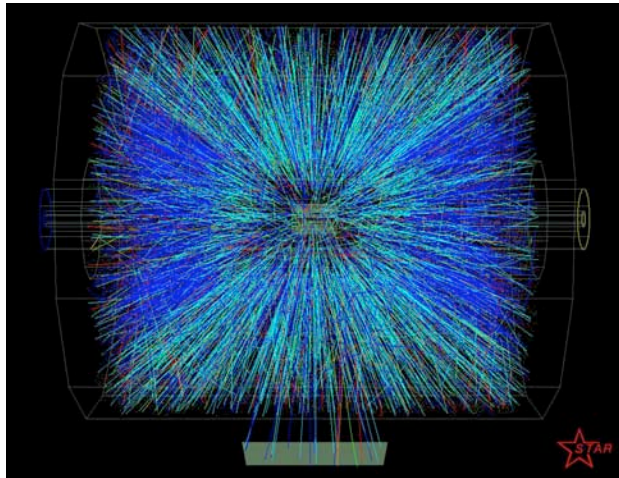


Edmond Iancu

Institut de Physique Théorique de Saclay



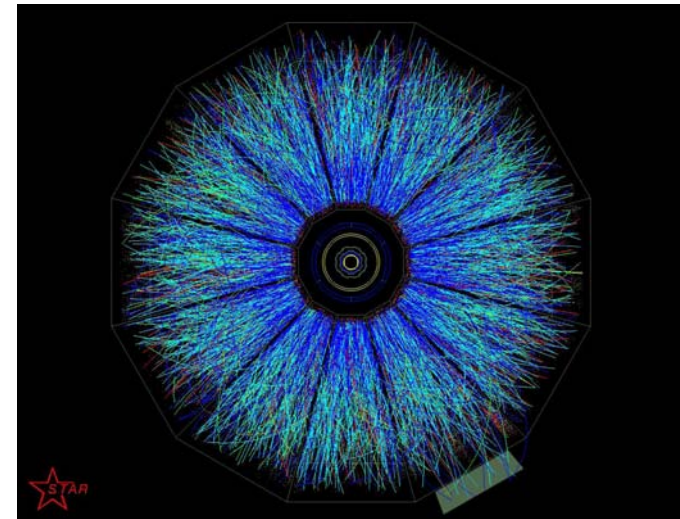
Au+Au collisions at RHIC



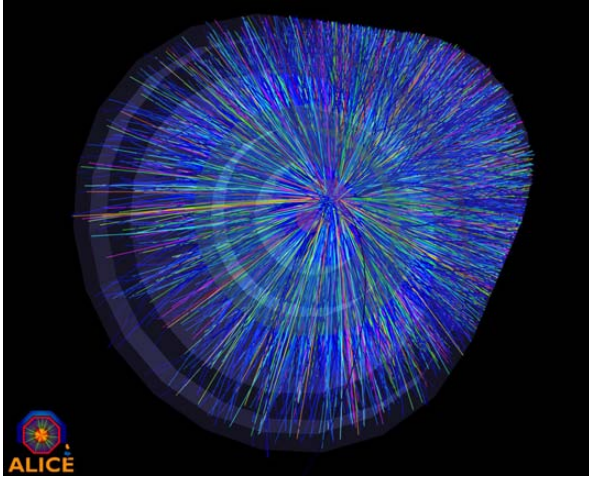
- Au+Au collision at STAR: longitudinal projection
- ~ 3000 produced particles streaming into the detector



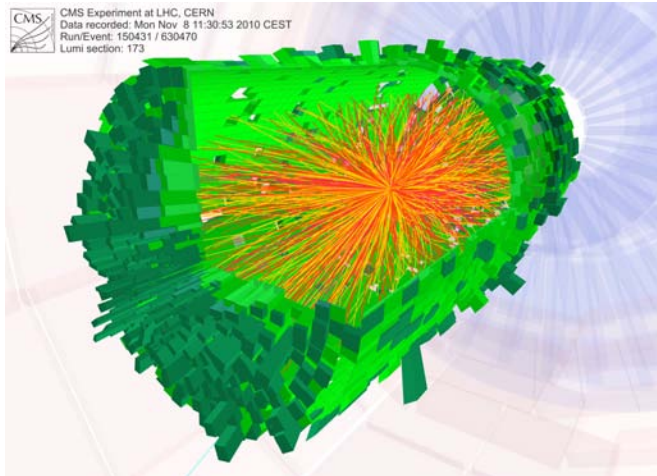
Au+Au collisions at RHIC



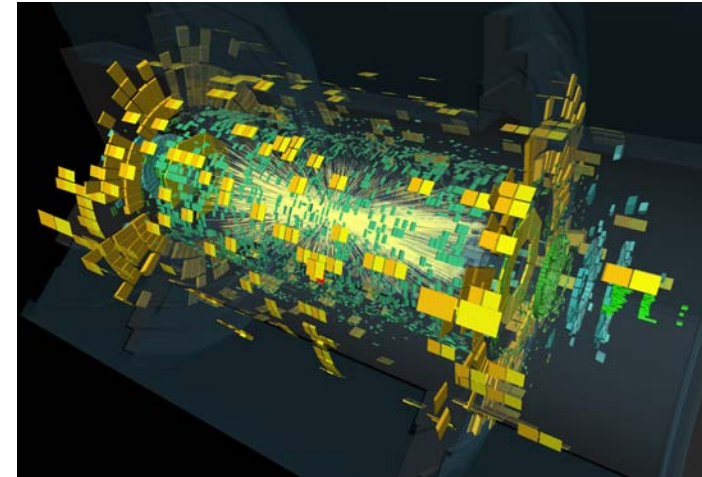
- Au+Au collision at STAR: transverse projection



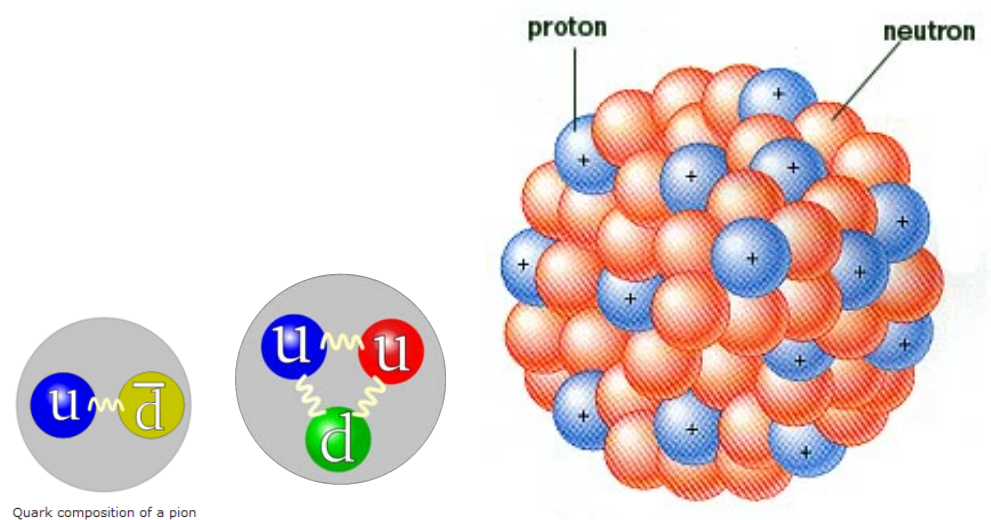
- Pb+Pb collision at ALICE: ~ 1600 hadrons per unit rapidity
- How to describe/understand such a complex system ?



- The concept of **particle** is not so useful anymore ...
- One should rather speak about **QCD matter**

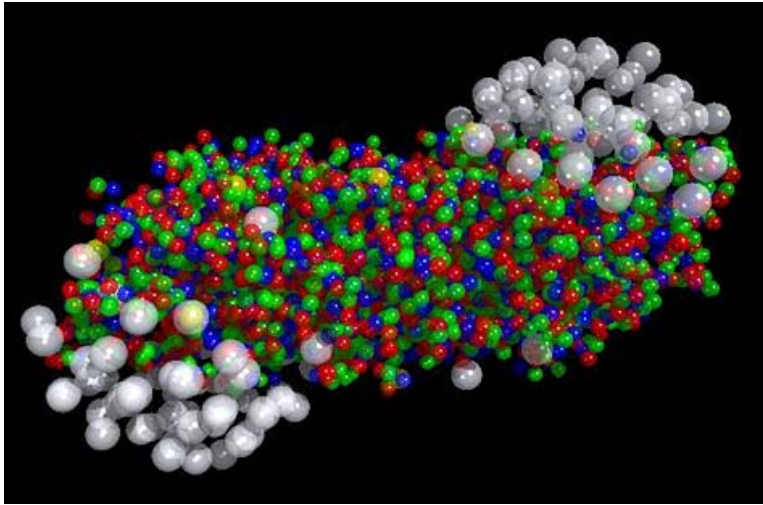


- Traditional **perturbative** methods become inappropriate (collective phenomena, multiple scattering ...)



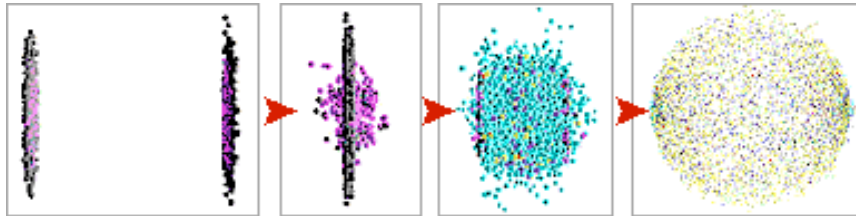
Quark composition of a pion

- At low energies, QCD matter exists only in the form of **hadrons** (mesons, baryons, nuclei)

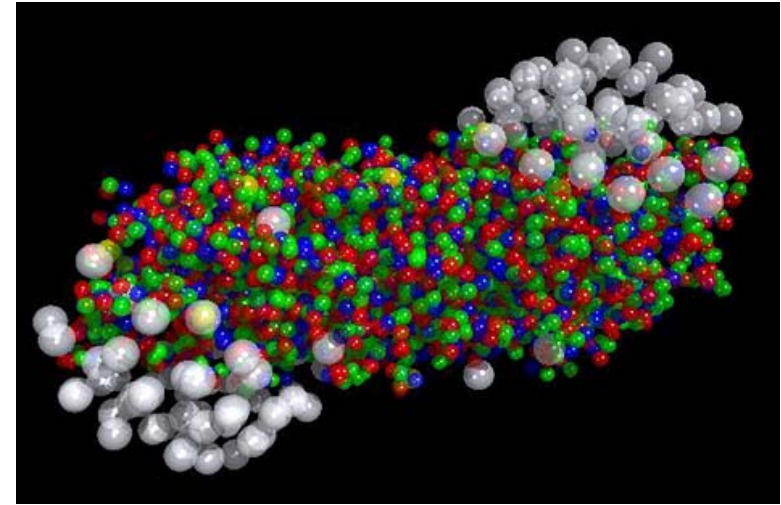


- At sufficiently high energies, the relevant degrees of freedom are **partonic** (quarks & gluons)
- True for both p+p collisions and A+A collisions ...

