{\text{part}}$): number of incoming nucleons (participants) in the overlap region

Color flux tubes

- Correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- Correlation length in rapidity ($y$ or $\eta$): $\Delta \eta \sim 1/\alpha_s$

The color fluxes eventually break into 'particles' (gluons)

Gluons emitted from different flux tubes are not correlated

Glasma

- Immediately after the collision, the chromo-electric and chromo-magnetic fields are purely longitudinal
- They form flux tubes extending between the projectiles

Glasma: the intermediate stage between the CGC and the Quark Gluon Plasma (McLerran and Lappi, 06)

The ridge in HIC at RHIC

- A natural explanation for the 'ridge'

Long-range correlations in rapidity $\Delta \eta$

Narrow correlation in azimuthal angle $\Delta \phi$

STAR Preliminary
Di–hadron correlations

- In a given even count the number of particles $N_1$ in a given bin centered at $(\eta_1, \phi_1)$ and similarly $N_2$.

$$ R \equiv \frac{\langle N_1 N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} $$

$$ \Delta \eta = \eta_1 - \eta_2, \quad \Delta \phi = \phi_1 - \phi_2 $$

- Recall: pseudo–rapidity

$$ \eta = \frac{1}{2} \ln \frac{p + p_z}{p - p_z} $$

$$ \eta = -\ln \tan \frac{\theta}{2}, \quad \theta = \frac{p_z}{p} $$

$$ \tau = \sqrt{t^2 - z^2} $$

**Long–range rapidity correlations probe early times**

- Generated at early stages, where particles with different longitudinal velocities were still causally connected

$$ \tau_{\text{correlation}} \leq \tau_{\text{freeze–out}} e^{-|\eta_A - \eta_B|/2} $$

**Di–hadron correlations: p+p vs Au+Au**

- p+p: peak around $\Delta \eta = 0$ & flat in $\Delta \phi$

  ▶ correlated particles make similar angles with the beam axis

- Au+Au: almost flat over $\Delta \eta \simeq 10$ & 2 peaks at $\Delta \phi = 0$ and $\Delta \phi = \pi$

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**The ridge in p+p at CMS (1)**

- A small ridge has been seen in p+p collisions at the LHC

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**CERN Summer School 2011**

**QCD in Heavy Ion Collisions**

Chile Gradistei, Romania 65 / 70

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**QCD in Heavy Ion Collisions**

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The ridge in p+p at CMS (2)

- ... but only in specially selected events!

(d) CMS $N \geq 110$, $1.0 \text{GeV/c} < p_T < 3.0 \text{GeV/c}$

- High-multiplicity (⇒ very central): $N \geq 110$ particles
- Narrow interval in transverse momentum: $1 \leq p_T \leq 3 \text{ GeV}$

... which look a lot like a heavy ion collision!!

▷ N.B. $1 \leq p_T \leq 3 \text{ GeV}$ is similar to the proton $Q_s$ at LHC