New forms of QCD matter produced in HIC



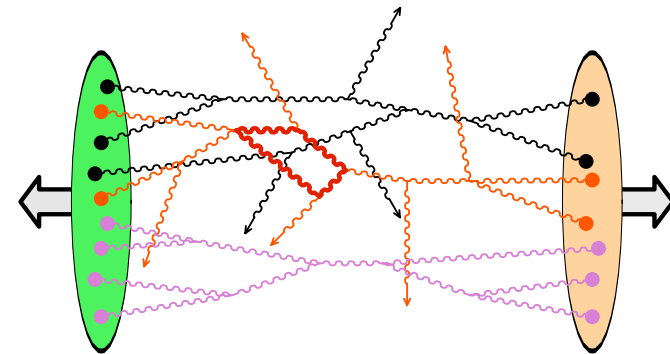
- Prior to the collision: 2 Lorentz-contracted nuclei ('pancakes')
 - 'Color Glass Condensate' (CGC)
- Right after the collision: non-equilibrium partonic matter
 - 'Glasma' (from 'Glass' + 'Plasma')
- At later stages ($\Delta t \gtrsim 1 \text{ fm}/c$) : local thermal equilibrium
 - 'Quark-Gluon Plasma' (QGP)
- Final stage ($\Delta t \gtrsim 6 \text{ fm}/c$) : hadrons
 - 'final event', or 'particle production'



- At sufficiently high energies, the relevant degrees of freedom are **partonic** (quarks & gluons)
- ... but HIC give us access to **new forms of partonic matter**

How to study these new forms of matter ?

- Standard **perturbation theory in QCD** (= expansion in powers of the coupling 'constant' α_s) fails **even at weak coupling**, because of the **high parton density**.



- High-density effects (**multiple scattering, parton saturation, Debye screening etc**) must be **resummed to all orders in α_s** .
- This results into **effective theories**.

The possibility of a strong coupling

- Besides, there is no guarantee that the coupling is weak !

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the [Relativistic Heavy Ion Collider](#) (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In [peer-reviewed papers](#) summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

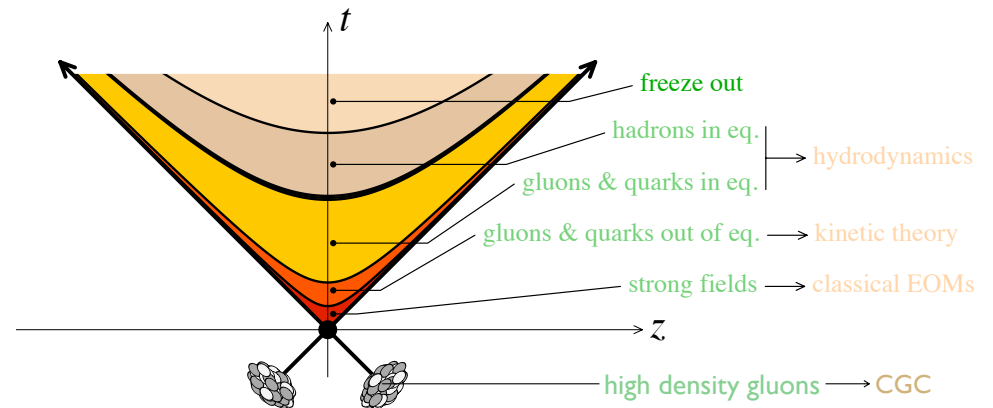
- 'Perfect fluid' = $\alpha_s \rightarrow \infty$
- Interesting connection with string theory ('AdS/CFT correspondence').

Lecture 0: A QCD Primer

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \sum_f \bar{\psi}_f (i\gamma^\mu D_\mu - m) \psi_f$$

Effective theories for Heavy Ion Collisions

- A space-time picture of a heavy ion collision



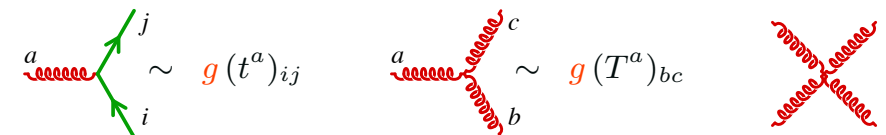
- Different effective theories apply at different stages.
- But they refer all to QCD !

QCD: Quarks & Gluons

- Electromagnetic interactions: **Quantum Electrodynamics (QED)**
 - matter : electron; interaction carrier : photon
 - interaction vertex :



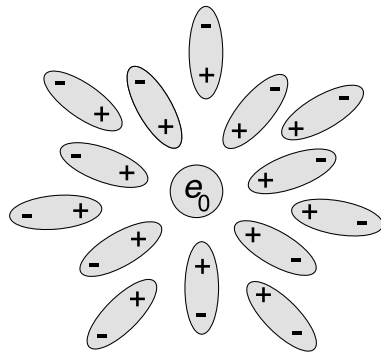
- Strong interactions: **Quantum Chromodynamics (QCD)**
 - matter : quarks; interaction carriers : gluons
 - interaction vertices :



- i, j : color indices of the quarks ($N_c = 3$ possible values)
- a, b, c : color indices of the gluons ($N_c^2 - 1 = 8$ possible values)

Running coupling: QED

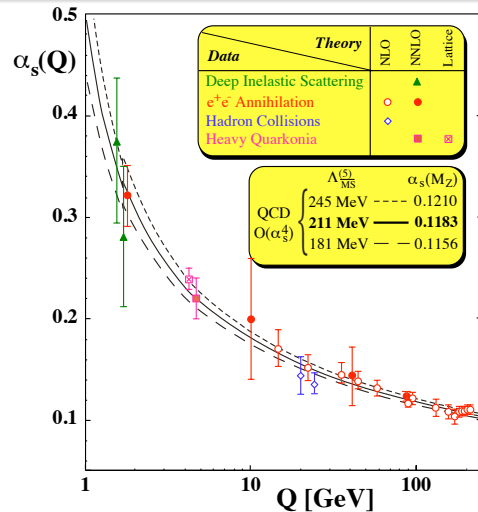
- An electric charge polarizes the surrounding medium:



$$V(R) = \frac{e_{\text{eff}}(R)}{R}$$

- The **effective charge** depends upon the distance R from the **bare** one.
- Normally this leads to **screening**: $e_{\text{eff}}(R)$ decreases with R .

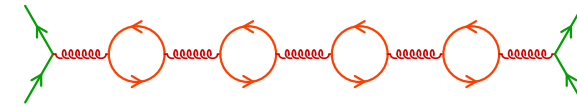
Asymptotic freedom



- The coupling is weak at **short distances**, or **large transferred momenta**:
 $Q \sim 1/R \gg \Lambda_{\text{QCD}} \simeq 200 \text{ MeV}$

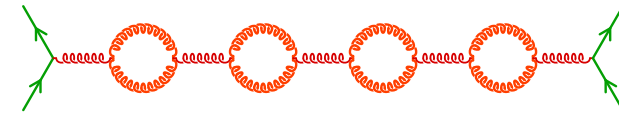
Running coupling: from QED to QCD

- The **vacuum** itself is a polarisable 'medium' !



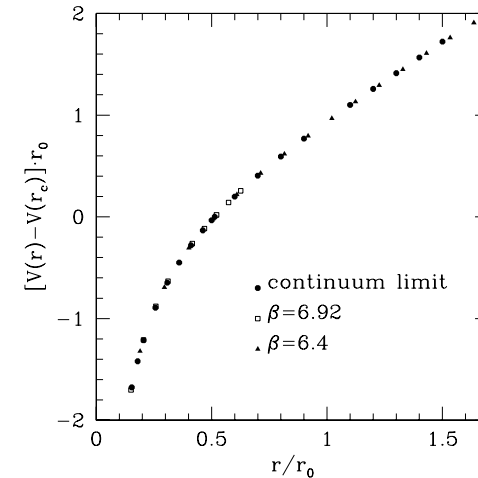
$$QED: \quad \alpha_{\text{eff}}(R) = \frac{\alpha}{1 - \frac{2\alpha}{3\pi} \ln(1/mR)}, \quad \alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- In QCD, the (longitudinal) gluons yield **antiscreening** !



$$QCD: \quad \alpha_s(R) \equiv \frac{g^2(R)}{4\pi} = \frac{2\pi N_c}{(11N_c - 2N_f) \ln(1/\Lambda_{\text{QCD}}R)}$$

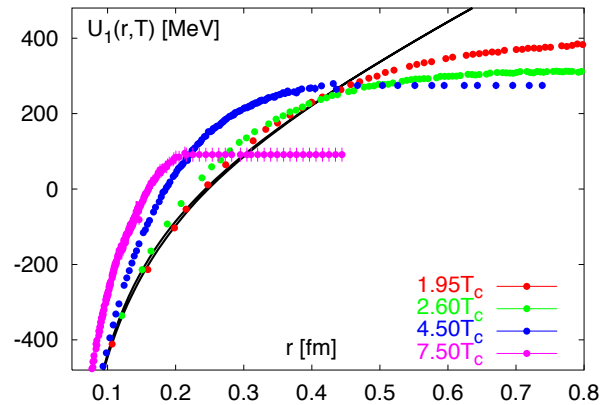
Confinement



- The quark-antiquark potential increases linearly with the distance.
- Quarks (and gluons) are confined into **colorless hadrons**

Quark–antiquark potential at finite T

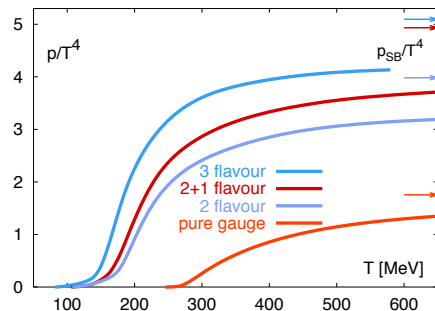
- With increasing the temperature T , the potential flattens at shorter and shorter distances.



- This eventually leads to a **phase transition** at some **critical temperature T_c**

Quark–Gluon Plasma

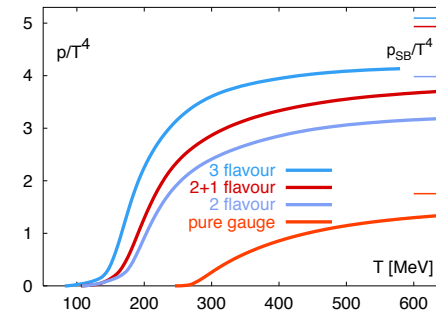
- Lattice calculations of the pressure in QCD at finite T



- Rapid increase of the pressure
 - at $T < T_c$: 3 light mesons (π^0, π^\pm)
 - at $T < T_c$: 52 d.o.f. (gluons: $8 \times 2 = 16$; quarks: $3 \times 3 \times 2 \times 2 = 36$)
- Interpreted as a rise in the number of active degrees of freedom due to the **liberation of quarks and gluons**

Quark–Gluon Plasma

- Lattice calculations of the pressure in QCD at finite T



- Rapid increase of the pressure
 - at $T \simeq 270$ MeV with gluons only
 - at $T \simeq 150$ to 180 MeV with light quarks
- Interpreted as a rise in the number of active degrees of freedom due to the **liberation of quarks and gluons**

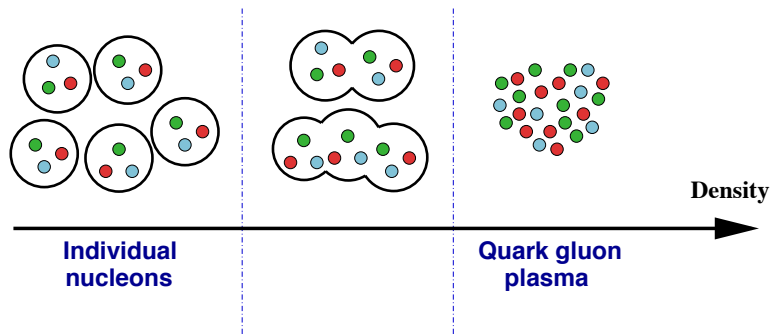
Debye screening

- **Quark–Gluon Plasma (QGP)** : a system of quarks and gluons which got free of confinement !
- How is that possible ???

$$V(r) = \frac{\exp(-m_{\text{Debye}} r)}{r}$$

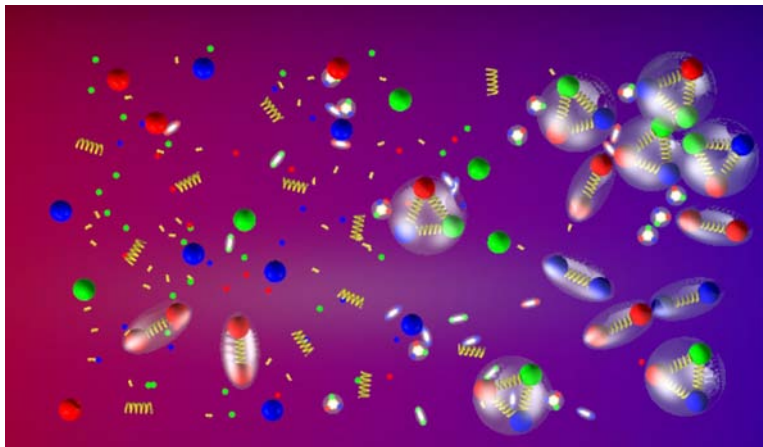
- In a dense medium, color charges are **screened by their neighbors**
- The interaction potential decreases exponentially beyond the **Debye radius $r_{\text{Debye}} = 1/m_{\text{Debye}}$**
- Hadrons whose sizes are larger than r_{Debye} cannot bind anymore

Deconfinement phase transition



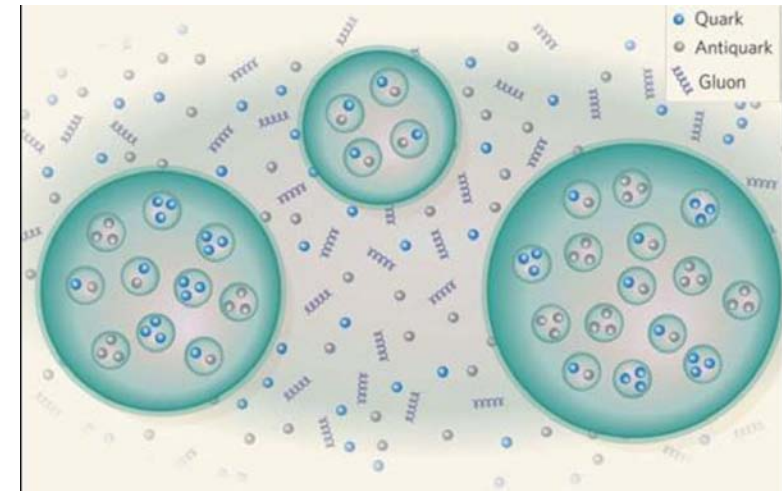
- When the nucleon density increases, **they merge**, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- This phenomenon extends to **the whole volume** when the phase transition ends
- Note: **if** the transition was **first-order**, it would go through a mixed phase containing a mixture of nucleons and plasma

The actual scenario is a 'cross-over'



▷ This was firmly established by the Wuppertal–Budapest lattice group (*Aoki et al., Nature, 443 (2006) 675*)

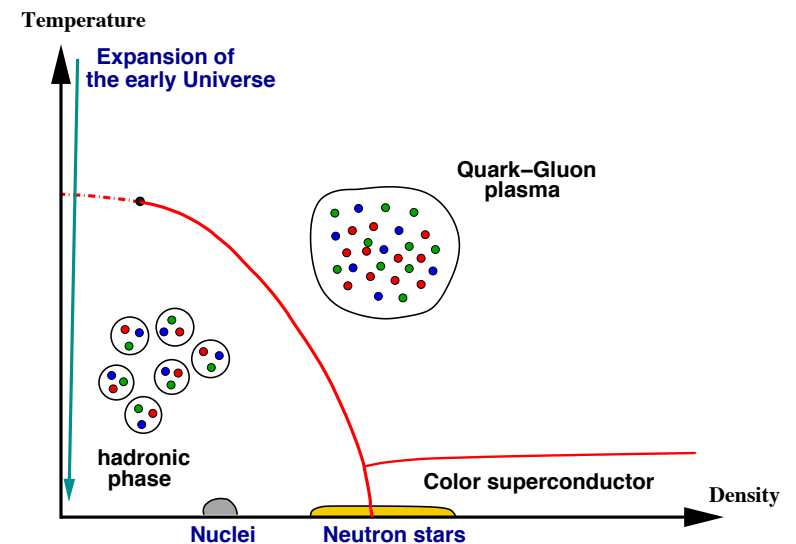
Possible first-order scenario with critical bubbles



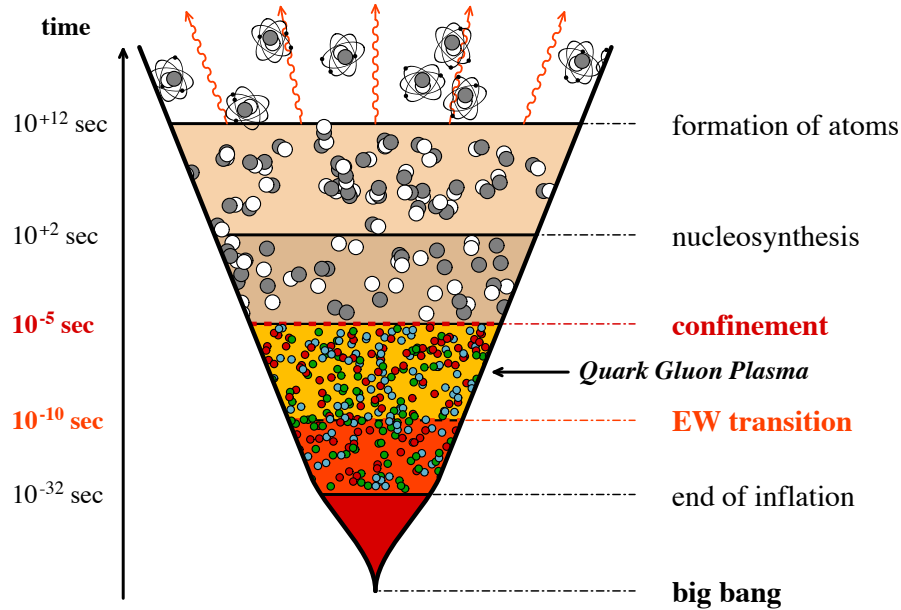
... but this is not what really happens !

Phase-diagram for QCD

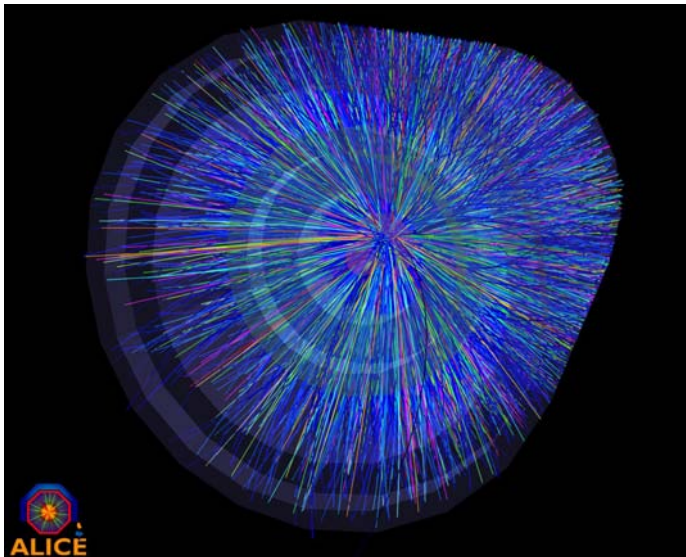
- ... as explored by the expansion of the Early Universe ...



The Big Bang



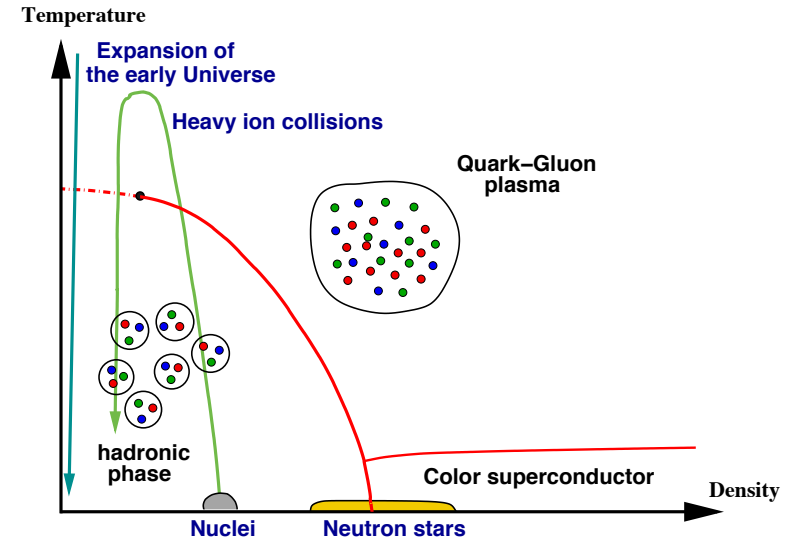
The Little Bang



- The subject of these lectures

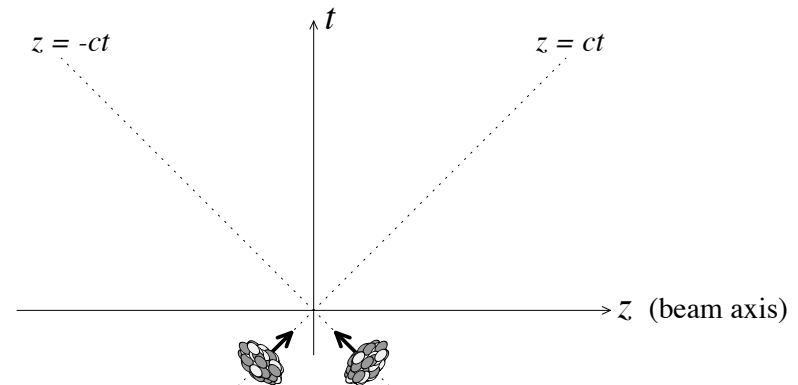
Phase-diagram for QCD

- ... as explored by the expansion of the Early Universe ...



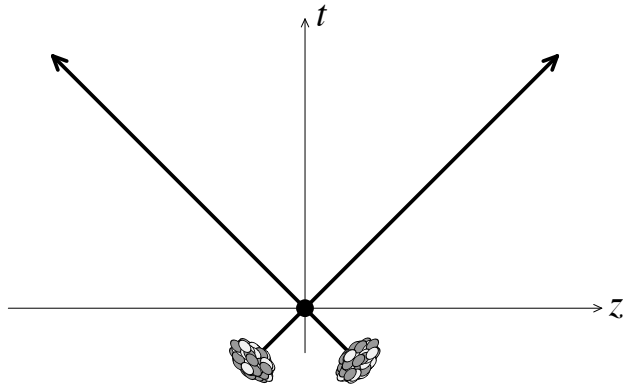
- ... and in the ultrarelativistic heavy ion collisions.

Lecture I: Initial conditions



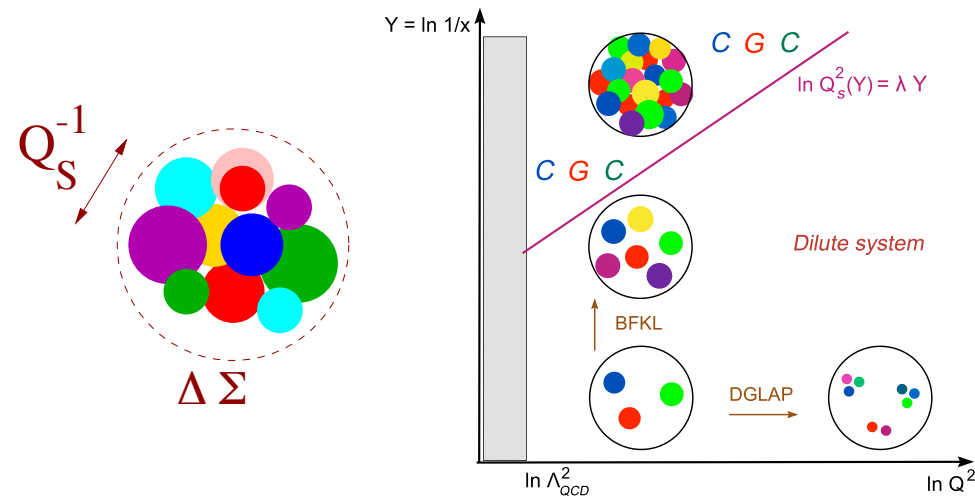
- $\tau < 0$: hadronic wavefunctions prior to the collision
 - high-energy evolution & the Color Glass Condensate
 - it applies to any highly energetic hadron (proton or nucleus)

Lecture I: Initial conditions

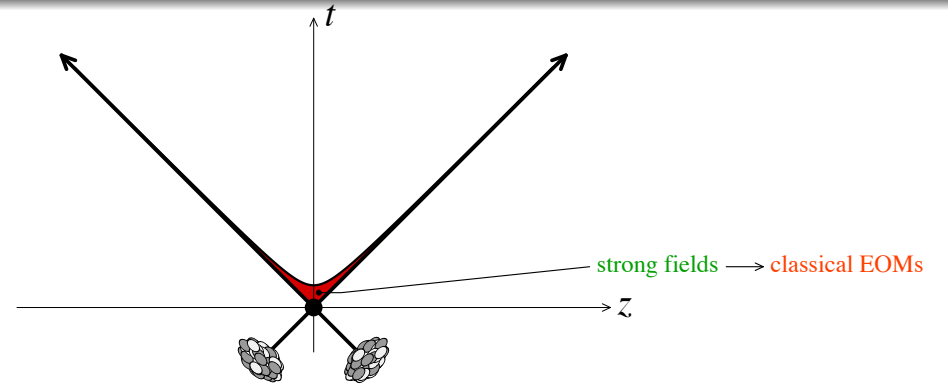


- $\tau < 0$: hadronic wavefunctions prior to the collision
- $\tau \sim 0 \text{ fm}/c$: the hard scattering
 - production of hard particles: jets, direct photons, heavy quarks
 - calculable within (standard) perturbative QCD ('leading twist')

Color Glass Condensate



Lecture I: Initial conditions

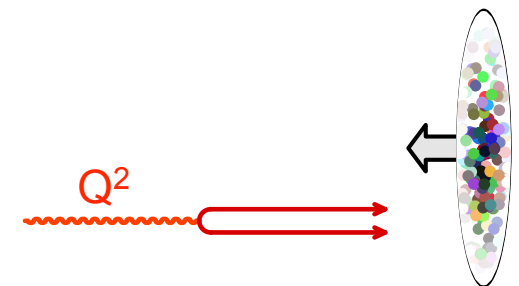


- $\tau < 0$: hadronic wavefunctions prior to the collision
- $\tau \sim 0 \text{ fm}/c$: the hard scattering
- $\tau \sim 0.2 \text{ fm}/c$: strong color fields (or 'glasma')
 - semi-hard quanta ($p_{\perp} \lesssim 2 \text{ GeV}$): gluons, light quarks
 - make up for most of the multiplicity
 - sensitive to the physics of saturation ('higher twist')

Parton picture

- When an energetic hadron is probed on a hard resolution scale (momentum transfer $Q^2 \gg \Lambda_{\text{QCD}}^2$), one sees a bunch of partons ...

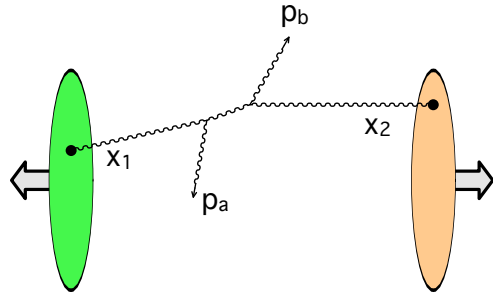
- with transverse area $\sim 1/Q^2$...
- and longitudinal momentum fraction $x = k_z/P$ fixed by the kinematics



- E.g. : in Deep Inelastic Scattering (DIS)

$$x = \frac{Q^2}{s} \quad s = \text{center-of-mass energy squared}$$

- N.B.: high energy \iff small x

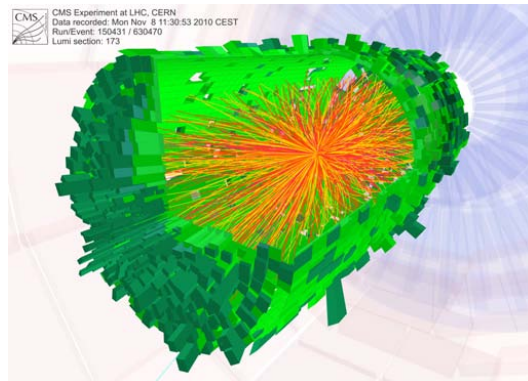
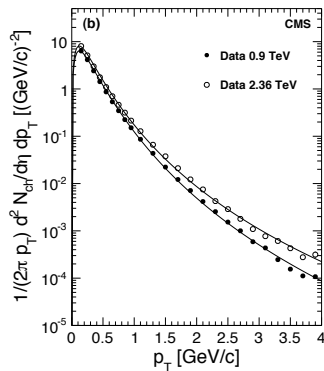


- The partons relevant for the process under consideration carry the longitudinal momentum fractions

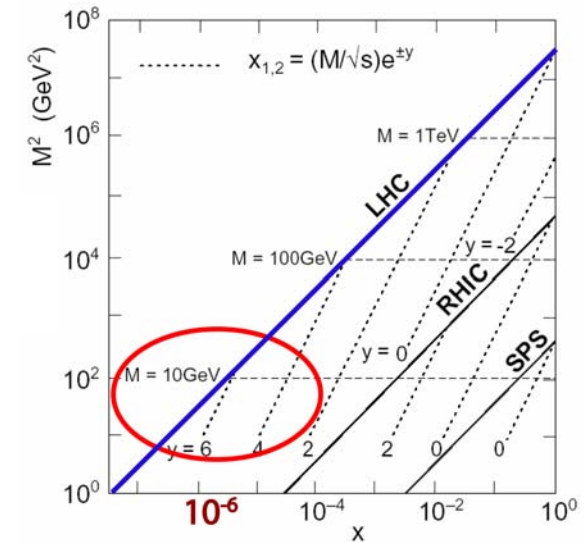
$$x_1 = \frac{p_{a\perp}}{\sqrt{s}} e^{Y_a} + \frac{p_{b\perp}}{\sqrt{s}} e^{Y_b}, \quad x_2 = \frac{p_{a\perp}}{\sqrt{s}} e^{-Y_a} + \frac{p_{b\perp}}{\sqrt{s}} e^{-Y_b}$$

- p_{\perp} : transverse momenta of the produced particles
- Y : their rapidities
- \sqrt{s} : collision energy

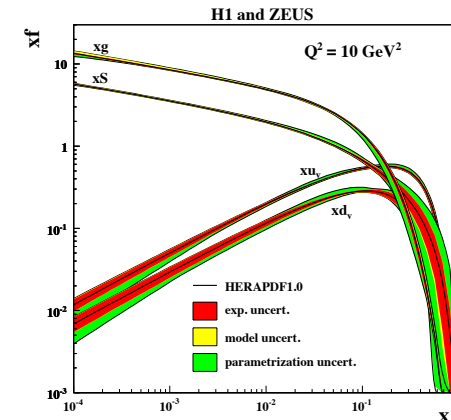
AA collisions at RHIC & LHC



- 99% of the total multiplicity lies below $p_{\perp} = 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4 \times 10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)
- ▷ partons at small x are the most important



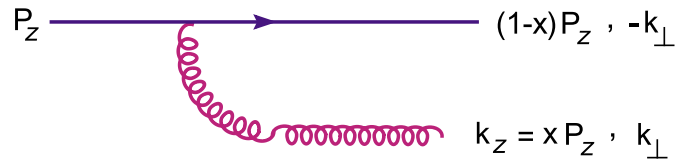
Parton distributions at HERA



The gluon distribution rises very fast with increasing energy

- Gluon distribution $xg(x, Q^2)$: # of gluons with transverse size $\Delta x_{\perp} \sim 1/Q$ and longitudinal momentum $k_z = xP$

Bremsstrahlung



$$d\mathcal{P}_{\text{Brem}} \sim \alpha_s(k_{\perp}^2) C_R \frac{d^2 k_{\perp}}{k_{\perp}^2} \frac{dx}{x}$$

- Phase-space enhancement for the emission of
 - collinear ($k_{\perp} \rightarrow 0$)
 - and/or soft (low-energy) ($x \rightarrow 0$) gluons

- The parent parton can be either a quark or a gluon

$$C_F = t^a t^a = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}, \quad C_A = T^a T^a = N_c = 3$$

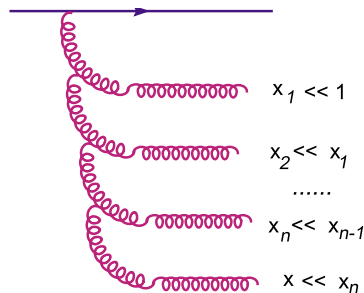
- The daughter gluon can in turn radiate an even softer gluon !

Gluon cascades

- n gluons strictly ordered in x
- The n -gluon cascade contributes

$$\frac{1}{n!} (\alpha_s Y)^n$$

- The sum of all the cascades exponentiates :

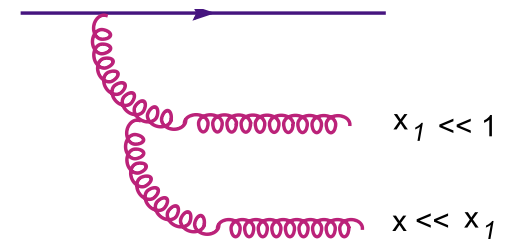


$$xg(x, Q^2) \propto e^{\omega \alpha_s Y} \quad \text{BFKL evolution}$$

(Balitsky, Fadin, Kuraev, Lipatov, 75-78)

- This evolution is linear :
the emitted gluons do not interact with each other

2 gluons



- The 'cost' of the addition gluon:

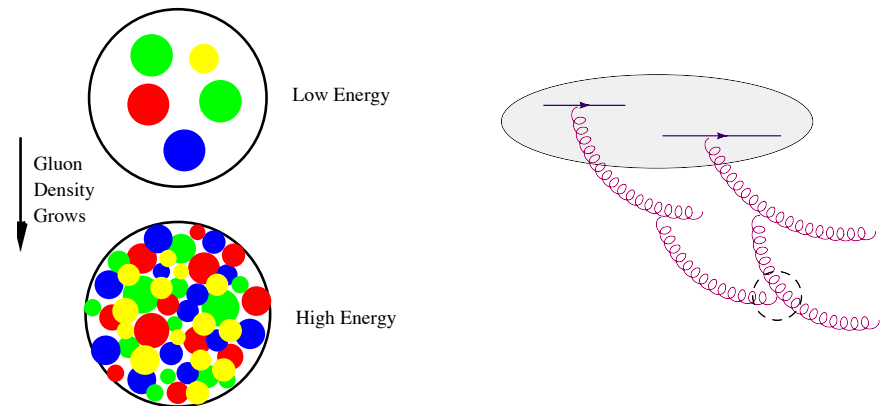
$$\alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

Formally, a process of higher order in α_s , but which is enhanced by the available rapidity interval

- $Y \equiv \ln(1/x)$: rapidity difference between the parent quark and the last emitted gluon
- When $\alpha_s Y \gtrsim 1 \implies$ need for resummation !

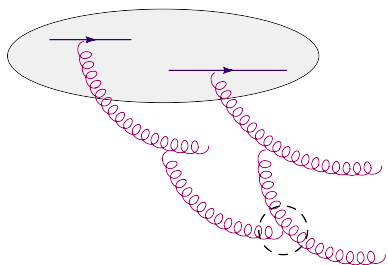
Gluon recombination

- The gluon density rises with decreasing x (increasing energy)



- Eventually gluons start overlapping with each other and then they interact: $2 \rightarrow 1$ gluon recombination
- These interactions stop the growth: saturation

Saturation momentum



- Number of gluons per unit area:

$$N \sim \frac{x g_A(x, Q^2)}{\pi R_A^2}$$

- Recombination cross-section

$$\sigma \sim \frac{\alpha_s}{Q^2}$$

- Recombination happens if $N\sigma \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with

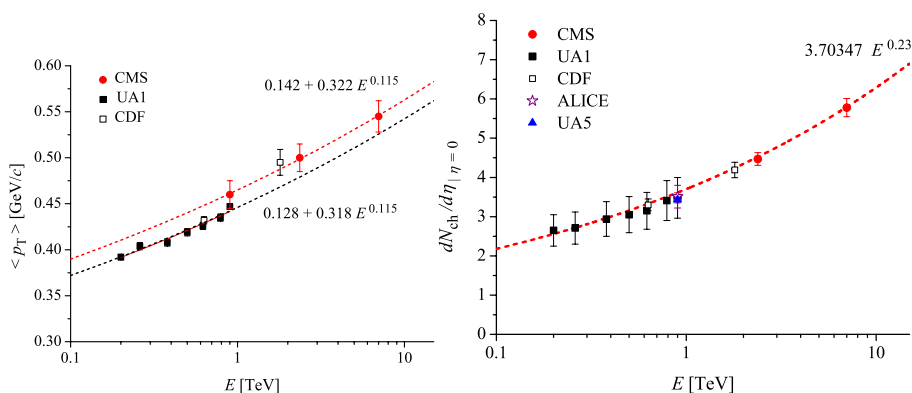
$$Q_s^2(x, A) \simeq \alpha_s \frac{x g_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.25}}$$

- Low $Q^2 \Rightarrow$ large area $\sim 1/Q^2 \Rightarrow$ strong overlapping

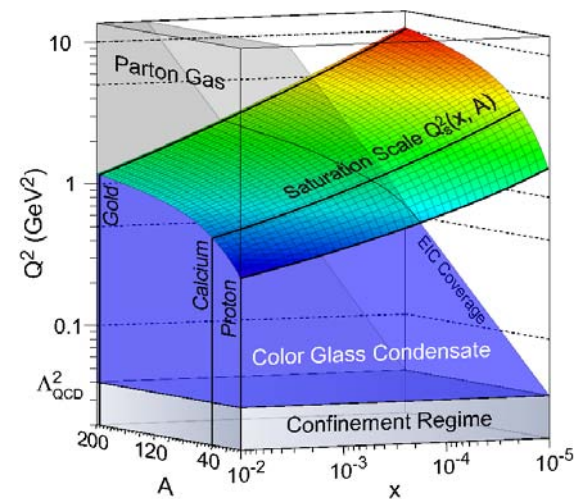
Multiplicities at the LHC: p+p

- In a high-energy scattering, the saturated gluons are released in the final state

- typical transverse momentum $\langle p_T \rangle \sim Q_s(E)$
- average multiplicity $dN/d\eta \sim Q_s^2(E)$

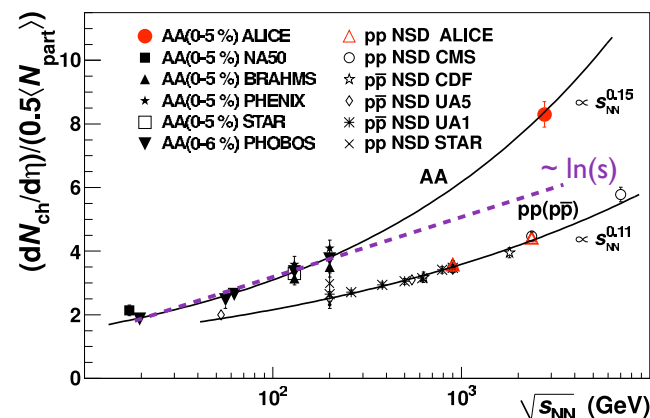


Saturation scale as a function of x and A



- $x \sim 10^{-5}$: $Q_s \sim 1$ GeV for proton and ~ 3 GeV for Pb or Au

Multiplicities in HIC: RHIC & LHC



- Logarithmic growth ($\ln s$) excluded by the LHC data
- Larger energy exponent (E^λ) for A+A than for p+p
- ▷ this difference is theoretically understood

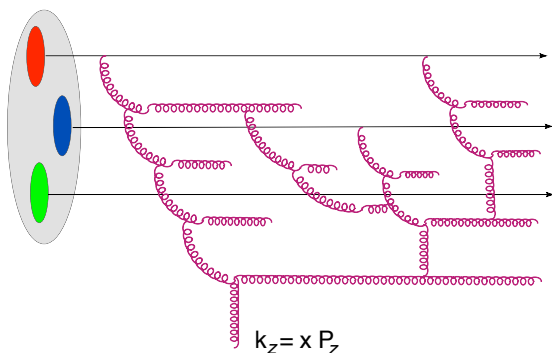
- A very robust, qualitative, prediction of saturation: DIS at HERA, Au+Au at RHIC, p+p at the LHC ... (looking forward to the relevant Pb+Pb data at the LHC)
- The single-inclusive spectra for particle production depend...
 - ... upon the particle transverse momentum p_T
 - ... and the COM energy of the collision \sqrt{s}
- ... only via the ratio of p_T to the saturation momentum Q_s :

$$\frac{dN}{d\eta d^2p_T} \simeq F(\tau) \quad \text{with} \quad \tau \equiv \frac{p_T^2}{Q_s^2(p_T/\sqrt{s})}$$

- At high energy, Q_s is the only intrinsic scale in the problem !

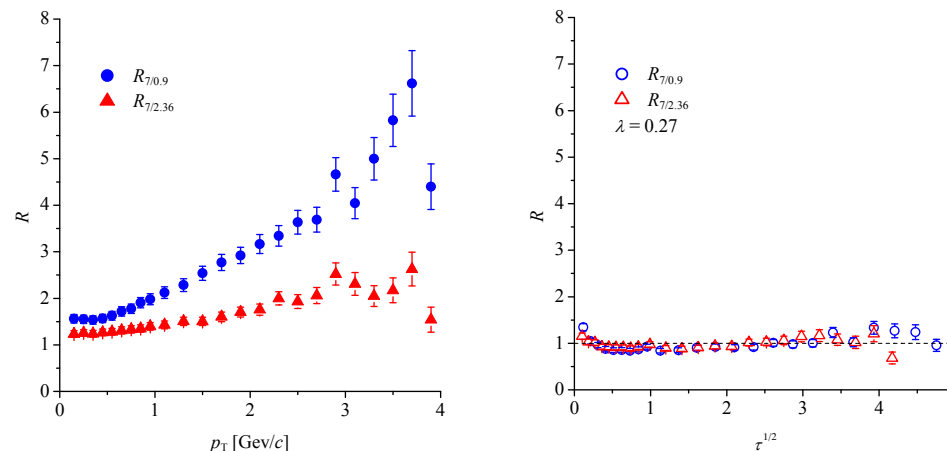
The need for an effective theory

- How to compute the saturation scale from first principle ?

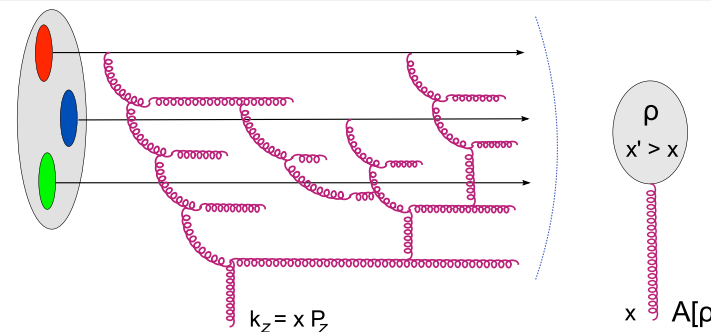


- Relatively hard scale ($Q_s \gg \Lambda_{\text{QCD}}$) \implies weak coupling !
- ... but high density \implies strong non-linear effects
- Solution: a reorganization of perturbation theory ! (McLerran and Venugopalan, 94; E.I., McLerran, and Leonidov, 00)

$$R_{s_1/s_2} = \frac{(dN/d\eta d^2p_T)|_{s_1}}{(dN/d\eta d^2p_T)|_{s_2}} \rightarrow 1 \quad \text{as a function of } \tau \dots \text{ if scaling}$$



Color Glass Condensate



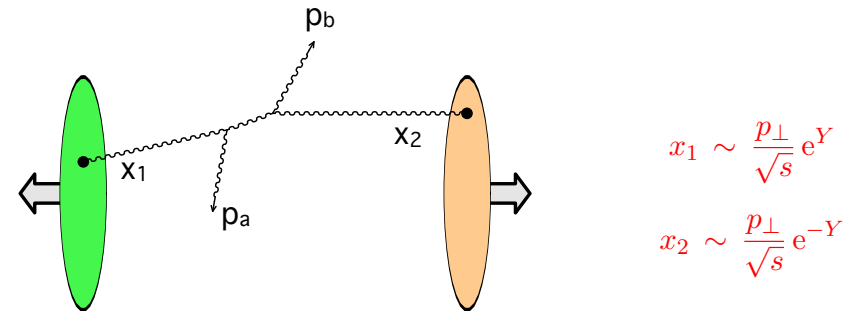
- **Small- x gluons:** classical color fields A_a^μ radiated by fast color charges ρ_a with $x' \gg x$, frozen in some random configuration
- $W_Y[\rho]$: probability distribution for the charge density at Y
- **Evolution equation** for $W_Y[\rho]$ with increasing $Y = \ln 1/x$

$$\frac{\partial}{\partial Y} W_Y[\rho] = H W_Y[\rho] \quad (\text{JIMWLK})$$

- A heavy ion collision at high energy

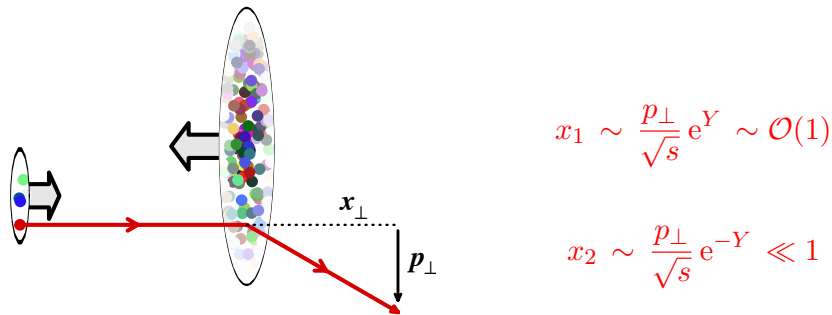


- Main difficulty: How to treat collisions involving a large number of partons ?



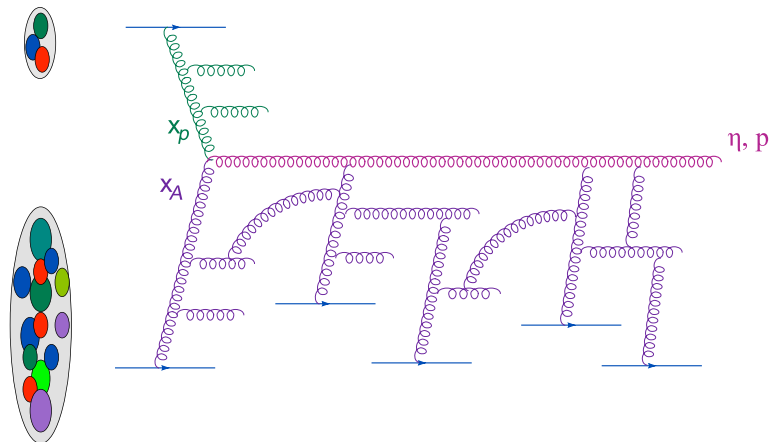
- Dilute-Dilute: one parton from each projectile interact
- Collinear factorization scheme of perturbative QCD
 - usual pdf's + DGLAP evolution
 - partonic cross-sections
- ▷ Caution: forward rapidity ($Y \gg 1$) & not too hard $p_{\perp} \Rightarrow x_2 \ll 1$

Proton-nucleus collisions (1)



- Most interesting situation: forward particle production ($Y \gtrsim 3$) at 'semi-hard' momenta ($p_{\perp} \sim 1 \div 5$ GeV)
 - very small $x_2 \ll 1$ in the nucleus
 - p_{\perp} comparable to $Q_s(A, x_2)$
- Dilute-Dense: new factorization scheme needed
 - ▷ similar to deep inelastic scattering at small x

Proton-nucleus collisions (2)



- How to include both multiple scattering and saturation ?
 - proton = collinear factorization (large x_1)
 - nucleus = described as a CGC
 - parton-CGC cross-section to all orders in the gluon density

- The color charges in the target (ρ_a) are 'frozen' during the collision (by Lorentz time dilation)
 - compute the scattering between the parton and a fixed configuration of color charges
 - average over all the configurations by integrating over ρ_a with the CGC weight function

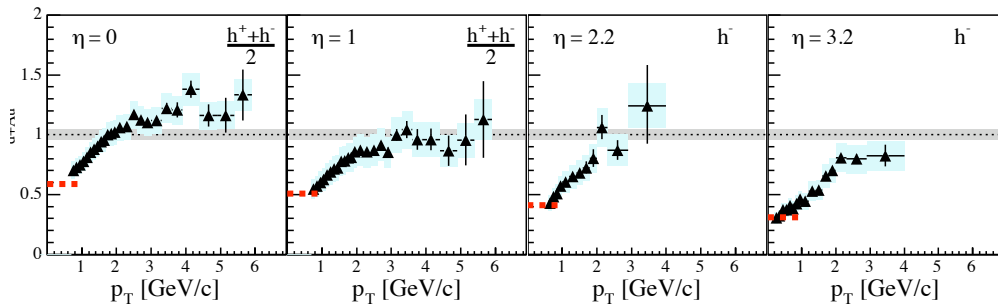
$$\left\langle \frac{dN}{dY d^2p_\perp} \right\rangle_Y = \int [D\rho] W_Y[\rho] \frac{dN}{dY d^2p_\perp}[\rho]$$

- The target color field A_a^μ (as generated by ρ_a) is **strong** and must be resummed **to all orders**

Nuclear modification factor in d+Au at RHIC

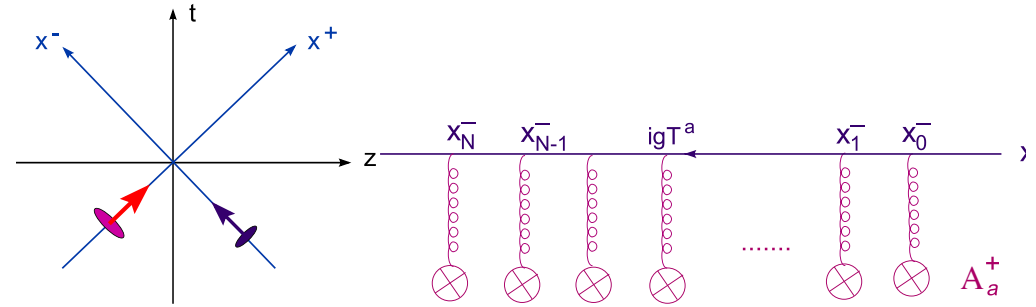
$$R_{d+Au} \equiv \frac{1}{2A} \frac{dN_{d+Au}/d^2p_\perp d\eta}{dN_{pp}/d^2p_\perp d\eta}$$

- R_{d+Au} would be one in the absence of nuclear effects



- R_{d+Au} decreases with increasing rapidity
- Strong suppression ($R \sim 0.5$) for $\eta = 3$: **coherent scattering**

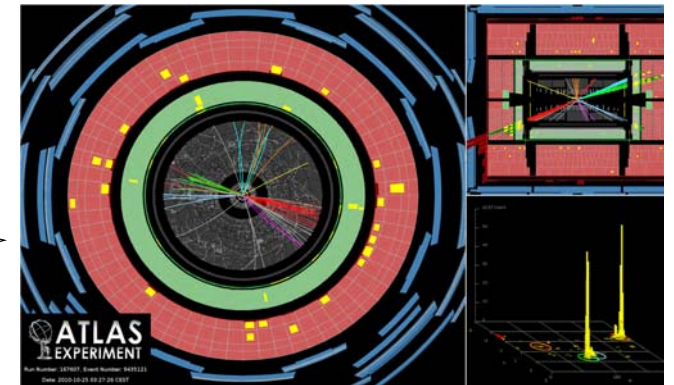
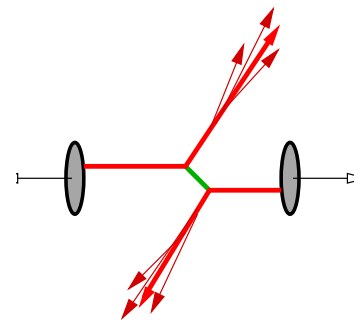
- A very energetic particle is **not deflected** by its interactions



- The sum of all the interactions simply **exponentiates**
- The single-particle state gets multiplied by a complex exponential known as **Wilson line**

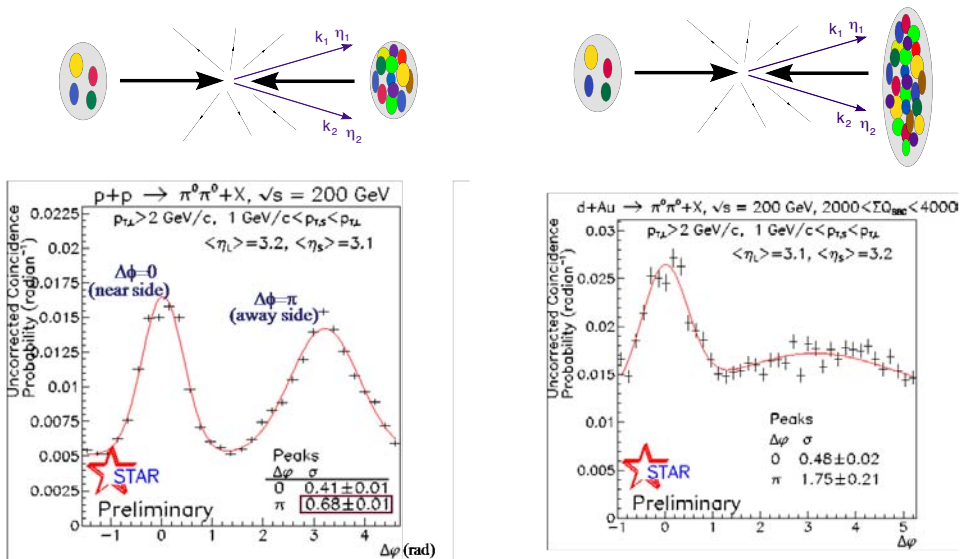
$$\Psi_i(x_\perp) \rightarrow U_{ij}(x_\perp) \Psi_j(x_\perp), \quad U(x_\perp) = T \exp \left\{ i \int dx^- A_a^+(x^-, x_\perp) t^a \right\}$$

Jets



- Two back-to-back jets in the transverse plane: visible via **2-particle azimuthal correlations**

Di-jet correlations at RHIC: p+p vs. d+Au

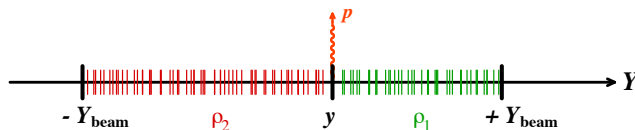


- d+Au : the 'away jet' gets smeared out = saturation in Au

The CGC factorization

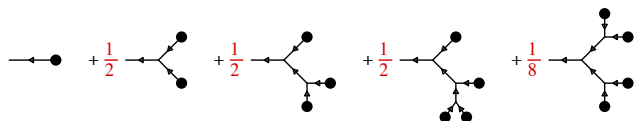
- Gluon production in the scattering between 2 CGC's :

$$\left\langle \frac{dN}{dY d^2p_{\perp}} \right\rangle = \int [D\rho_1 D\rho_2] W_{Y_{beam-y}[\rho_1]} W_{Y_{beam+y}[\rho_2]} \frac{dN}{dY d^2p_{\perp}} \Big|_{\text{class}}$$



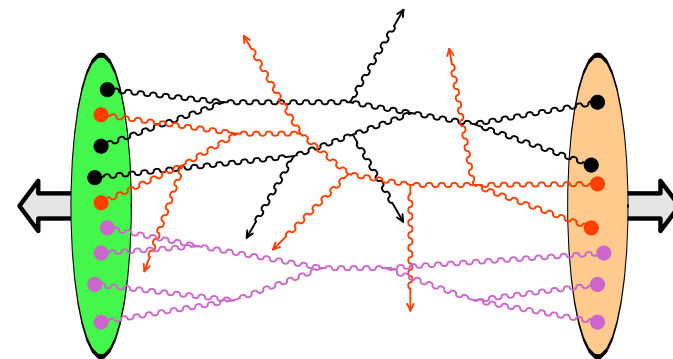
- The classical solution is non-linear to all orders in ρ_1 and ρ_2 :

$$D_{\nu} F^{\nu\mu}(x) = \delta^{\mu+} \rho_1(x) + \delta^{\mu-} \rho_2(x)$$



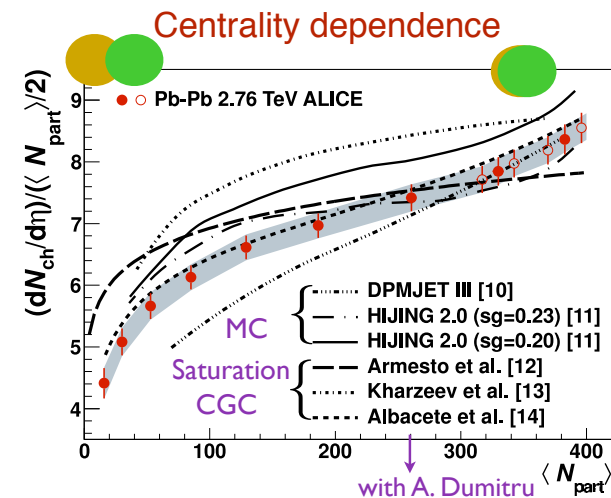
- All the leading logs of $1/x_{1,2}$ are absorbed in the W 's.

Nucleus-nucleus collisions



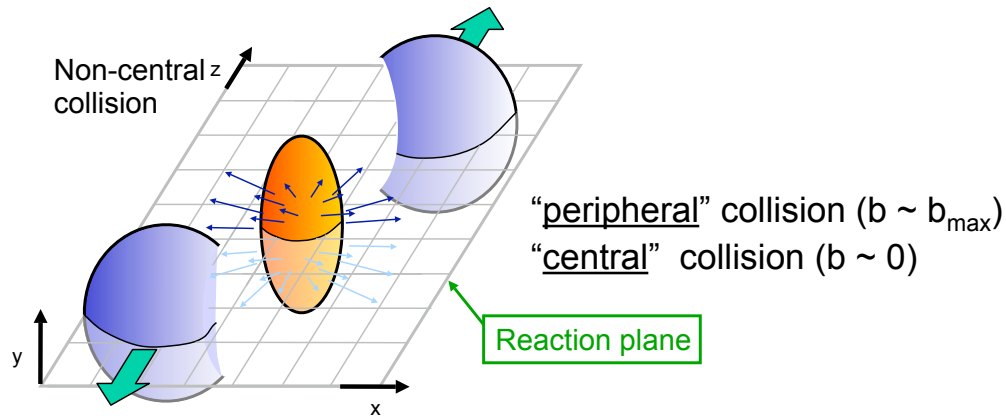
- Non-linear effects in the wavefunctions: gluon saturation
 - 2 CGC weight functions: $W_{Y_1}[\rho_1], W_{Y_2}[\rho_2]$
 - generalized pdf's : multi-parton correlations
- ... and in the scattering: multiple interactions
 - classical Yang-Mills equations with 2 sources

Multiplicity in HIC at the LHC



- Excellent fit by the CGC approach
- All the models include some form of saturation
 - ▷ HIJING : energy dependent low- p_T cutoff

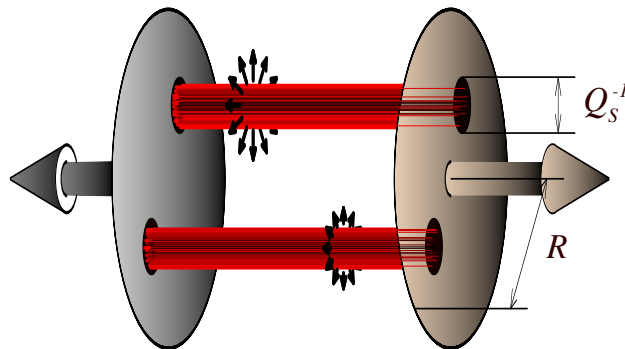
The geometry of a HIC



Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region

Color flux tubes

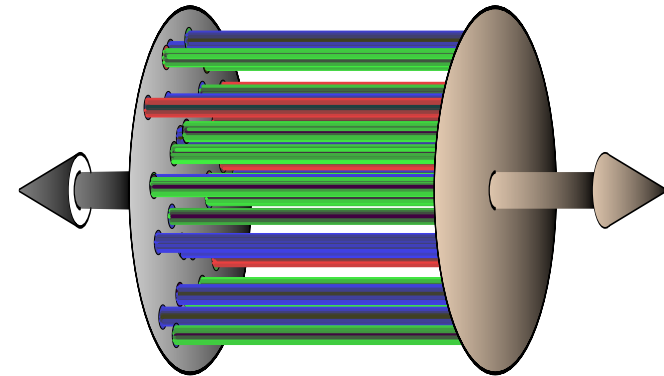
- Correlation length in the transverse plane: $\Delta r_{\perp} \sim 1/Q_s$
- Correlation length in rapidity (y or η): $\Delta \eta \sim 1/\alpha_s$



- The color fluxes eventually **break into 'particles'** (gluons)
- Gluons emitted from **different** flux tubes are **not** correlated

Glasma

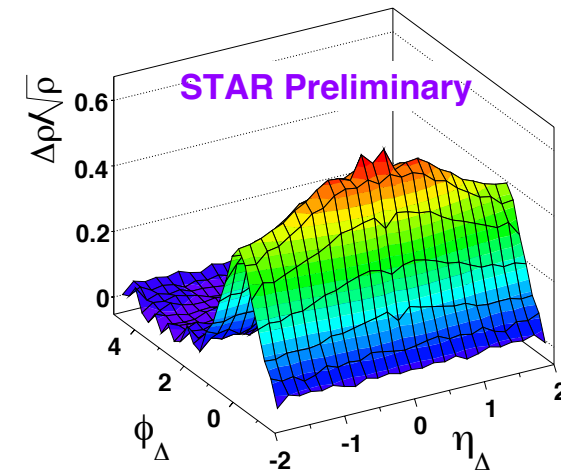
- Immediately after the collision, the **chromo-electric** and **chromo-magnetic** fields are **purely longitudinal**
- They form **flux tubes** extending between the projectiles



- **Glasma** : the intermediate stage between the **CGC** and the Quark Gluon Plasma (*McLerran and Lappi, 06*)

The ridge in HIC at RHIC

- A natural explanation for the **the 'ridge'**

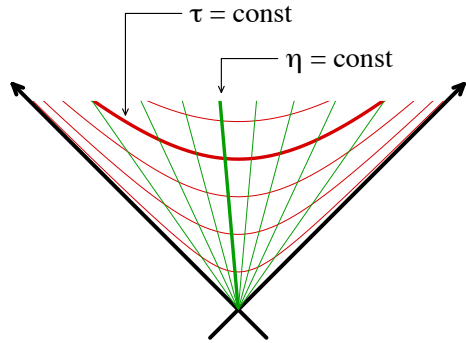


- Long-range correlations in rapidity $\Delta \eta$
- Narrow correlation in azimuthal angle $\Delta \phi$

Di-hadron correlations

- In a given event count the number of particles N_1 in a given bin centered at (η_1, ϕ_1) and similarly N_2 .

$$\mathcal{R} \equiv \frac{\langle N_1 N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \quad \Delta\eta = \eta_1 - \eta_2, \quad \Delta\phi = \phi_1 - \phi_2$$



- Recall: pseudo-rapidity

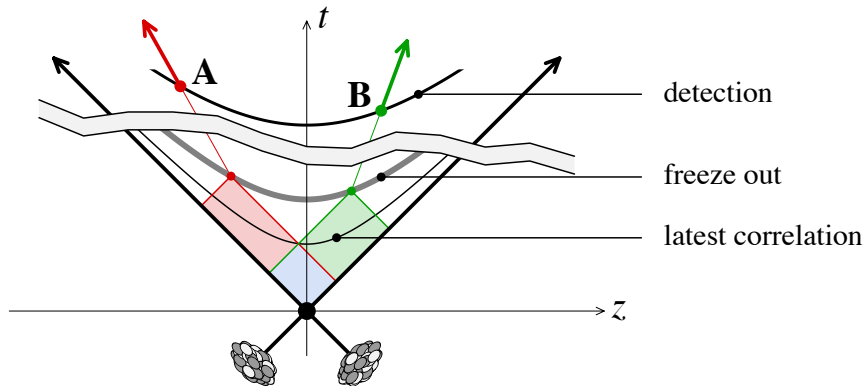
$$\eta = \frac{1}{2} \ln \frac{p + p_z}{p - p_z}$$

$$\eta = -\ln \tan \frac{\theta}{2}, \quad \theta = \frac{p_z}{p}$$

$$\tau = \sqrt{t^2 - z^2}$$

Long-range rapidity correlations probe early times

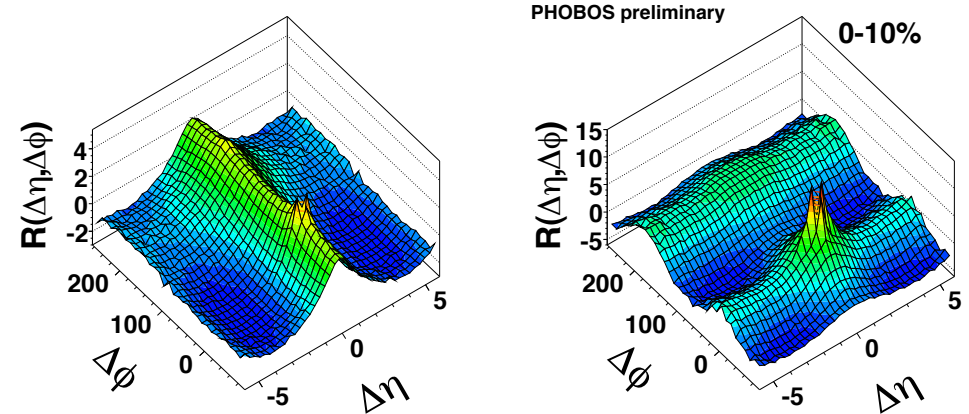
- Generated at **early stages**, where particles with different longitudinal velocities were still **causally connected**



$$\tau_{\text{correlation}} \leq \tau_{\text{freeze-out}} e^{-|\eta_A - \eta_B|/2}$$

Di-hadron correlations: p+p vs Au+Au

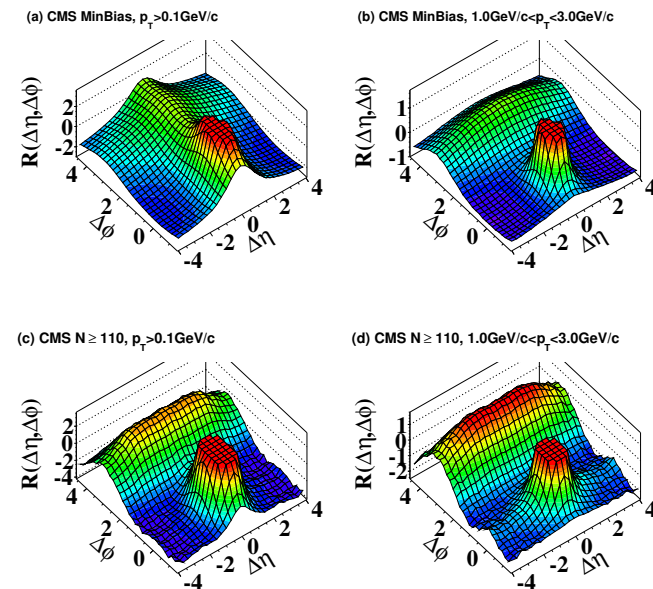
- p+p** : peak around $\Delta\eta = 0$ & flat in $\Delta\phi$
 ▷ correlated particles make similar angles with the beam axis



- Au+Au** :
 almost flat over $\Delta\eta \simeq 10$ & 2 peaks at $\Delta\phi = 0$ and $\Delta\phi = \pi$

The ridge in p+p at CMS (1)

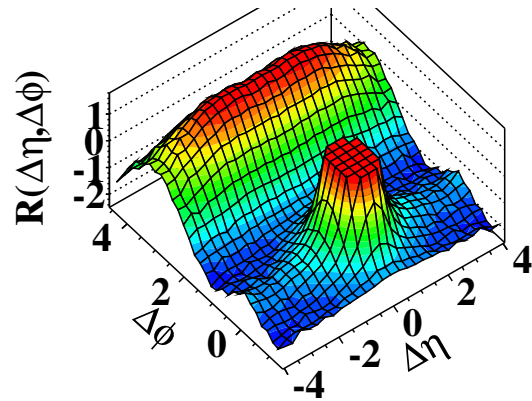
- A small ridge has been seen in **p+p collisions** at the LHC



The ridge in p+p at CMS (2)

- ... but only in **specially selected events** !

(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_{\perp} < 3.0 \text{ GeV}/c$

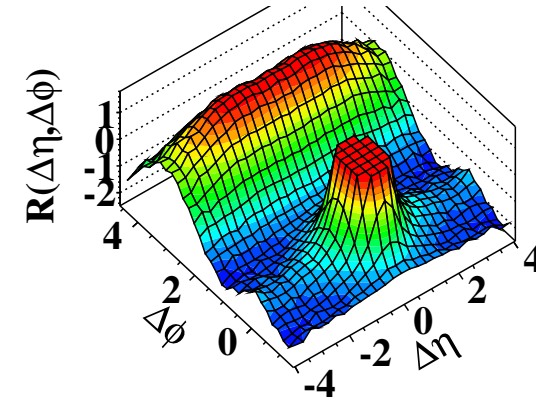


- High-multiplicity (\implies very central): $N \geq 110$ particles
- Narrow interval in transverse momentum: $1 \leq p_{\perp} \leq 3 \text{ GeV}$

The ridge in p+p at CMS (2)

- ... but only in **specially selected events** !

(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_{\perp} < 3.0 \text{ GeV}/c$



- ... which look a lot like a heavy ion collision !!
 - ▷ N.B. $1 \leq p_{\perp} \leq 3 \text{ GeV}$ is similar to the proton Q_s at LHC